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School of Engineering & Applied Science

TOPOLOGY-CONTROLLED RECONSTRUCTION OF MULTI-LABELLED DOMAINS FROM CROSS-SECTIONS

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Nathan Carr *Adobe*

Tao Ju *Washington University in St. Louis*

Introduction

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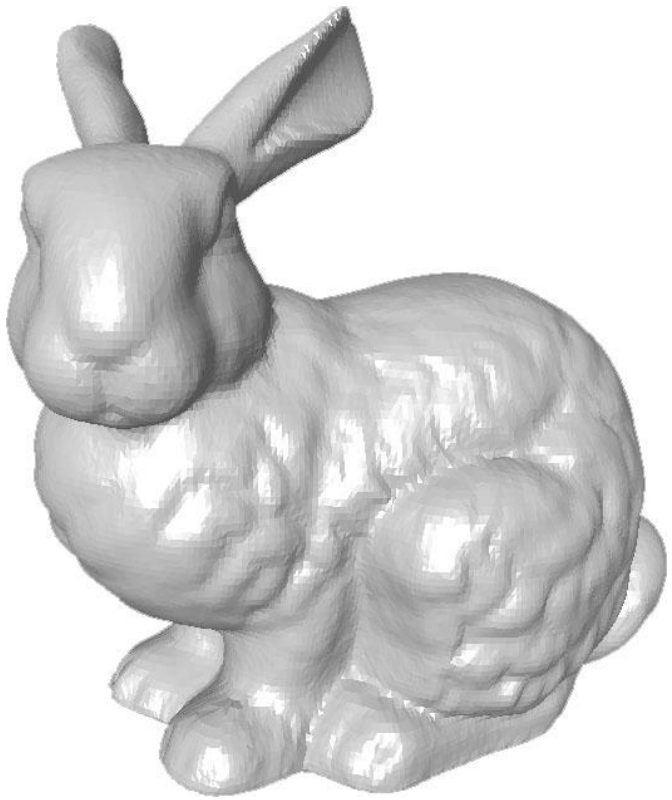
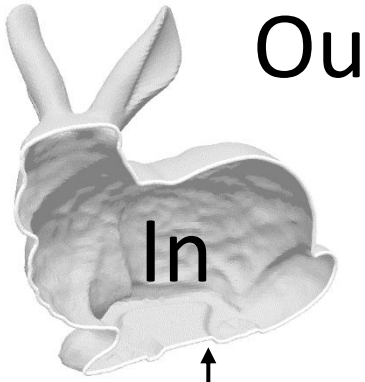
- Two-labelled domains





Boundary surface (manifold)

Introduction

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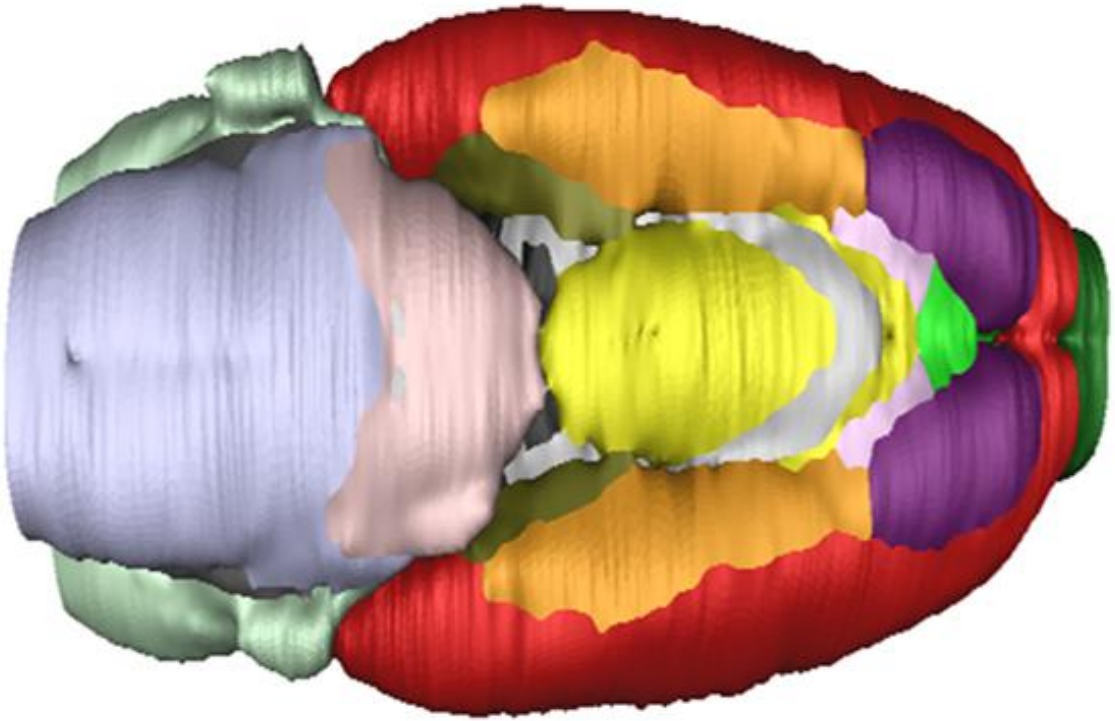
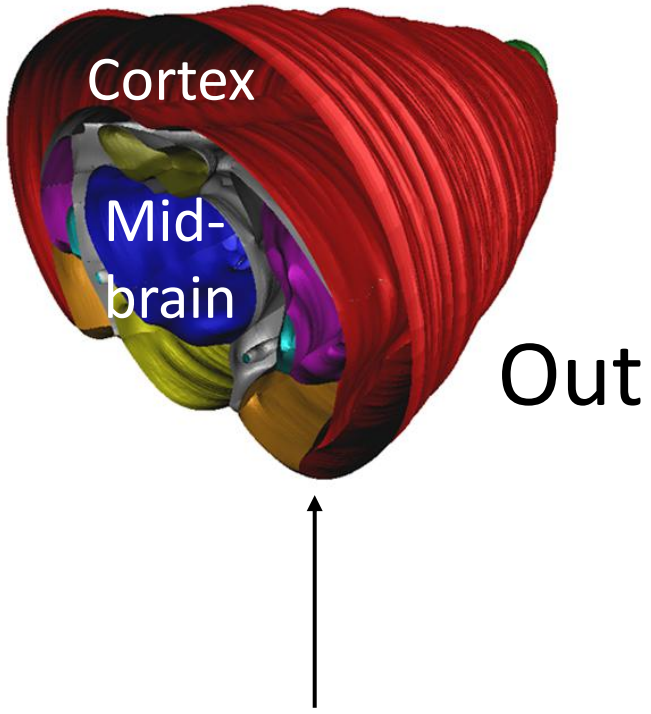
- Multi-labelled domains





Introduction

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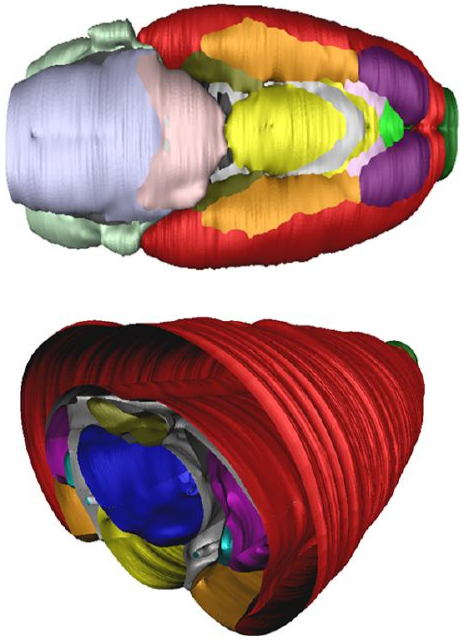
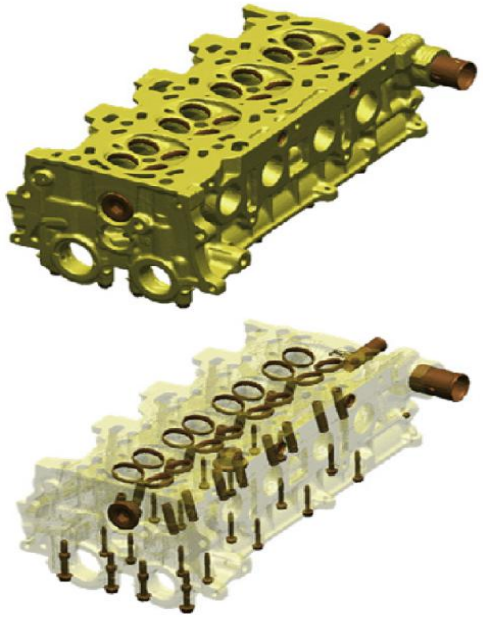
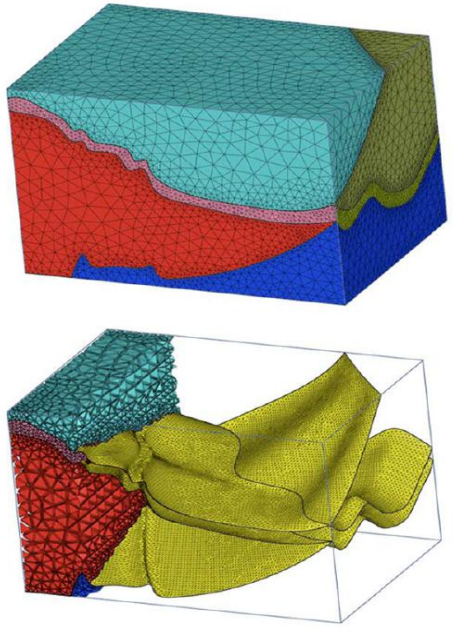
- Applications of multi-labelled domains



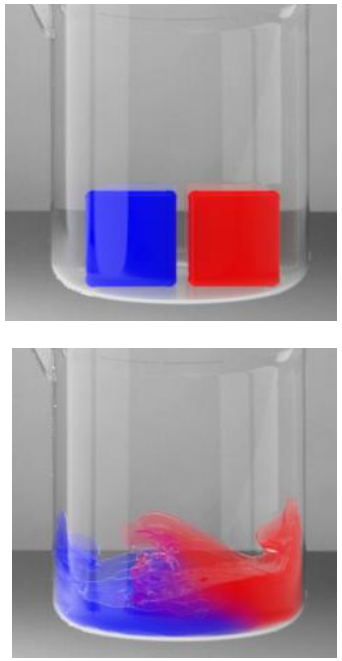
Biomedicine



Manufacturing



Geology



Fluid Dynamics

Introduction

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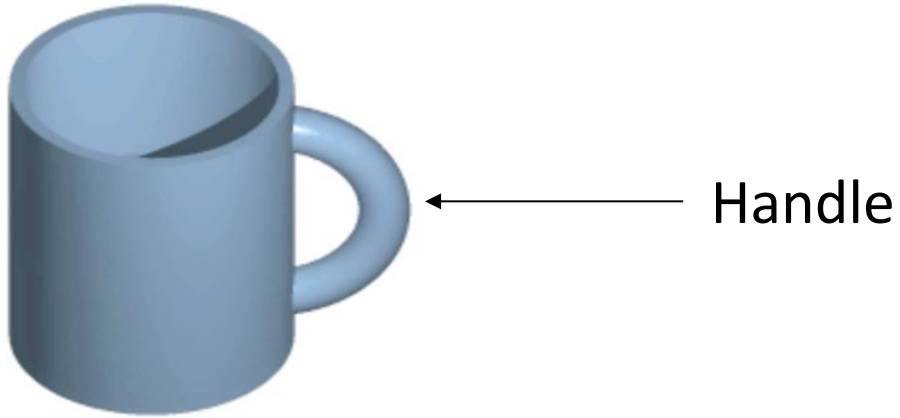
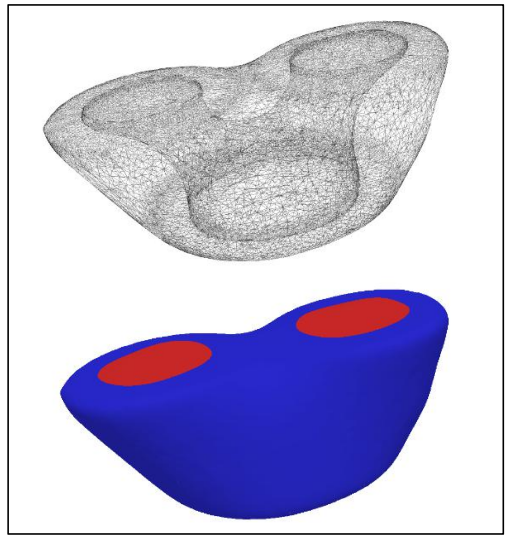
- Topology of material interfaces
  - 2-labels: number and genus (# handles) of boundary surface



Introduction

- Topology of material interfaces
  - 2-labels: number and genus (# handles) of boundary surface
  - Multiple labels: number/genus of surfaces bounding each label



Surface components	1	1	1
genus	2	0	0

Related works

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- Reconstructing material interfaces
  - Surface tracking
  - From labelled volumes: Iso-surfacing, Delaunay meshing, Particle diffusion
  - From cross-sections: Implicit functions, Projection

Related works

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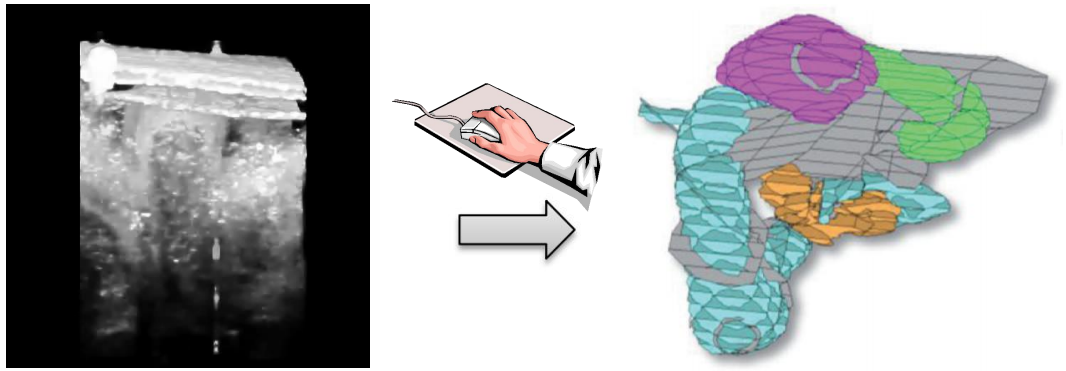
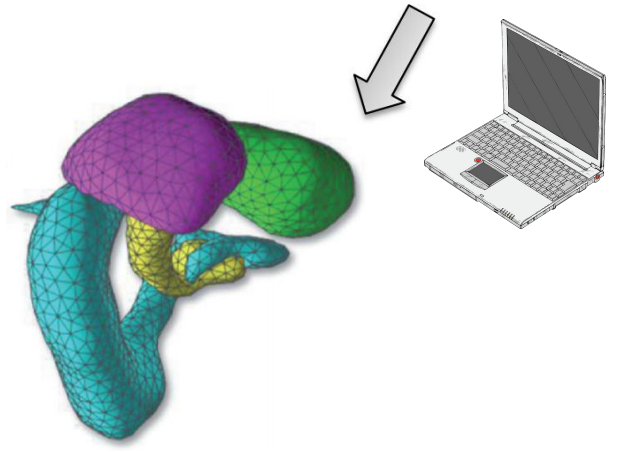


3D MRI/CT volume

Segmented 2D slices



Reconstructed material interface [Bermano 11]

Related works

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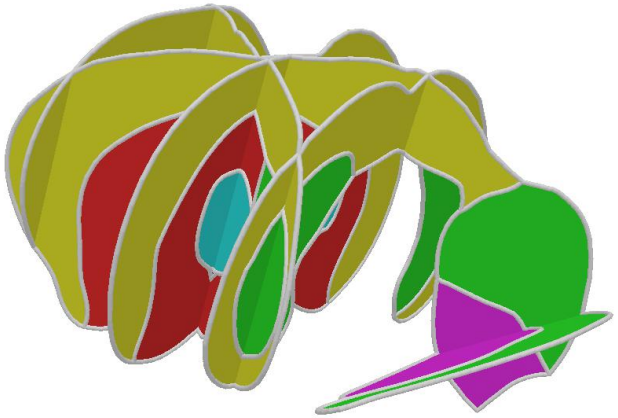
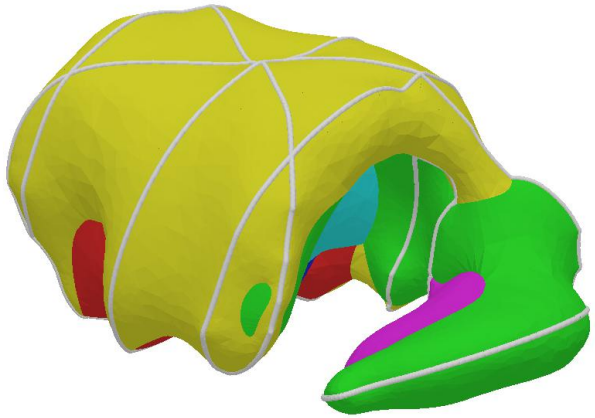
- Reconstructing material interfaces
  - Surface tracking
  - From labelled volumes: Iso-surfacing, Delaunay meshing, Particle diffusion
  - From cross-sections: Implicit functions, Projection
- Geometrically correct, but no topological guarantees

Related works

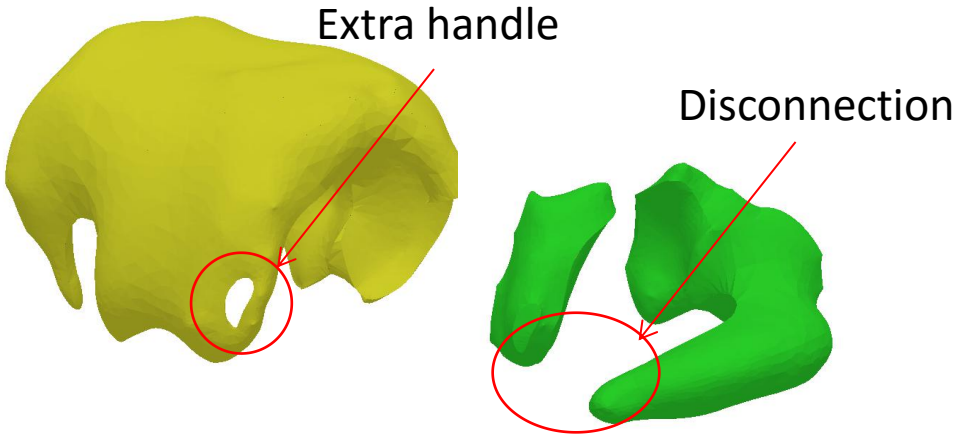
- Reconstructing material interfaces from cross-sections [Bermano 11]



Cross-sections



Reconstruction



Topological errors

Related works

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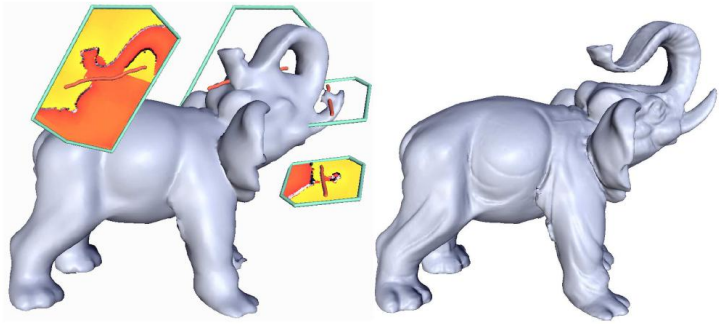
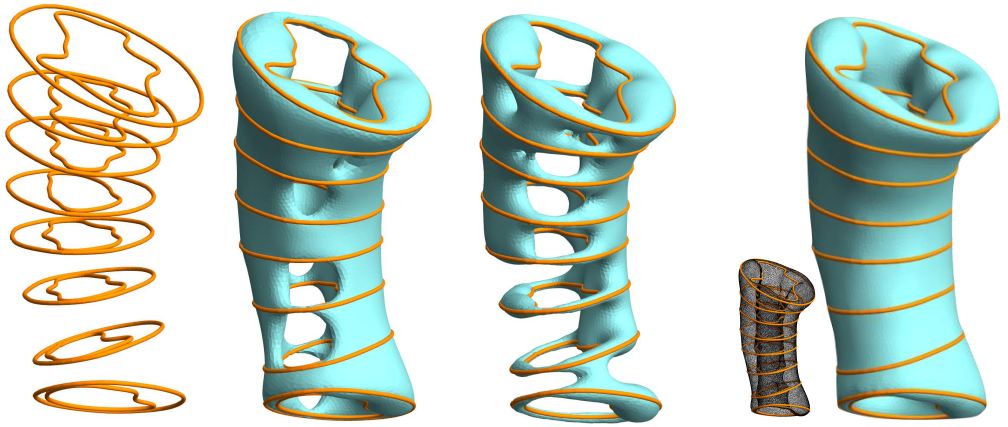
- Topology-aware modeling of 2-labelled domains
  - Topology repair
  - Reconstruction with topology control



[Sharf 07]

- Do not handle multiple labels
  - Independent reconstruction of individual labels leads to intersecting material interface



Input [Liu 08] [Bermano 11] Genus-1 [Zou 15]

Our work

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- Reconstructs material interfaces from cross-sections
  - Allowing any number of labels and non-parallel planes
- Automatic and interactive topology control
  - Global constraints (components and genus per label)
  - Interactive sketching

Our work

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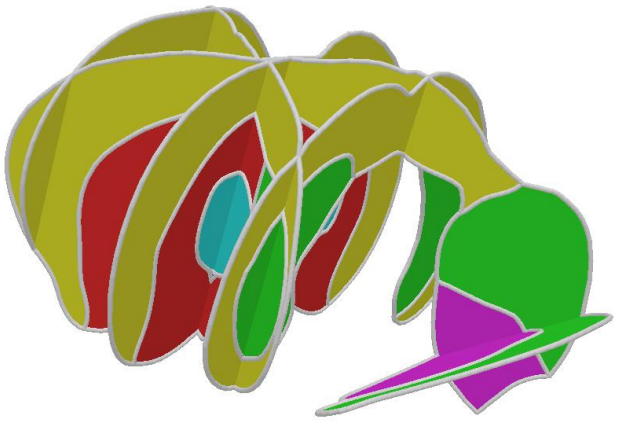
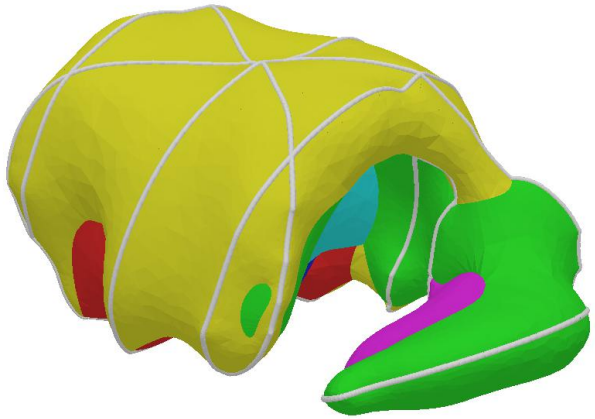
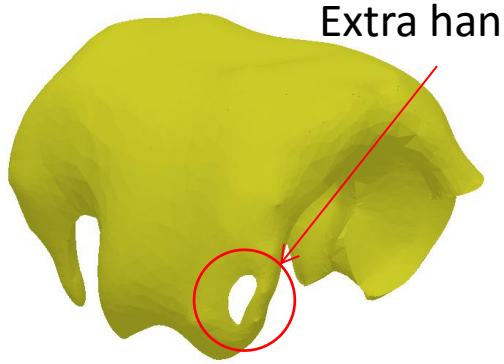
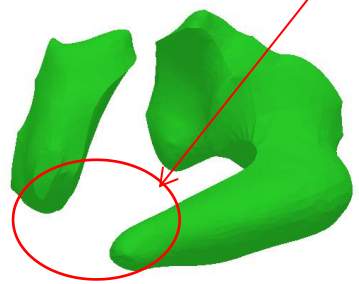
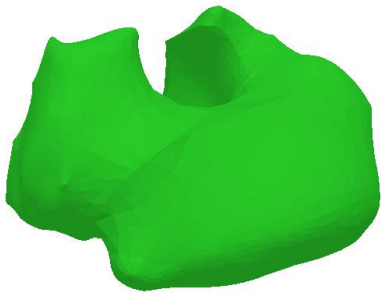








w/o topology constraints



with topology constraints (yellow/green: genus 0)

Technical contributions

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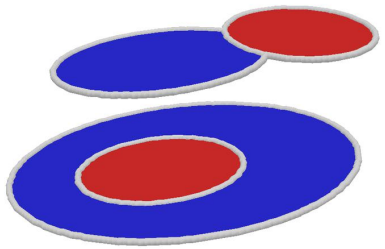
- Extending the divide-and-conquer paradigm of [Zou *et al.* 15]
  - From 2-labels to multiple labels
- Introducing a new implicit definition of material interfaces
  - Allowing systematic exploration of topological variations

Divide-and-conquer

- Cross-section planes divide space into *cells*



Divide-and-conquer

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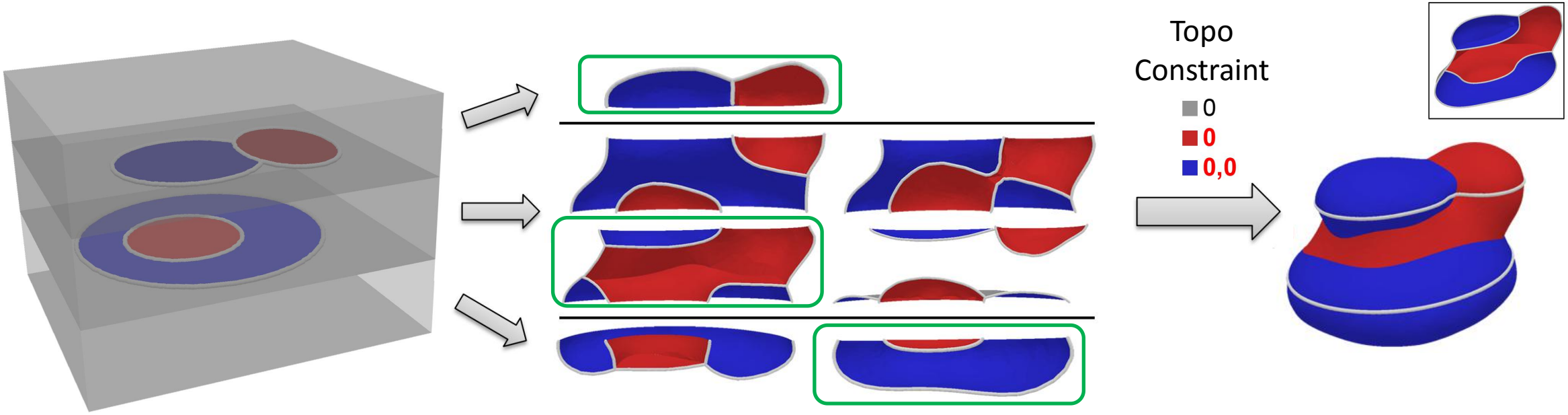
- Cross-section planes divide space into *cells*
  1. Within each cell, explore and score candidate surface topologies
  2. Pick one per cell to meet the topological constraint while maximizing score



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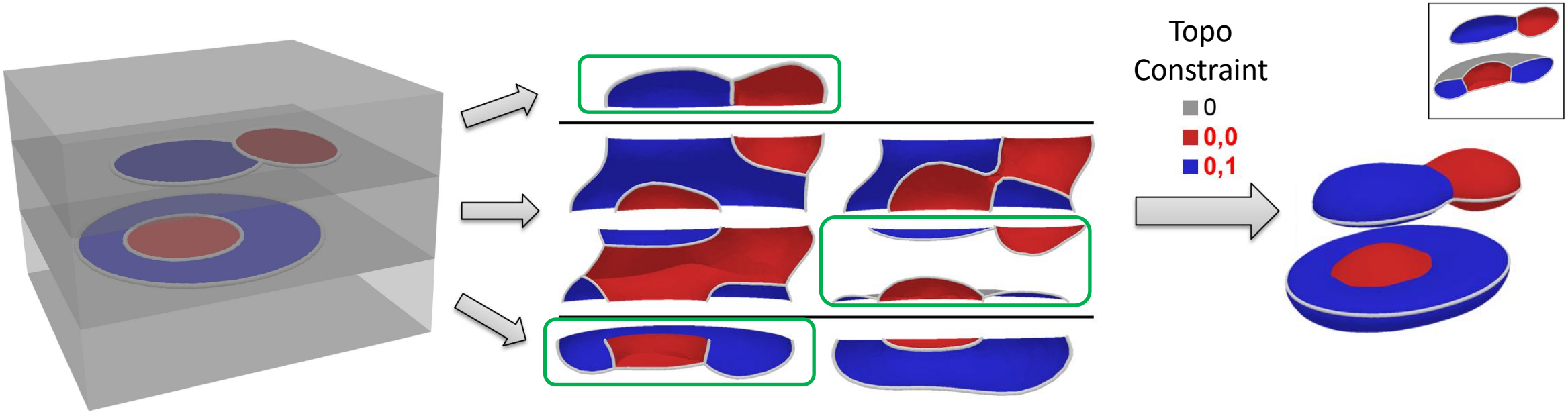

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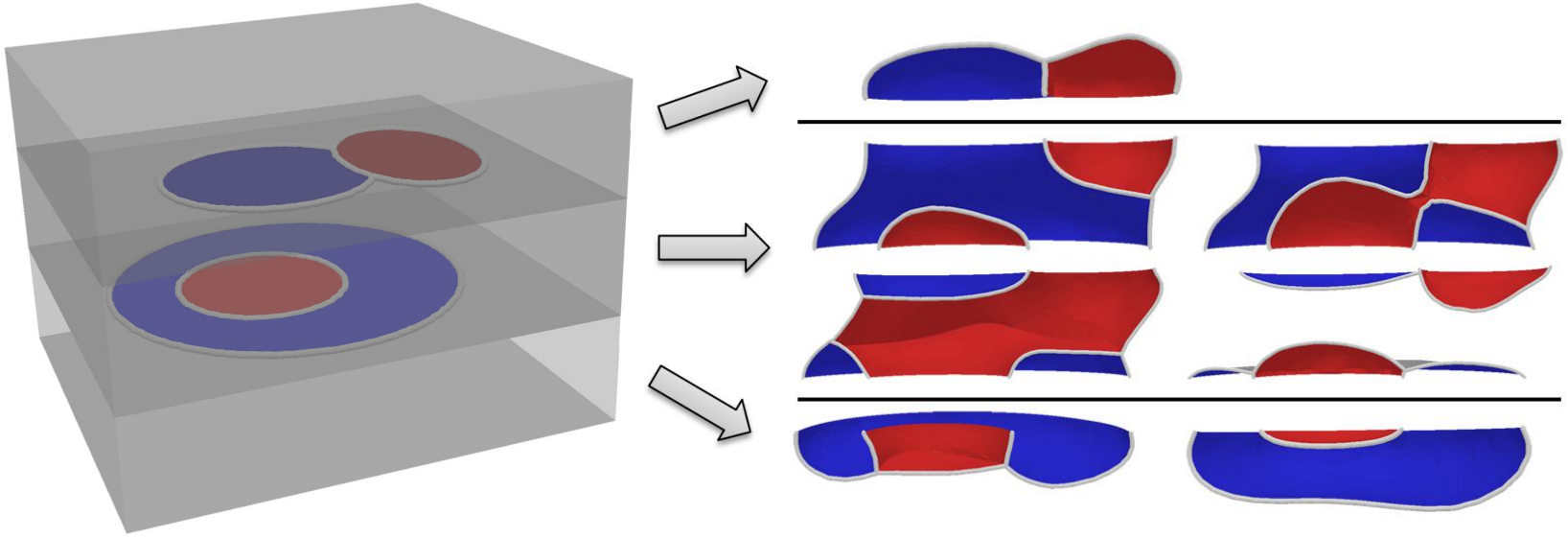

2-Labels: Level sets

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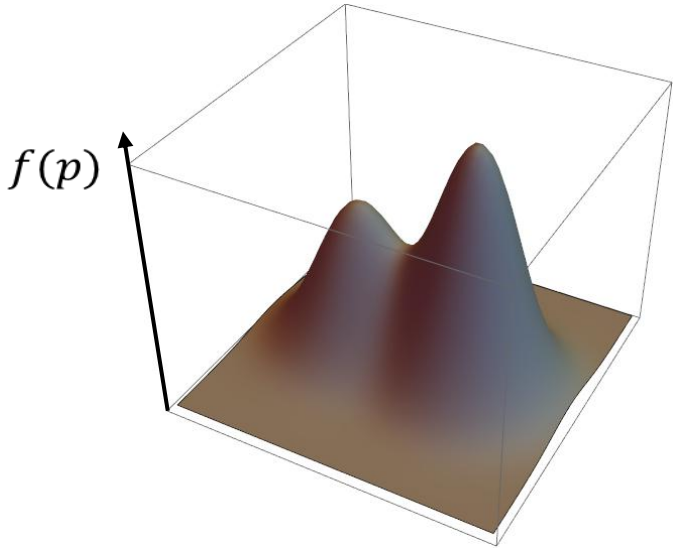
- Scalar function $f(p)$ for $p \in R^d$
- Scalar "level" c
  - $f(p) > c$: p is inside
  - $f(p) < c$: p is outside
  - $f(p) = c$: p is on the level set



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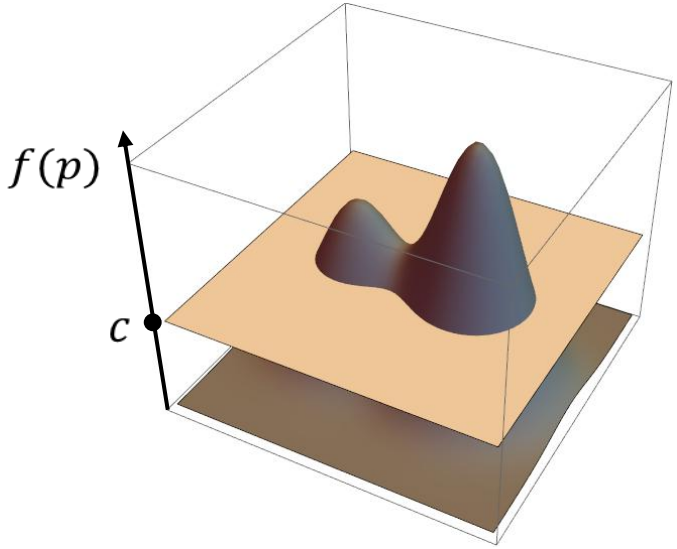

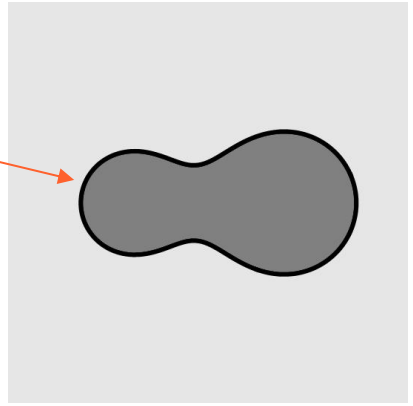

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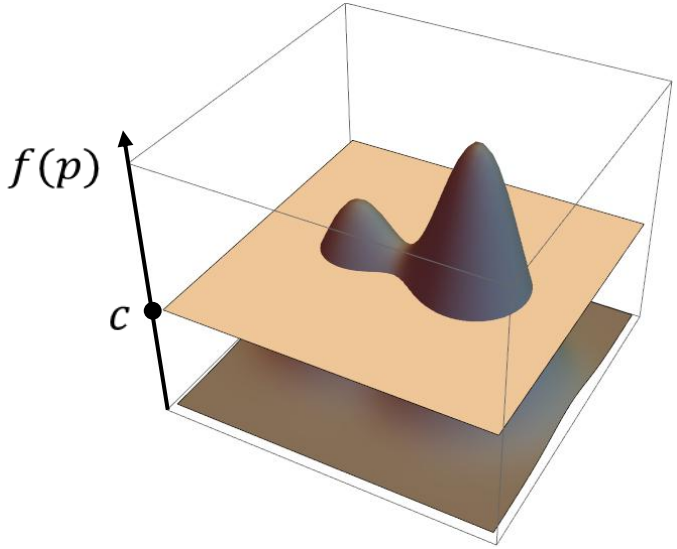

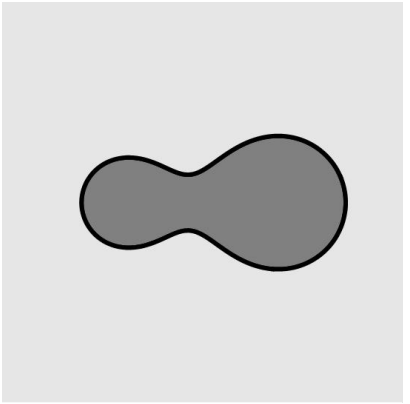

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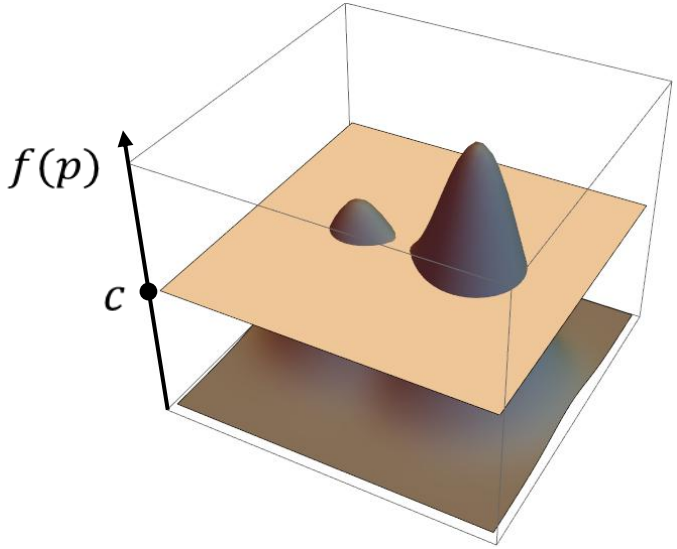

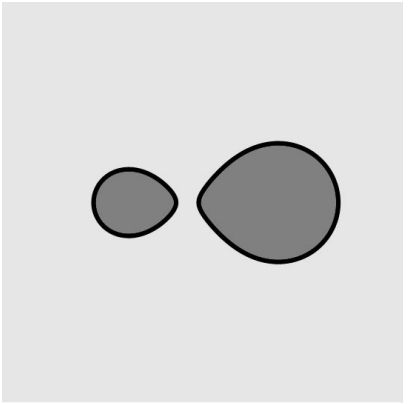

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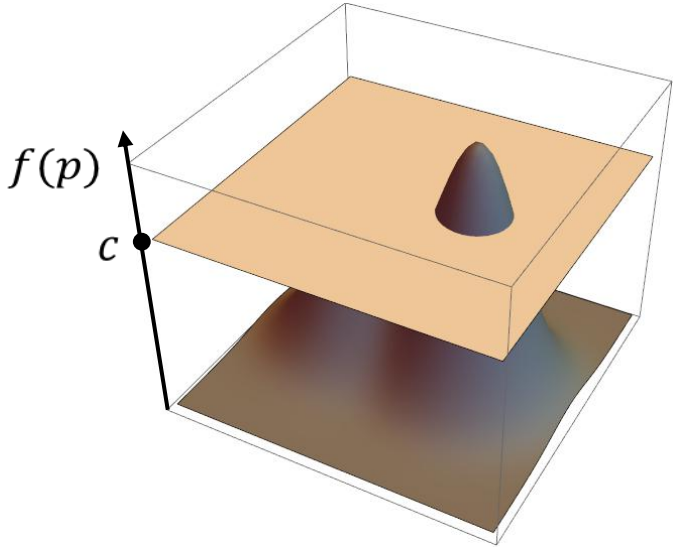

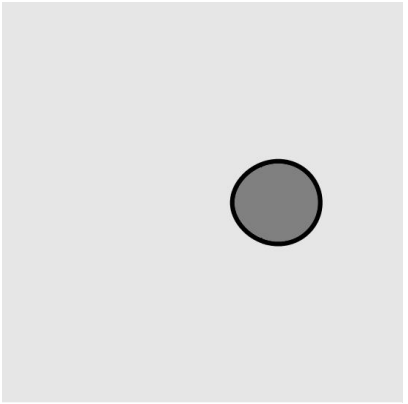

2-Labels: Level sets

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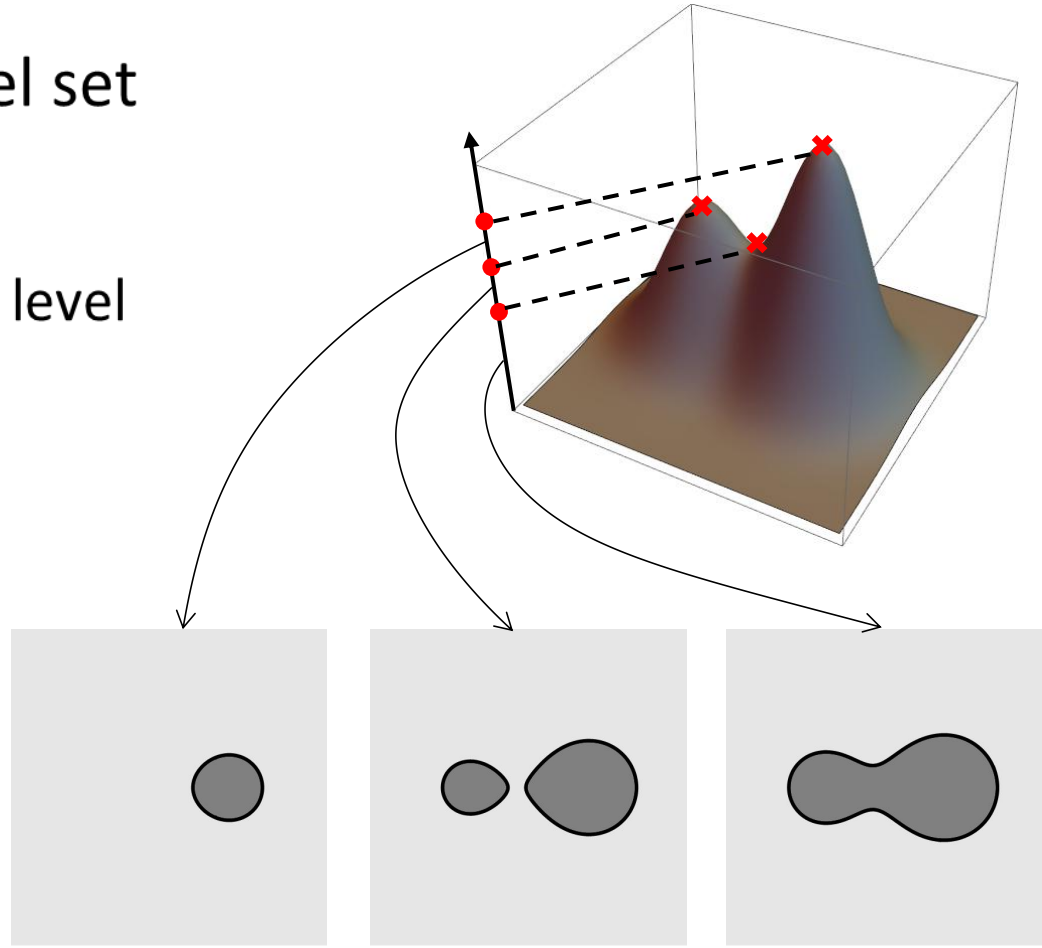
- Critical values: levels c at which level set changes topology
  - Divides levels into ranges with distinct level set topology
  - Associated with critical points of f



n-labels: Interface sets

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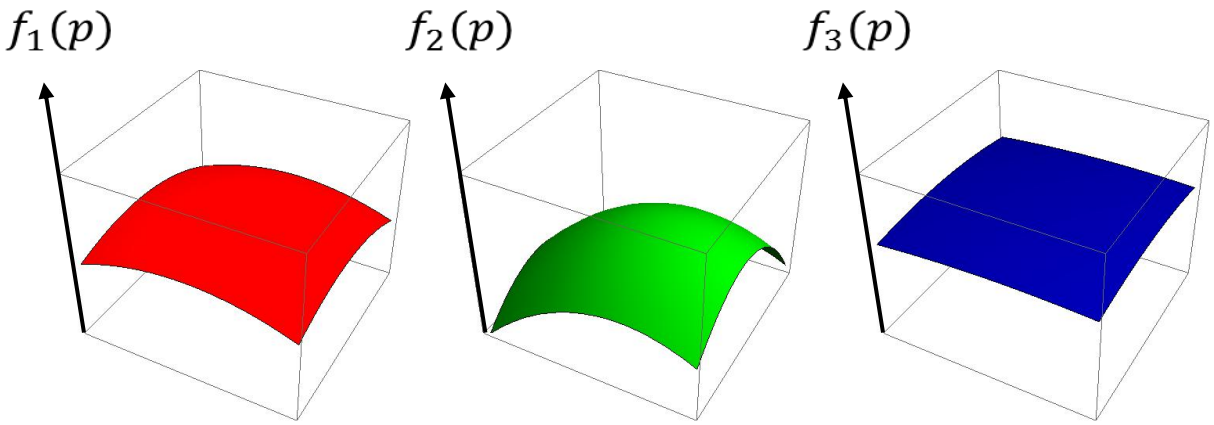
- Vector function $\vec{f}(p) = \{f_1(p), \ldots, f_n(p)\}$
  - One scalar function f_i for each label



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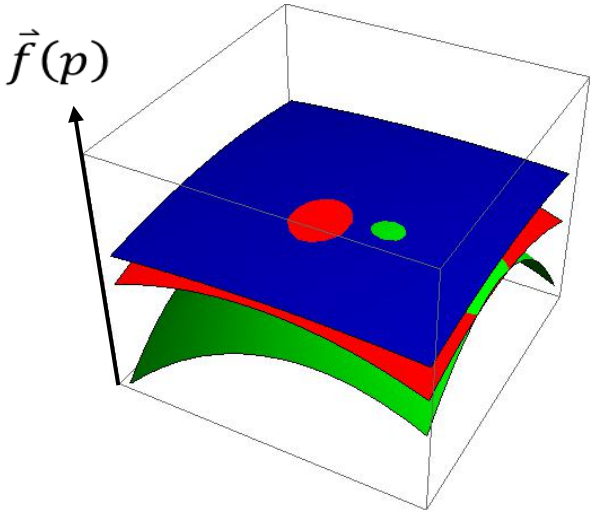
  - $Label(p) = \underset{i}{\text{argmax}}(f_i(p) + c_i)$
  - $|Label(p)| > 1$: p is on the interface set



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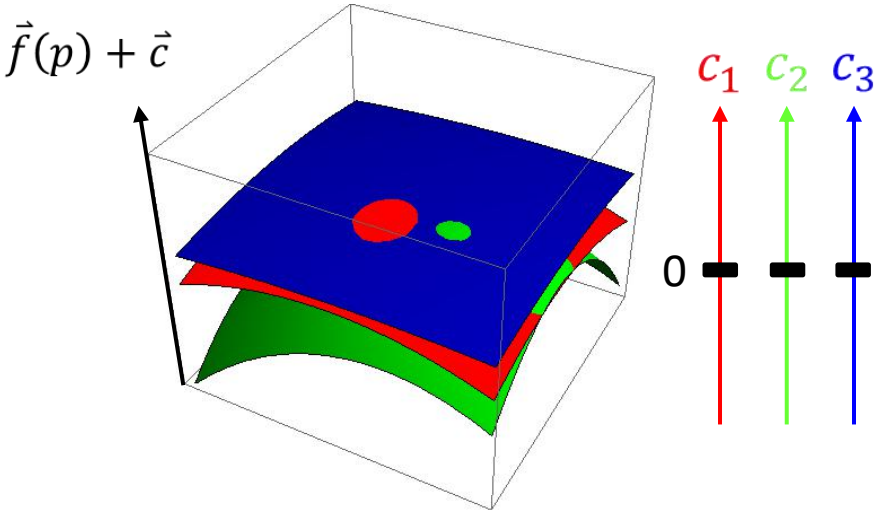

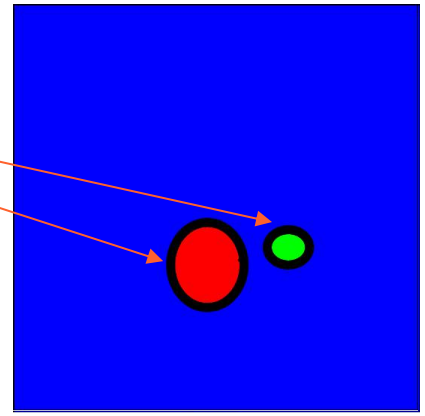

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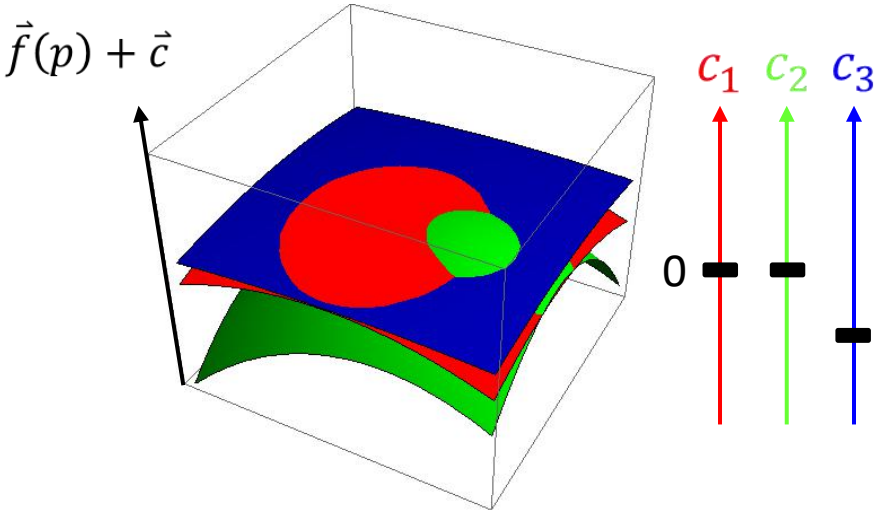

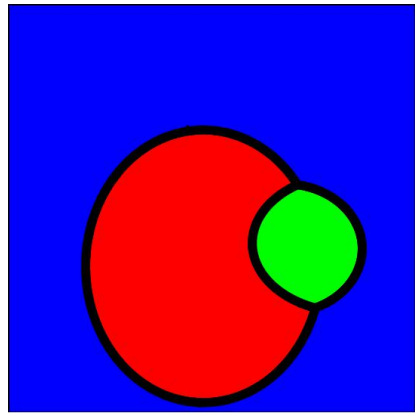

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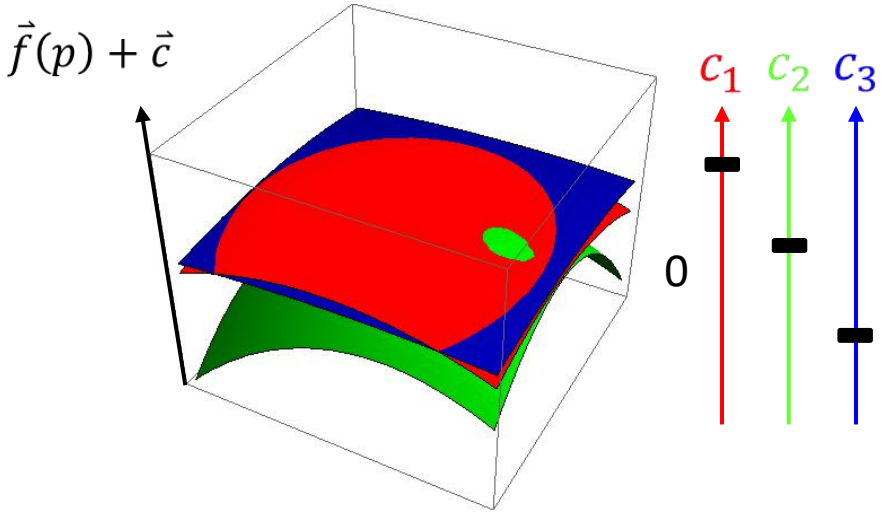

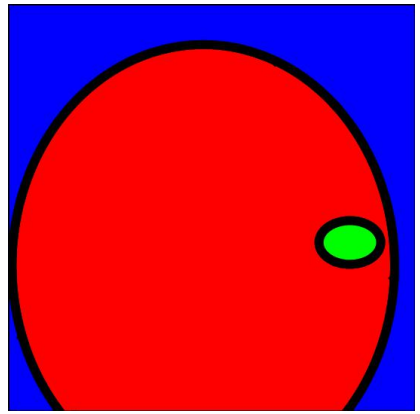

n-labels: Interface sets

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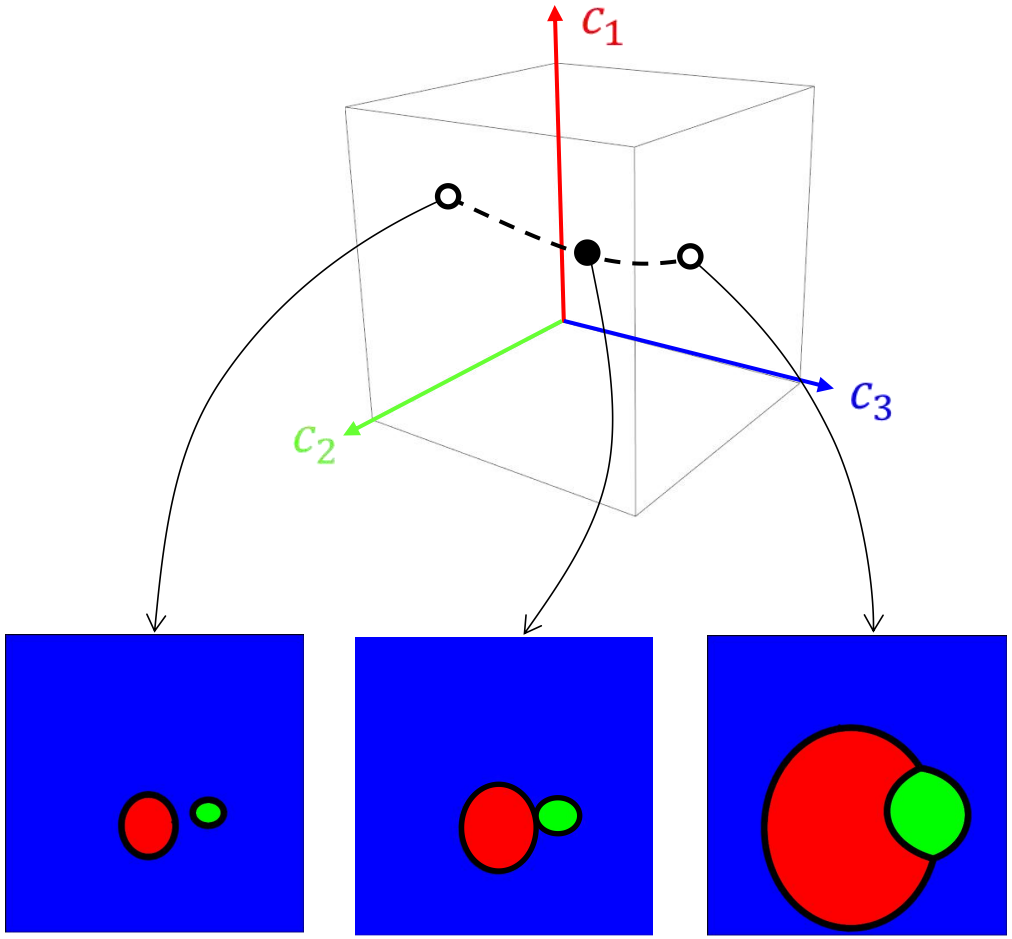
- Critical offset: vectors $\vec{c}$ at which interface set changes topology



n-labels: Interface sets

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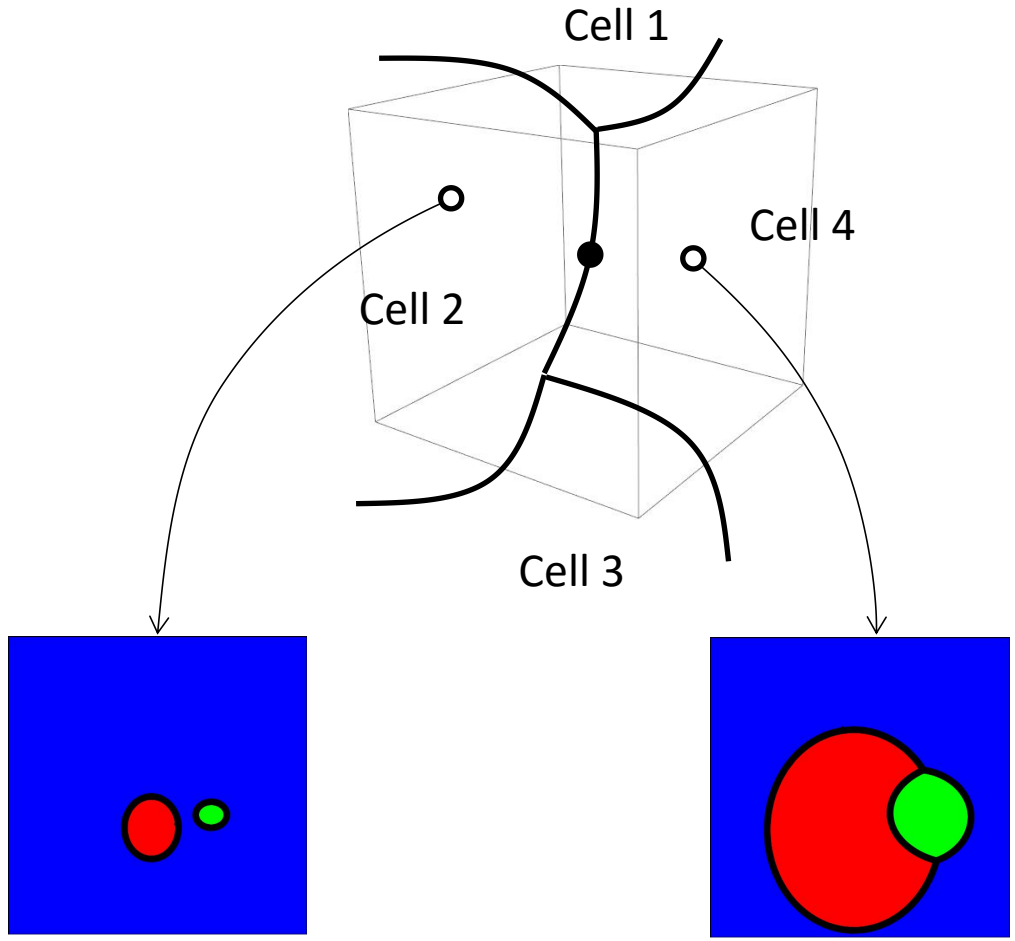
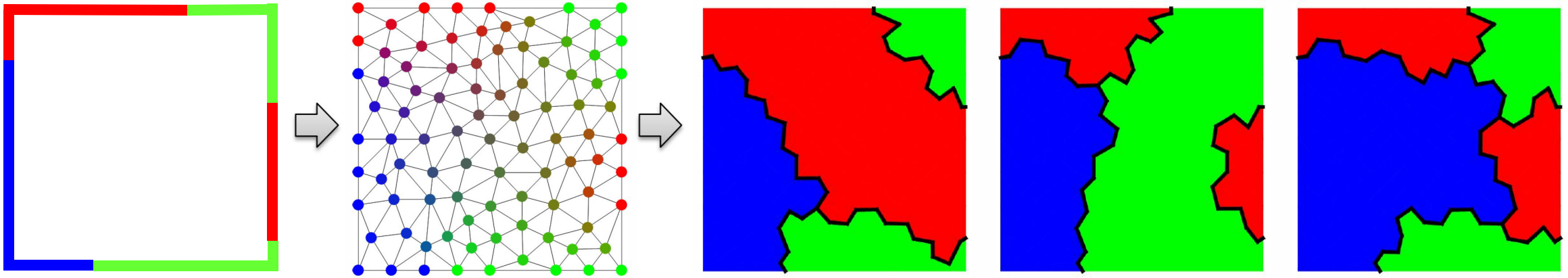
- Critical offset: vectors $\vec{c}$ at which interface set changes topology
  - Divides *n*-D space into cells with distinct topologies of interface sets
- We give a discrete algorithm for exploring topological cells
  - Assuming $\vec{f}$ is piece-wise constant



Vector function

- Harmonic vector function within each cell
  - Interpolates labelling on cell boundary



$\vec{f} = \{f_1, f_2, f_3\}$

Explored topologies

Examples

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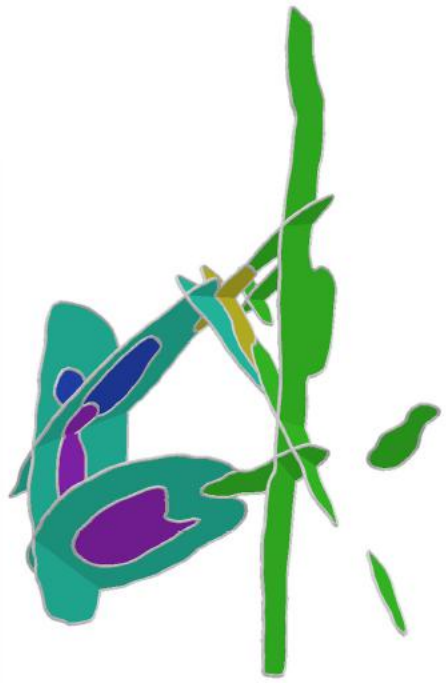
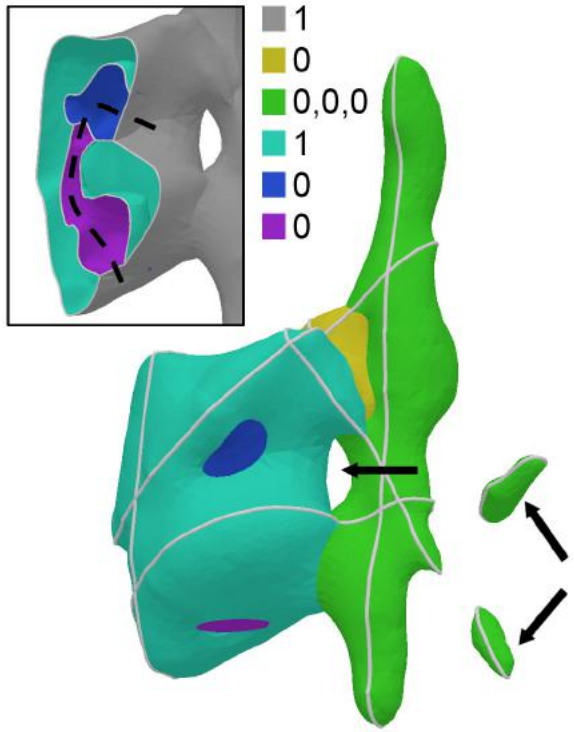
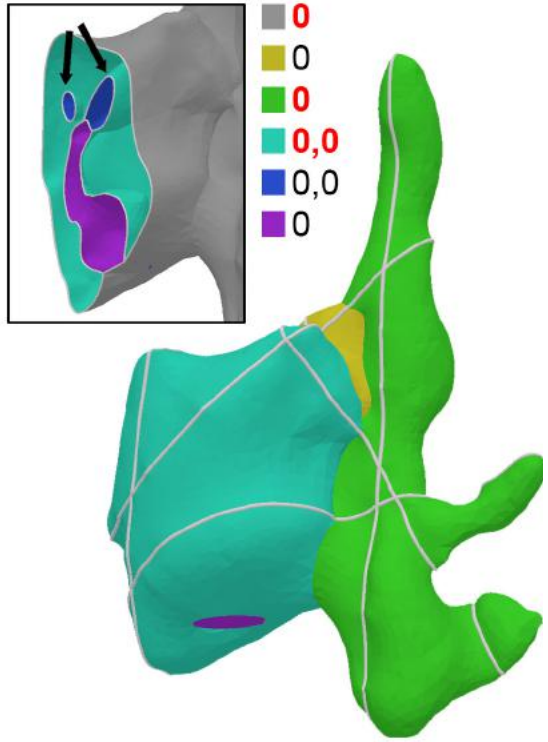


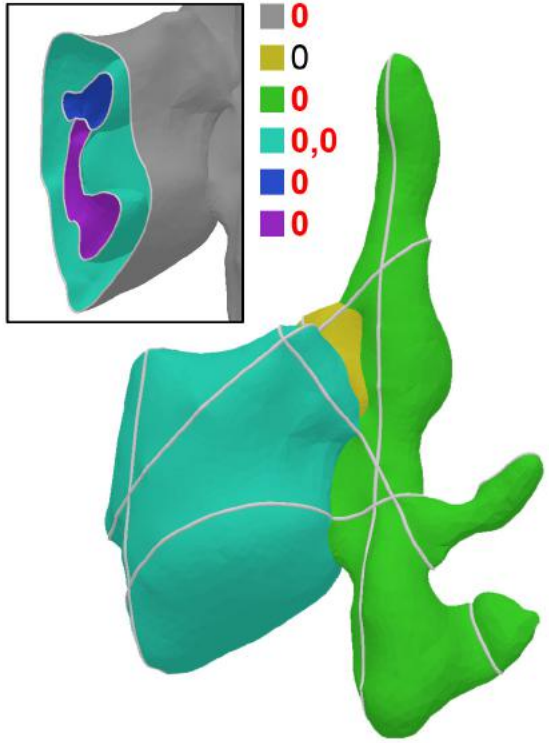
Input
(5 cross-sections, 6 labels)



Reconstruction
w/o constraints



Reconstruction with
constraints



More constraints
(5 labels, 212 secs)

Examples

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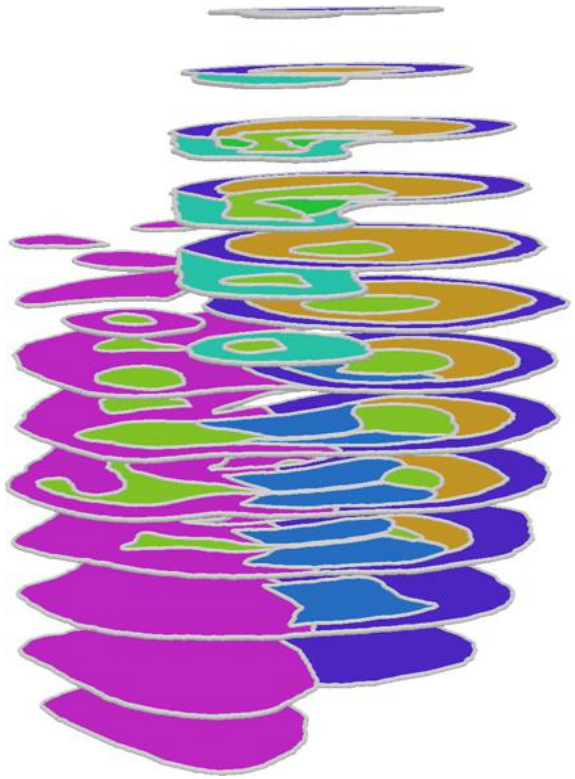
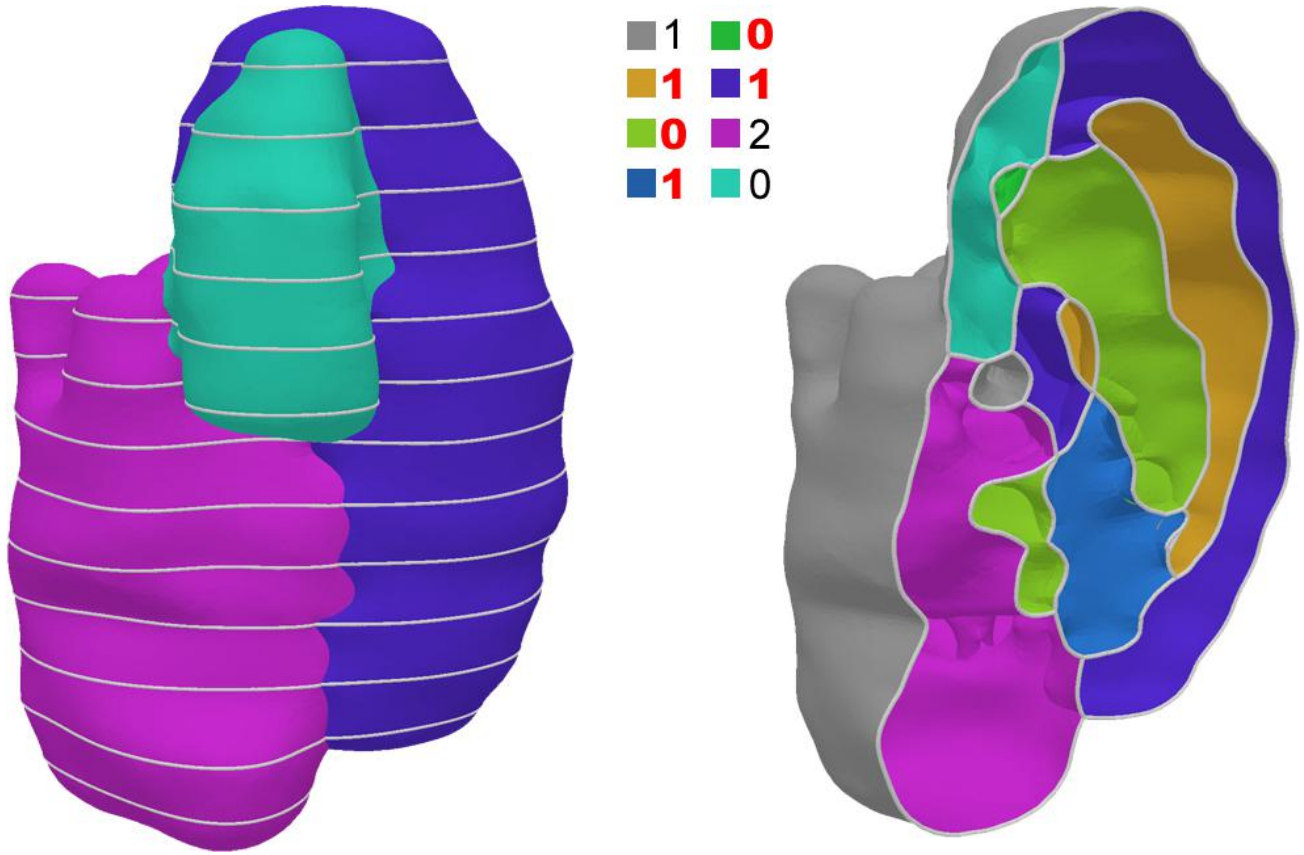
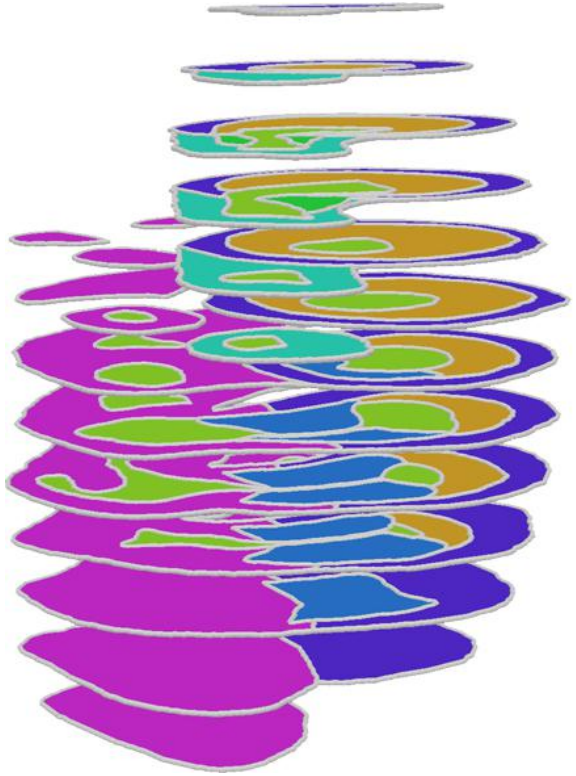


Input cross-sections
(13 planes, 8 labels)

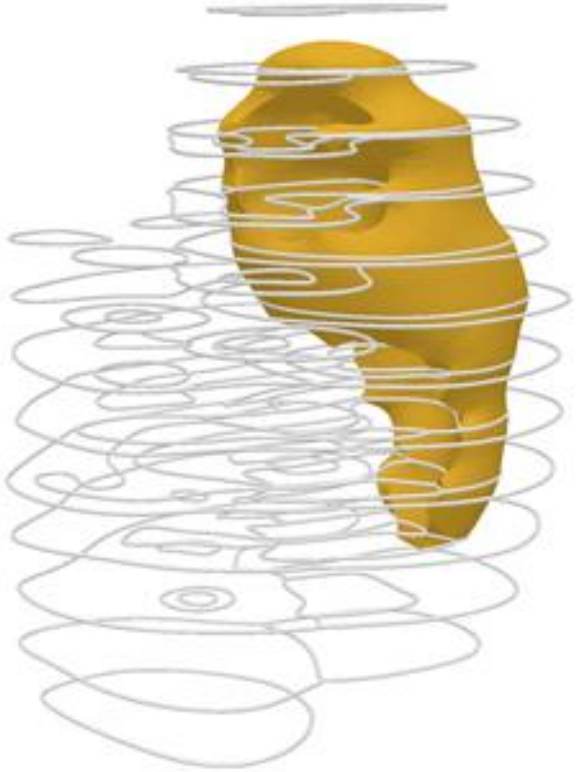


Reconstruction
(5 constrained labels, 2758s)

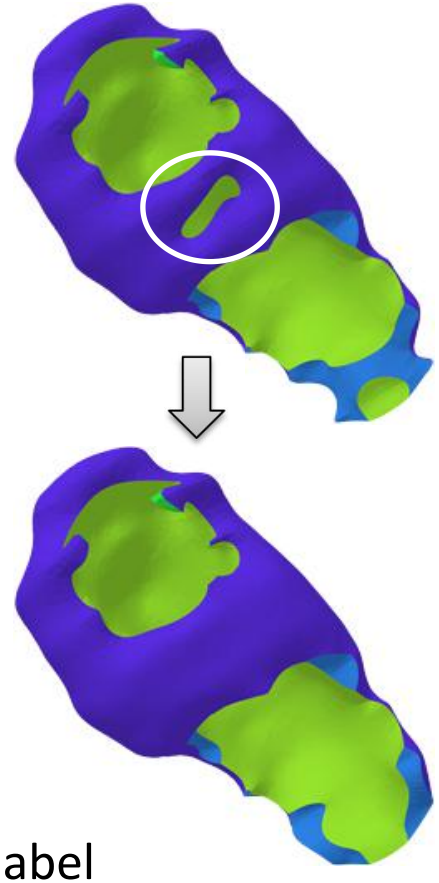
Examples



Input cross-sections
(13 planes, 8 labels)



Yellow label



w/o constraint:
Genus 2

w constraint:
Genus 1

Interaction

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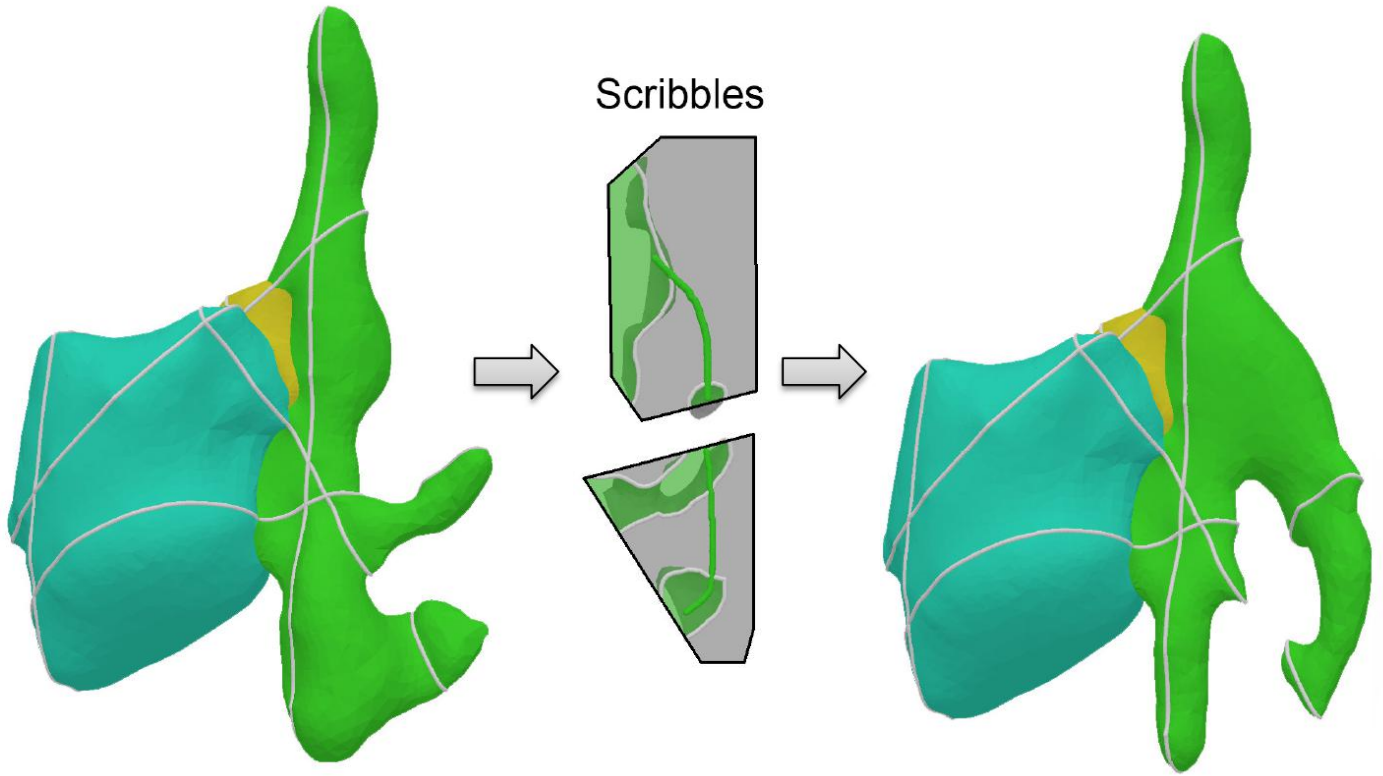
- Selecting candidate topology
- Sketching new topology
  - Modifying the vector function



Summary

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- First algorithm for modeling multi-labelled domains with topology control
- Interface sets for topology exploration of material interfaces

- Limitations
  - Topology exploration is computational expensive
  - The space of explored topologies is often insufficient

Future work

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- Analysis of interface sets topology
  - Critical points/offsets, their types, connectivity
- Consider other topological properties
  - Adjacency of labels, topology of non-manifold junctions
- Extension to other inputs and to topology repair

Thank you!

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