





libWetHair

A Multi-Scale Model for Simulating Liquid-Hair Interactions

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COLUMBIA UNIVERSITY AND UNIVERSITY OF WATERLOO



<http://libwethair.info>

Paper, Executables, Code, and More



cohesion

<http://libwethair.info>

Paper, Executables, Code, and More



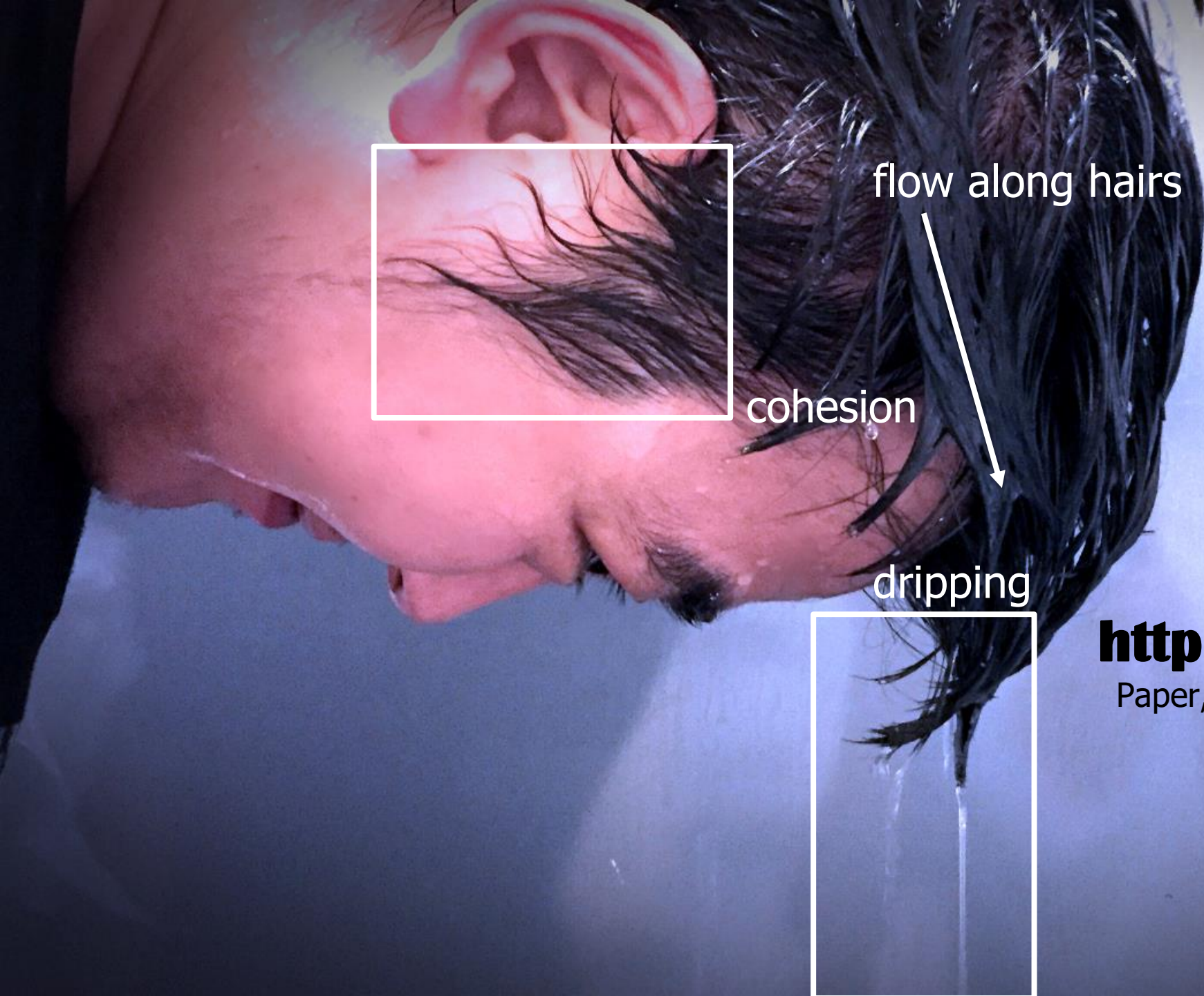
flow along hairs



cohesion

<http://libwethair.info>

Paper, Executables, Code, and More

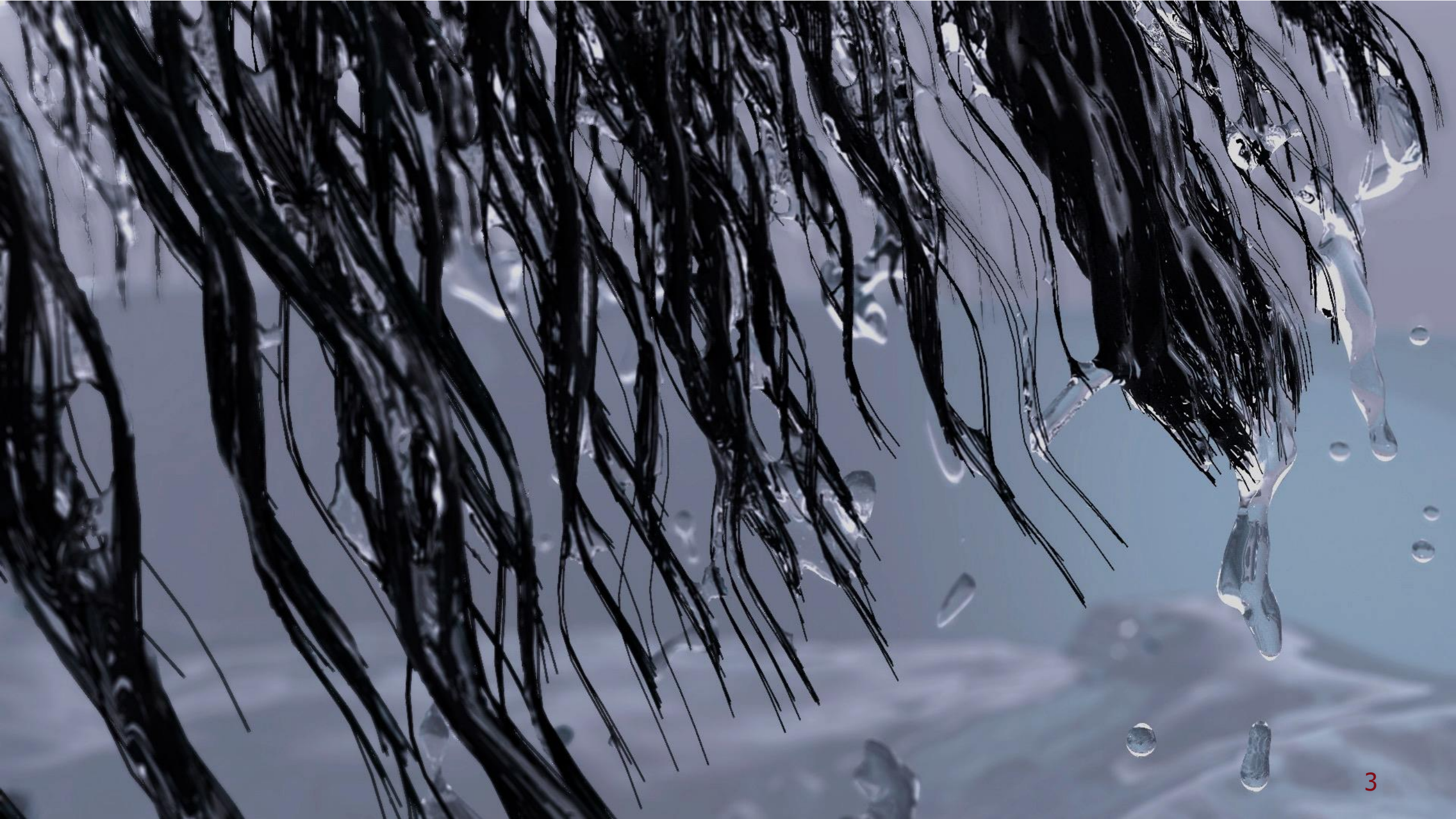


flow along hairs

cohesion

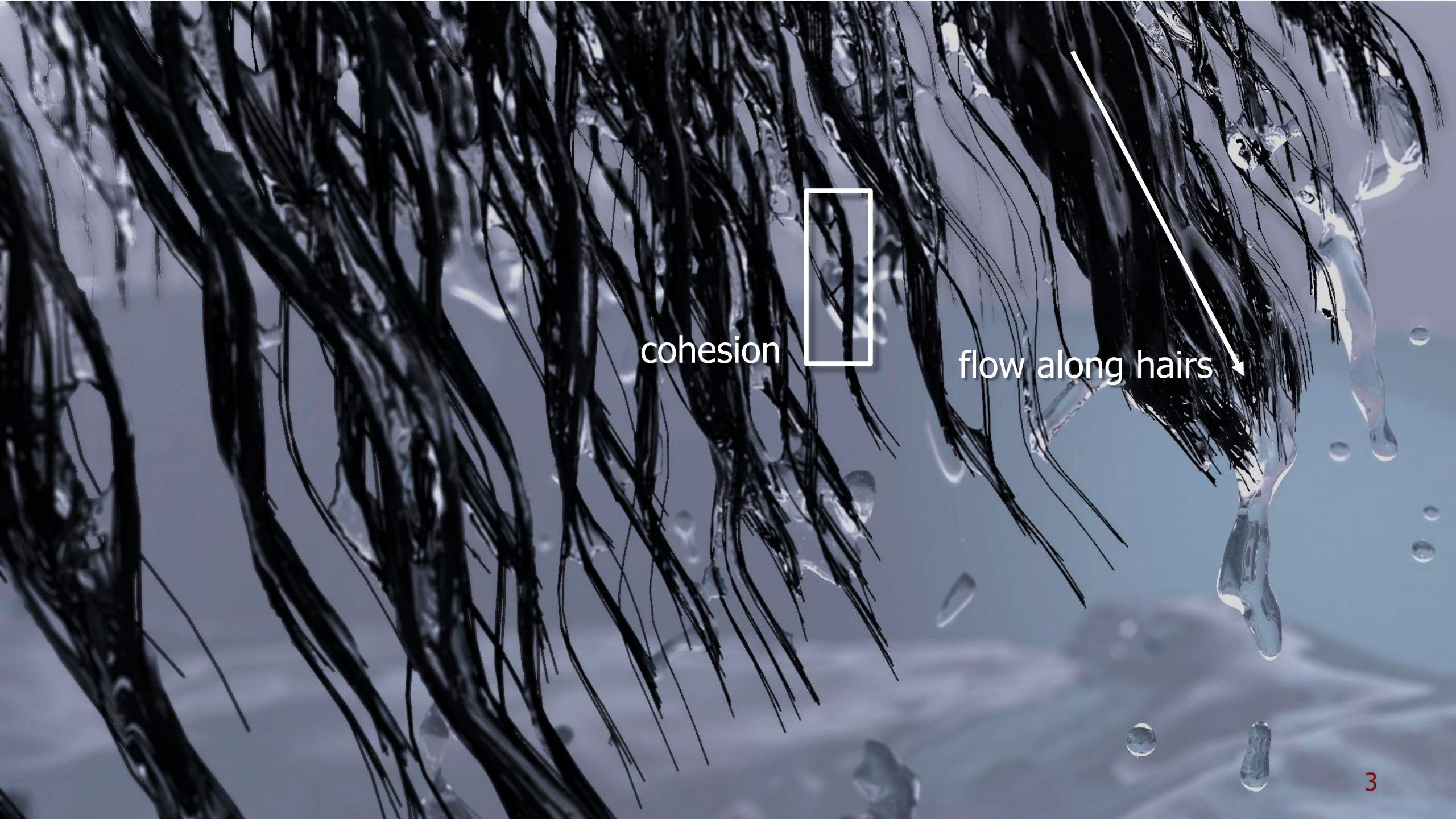
dripping

<http://libwethair.info>
Paper, Executables, Code, and More





cohesion



cohesion



flow along hairs





cohesion

flow along hairs

dripping

Related Work



[Ward et al. 2004]



[Bertails-Descoubes et al. 2005]

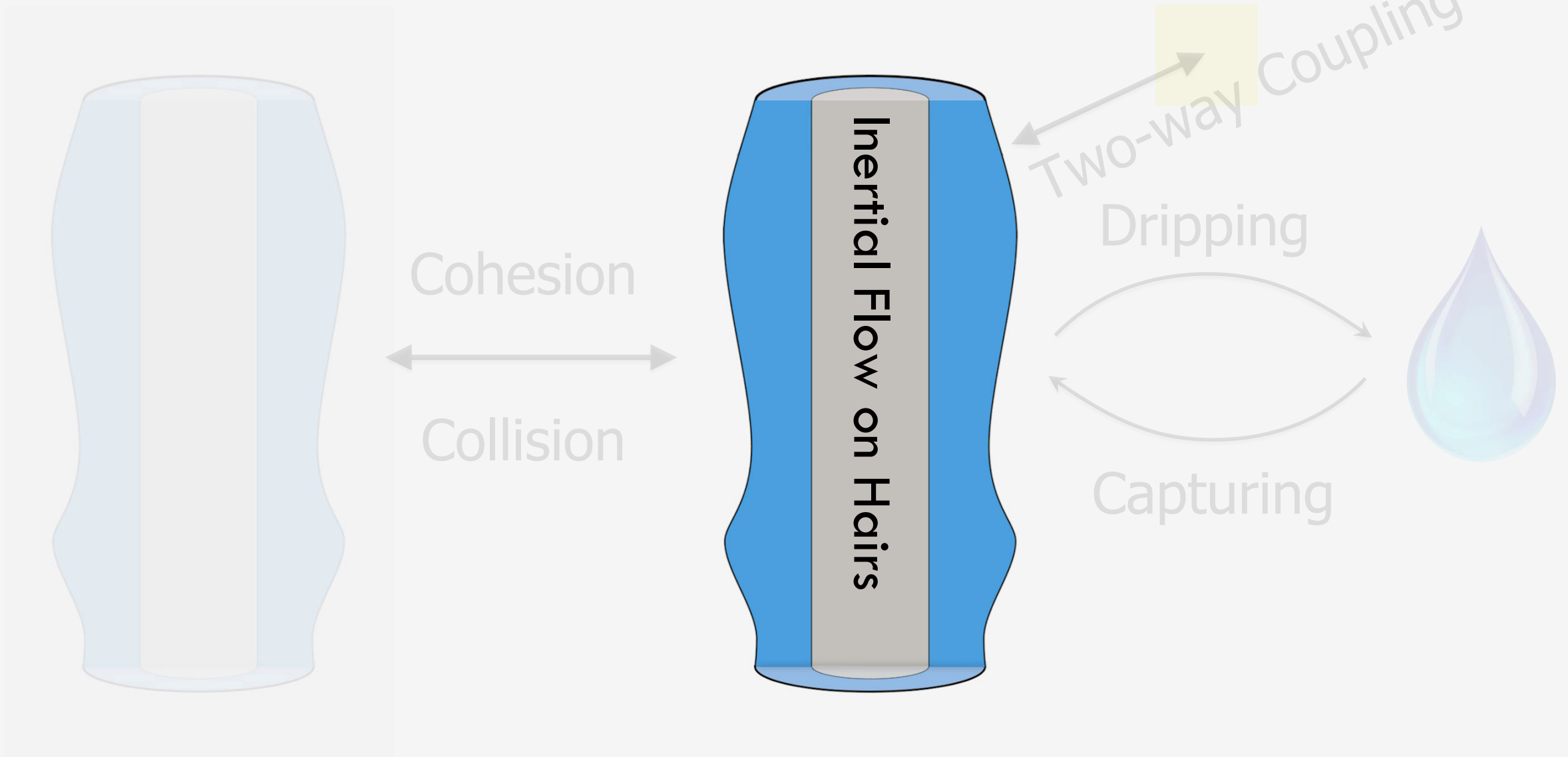
Related Work



[Rungjiratananon et al. 2012]

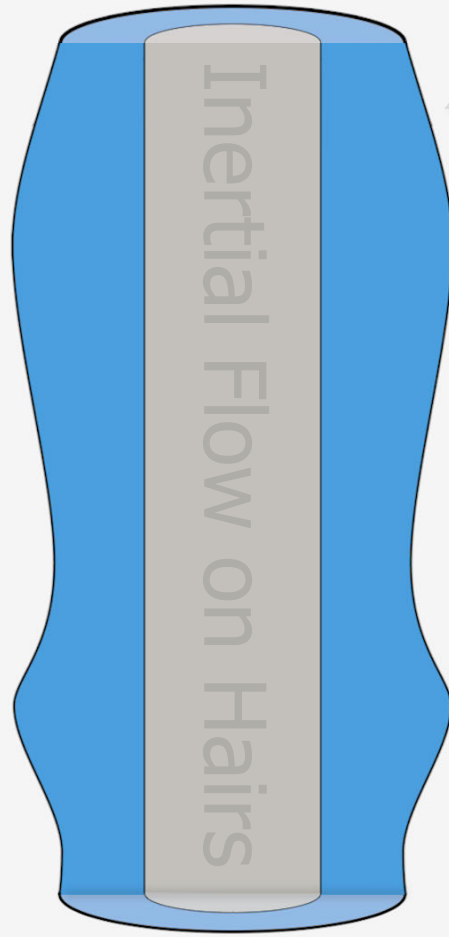


[Lin 2015]





Cohesion
Collision



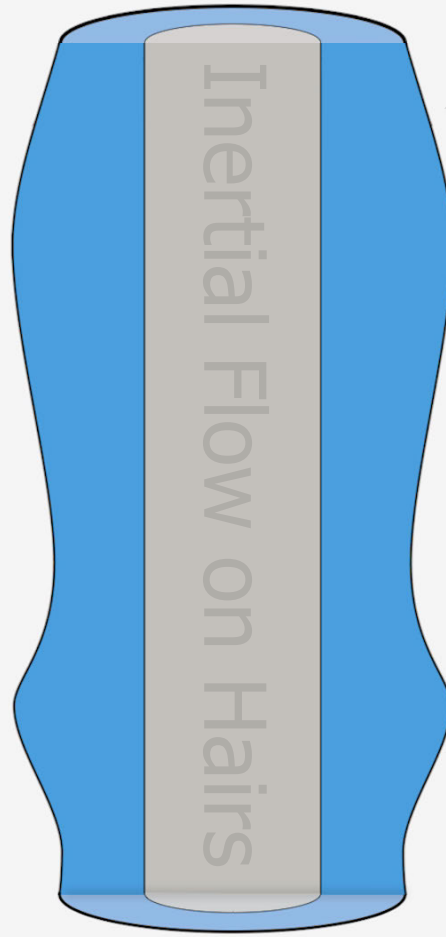
Two-way Coupling

Dripping
Capturing





Cohesion
↔
Collision



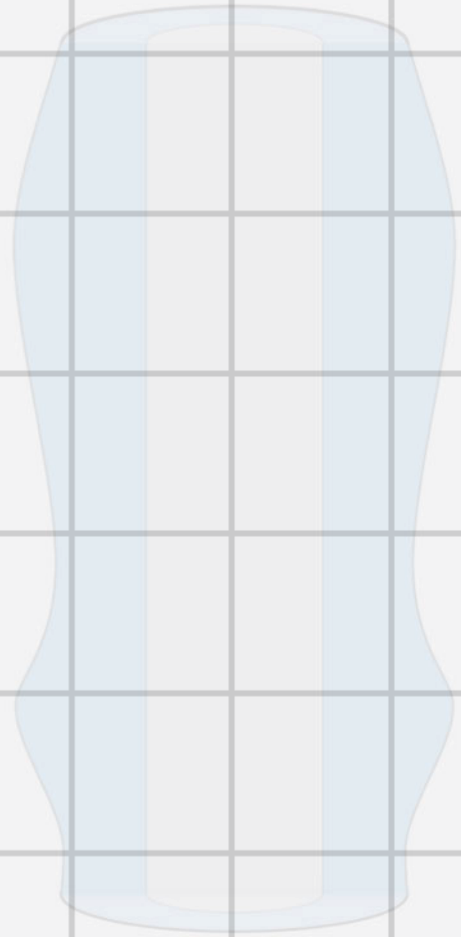
Two-way Coupling

Dripping



Capturing

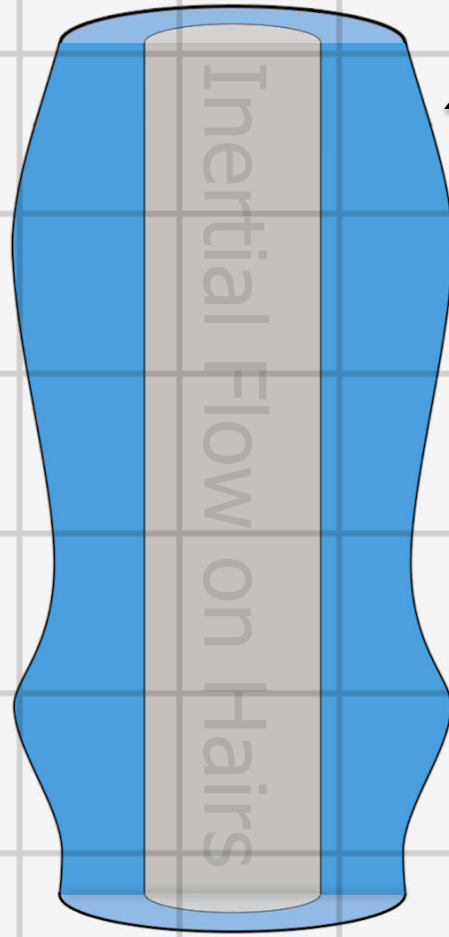




Cohesion

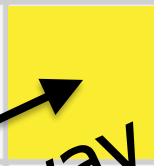


Collision



Inertial Flow on Hairs

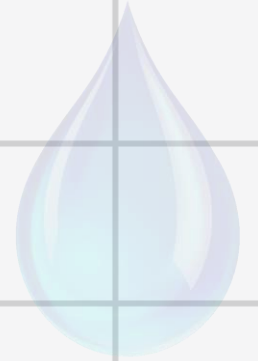
Two-way Coupling

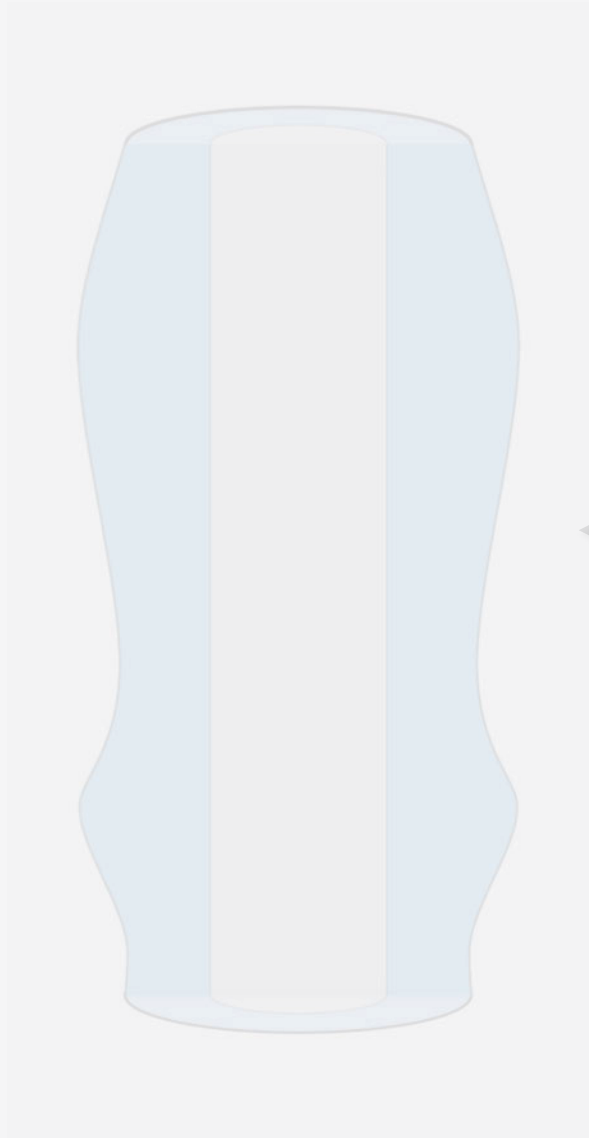


Dripping

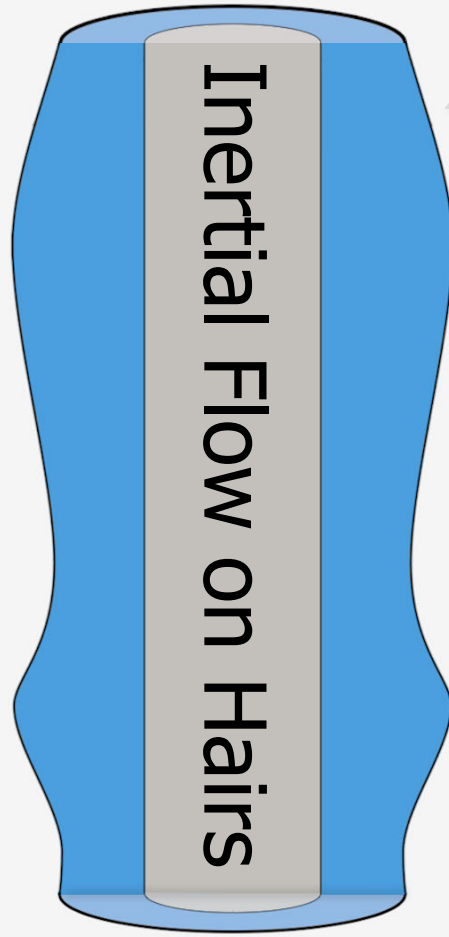


Capturing





Cohesion
Collision



Two-way Coupling

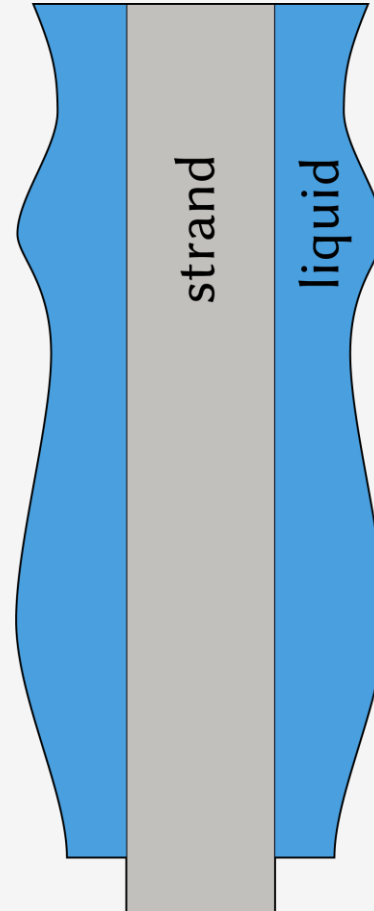
Dripping
Capturing



Where does the liquid live?



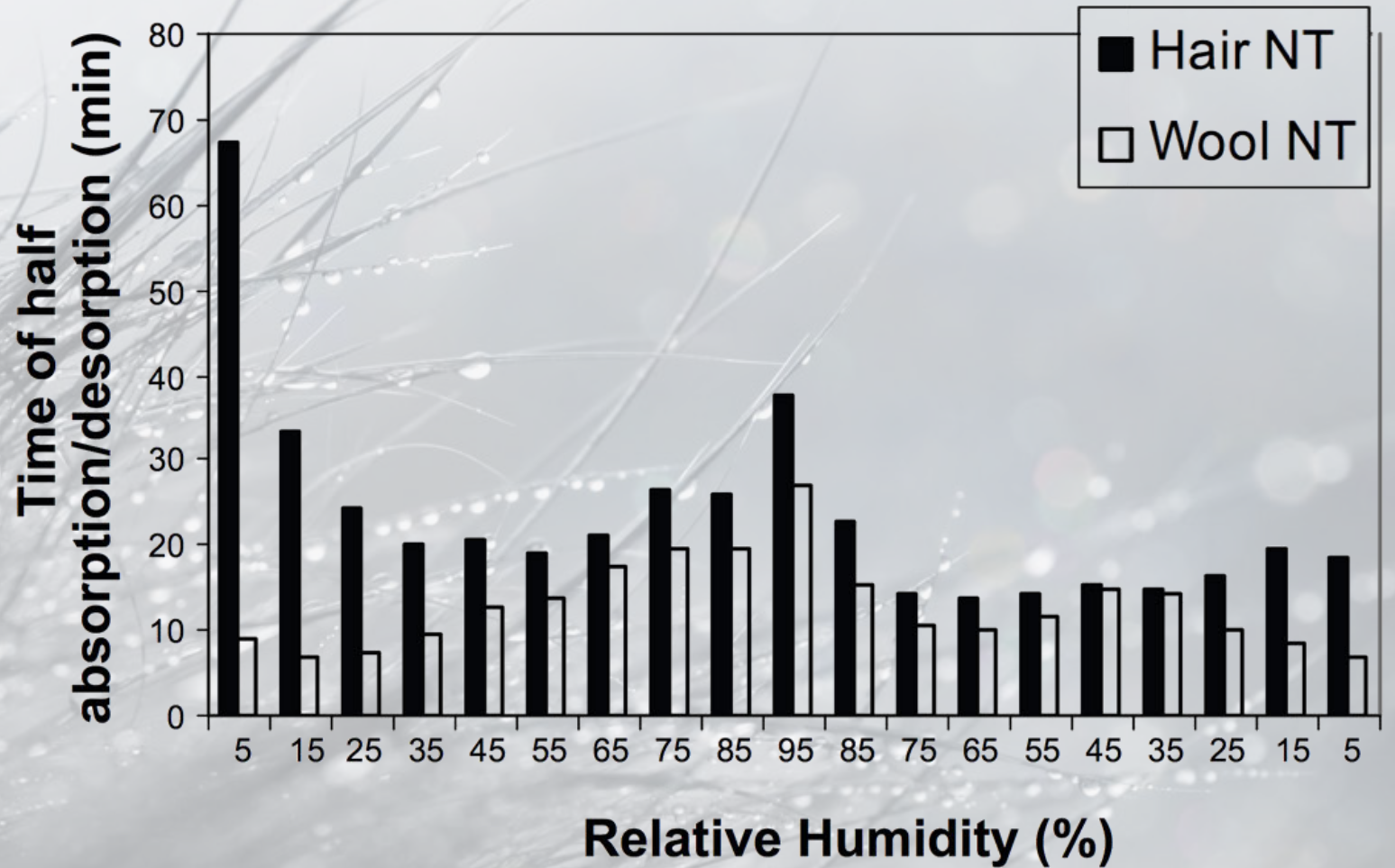
Liquid Inside Hair



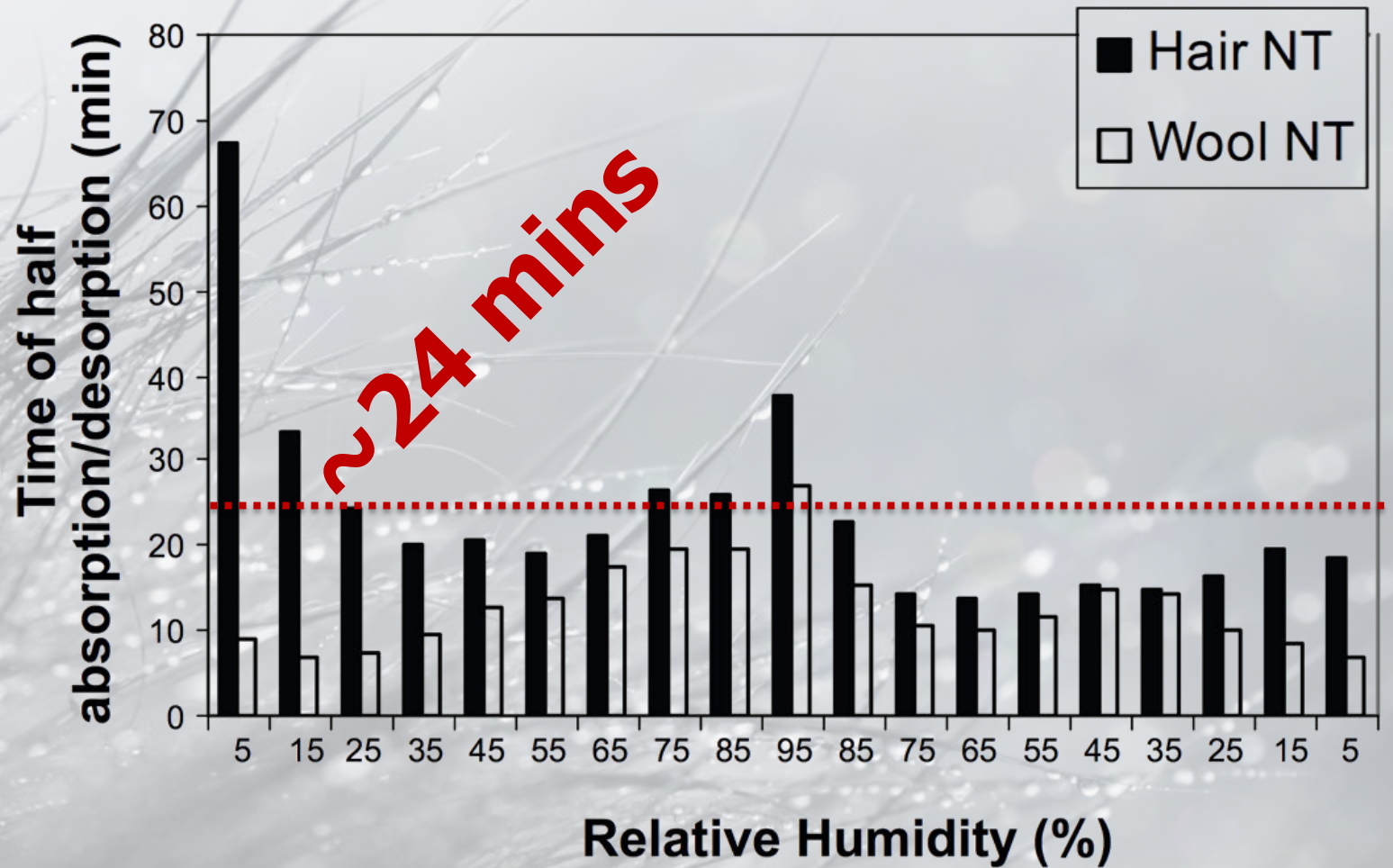
On Hair Surface



Image Courtesy Of La Corivo

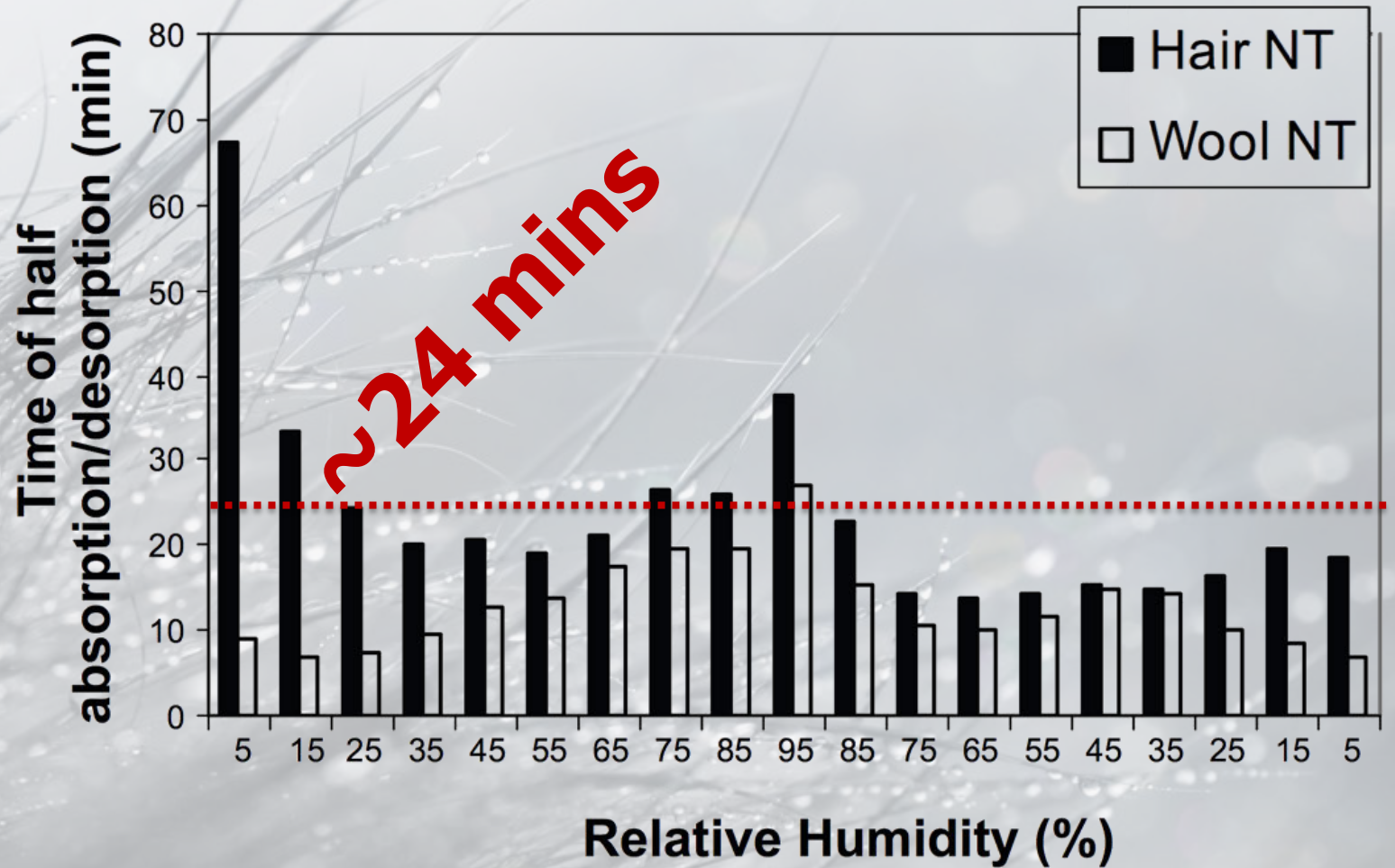


Barba, C., et al. "Moisture sorption/desorption of protein fibres." *Thermochimica acta* 552 (2013): 70-76.



Barba, C., et al. "Moisture sorption/desorption of protein fibres." *Thermochimica acta* 552 (2013): 70-76.

~Seconds

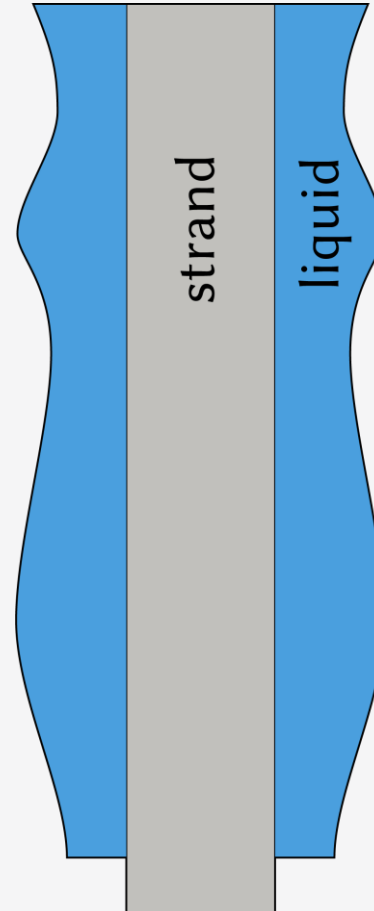


Barba, C., et al. "Moisture sorption/desorption of protein fibres." *Thermochimica acta* 552 (2013): 70-76.

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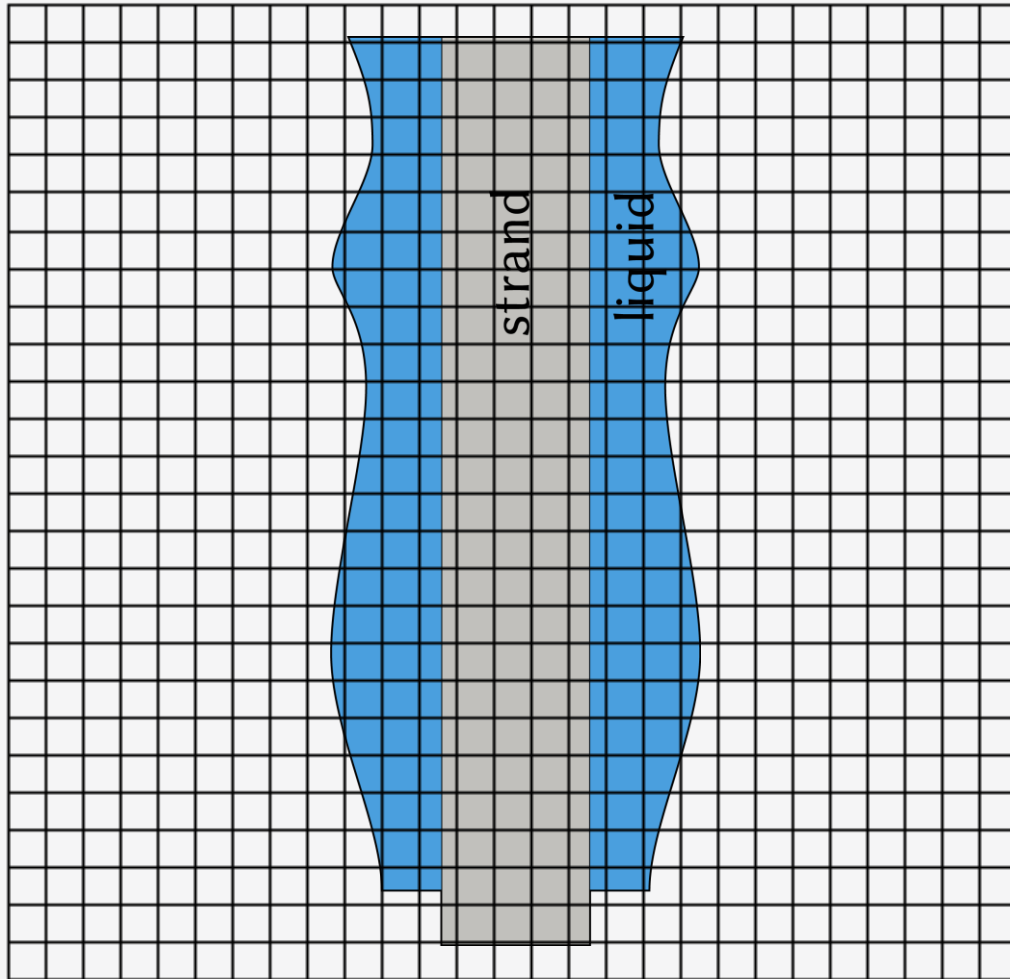
Liquid Inside Hair



On Hair Surface



Simulating Liquid on Hair Surface



Full discretized simulation is too costly ✘

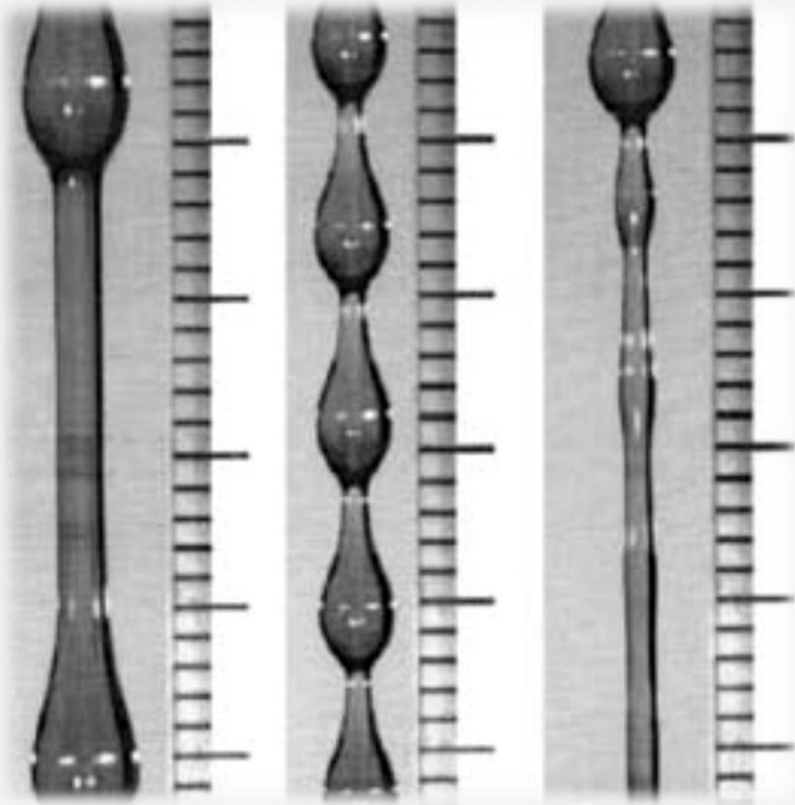


Support Inertia & Sudden Accelerations

Need Surface Tension

Specifically Designed for Thin Liquid

Image Courtesy of Alexander Demianchuk / Reuters



- ✓ Support Inertia & Sudden Accelerations
 - ✓ Need Surface Tension
- Specifically Designed for Thin Liquid

Craster, R. V., and O. K. Matar. "On viscous beads flowing down a vertical fibre." *Journal of Fluid Mechanics* 553 (2006): 85-105.



Hair

Craster, R. V., and O. K

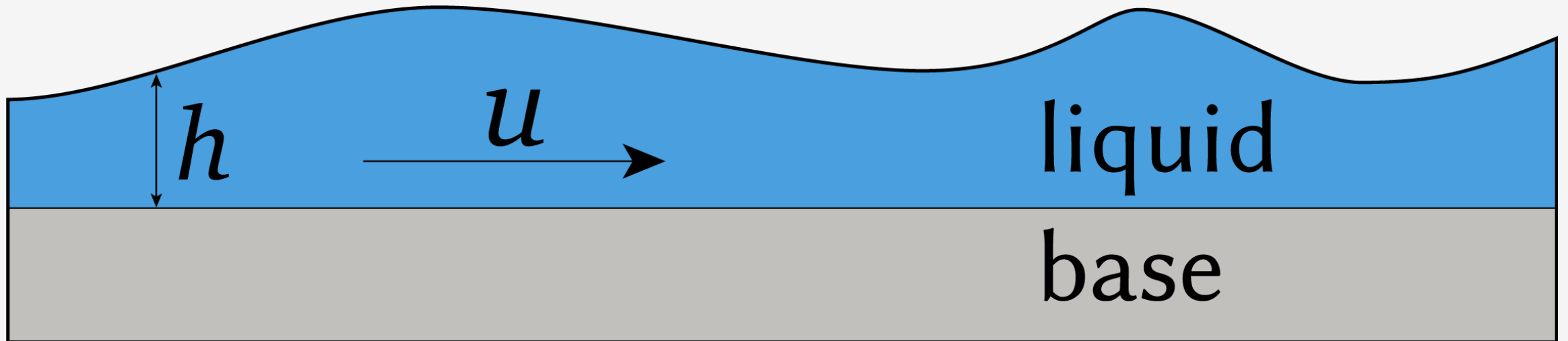
flowing down a vertical fibre." *Journal of Fluid Mechanics* 553 (2006): 85-105.

- ✓ Support Inertia & Sudden Accelerations
- ✓ Need Surface Tension
- ✓ Specifically Designed for Thin Liquid

Shallow Water Equation

Momentum Equation

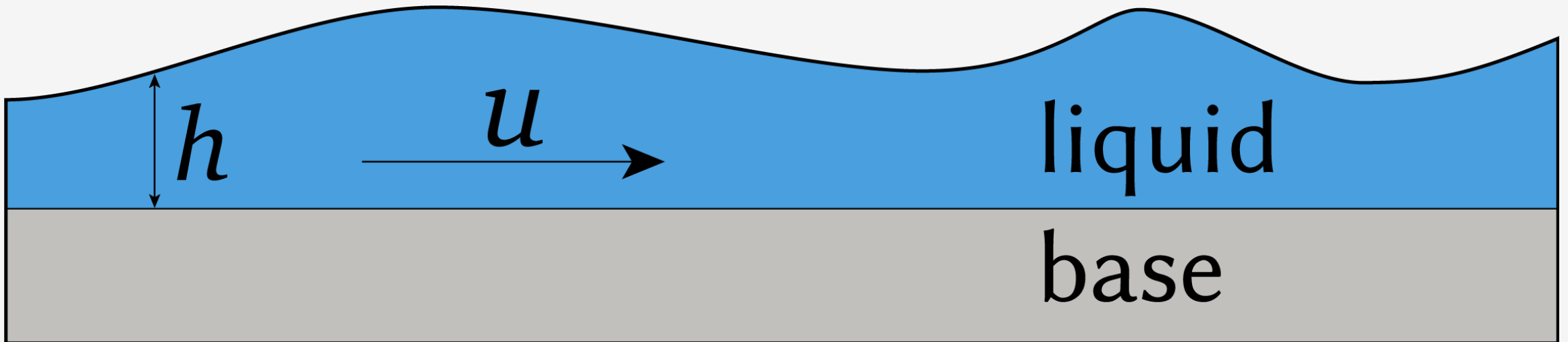
$$\underbrace{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}}_{\text{Inertia}} = \underbrace{-\frac{1}{\rho} \frac{\partial p}{\partial x}}_{\text{Surface Tension}} + \underbrace{a_{\text{ext}}}_{\text{External Force (Gravity)}}$$



Shallow Water Equation

Momentum Equation

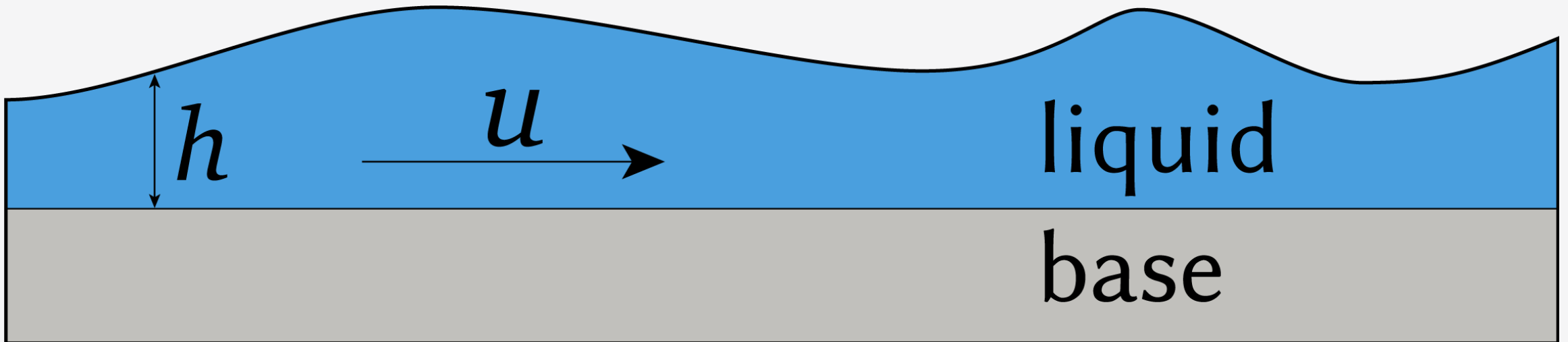
$$\underbrace{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}}_{\checkmark \text{ Inertia}} = - \underbrace{\frac{1}{\rho} \frac{\partial p}{\partial x}}_{\text{Surface Tension}} + \underbrace{a_{\text{ext}}}_{\text{External Force (Gravity)}}$$



Shallow Water Equation

Momentum Equation

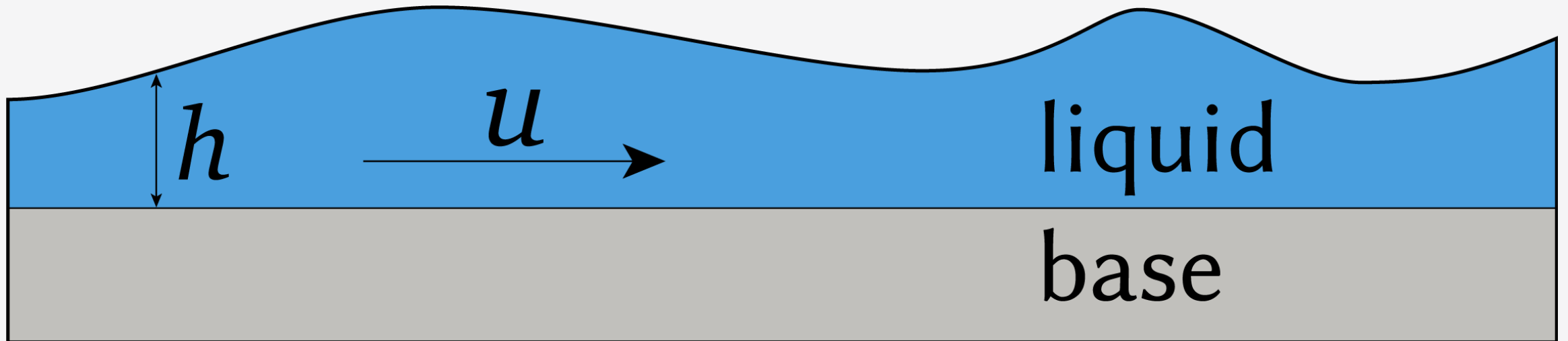
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Shallow Water Equation

Momentum Equation

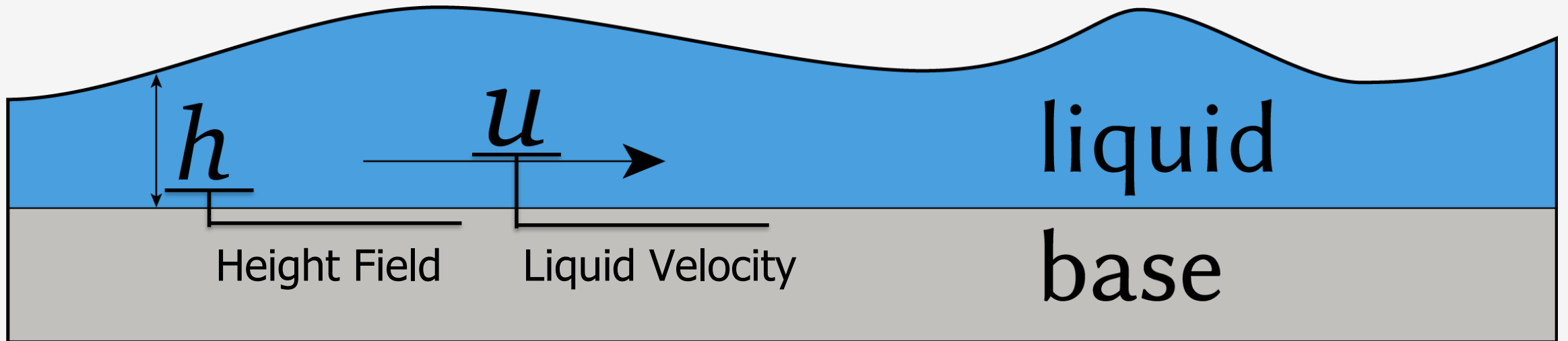
$$\underbrace{\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}}_{\checkmark \text{ Inertia}} = \underbrace{-\frac{1}{\rho} \frac{\partial p}{\partial x}}_{\checkmark \text{ Surface Tension}} + \underbrace{a_{\text{ext}}}_{\checkmark \text{ External Force (Gravity)}}$$



Shallow Water Equation

Continuity Equation for Mass

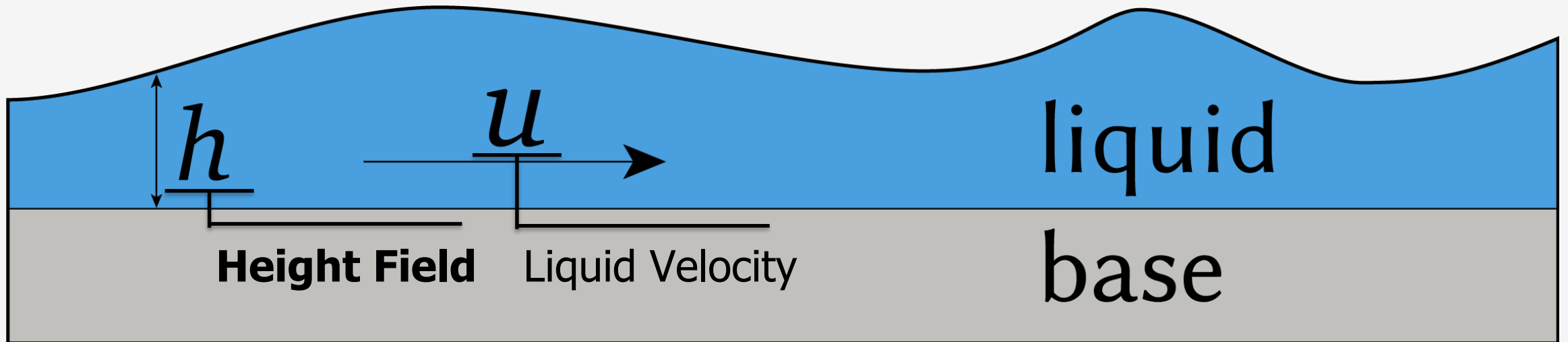
$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0$$



Shallow Water Equation

Continuity Equation for Mass

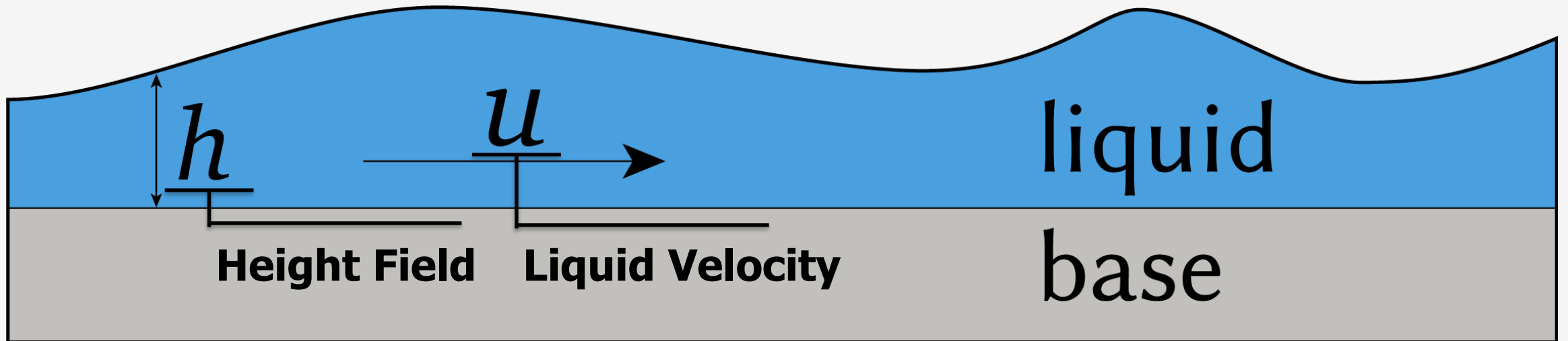
$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0$$



Shallow Water Equation

Continuity Equation for Mass

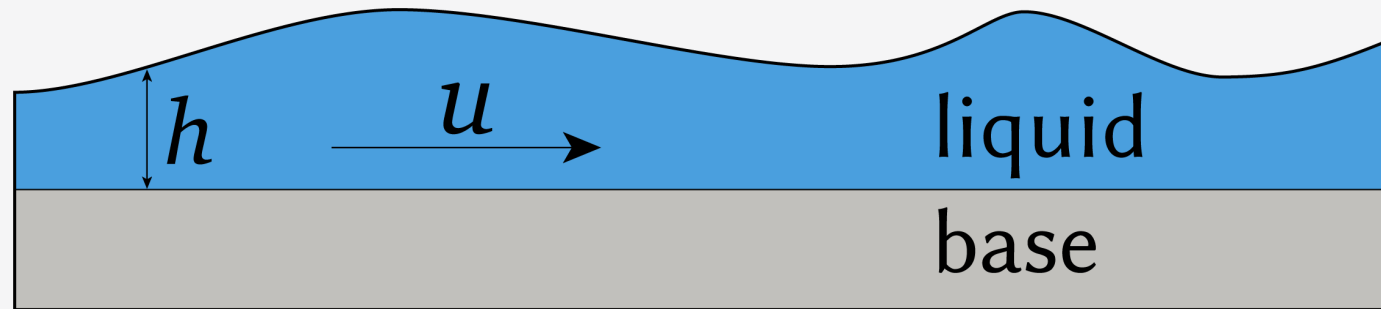
$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0$$



Shallow Water Equation

Continuity Equation for Mass

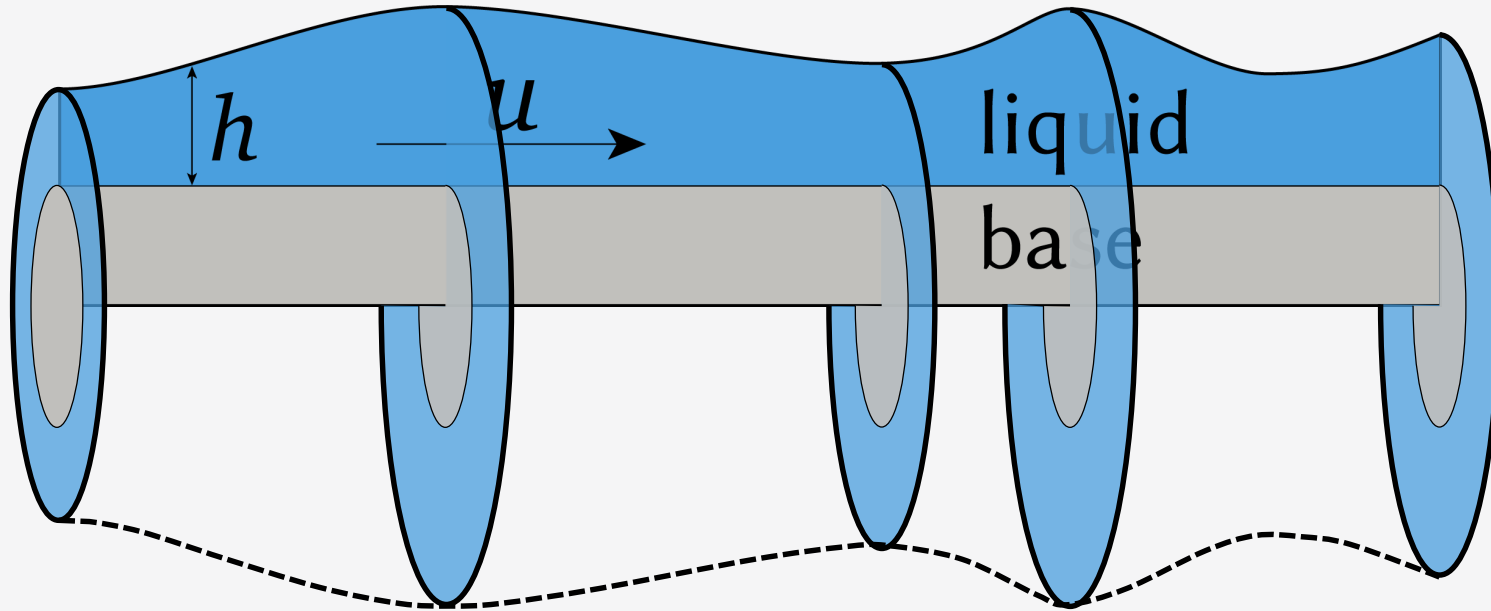
$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0$$



Reduced-Liquid Equation on Hair

Continuity Equation for Mass

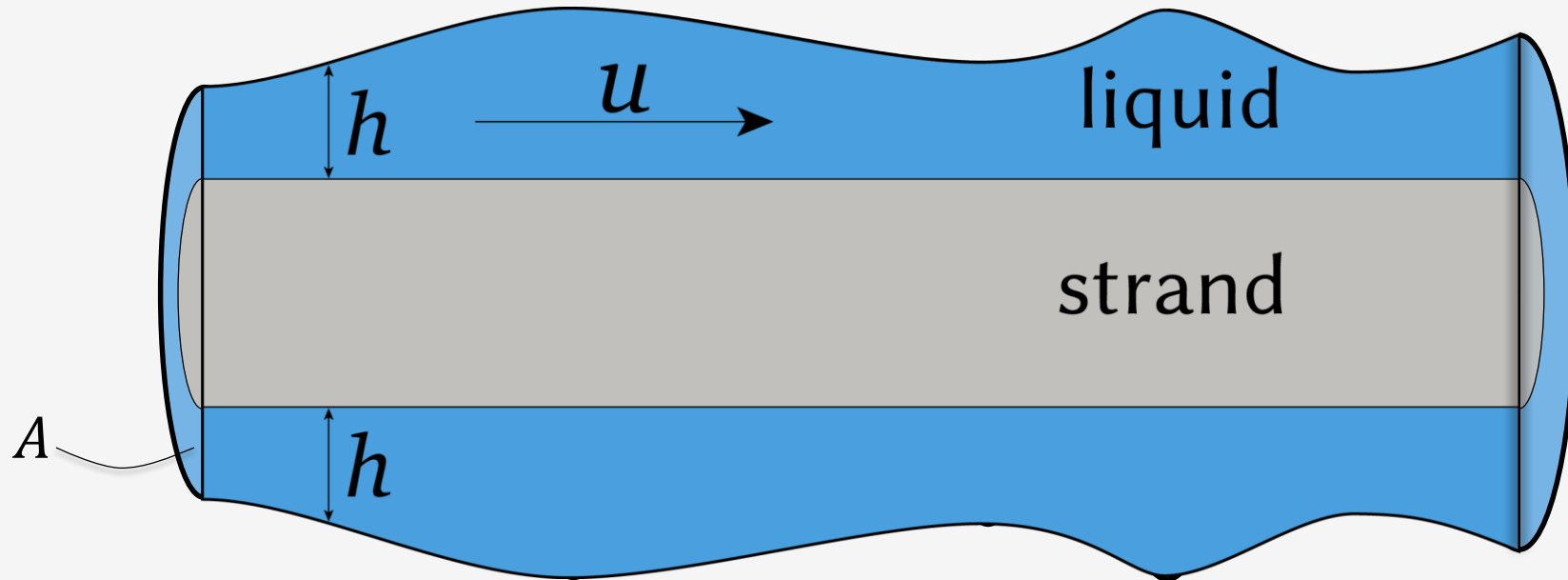
$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0$$



Reduced-Liquid Equation on Hair

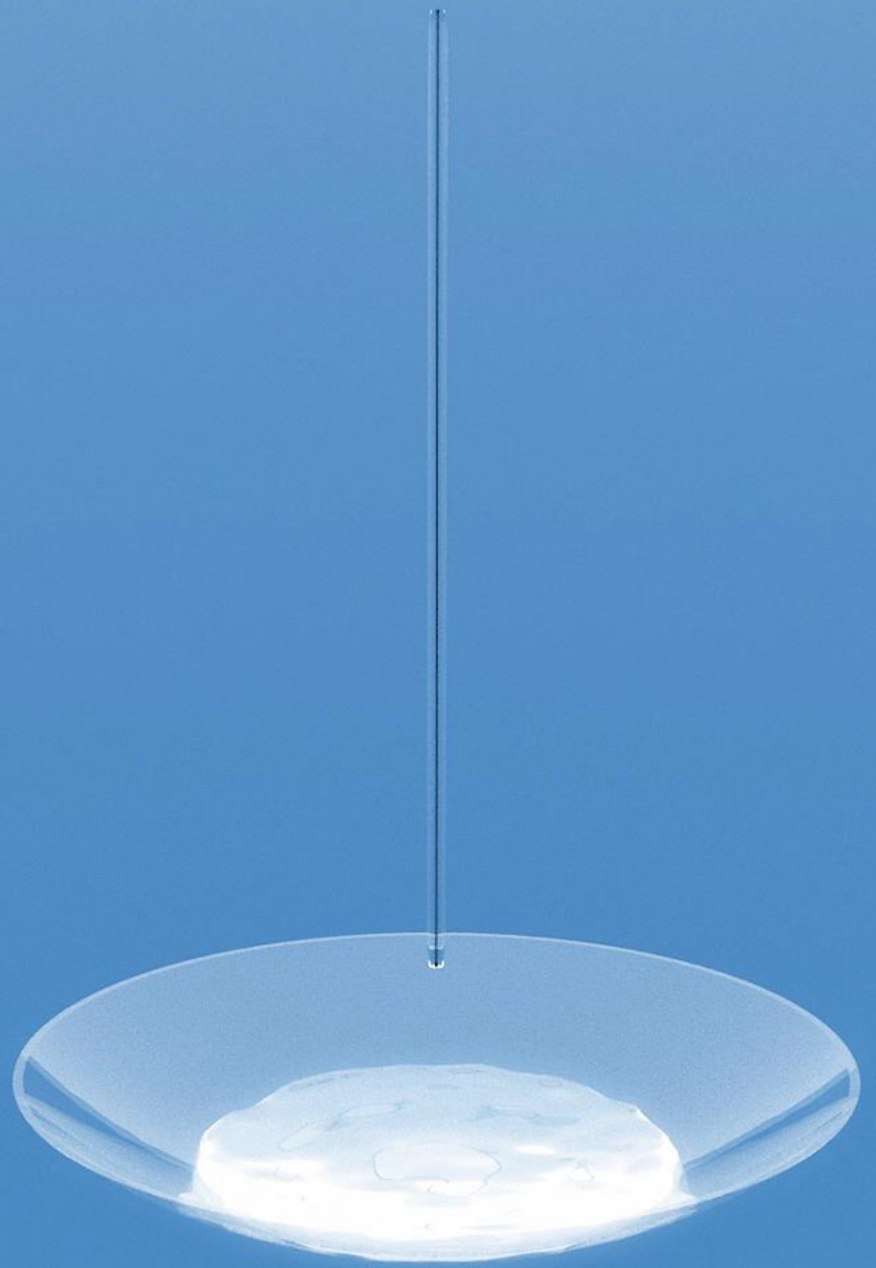
Continuity Equation for Mass

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0 \rightarrow \frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} = 0$$





Flow OFF

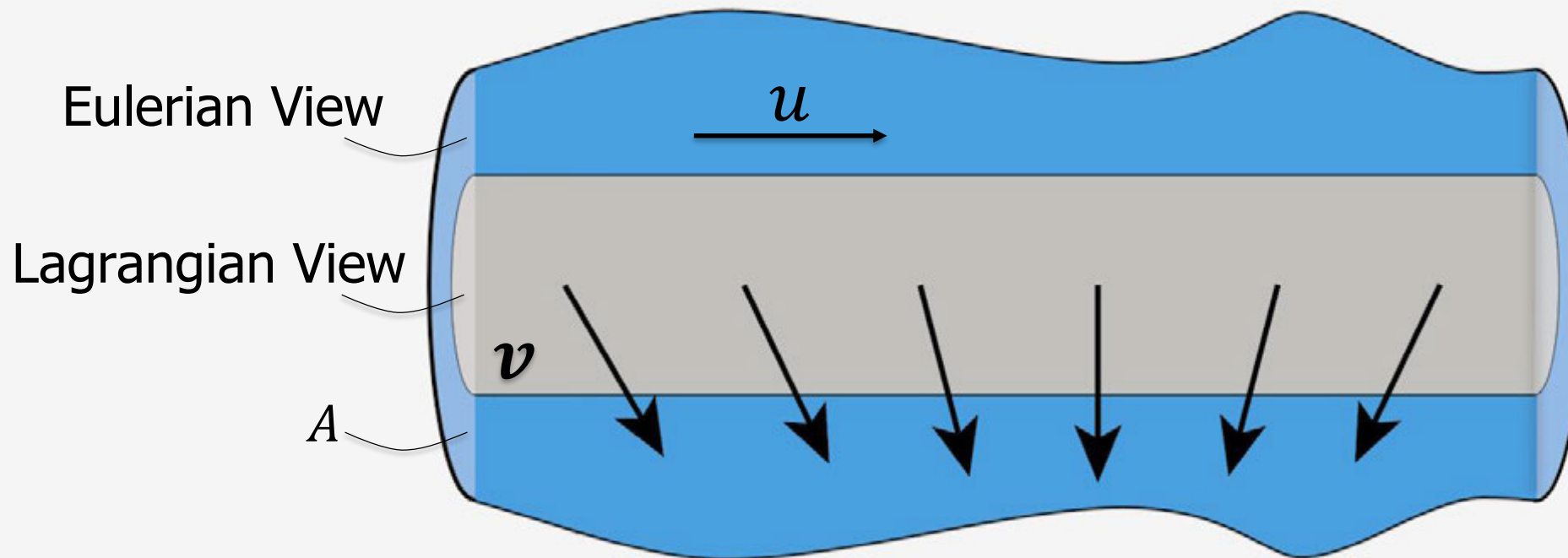


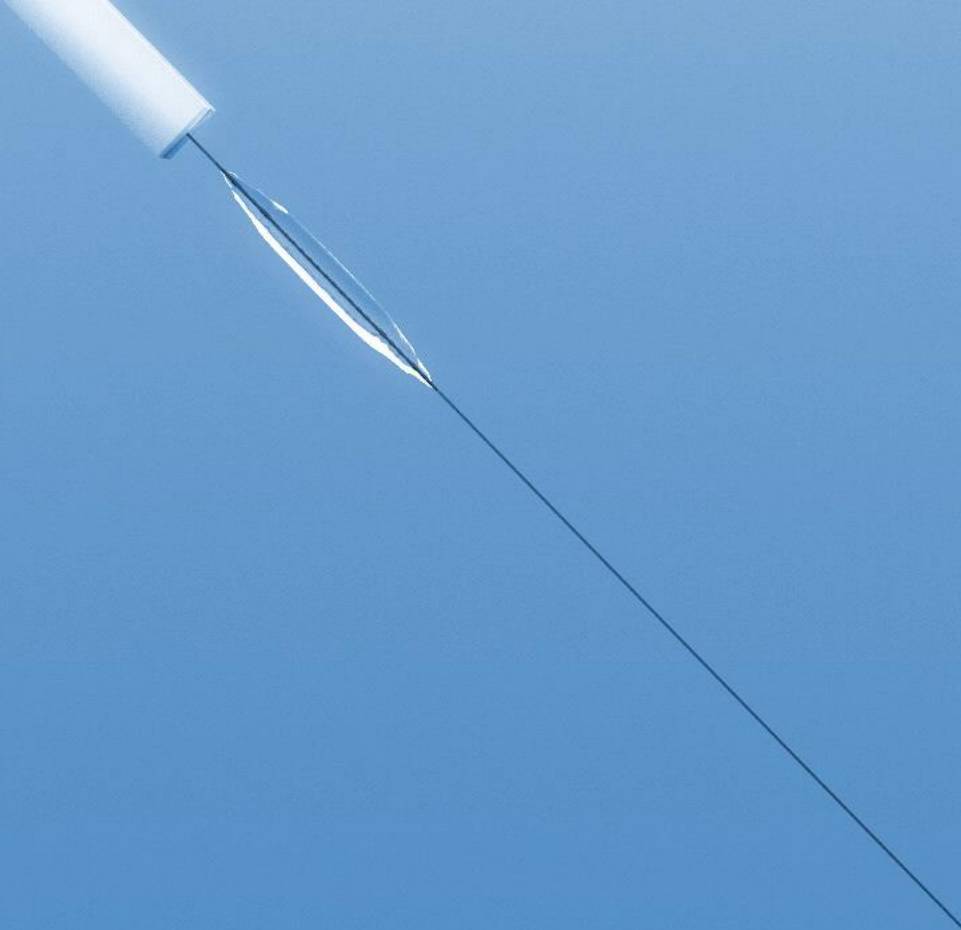
Flow ON

Momentum Conservation for Eulerian-on-Lagrangian Flow

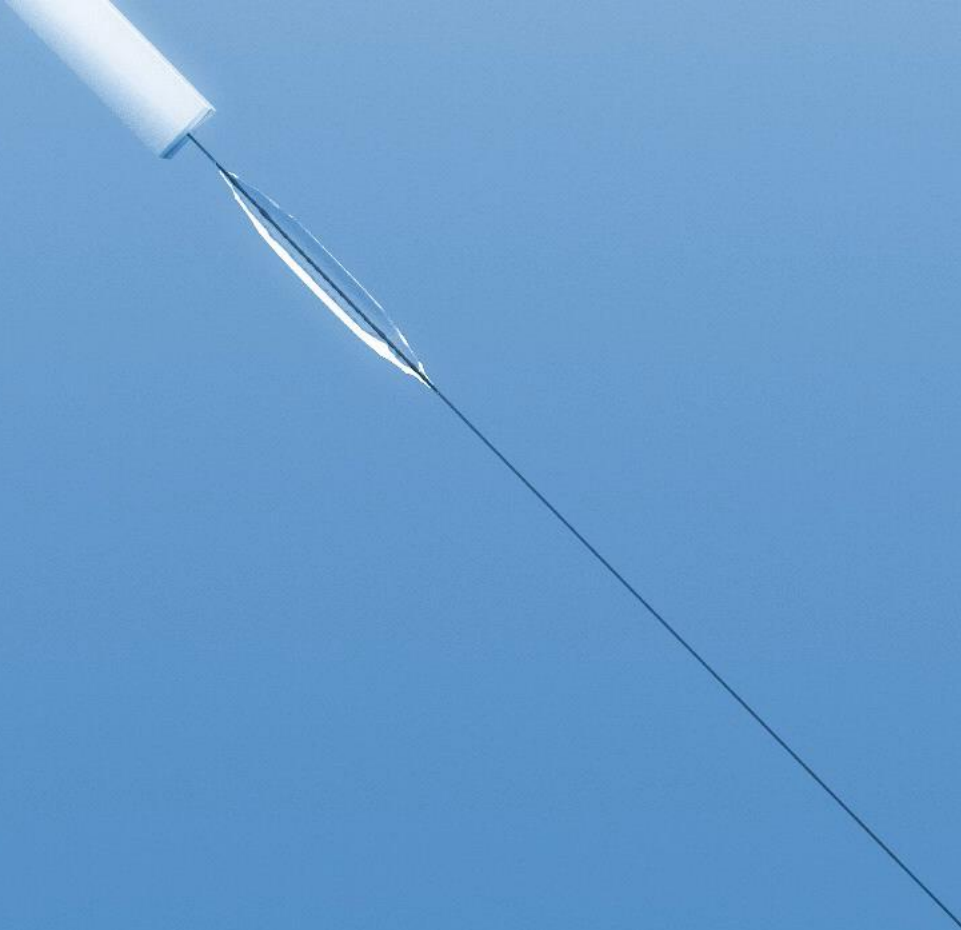
Continuity Equation for Extrinsic Momentum

$$\frac{\partial}{\partial t}(A\mathbf{v}) = -\frac{\partial}{\partial x}(A\mathbf{u}\mathbf{v})$$



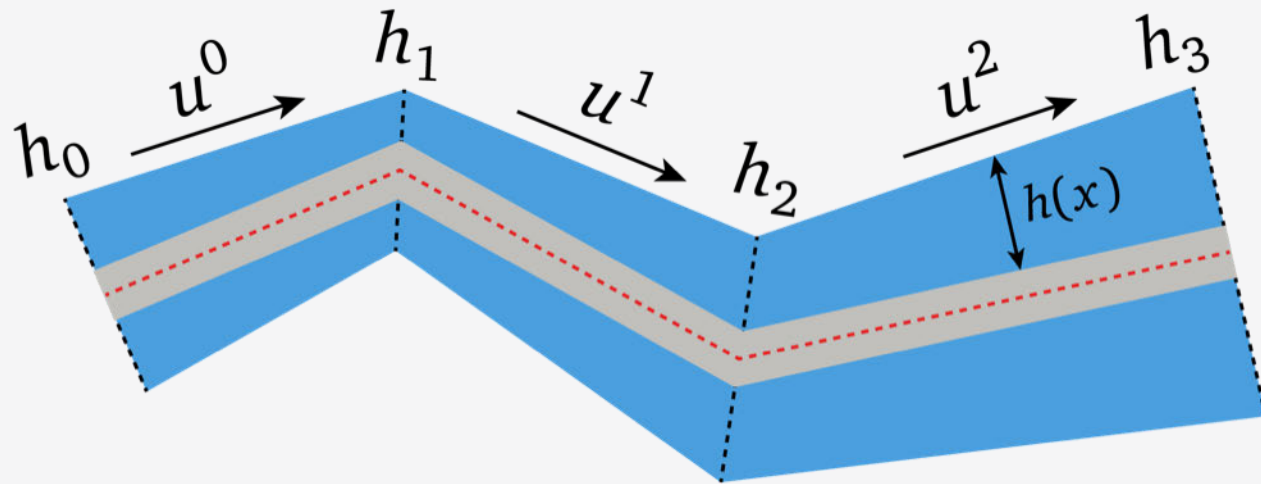


Ignore Hair Momentum Change



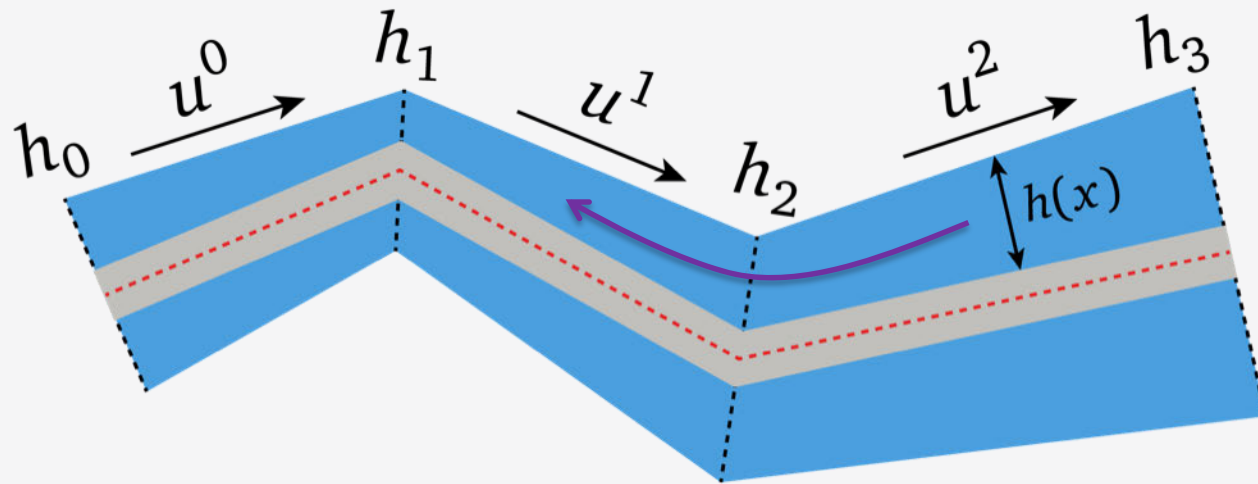
Our Method

Discretization & Pipeline



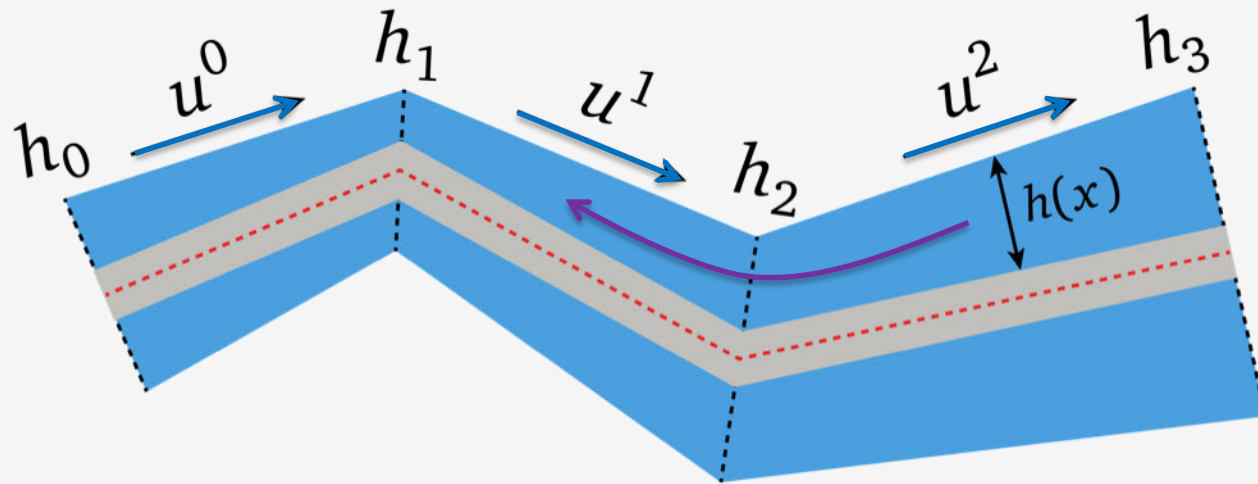
1. Semi-Lagrangian advection
2. Add external force, pressure, and update flow velocity u
3. Solve for the continuity equations
4. Update hair velocity.

Discretization & Pipeline



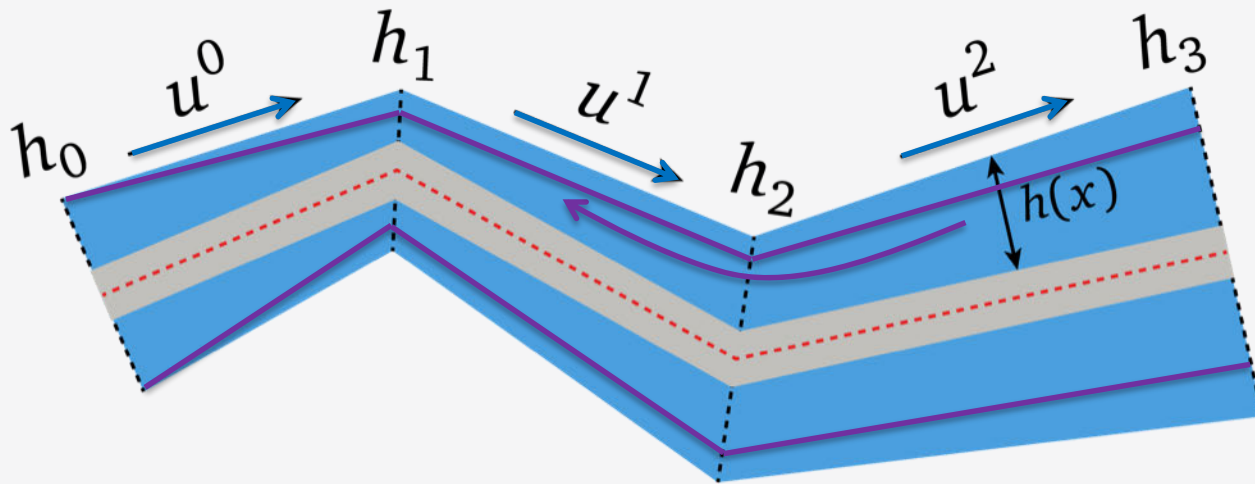
- 1. Semi-Lagrangian advection**
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Discretization & Pipeline



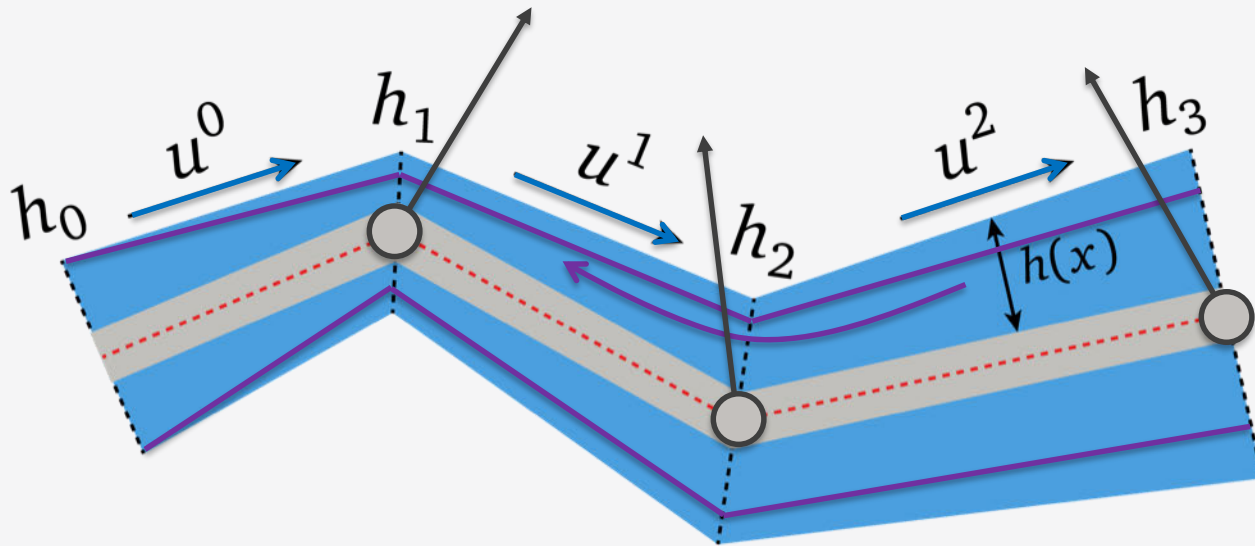
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Discretization & Pipeline

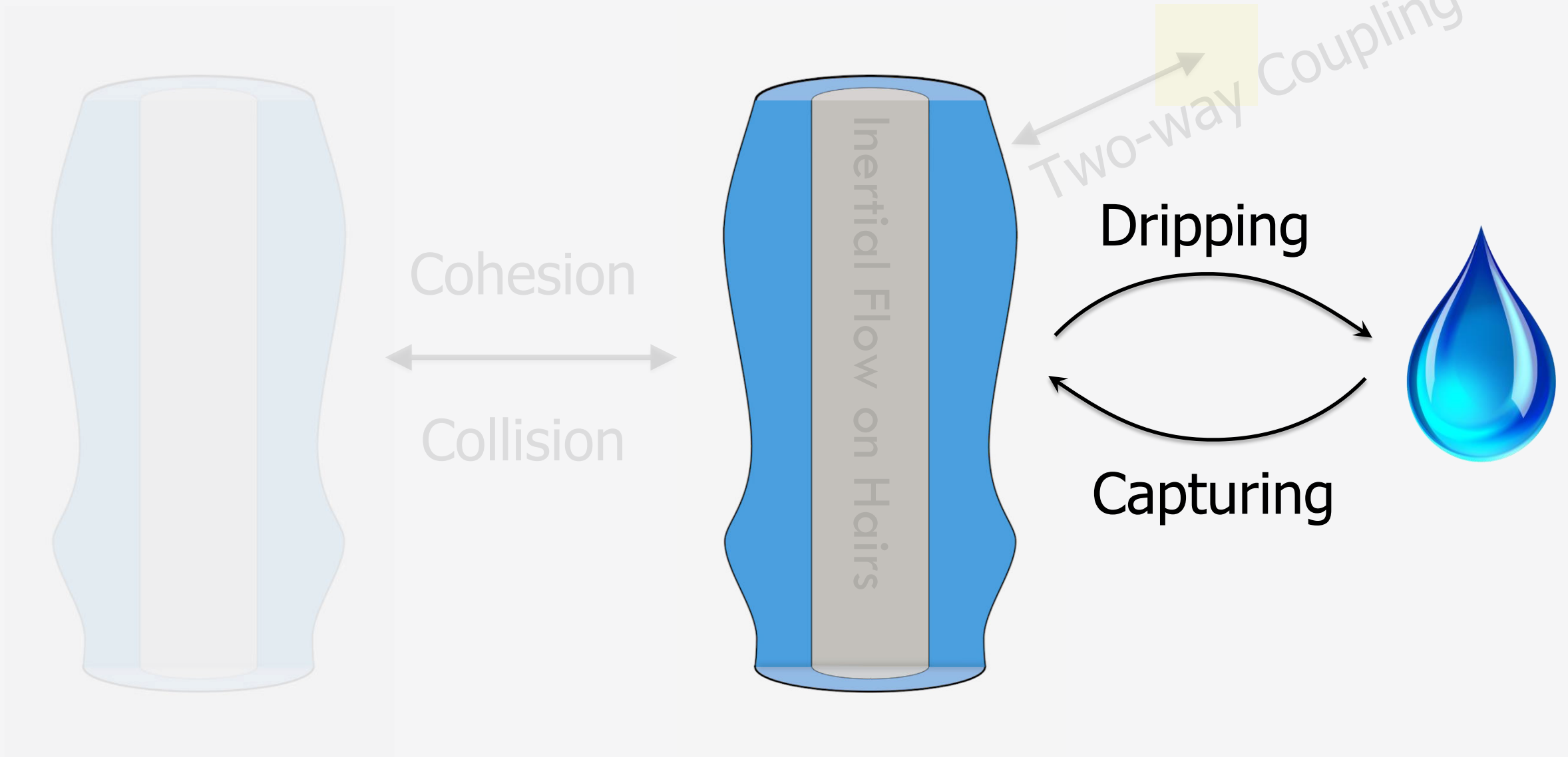


- 1. Semi-Lagrangian advection**
- 2. Add external force, pressure, and update flow velocity u**
- 3. Solve for the continuity equations**
4. Update hair velocity.

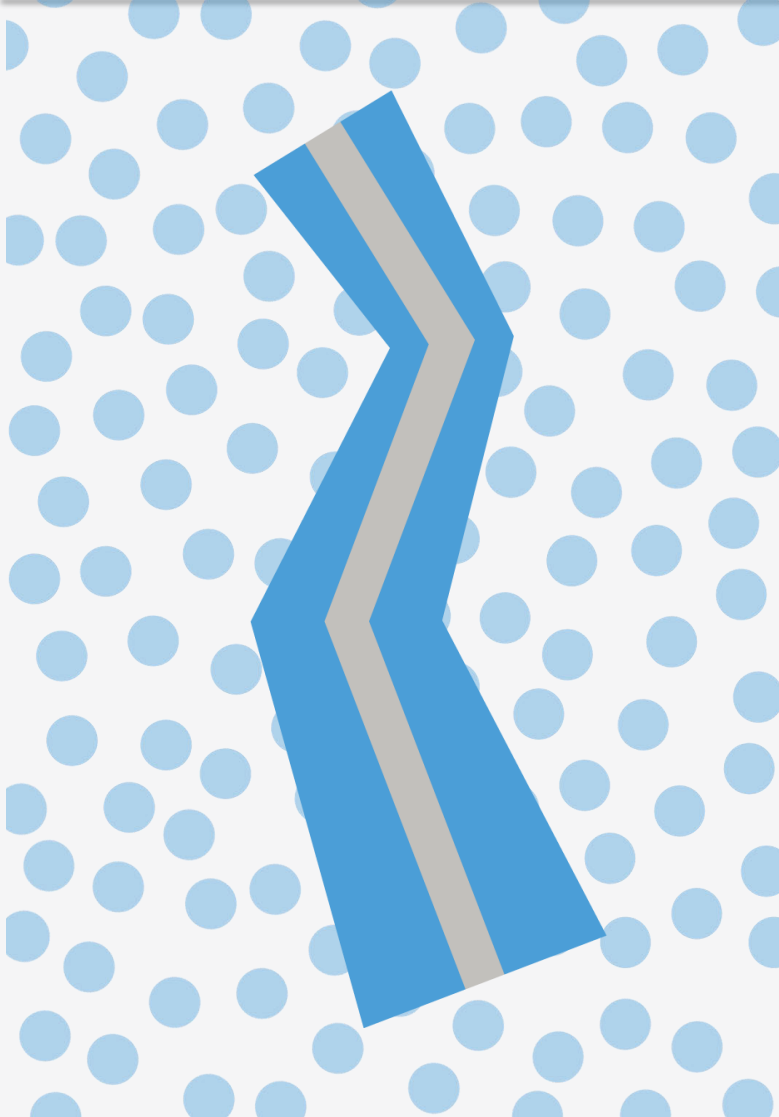
Discretization & Pipeline



- 1. Semi-Lagrangian advection**
- 2. Add external force, pressure, and update flow velocity u**
- 3. Solve for the continuity equations**
- 4. Update hair velocity.**



Capturing Reduced-Liquid



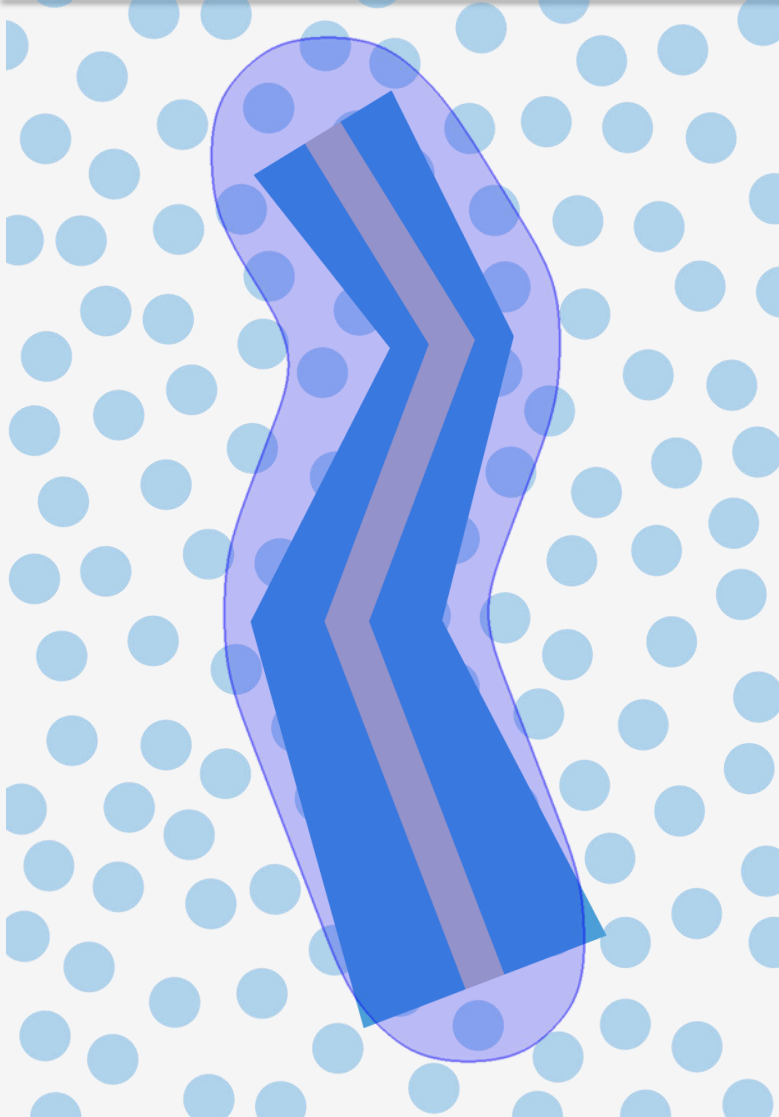
New Volume \leftarrow Original + Gathered

New Momentum \leftarrow Hair + Reduced + Gathered

New Mass \leftarrow Original + Gathered

New Velocity \leftarrow Original + Gathered

Capturing Reduced-Liquid



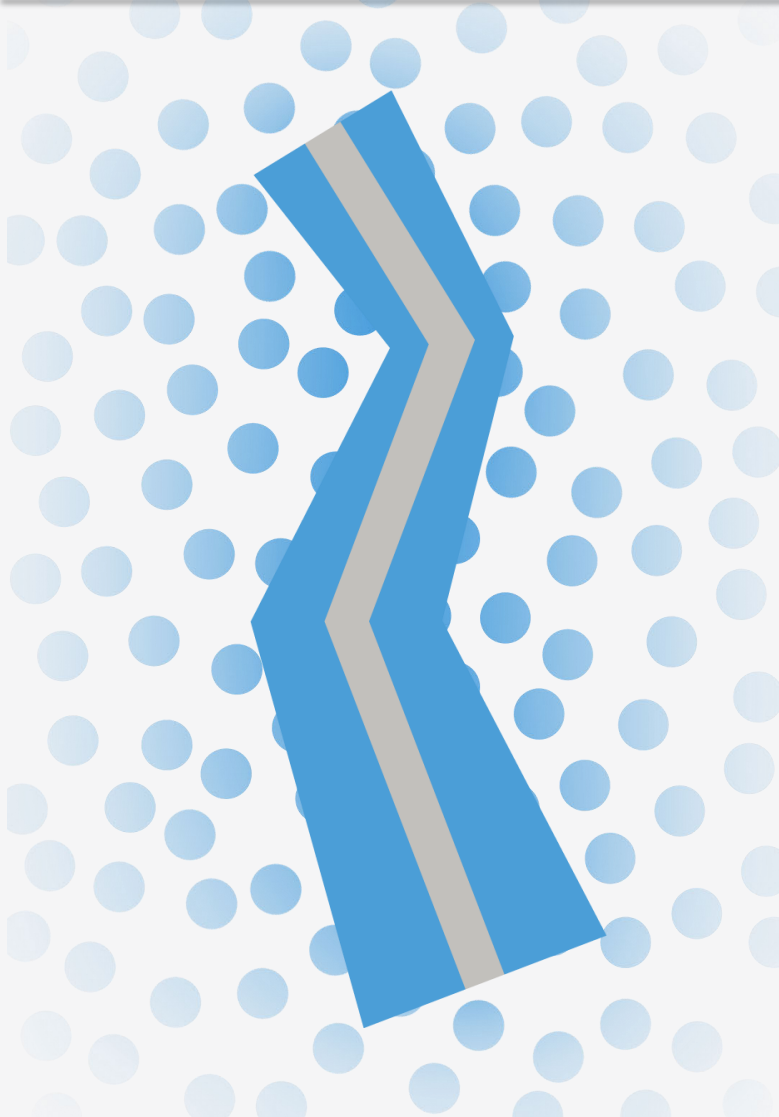
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Capturing Reduced-Liquid



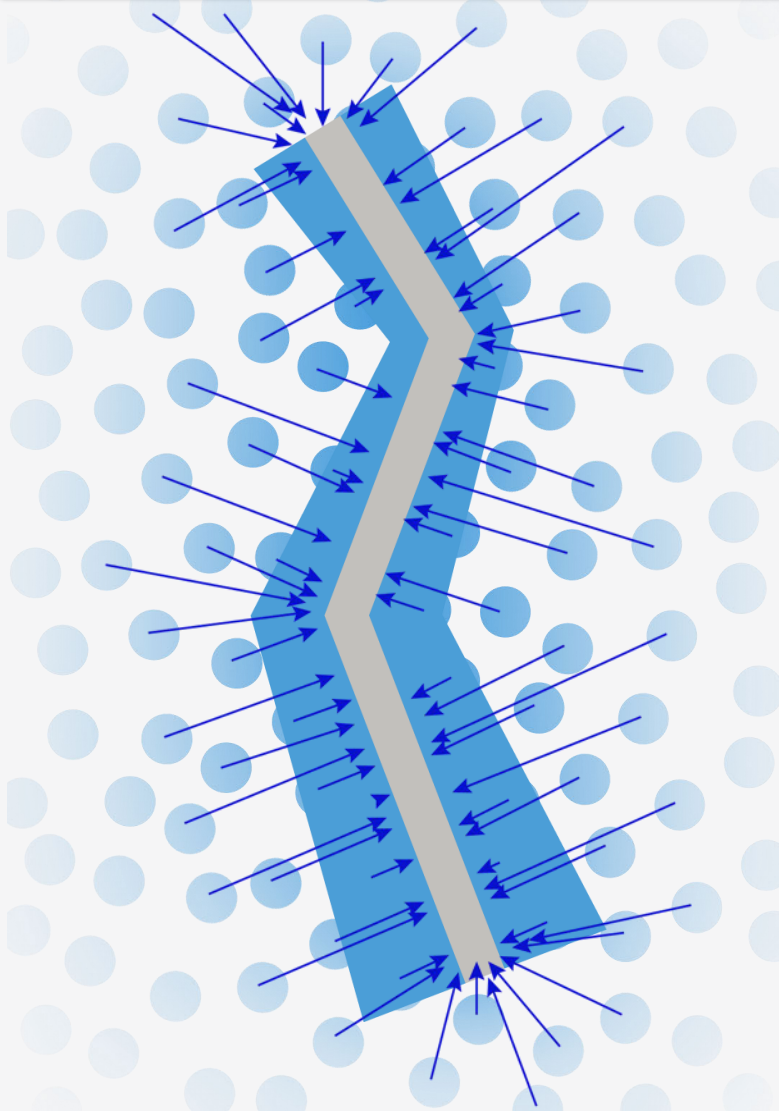
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Capturing Reduced-Liquid



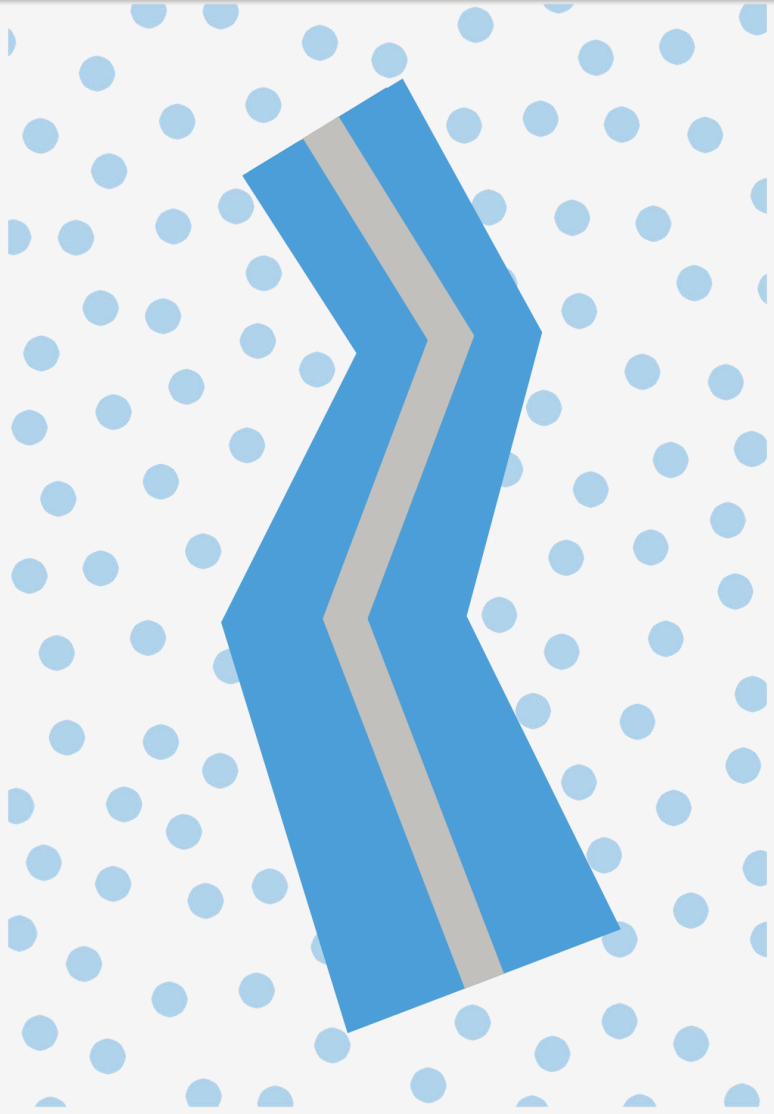
New Volume \leftarrow Original + Gathered

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New Velocity \leftarrow Original + Gathered

Capturing Reduced-Liquid



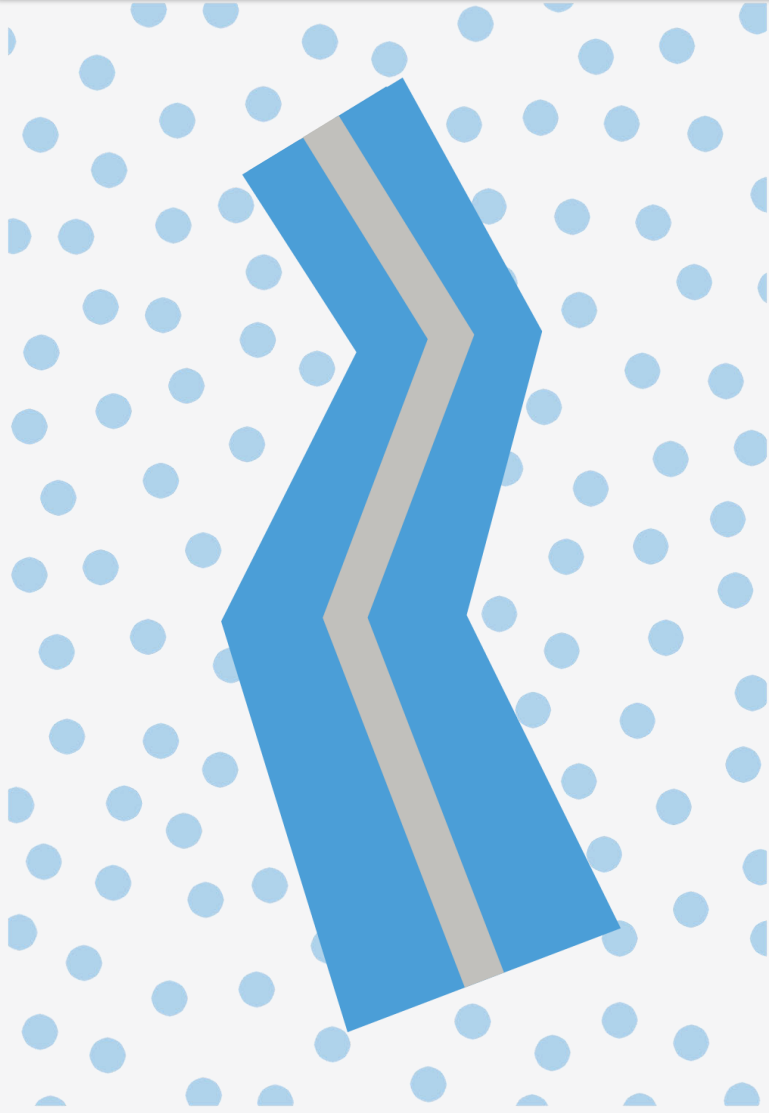
New Volume \leftarrow **Original + Gathered**

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Capturing Reduced-Liquid



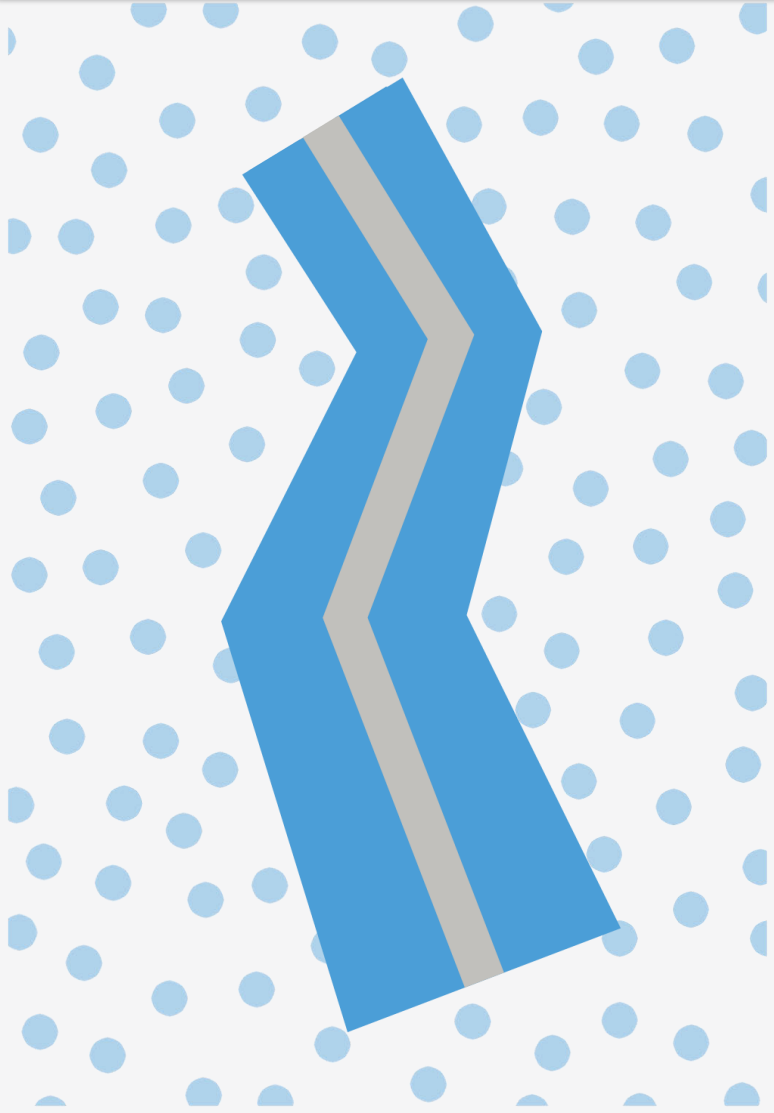
New Volume ← **Original + Gathered**

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Capturing Reduced-Liquid



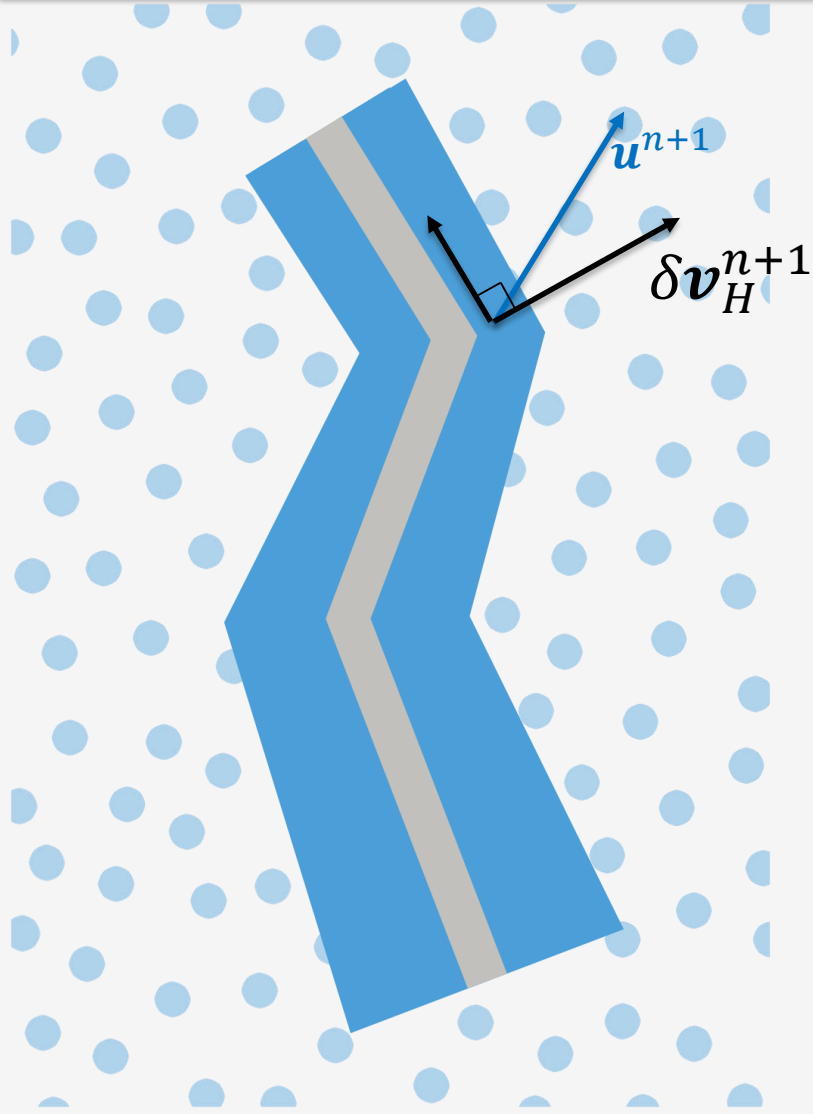
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Capturing Reduced-Liquid



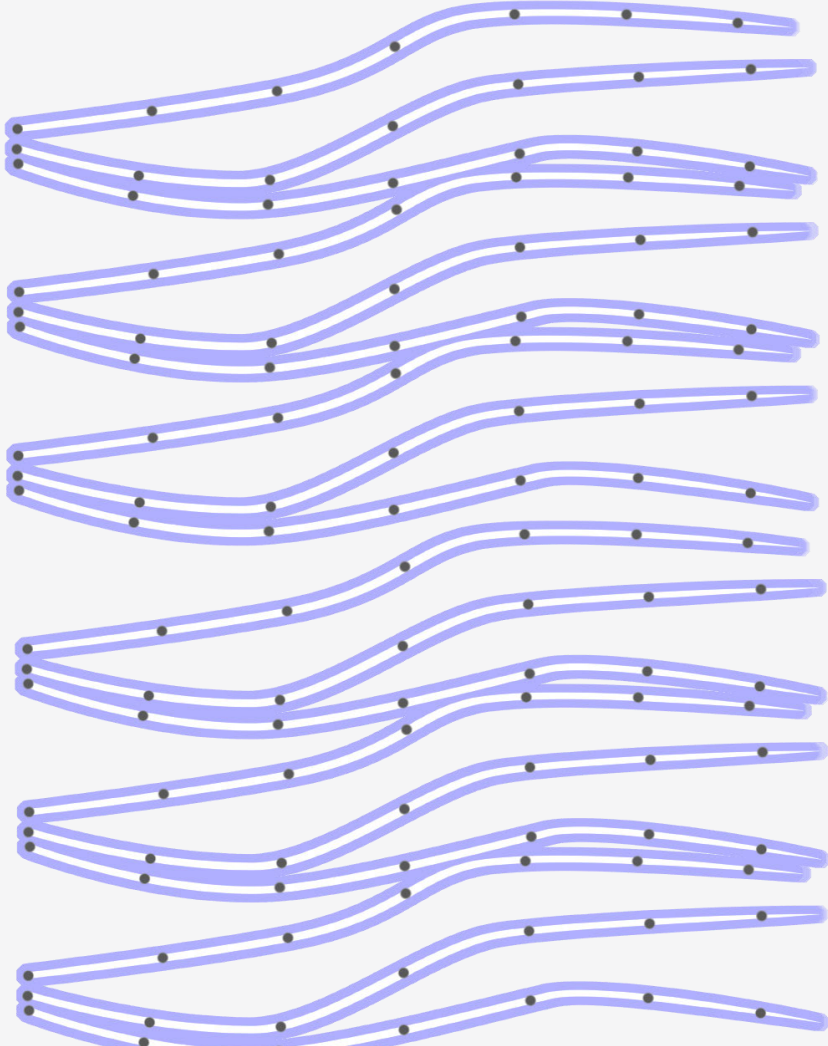
New Volume ← Original + Gathered

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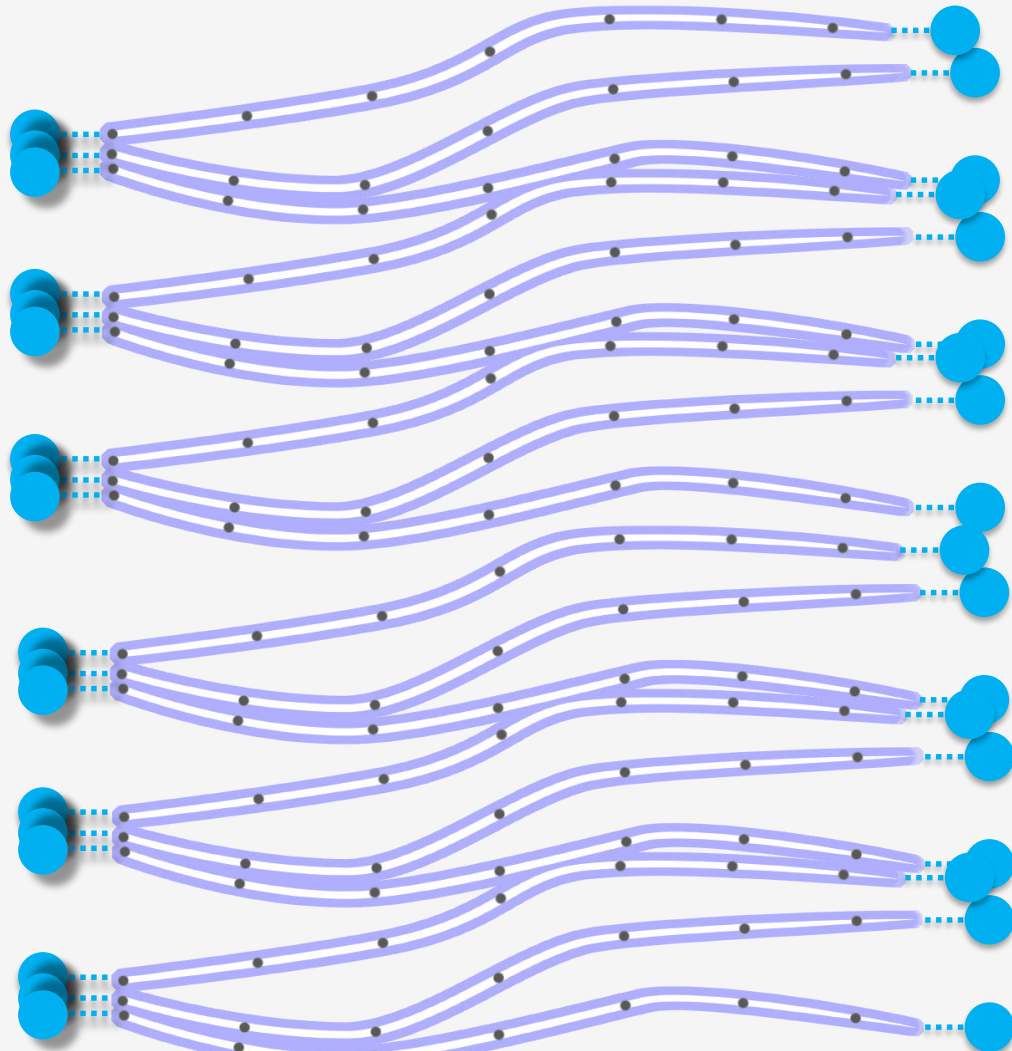
New Mass ← Original + Gathered

New Velocity ← Original + Gathered

Dripping of Reduced-Liquid

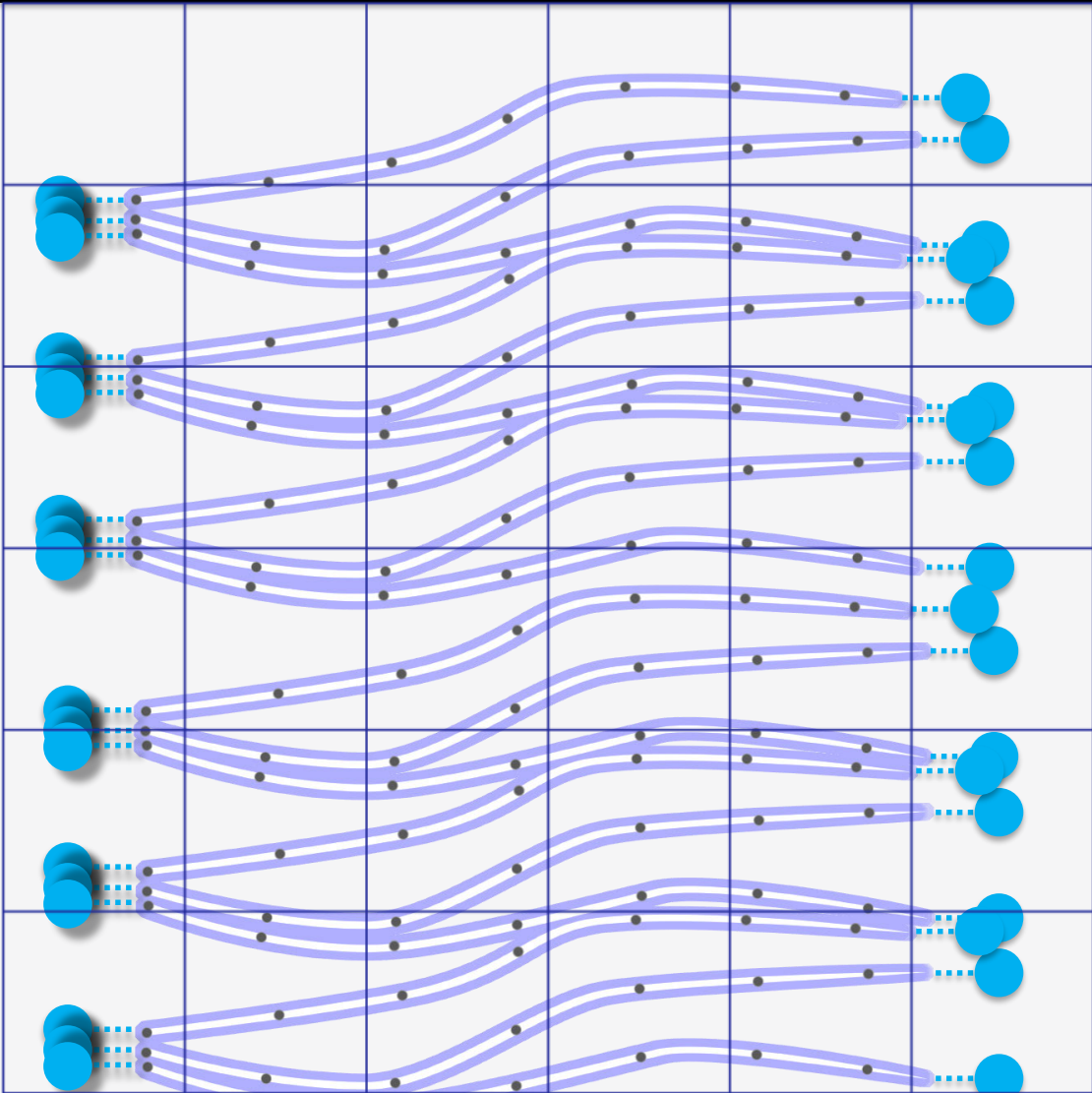


Dripping of Reduced-Liquid

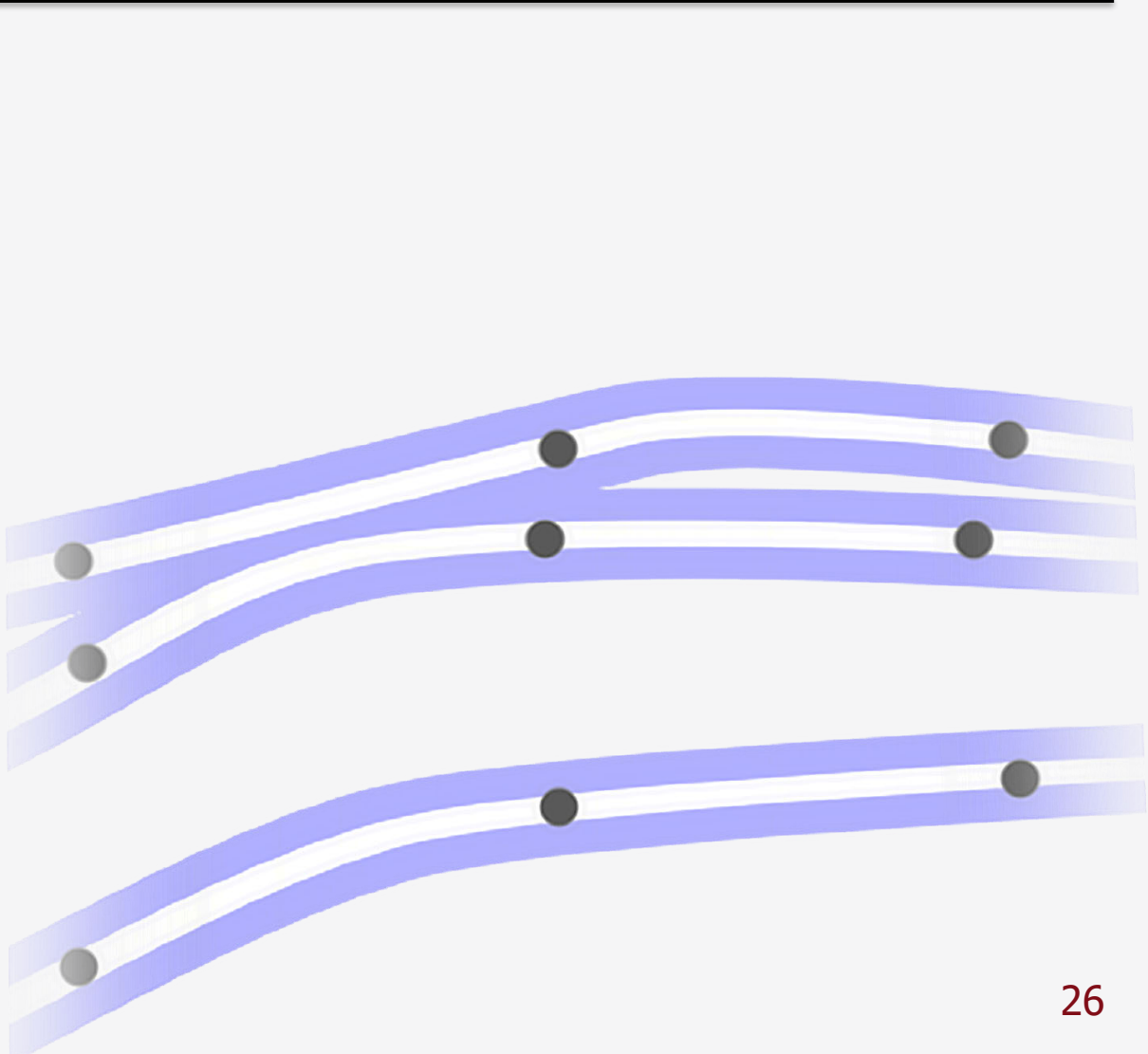
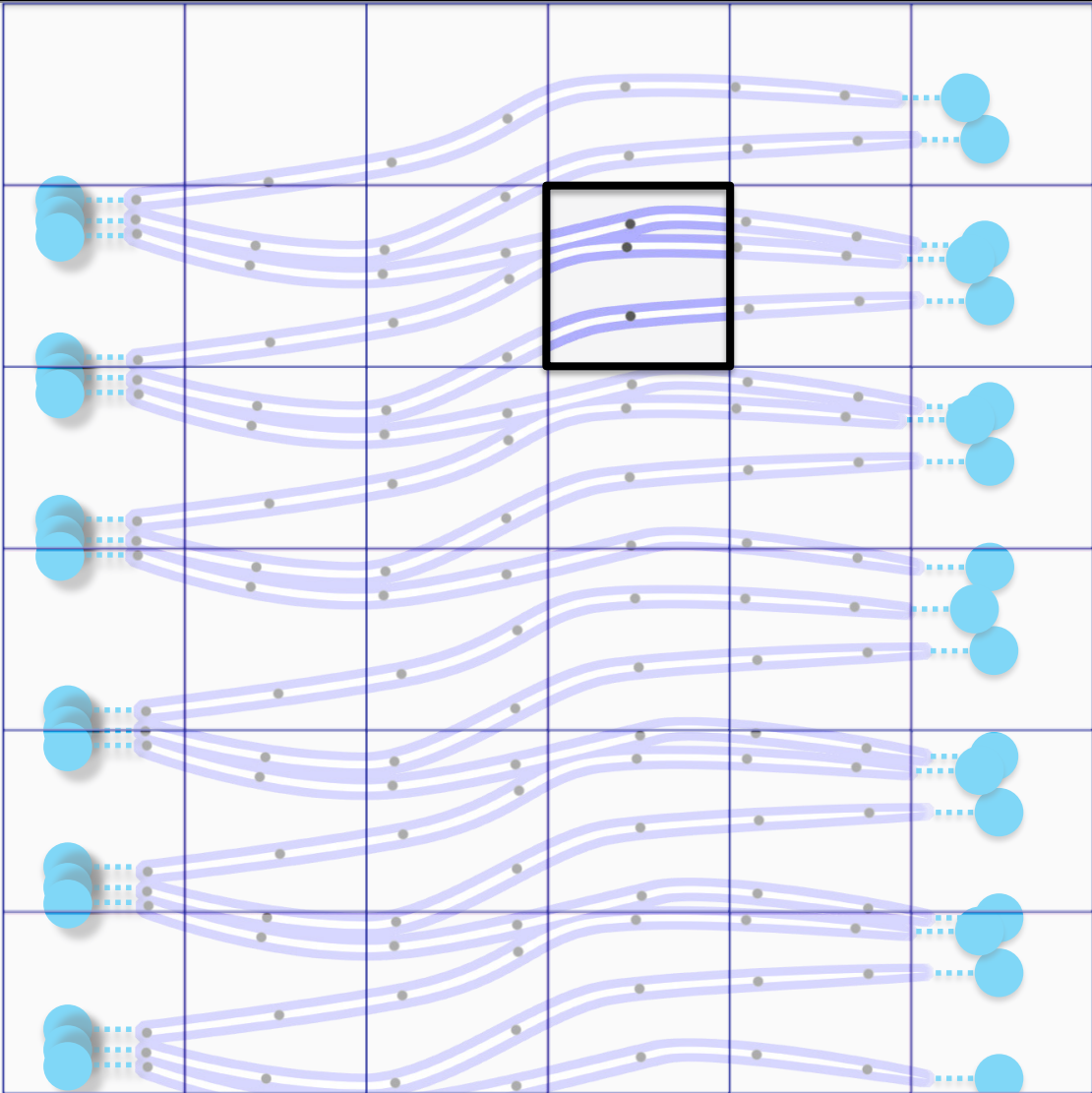


$$V_R = -\pi \int_t \int_s \frac{\partial}{\partial t} [(h + r)^2 - r^2] ds dt$$

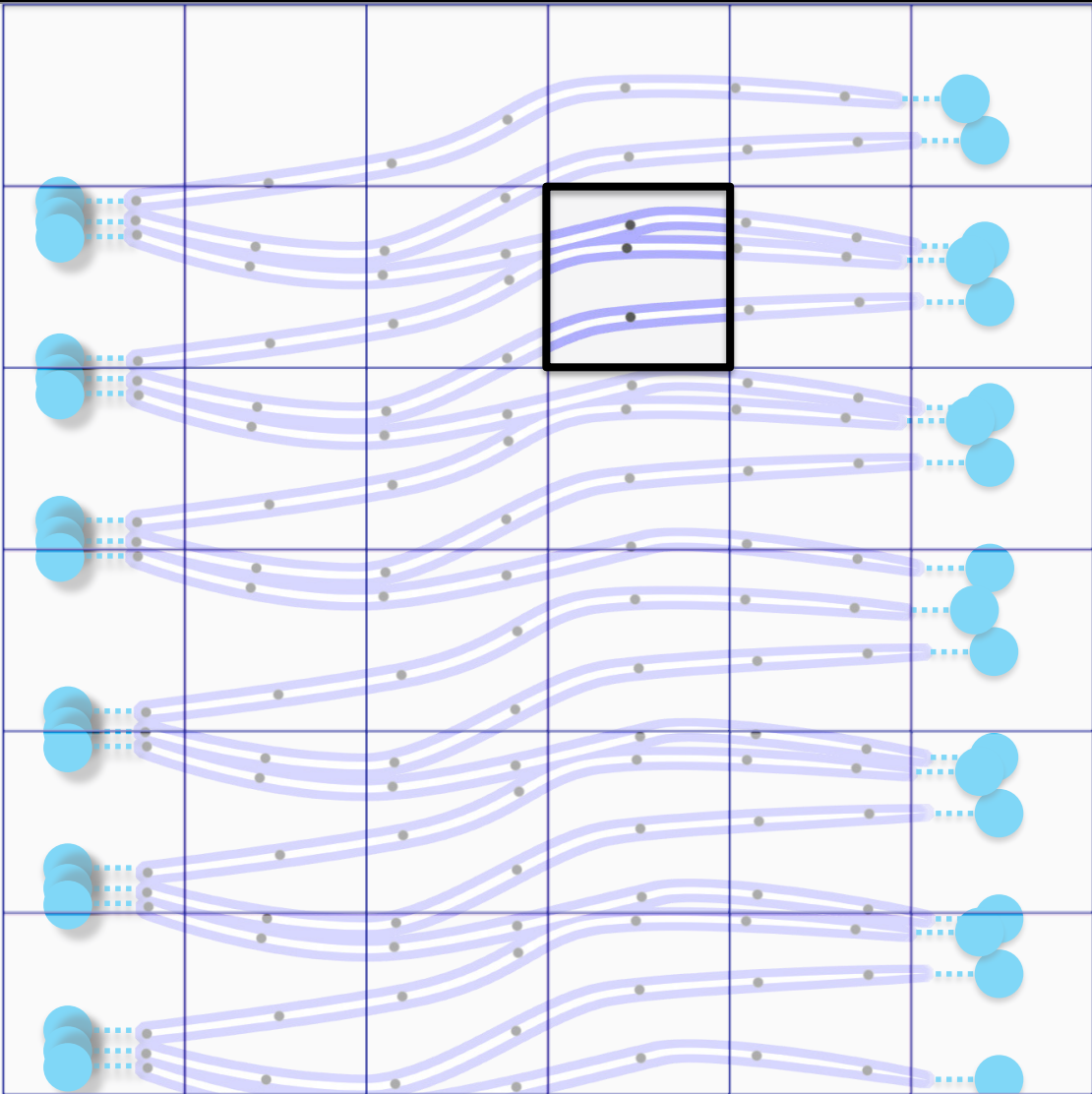
Dripping of Reduced-Liquid



Dripping of Reduced-Liquid



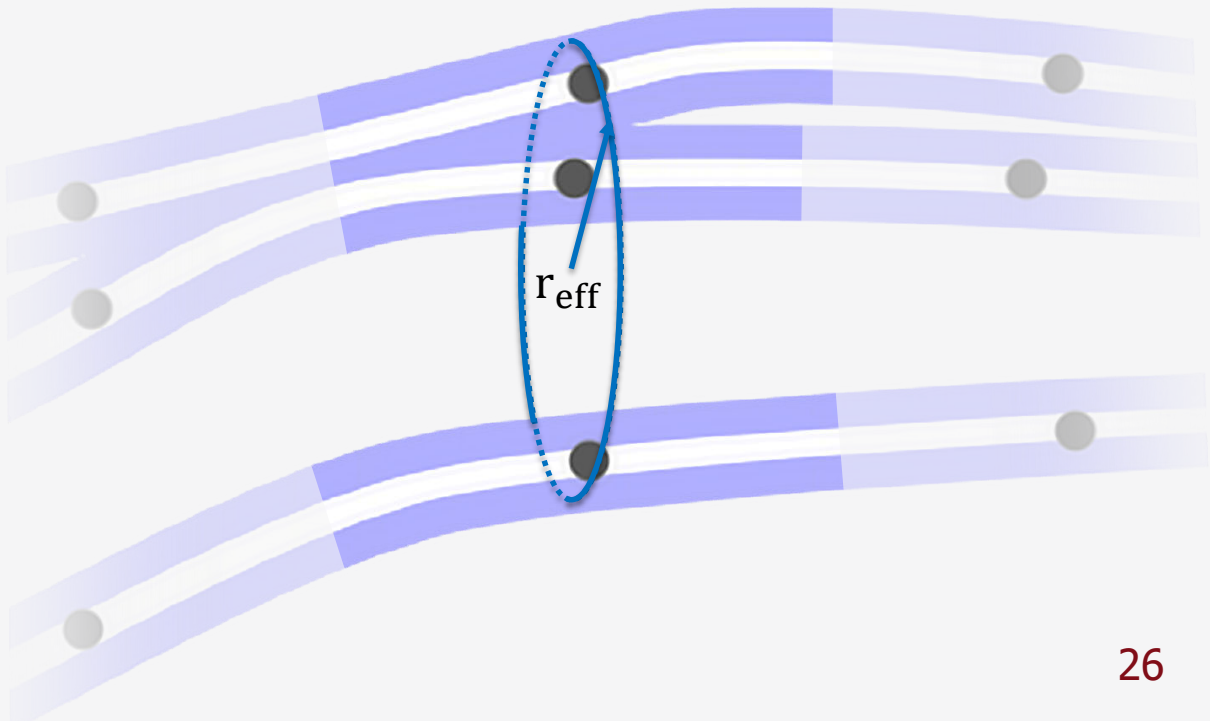
Dripping of Reduced-Liquid



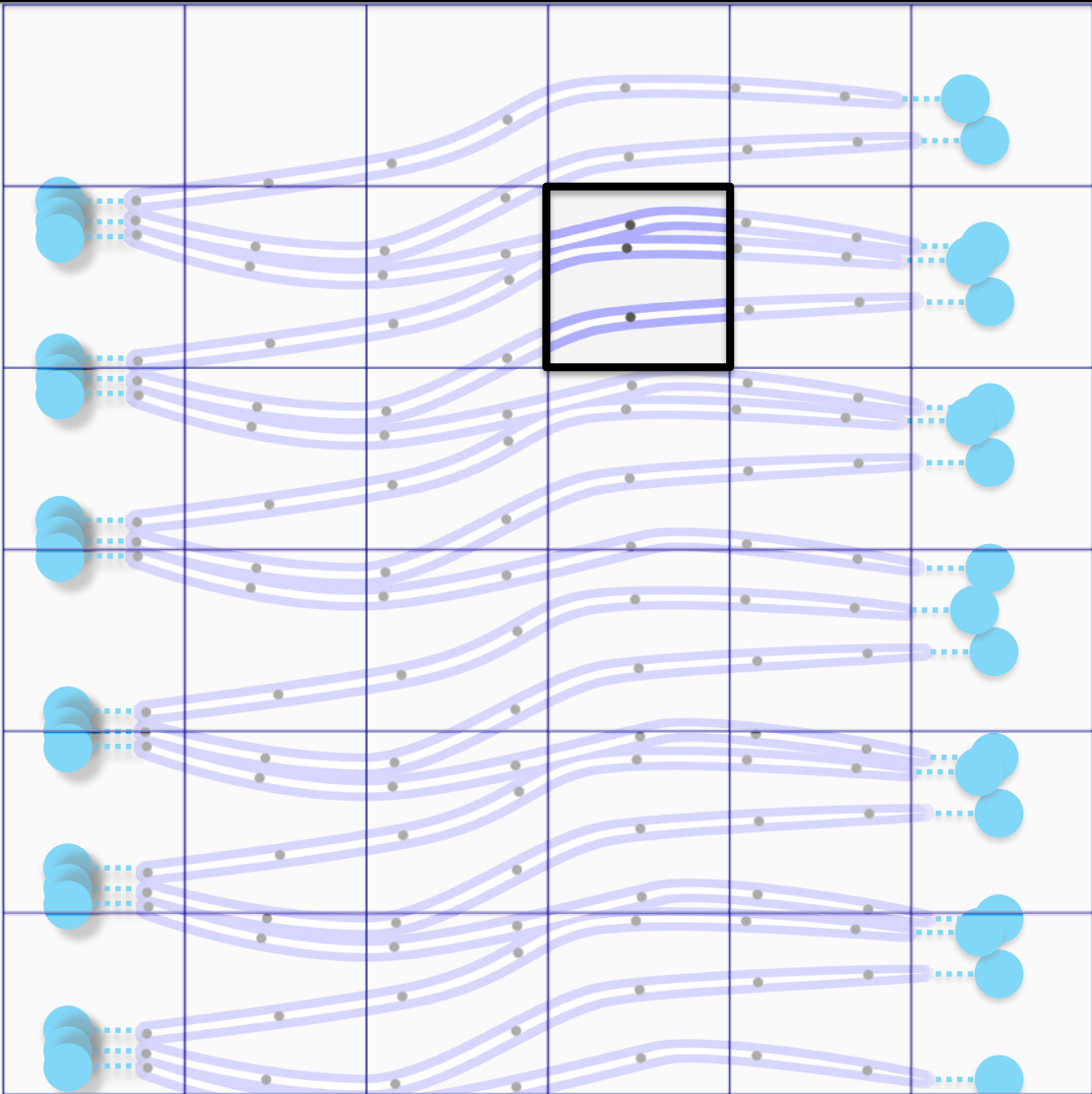
Convert redundant liquid

$$\text{if } r_{\text{eff}} > r_{\text{max}} = \left(\frac{3r\sigma\sqrt{N}}{\rho_L a \hat{n}} \right)^{\frac{1}{3}}$$

[Lorenceanu et al. 2004]



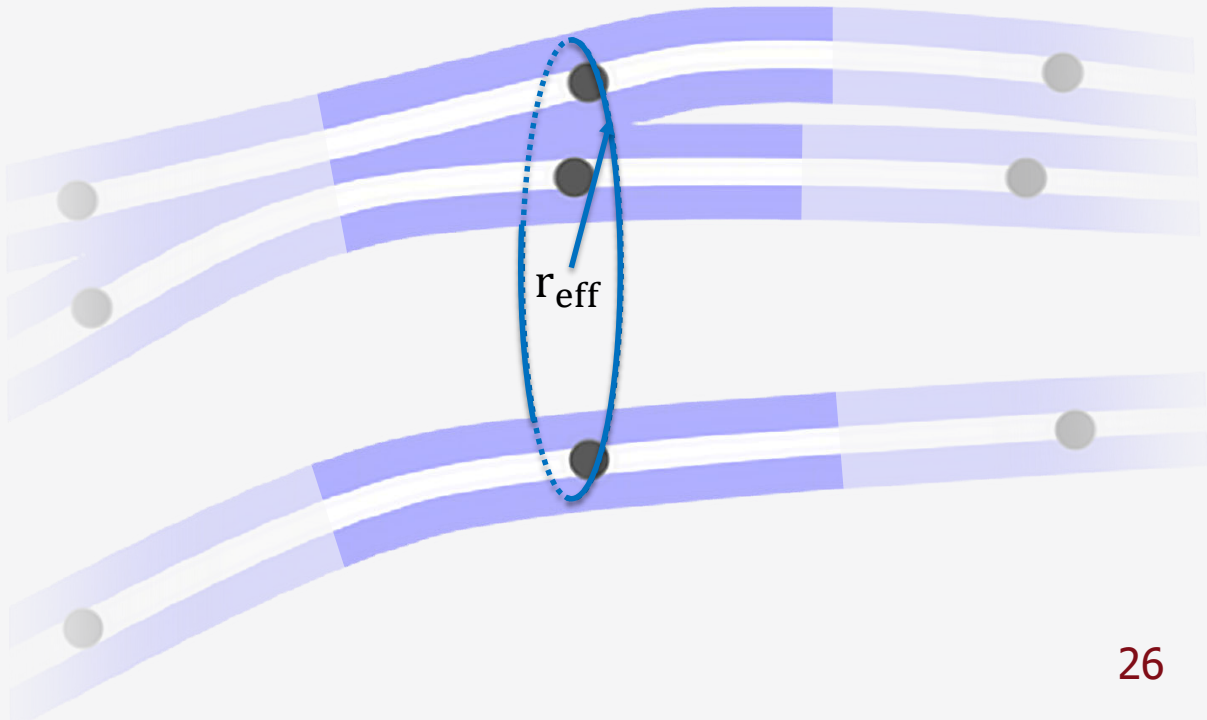
Dripping of Reduced-Liquid



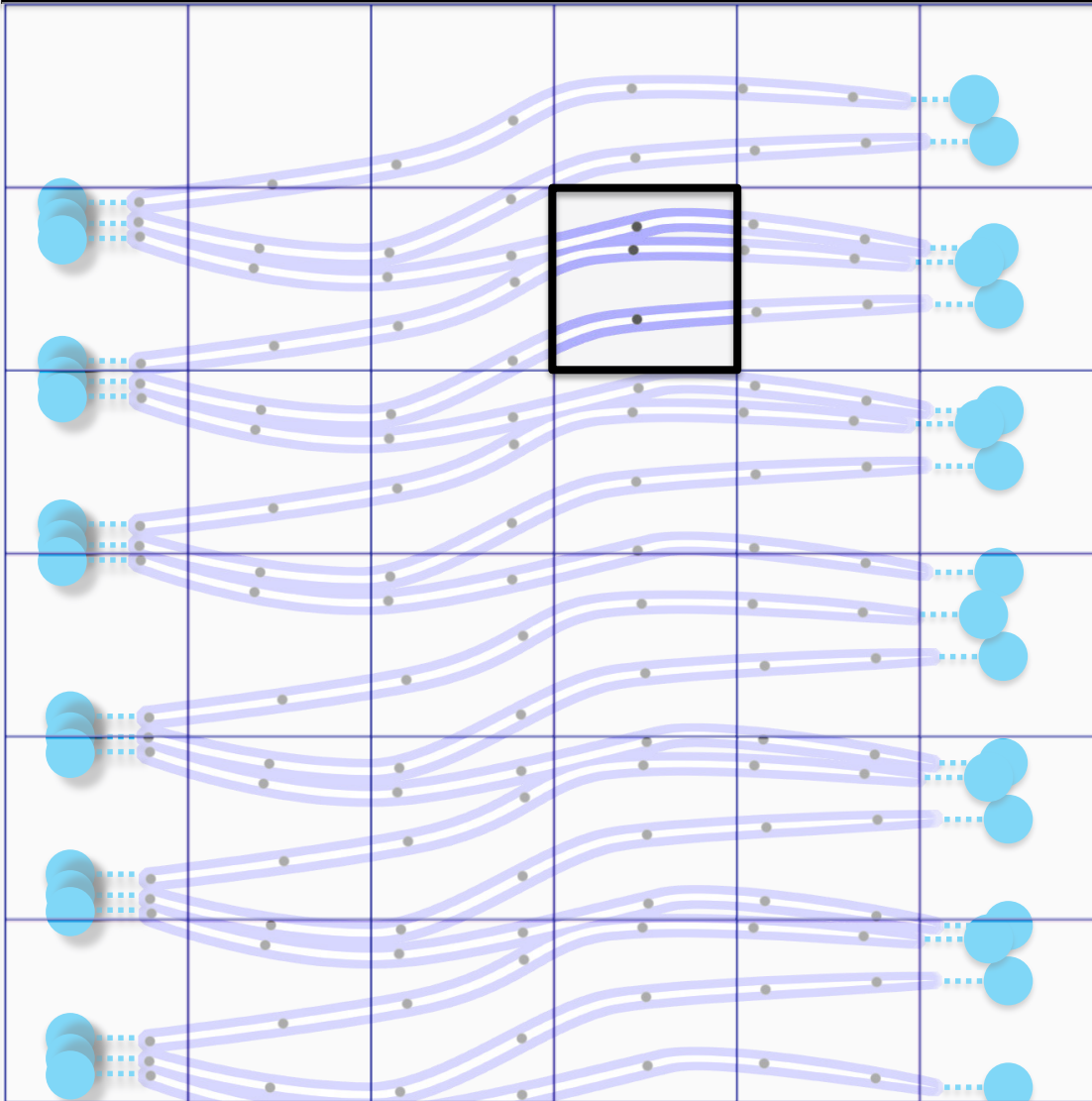
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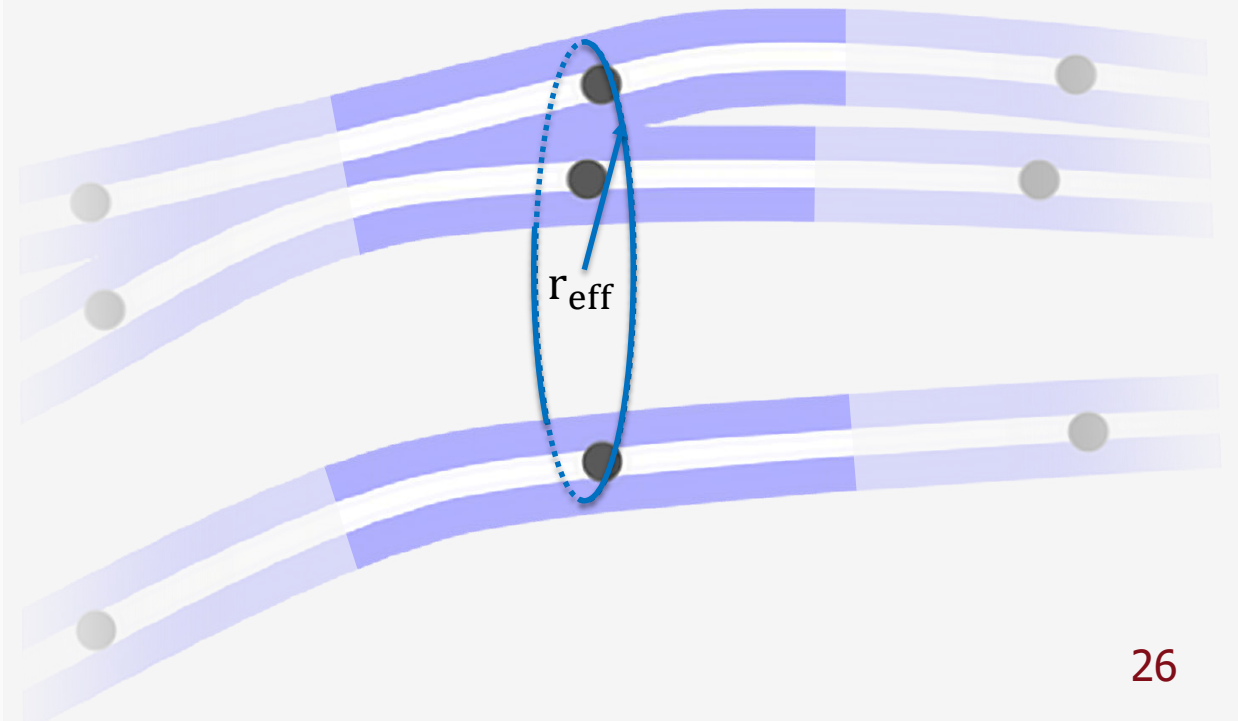
Dripping of Reduced-Liquid



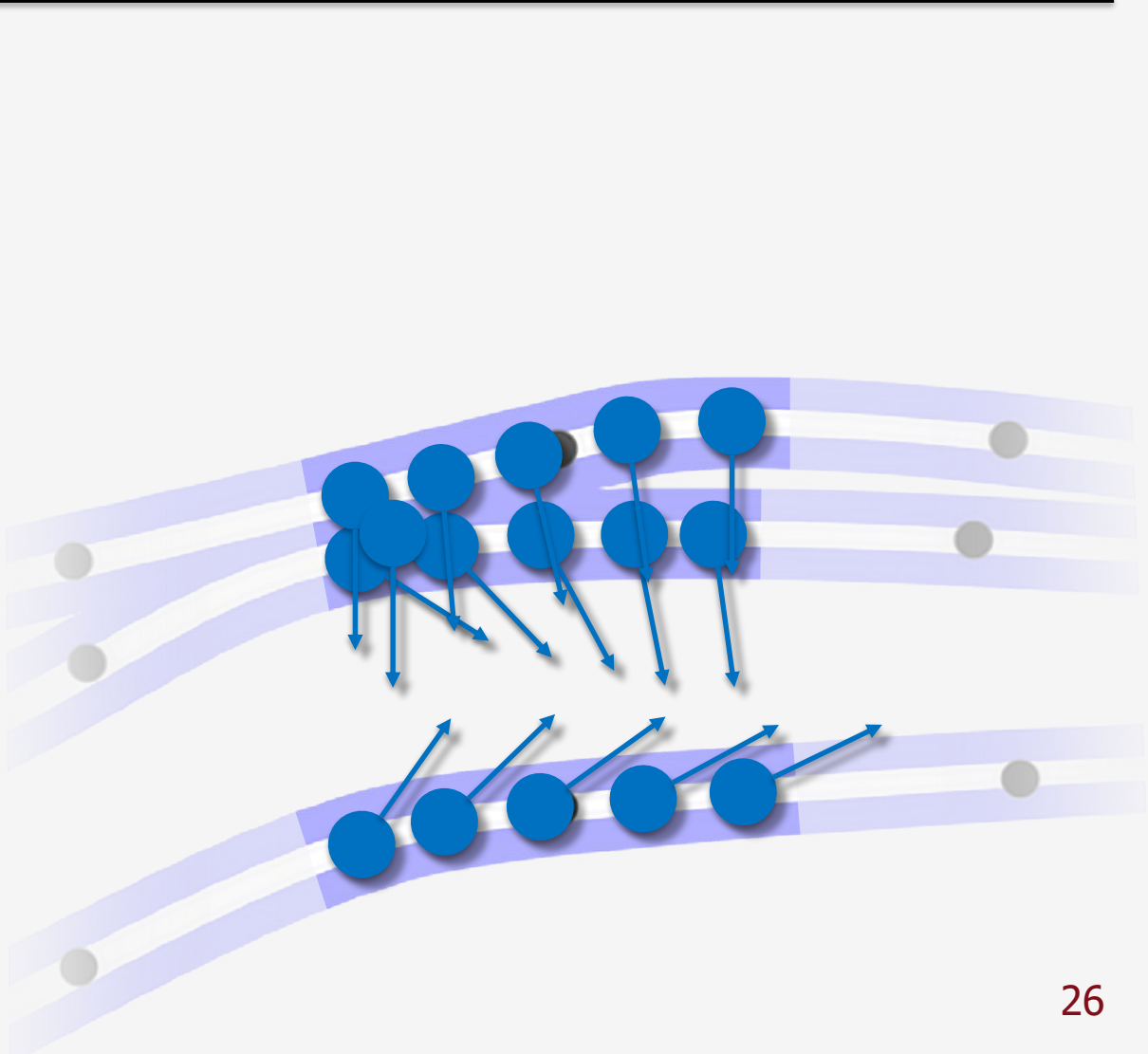
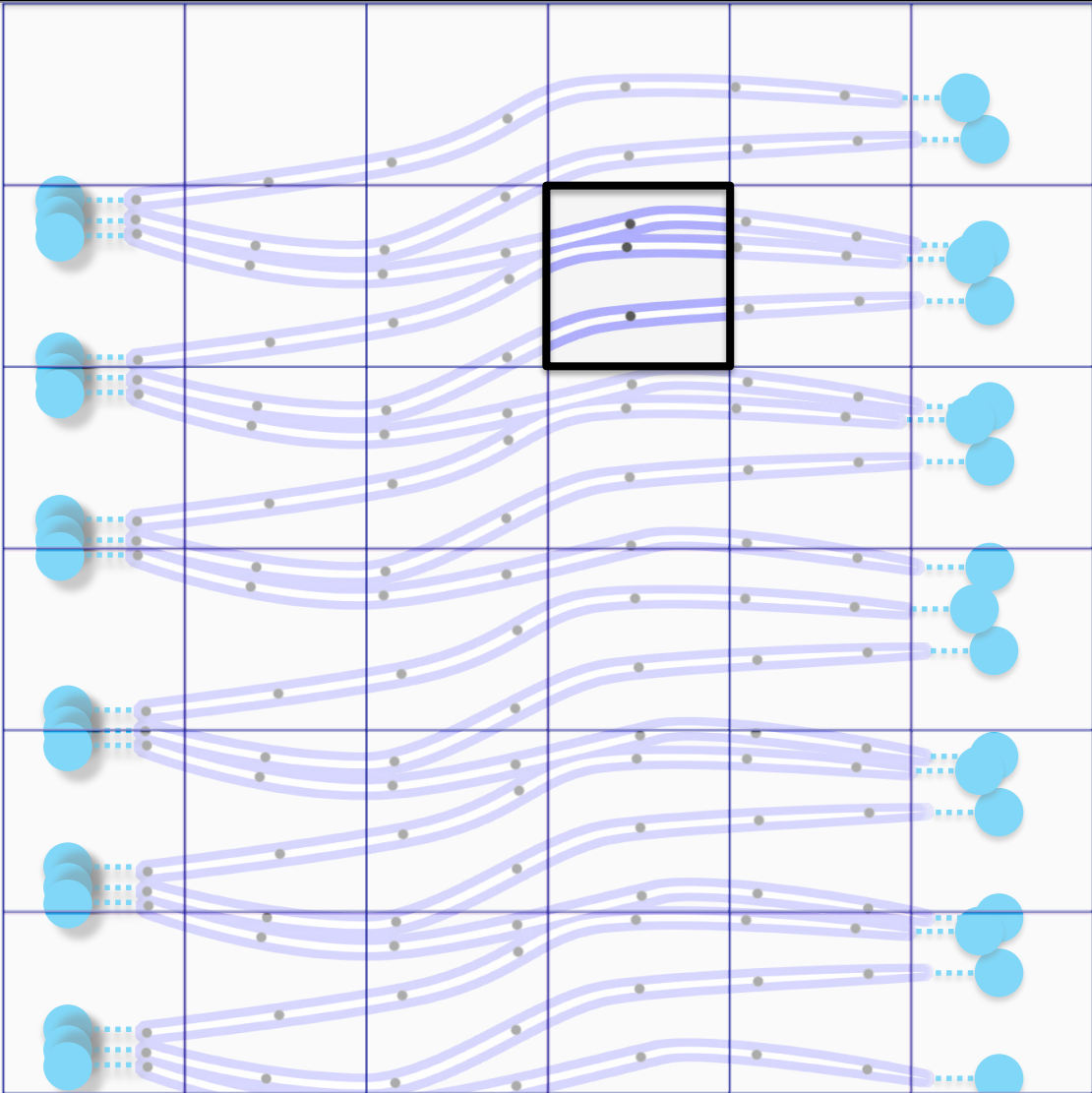
Convert redundant liquid

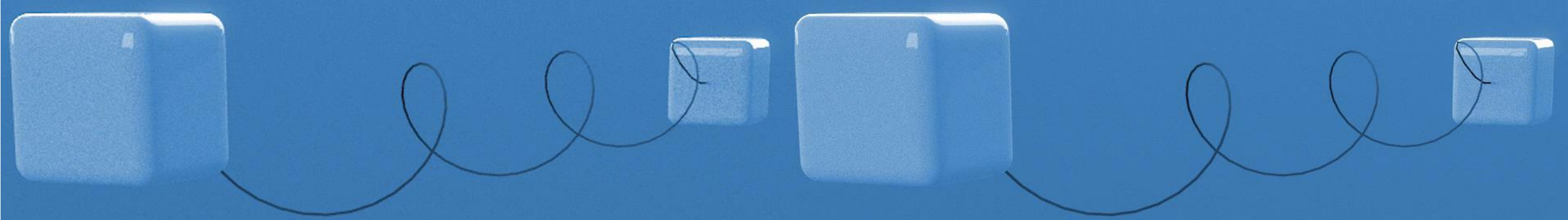
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[Lorenceanu et al. 2004]



Dripping of Reduced-Liquid





Capture OFF

Capture ON

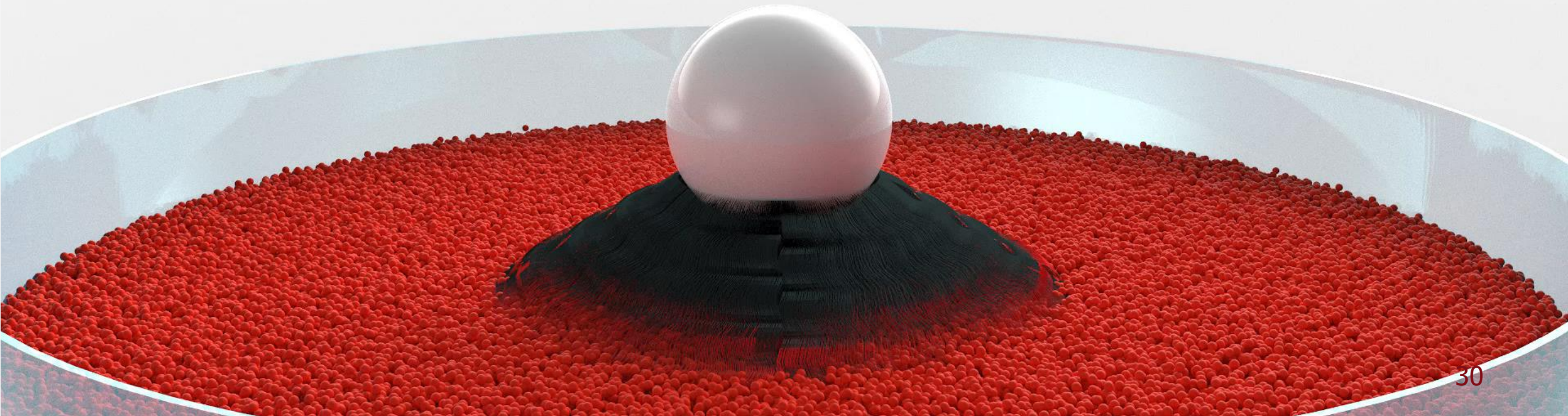
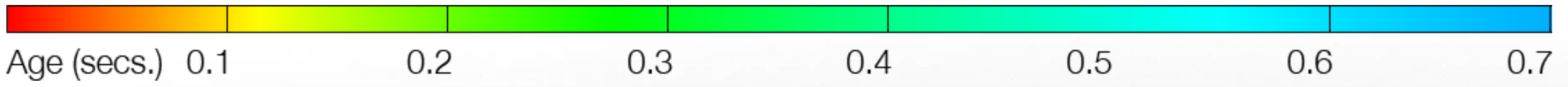


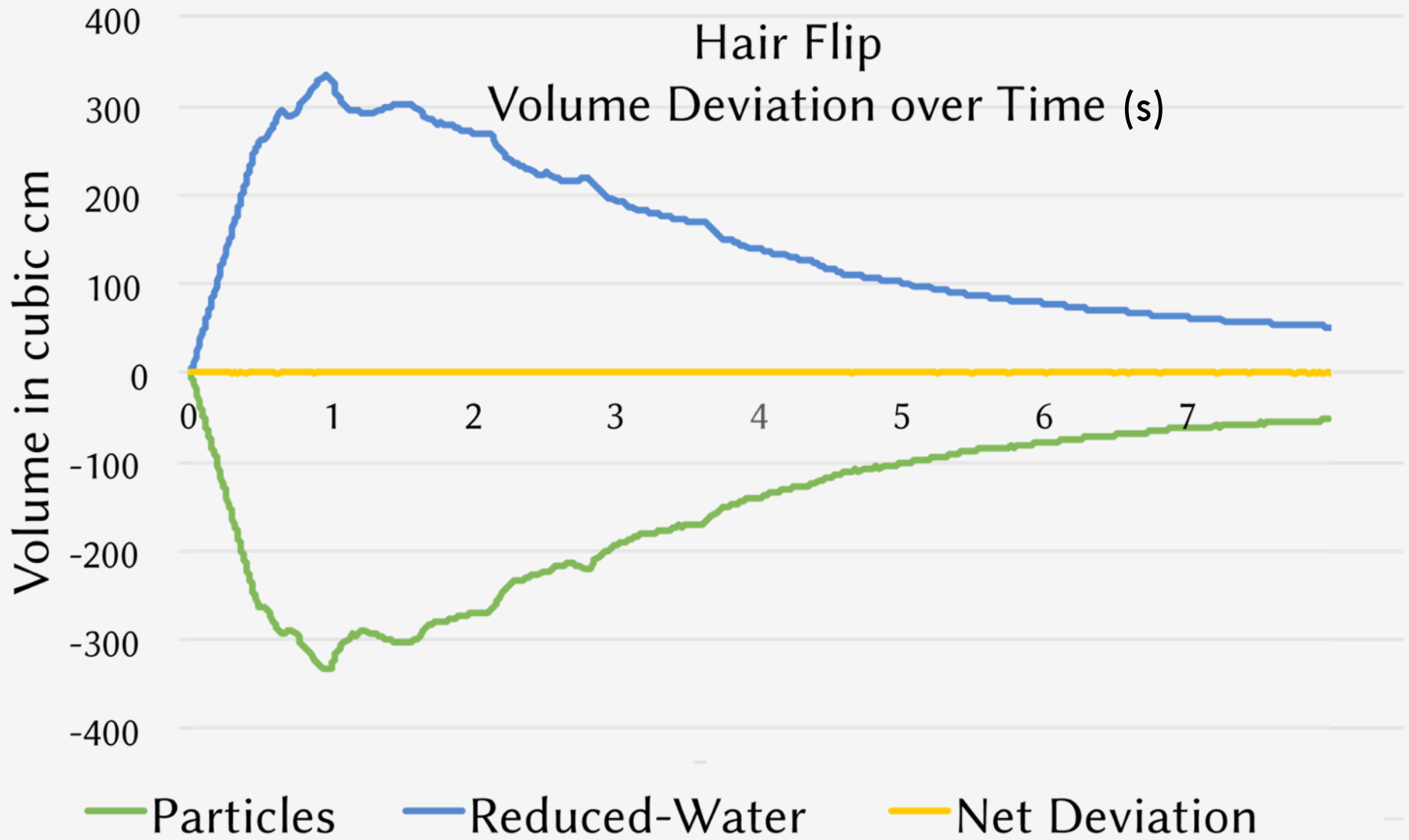
Dripping OFF



Dripping ON

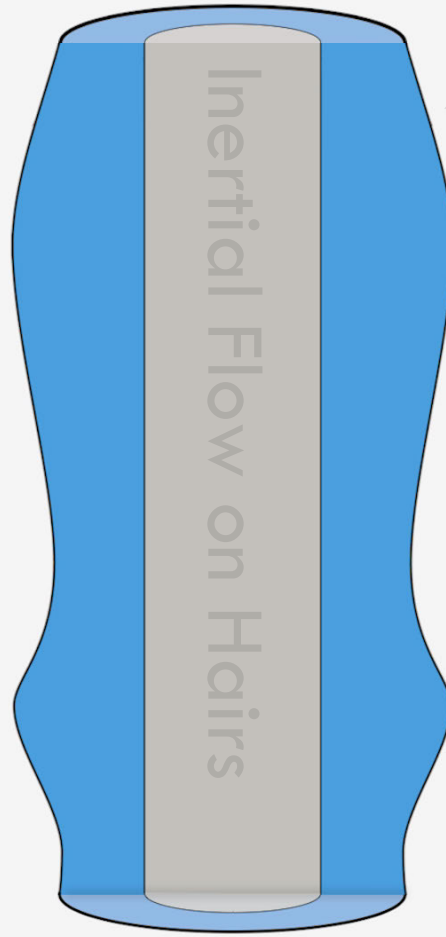








Cohesion
↔
Collision



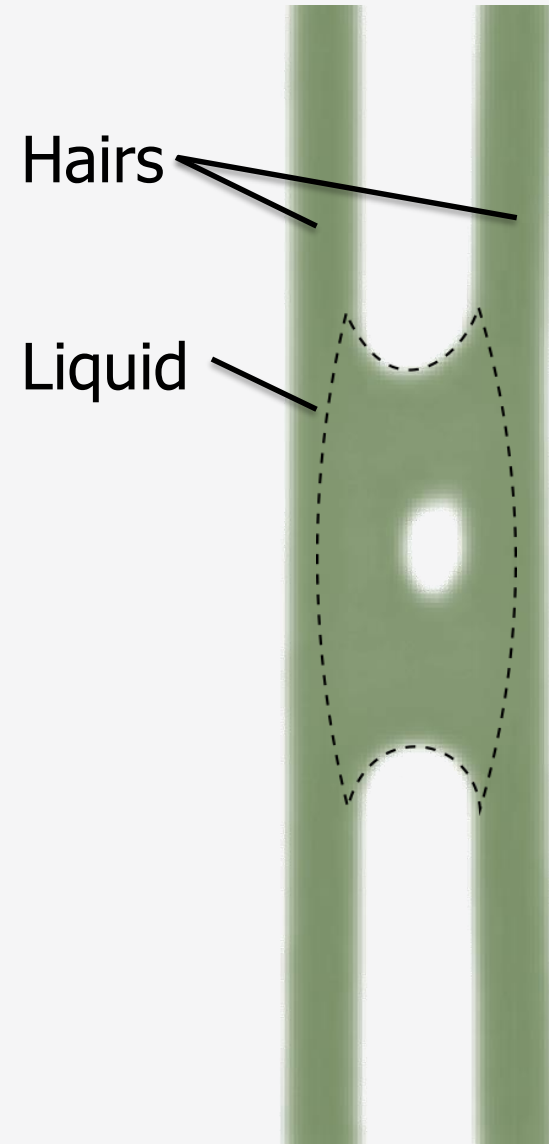
Two-way Coupling

Dripping

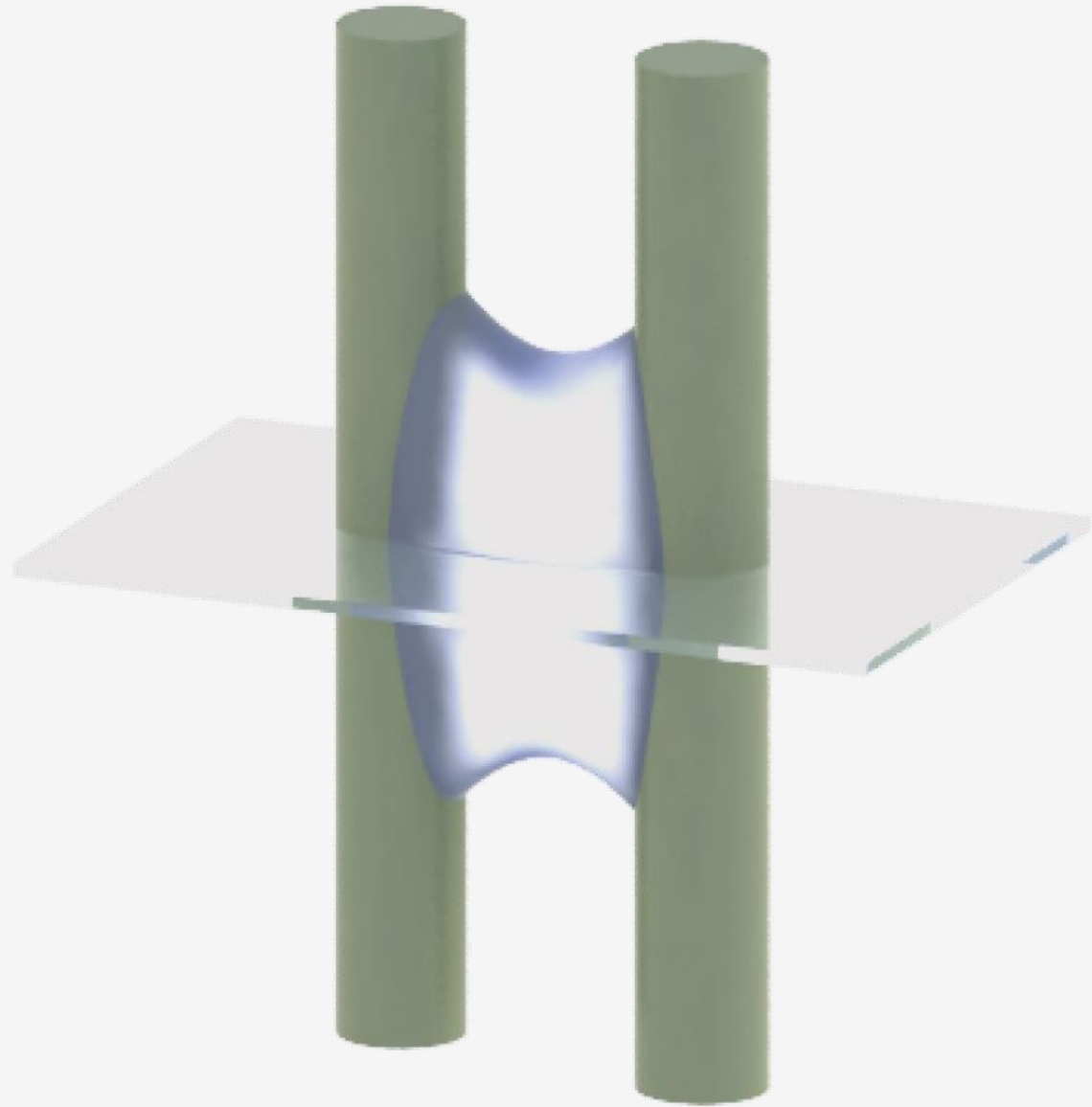


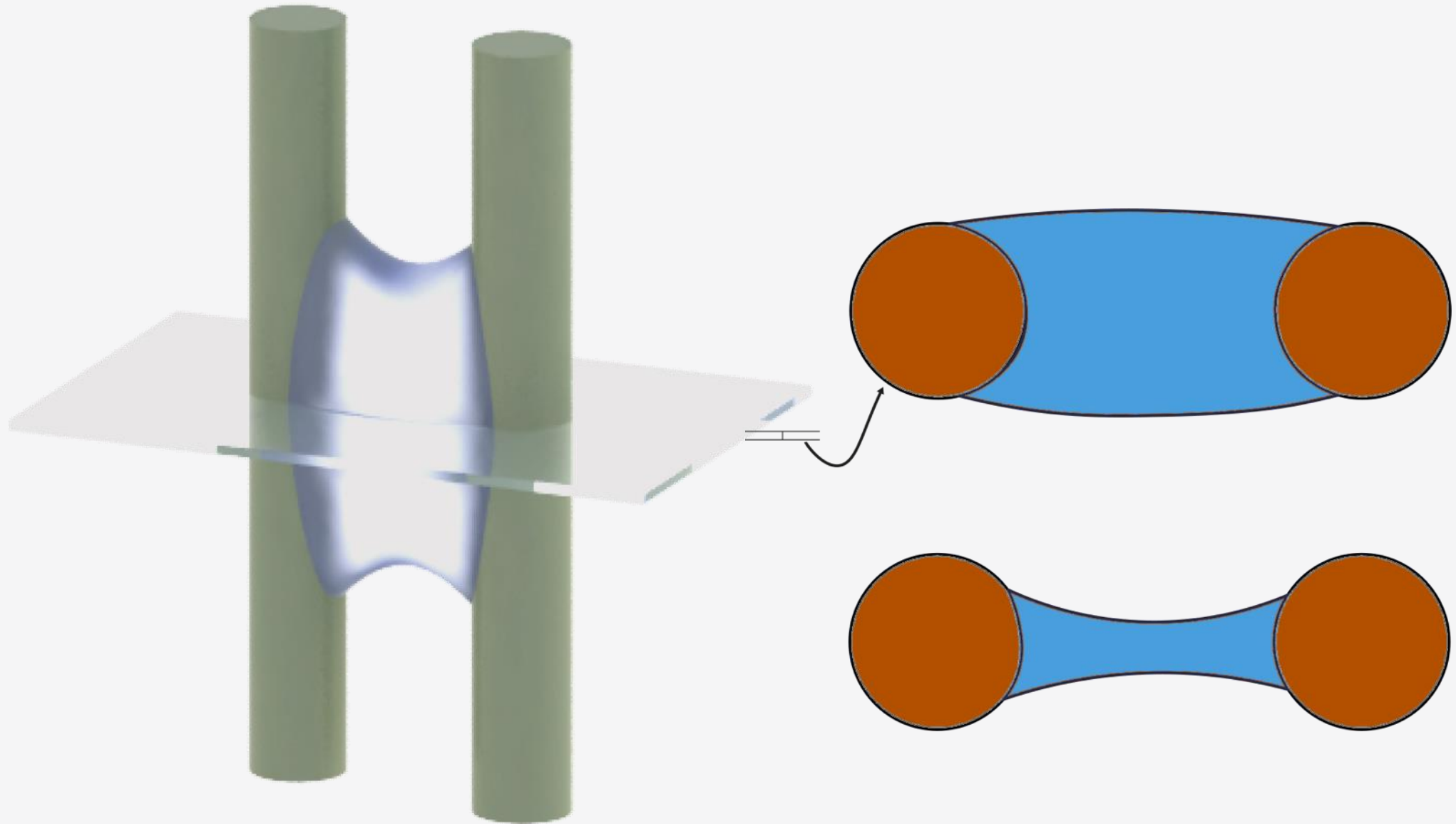
Capturing

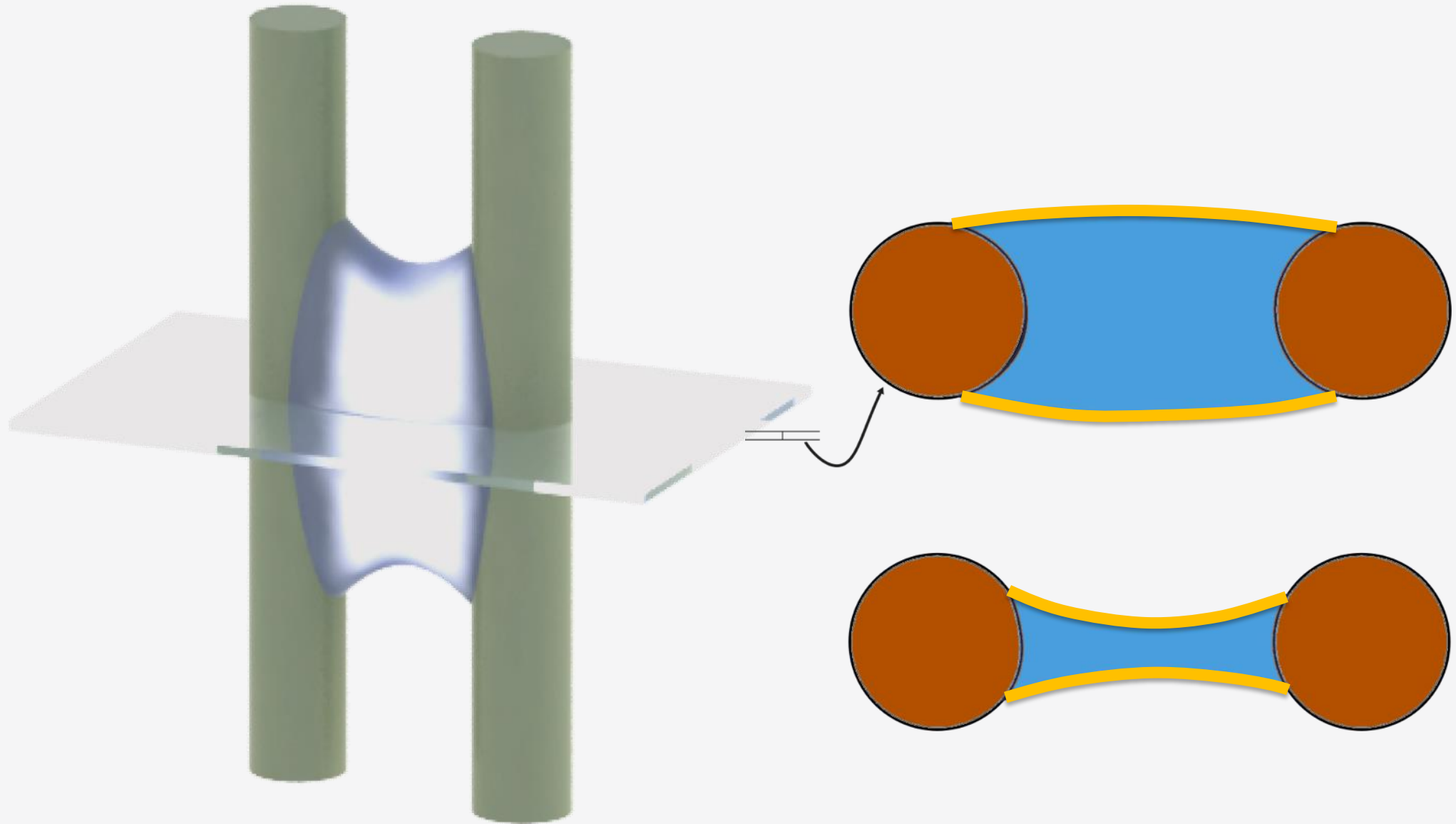


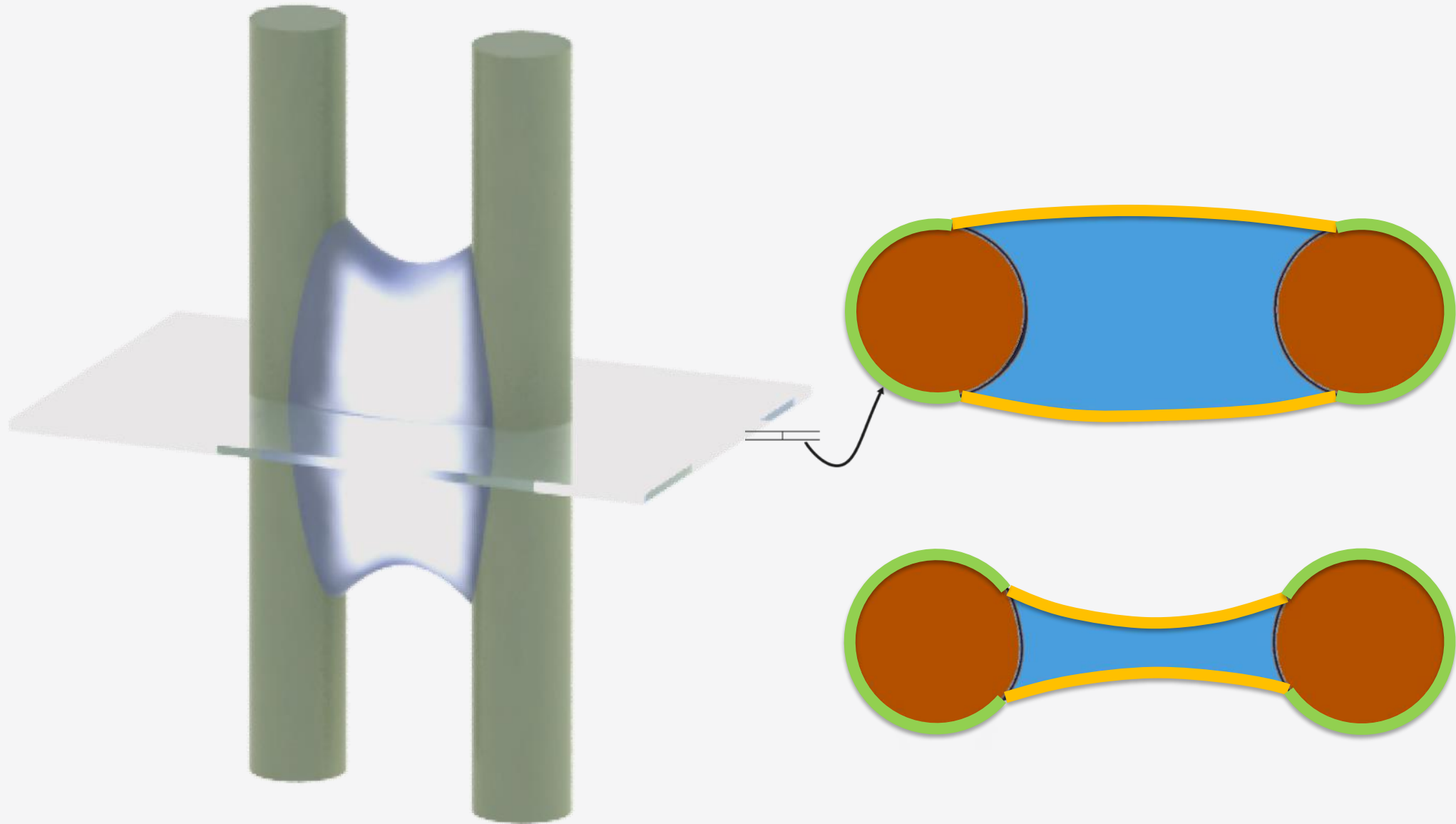


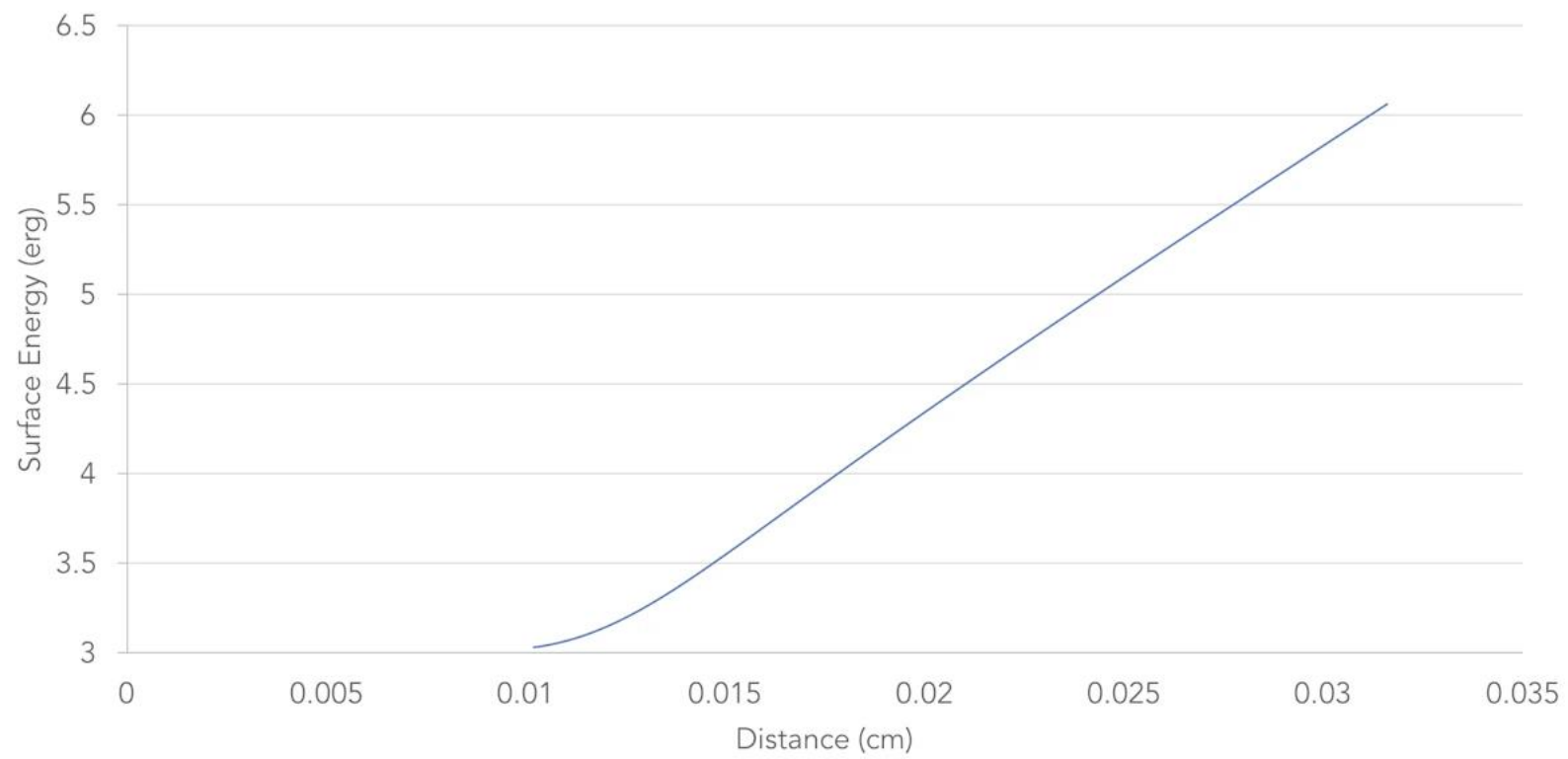
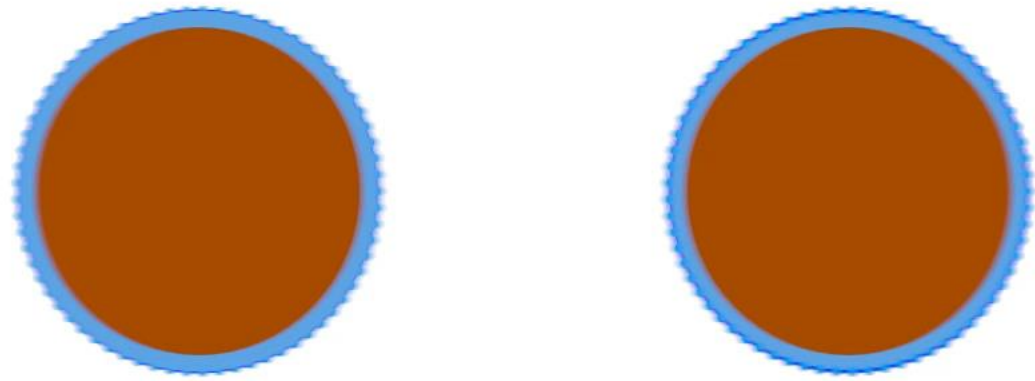
Real Experiment [Wang et al. 2014]

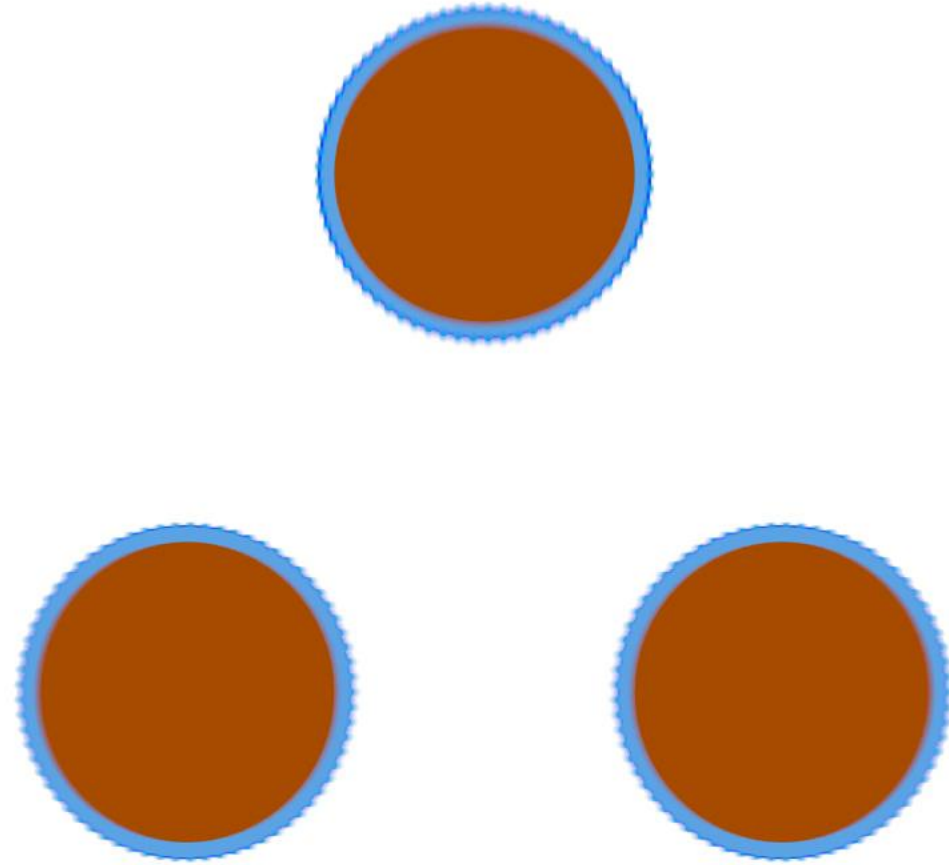


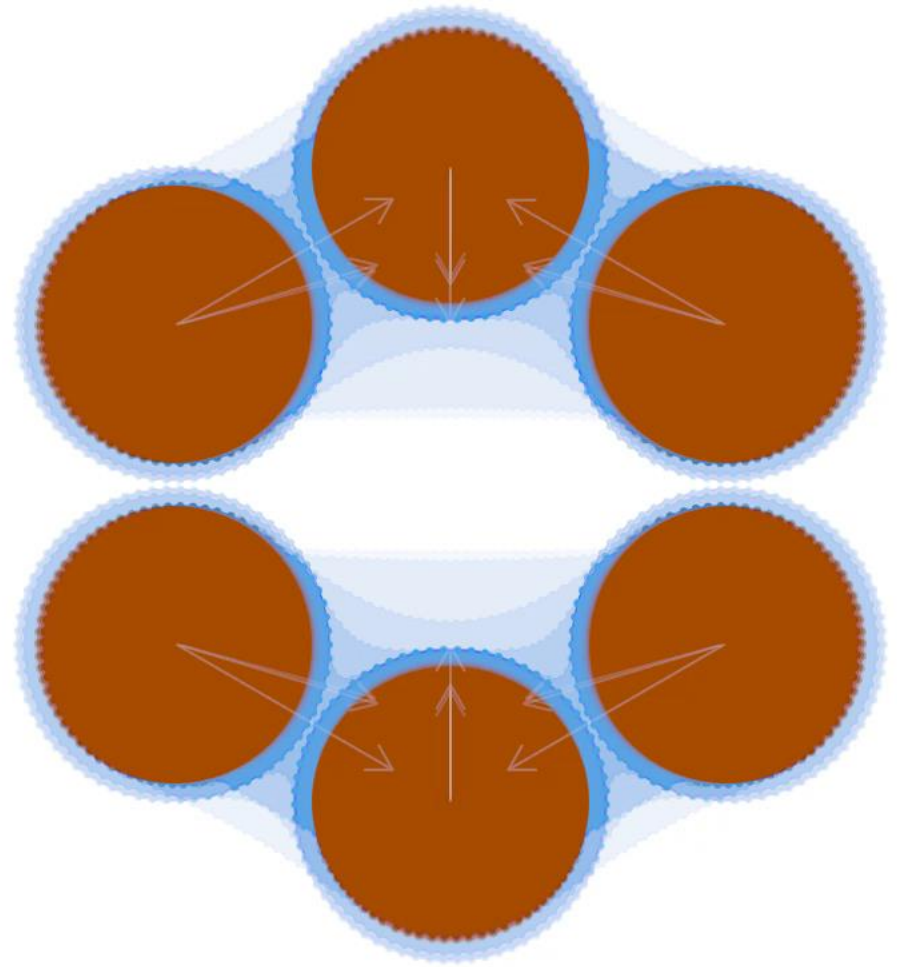


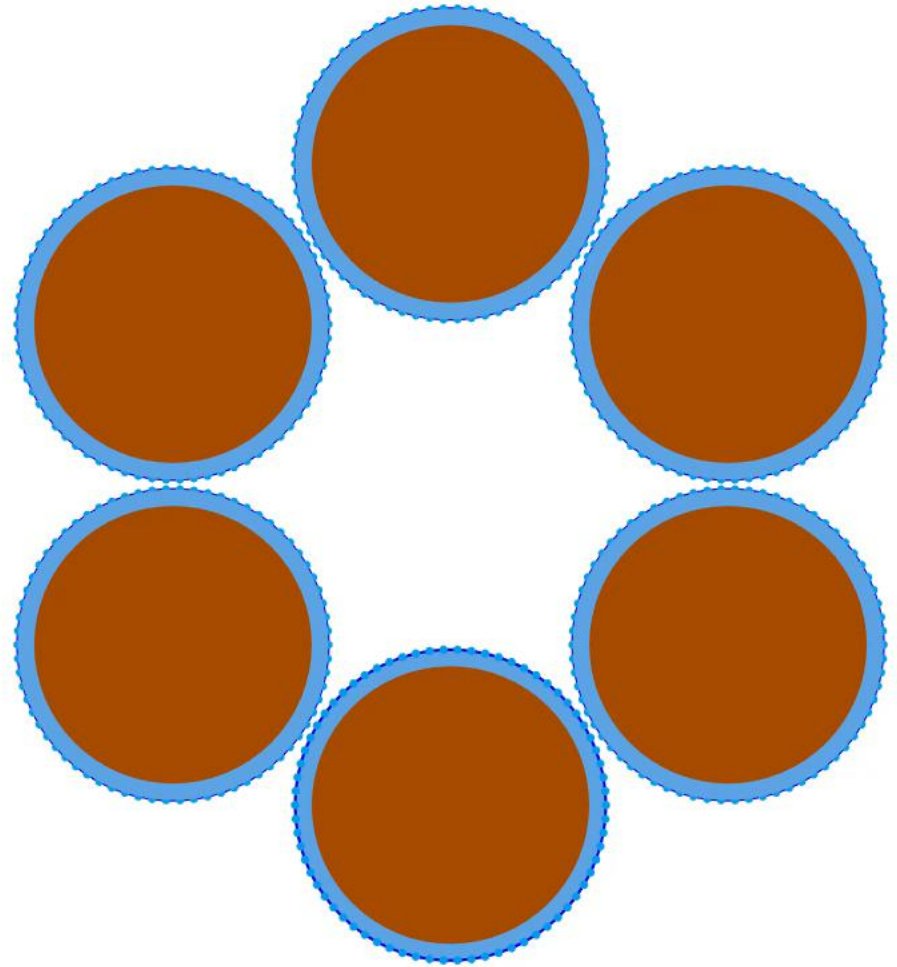




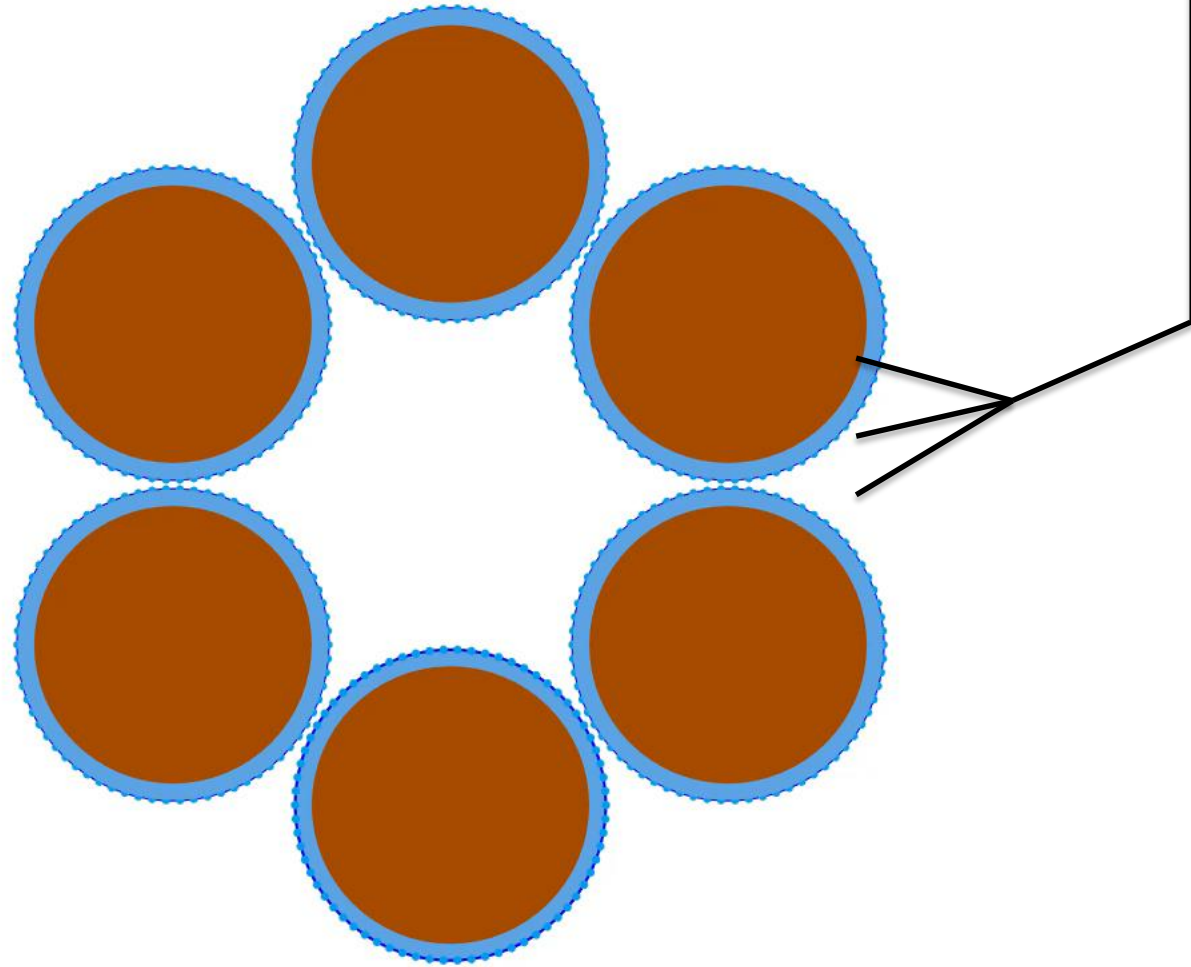




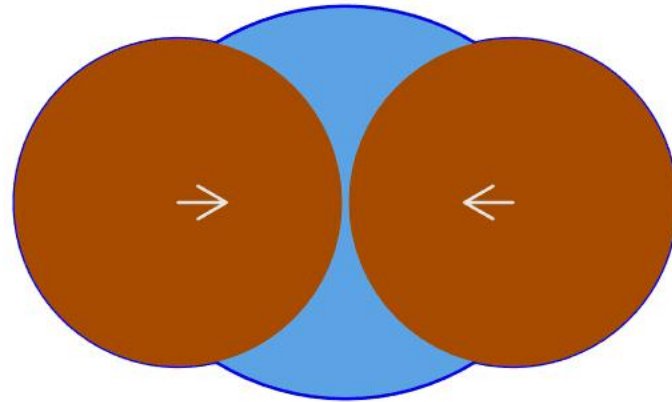




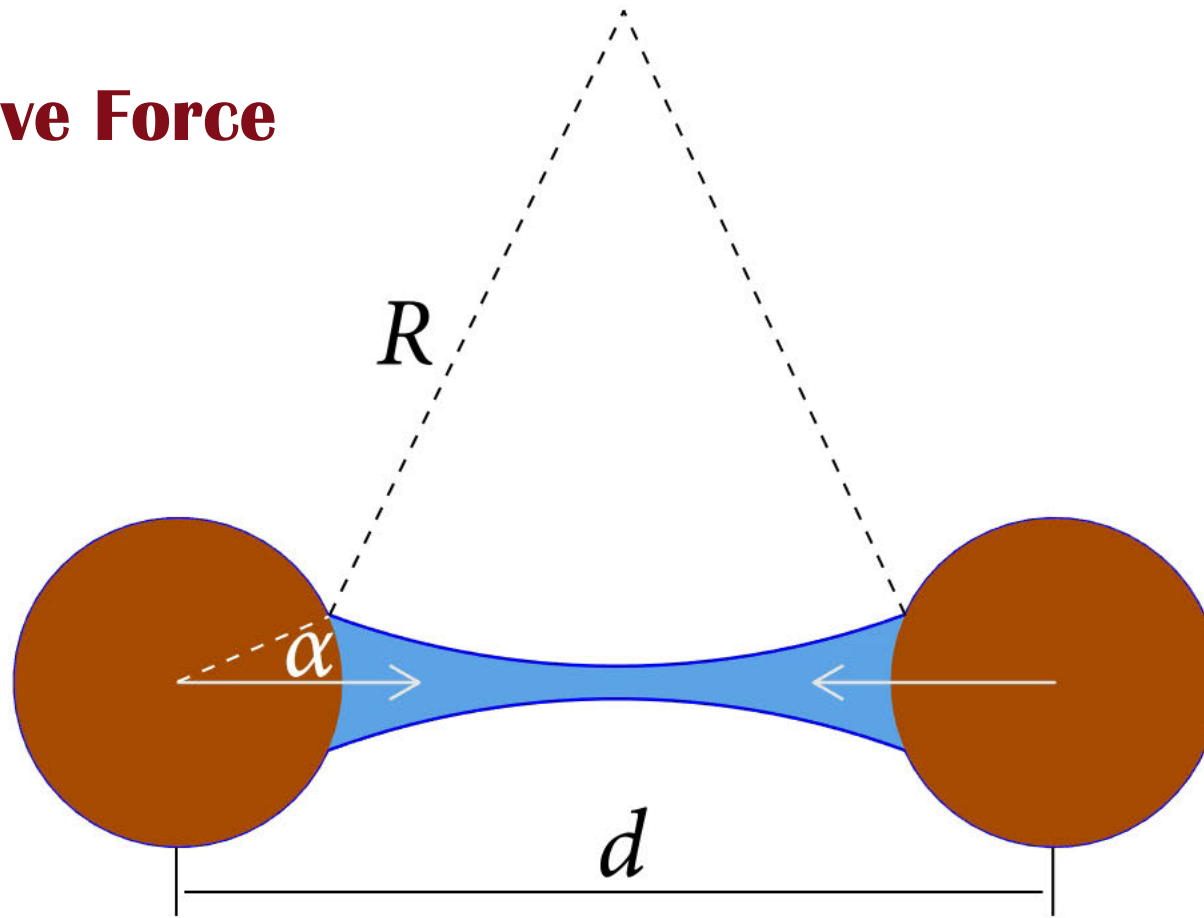
Too Costly for Practical Applications



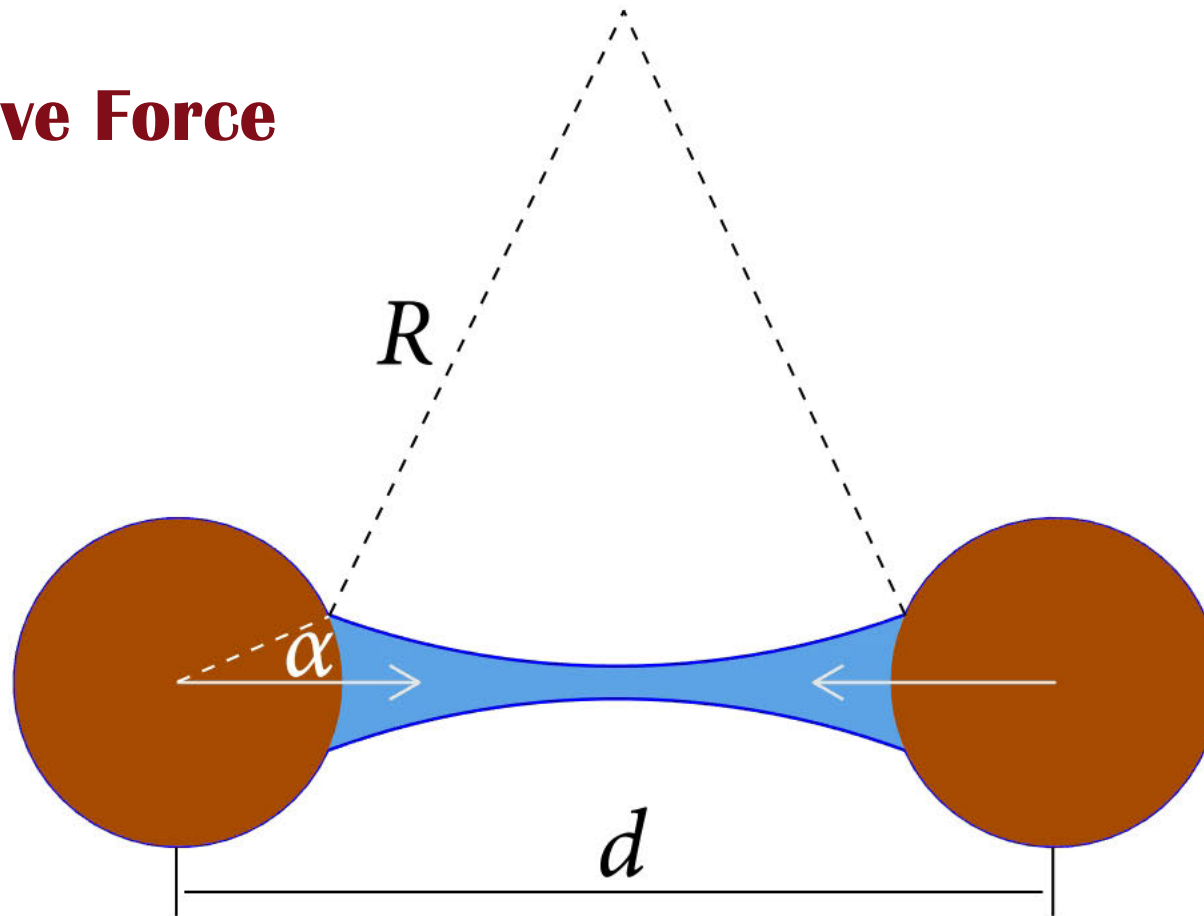
Analytical Model



Compute Cohesive Force

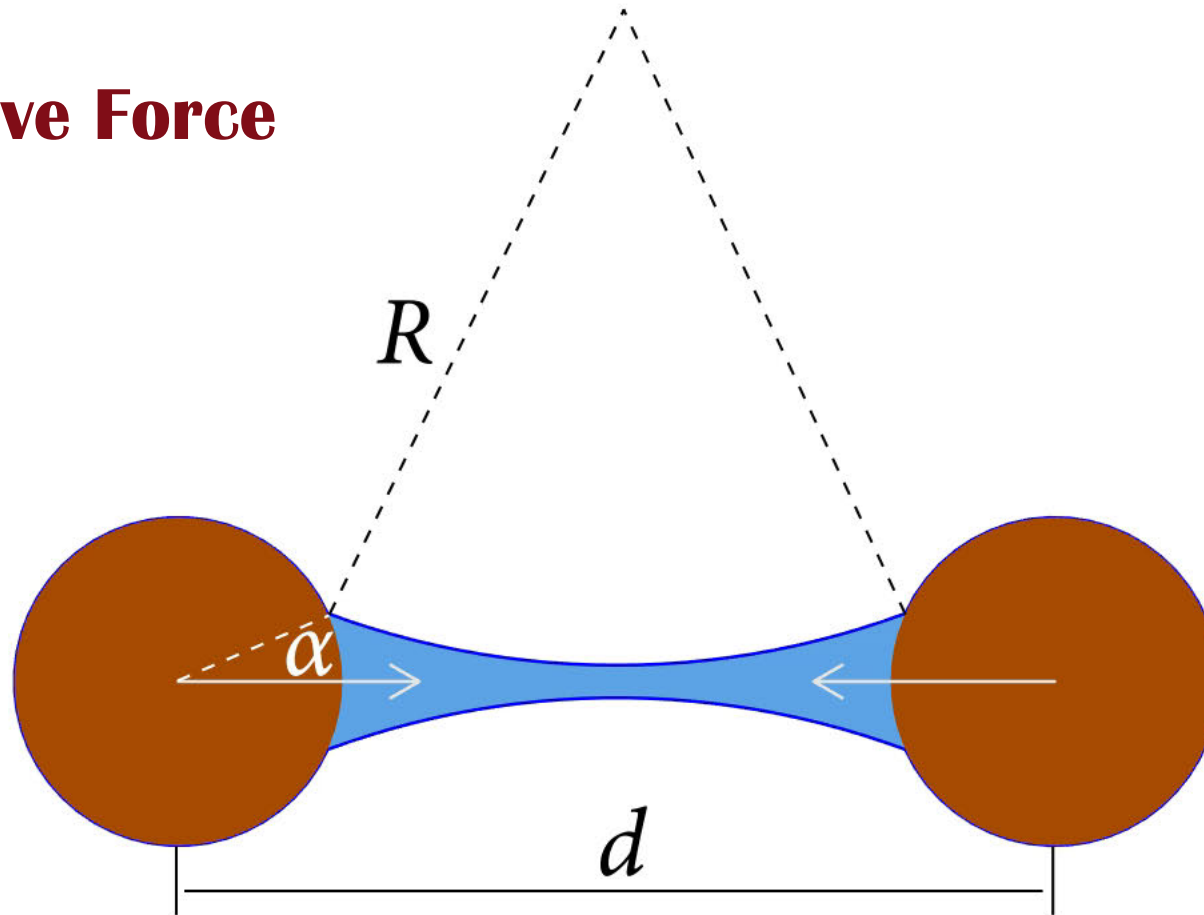


Compute Cohesive Force



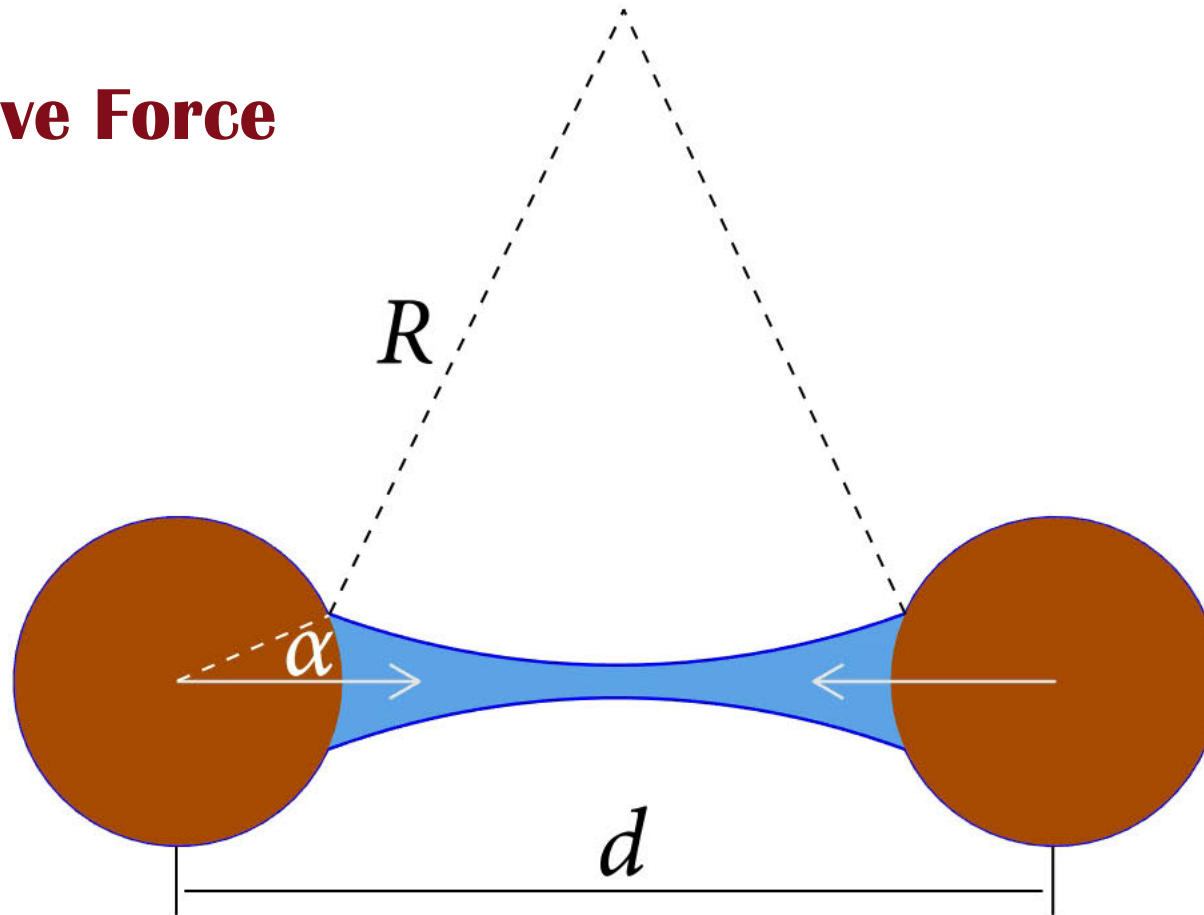
$$f ds = -\frac{\partial d E_s}{\partial d} = -\left(\frac{\partial d E_s}{\partial R} \cdot \frac{\partial R}{\partial d} + \sum_{i=1,2} \frac{\partial d E_s}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d} \right)$$

Compute Cohesive Force



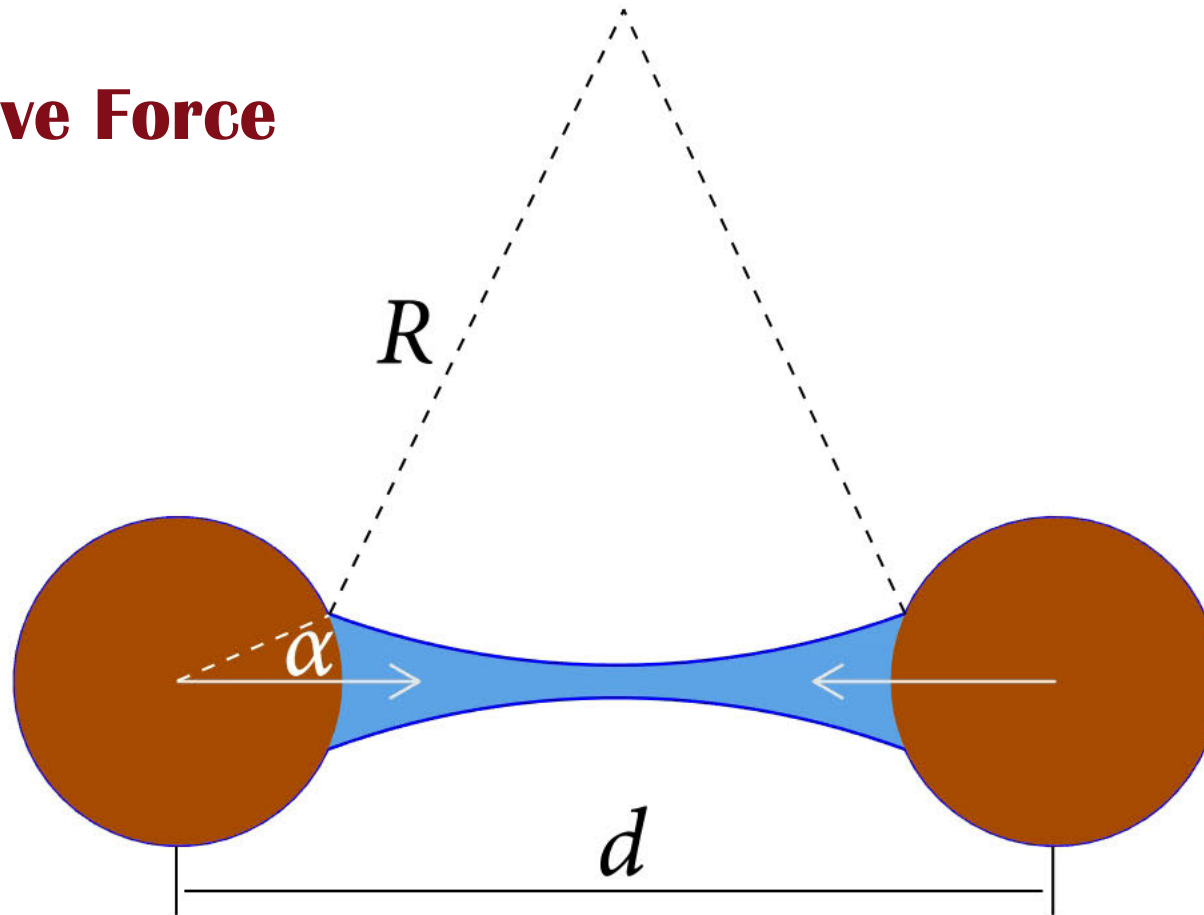
$$f ds = -\frac{\partial d E_s}{\partial d} = -\left(\frac{\partial d E_s}{\partial R} \cdot \frac{\partial R}{\partial d} + \sum_{i=1,2} \frac{\partial d E_s}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d} \right)$$

Compute Cohesive Force



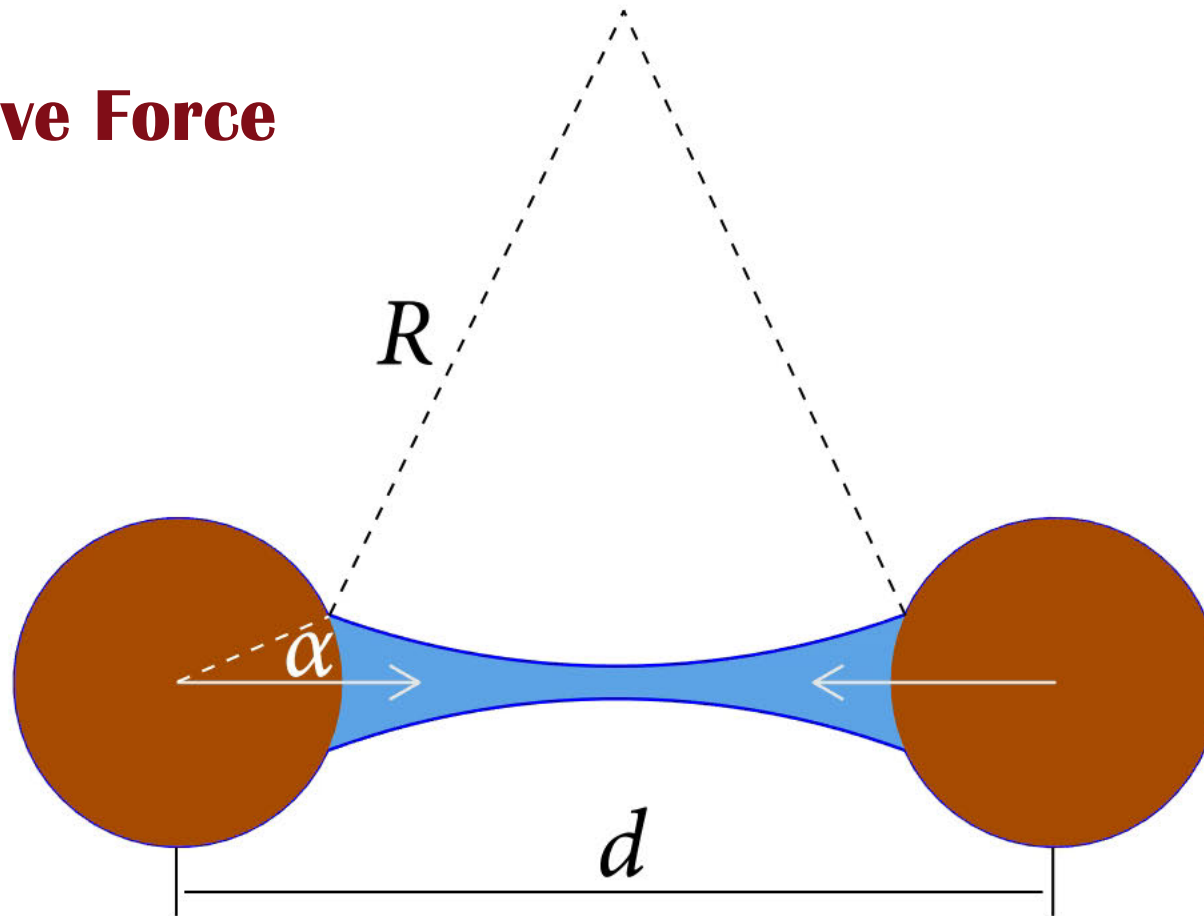
$$f ds = -\frac{\partial d E_s}{\partial d} = -\left(\frac{\partial d E_s}{\partial R} \cdot \frac{\partial R}{\partial d} + \sum_{i=1,2} \frac{\partial d E_s}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d} \right)$$

Compute Cohesive Force



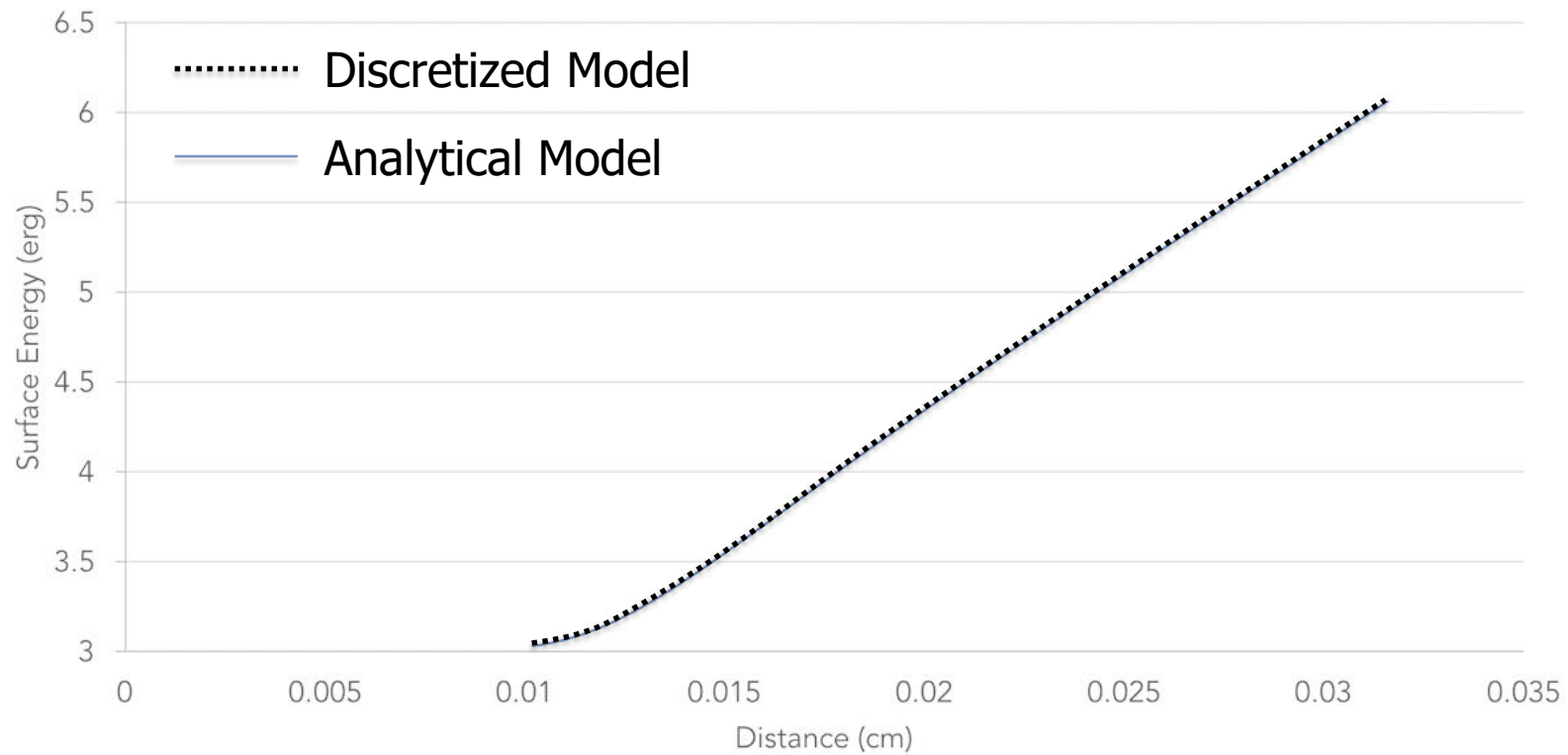
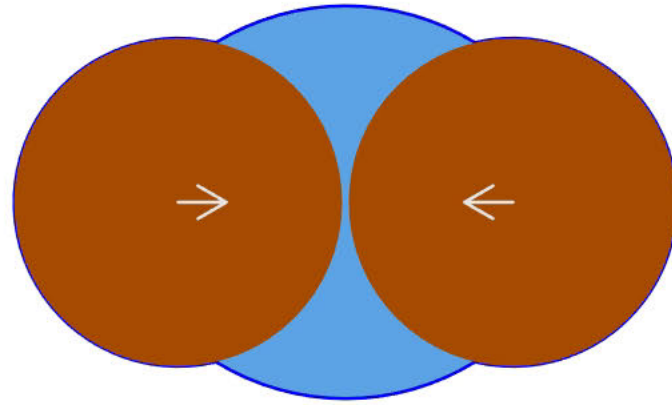
$$f ds = - \boxed{\frac{\partial d E_s}{\partial d}} = - \left(\frac{\partial d E_s}{\partial R} \cdot \frac{\partial R}{\partial d} + \sum_{i=1,2} \frac{\partial d E_s}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d} \right)$$

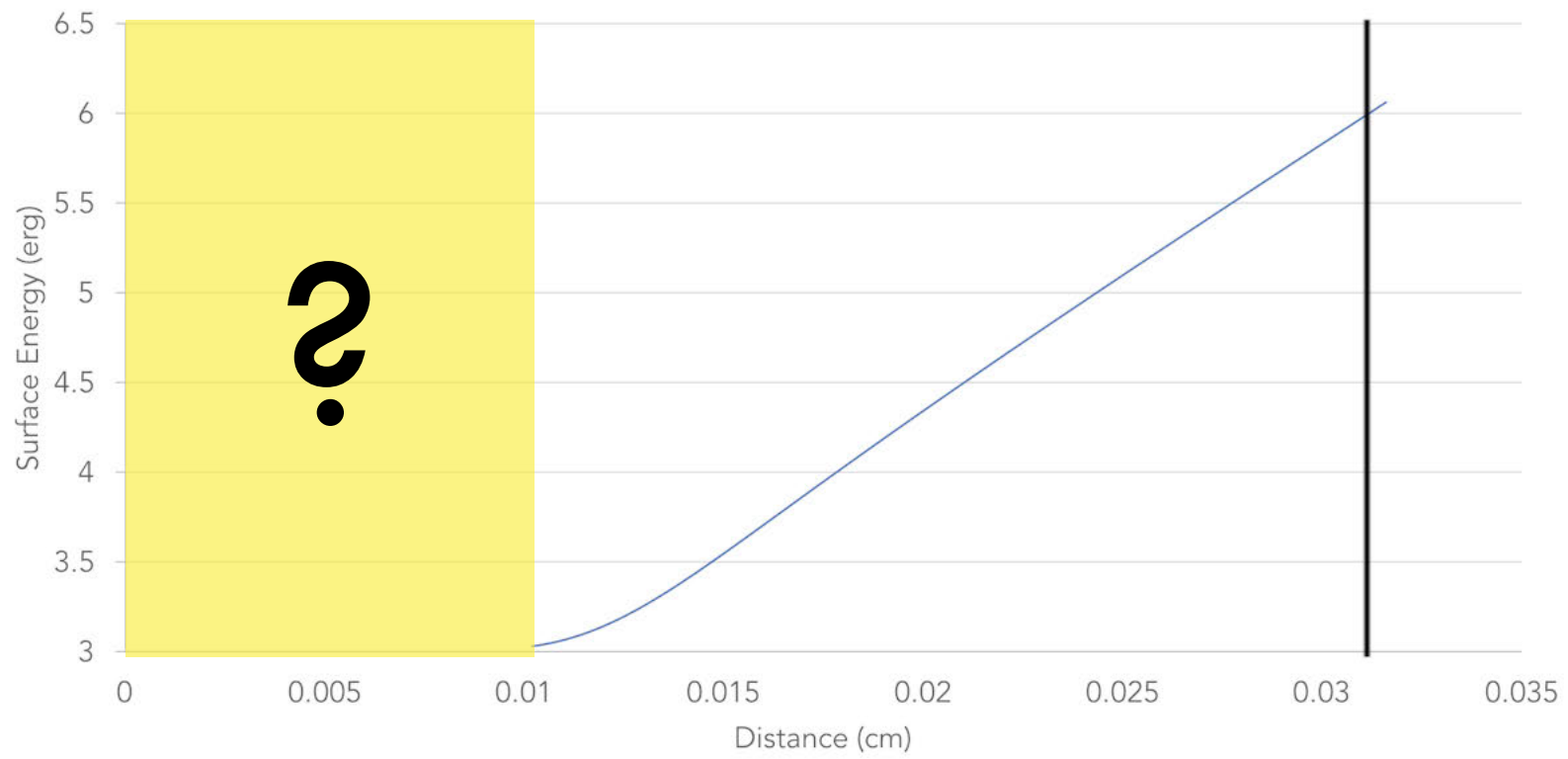
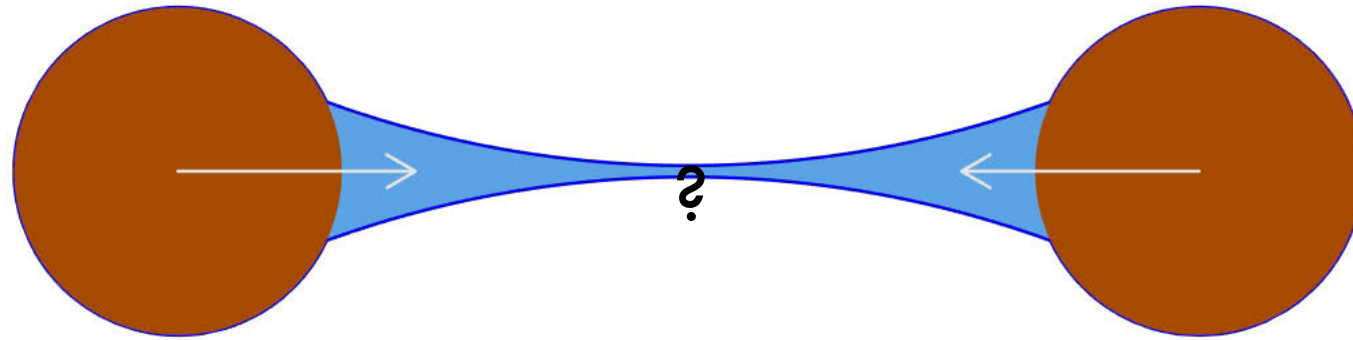
Compute Cohesive Force



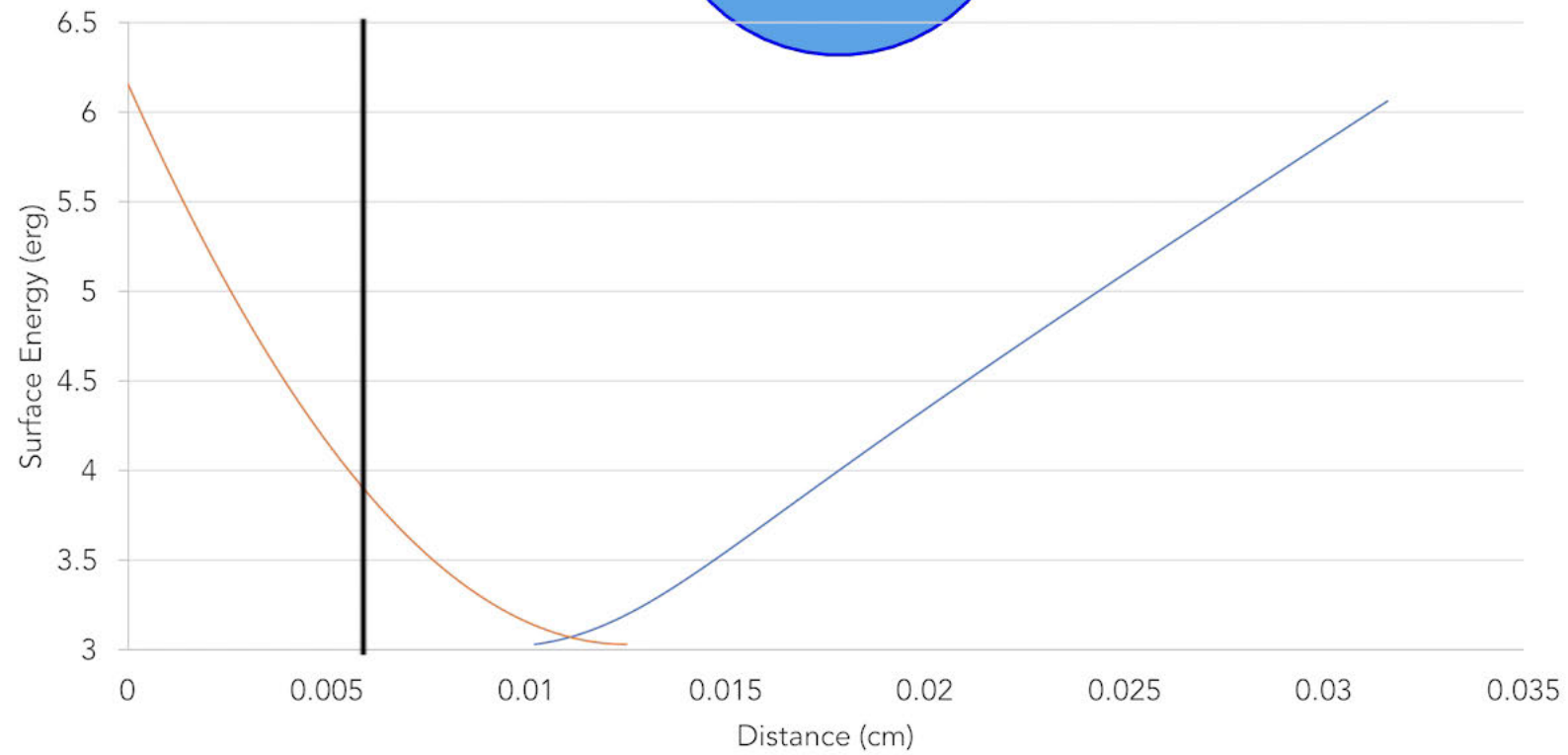
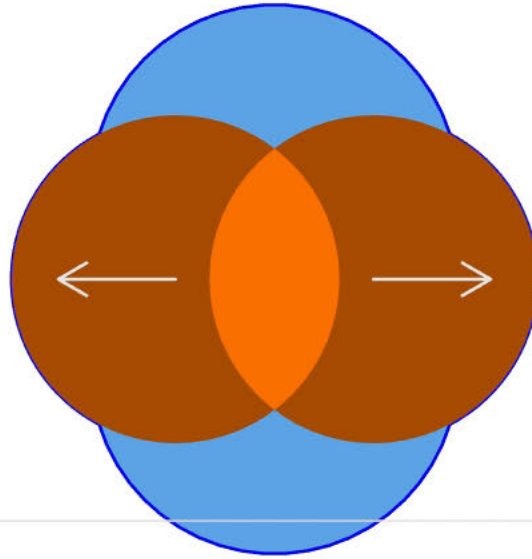
$$\boxed{f_{ds}} = -\frac{\partial dE_s}{\partial d} = -\left(\frac{\partial dE_s}{\partial R} \cdot \frac{\partial R}{\partial d} + \sum_{i=1,2} \frac{\partial dE_s}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial d} \right)$$

Analytical Model

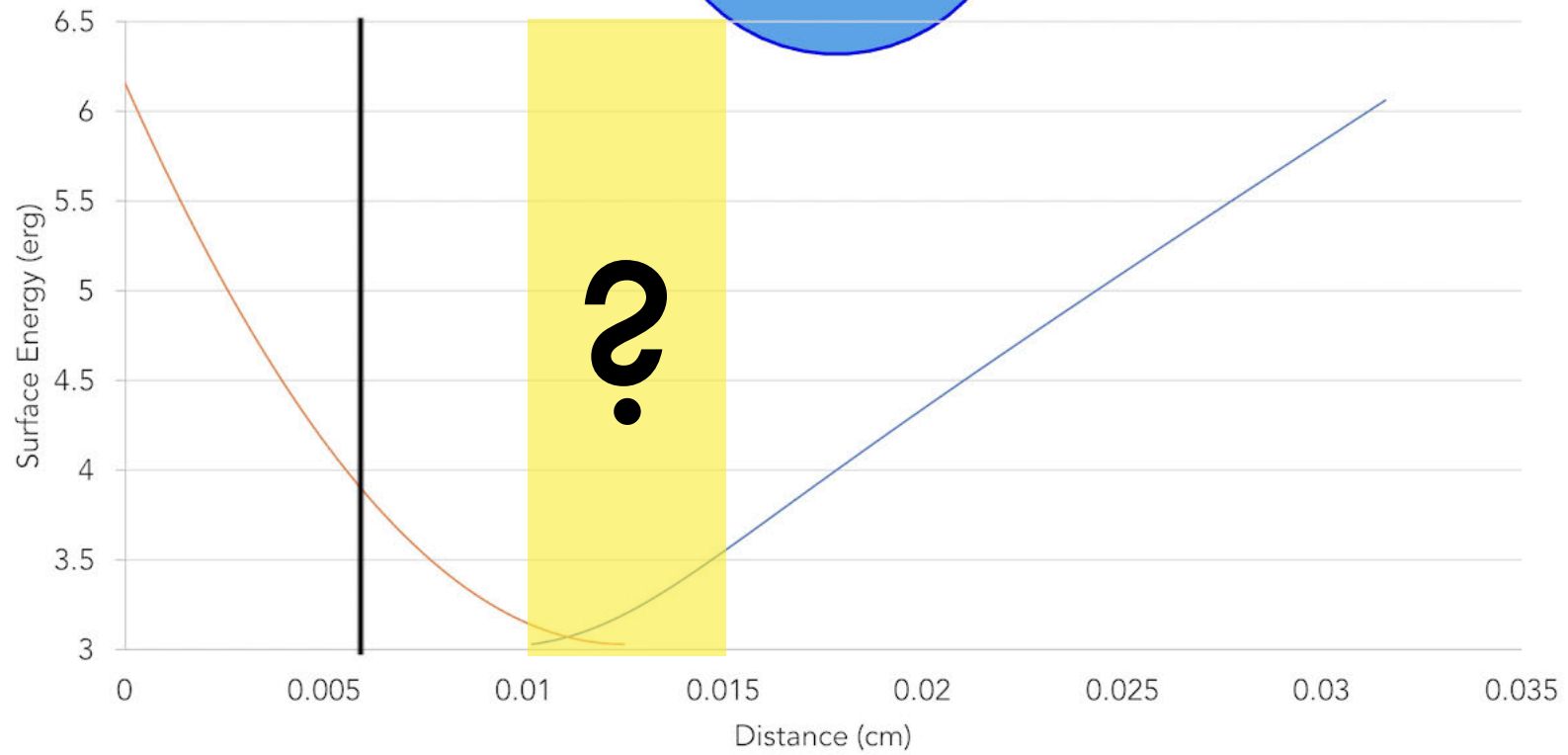
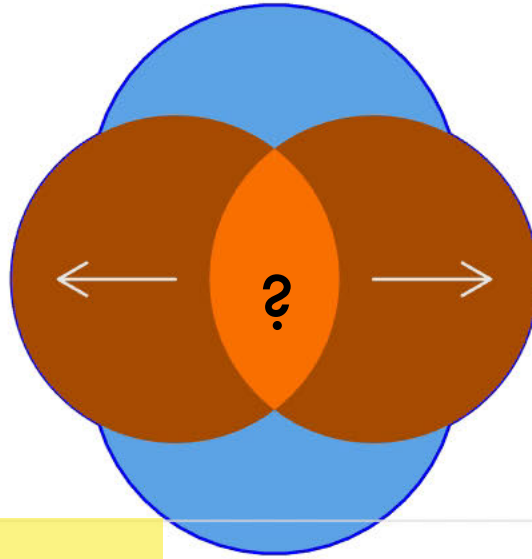




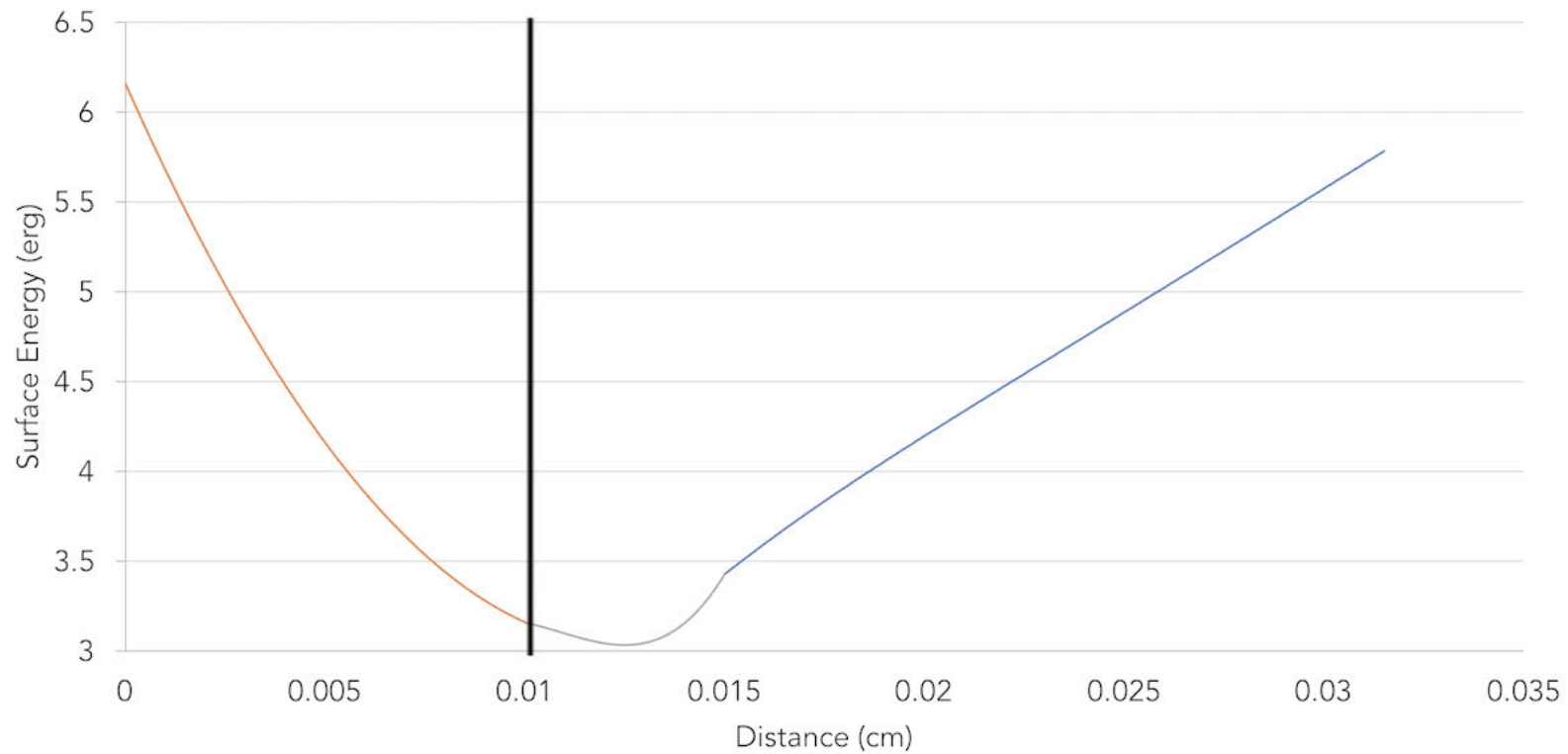
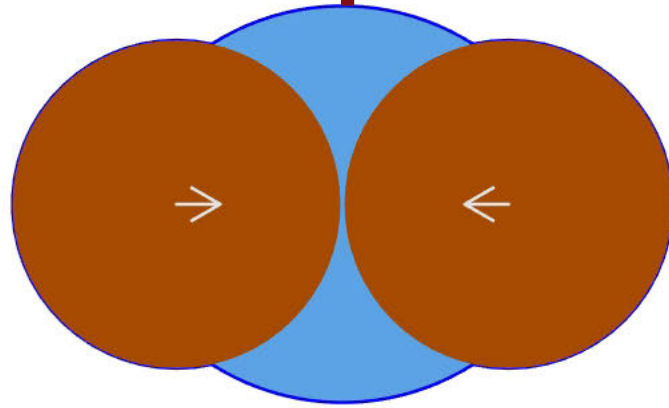
Handle Collisions



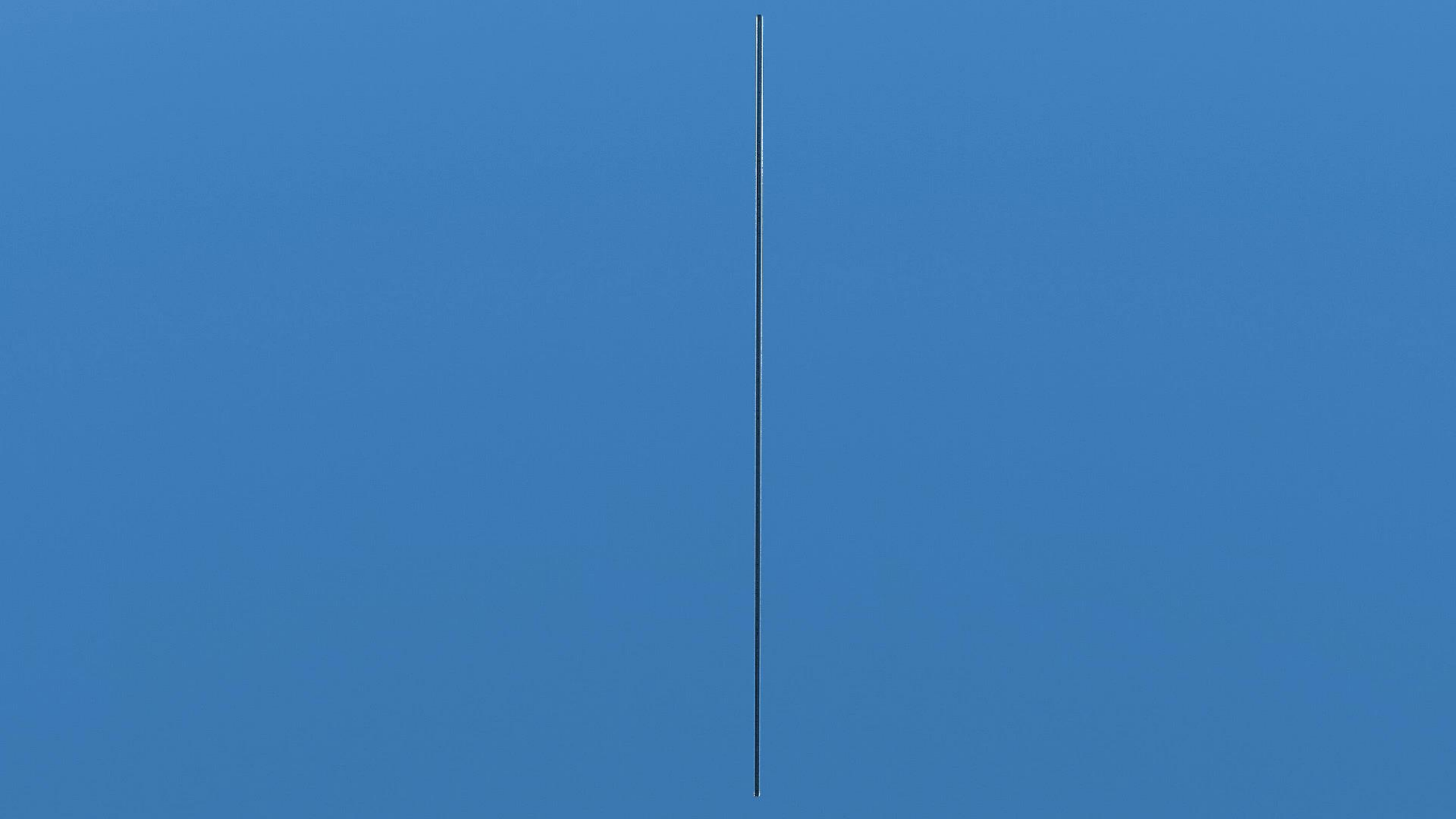
Handle Collisions



Interpolate between Cohesive & Repulsive Effect

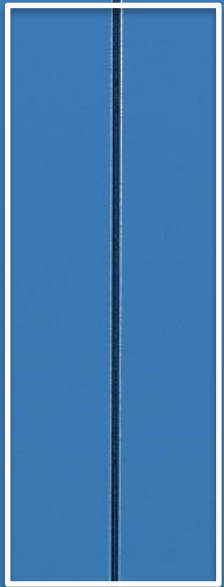


Discretized Cohesive Force

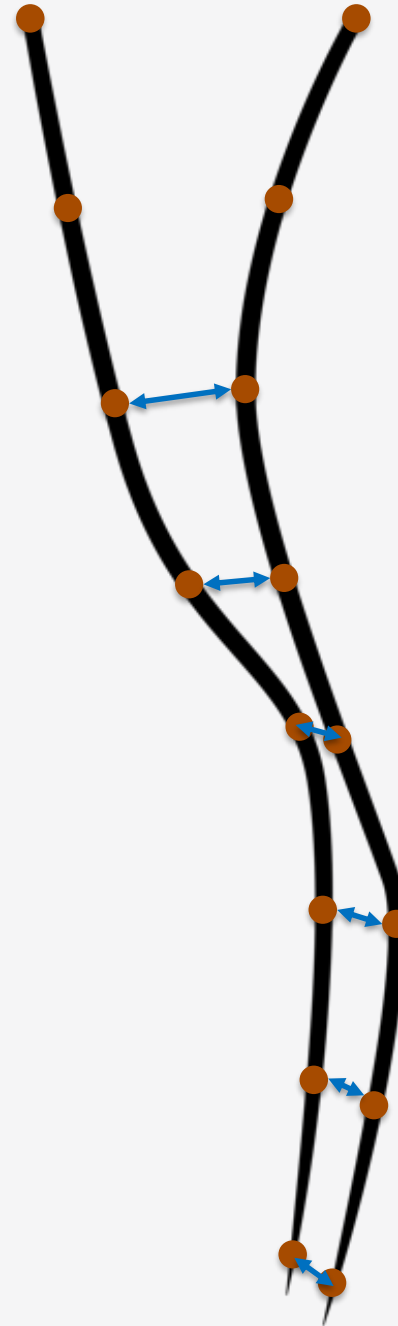




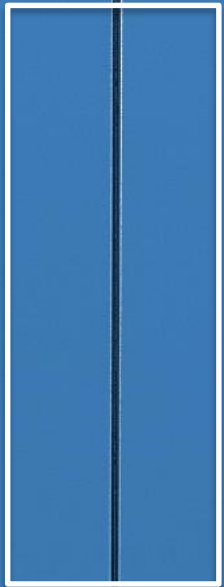
Discontinuous Motion



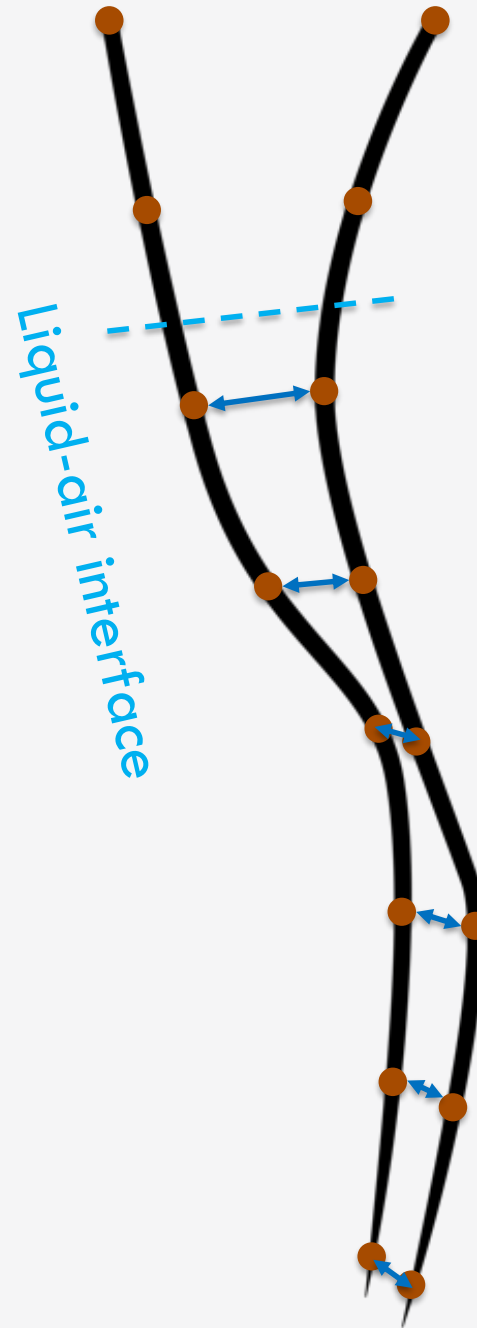
Discontinuous
Motion



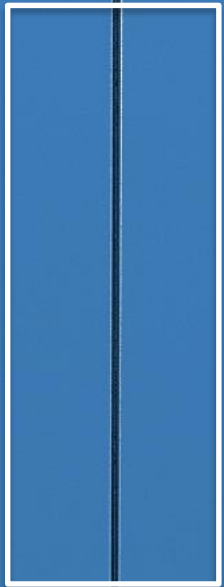
Naive Solution



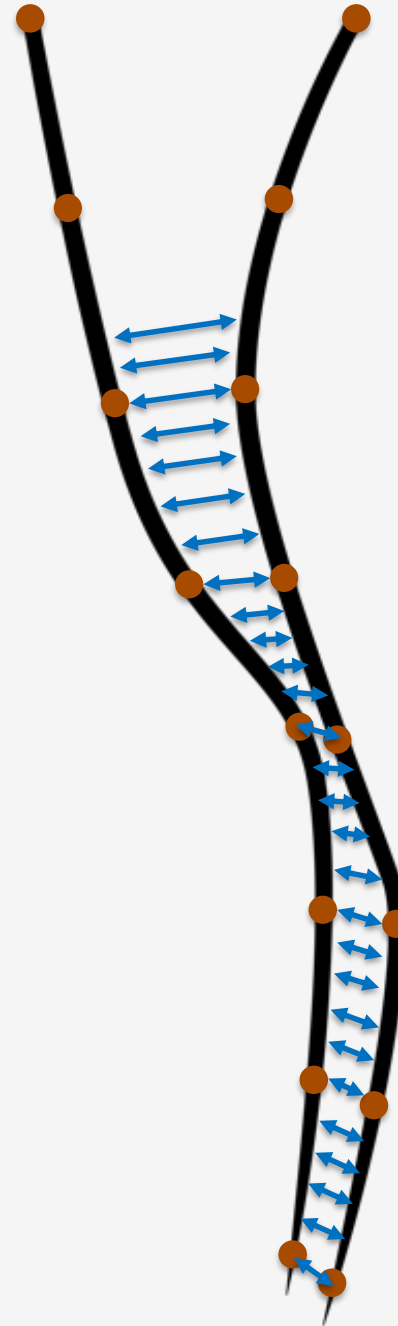
Discontinuous
Motion



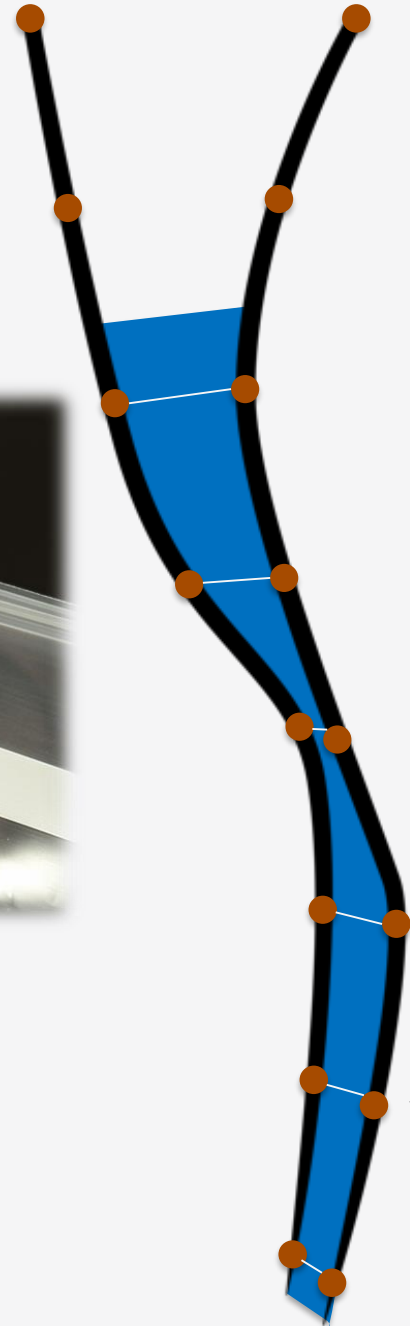
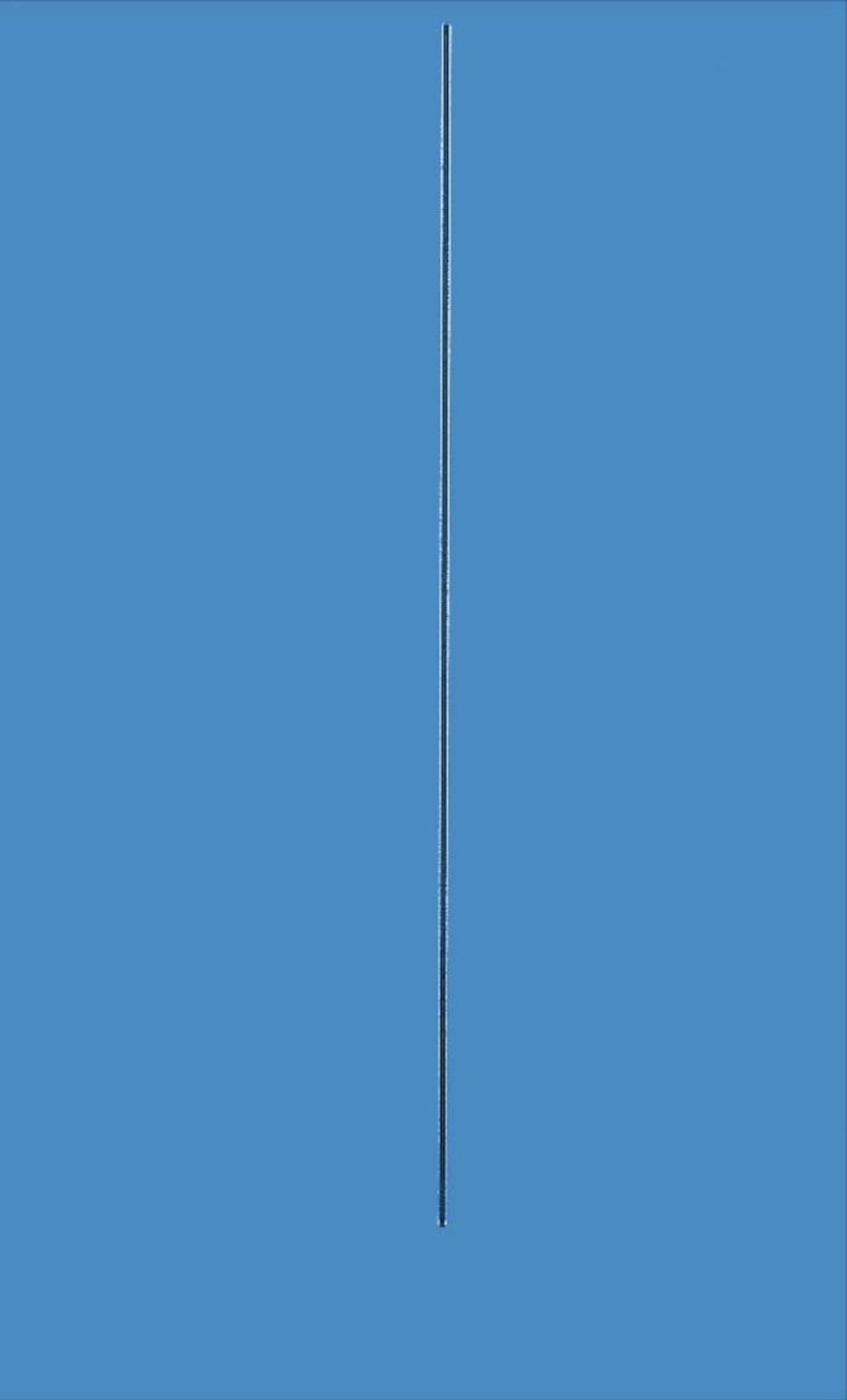
Naive Solution



Discontinuous
Motion



Over-Sampling



Variable Quadrature



Cohesion OFF

Constant Quadrature

Variable Quadrature₄₈

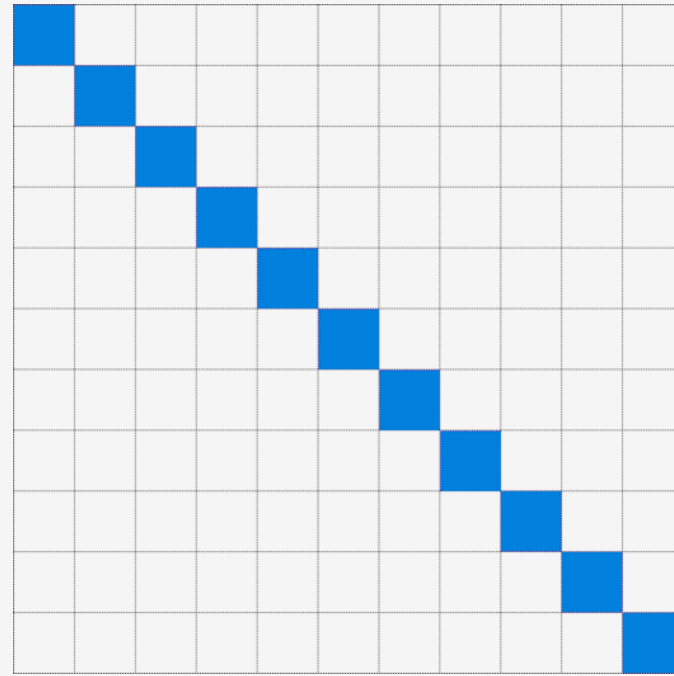
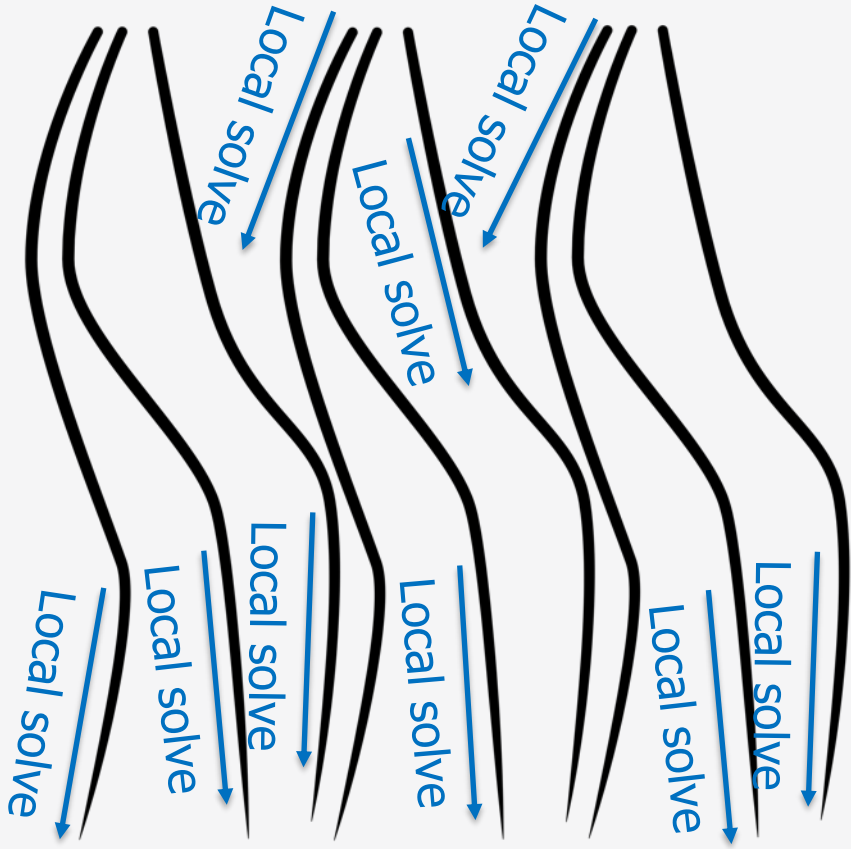


Our Simulation

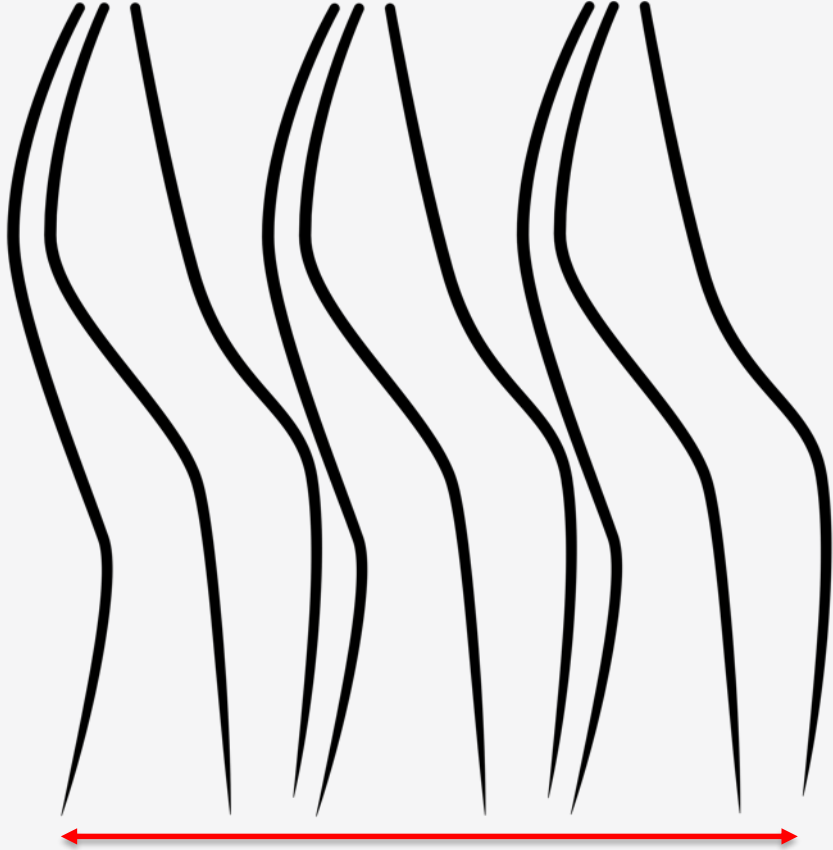
Real Experiment [Bico et al. 2004]



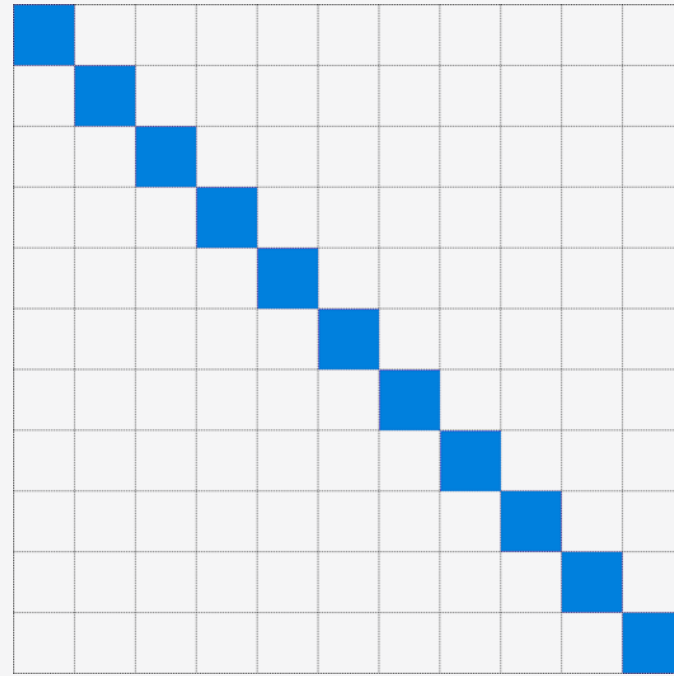
Preconditioning with Local Solves



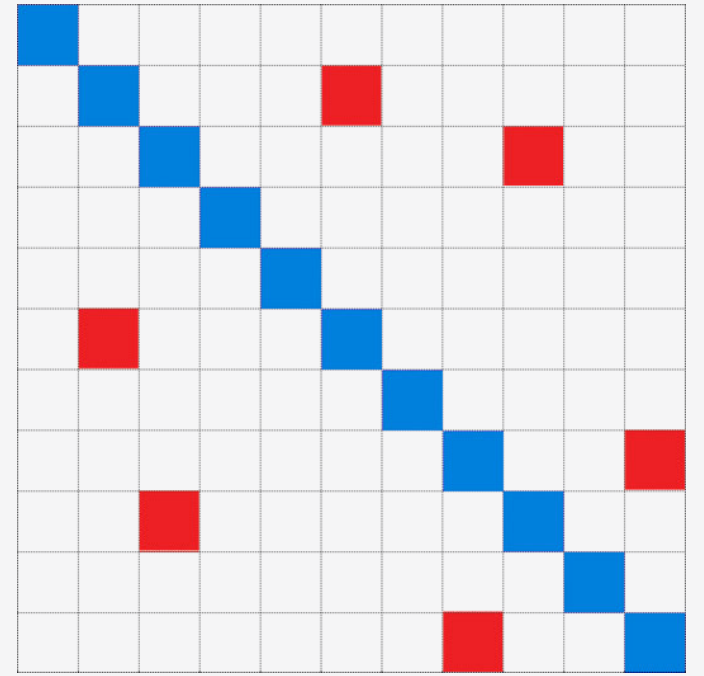
Pre-factorized LDLT in Parallel



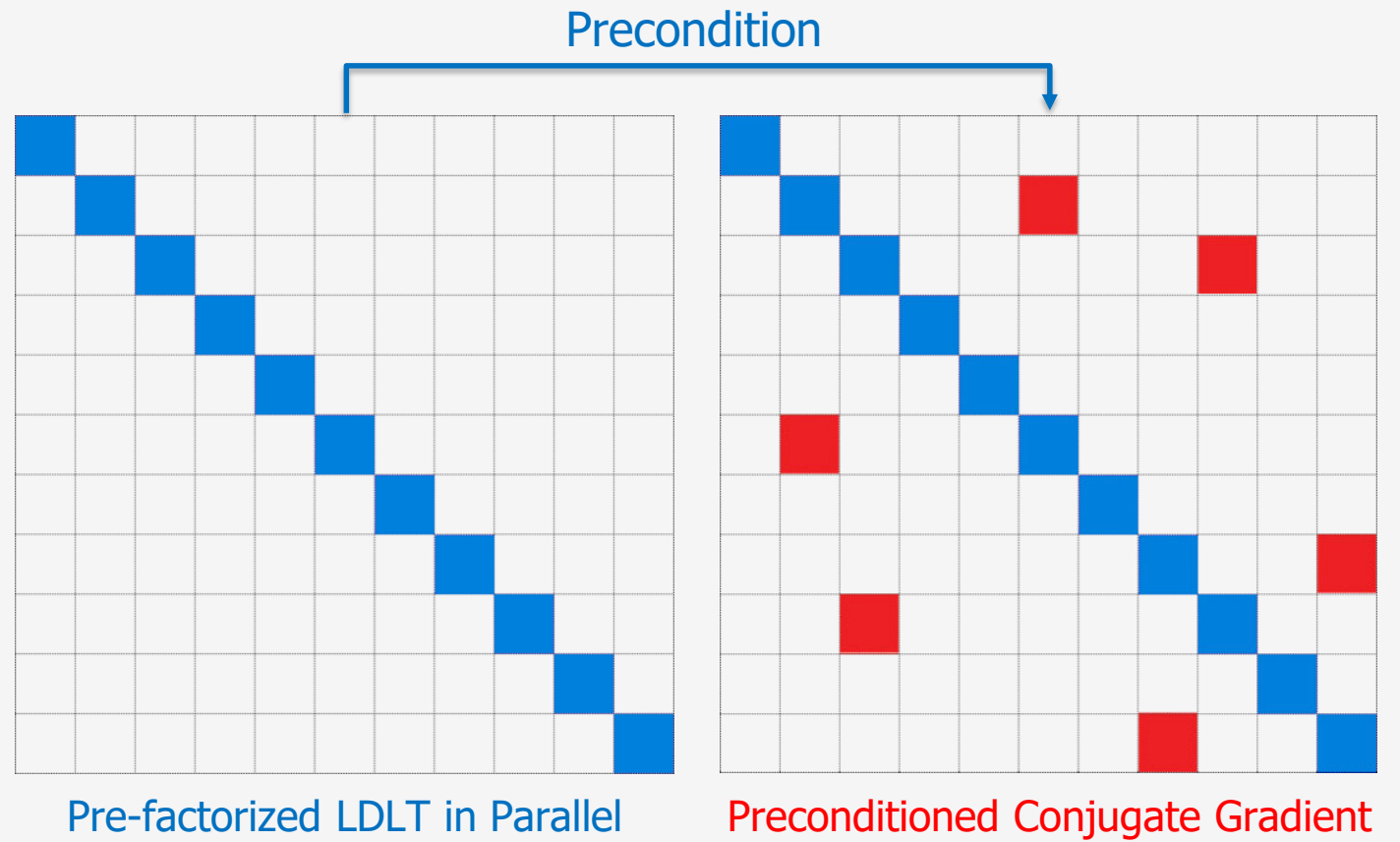
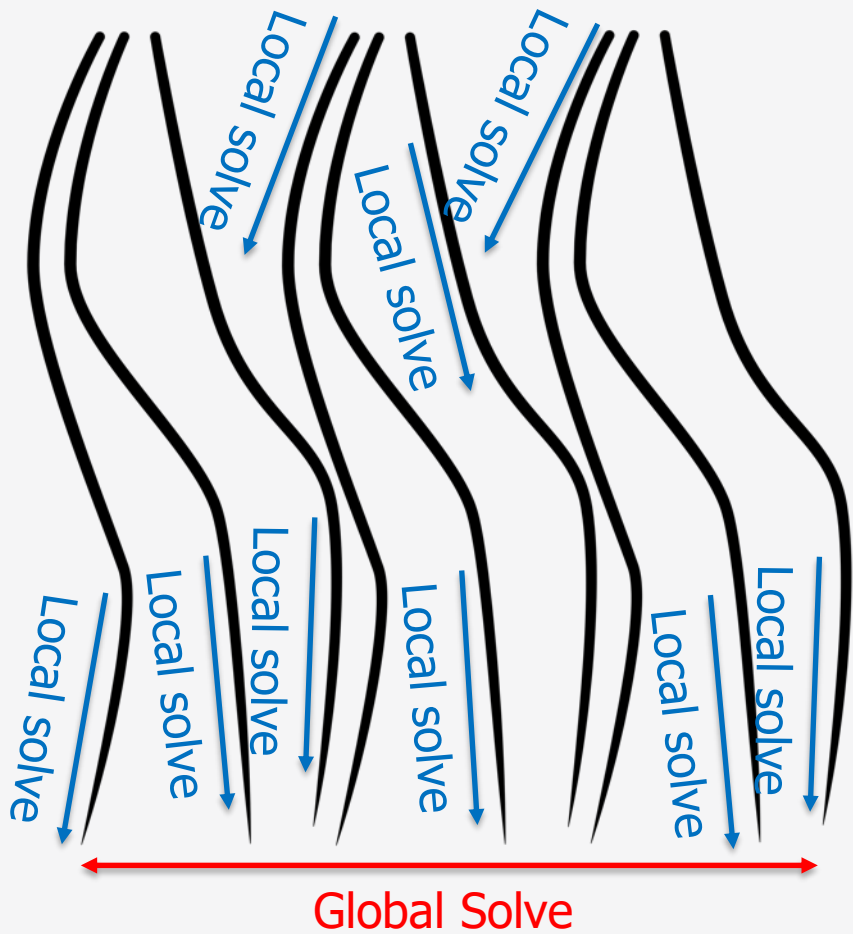
Global Solve

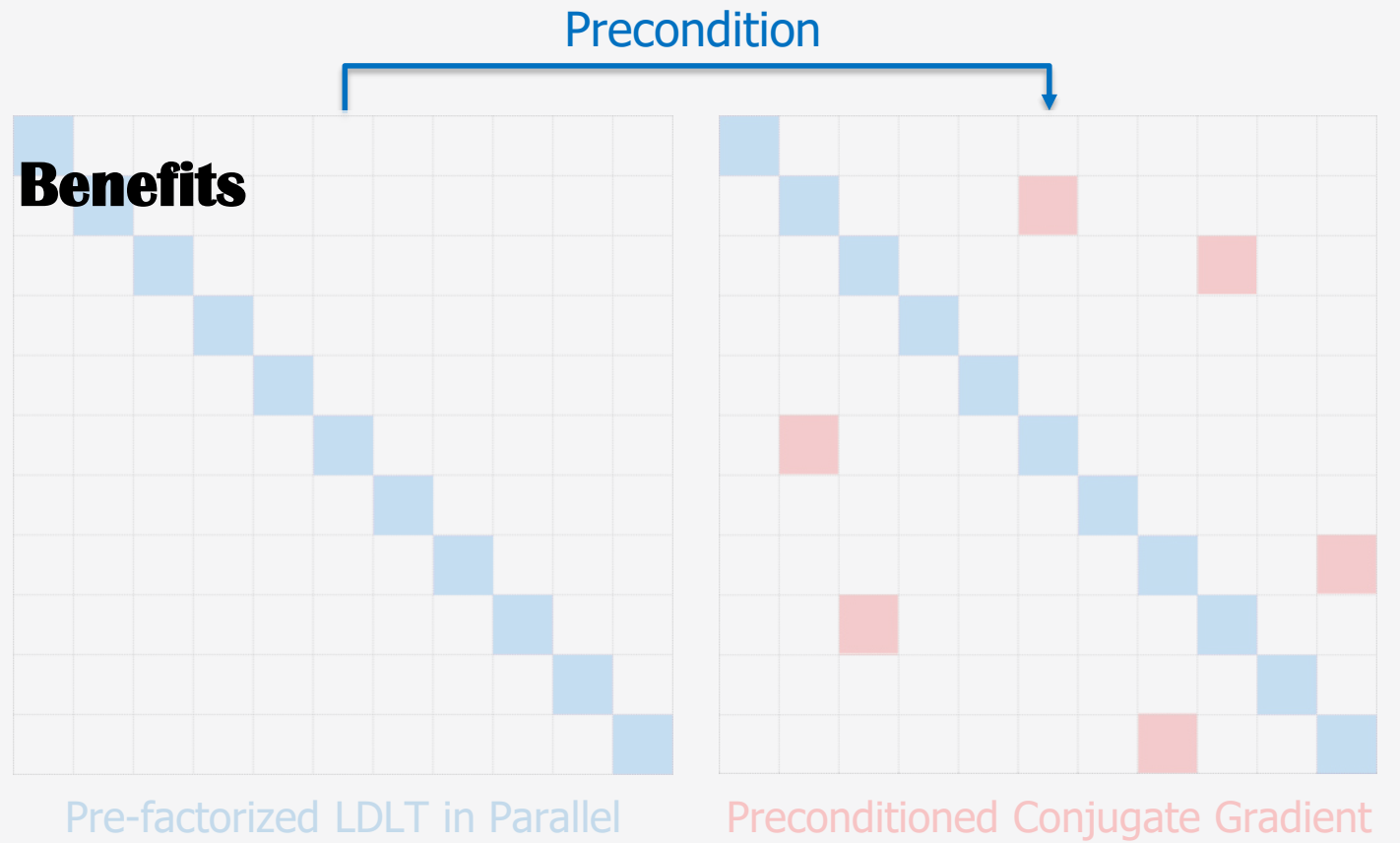
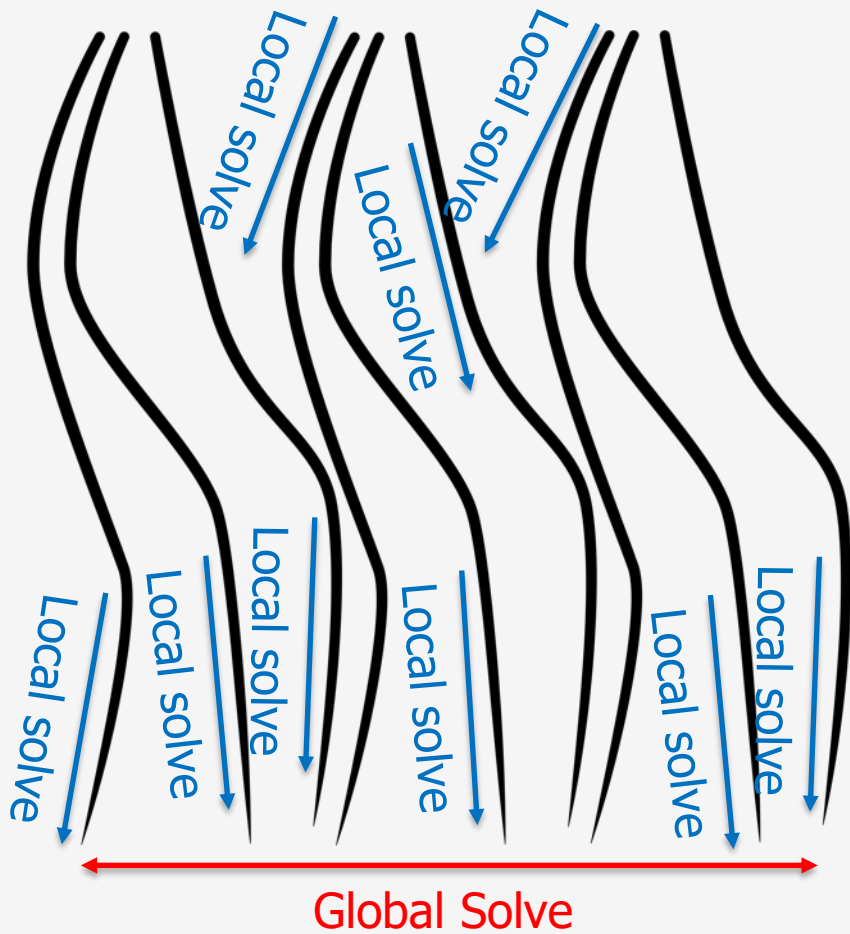


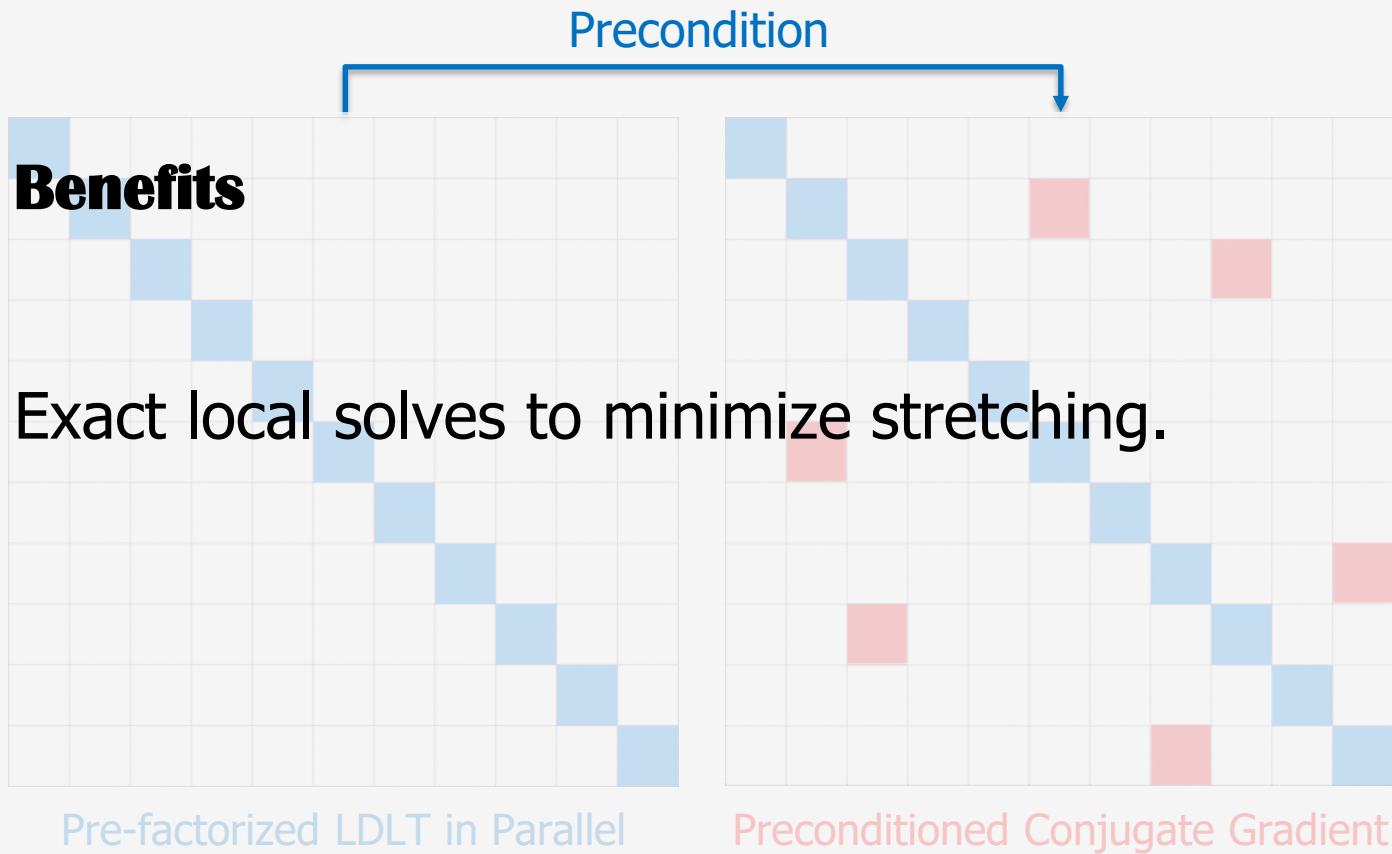
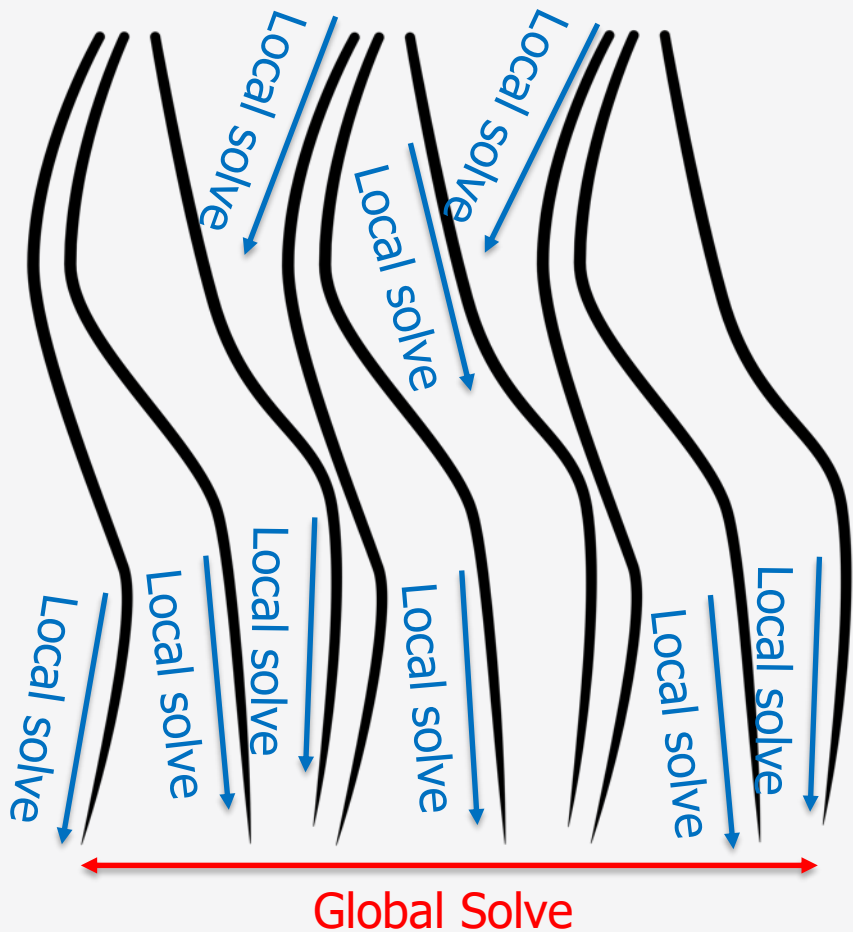
Pre-factorized LDLT in Parallel

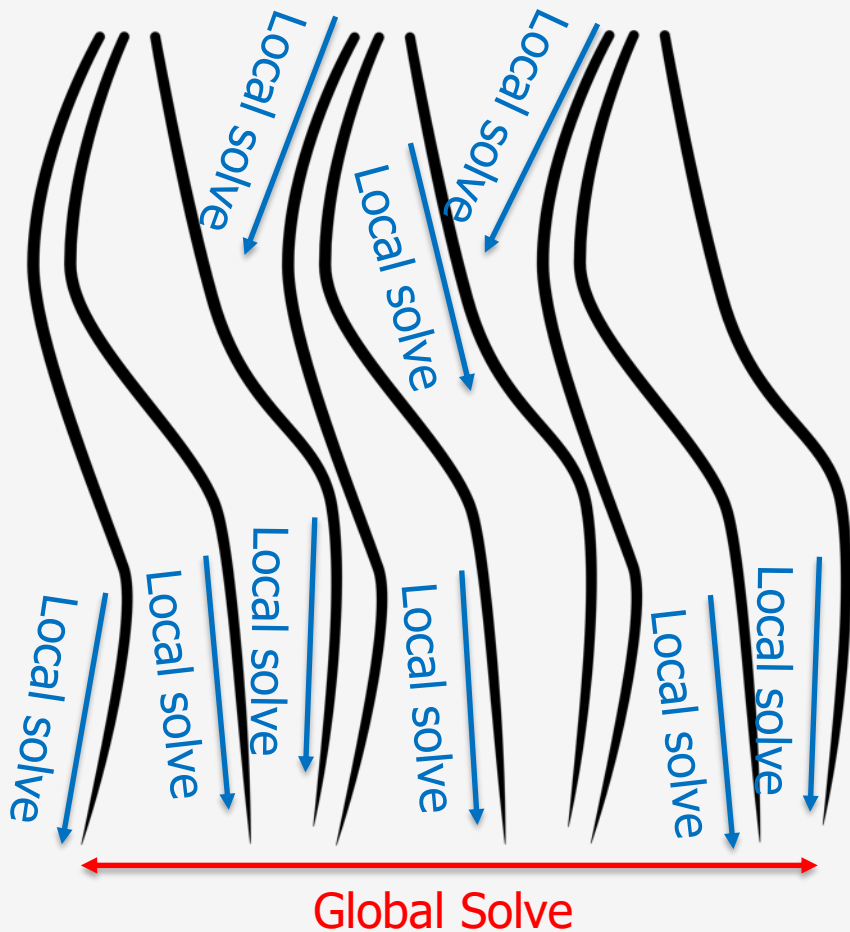


Preconditioned Conjugate Gradient







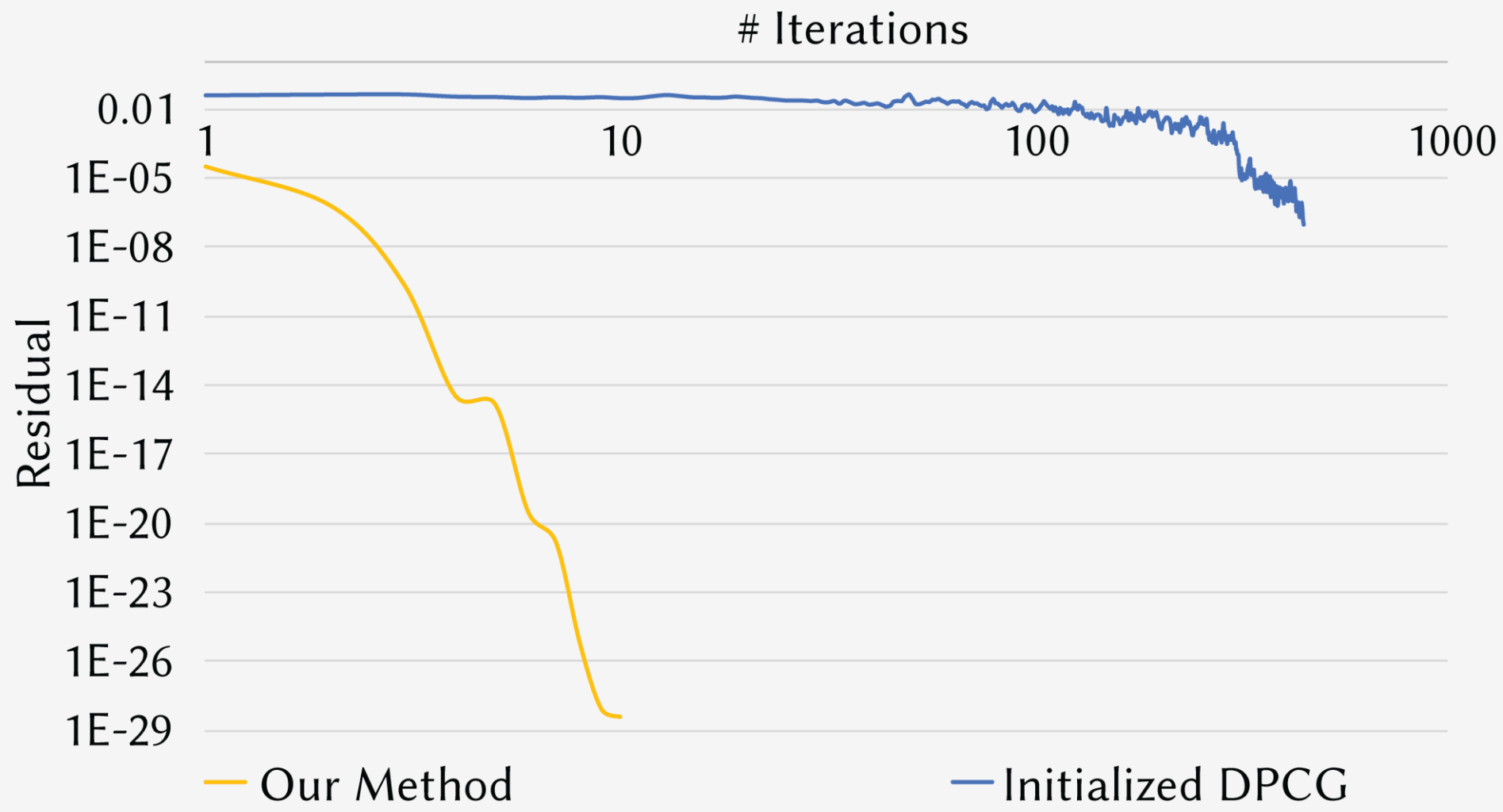


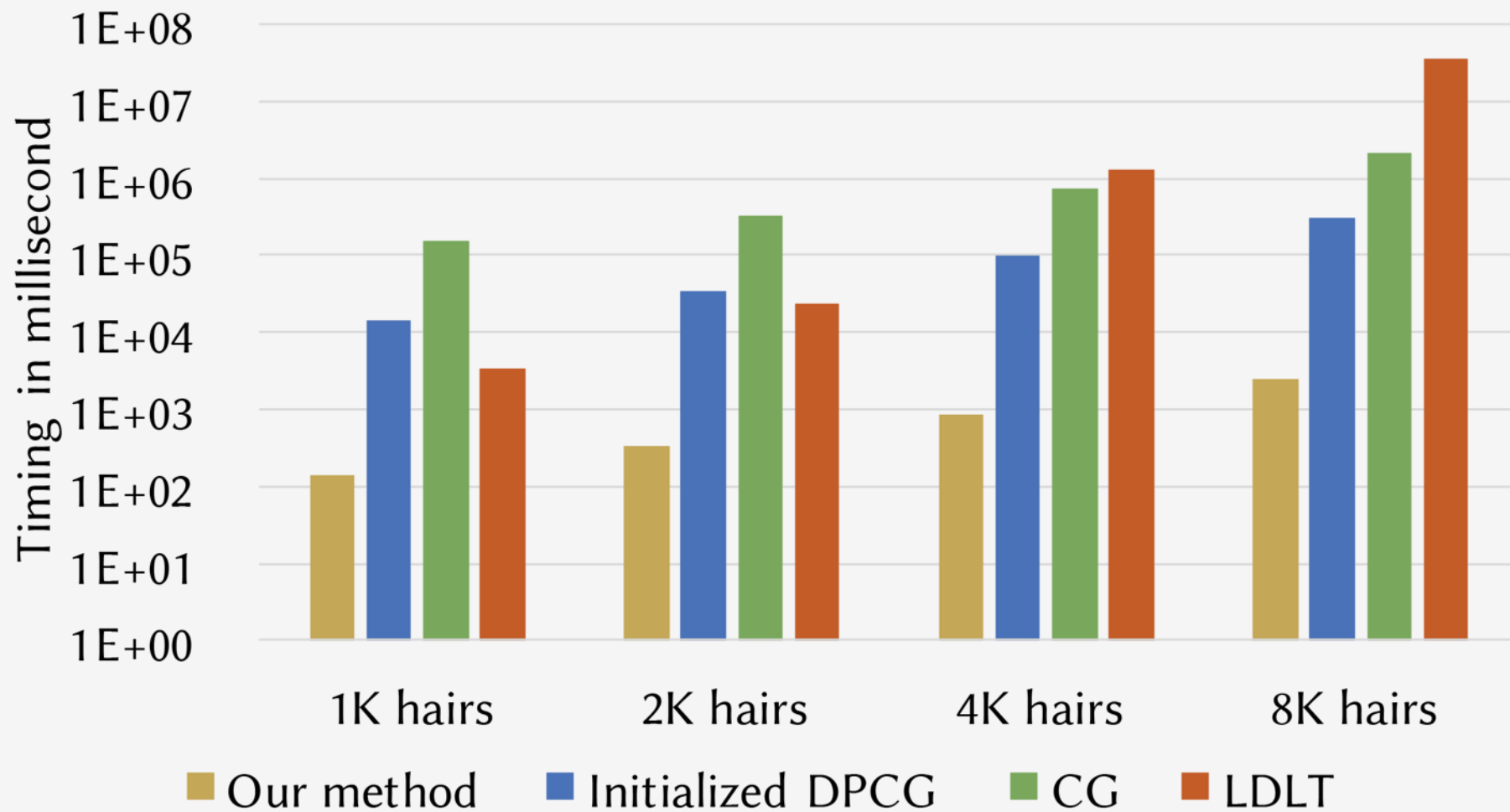
Benefits

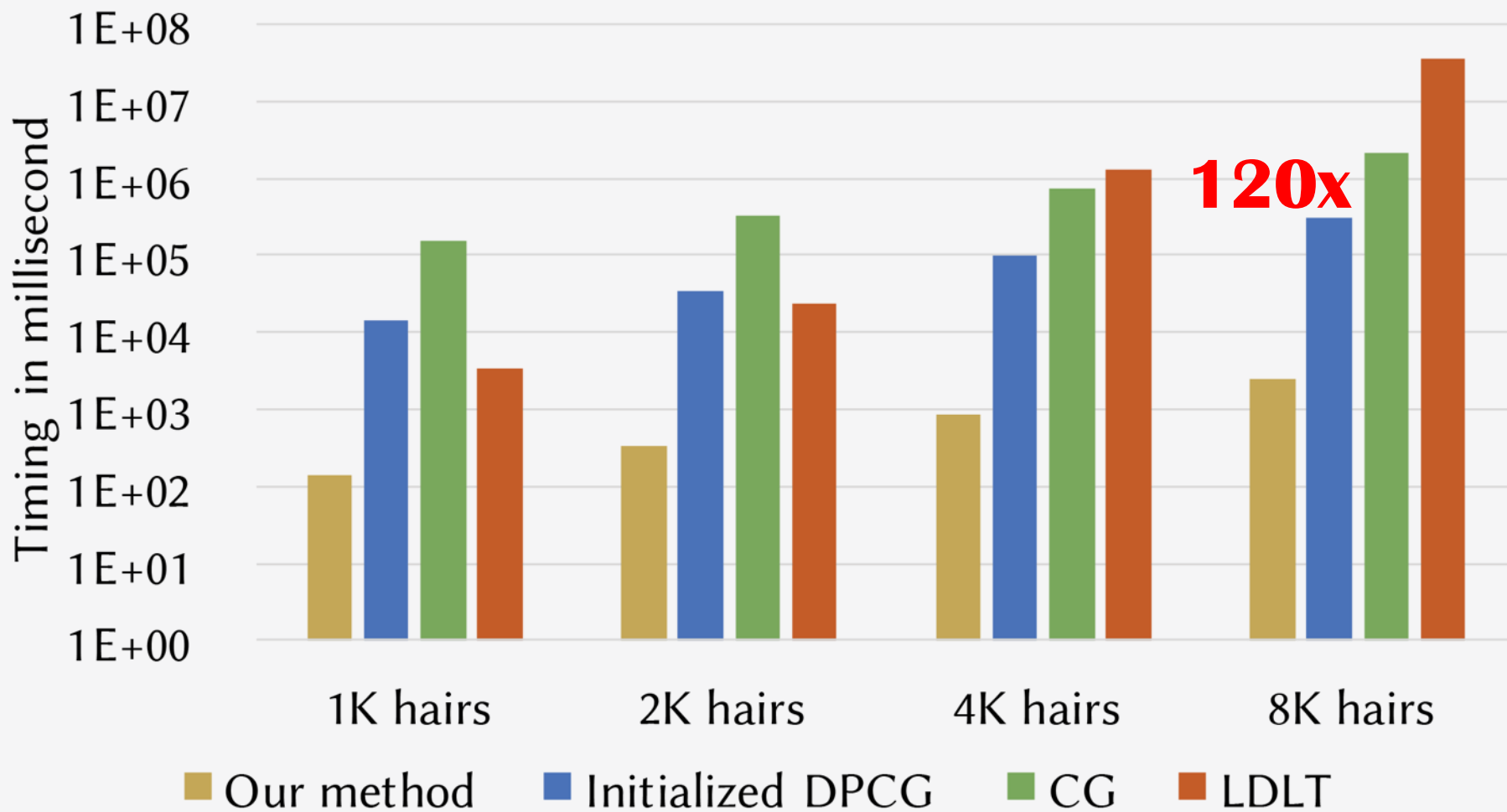
Exact local solves to minimize stretching.

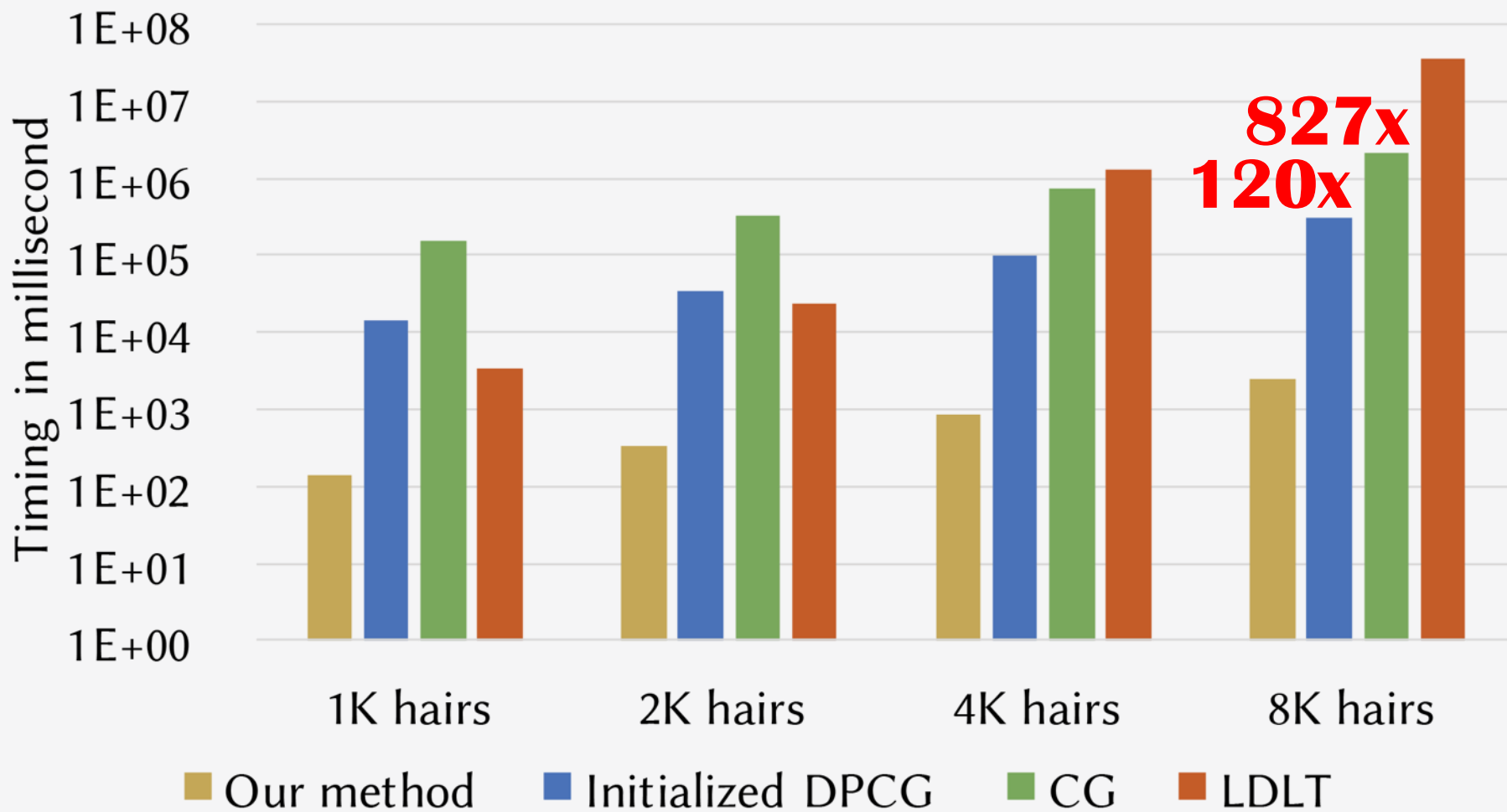
Similarity between the local and global matrices increases the rate of convergence.

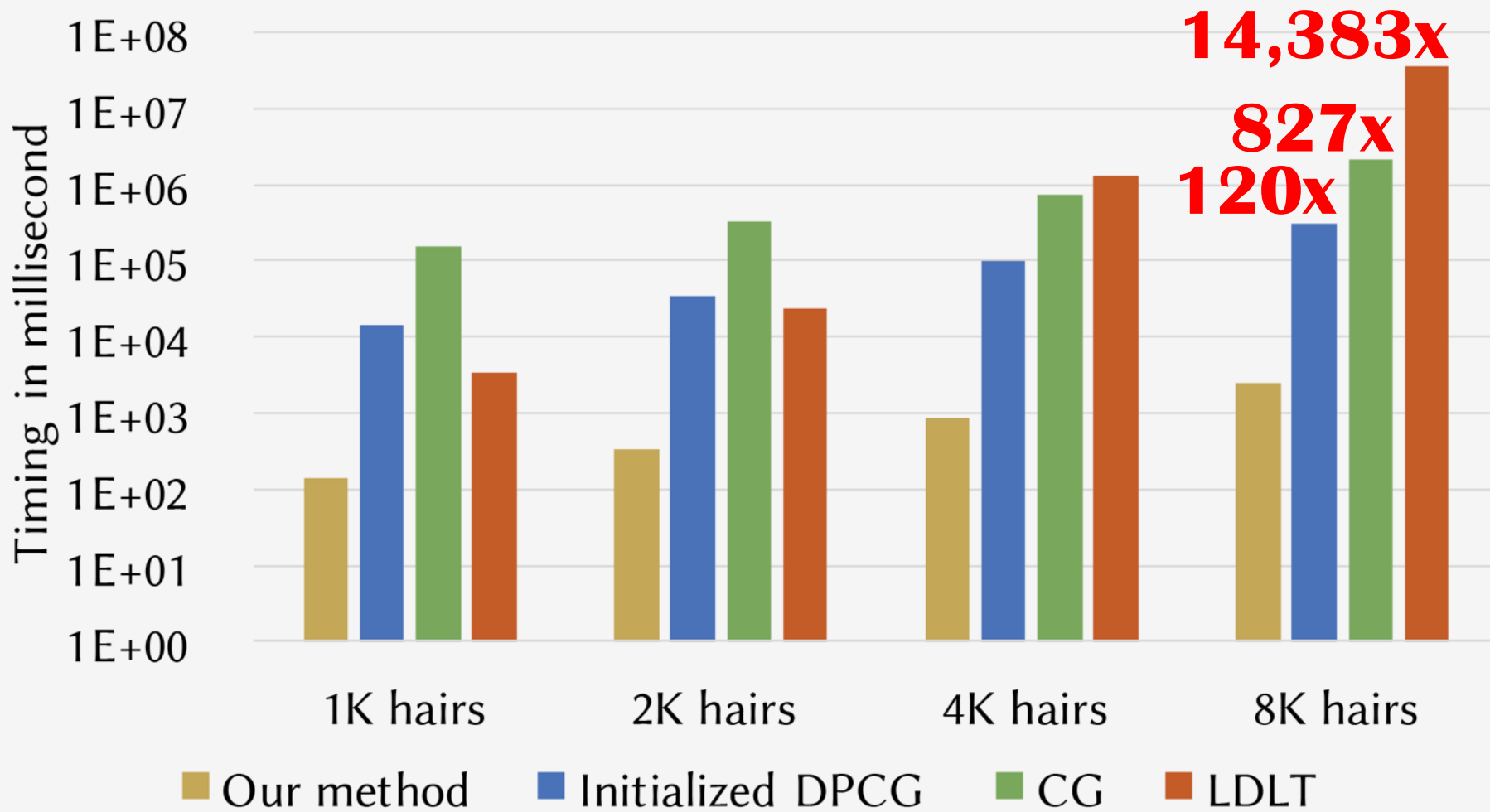
Pre-factorized LDLT in Parallel Preconditioned Conjugate Gradient



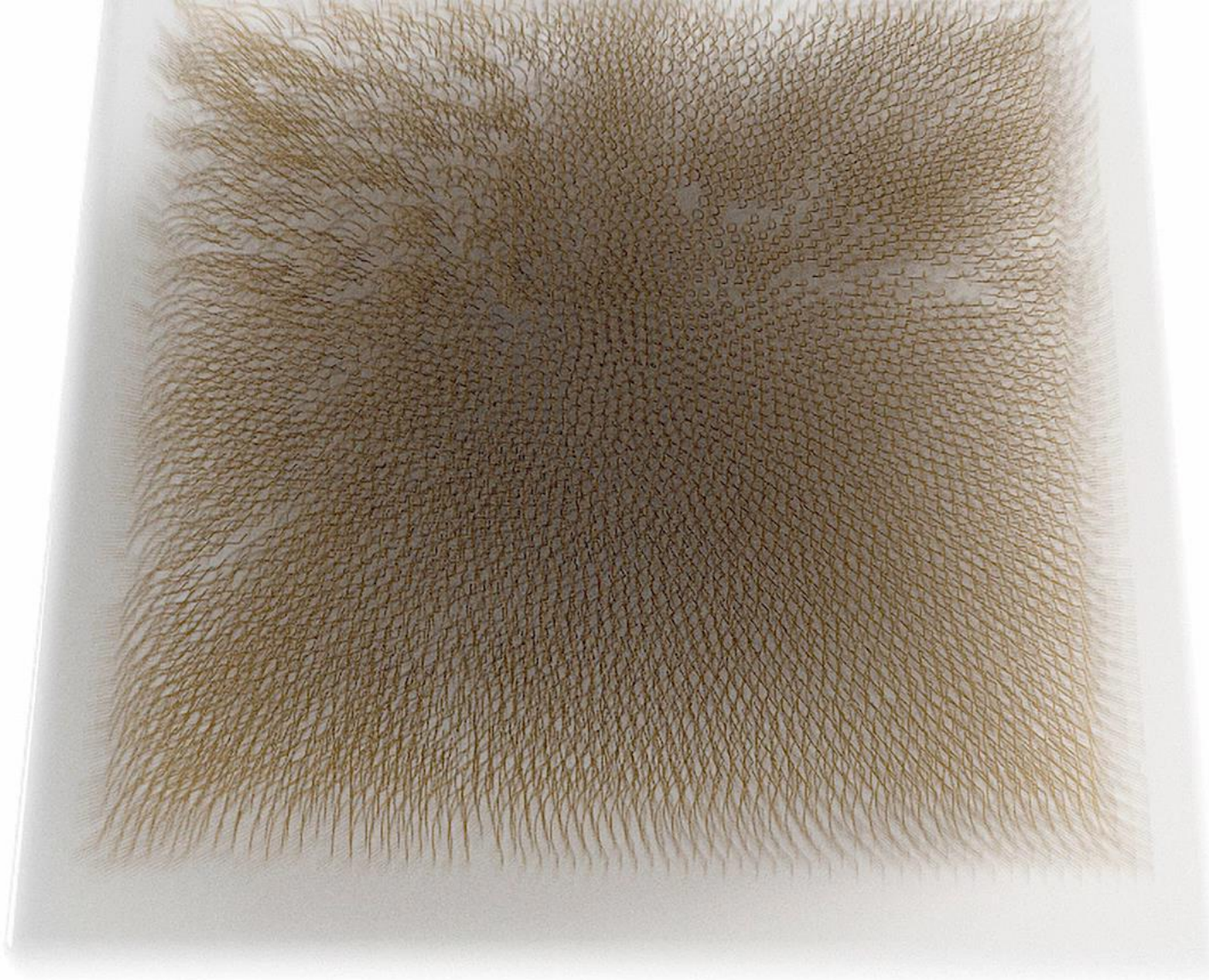








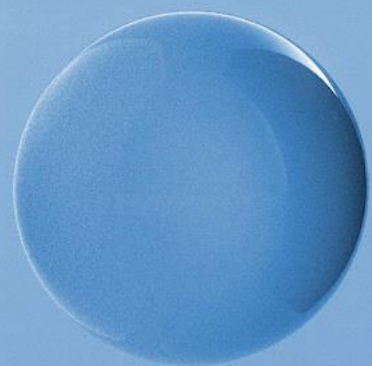
Complicated Cohesion Effects

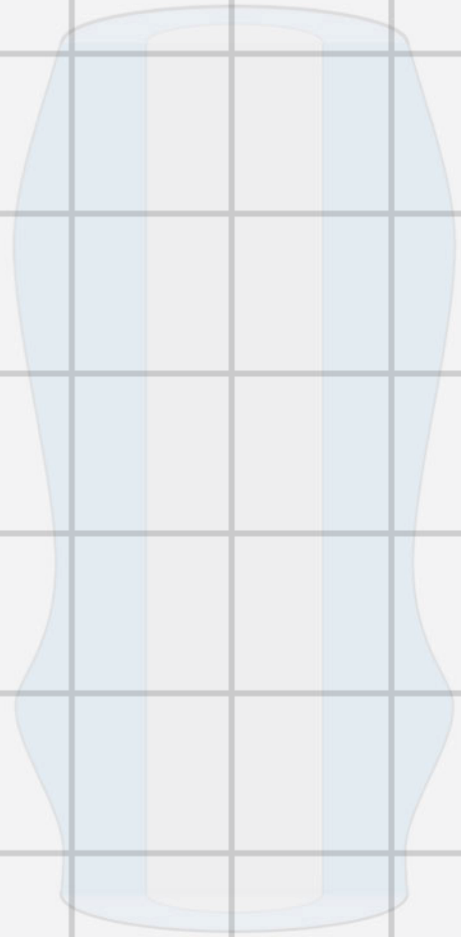


Water on
Mat of Fur

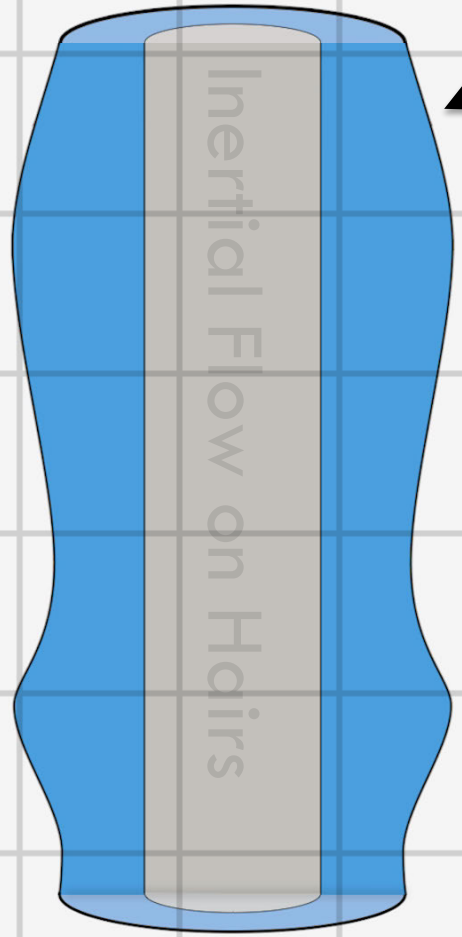
1/2x replay

Whipping
Wet Hairs





Cohesion
Collision



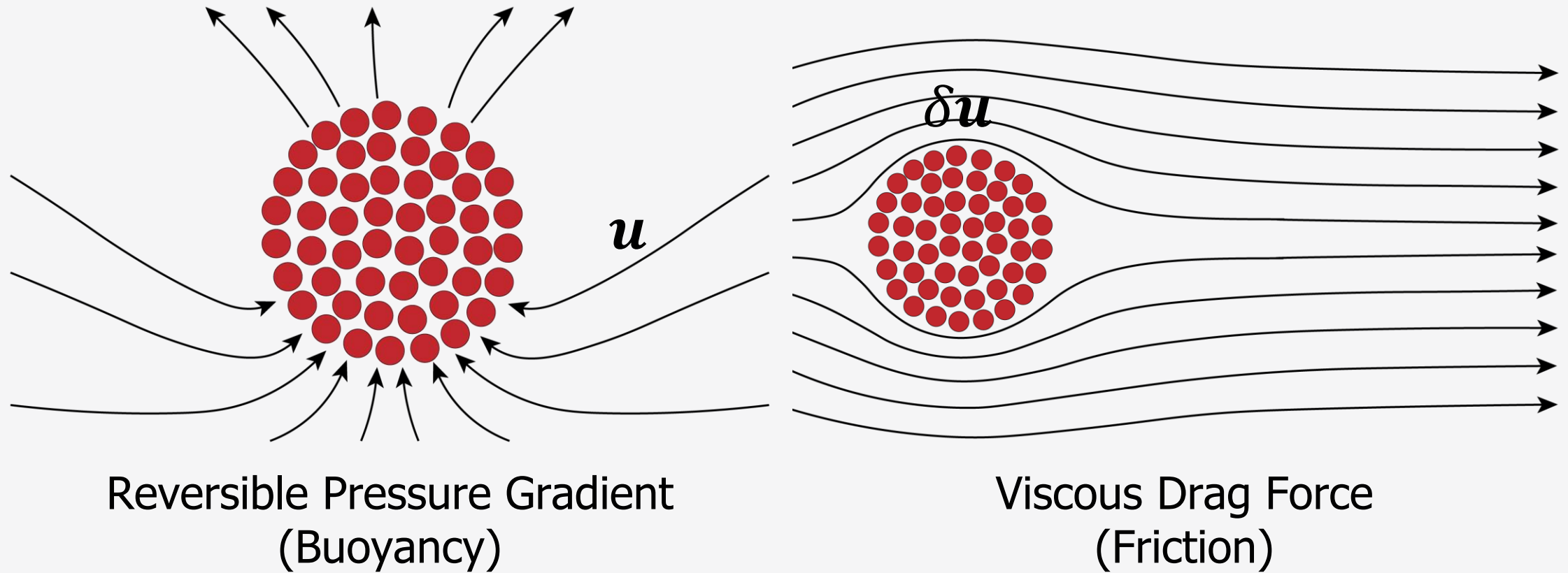
Two-way Coupling

Dripping
Capturing



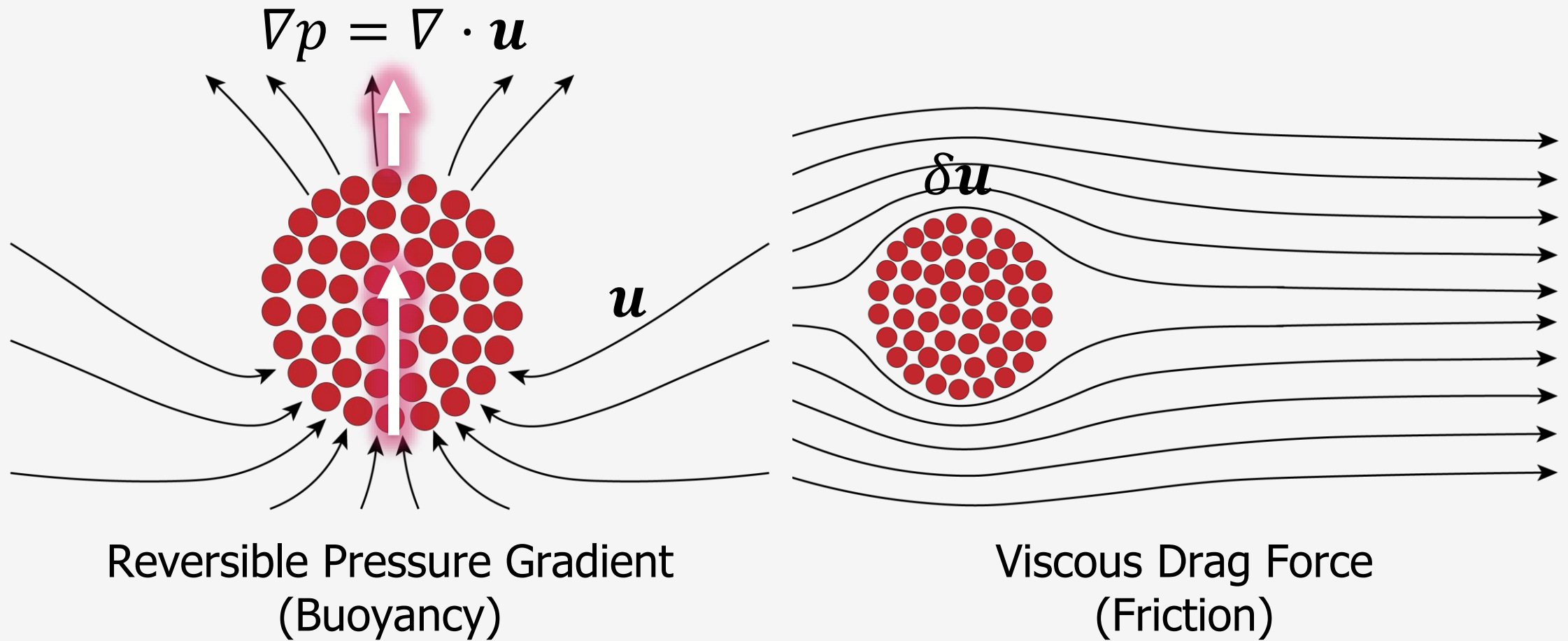
Hairs and Liquid as a Continuum

Related to [Daviet & Bertails-Descoubes 2017, Tampubolon et al. 2017]



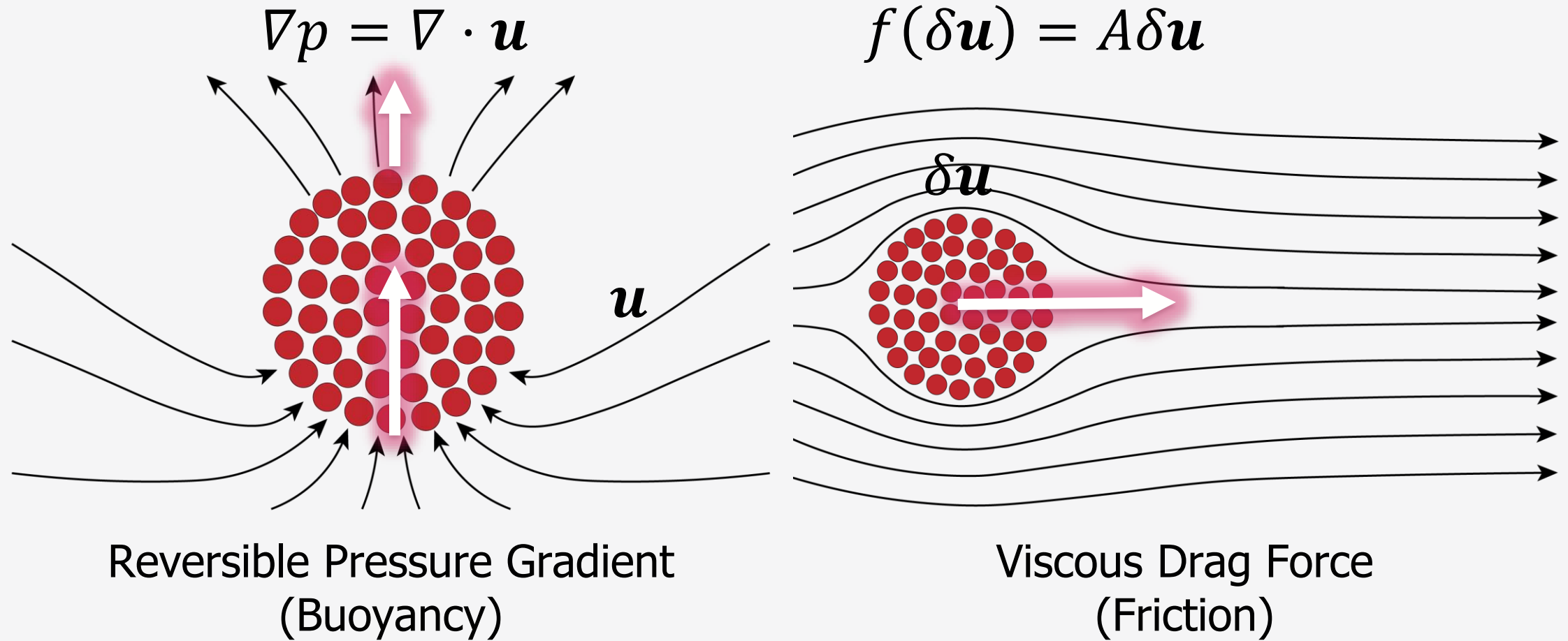
Hairs and Liquid as a Continuum

Related to [Daviet & Bertails-Descoubes 2017, Tampubolon et al. 2017]



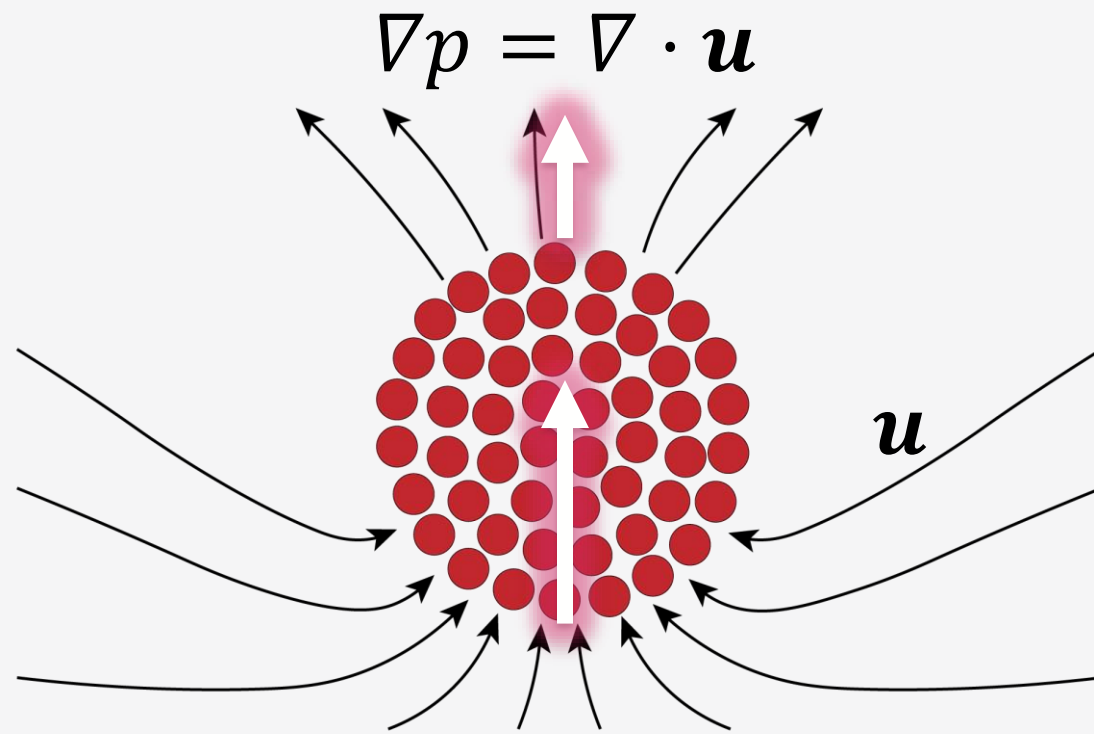
Hairs and Liquid as a Continuum

Related to [Daviet & Bertails-Descoubes 2017, Tampubolon et al. 2017]



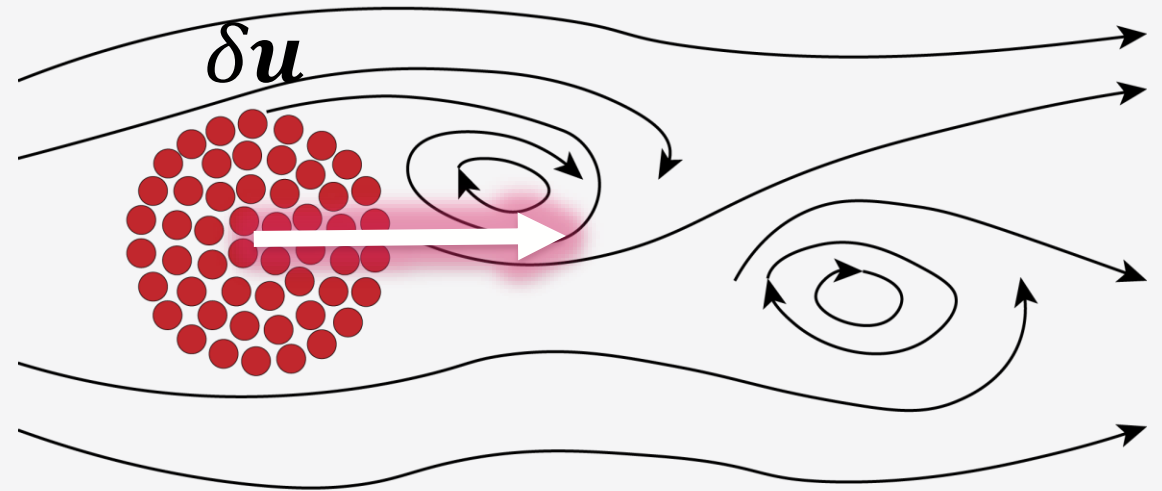
Hairs and Liquid as a Continuum

Related to [Daviet & Bertails-Descoubes 2017, Tampubolon et al. 2017]



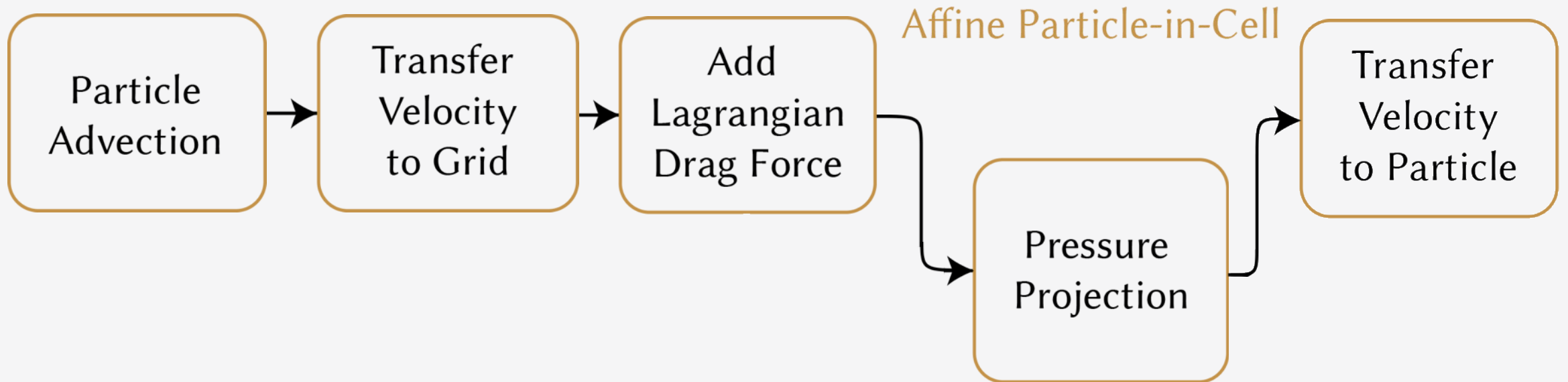
Reversible Pressure Gradient
(Buoyancy)

$$f(\delta \mathbf{u}) = A\delta \mathbf{u} + B|\delta \mathbf{u}|\delta \mathbf{u}$$



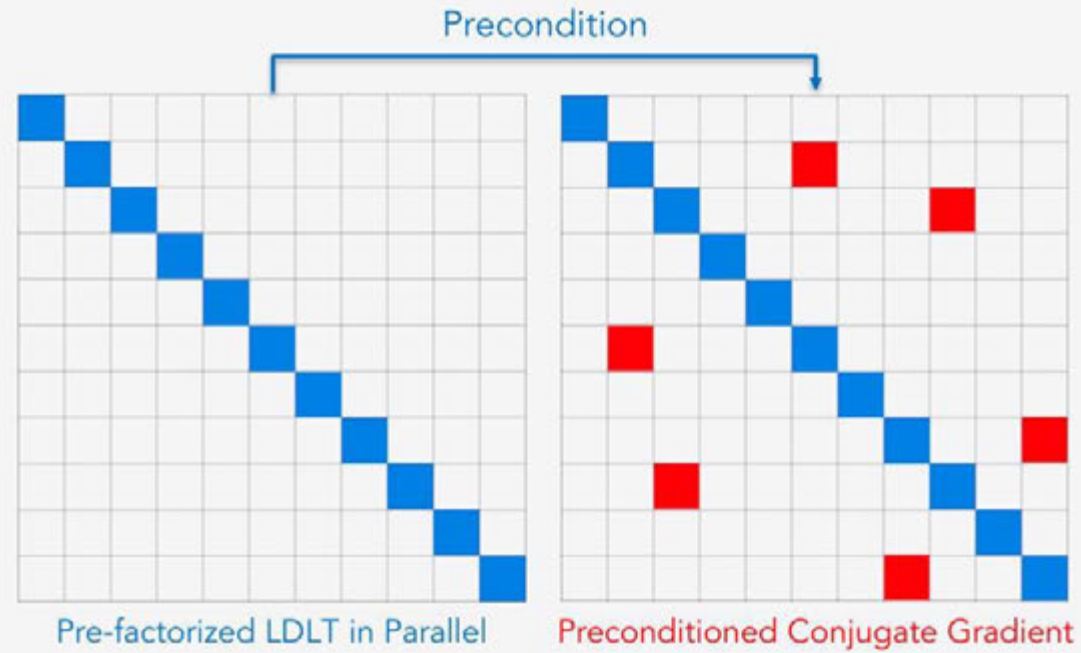
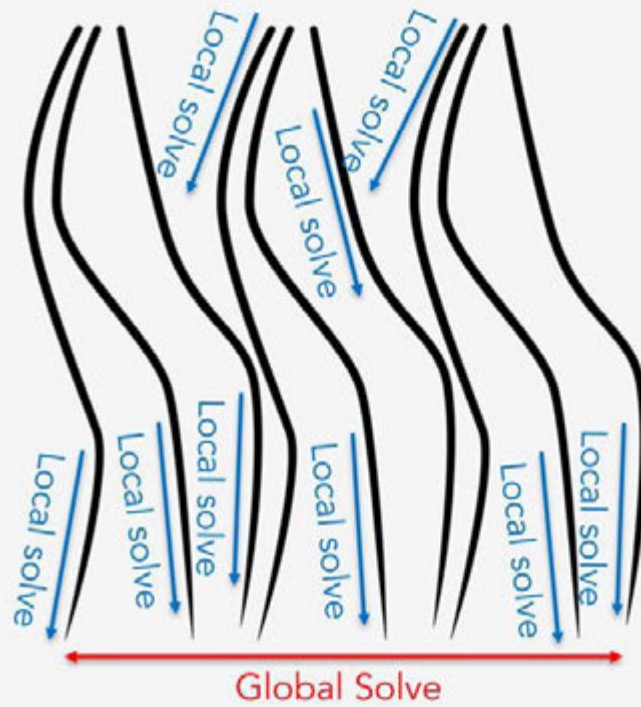
Quadratic Drag Force
(Friction + Wake Turbulence)

Pipeline



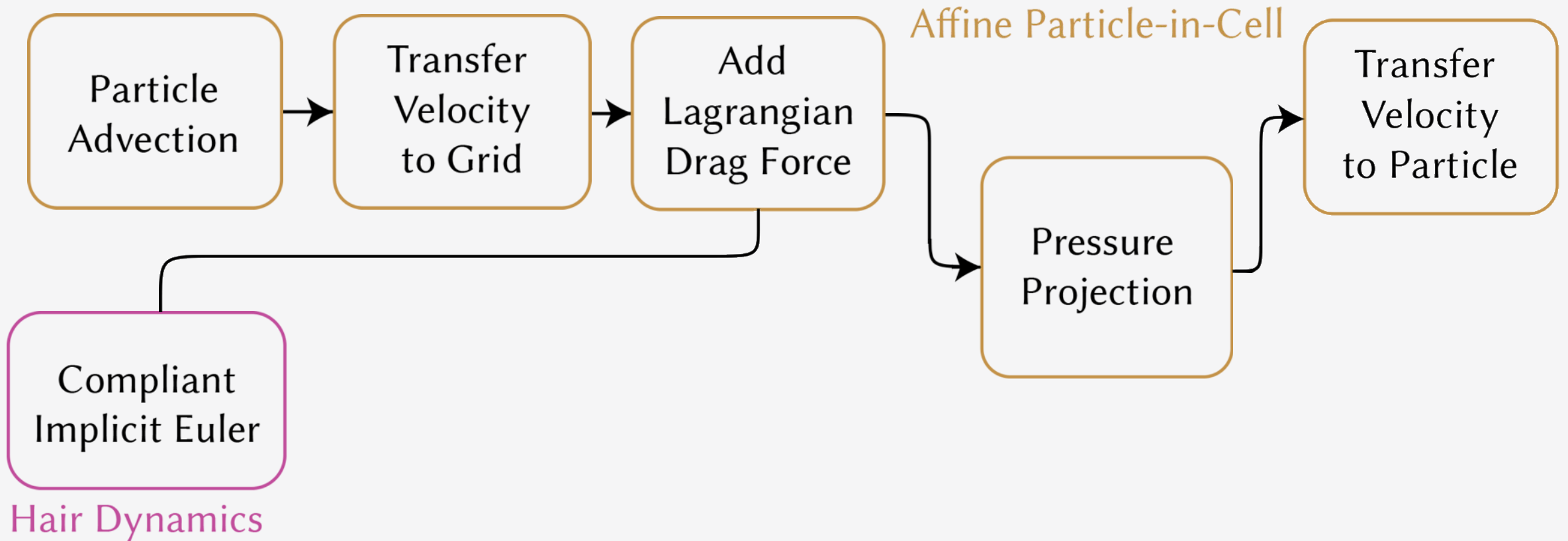
Pipe

Par
Adve



fer
city
article

Pipeline



Shallow Water Equation

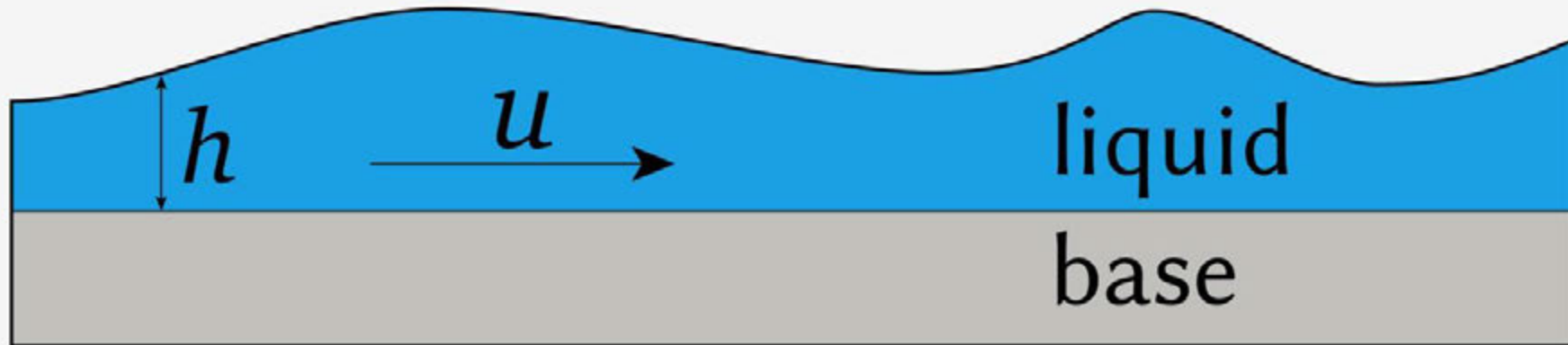
Momentum Equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + a_{\text{ext}}$$

✓ Inertia

✓ Surface Tension

✓ External Force (Gravity)



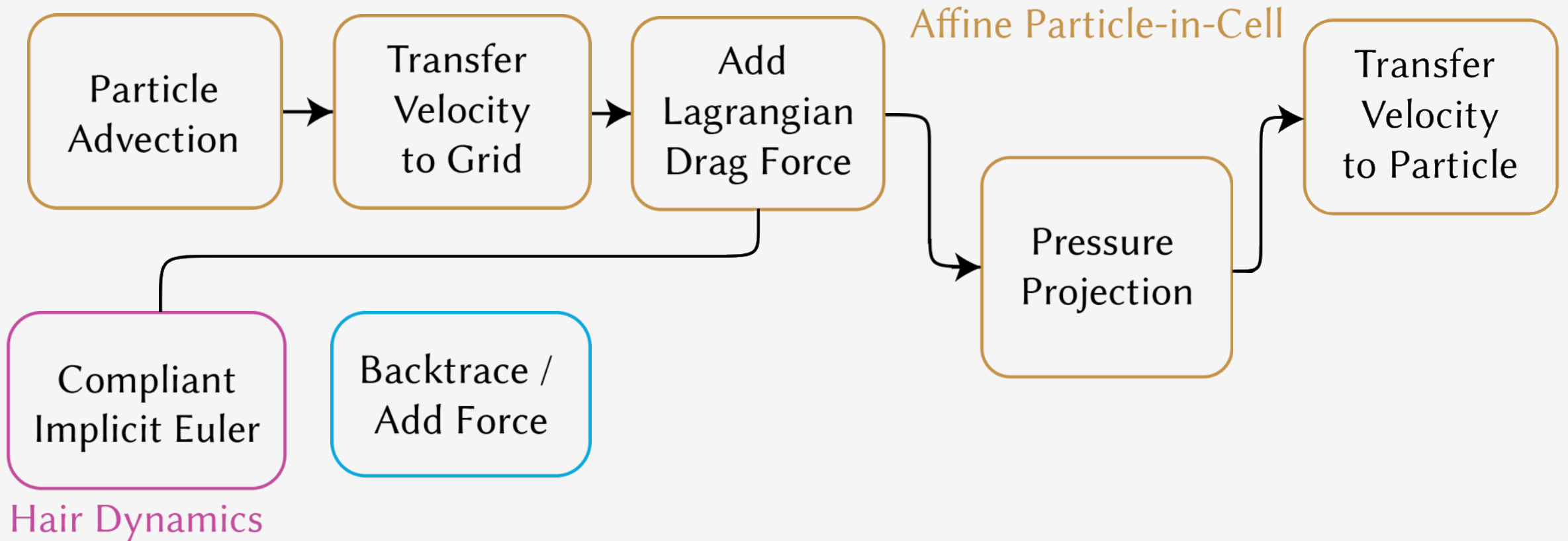
Part
Adve

Transfer
particle

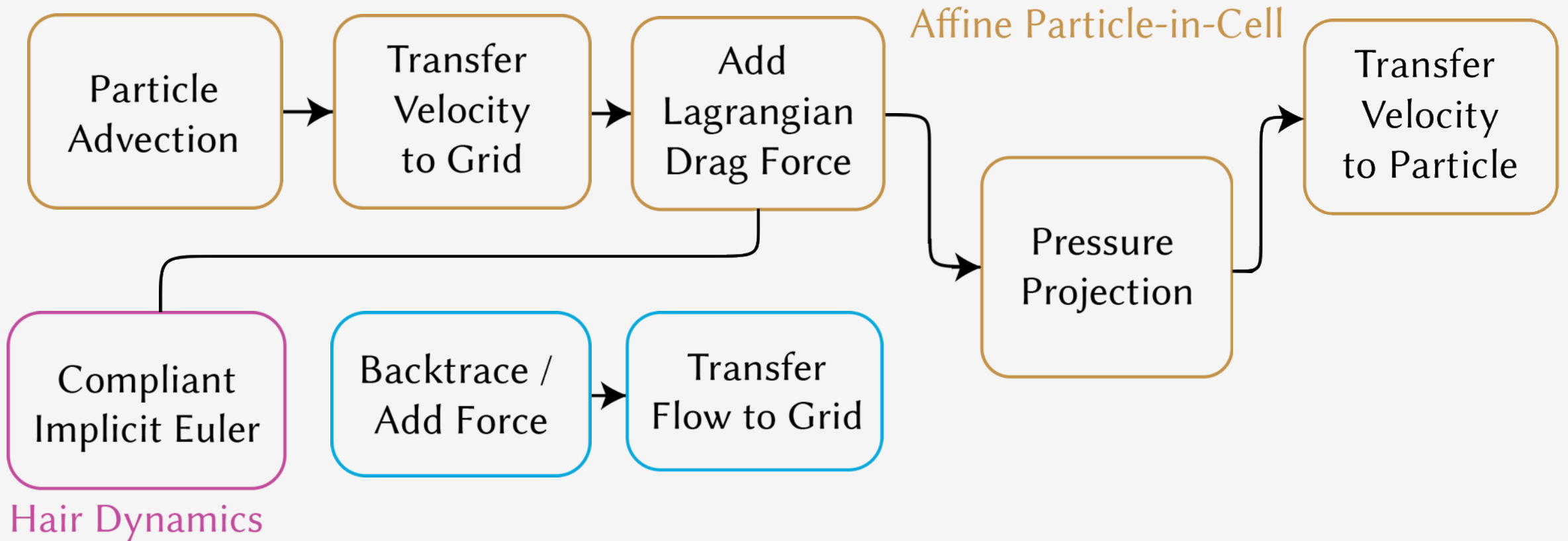
Comp
Implici

Hair Dy

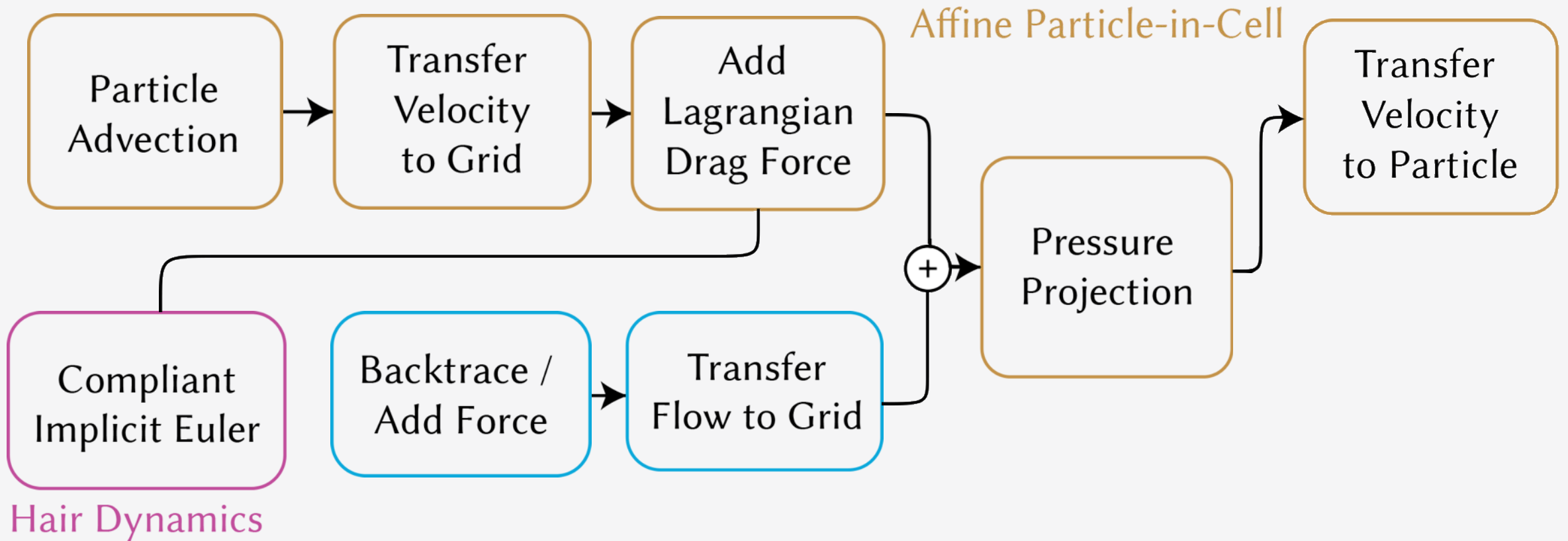
Pipeline



Pipeline



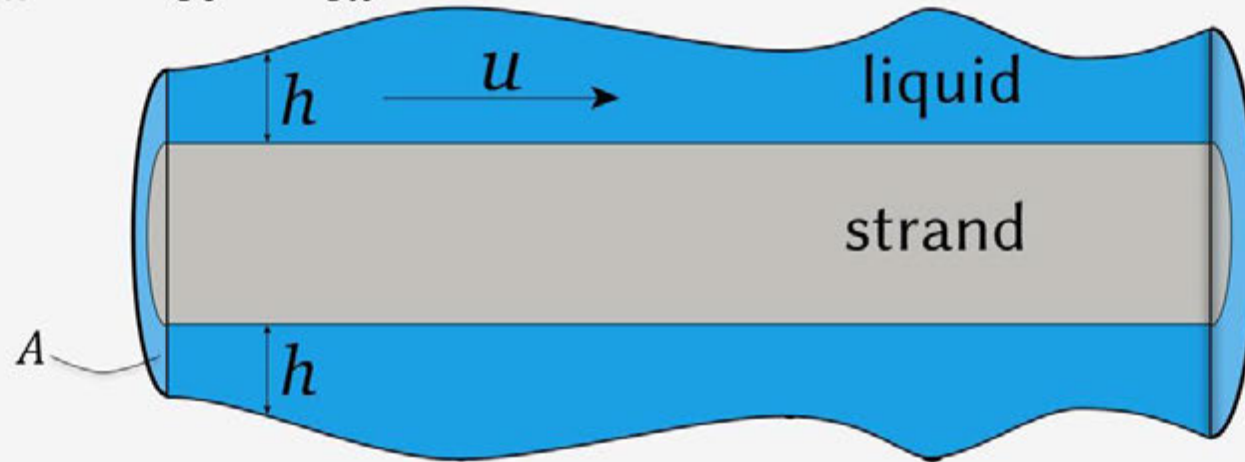
Pipeline



Reduced-Liquid Equation on Hair

Continuity Equation for Mass

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = 0 \quad \frac{\partial A}{\partial t} + u \frac{\partial A}{\partial x} = 0$$



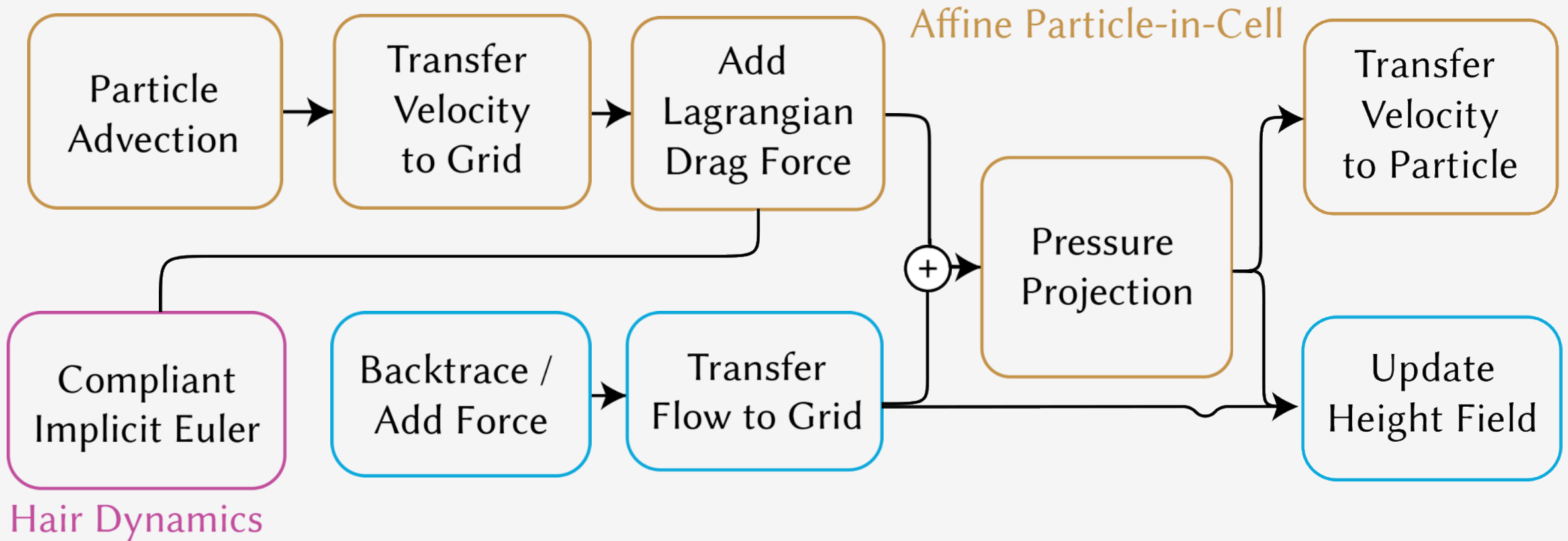
Part
Adve

Transfer
Particle

Comp
Implici

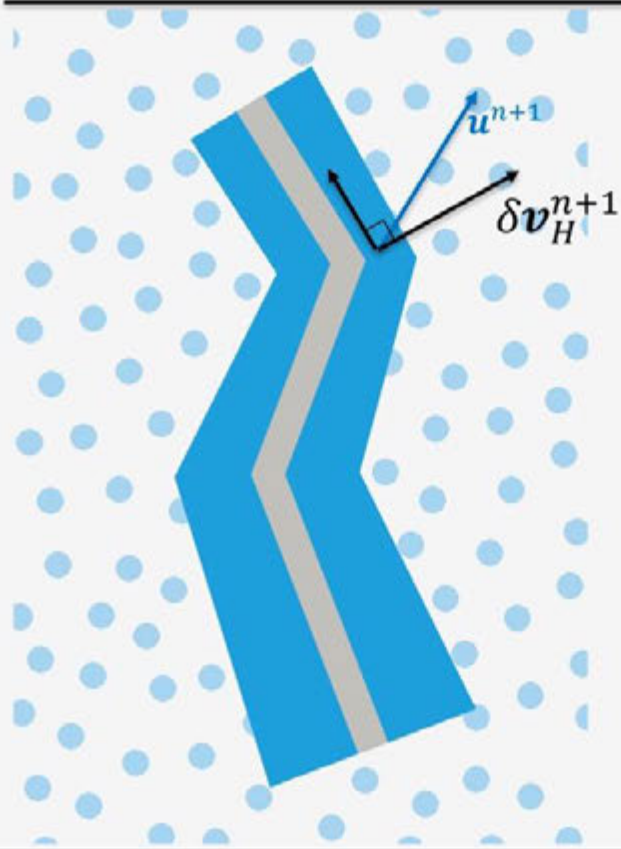
Hair Dyn

Pipeline



Pipe

Capturing Reduced-Liquid



New Volume \leftarrow Original + Gathered

New Momentum \leftarrow Hair + Reduced + Gathered

New Mass \leftarrow Original + Gathered

New Velocity \leftarrow Original + Gathered

Part
Adve

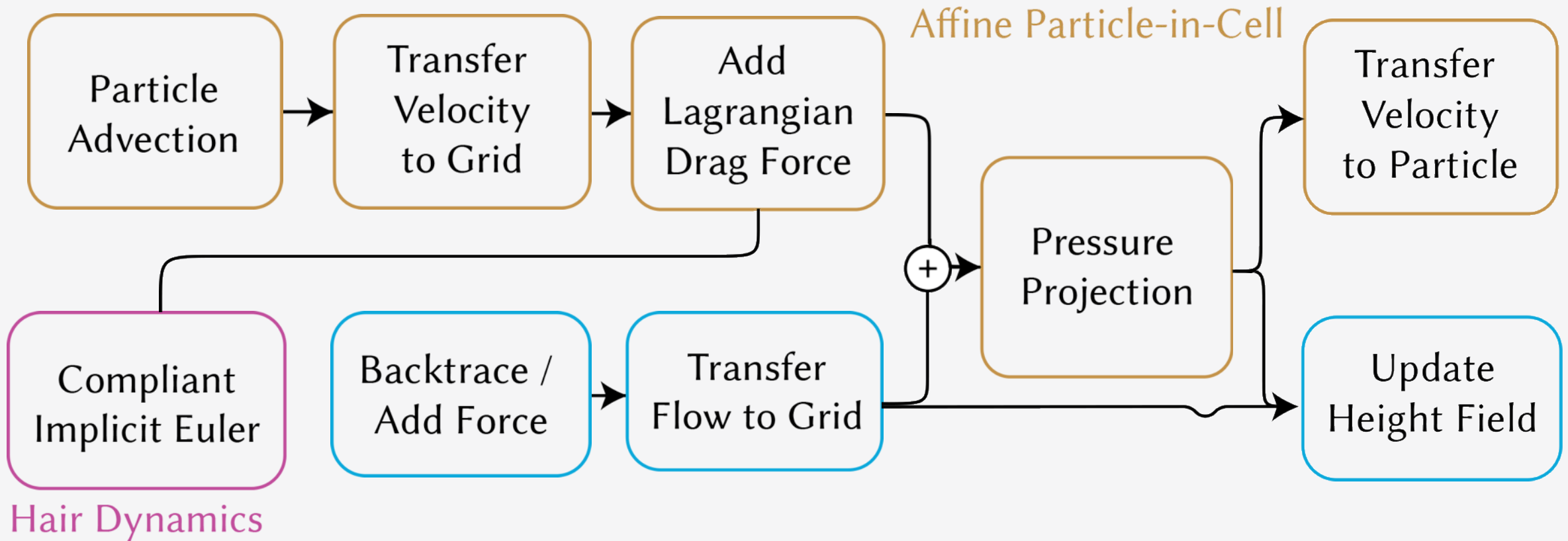
fer
city
article

Comp
Implici

ate
Field

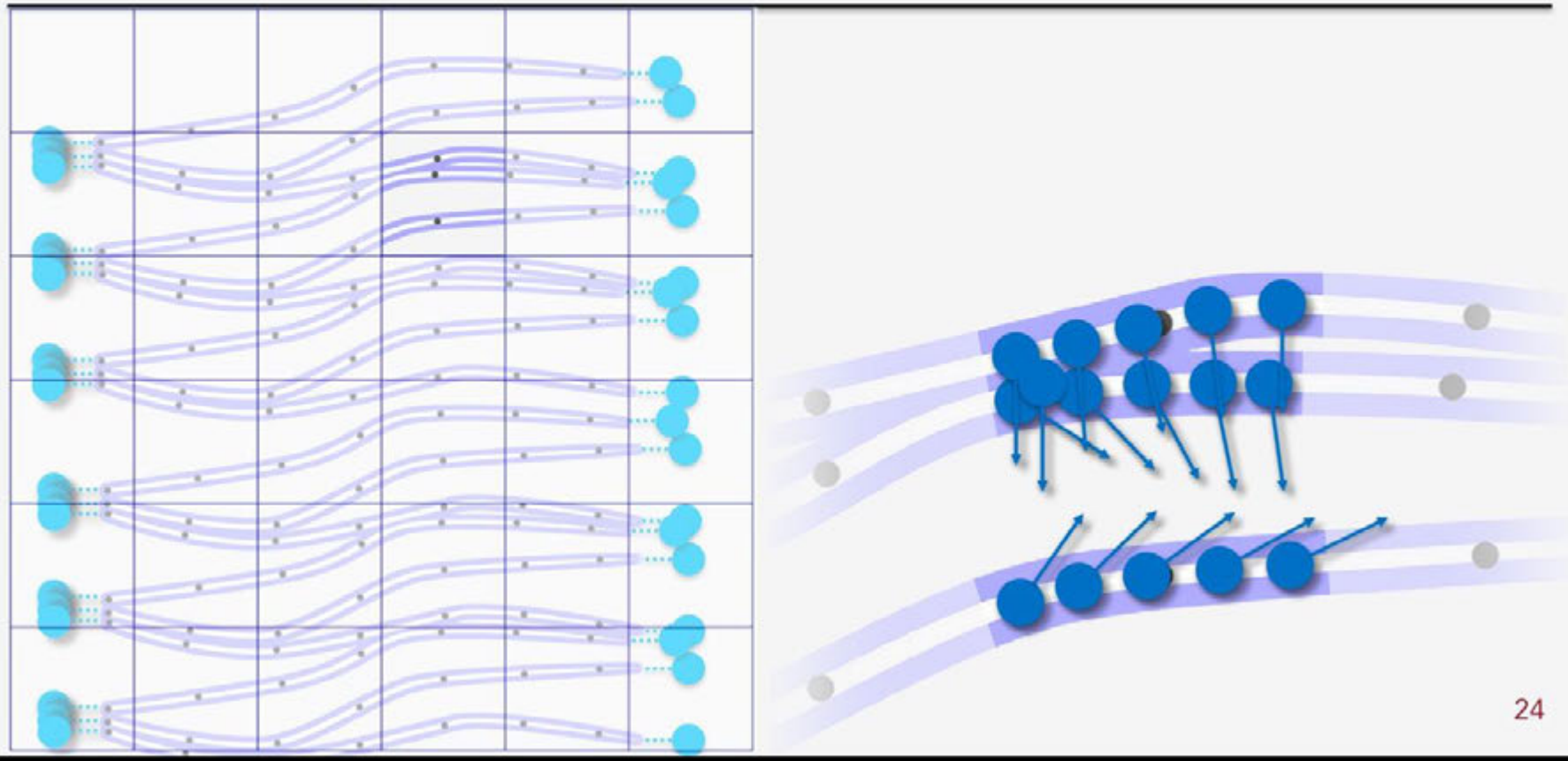
Hair Dyn

Pipeline



Pipe

Dripping of Reduced-Liquid



Par
Adve

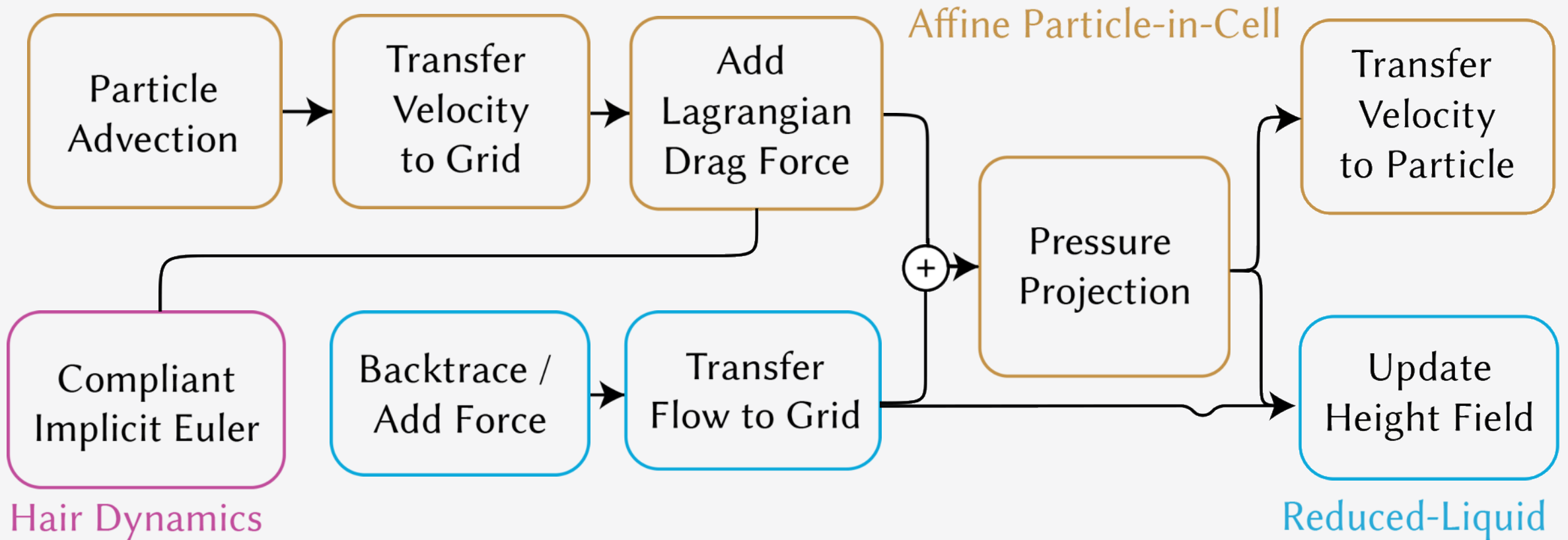
fer
city
particle

Comp
Implici

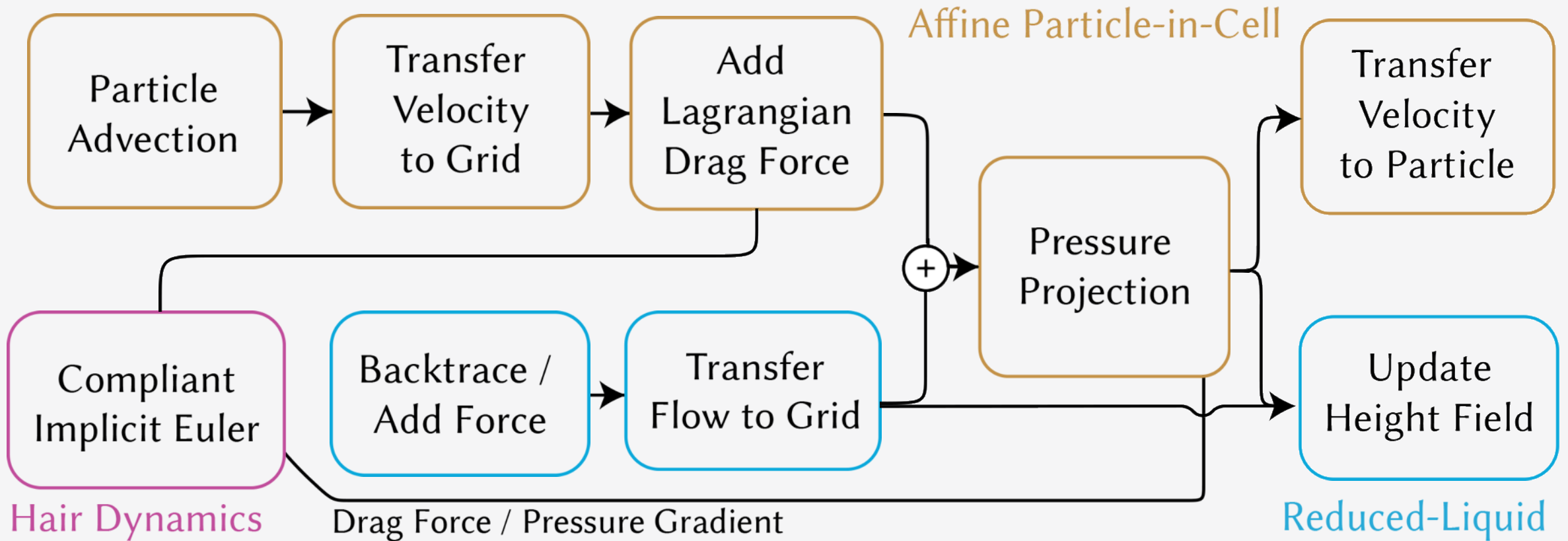
ate
Field

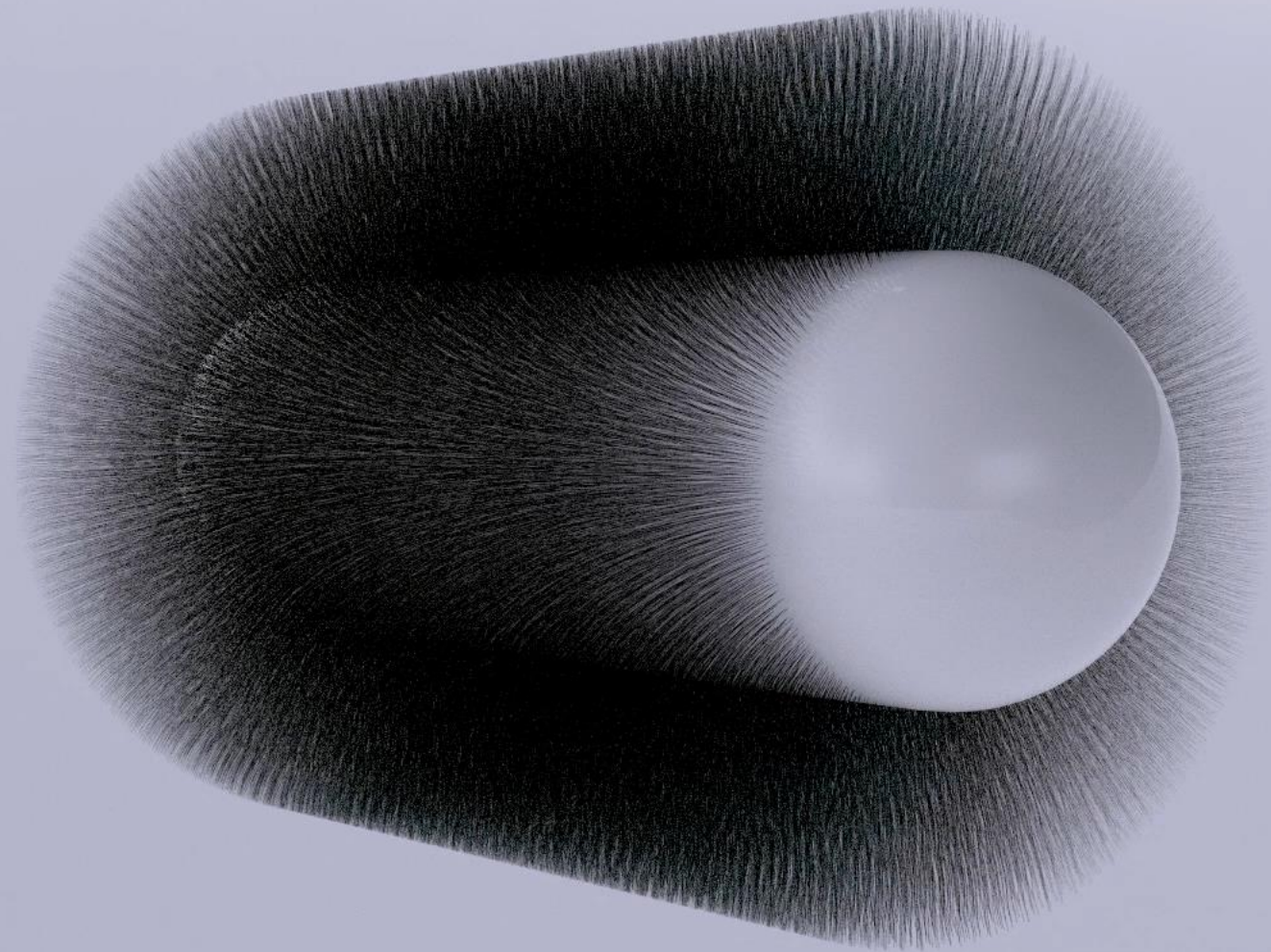
Hair Dy

Pipeline



Pipeline





Timing

Hairs

32,768 strands
63 DoFs / strand

Grid Res

128x128x256

Time Step

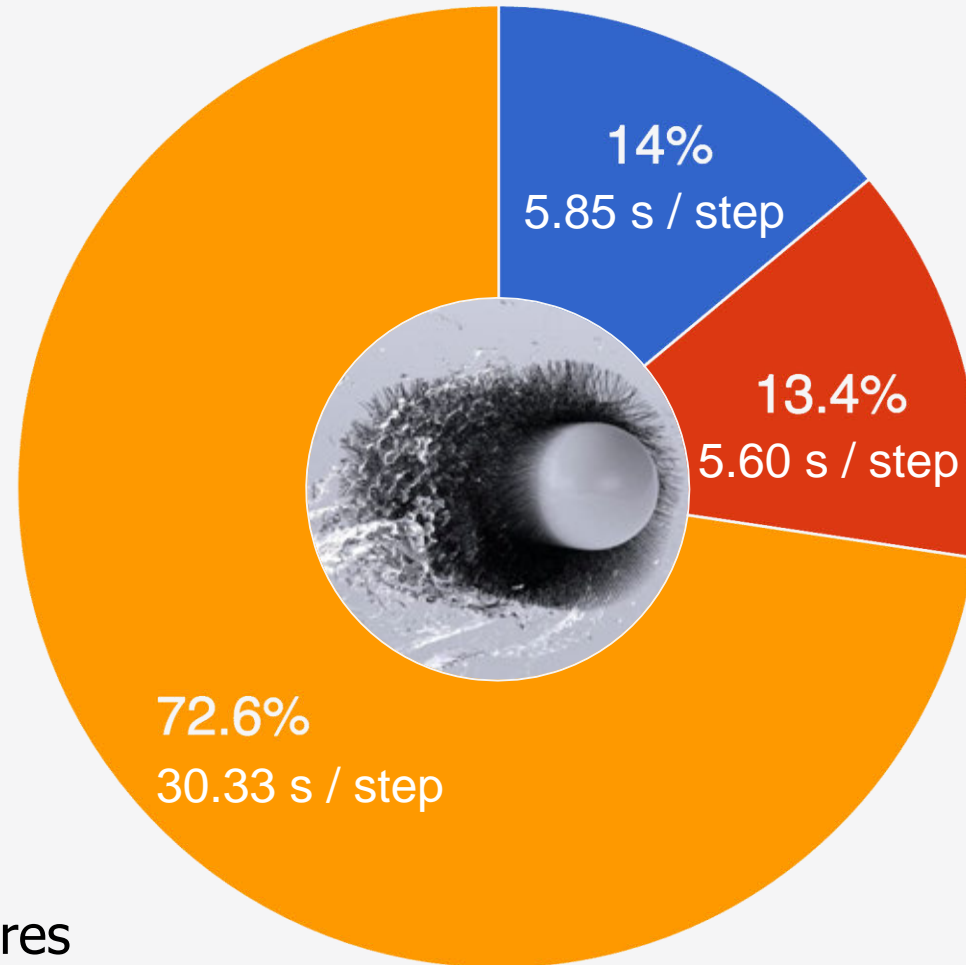
0.003 s

Computational

Time / Step

41.78 s

Xeon E5-2687W x 32 cores



- APIC
- Reduced-Liquid
- Hair Dynamics

Future Work

Quantitative validation w. r. t. real world.

More effective surface reconstruction.

Extension for hair skinning.

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Surface Reconstruction / Rendering: Houdini
Tree Structure Experiment

Bico, José, et al. "Adhesion: elastocapillary coalescence in wet hair." *Nature*, 432.7018 (2004): 690-690.

Executables & Code

<http://libwethair.info>