Design and Volume Optimization of Space Structures

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Outline

- Introduction
- Related work
- Overview
- Optimization framework
- Results
- Conclusion and future work

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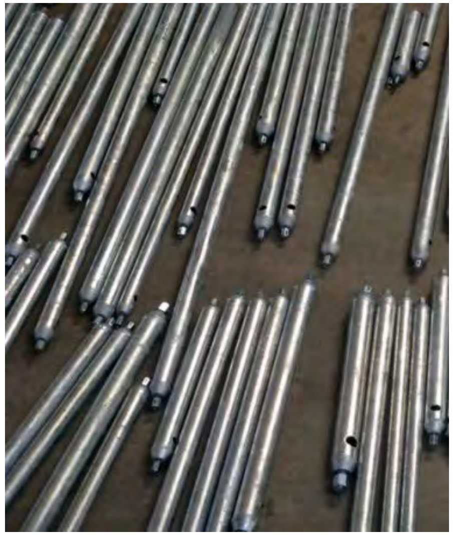
Space structures

- A **space frame** or **space structure** is a truss-like, lightweight rigid structure constructed from interlocking struts in a geometric pattern. (From Wikipedia)

nodes



struts (or bars)

The Heydar Aliyev Cultural Center
Designed by Zaha Hadid

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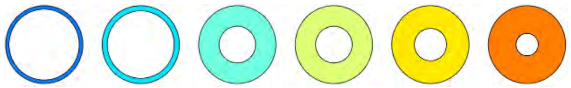
Designed by Zaha Hadid


Engineering goals in space structure design

- statically sound
- aesthetically pleasing
- approximating reference surfaces
- with minimized construction cost
  - minimizing the total volume of material used for beams
  - limited types of cross section areas 

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Structural design for the virtual worlds

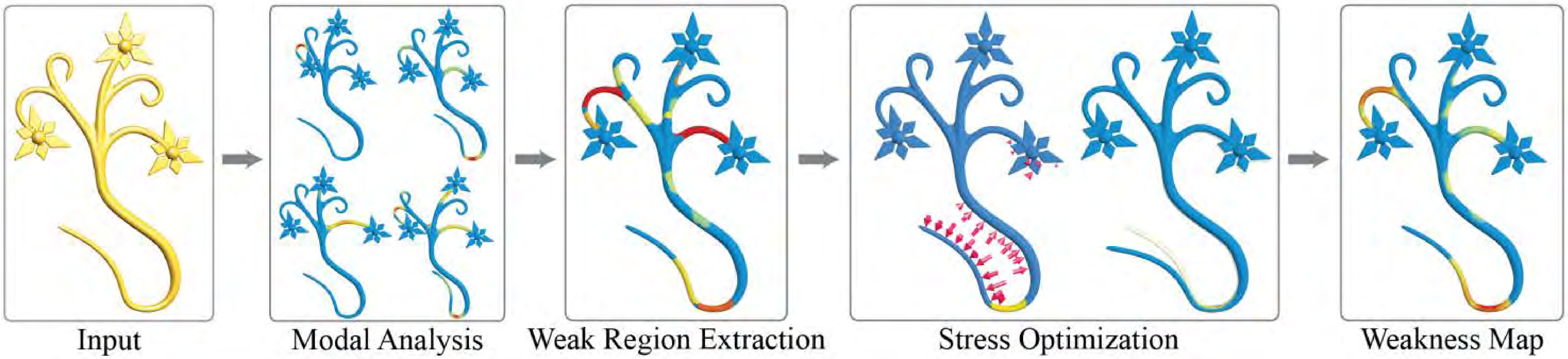
Smith, J., Hodgins, J., Oppenheim, I. and Witkin, A., 2002. **Creating models of truss structures with optimization**. *ACM Trans. Graph*.

Static-aware design

Umetani, N., Igarashi, T. and Mitra, N.J., 2012. **Guided exploration of physically valid shapes for furniture design.** *ACM Trans. Graph.*

Martínez, J., Dumas, J., Lefebvre, S. and Wei, L.Y., 2015. **Structure and appearance optimization for controllable shape design.** *ACM Trans. Graph.*

Static-aware 3D printing



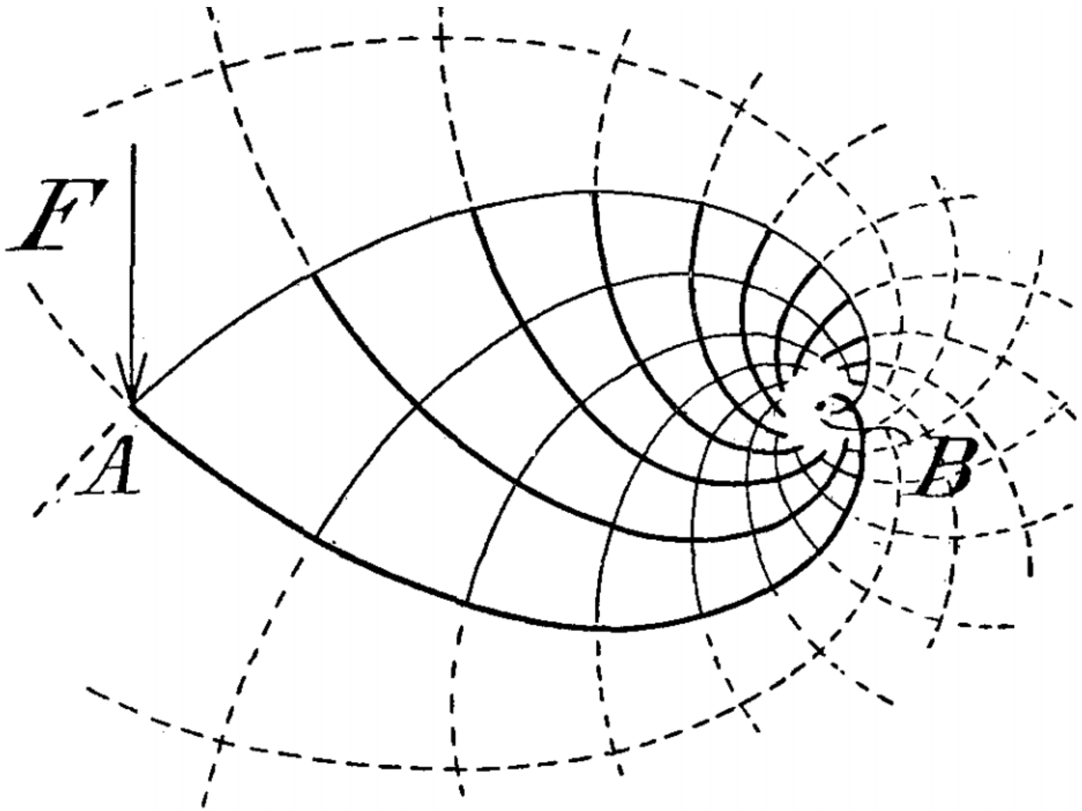
Zhou, Q., Panetta, J. and Zorin, D., 2013. **Worst-case structural analysis.** *ACM Trans. Graph*.

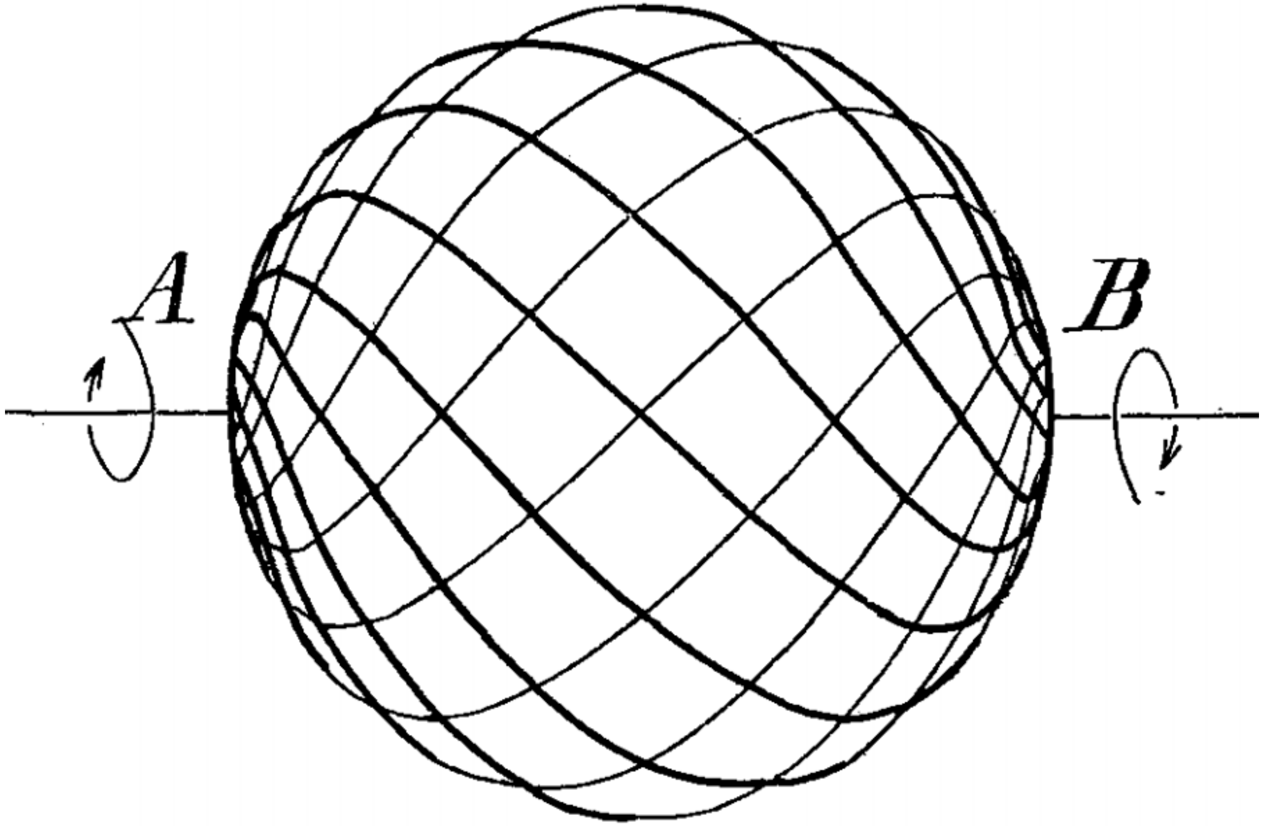
Static-aware 3D printing

(a) (b) (c) (d)

Wang, W., Wang, T.Y., Yang, Z., Liu, L., Tong, X., Tong, W., Deng, J., Chen, F. and Liu, X., 2013. **Cost-effective printing of 3D objects with skin-frame structures.** *ACM Trans. Graph.*

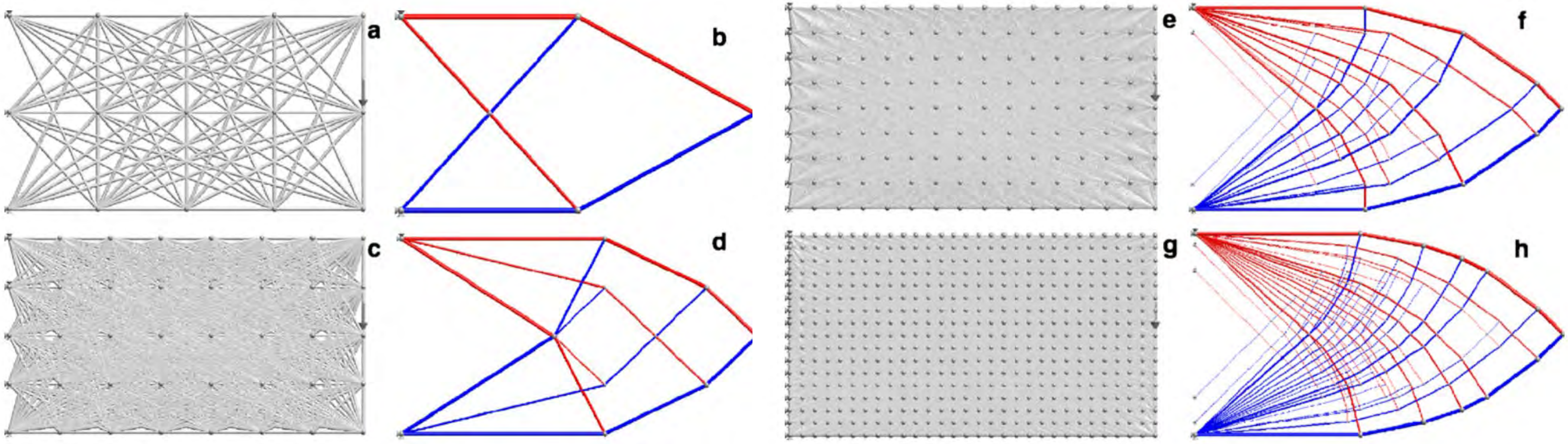
Related work: optimal trusses





Michell, A.G.M., 1904. LVIII. **The limits of economy of material in frame-structures**. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, *8*(47).

Related work: the ground structure method



Martinez, P., Marti, P. and Querin, O.M., 2007. **Growth method for size, topology, and geometry optimization of truss structures**. *Structural and Multidisciplinary Optimization*, *33*(1).

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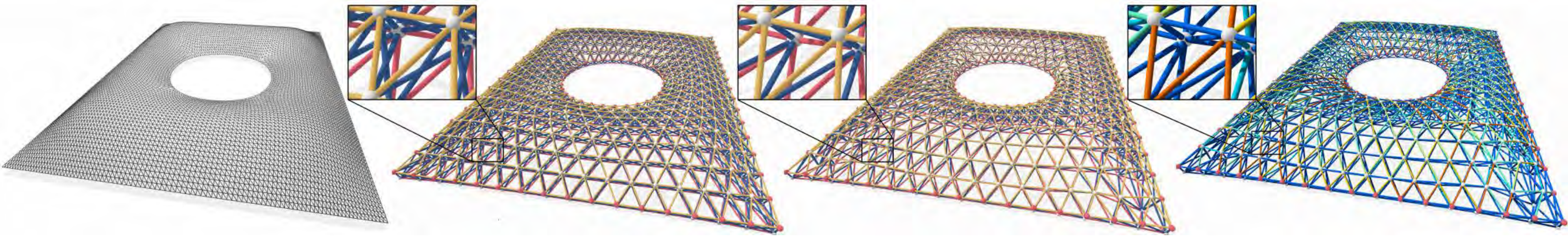
Framework overview



Input reference surface

Initial structure

Joint optimization of node positions and connectivity

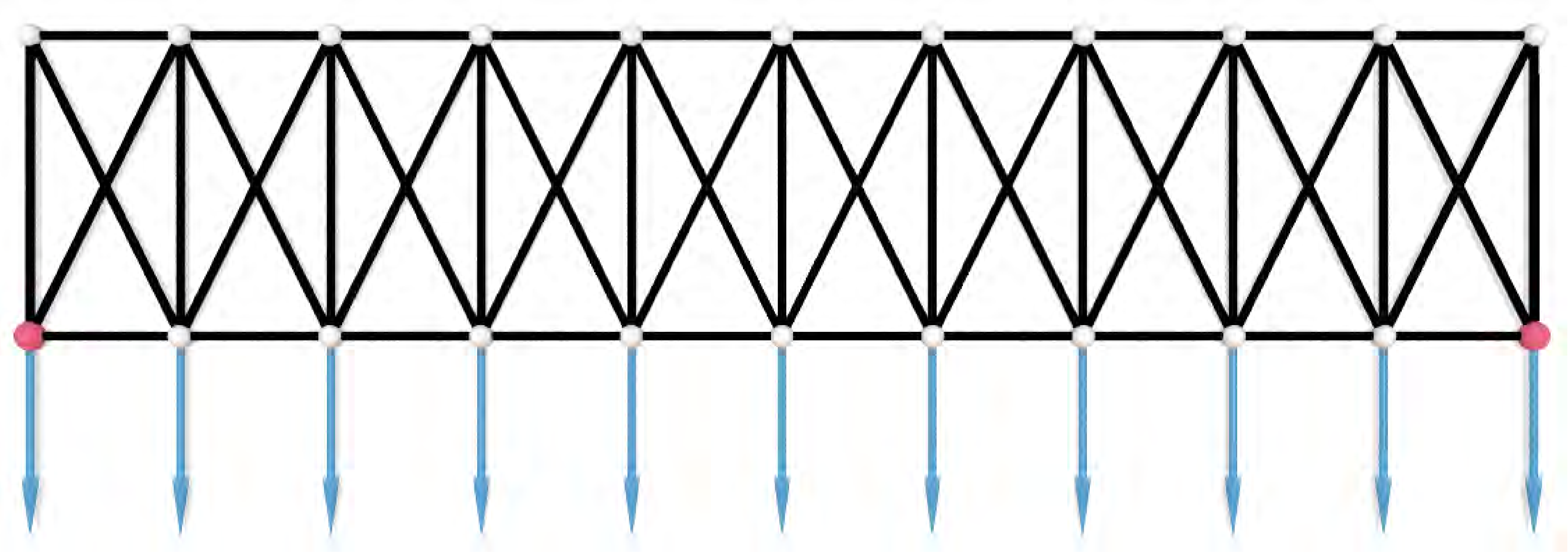
Discrete optimization of beam cross sections

1

2

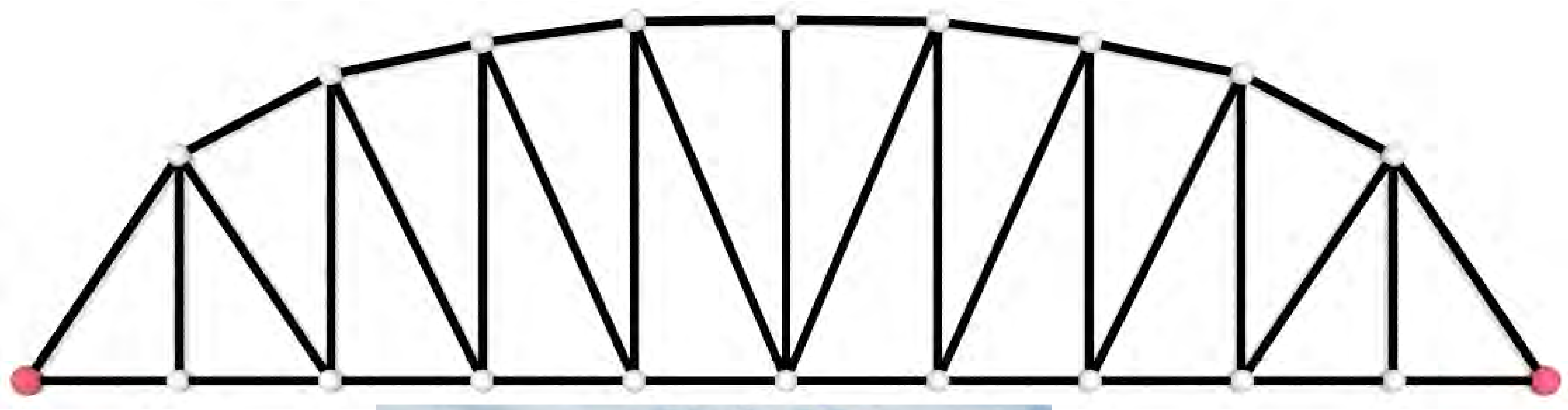
3

2D illustration: initial structure



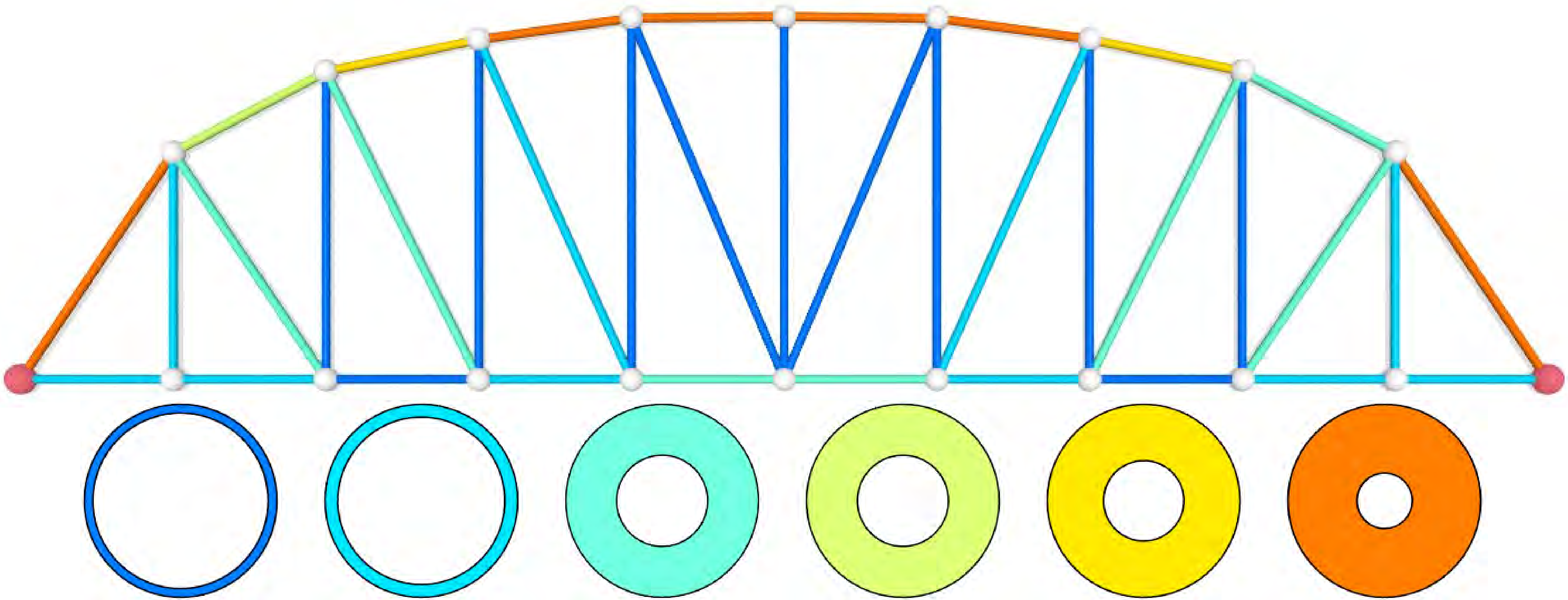
An initial over-complete 2D bridge configuration with uniform loads

2D illustration: joint optimization of connectivity and node positions



bridge design after joint optimization of node positions and connectivity

2D illustration: discrete optimization



Discrete optimization assigns customized beam cross section types to minimize material usage while ensuring static equilibrium.

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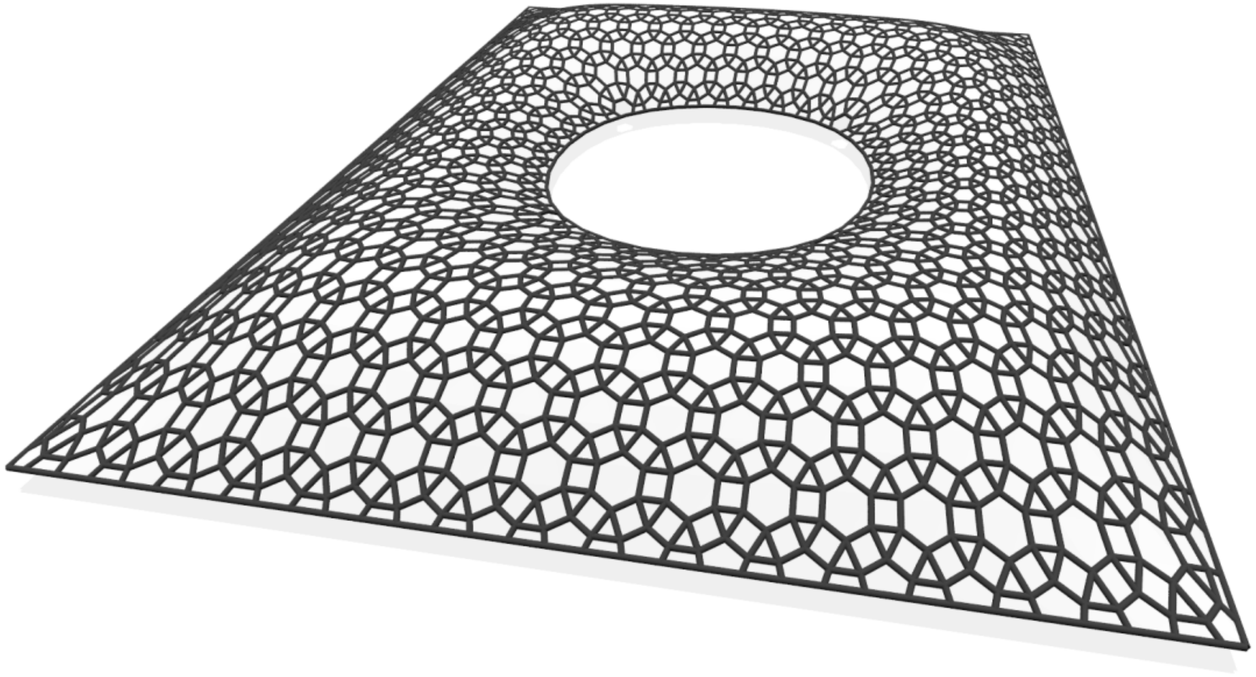
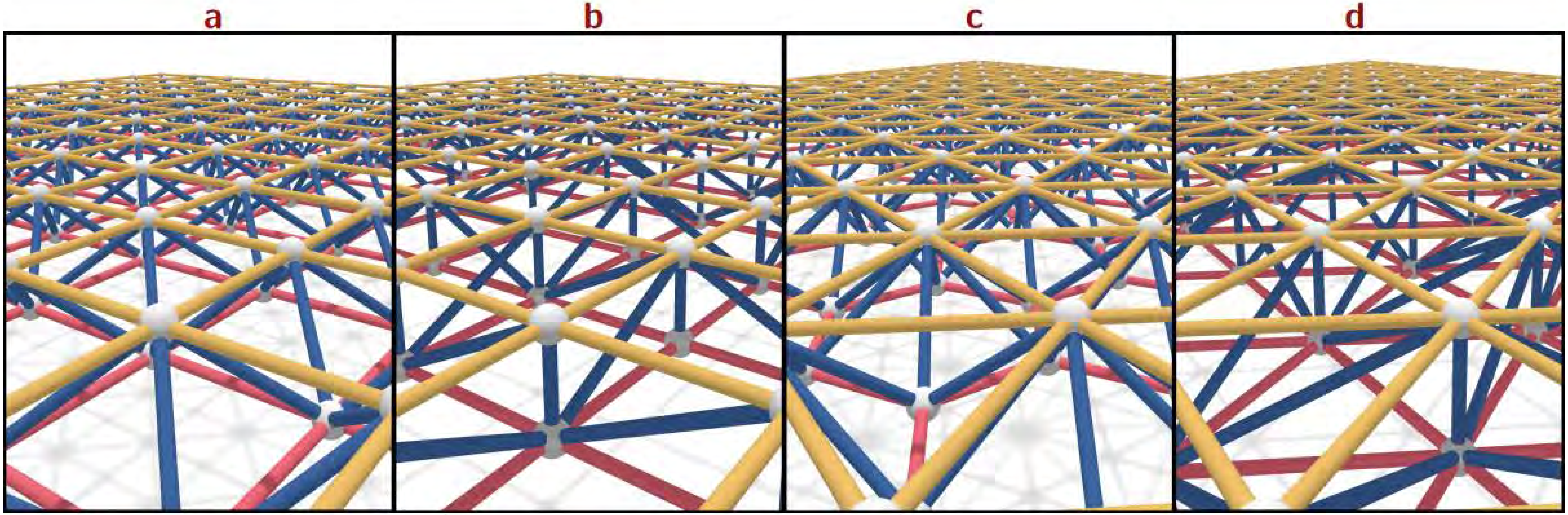
Connectivity initialization

- Coarse-mesh generation



Connectivity initialization

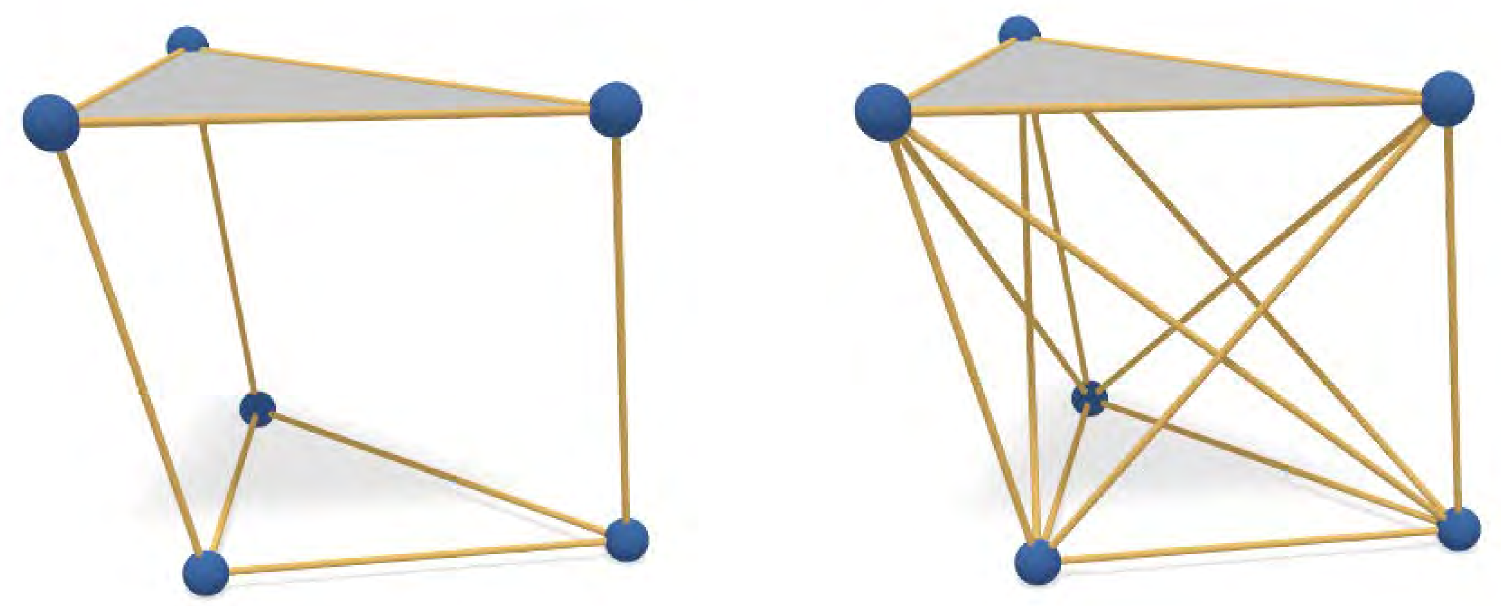
- Double-layer structures



Derivation of double-layer space structures based on offsetting a quad mesh with (a) and without (b) dualization and a triangle mesh with (c) or without (d) dualization.

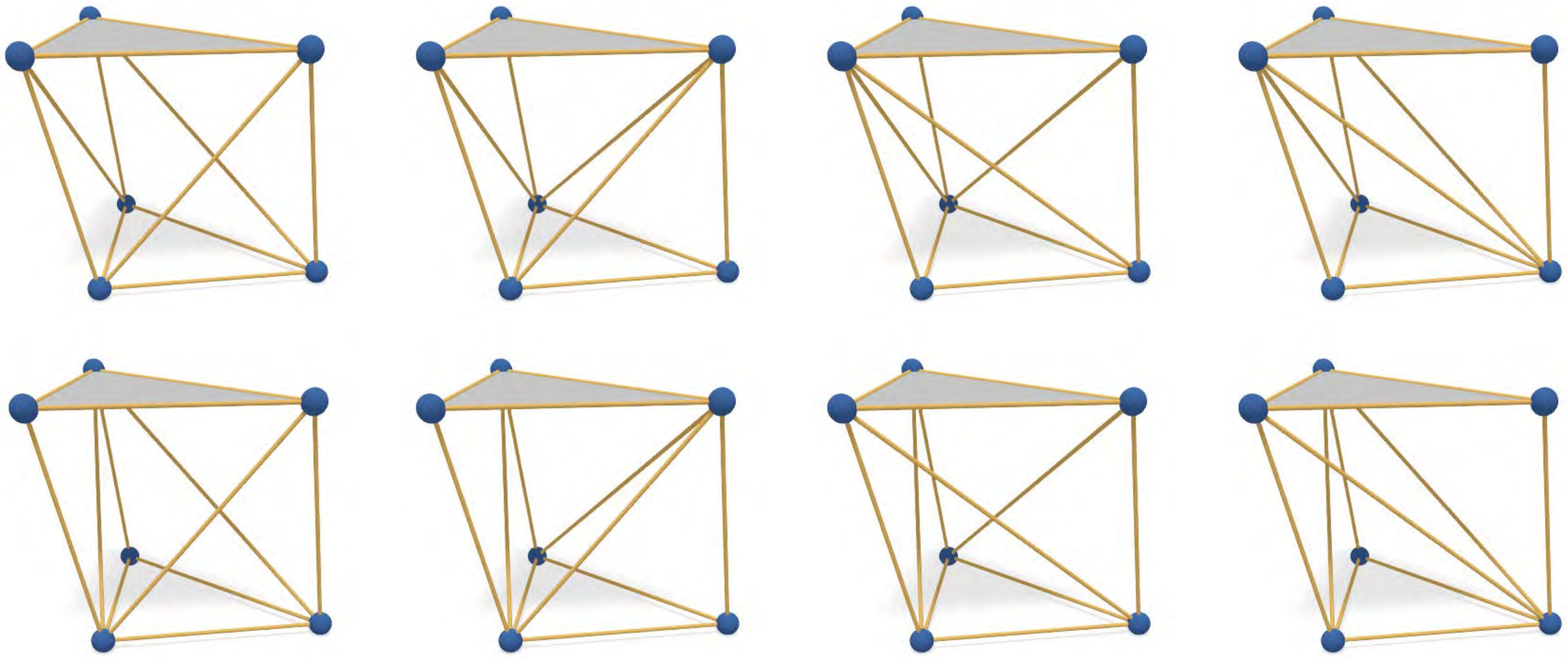
Connectivity initialization

- Adding additional beams

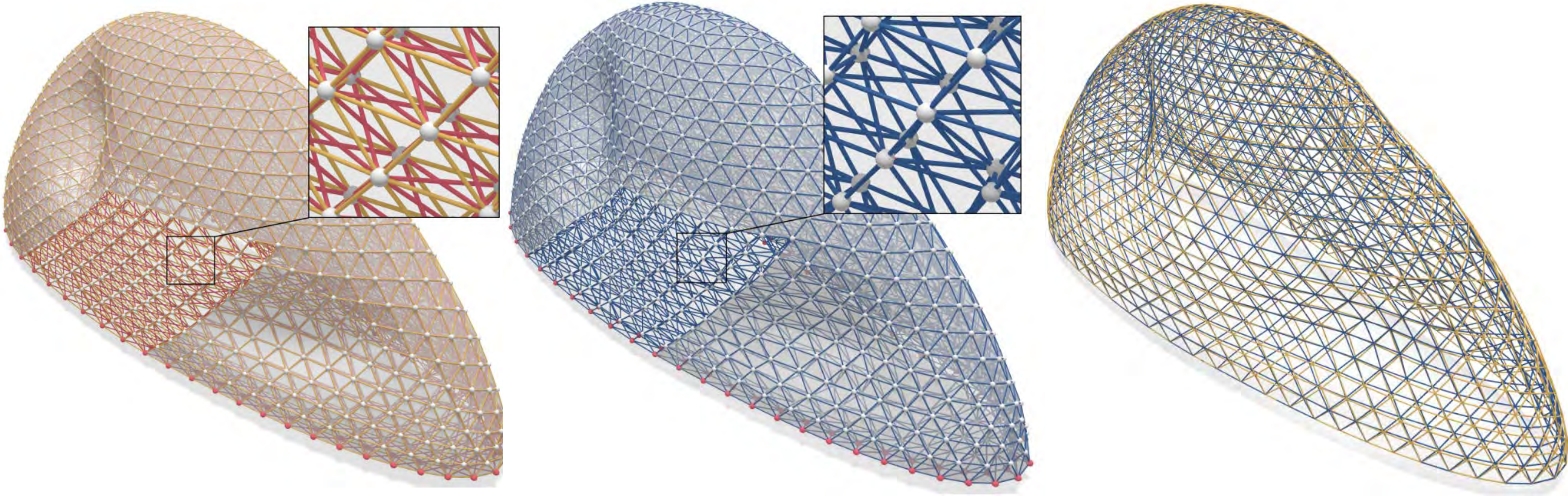


Connectivity initialization

- Adding additional beams: possible solutions after beams are removed

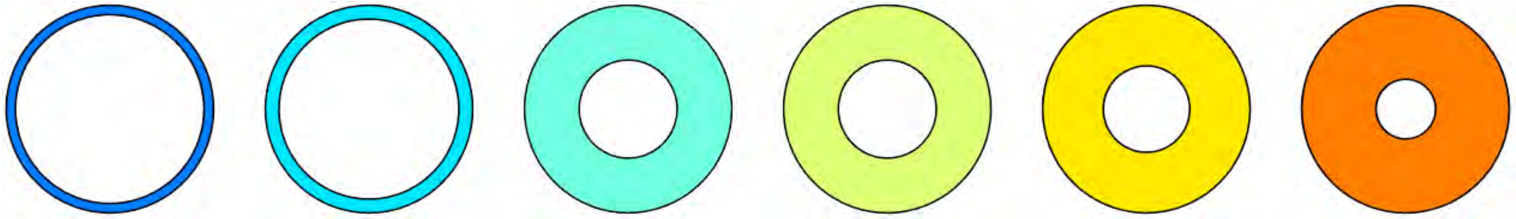


Joint optimization: overview



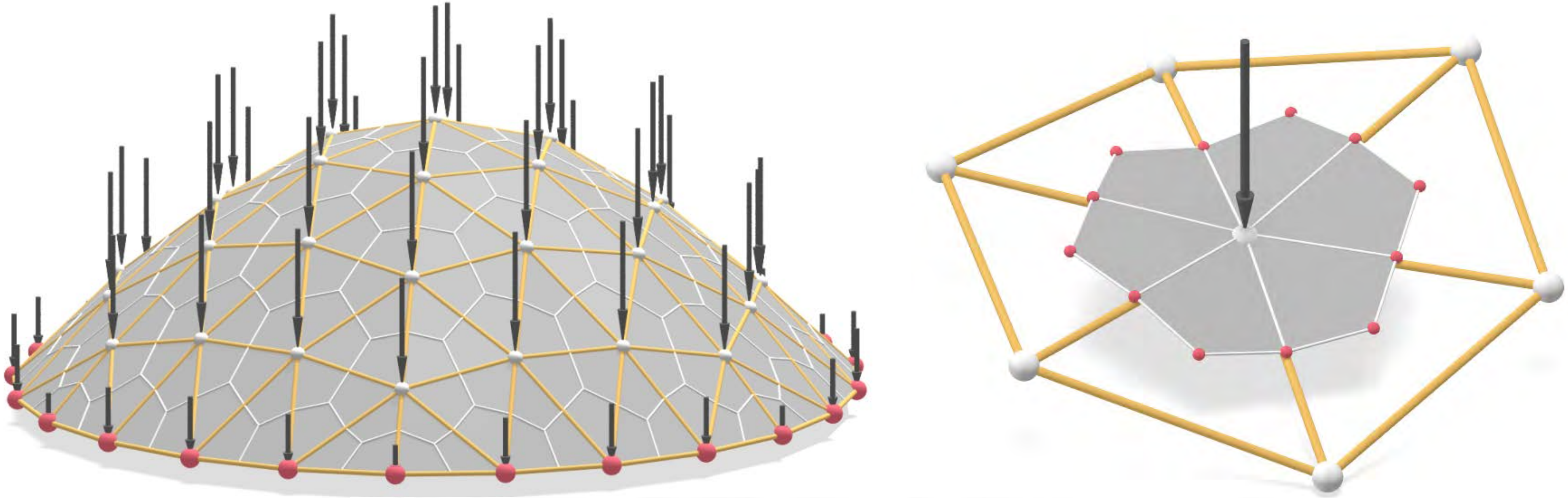
Joint optimization: overview

- A nonlinear continuous optimization
- Variables:
  - node positions
  - internal forces
- Assumptions
  - axial forces only
  - force limits in proportion to cross-section areas



Joint optimization: force initialization

- Internal force initialization



$$\sum_{j:\ \{i,j\}\in E} w_{ij}(\mathbf{v}_j - \mathbf{v}_i) = -\mathbf{l}_i,\ i = 1, \ldots, |V|.$$

w_{ij} the axial forces per unit length defined on each beam

Joint optimization: energy terms

- Static equilibrium

$$E_{static} = \sum_{i \in V} \left(\sum_{j:\ \{i,j\} \in E} w_{ij}(\mathbf{v}_j - \mathbf{v}_i) + \mathbf{l}_i \right)^2 .$$

- Closeness

$$E_{close} = \sum_{\mathbf{v}_i \in \overline{V}} ((\mathbf{v}_i - \mathbf{v}_i^*) \cdot \mathbf{n}_i^*)^2 .$$

- Total volume

$$E_{volume} = \sum_{i,j:\ \{i,j\} \in E} |w_{ij}| \cdot ||\mathbf{v}_j - \mathbf{v}_i||^2 .$$

- Combinatorial validity

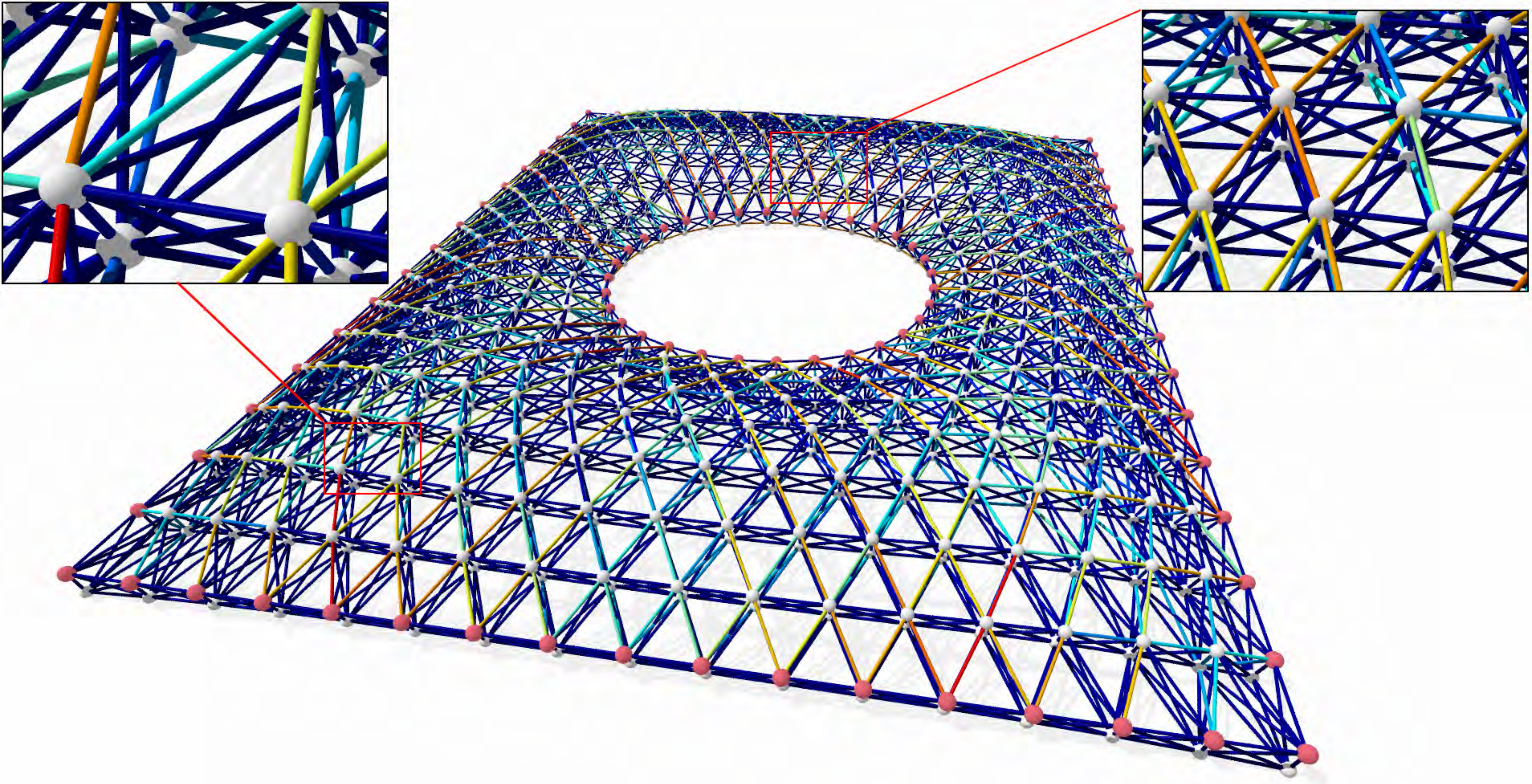
$$E_{comb.} = \sum_{i,j:\ \{i,j\} \in Ex} |w_i \cdot w_j| .$$

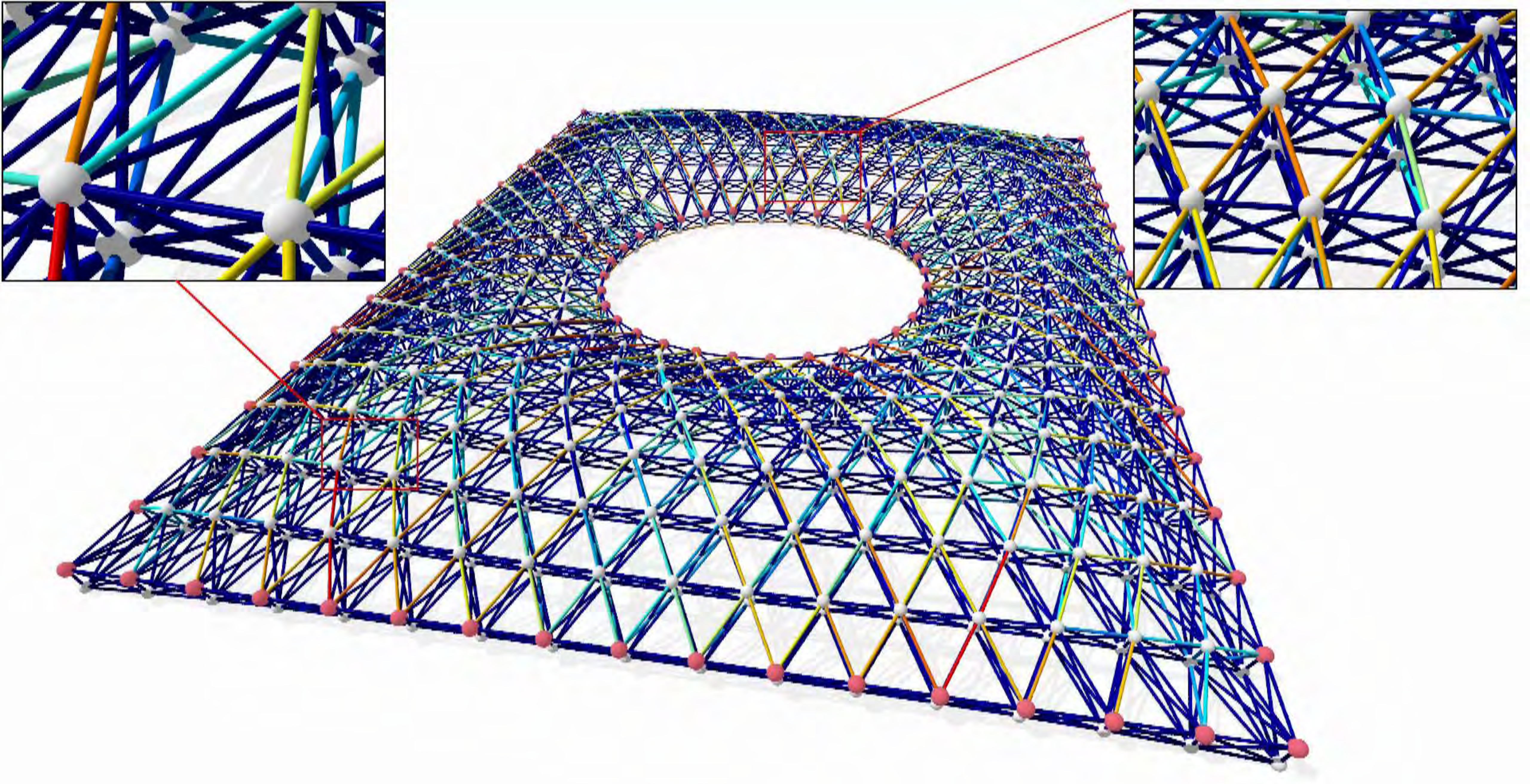
Joint optimization: energy terms

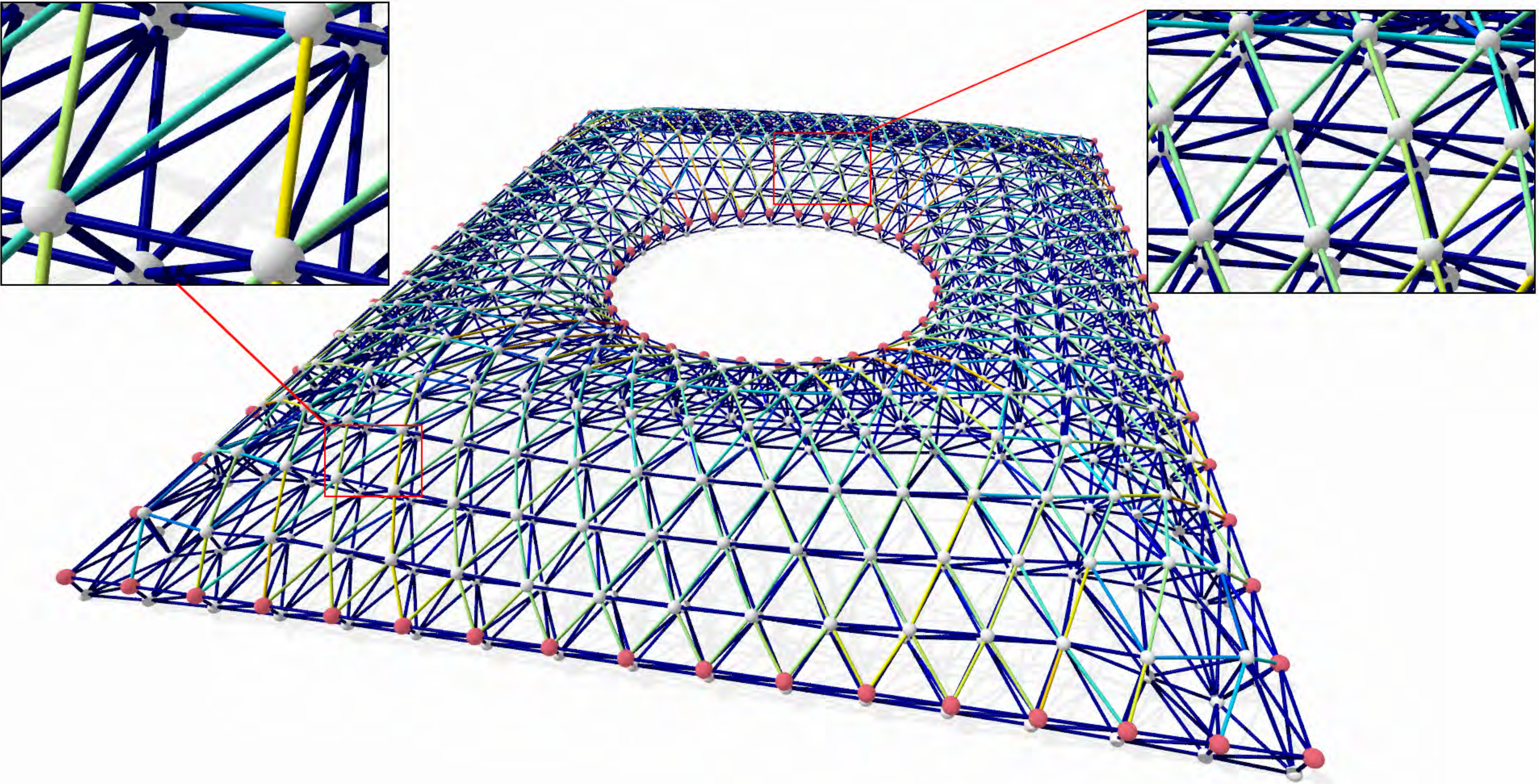
$$E_{sum} = \lambda_{static} E_{static} + \lambda_{close} E_{close} + \lambda_{volume} E_{volume} + \lambda_{comb} E_{comb} + \lambda_{reg} E_{reg}$$

higher weight (red): λ_{static}, λ_{close}, λ_{comb}

lower weight (blue): λ_{volume}, λ_{reg}







Discrete optimization: problem formulation

- Mixed integer programming formulation

$$\underset{x_{ij},\bar{a}_j,s_i}{\text{minimize}} \quad \sum_i l_i \sum_{j=1}^{k} \bar{a}_j x_{ij}$$

Total volume

$$\text{subject to} \quad B^T \mathbf{s} = -\mathbf{f}$$

Force equilibrium

$$\sum_{j=1}^{k} x_{ij}\bar{a}_j + s_i \geq 0; \qquad i = 1, \ldots, |E|$$

$$\sum_{j=1}^{k} x_{ij}\bar{a}_j - s_i \geq 0; \qquad i = 1, \ldots, |E|$$

$$\sum_{j=1}^{k} x_{ij} \leq 1; \qquad i = 1, \ldots, |E|$$

$$x_{ij} \in \{0, 1\}; \quad j = 1, \ldots, k, \quad i = 1, \ldots, |E|.$$

l_i lengths of the beams

$\bar{a}_j, \quad j = 1, \ldots, k$

k types of customized cross-section areas

$s_i, \quad i = 1, \ldots, |E|$

axial forces of beams

$x_{ij}, \quad j = 1, \ldots, k$

assignment of types for beams

Discrete optimization: sub-problems

- Sp-1: Fix $\overline{a}_j$, and solve for s_j and x_{ij}.
- Sp-2: Fix s_j, and solve for $\overline{a}_j$ and x_{ij}.
- Sp-3: Fix x_{ij}, and solve for s_j and $\overline{a}_j$.

$\overline{a}_j, \quad j = 1, \ldots, k$

k types of customized cross-section areas

$s_i, \quad i = 1, \ldots, |E|$

axial forces of beams

$x_{ij}, \quad j = 1, \ldots, k$

assignment of types for beams

Discrete optimization: sub-problem 1

- Sp-1: Fix $\overline{a}_j$, and solve for s_j and x_{ij}.

$$\begin{aligned}
\underset{x_{ij},\overline{a}_j,s_i}{\text{minimize}} \quad & \sum_i l_i \sum_{j=1}^{k} \overline{a}_j x_{ij} \\
\text{subject to} \quad & B^T \mathbf{s} = -\mathbf{f} \\
& \sum_{j=1}^{k} x_{ij}\overline{a}_j + s_i \geq 0; \qquad i = 1, \ldots, |E| \\
& \sum_{j=1}^{k} x_{ij}\overline{a}_j - s_i \geq 0; \qquad i = 1, \ldots, |E| \\
& \sum_{j=1}^{k} x_{ij} \leq 1; \qquad i = 1, \ldots, |E| \\
& x_{ij} \in \{0, 1\}; \quad j = 1, \ldots, k, \quad i = 1, \ldots, |E|.
\end{aligned}$$

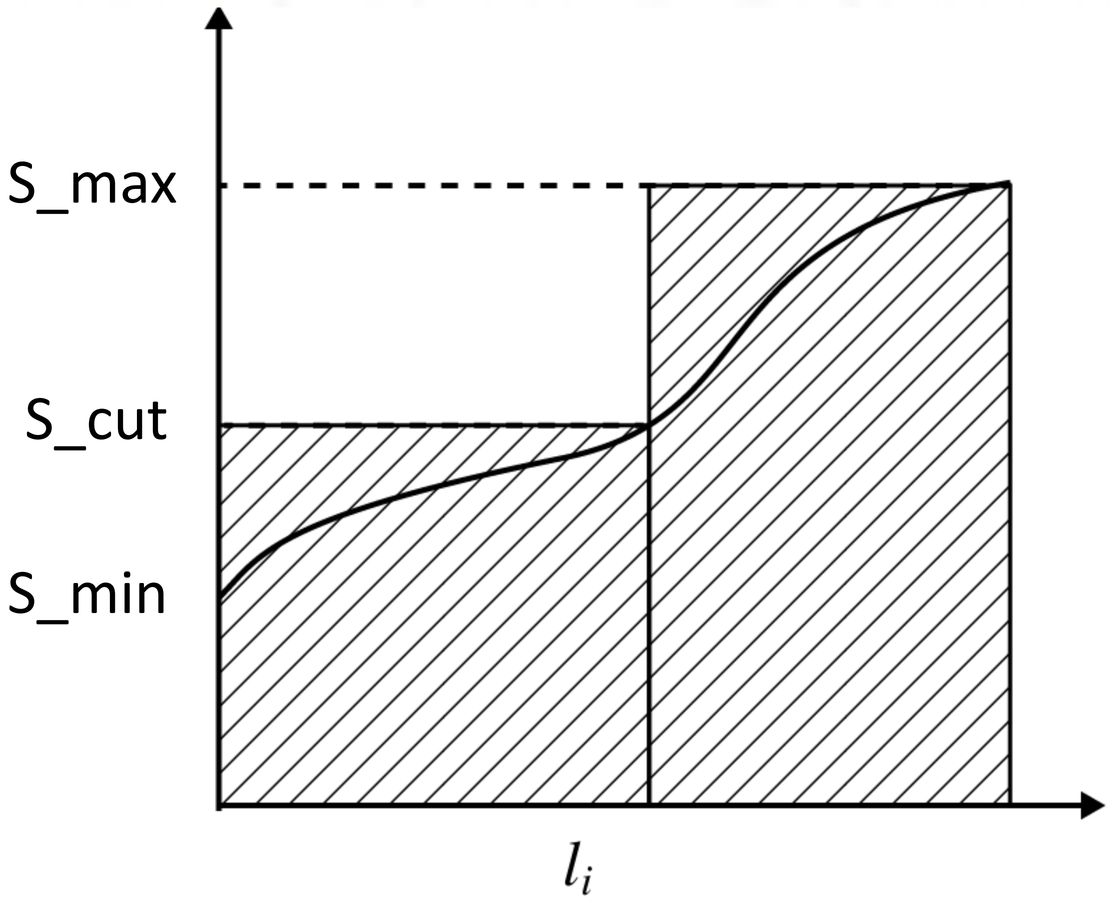
$$\begin{aligned}
\underset{a_i,s_i}{\text{minimize}} \quad & \sum_{i=1}^{|E|} l_i a_i \\
\text{subject to} \quad & B^T \mathbf{s} = -\mathbf{f} \\
& a_i + s_i \geq 0; \\
& a_i - s_i \geq 0; \\
& a_i \leq a_{max};
\end{aligned}$$

$\max(\overline{a}_j, j = 1, \ldots, k)$

The ground structure method

Discrete optimization: sub-problem 2

- Sp-2: Fix s_j, and solve for $\overline{a}_j$ and x_{ij}.

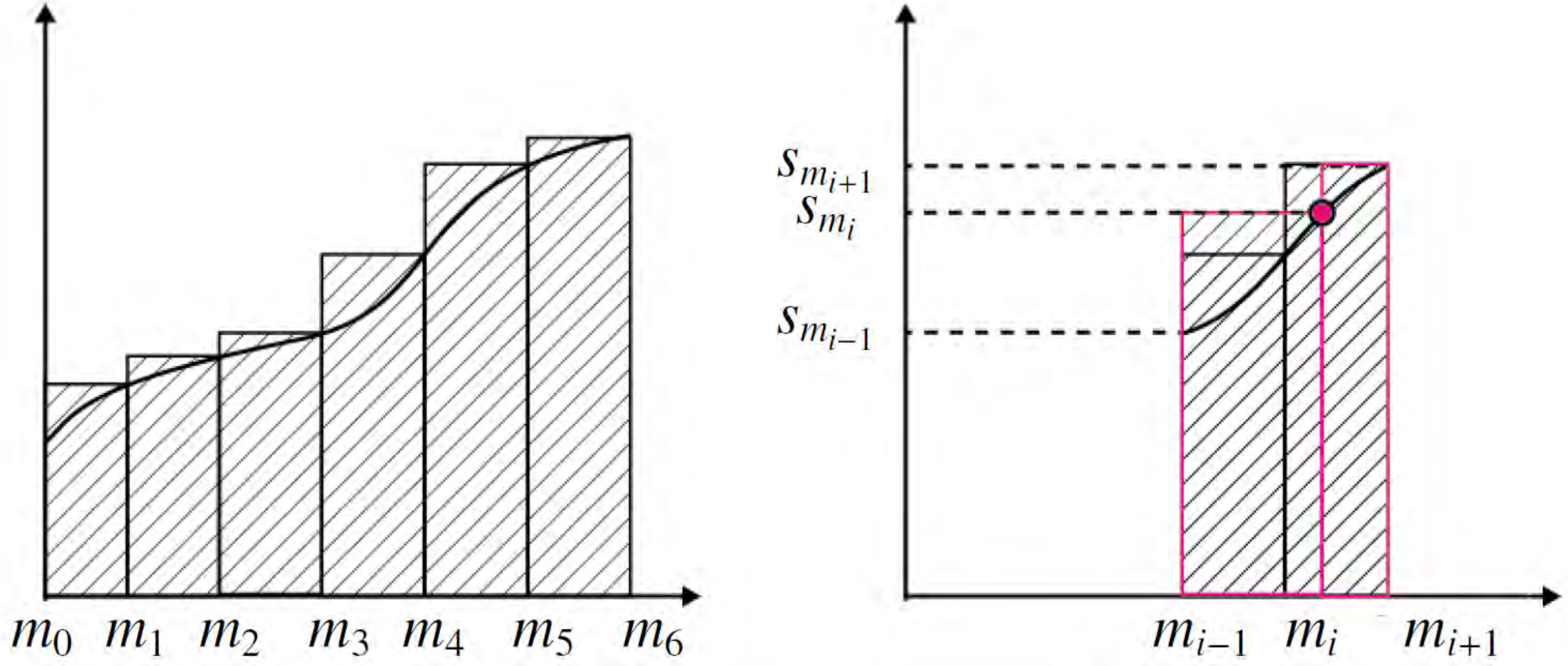


X: accumulation of edge lengths

Y: absolute values of axial forces

Discrete optimization: sub-problem 2

- Sp-2: Fix s_j, and solve for $\overline{a}_j$ and x_{ij}.



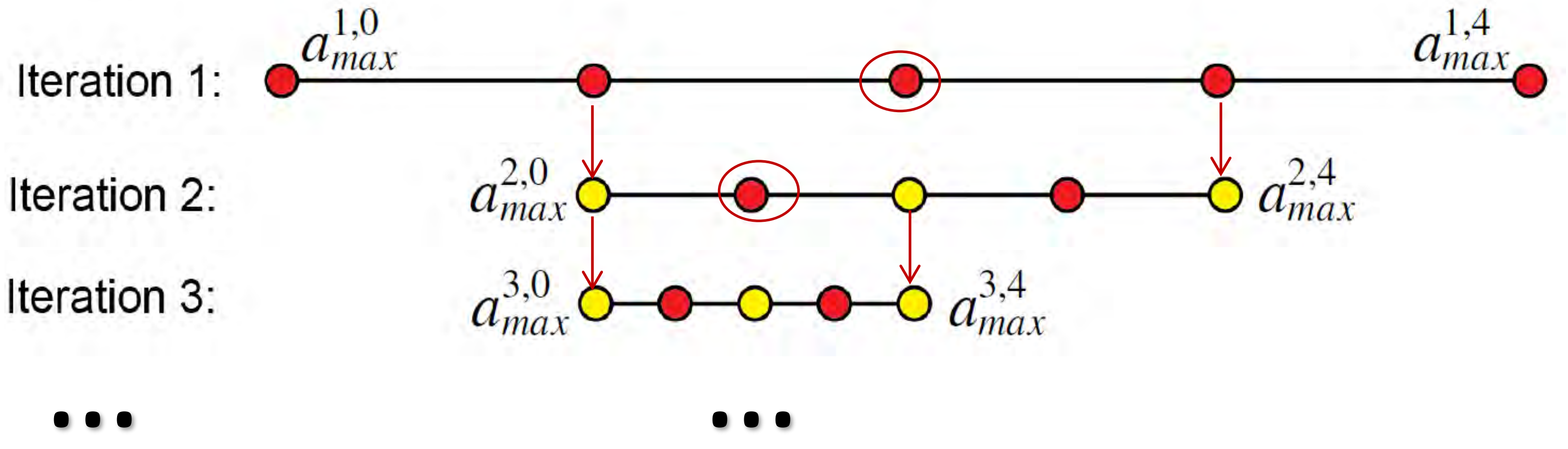
Discrete optimization: sub-problem 3

- Sp-3: Fix x_{ij}, and solve for s_j and $\overline{a}_j$.

$$
\begin{aligned}
\underset{s_i,\overline{a}_j}{\text{minimize}} \quad & \sum_{j=1}^{k}\left(\sum_{i\in\Phi_j} l_i\right)\overline{a}_j \\
\text{subject to} \quad & B^T\mathbf{s} = -\mathbf{f} \\
& \overline{a}_1 + s_i \geq 0,\ \overline{a}_1 - s_i \geq 0;\ i \in \Phi_1 \\
& \dots \\
& \overline{a}_k + s_i \geq 0,\ \overline{a}_k - s_i \geq 0;\ i \in \Phi_K
\end{aligned}
$$

Discrete optimization: overall algorithm

- First, as a preprocessing step, we determine the bounds of a_{max}, which could be used as input for Sp-1 (fix a_j)
- Next, as the main procedure, the algorithm alternates between Sp-1 (fix a_j) and Sp-2 (fix s_i) to find the optimal beam type assignment, x_{ij} .



- Finally, as a post-processing step, we use Sp-3 (fix x_{ij}) to further adjust the cross-section areas of each type, a_j.

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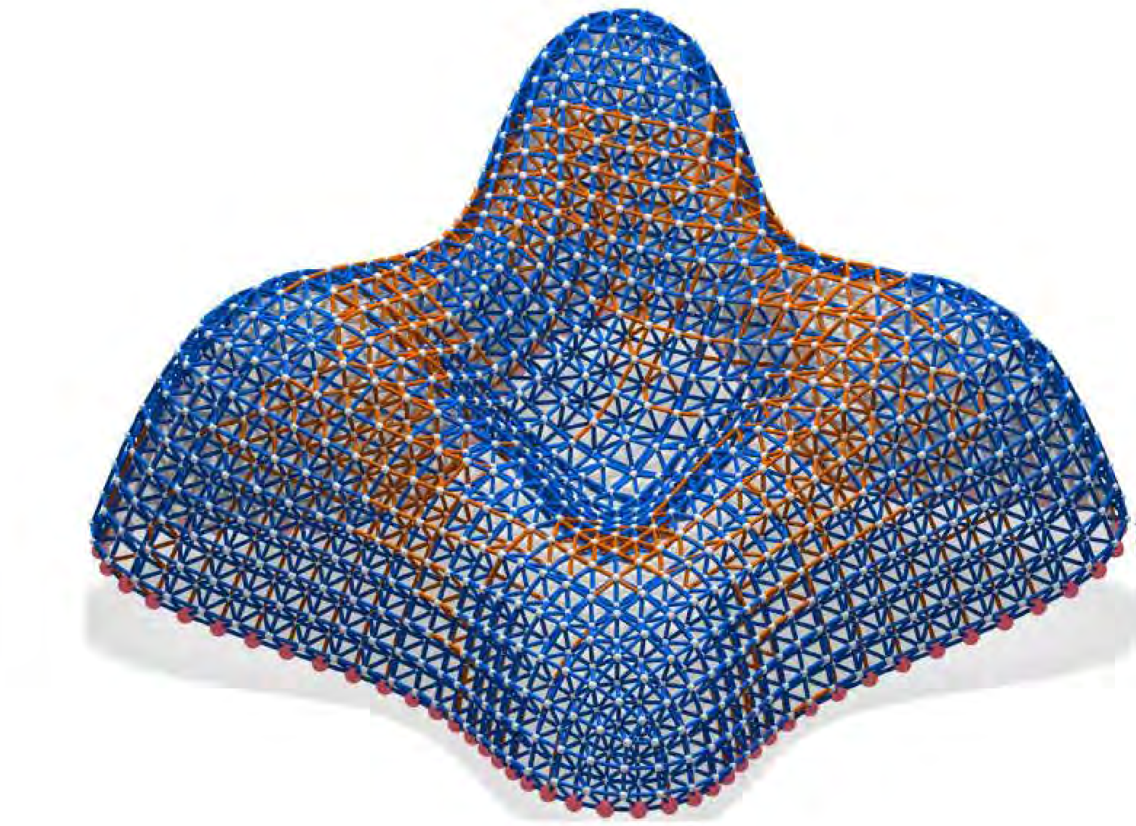
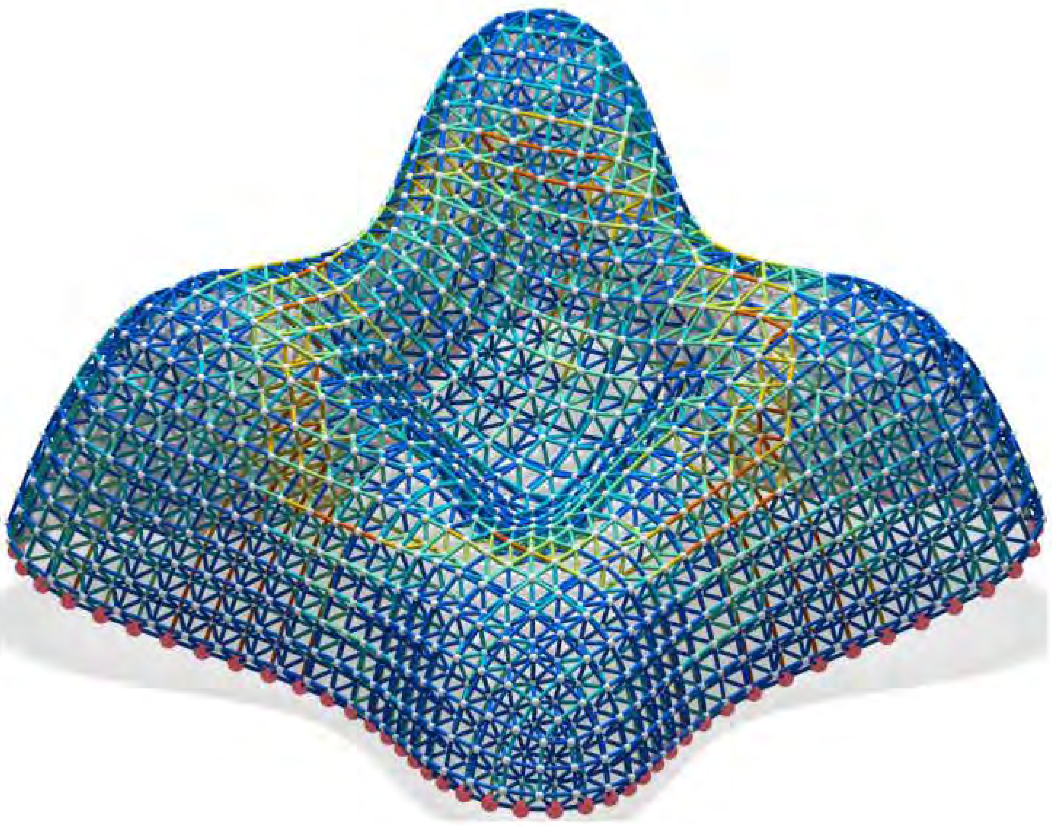
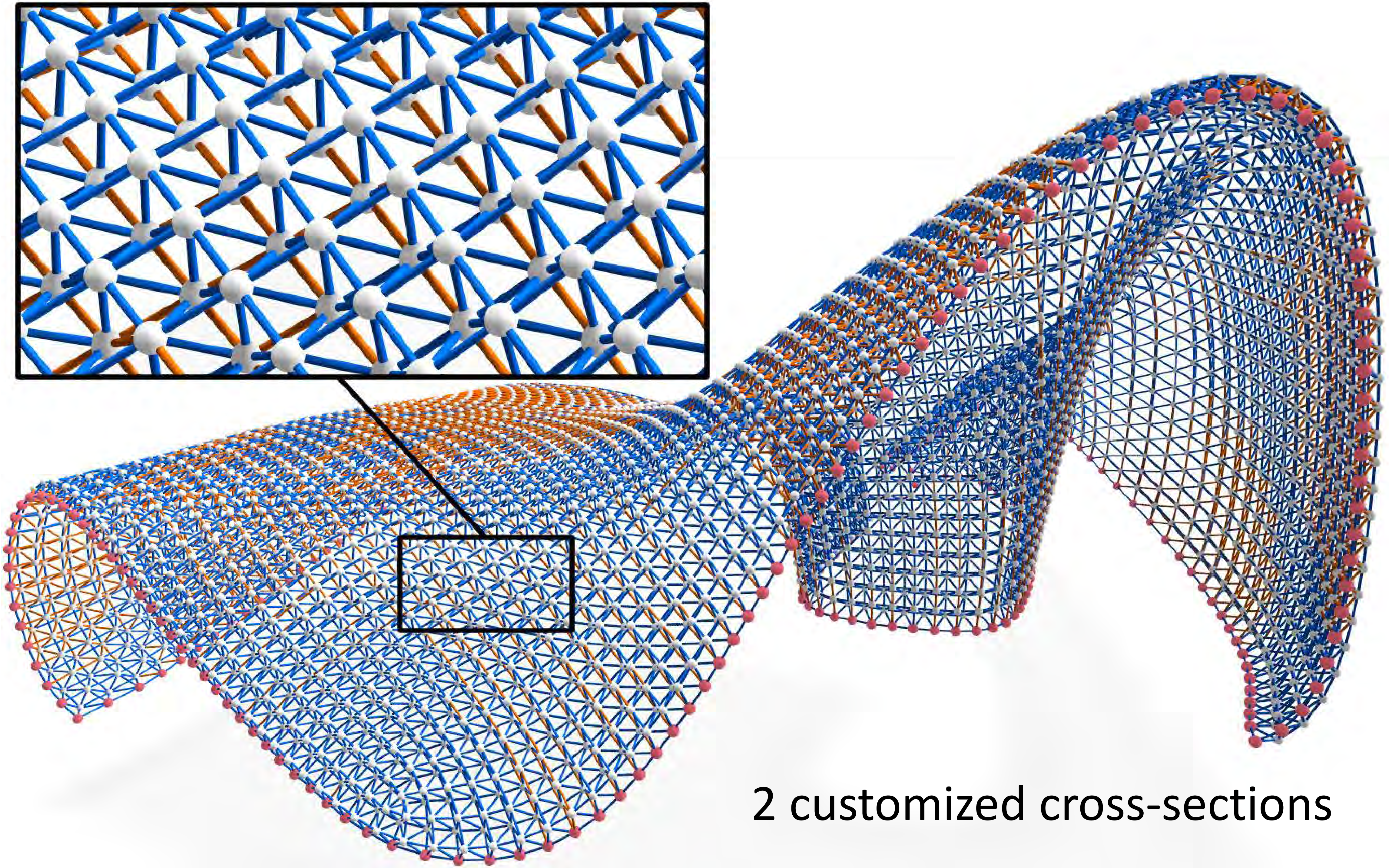
Results



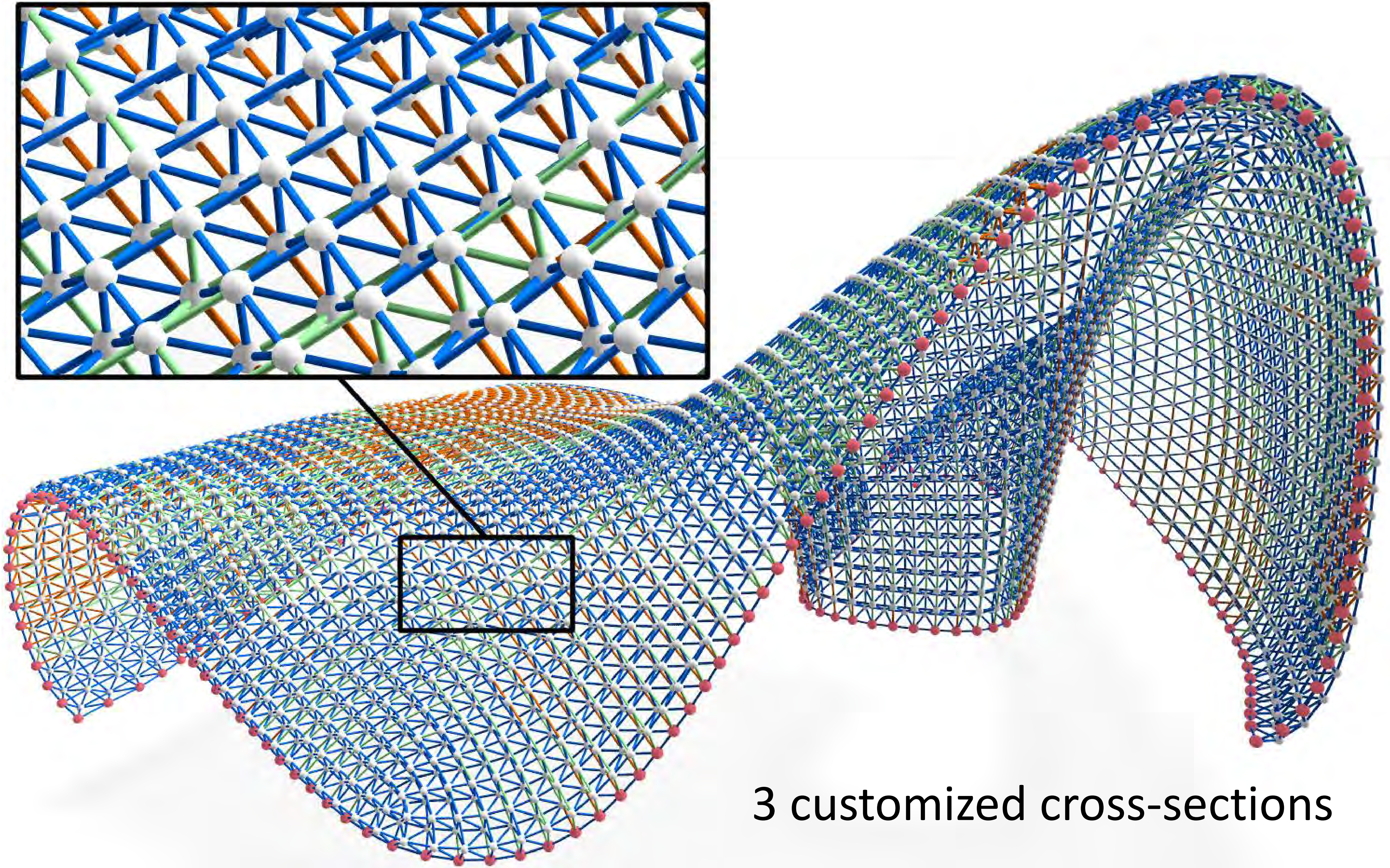
2 customized cross-sections



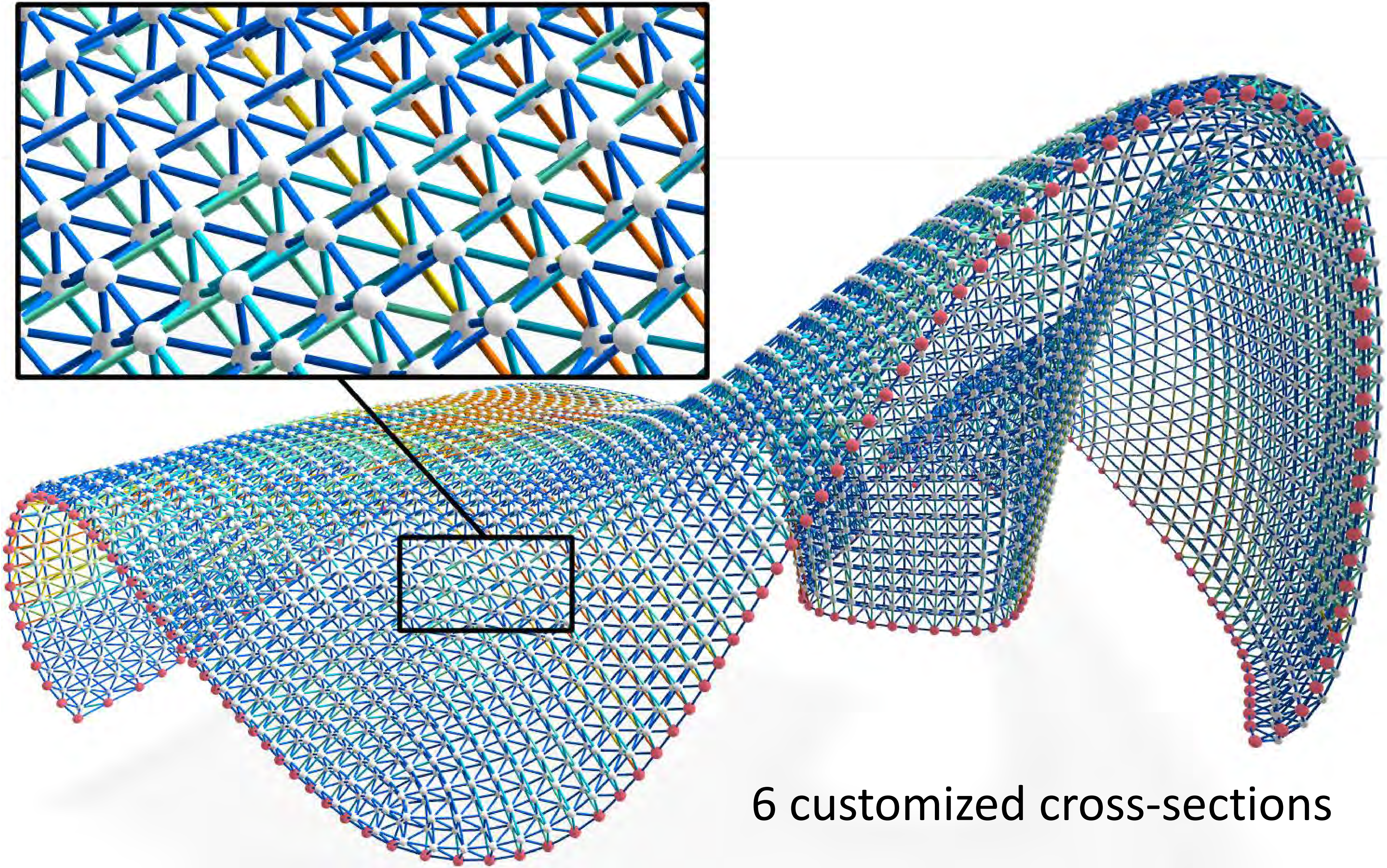
6 customized cross-sections



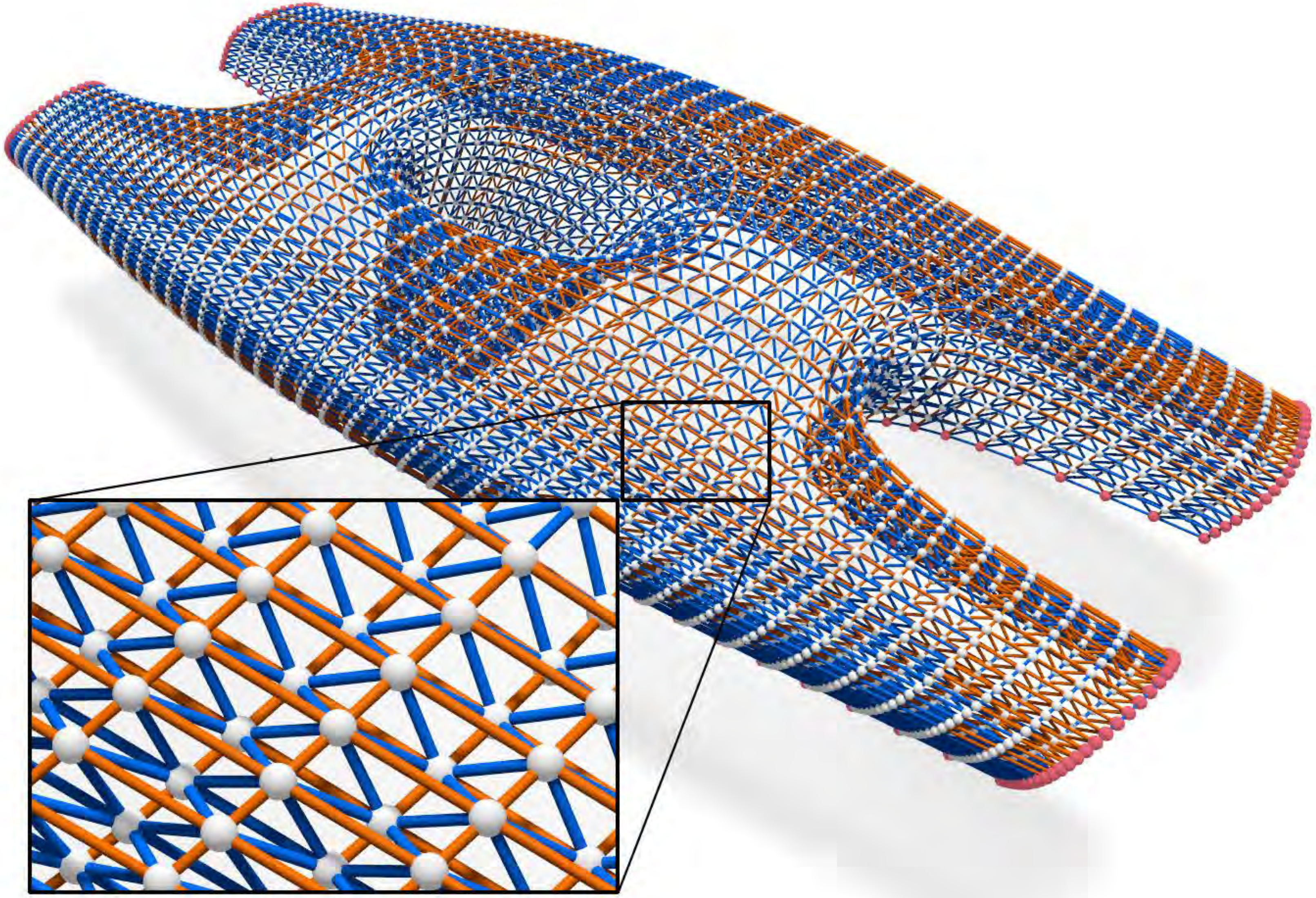
2 customized cross-sections



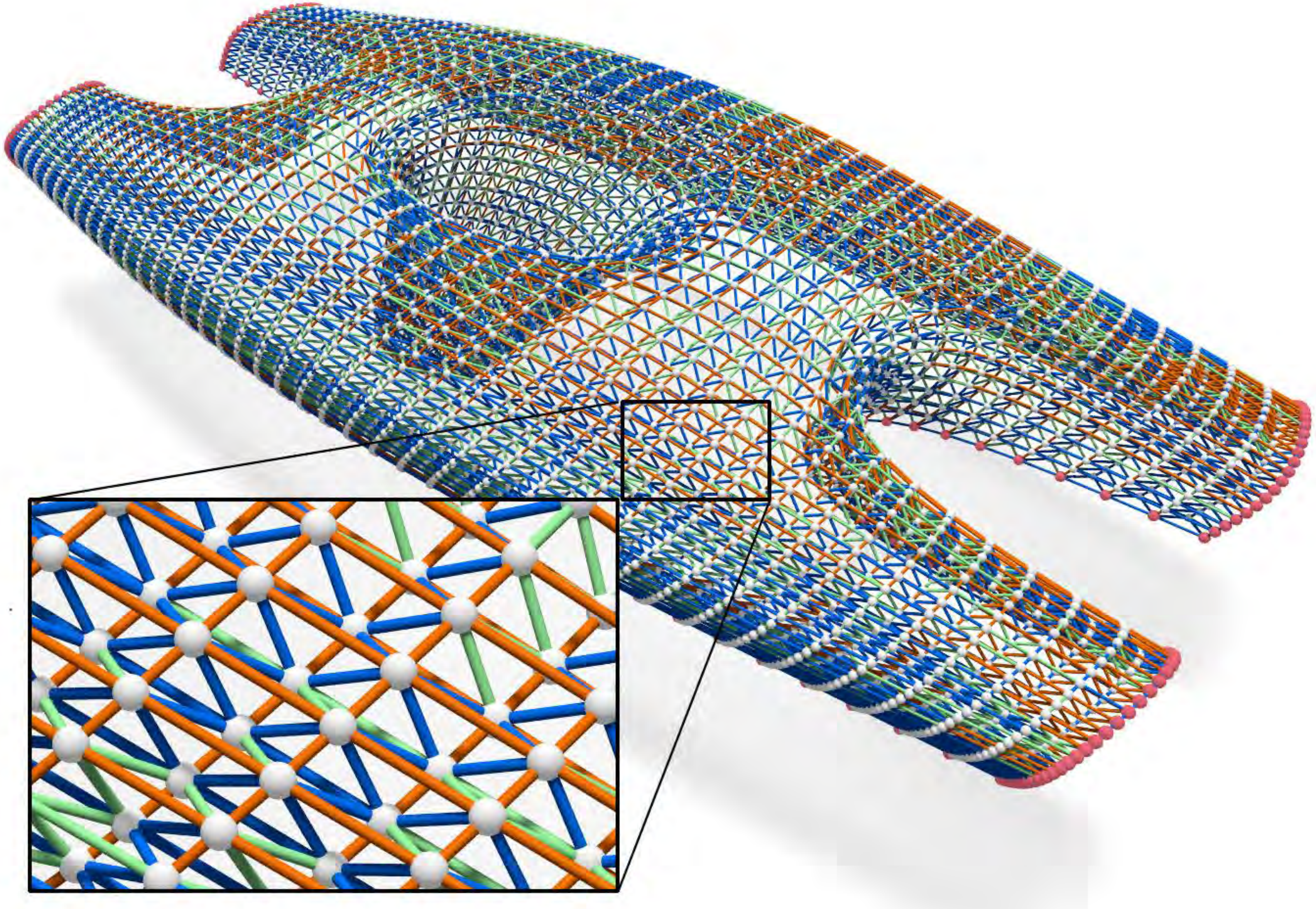
3 customized cross-sections



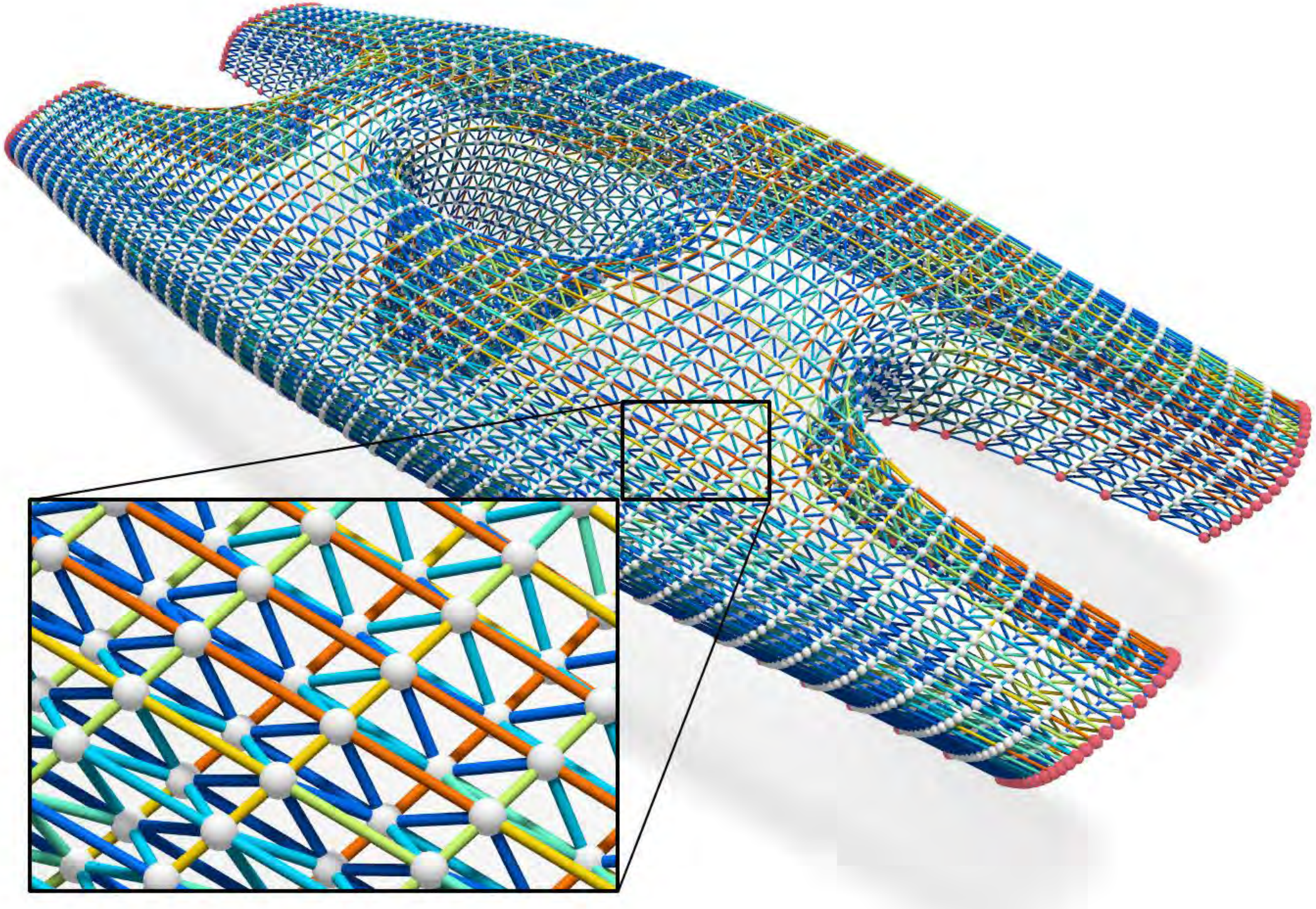
6 customized cross-sections



2 customized cross-sections

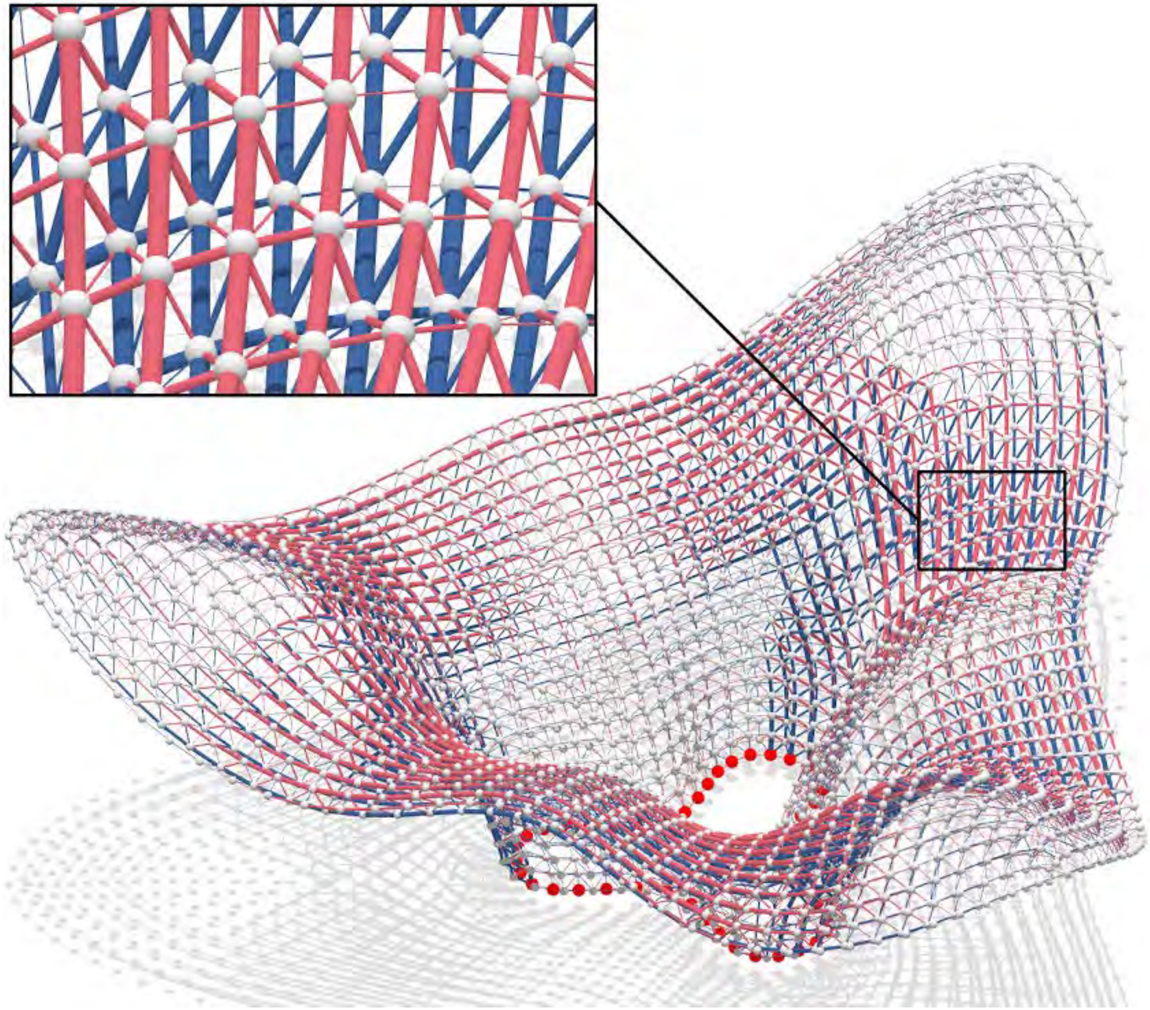


3 customized cross-sections



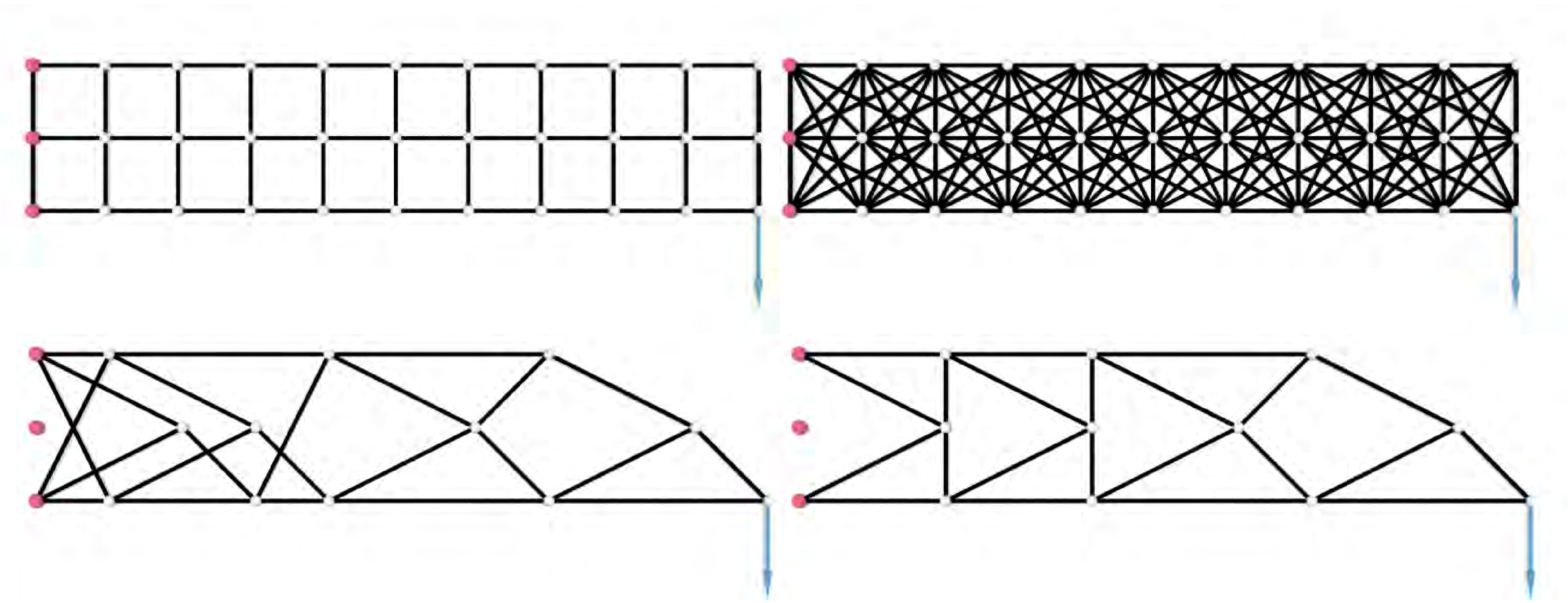
6 customized cross-sections

Results: tension and compression



Color coding: tension (red) and compression (blue)

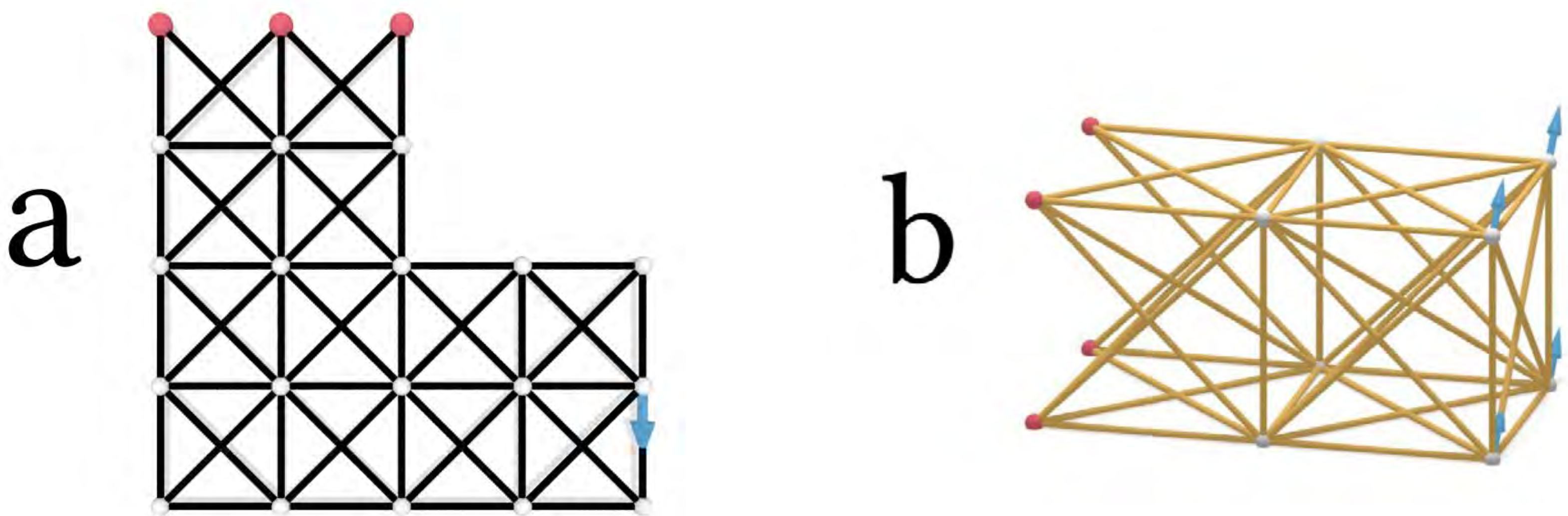
Results: comparisons

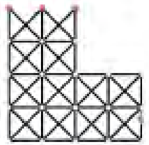
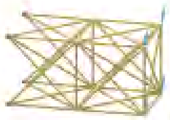


Ground structure method

Our method

Results: comparisons



Structure	Stress limit	[Rasmussen and Stolpe 2008]			Ours rounding-up			Ours		
		Vol(m^3)	T	CS($10^{-3}m^2$)	Vol(m^3)	T	CS($10^{-3}m^2$)	Vol(m^3)	T	CS($10^{-3}m^2$)
a 	170Mpa	0.0466	hours	10/5	0.0466	0.4s	10/5	**0.0322**	0.5s	5.30/2.65
	120MPa	0.0608	hours	10/5	0.0608	0.4s	10/5	**0.0456**	0.5	7.5/3.75
	90MPa	0.0608	hours	10/5	0.0608	0.4s	10/5	**0.0608**	0.5	10/5
b 	170MPa	0.438	hours	10/7.5/5/2.5	0.455	0.5s	10/7.5/5/2.5	**0.294**	0.6s	4.71/2.04/1.66/0.59
	120MPa	0.524	hours	10/7.5/5/2.5	0.524	0.5s	10/7.5/5/2.5	**0.417**	0.6s	6.67/2.89/2.34/0.83
	90MPa	0.656	hours	10/7.5/5/2.5	0.698	0.5s	10/7.5/5/2.5	**0.555**	0.6s	8.89/3.85/3.14/1.11

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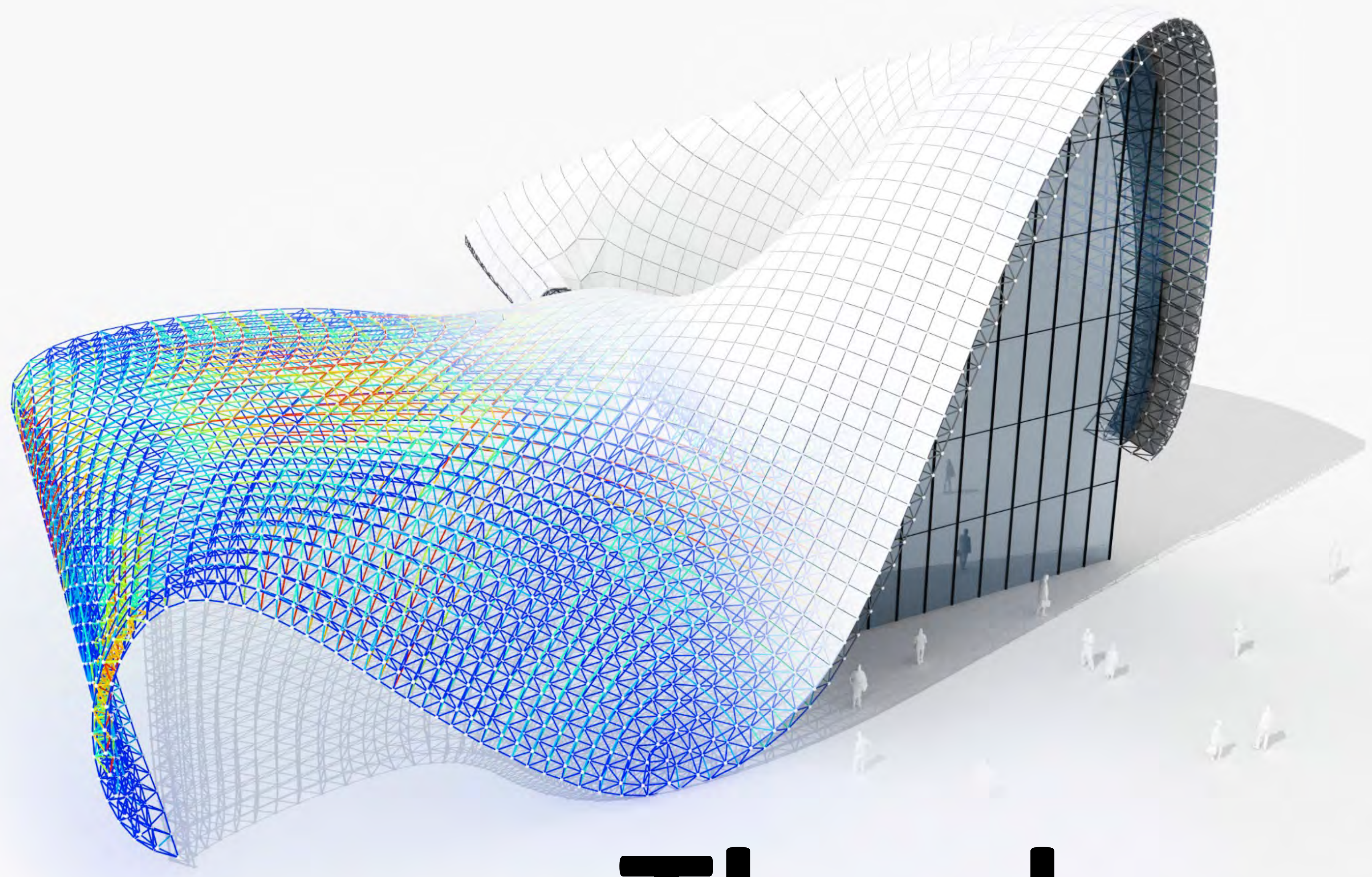
Limitations and future work

Limitations

- Global convergence
- Global buckling

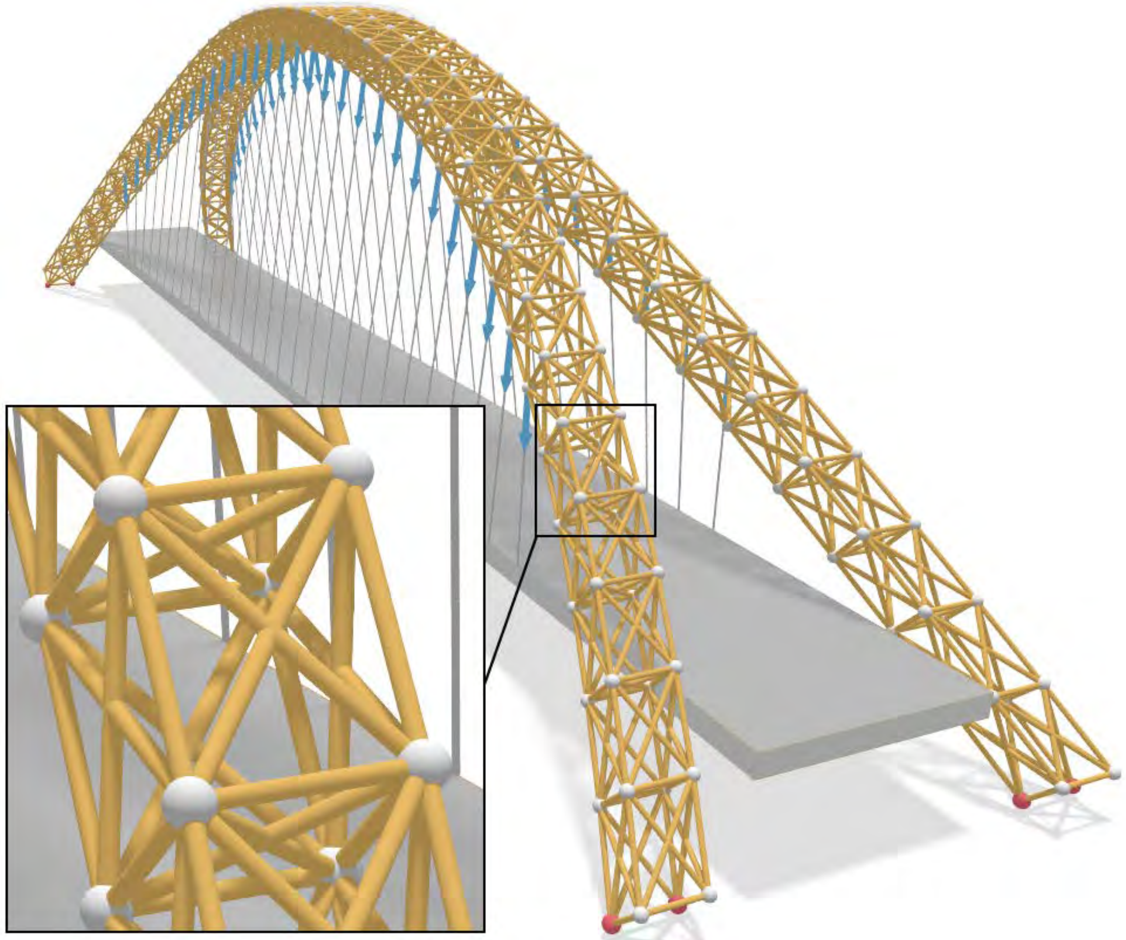
Future work

- Design of other types of statically sound structures, e.g., bikes
- Field generation for optimal structures
- Structural optimization for dynamic structures



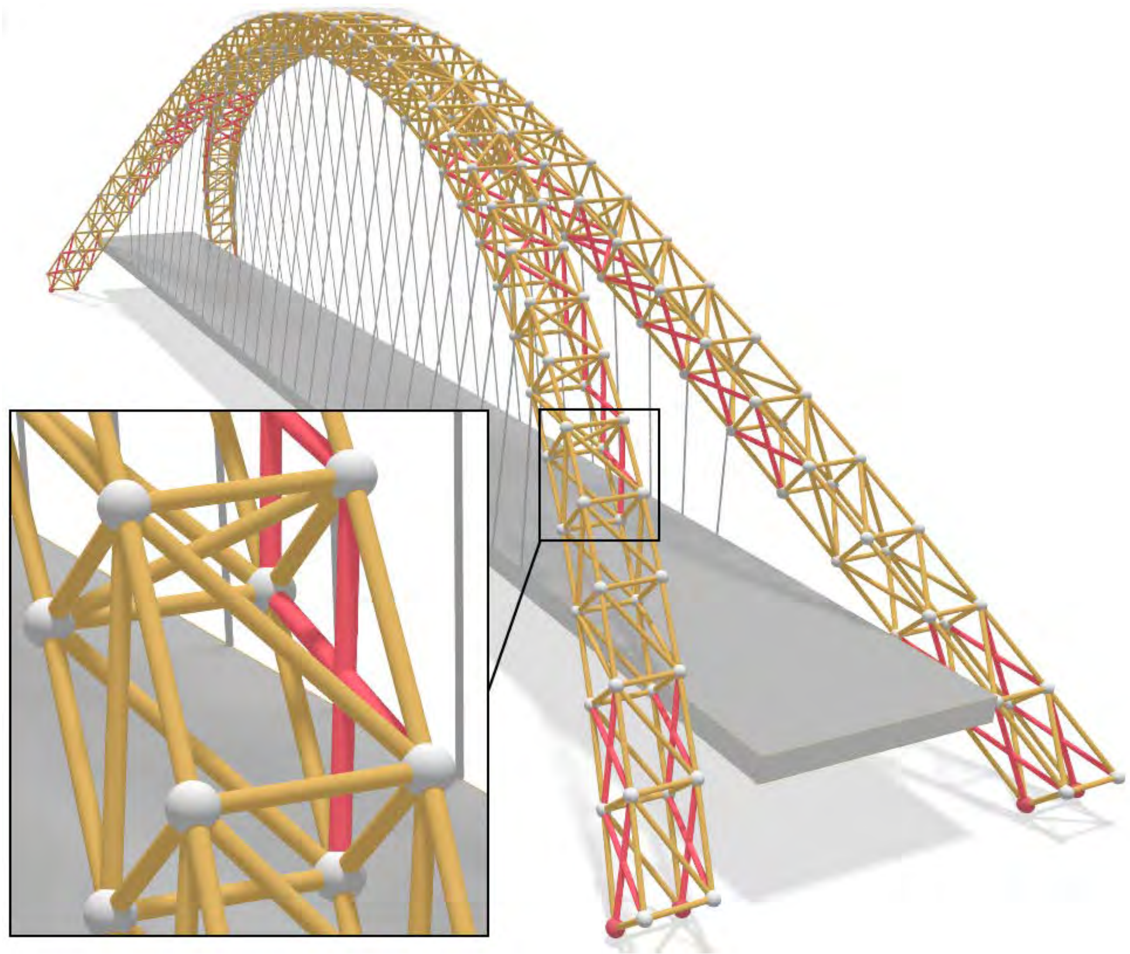
Thank you!

3D bridge



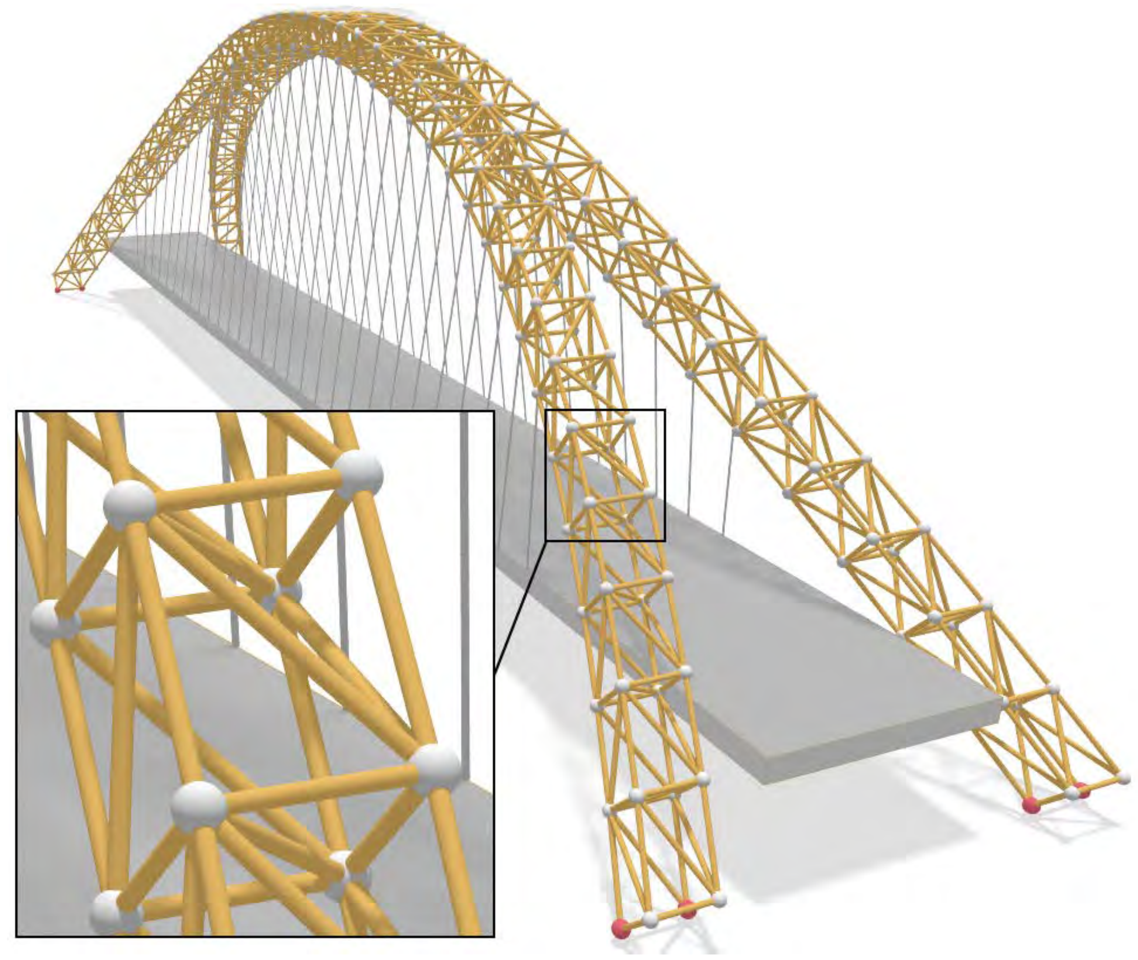
initial structure

3D bridge



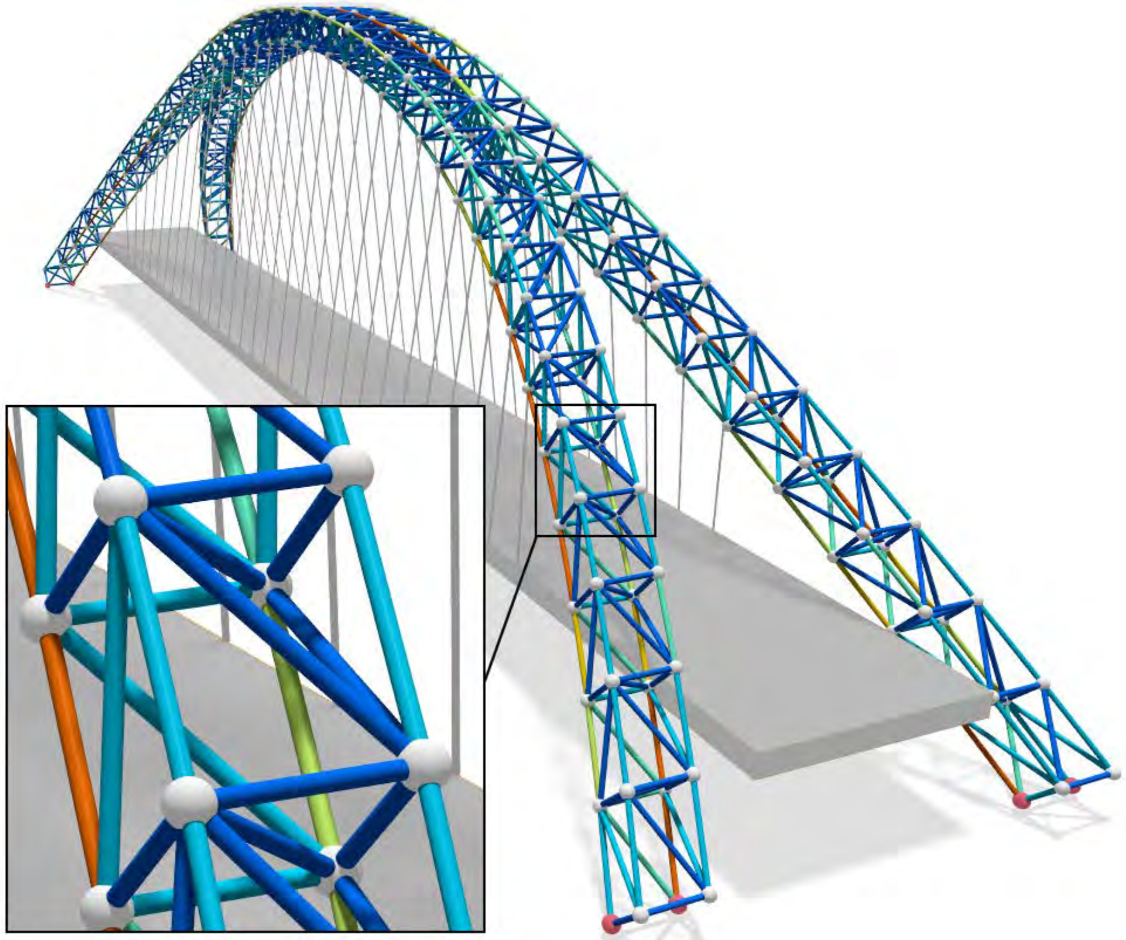
optimized connectivity (the ground structure method)

3D bridge



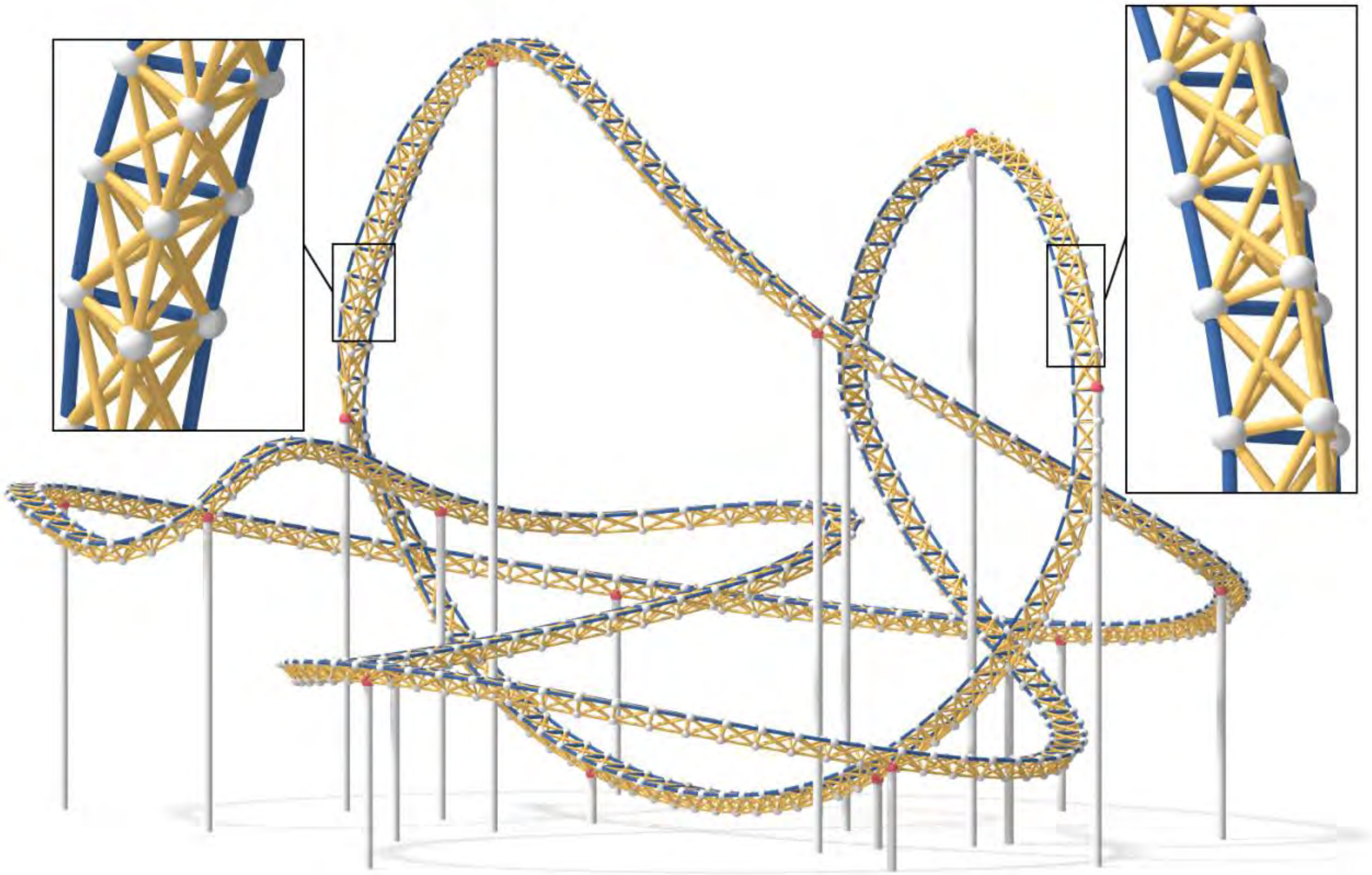
optimized connectivity (our method)

3D bridge



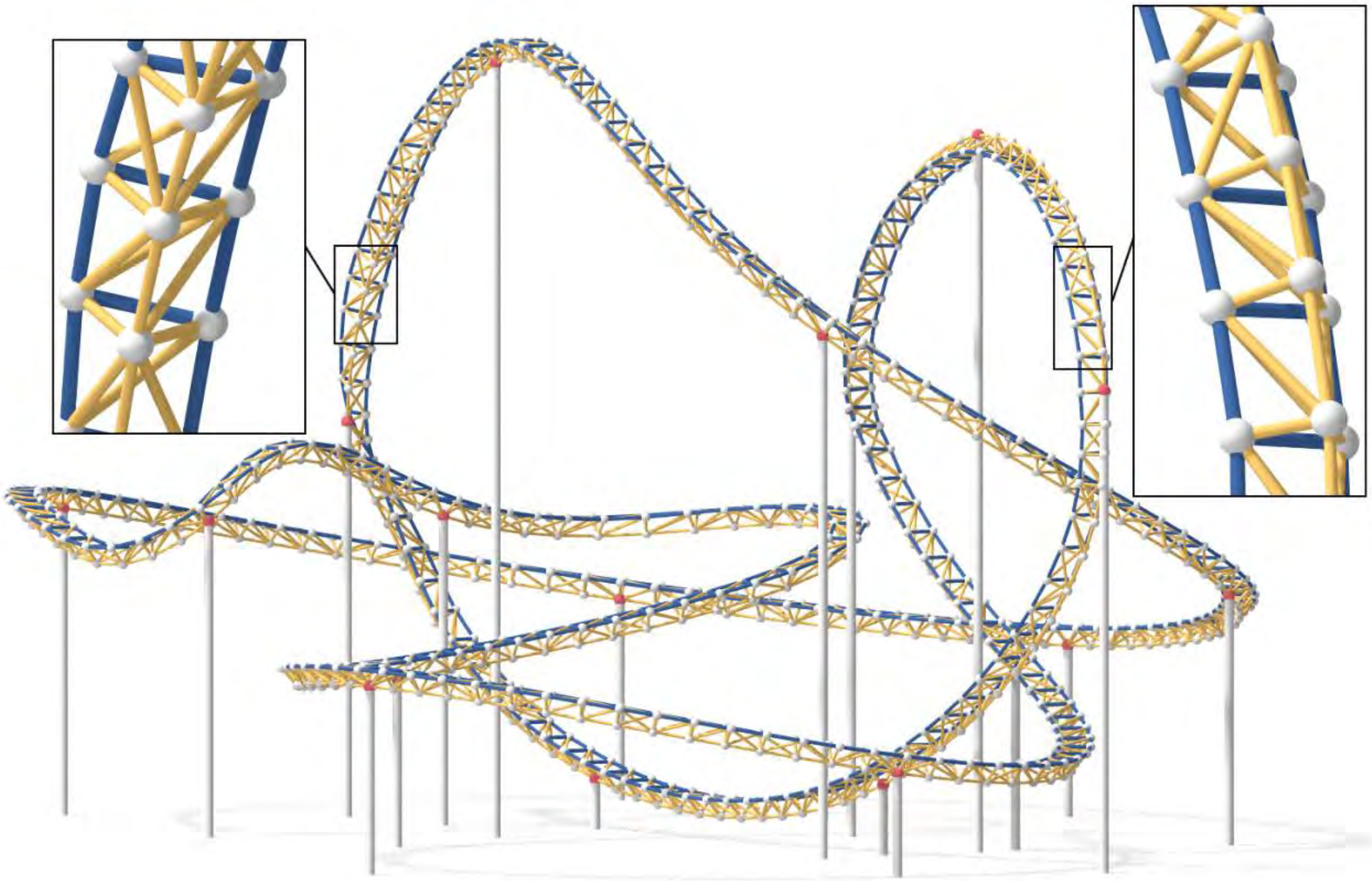
6 customized cross-sections

Roller coaster



initial structure

Roller coaster

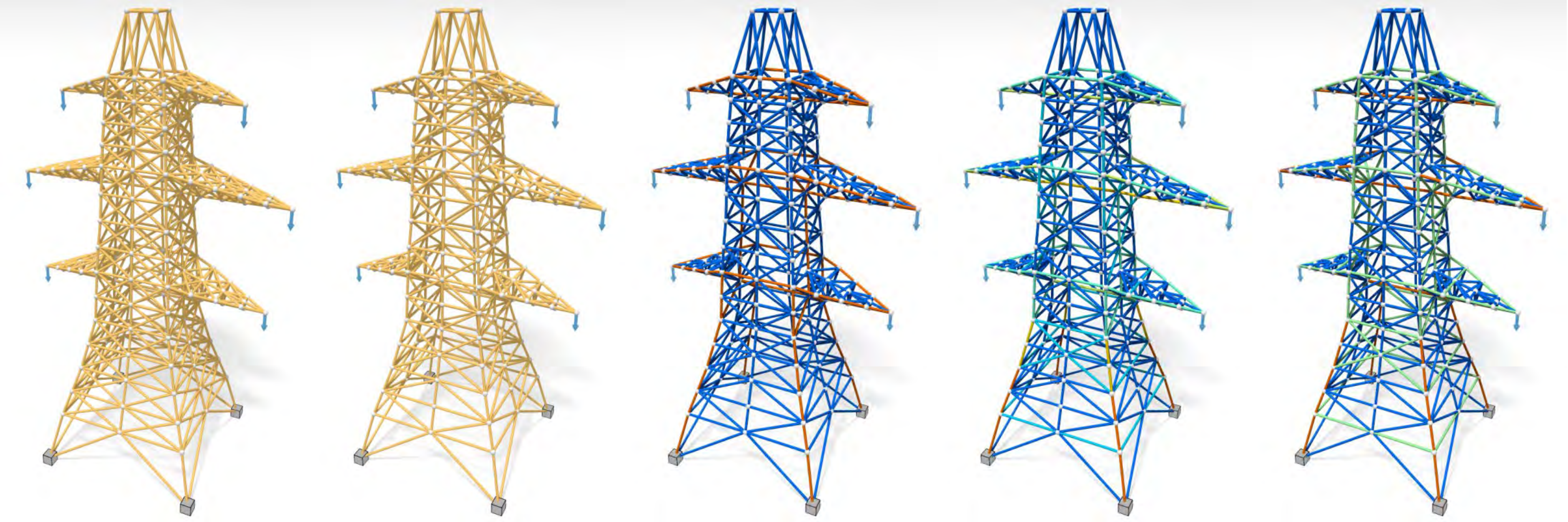


optimized connectivity (our method)

Roller coaster

6 customized cross-sections

Electrical transmission tower



Blob