Complex Fluids and Soft Materials: A Numerical Perspective
复杂流体和柔性材料的计算方法

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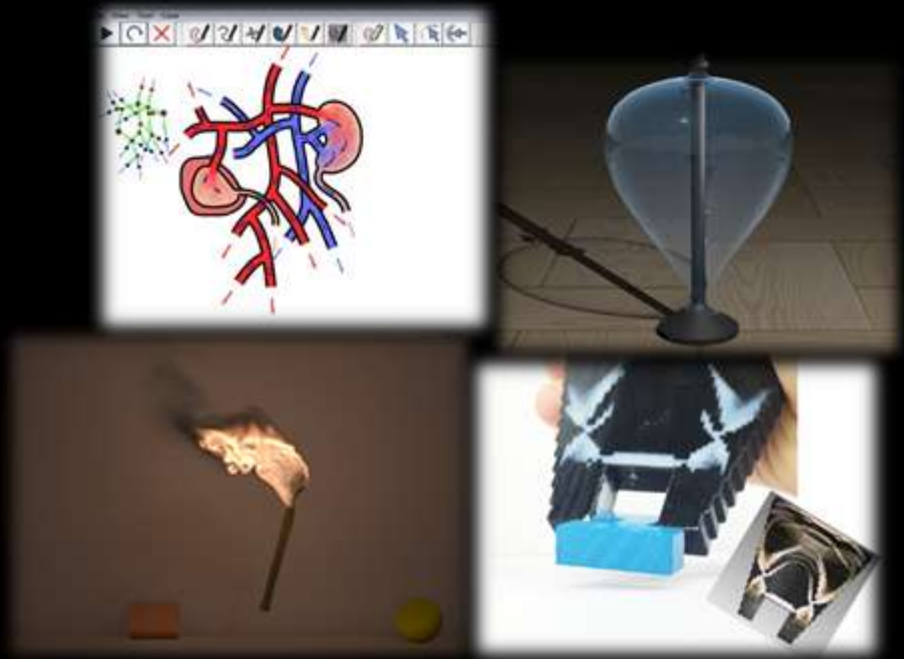


Complex Physical Systems

Geometry, Topology, Dynamics

Material, Structure, Codimension, Transition

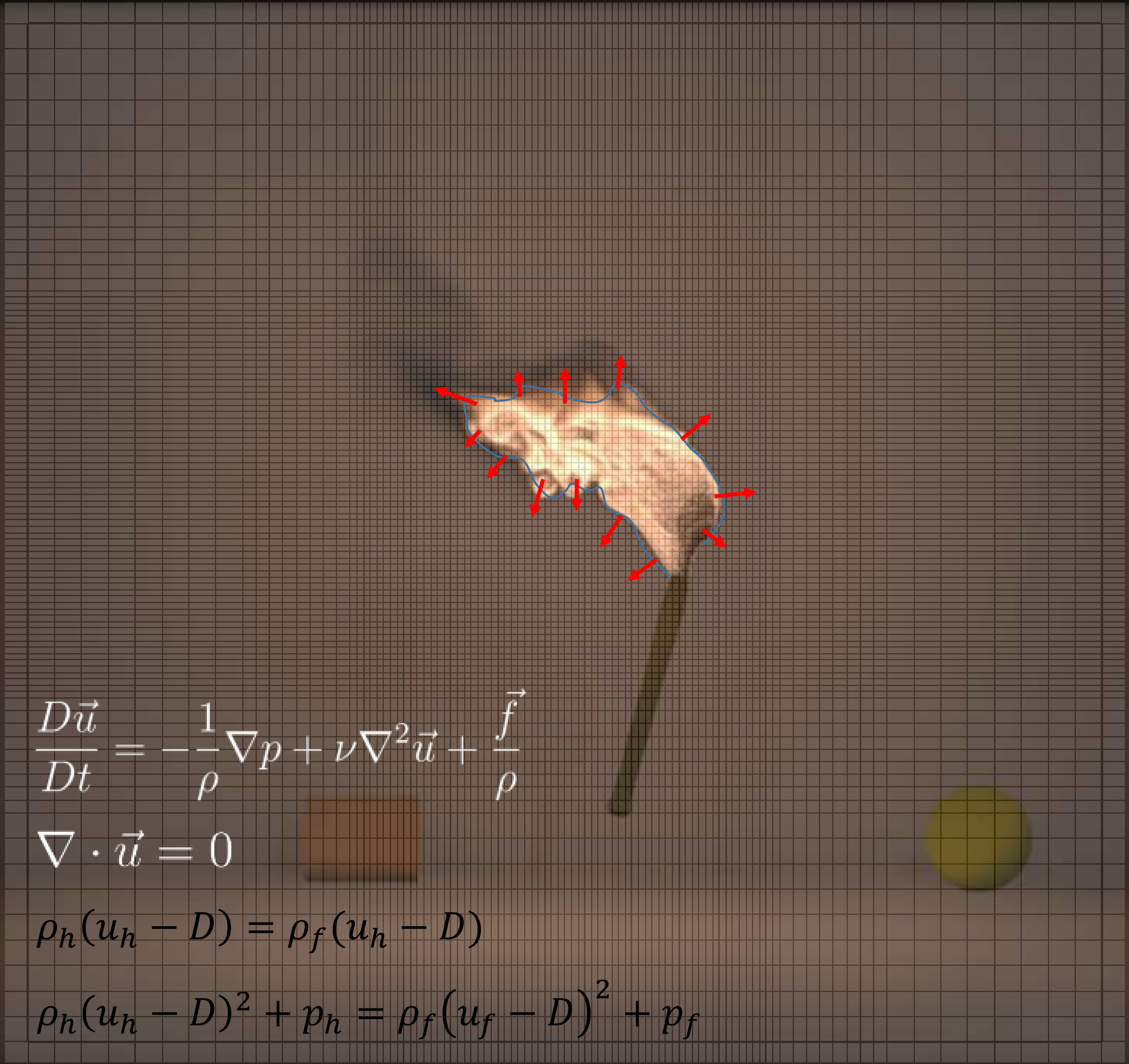
Computer Graphics,

Computational Fluid Dynamics,

Computational Fabrication, 3D Printing,

Biomedical Engineering, Robotics

Flame

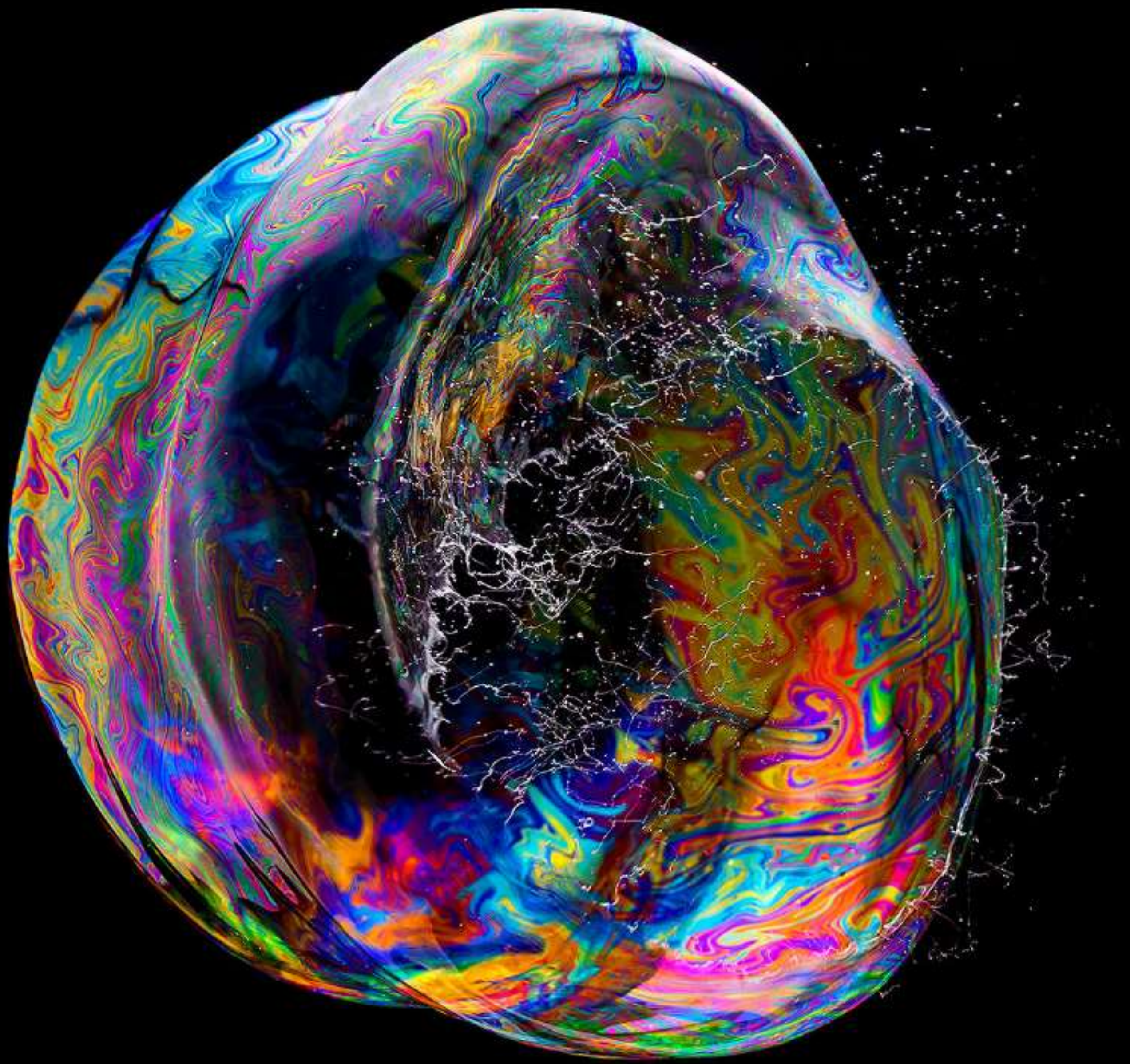


$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{\vec{f}}{\rho}$$

$$\nabla \cdot \vec{u} = 0$$

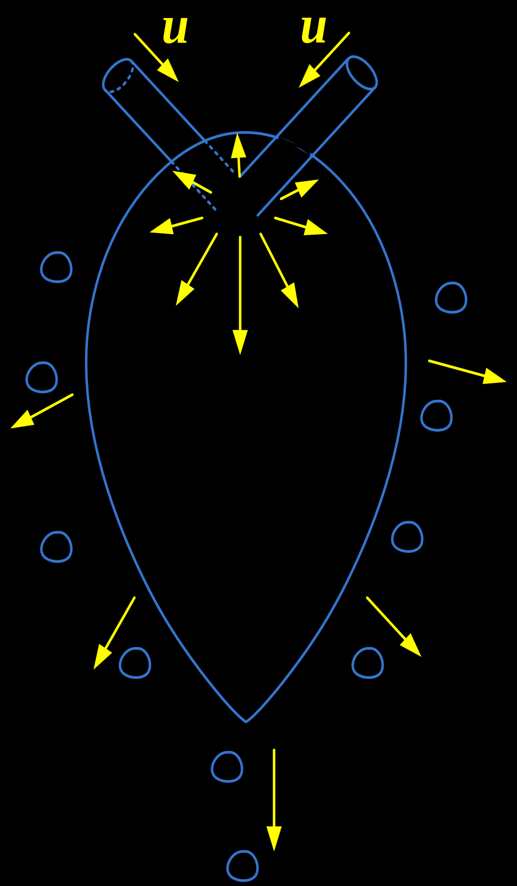
$$\rho_h(u_h - D) = \rho_f(u_h - D)$$

$$\rho_h(u_h - D)^2 + p_h = \rho_f\left(u_f - D\right)^2 + p_f$$

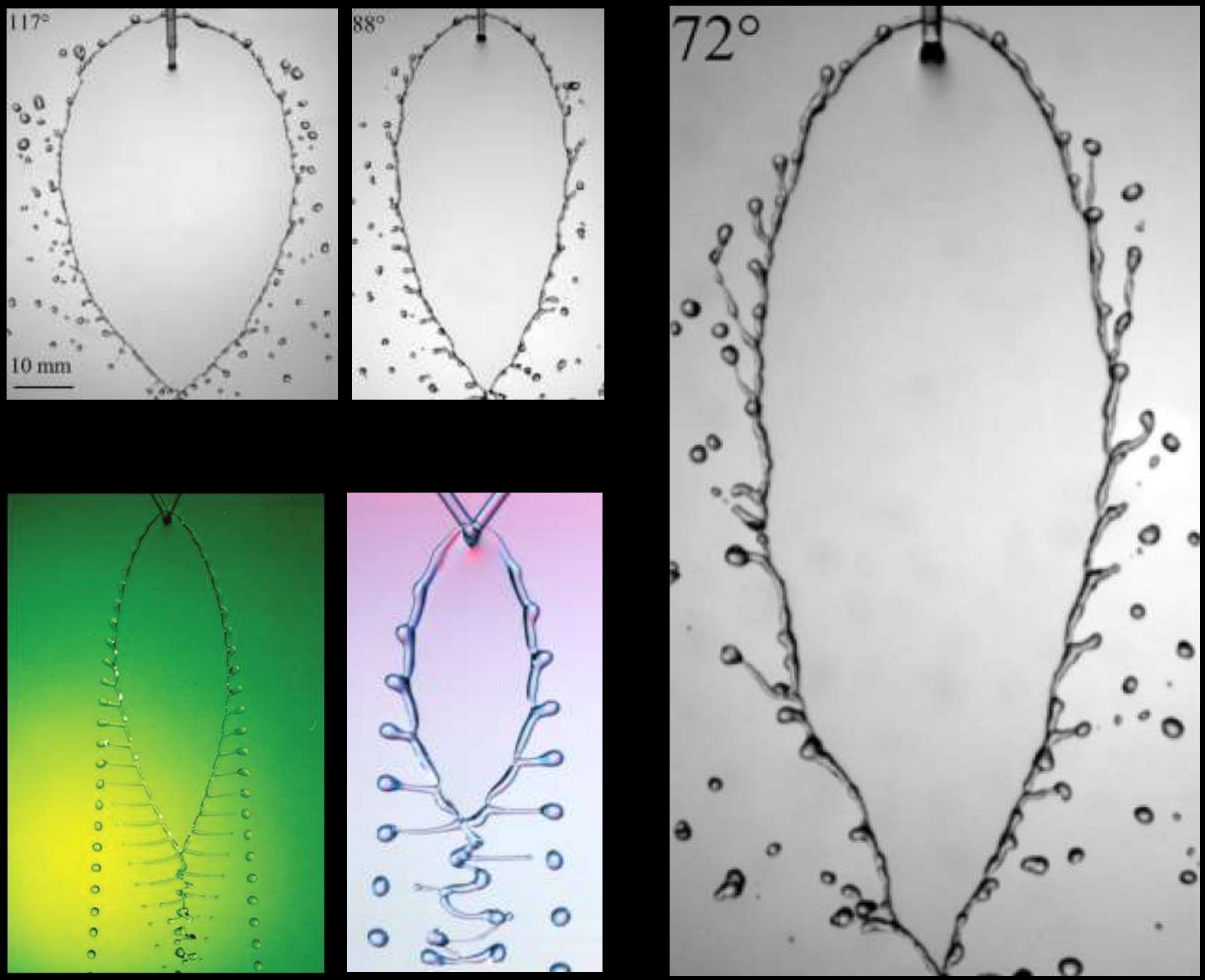


Bubbles

[Fabian Oefner]



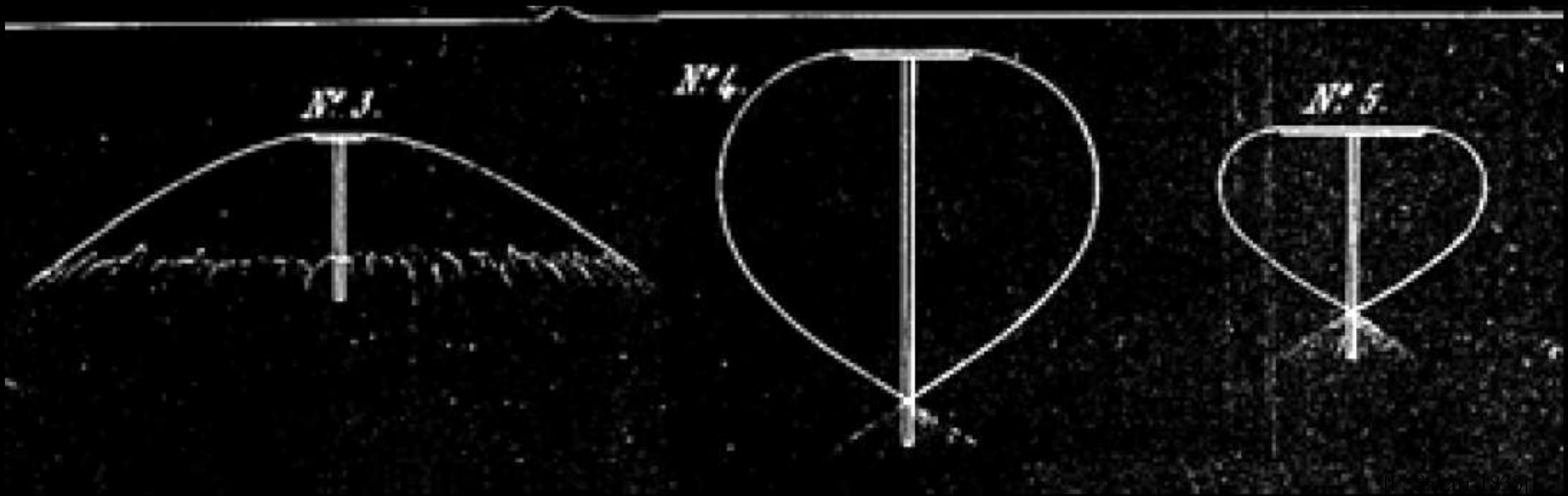
Two liquid jets
collide with each other



[John Bush Lab, MIT, 2004]

[Bremond N and Villermaux E 2006]

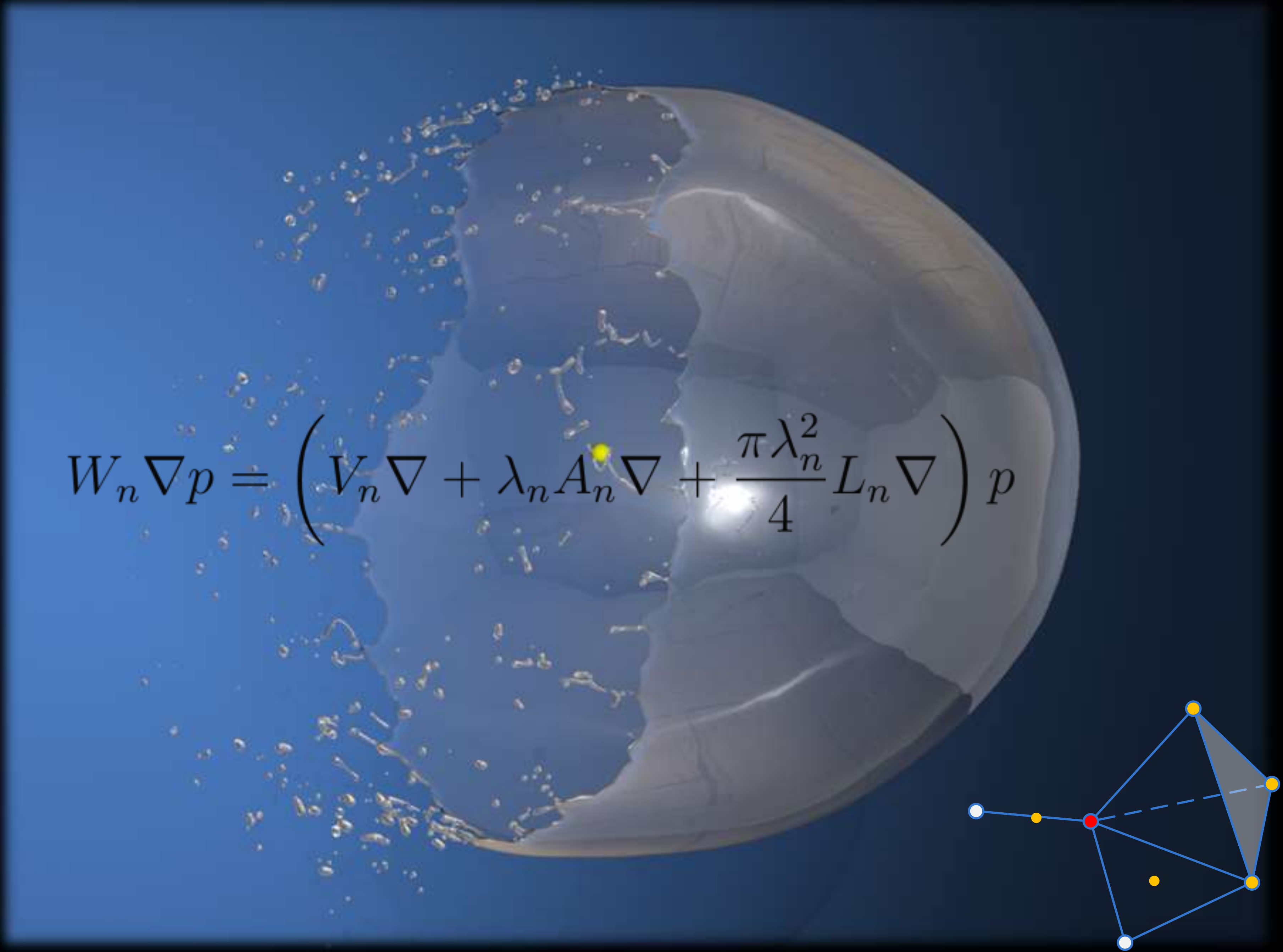
Impinging Jets



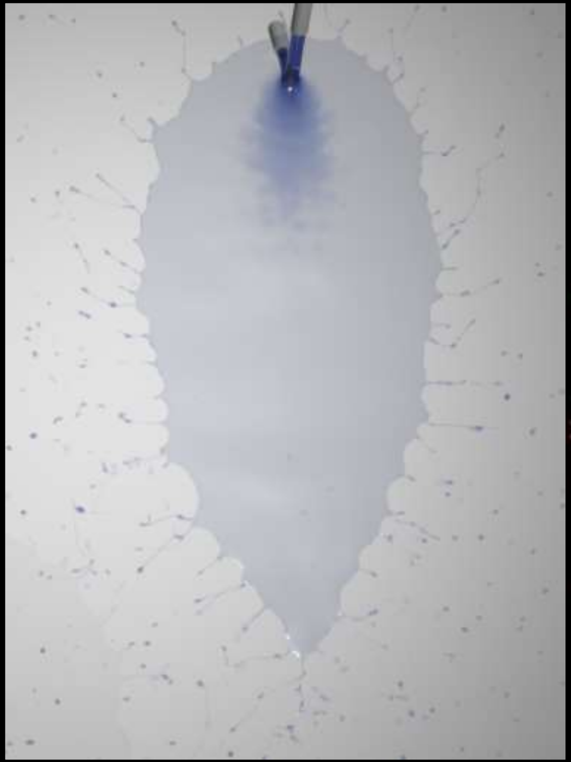
http://www.phikwadraat.nl/

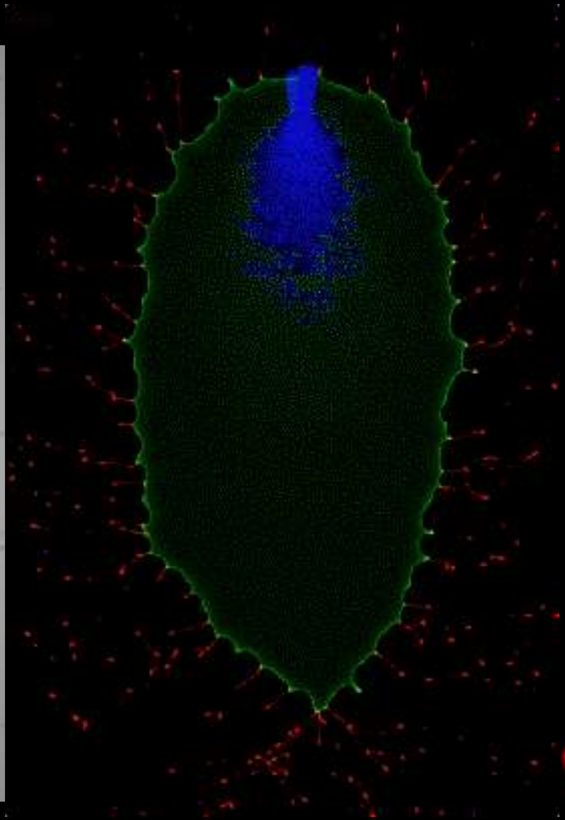
[John Bush Lab, MIT, 2004]

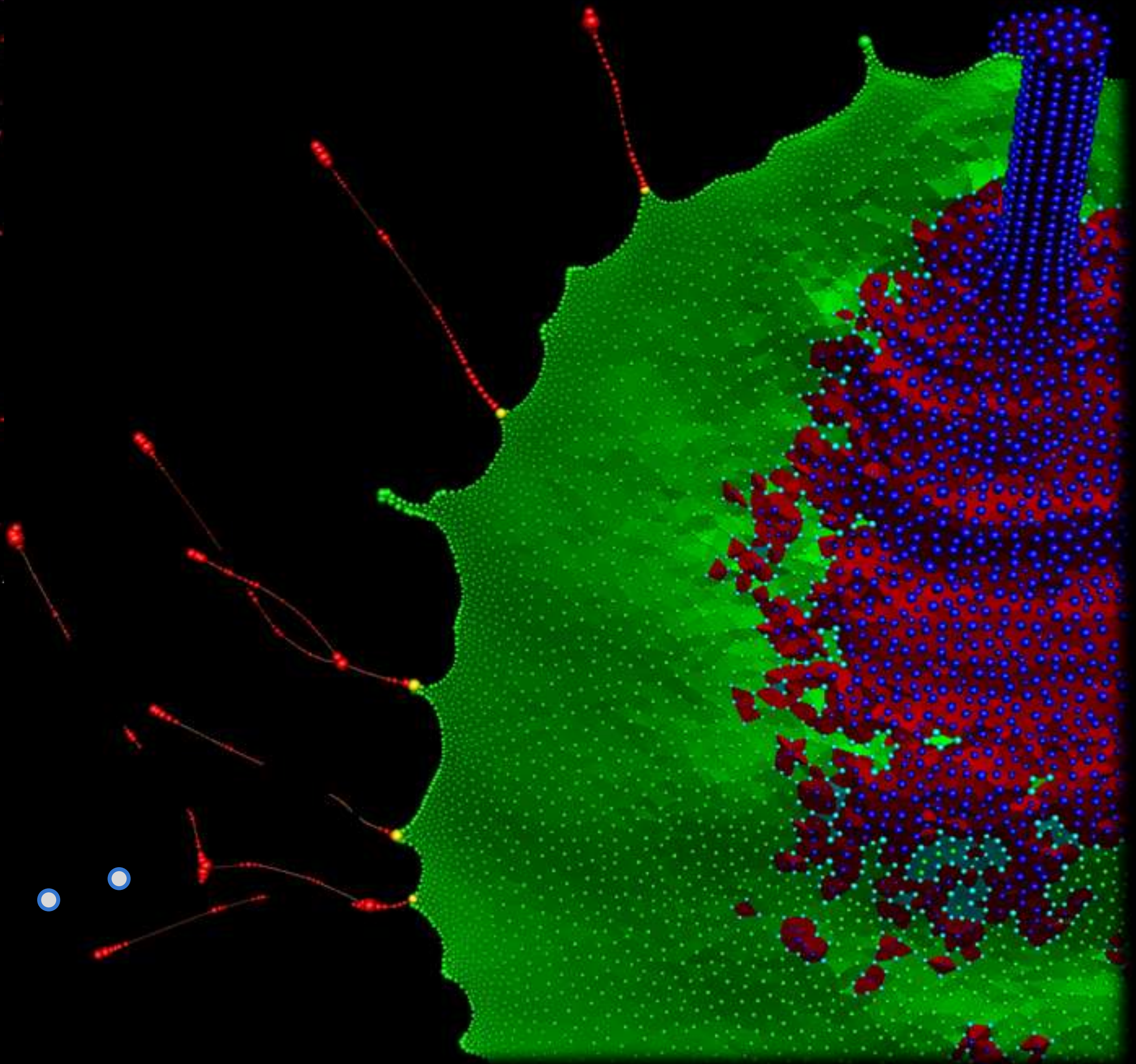
Water Bell

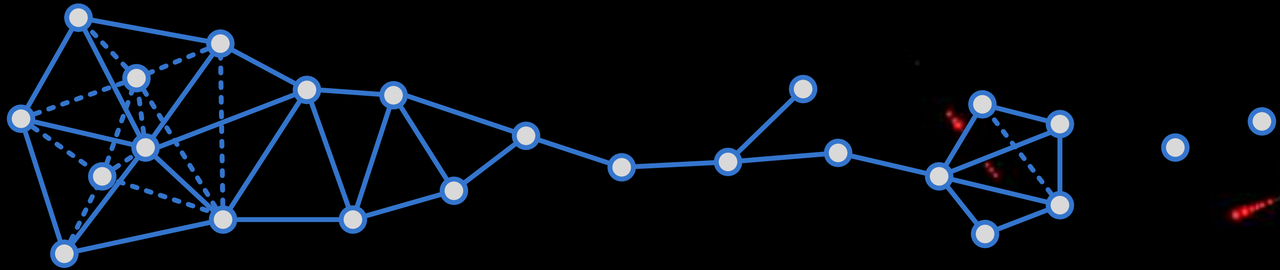


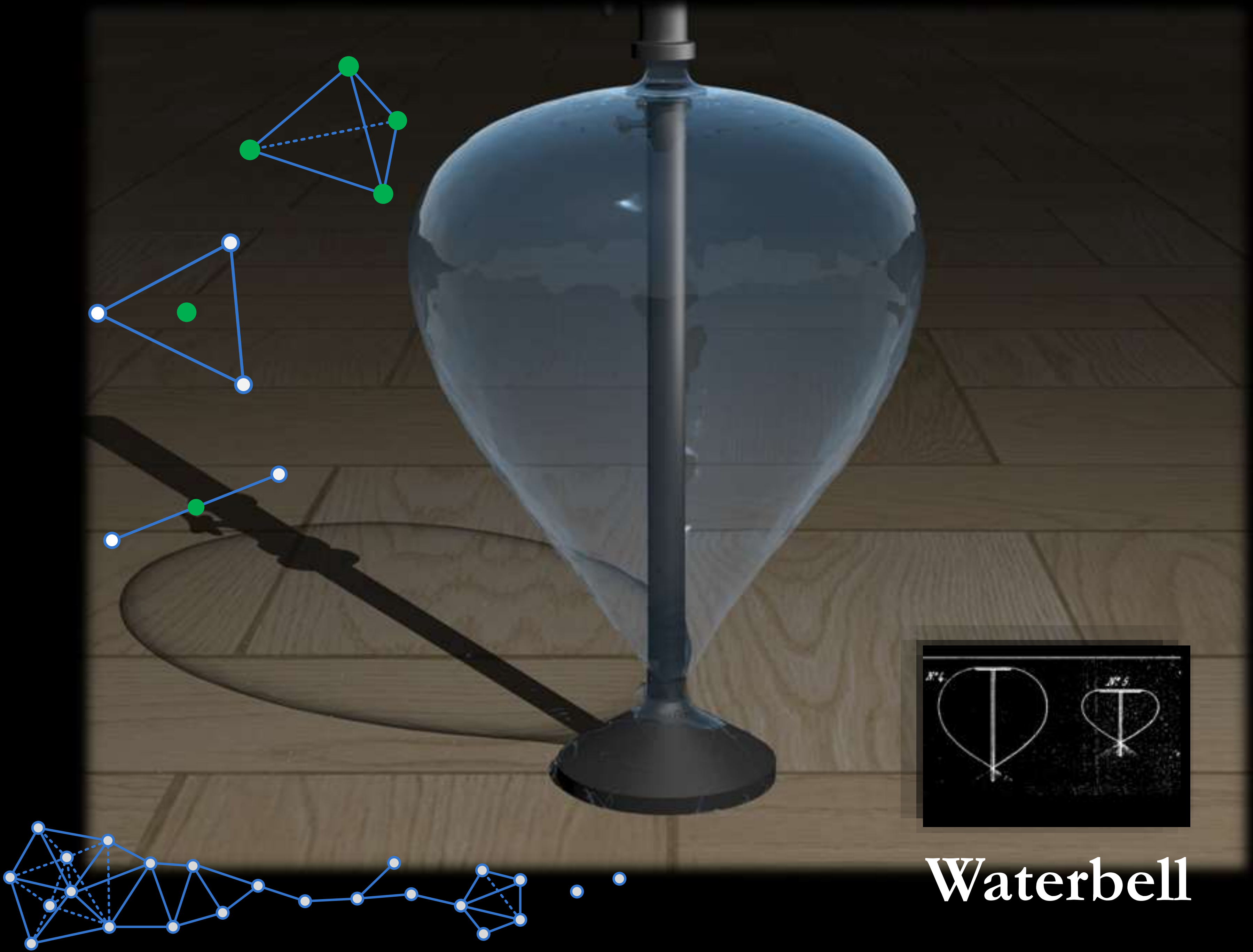
$$W_n \nabla p = \left(V_n \nabla + \lambda_n A_n \nabla + \frac{\pi \lambda_n^2}{4} L_n \nabla \right) p$$



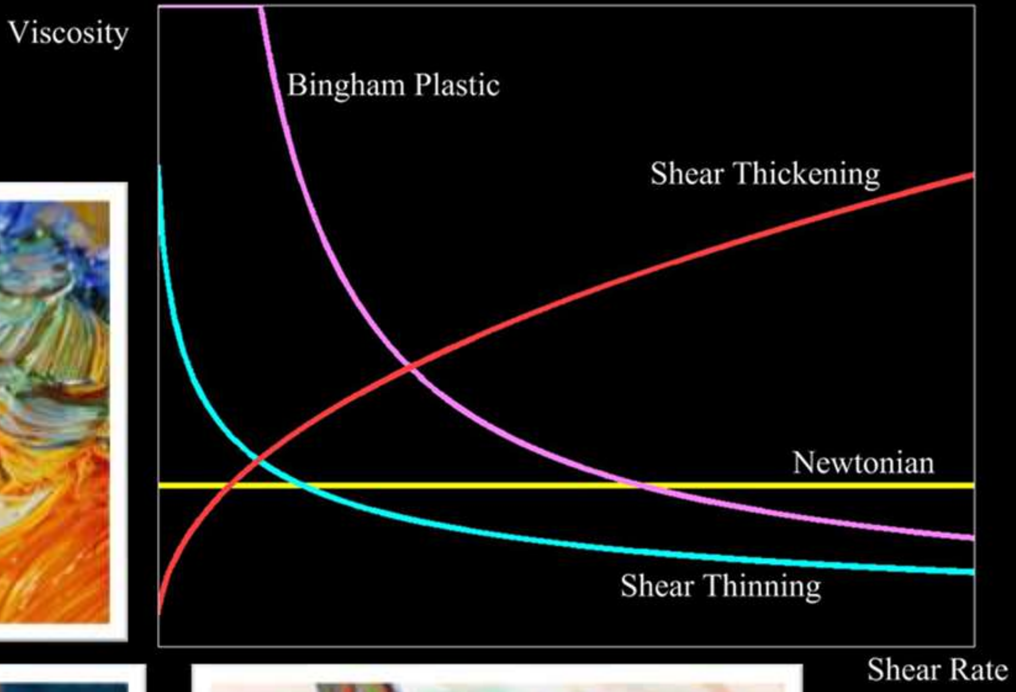






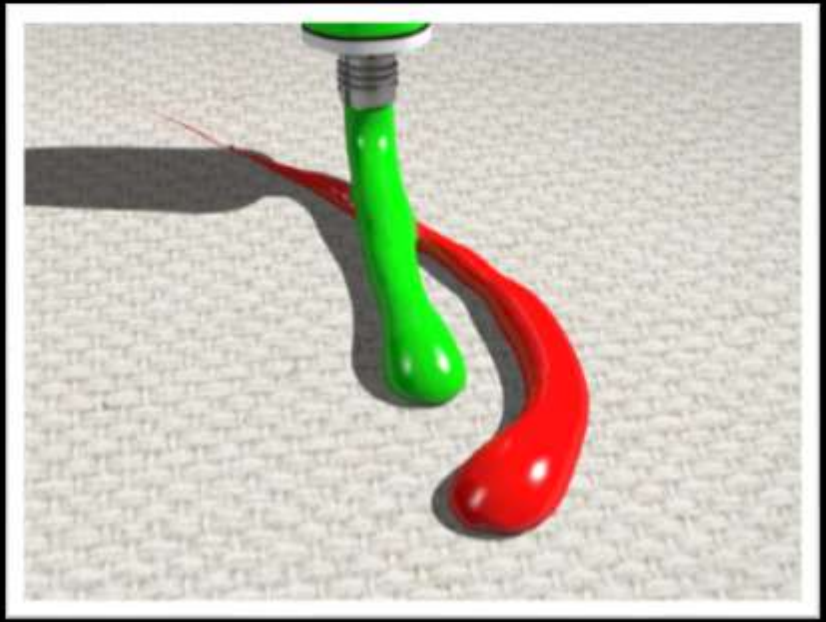


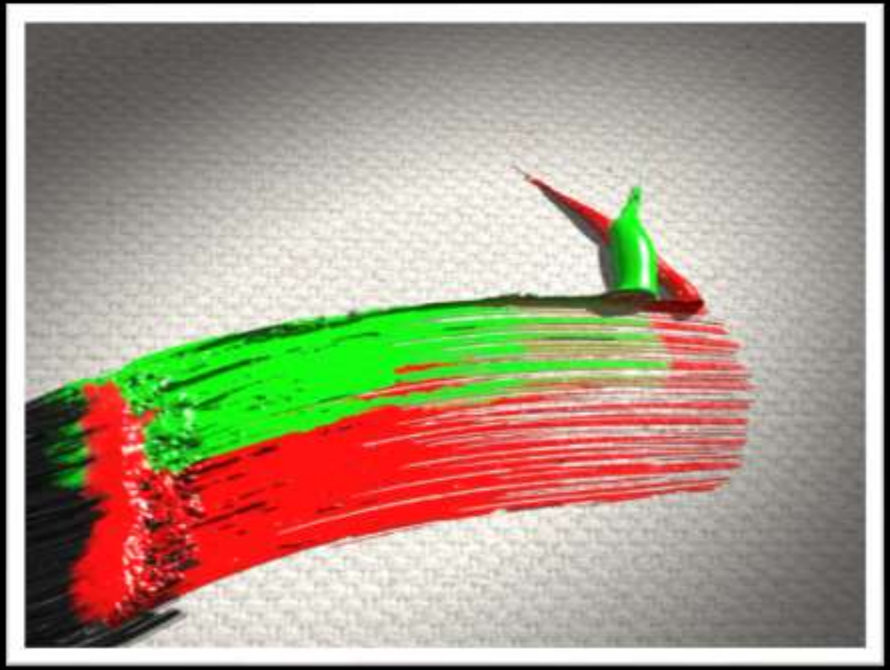
Waterbell



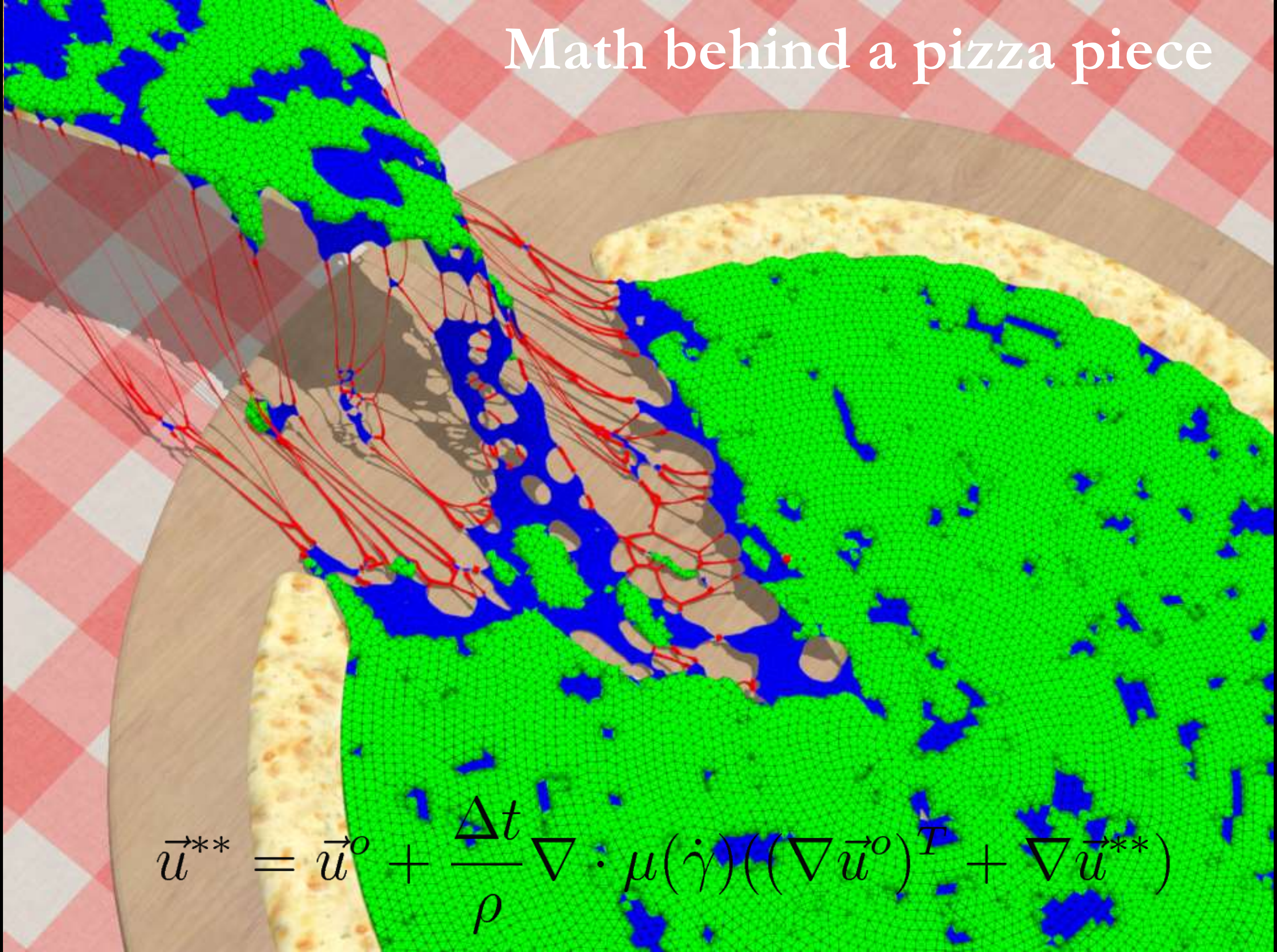
Non-Newtonian Flow

Viscosity matters!

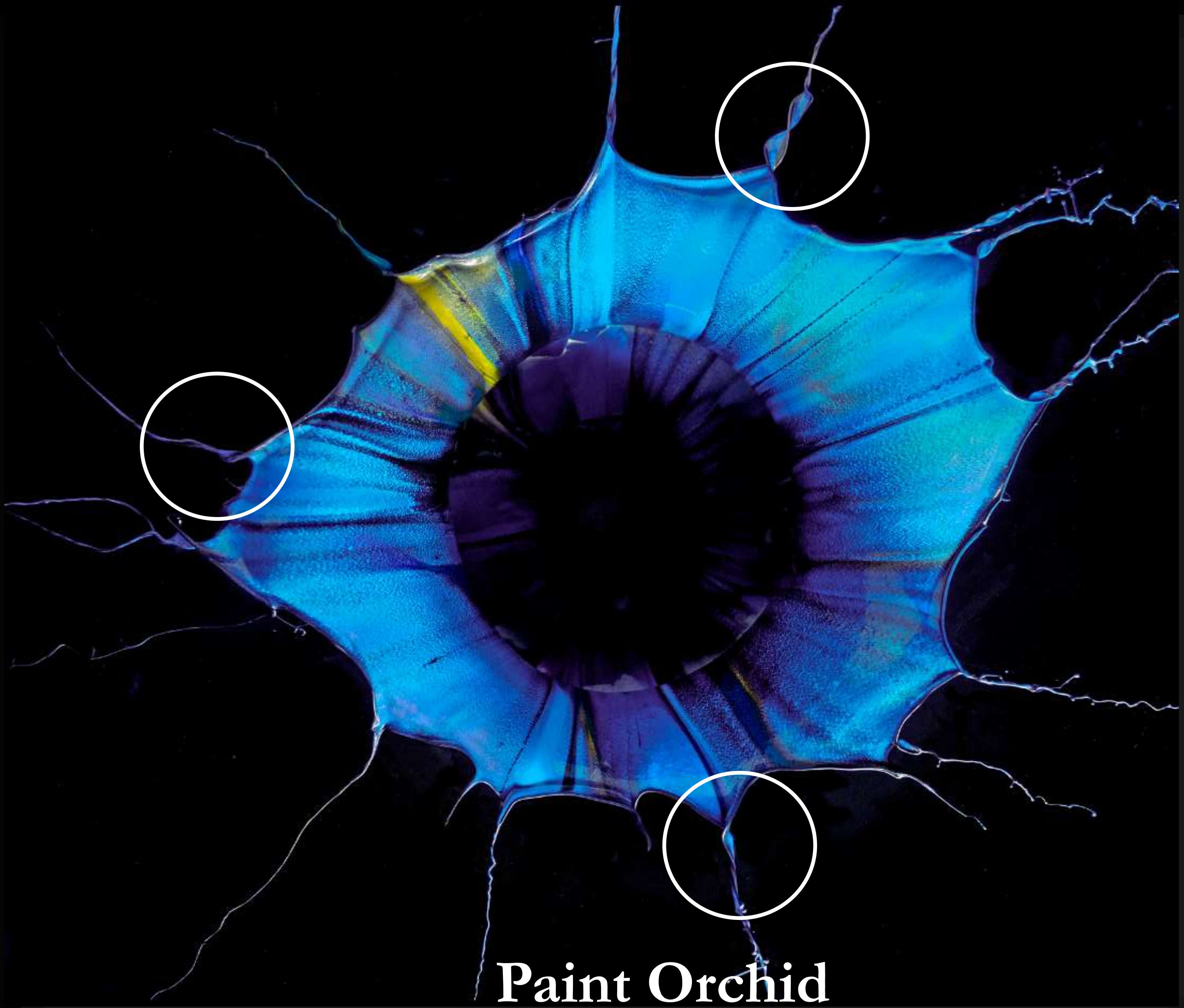




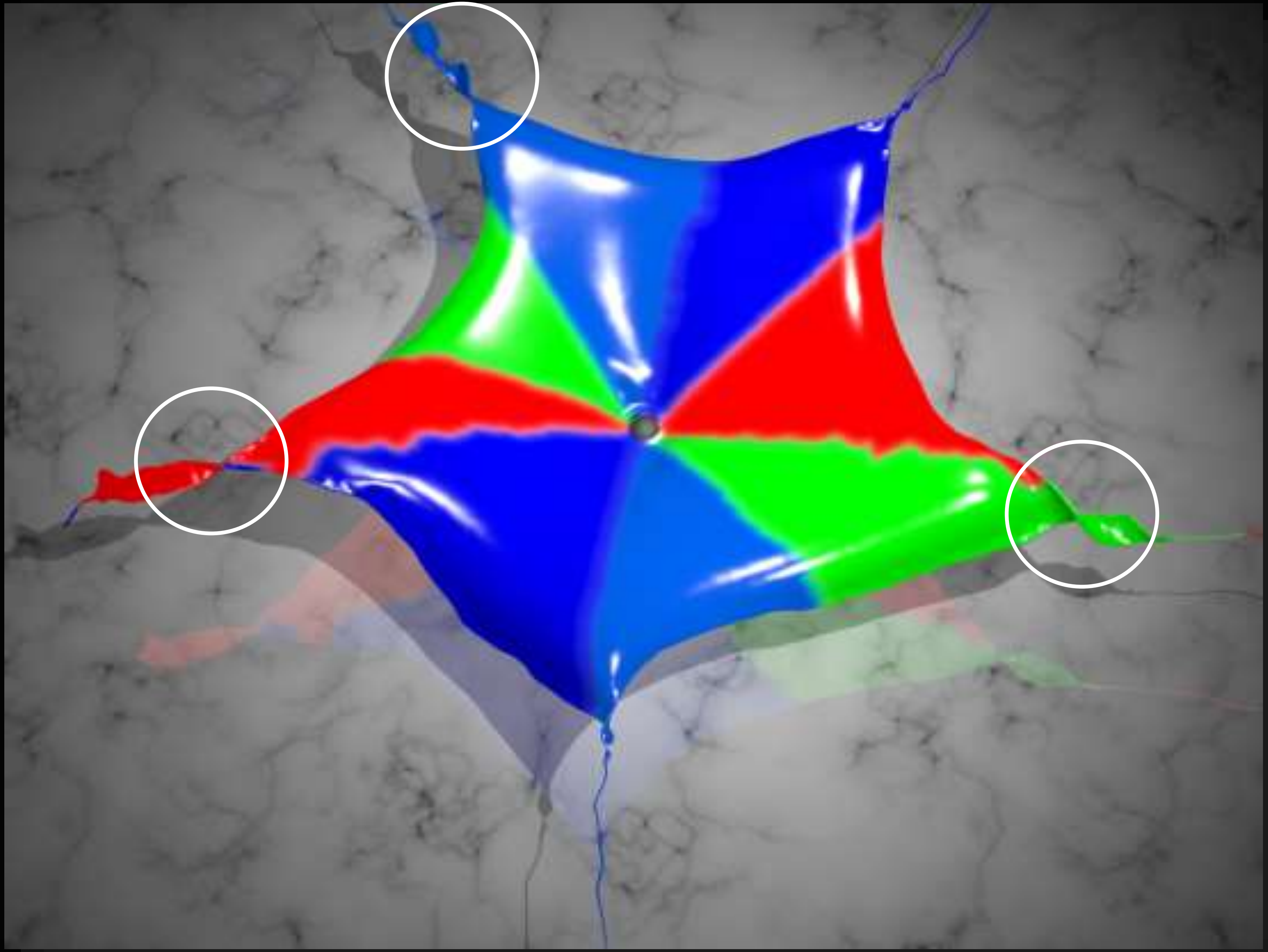
Math behind a pizza piece



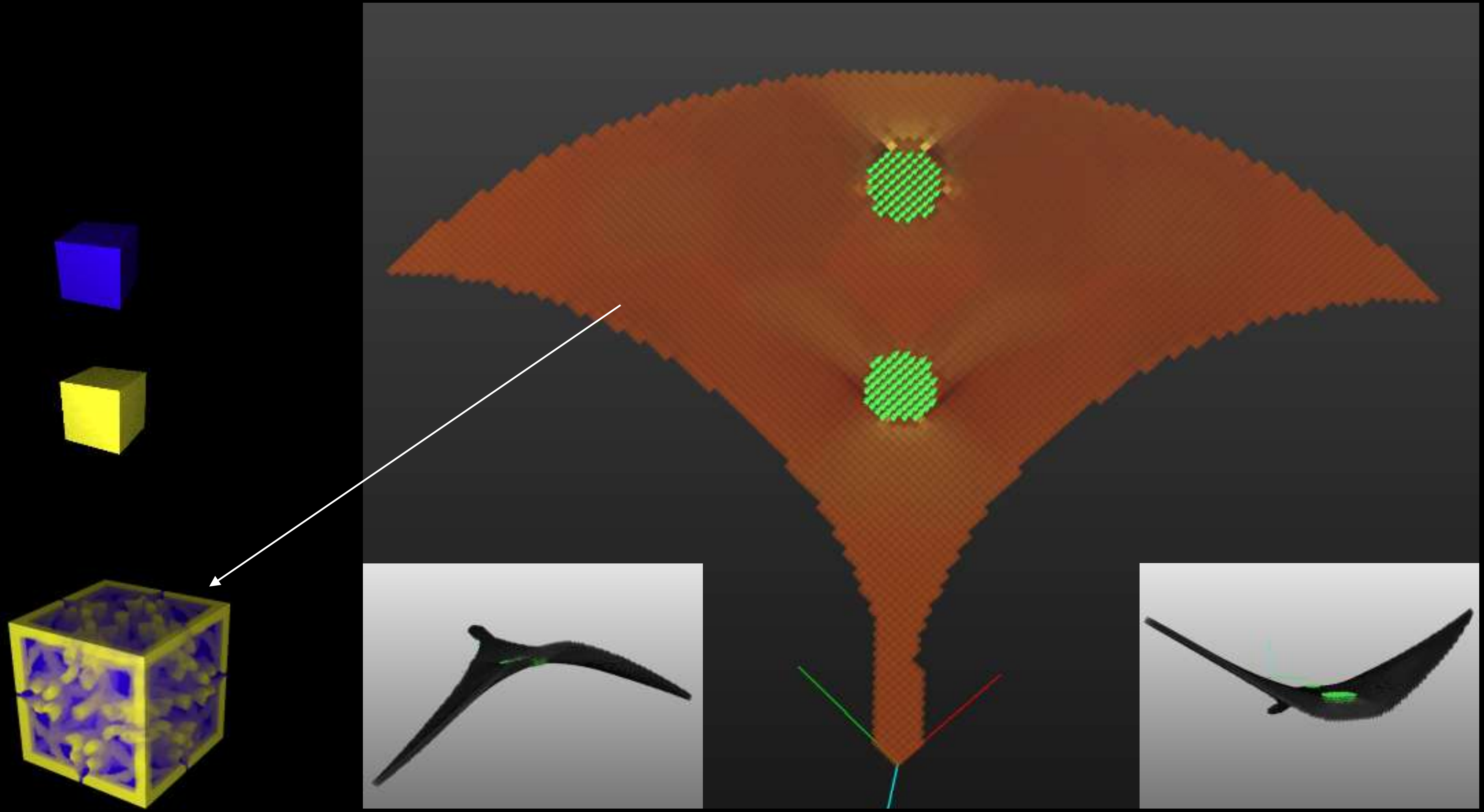
$$\vec{u}^{**} = \vec{u}^{o} + \frac{\Delta t}{\rho}\nabla \cdot \mu(\dot{\gamma})((\nabla \vec{u}^{o})^{T} + \nabla \vec{u}^{**})$$

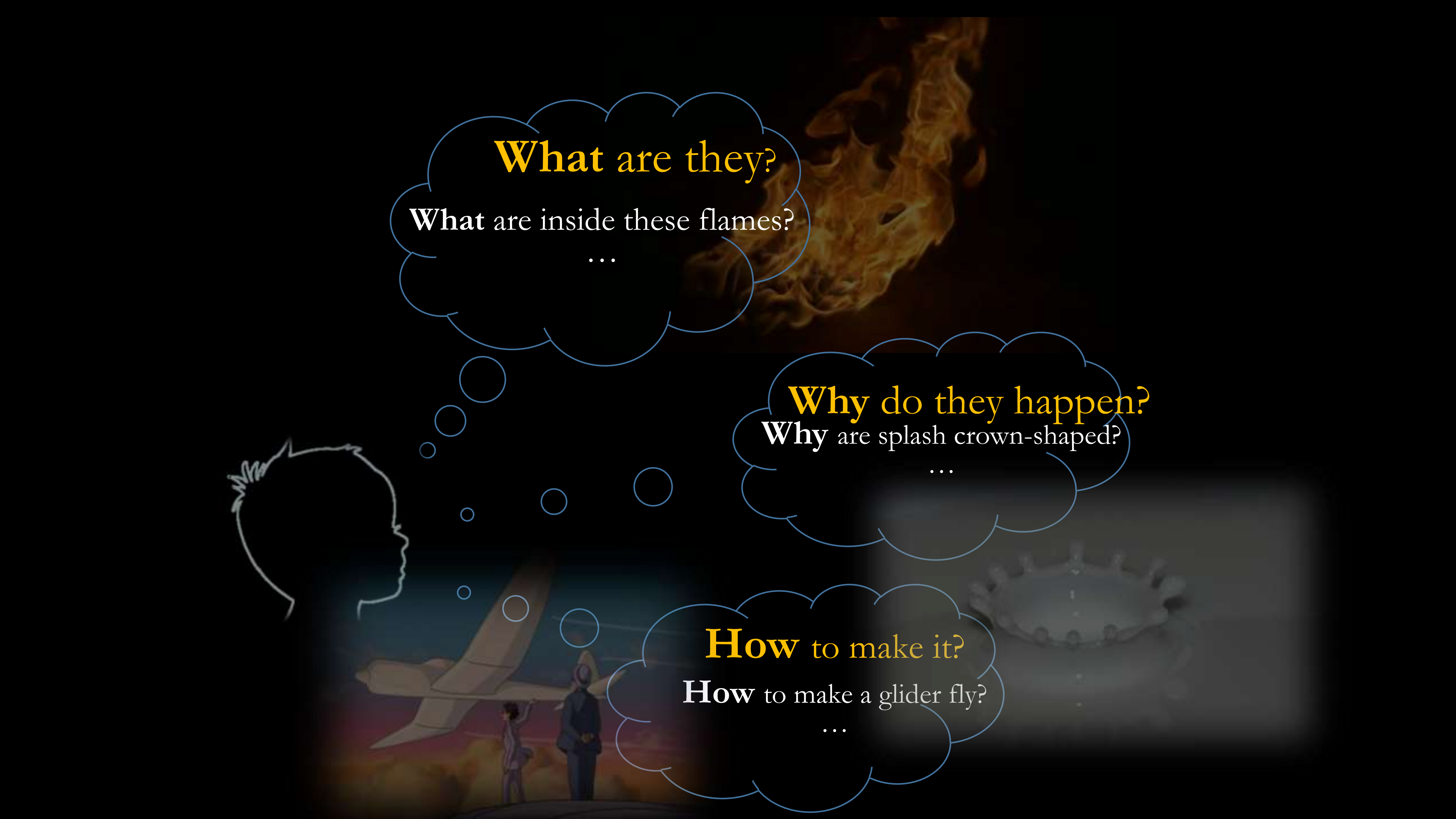


Paint Orchid

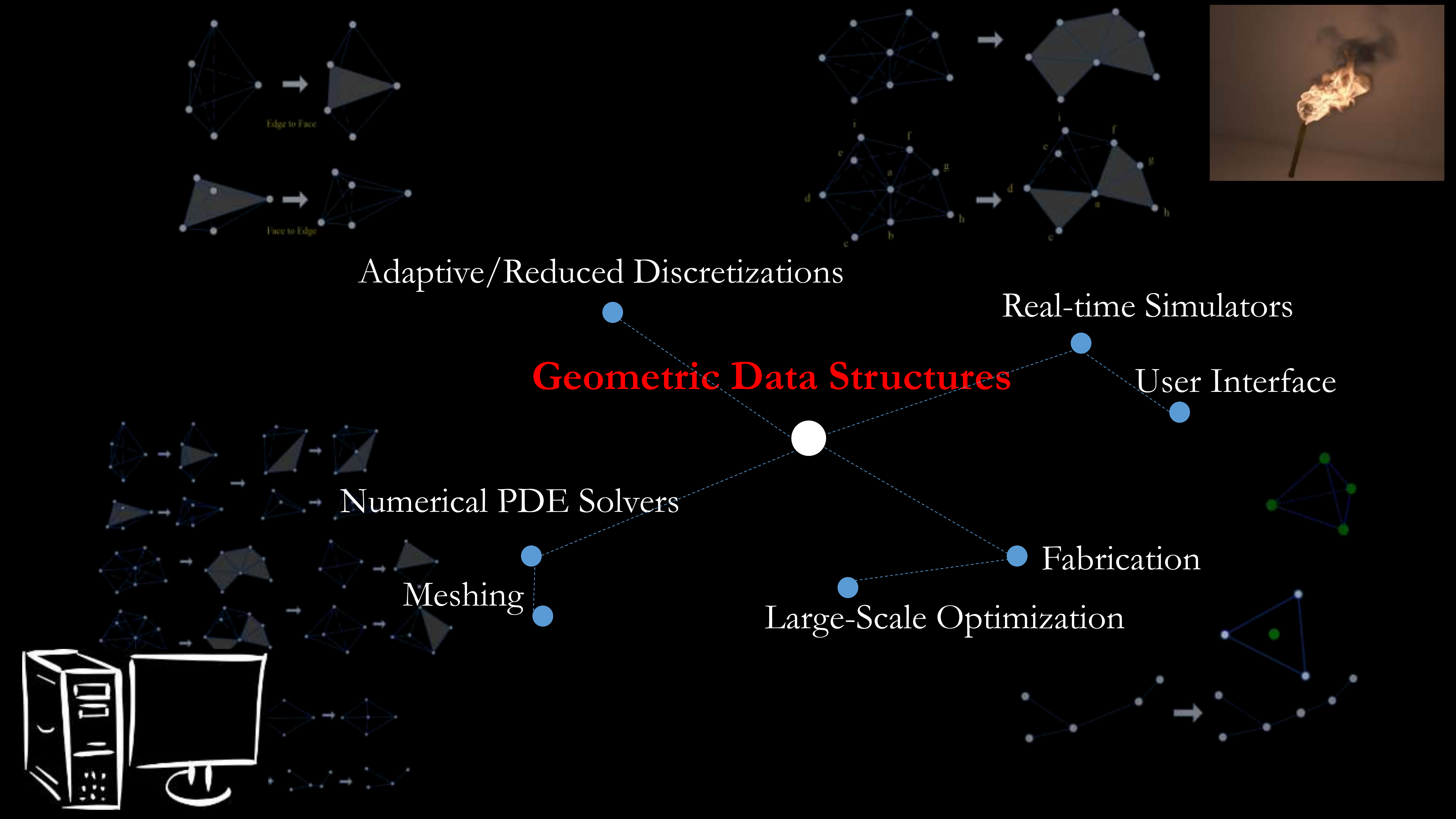


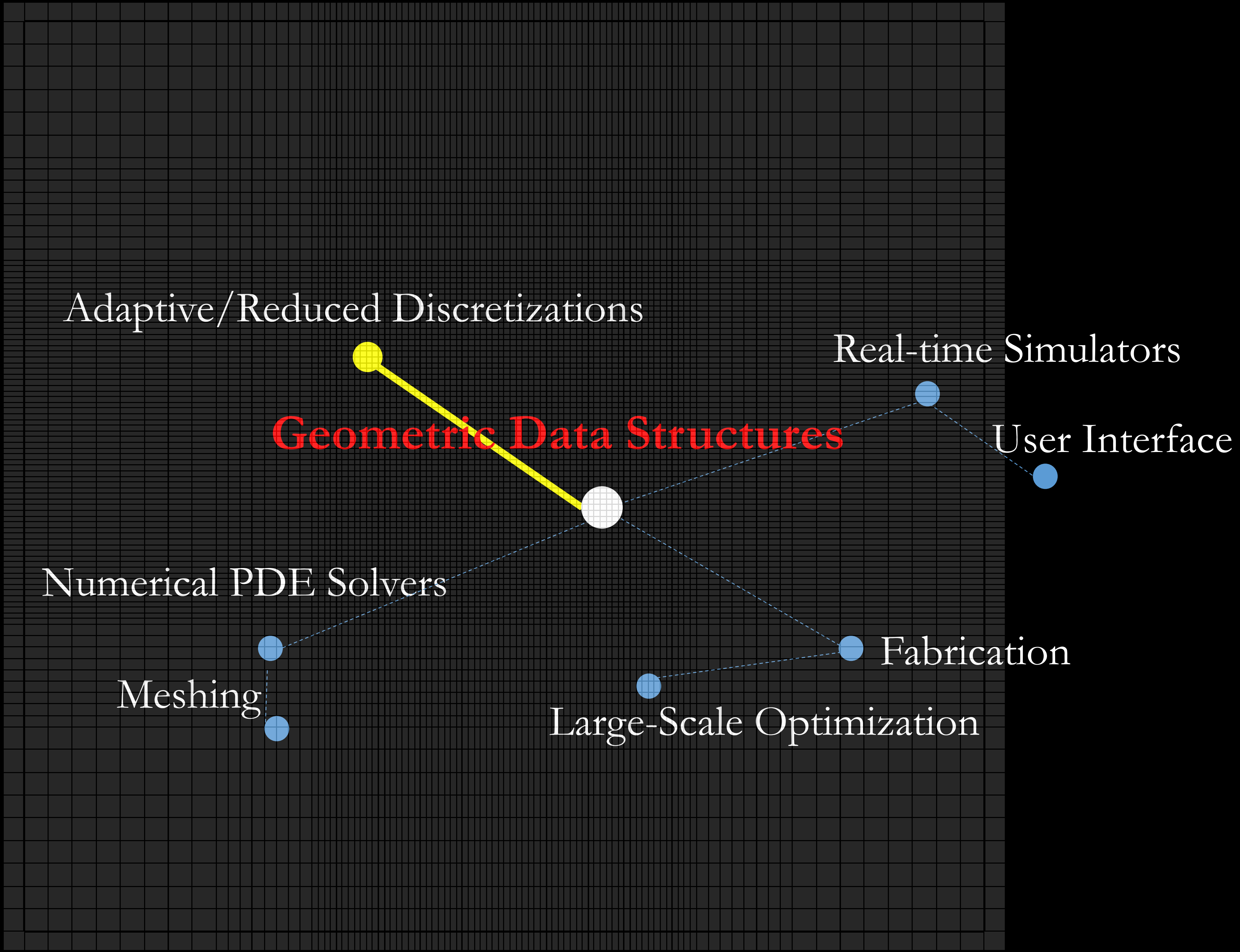
Functional Soft Bodies





What are they?
What are inside these flames?
…
Why do they happen?
Why are splash crown-shaped?
…
How to make it?
How to make a glider fly?
…


Edge to Face
Face to Edge
Adaptive/Reduced Discretizations
Real-time Simulators
Geometric Data Structures
User Interface
Numerical PDE Solvers
Meshing
Fabrication
Large-Scale Optimization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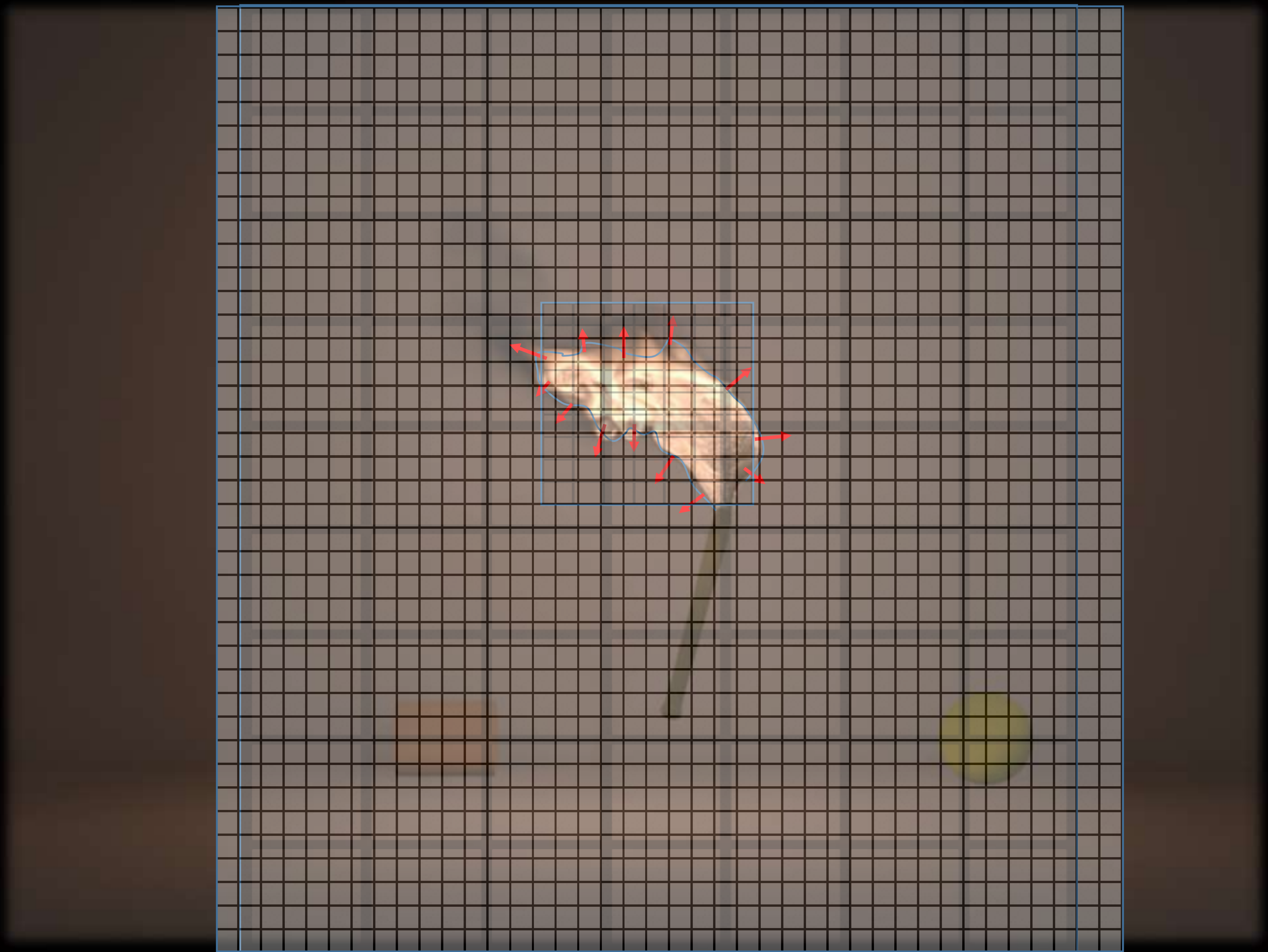
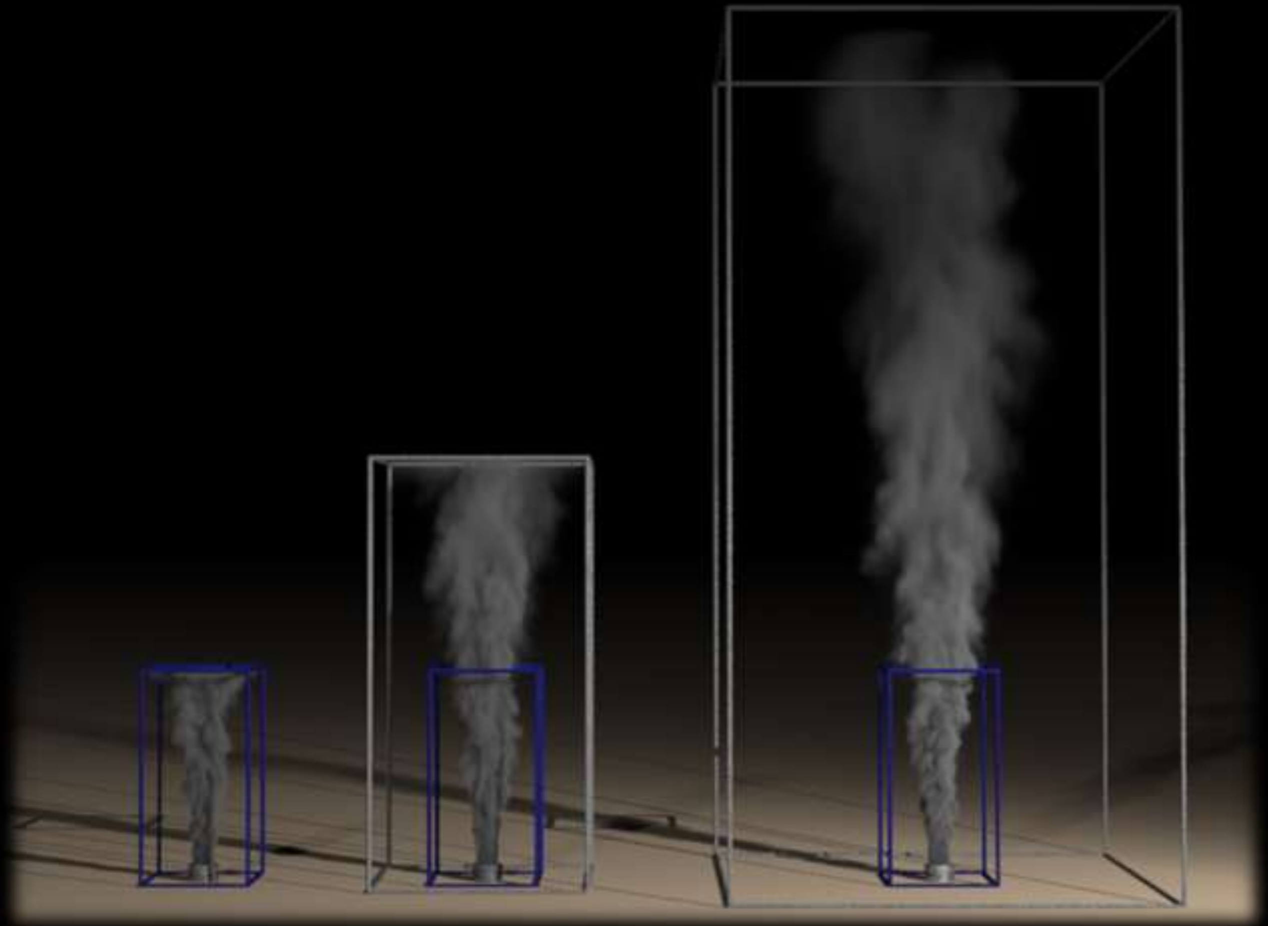
Large-scale Simulation for Film Visual Effects

Bo Zhu, Wenlong Lu, Matthew Cong, Byungmoon Kim, and Ron Fedkiw.
A New Grid Structure for Domain Extension.
ACM Trans. Graph. (SIGGRAPH 2013), 32, 63.1-63.8.



Domain Extension



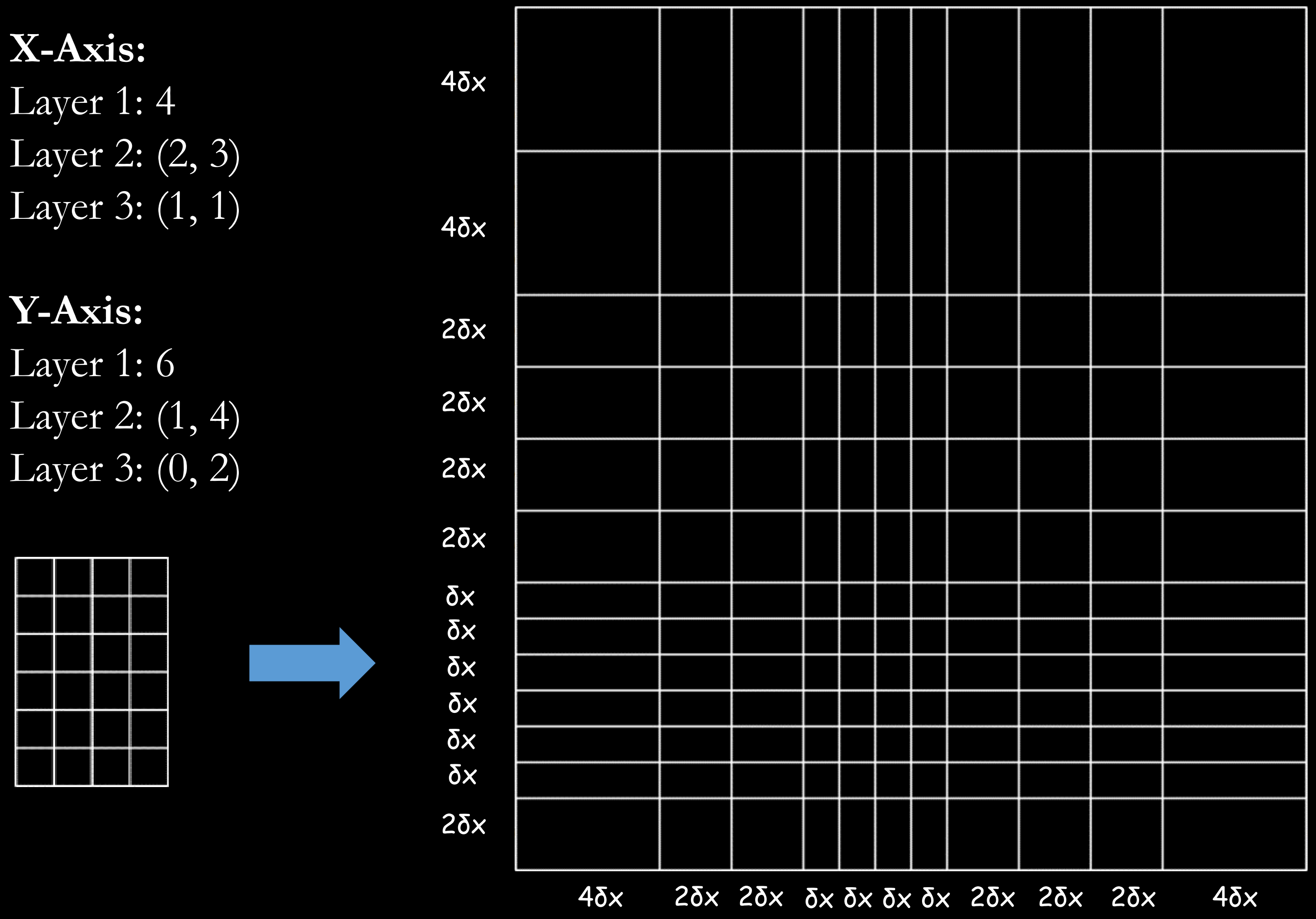
Sim time on a far-field grid:

1x	~~12x~~ 3.1x	~~160x~~ 6.1x

New Grid Structure

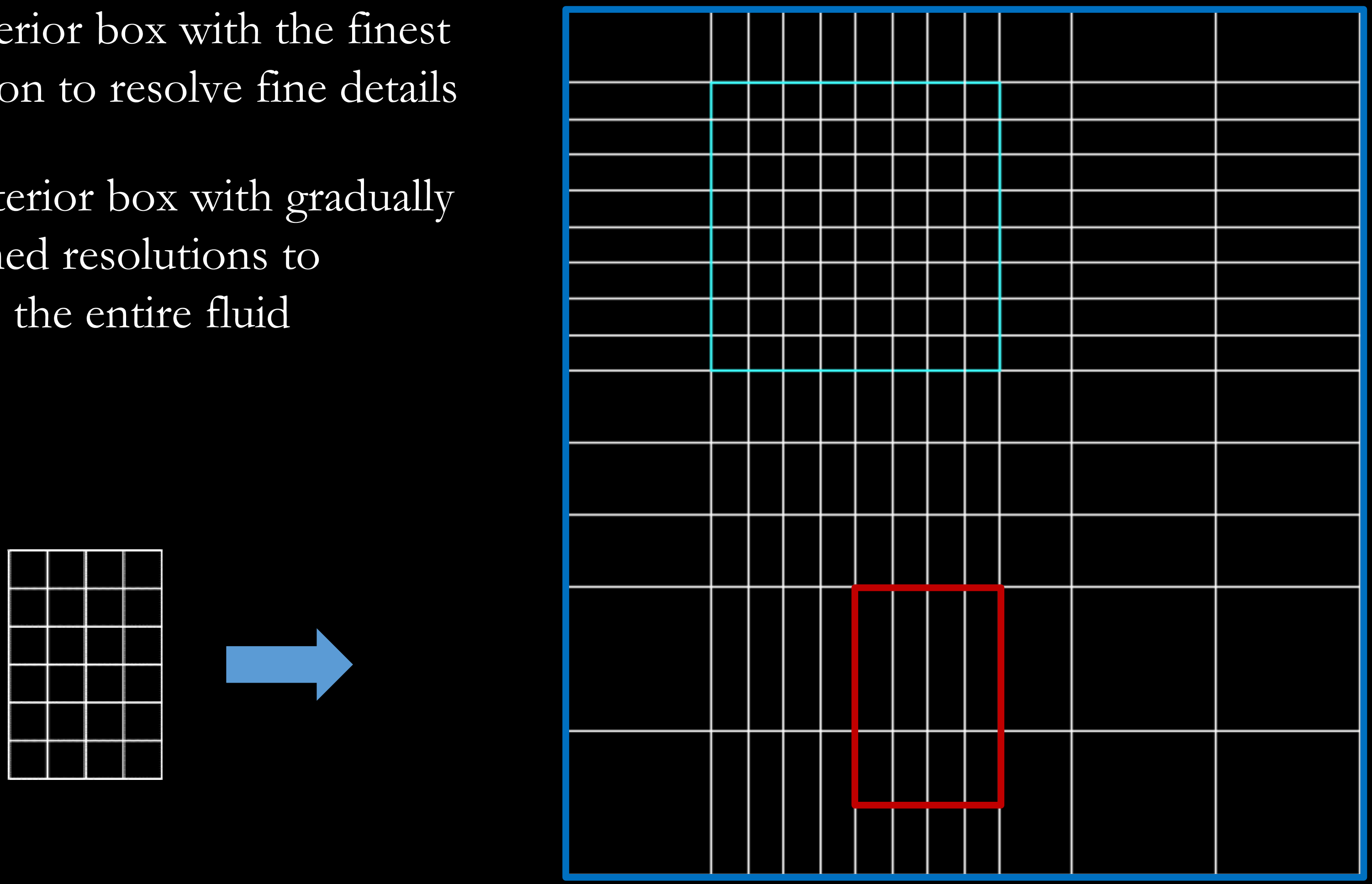
X-Axis:
Layer 1: 4
Layer 2: (2, 3)
Layer 3: (1, 1)

Y-Axis:
Layer 1: 6
Layer 2: (1, 4)
Layer 3: (0, 2)

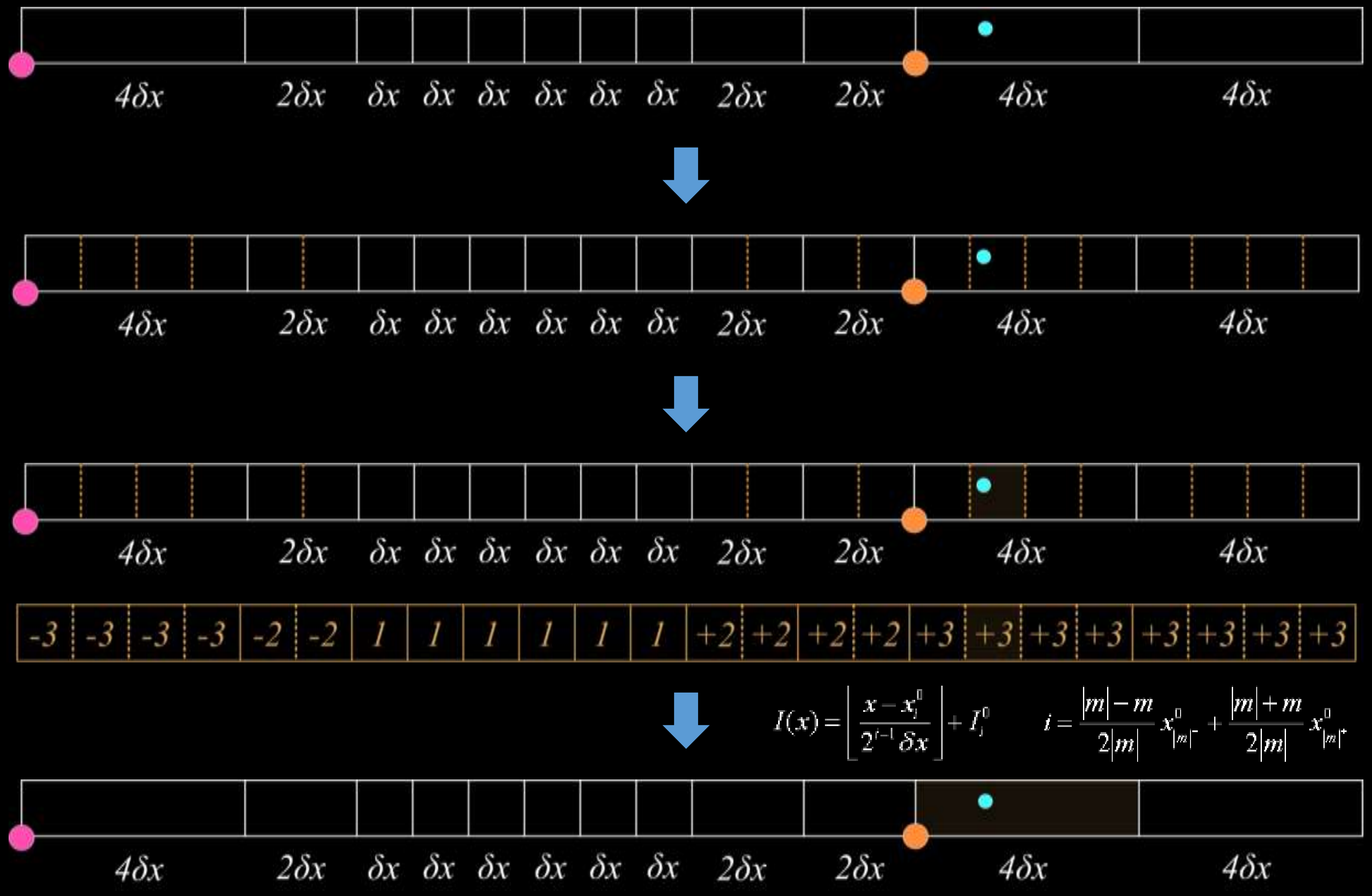


Two Grid Boxes

- The interior box with the finest resolution to resolve fine details
- The exterior box with gradually coarsened resolutions to enclose the entire fluid



Fast Index Access



1D Array for Layer Information

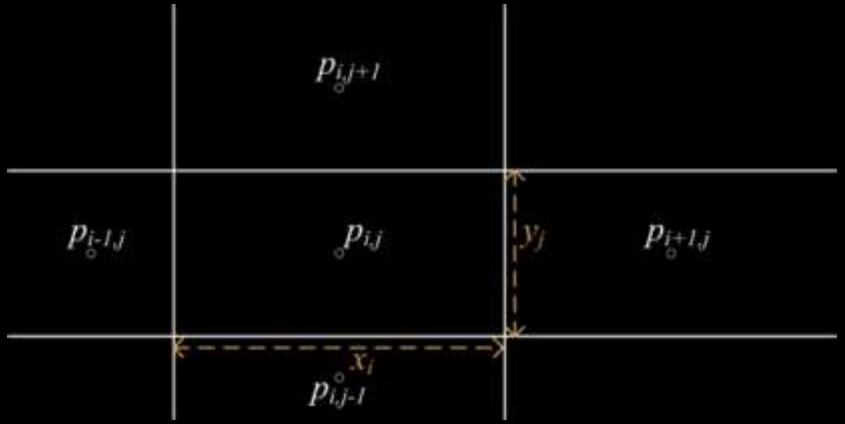
$$I(x) = \left\lfloor \frac{x - x_i^0}{2^{i-1}\delta x} \right\rfloor + I_i^0 \qquad i = \frac{|m|-m}{2|m|} x^0_{|m|^-} + \frac{|m|+m}{2|m|} x^0_{|m|^+}$$

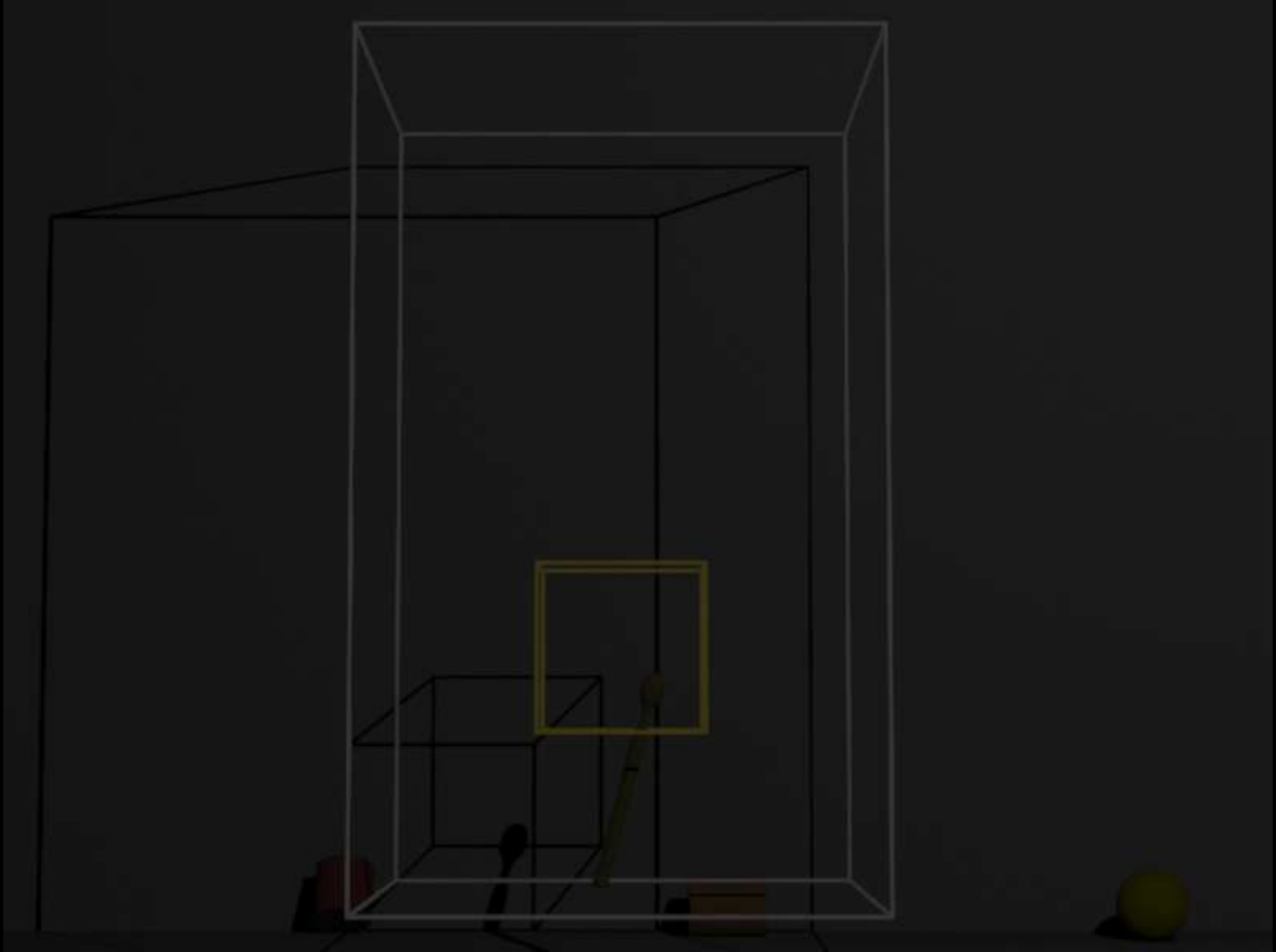
Solving Incompressible Flow on Stretched Grid Cells

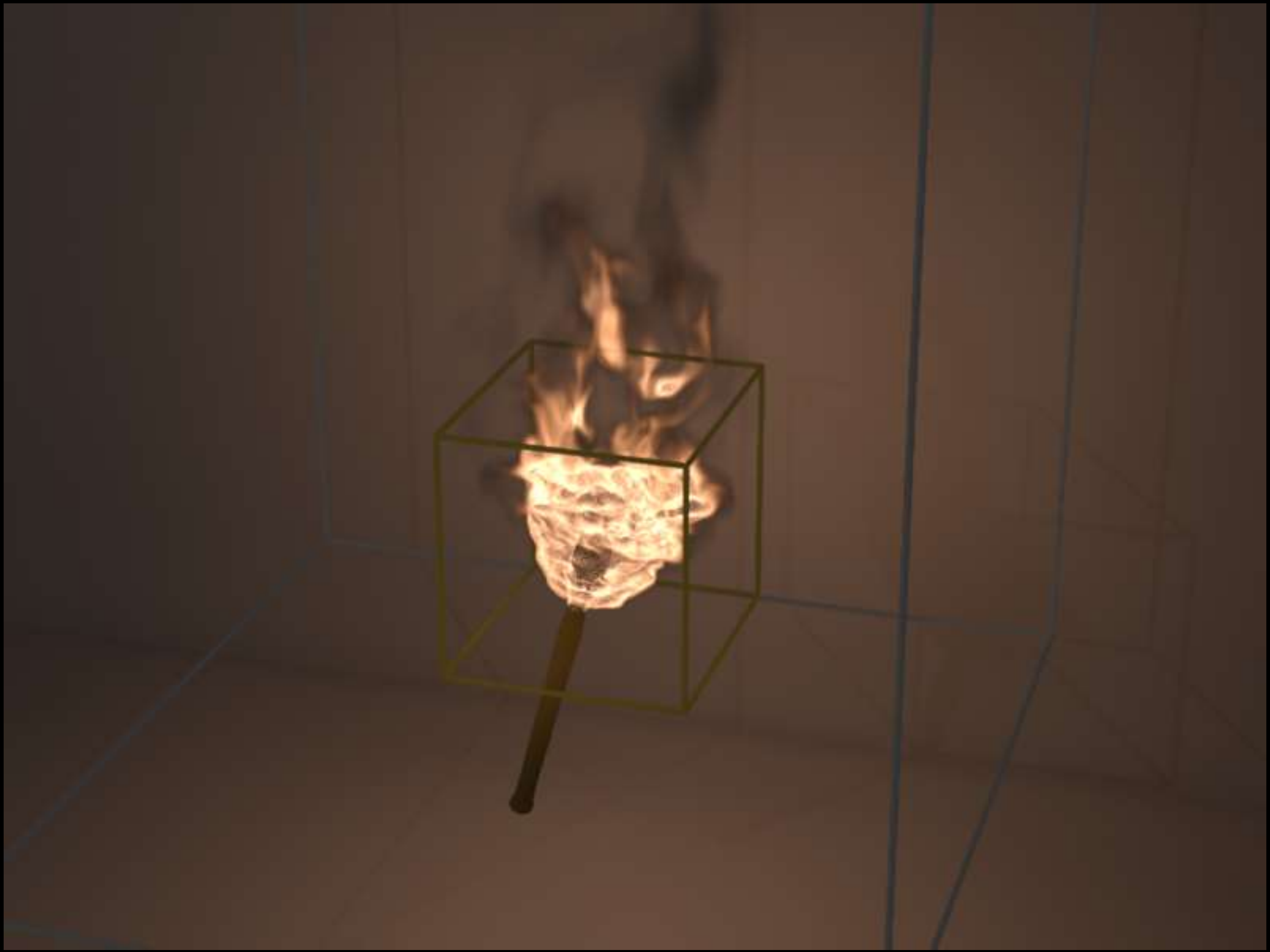
- Use the volume weighted divergence to solve the Poisson equation for pressure on stretched cells in order to obtain a SPD system

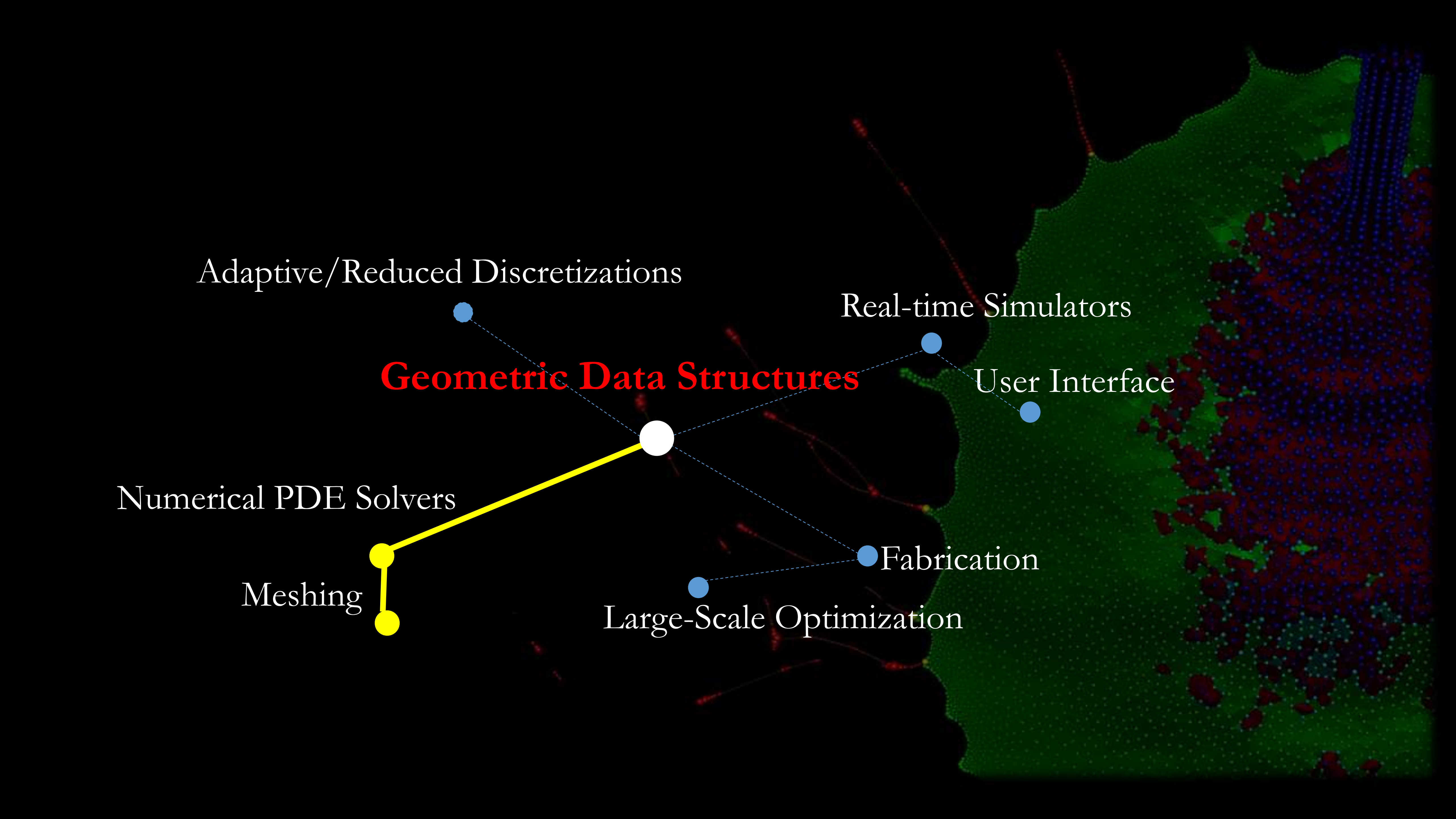
$$V_{\text{cell}} \nabla \cdot \left(\frac{\nabla \hat{p}}{\rho} \right) = V_{\text{cell}} \nabla \cdot \mathbf{u}^*$$

$$\sum_{\text{faces}} \frac{\nabla \hat{p}}{\rho} \cdot \mathbf{dA}_{\text{face}} = \sum_{\text{faces}} \mathbf{u}^*_{\text{face}} \cdot \mathbf{dA}_{\text{face}}$$





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Large-Scale Optimization

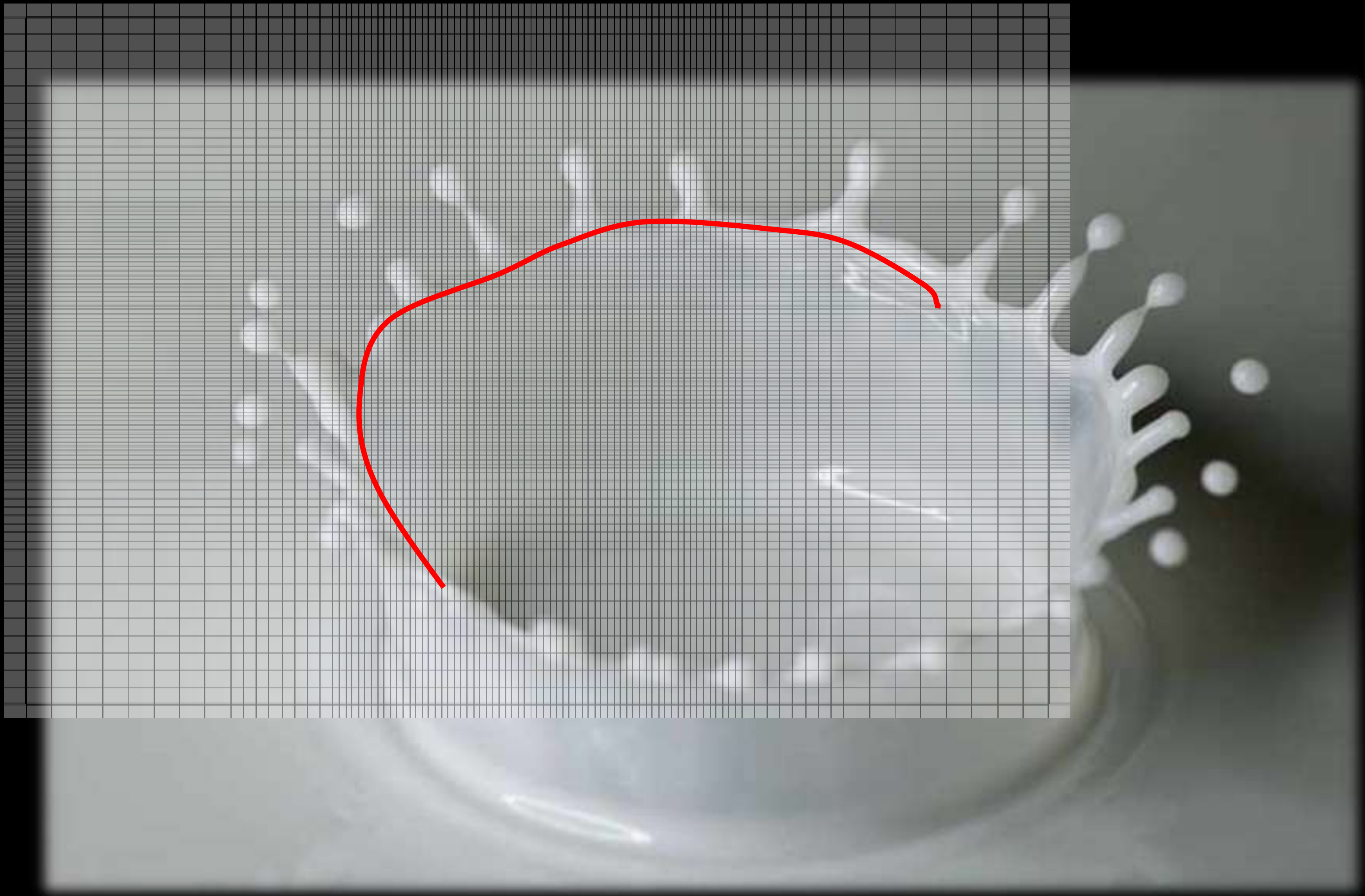
Computational Tools for Exploring Fundamental Sciences

Bo Zhu, Ed Quigley, Matthew Cong, Justin Solomon, and Ron Fedkiw.
Codimensional Surface Tension Flow on Simplicial Complexes.
ACM Trans. Graph. (SIGGRAPH 2014).

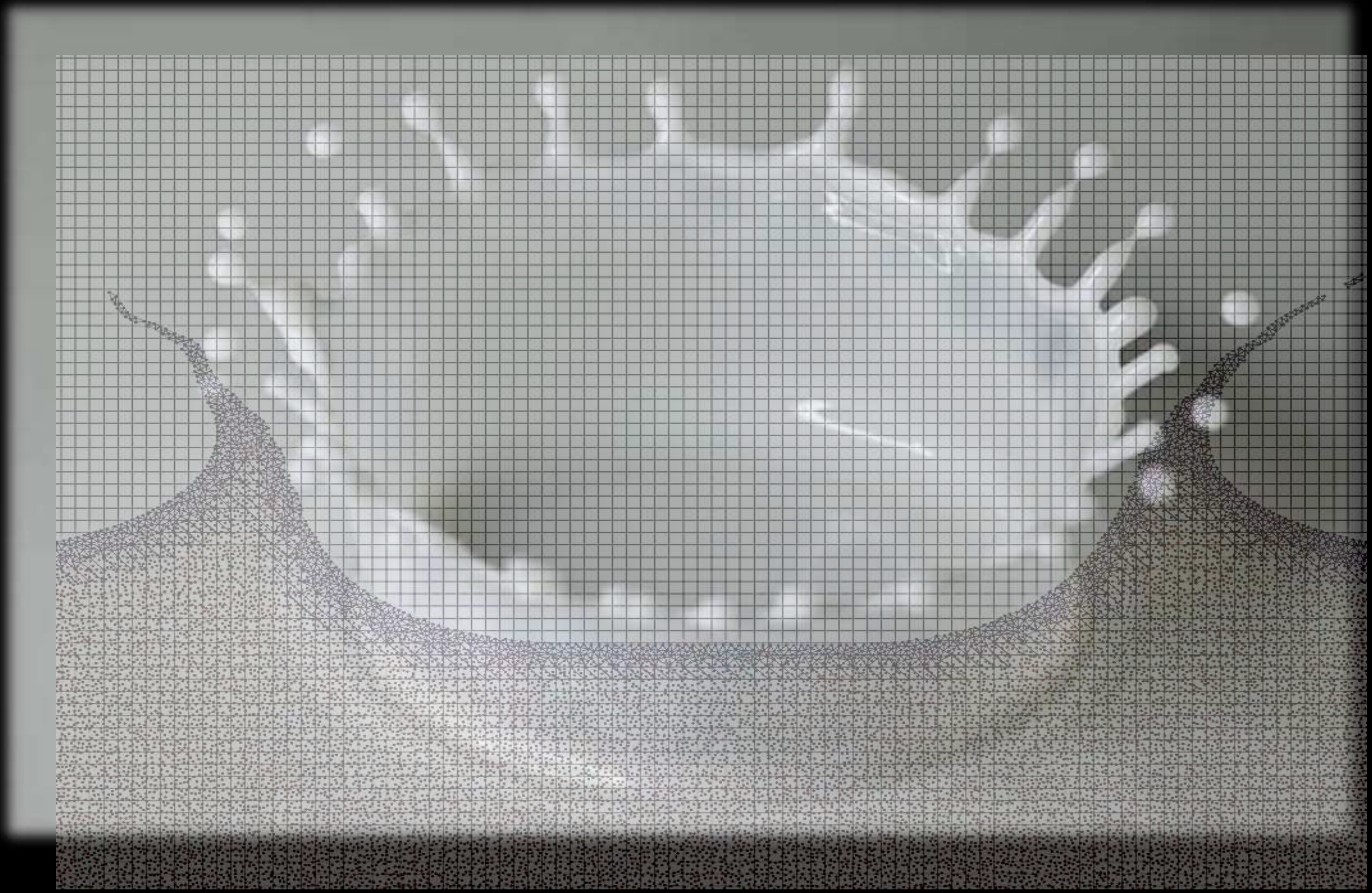
Bo Zhu, Minjae Lee, Ed Quigley, and Ron Fedkiw.
Codimensional Non-Newtonian Fluids.
ACM Trans. Graph. (SIGGRAPH 2015).

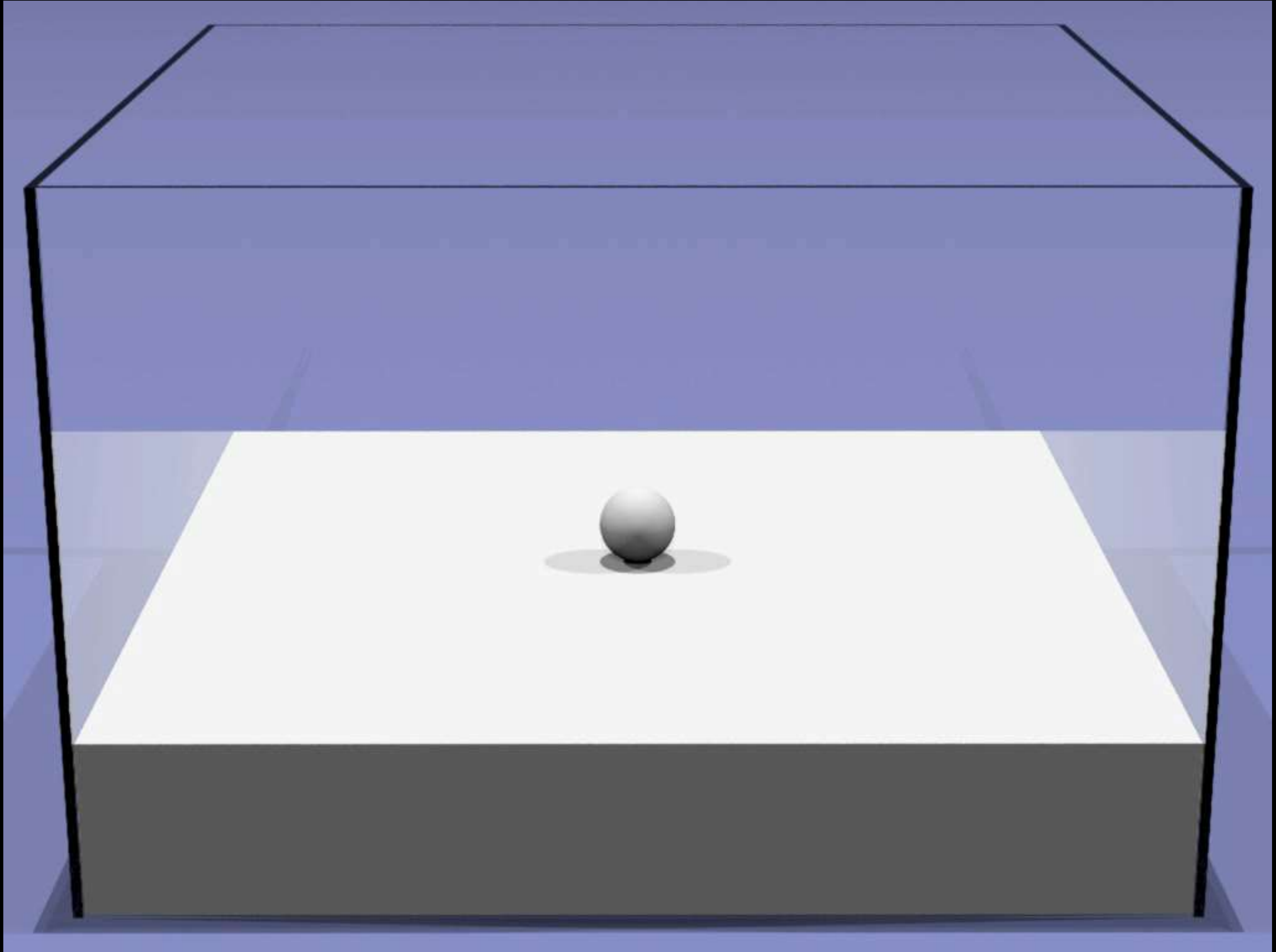
Wen Zheng, Bo Zhu, Byungmoon Kim, and Ron Fedkiw.
A New Incompressibility Discretization for a Hybrid Particle MAC Grid Representation with Surface Tension. J. Comp. Phys., 280, 94-142, 2015.

Anisotropic Thin Features

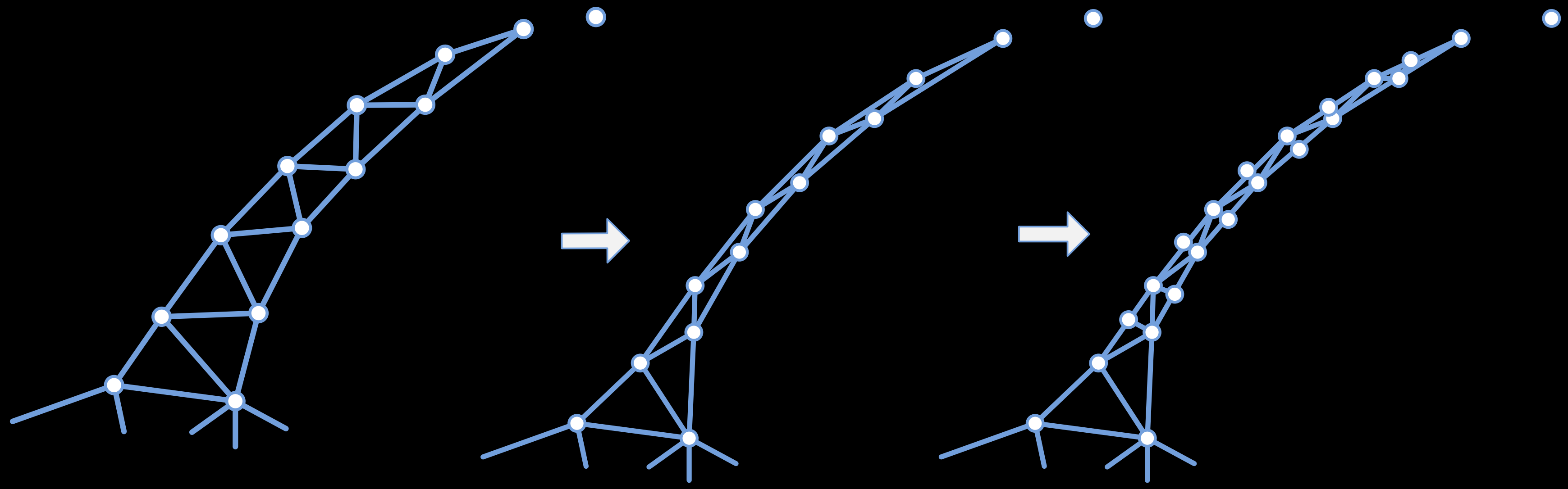


Embed a Lagrangian mesh in a grid

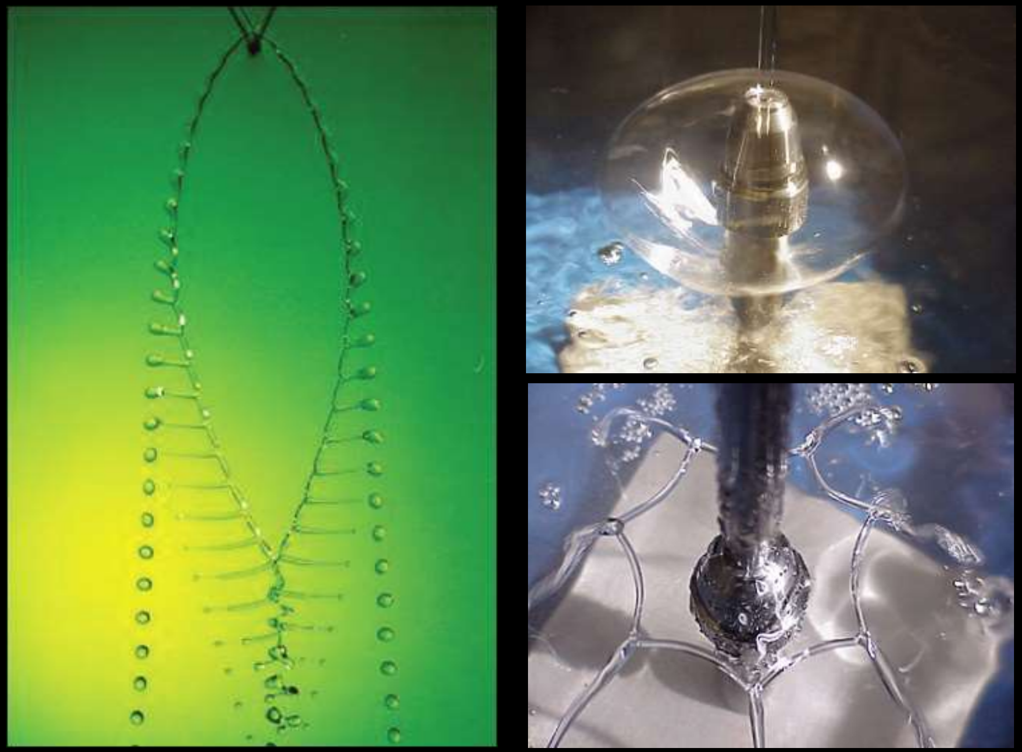




What will happen if the features get even thinner? Vanishingly thin?



These phenomena are not rare…



Membrane:

Oefner's photography

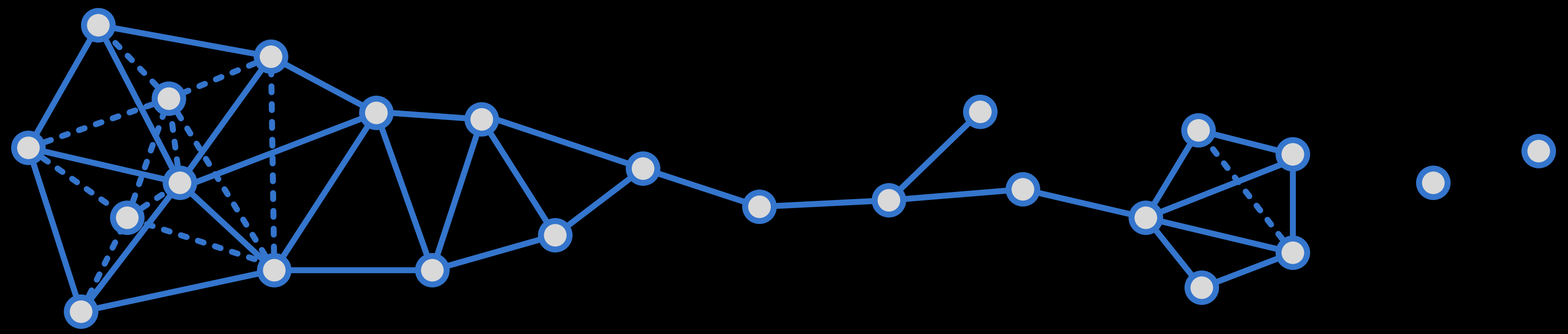
fabianoefner.com

Jets and sheets:

Bush's experiments,
MIT Applied Math Lab

Simplicial Complex

A geometric structure that consists of points, segments, triangles, and tetrahedra



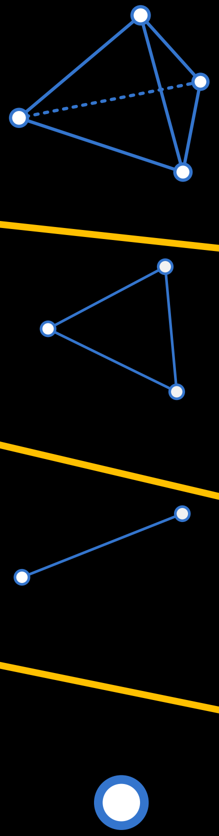
$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{\vec{f}}{\rho}$$

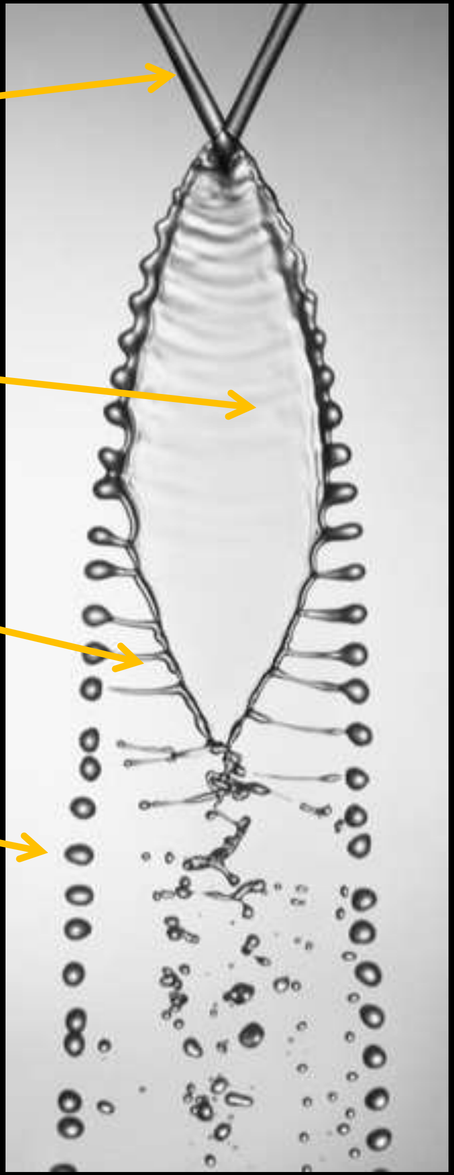
$$\nabla \cdot \vec{u} = 0$$

surface tension, adhesion, gravity, etc.

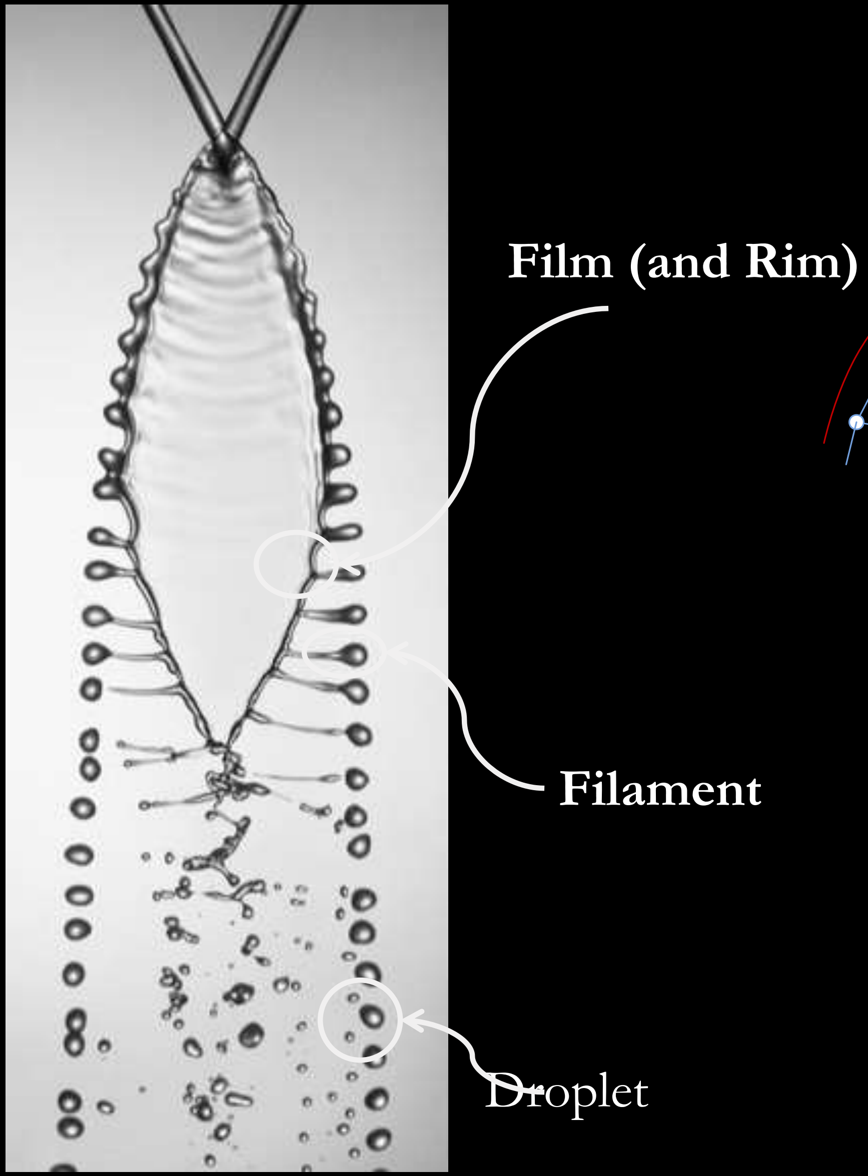
Discrete Geometric Analogues

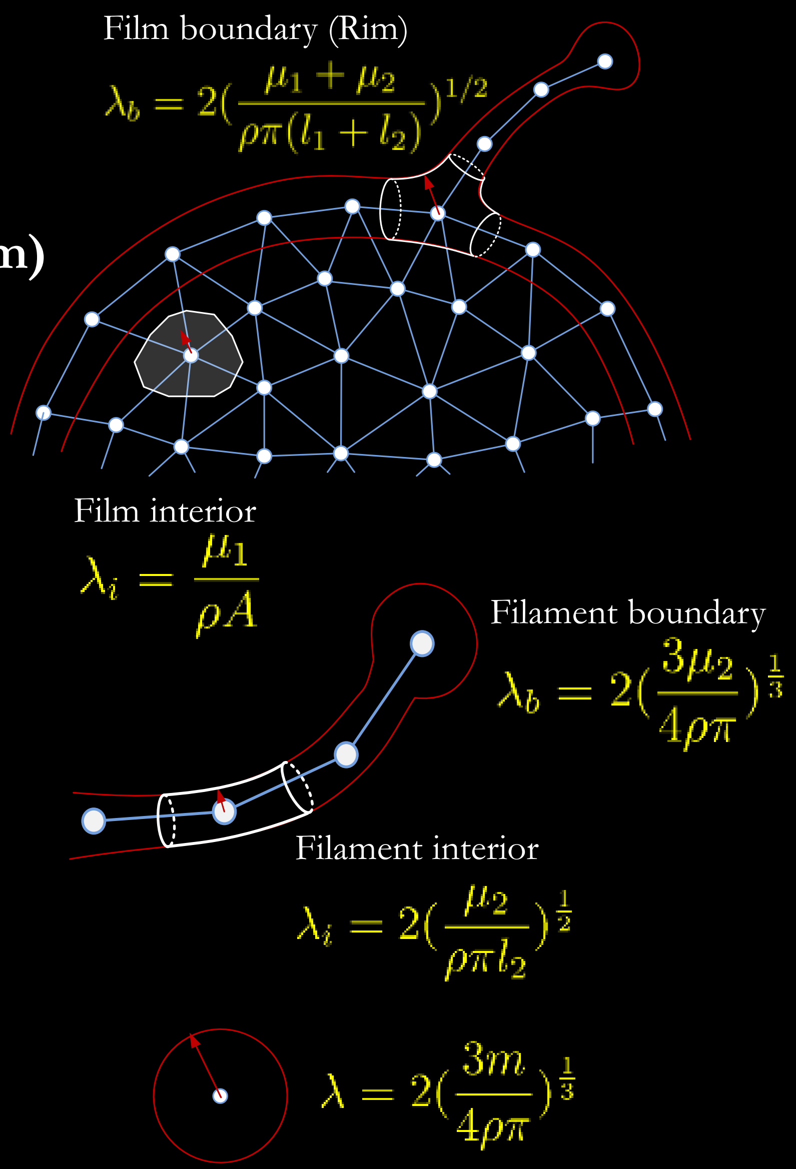
Codimension-0	**Tetrahedra**
Codimension-1	**Triangles**
Codimension-2	**Segments**
Codimension-3	**Points**





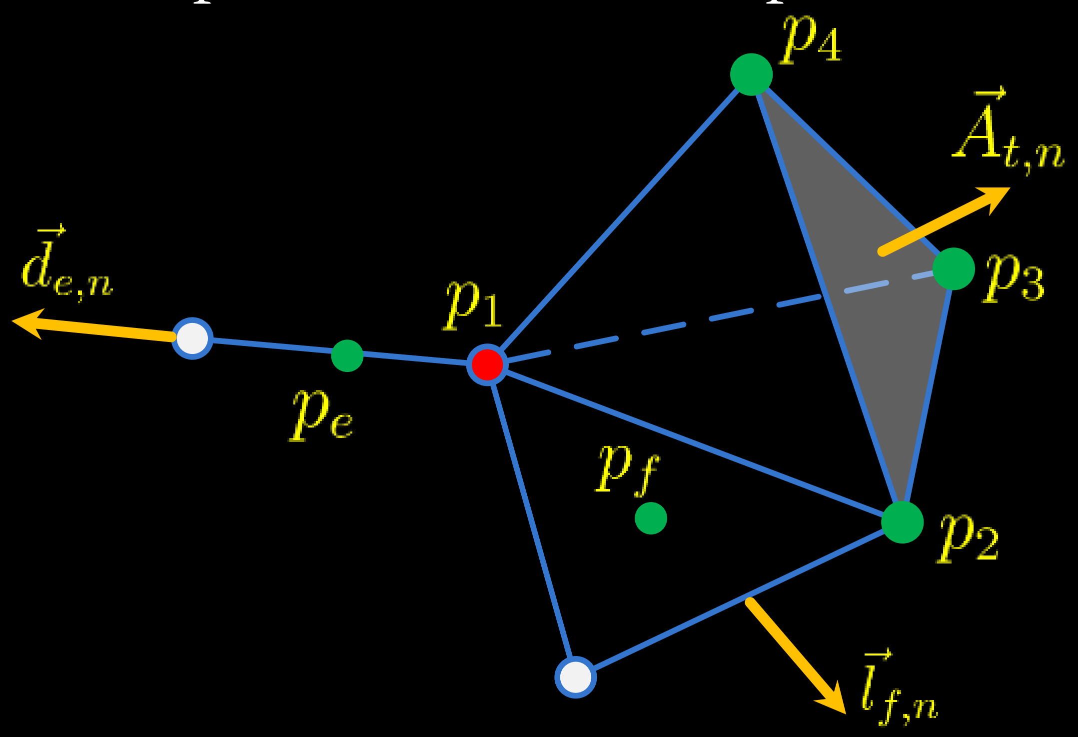
Reduced Geometry





Codimensional Volume-Weighted Gradient

- For all the simplexes incident to a particle:



$$W_n \nabla p = \left(V_n \nabla + \lambda_n A_n \nabla + \frac{\pi \lambda_n^2}{4} L_n \nabla \right) p$$

Discretized Poisson Equation

- Poisson equation:

$$\nabla \cdot \frac{1}{\rho} \nabla p = \frac{1}{\Delta t} \nabla \cdot \vec{u}^{**}$$

- Volume weighted formula:

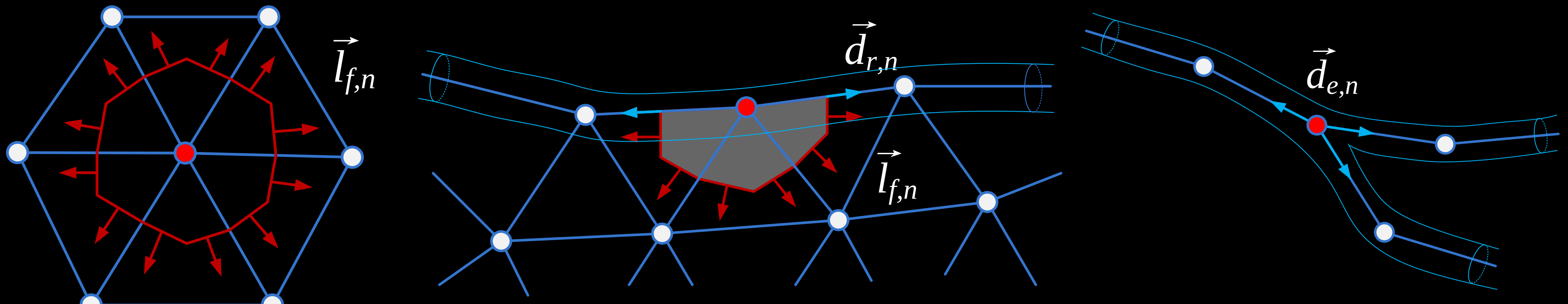
$$V \nabla \cdot \frac{1}{\rho V} V \nabla p = \frac{1}{\Delta t} V \nabla \cdot \vec{u}^{**}$$

Discretizing

$$-\mathbf{G}^T \mathbf{M}^{-1} \mathbf{G} \mathbf{p} = -\frac{1}{\Delta t} \mathbf{G}^T \mathbf{u}$$

Surface Tension

- Discretization:



$$\vec{f}_{t,n} = \sigma \vec{l}_{t,n}/2$$

$$\vec{f}_n = \sum_{t \in F_n} \vec{f}_{t,n}$$

$$\vec{f}_{r,n} = c\hat{\lambda}\sigma \vec{d}_{r,n}$$

$$\vec{f}_n = \sum_{f \in F_n} \vec{f}_{f,n} + \sum_{r \in R_n} \vec{f}_{r,n}$$

$$\vec{f}_{e,n} = \pi \lambda_n \sigma \vec{d}_{e,n}$$

$$\vec{f}_n = \sum_{e \in E_n} \vec{f}_{e,n}$$

Meshing Algorithm

For each timestep

//Volumetric meshing

Tetrahedron edge/face flip

Tetrahedron edge split

Skinny tetrahedron collapse

Tetrahedron edge/face flip

//Thin film meshing

Triangle edge split

Triangle edge collapse

Triangle edge flip

Triangle crumple merge

//Filament meshing

Segment edge split

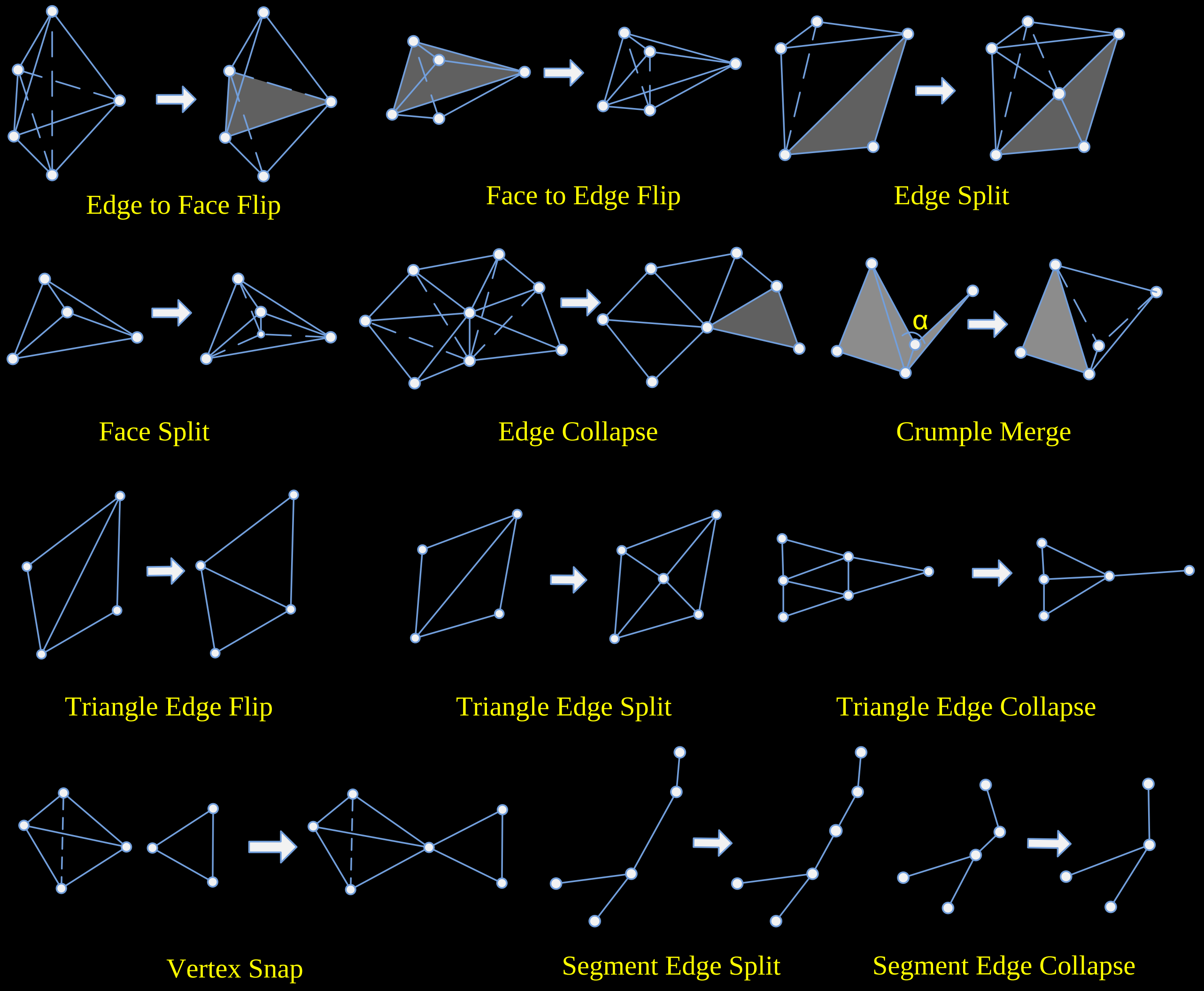
Segment edge collapse

//Topological merging/breaking

Boundary vertex snap

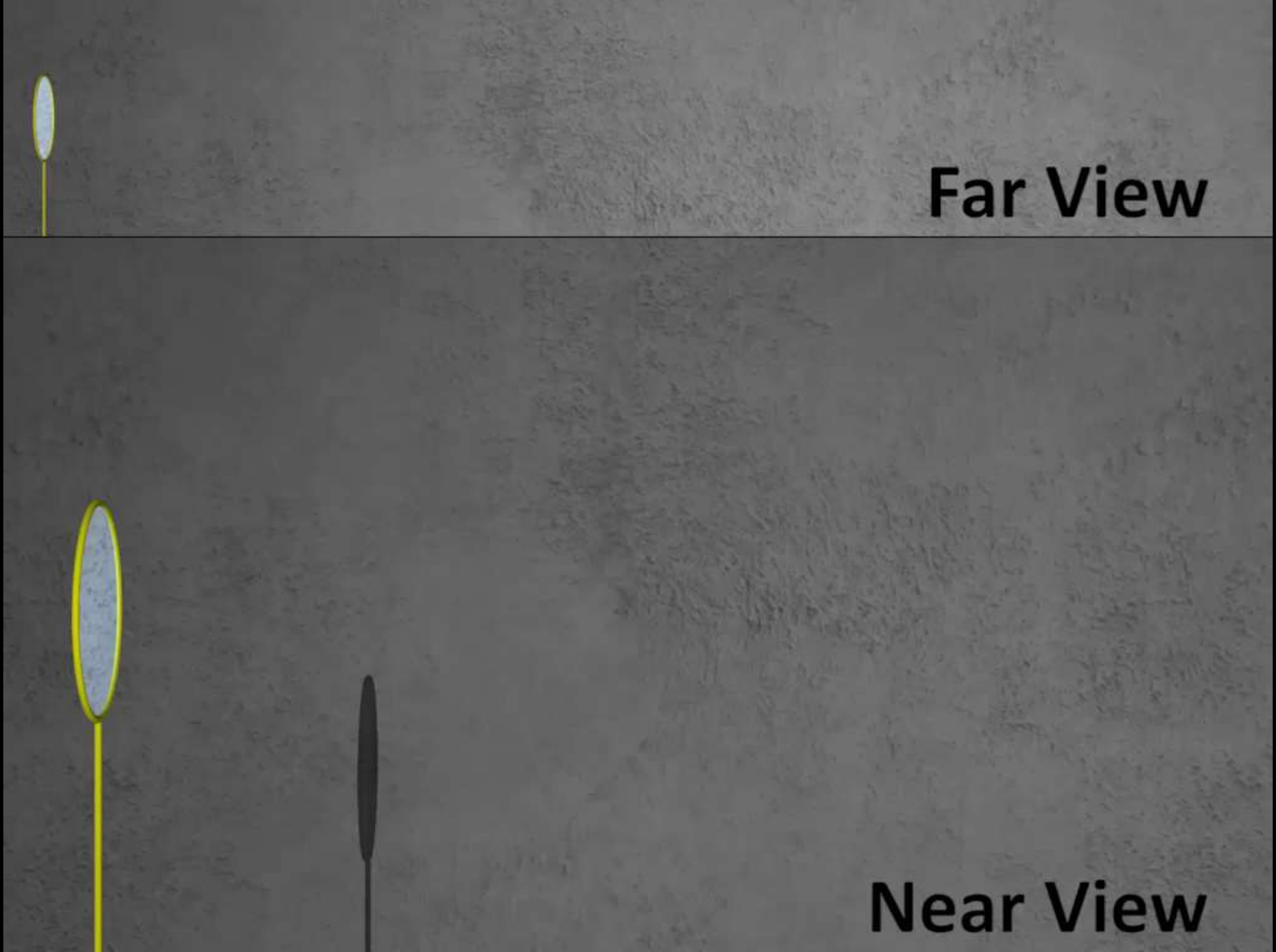
Thin triangle break

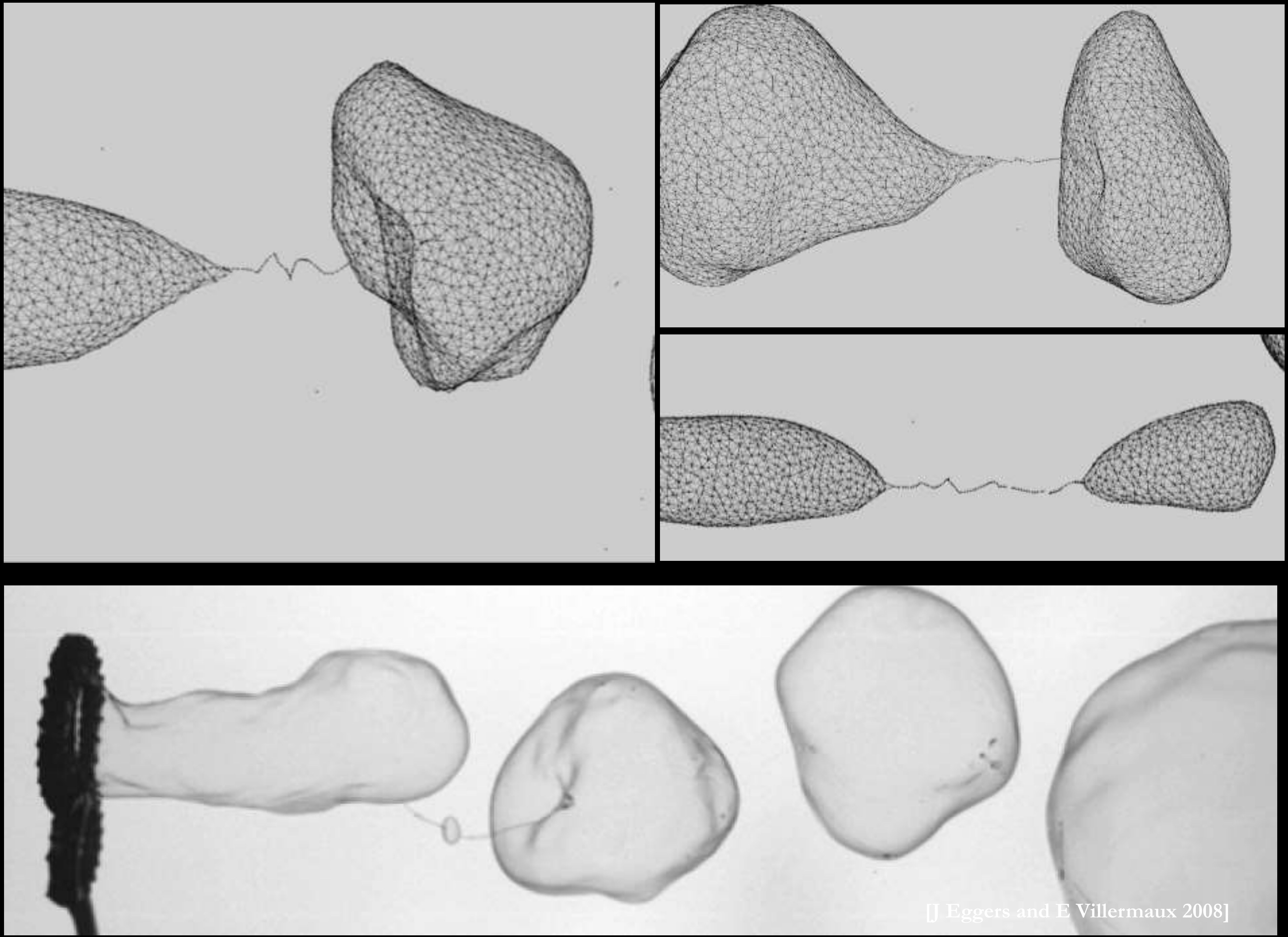
Thin segment break



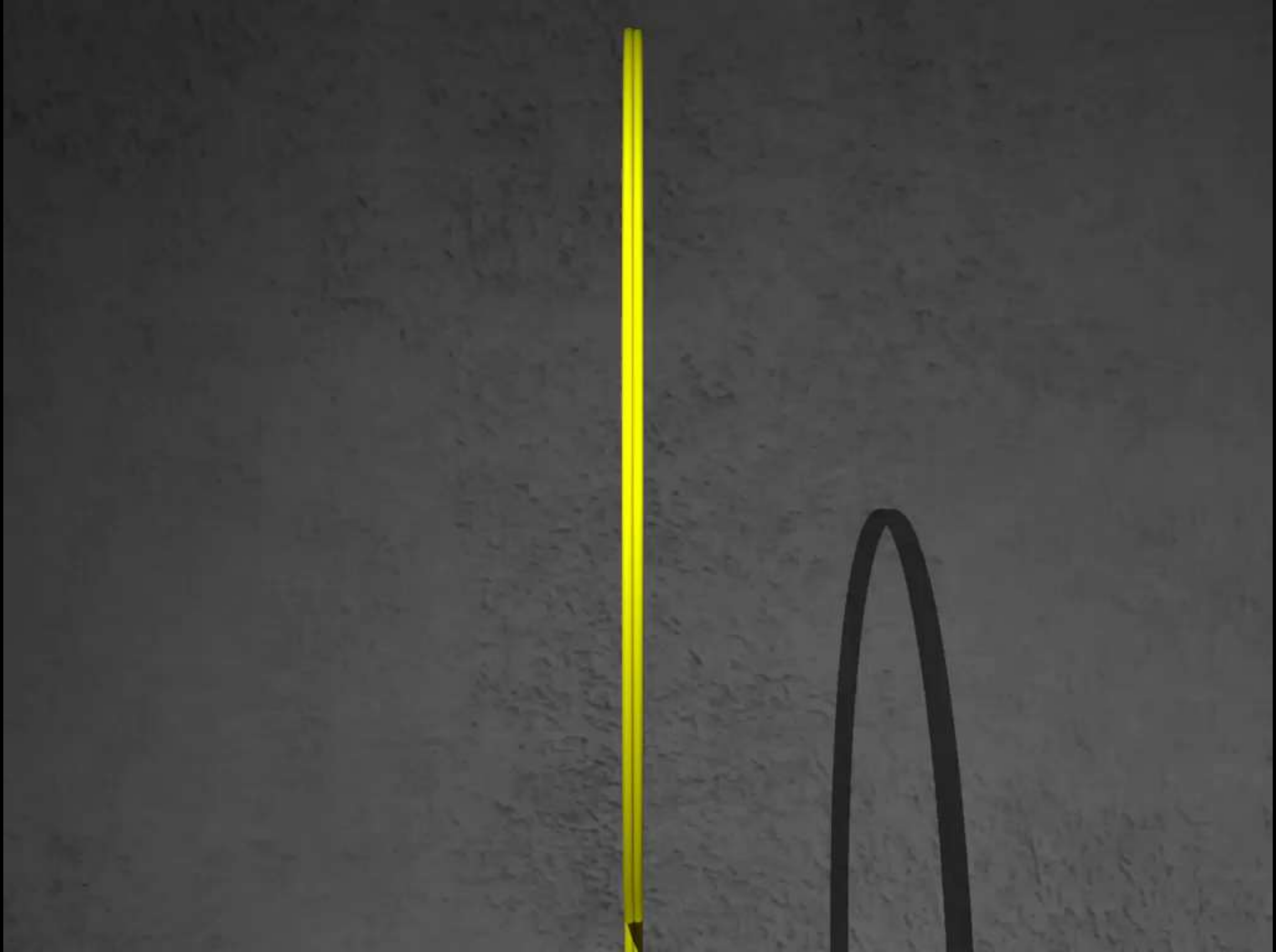
Example: Blowing Bubbles

http://www.soapbubble.dk/

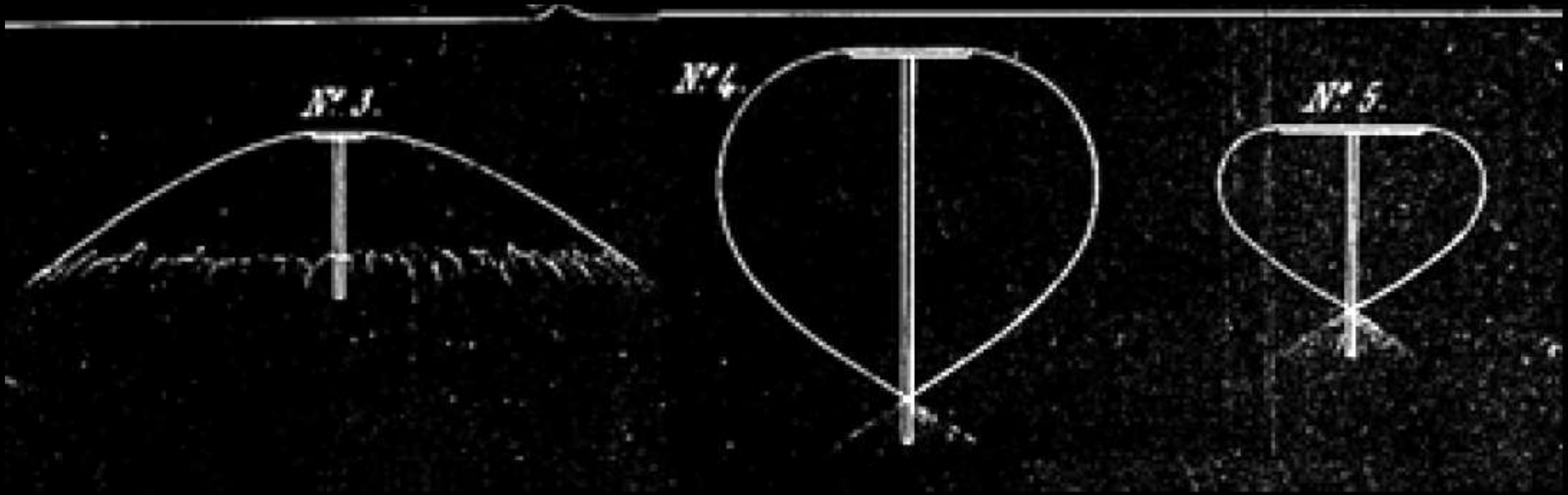

Far View
Near View


[J Eggers and E Villermaux 2008]

Example: Film Catenoid



Example: Waterbell

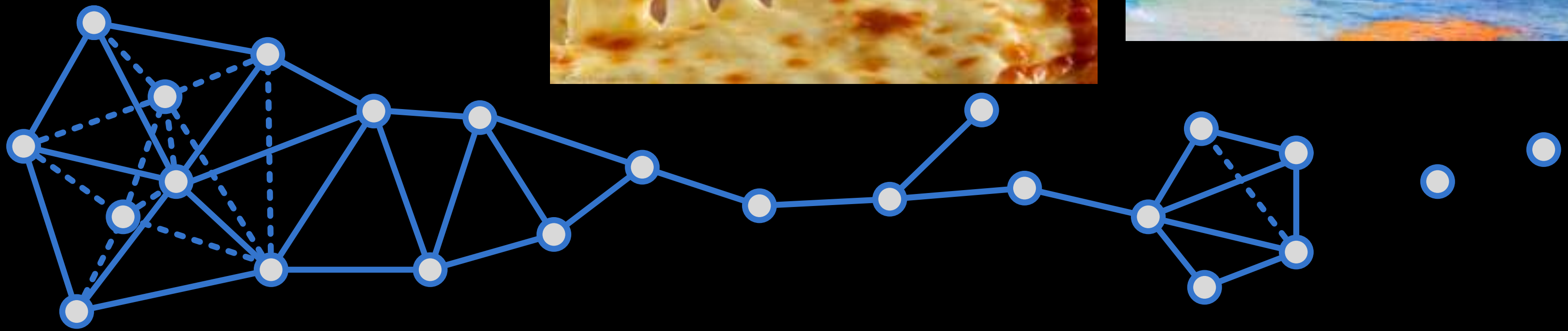


http://www.phikwadraat.nl/

Numerical Simulation of Non-Newtonian Fluids







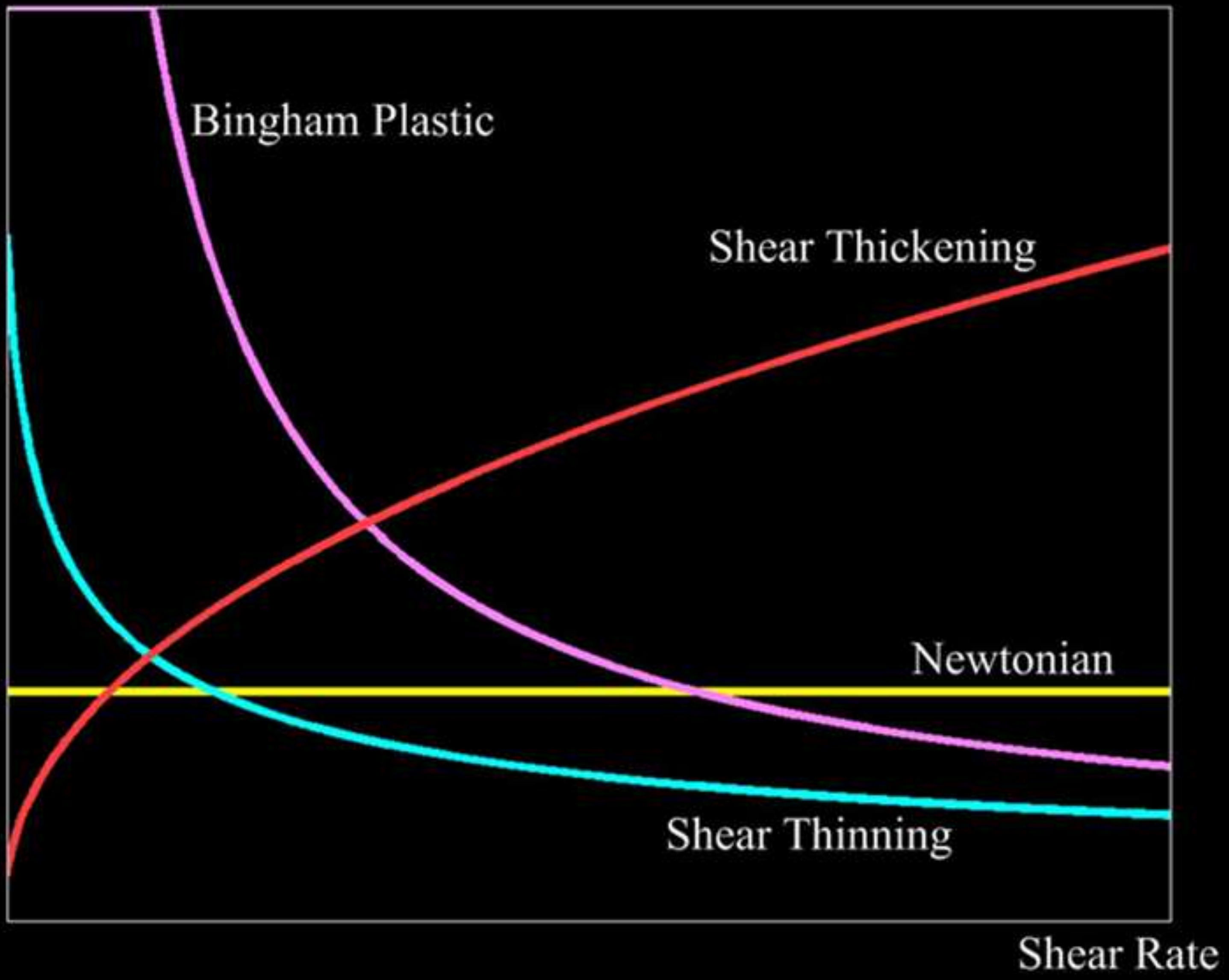
Bo Zhu, Minjae Lee, Ed Quigley, and Ron Fedkiw.
Codimensional Non-Newtonian Fluids. *ACM Trans. Graph. (SIGGRAPH 2015).*

Different Material Models

Paint:
Shear Thinning

Quicksand:
Shear Thickening

Mud:
Bingham Plastic



Variable Viscosity

- Non-Newtonian flow: $\mu = \mu(\dot{\gamma})$

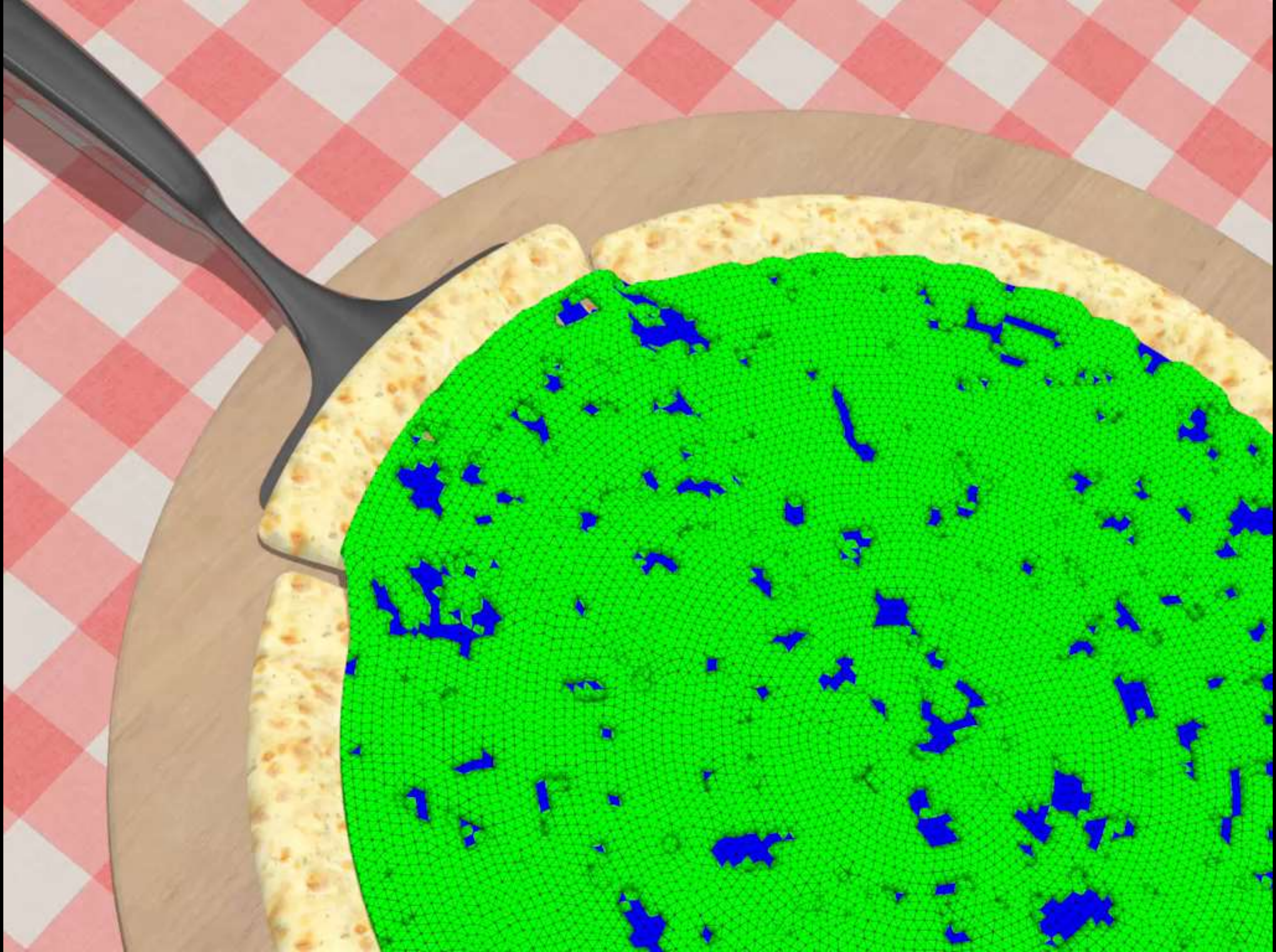
- Semi-Implicit viscosity force:

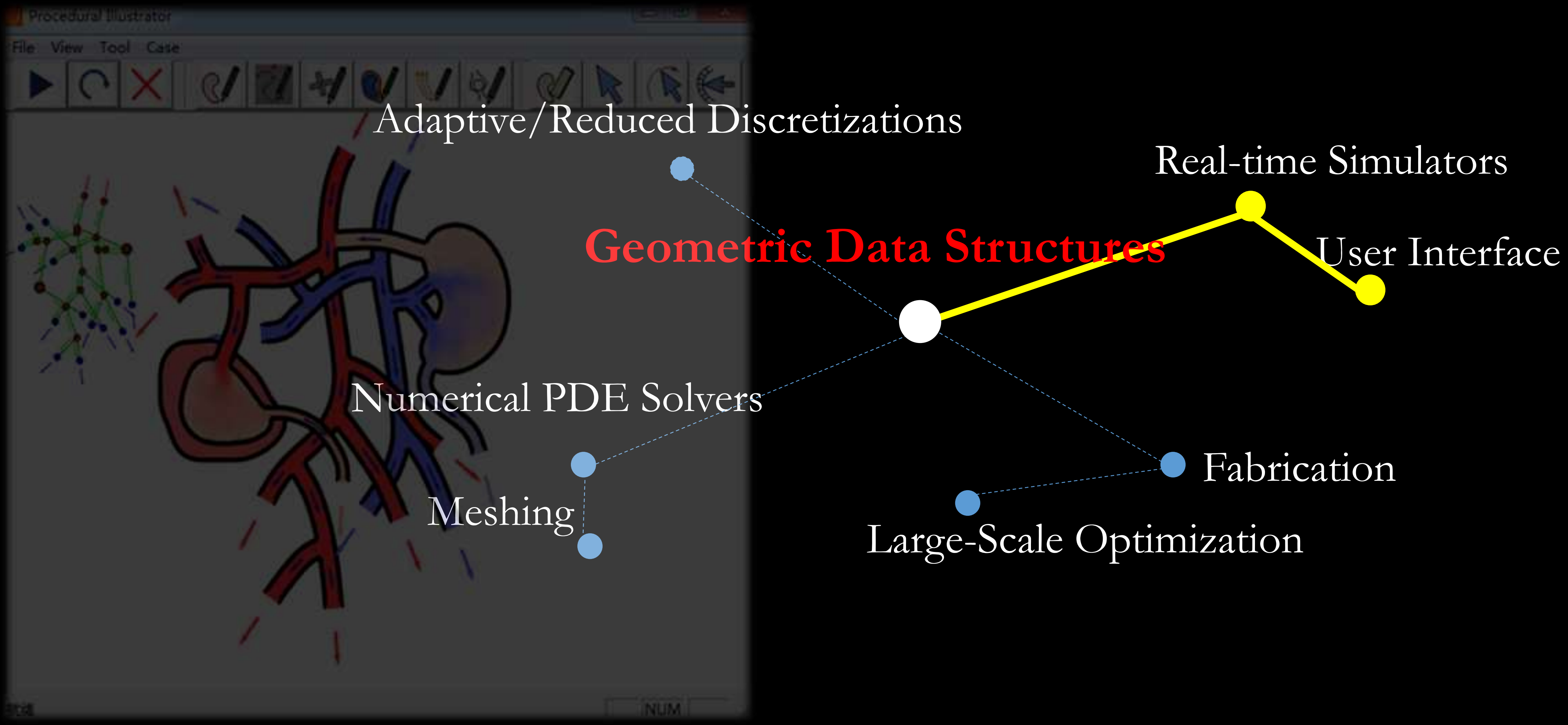
$$\vec{u}^{**} = \vec{u}^{o} + \frac{\Delta t}{\rho}\nabla \cdot \mu(\dot{\gamma})\Big(\underbrace{\boxed{(\nabla \vec{u}^{o})^T}}_{\text{Explicit part}} + \underbrace{\boxed{\nabla \vec{u}^{**}}}_{\text{Implicit part}}\Big)$$

- Volume weighted formula for the implicit part:

$$(\boldsymbol{W} + \frac{\Delta t}{\rho}\boldsymbol{G}^T\hat{\boldsymbol{W}}^{-1}\boldsymbol{G})\vec{u}^{**} = \boldsymbol{W}\vec{u}^{*}$$


SIGGRAPH 2015
Xroads of Discovery
Los Angeles
PYRROLE
RED LIGHT
ARTIST'S OIL COLOR

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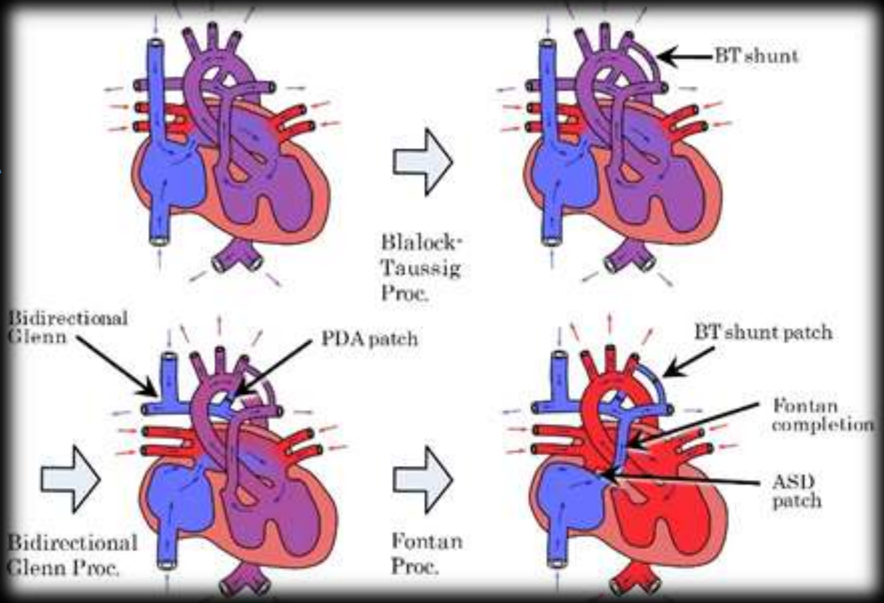
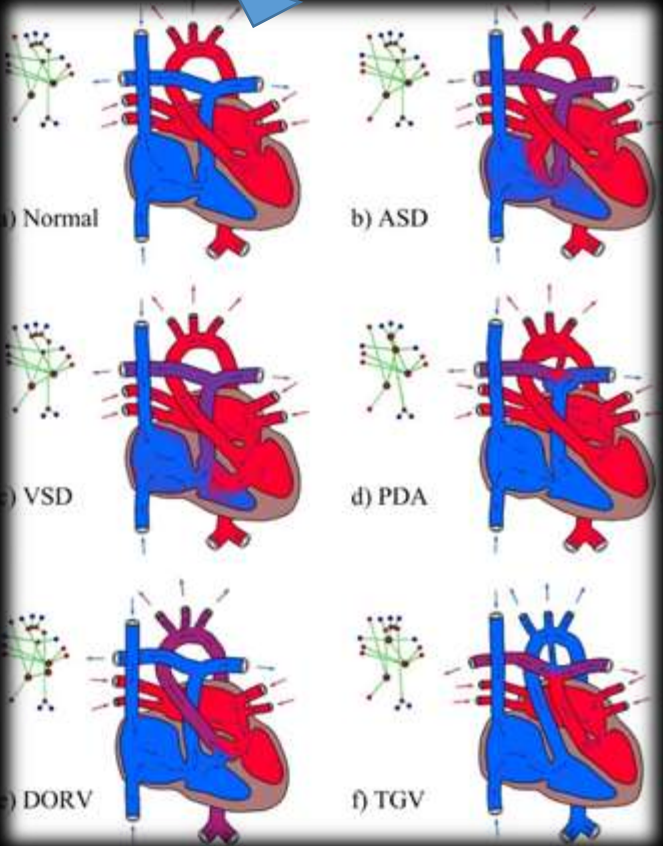
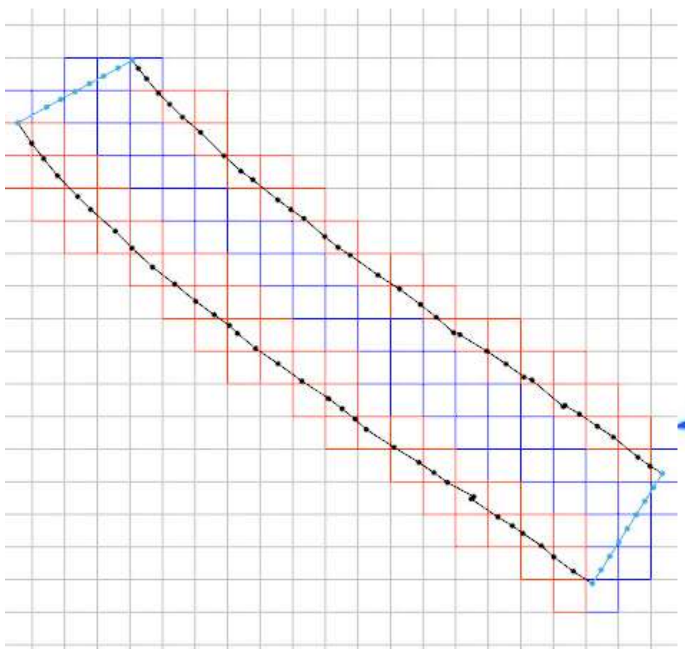
An interactive system for cardiovascular surgeons

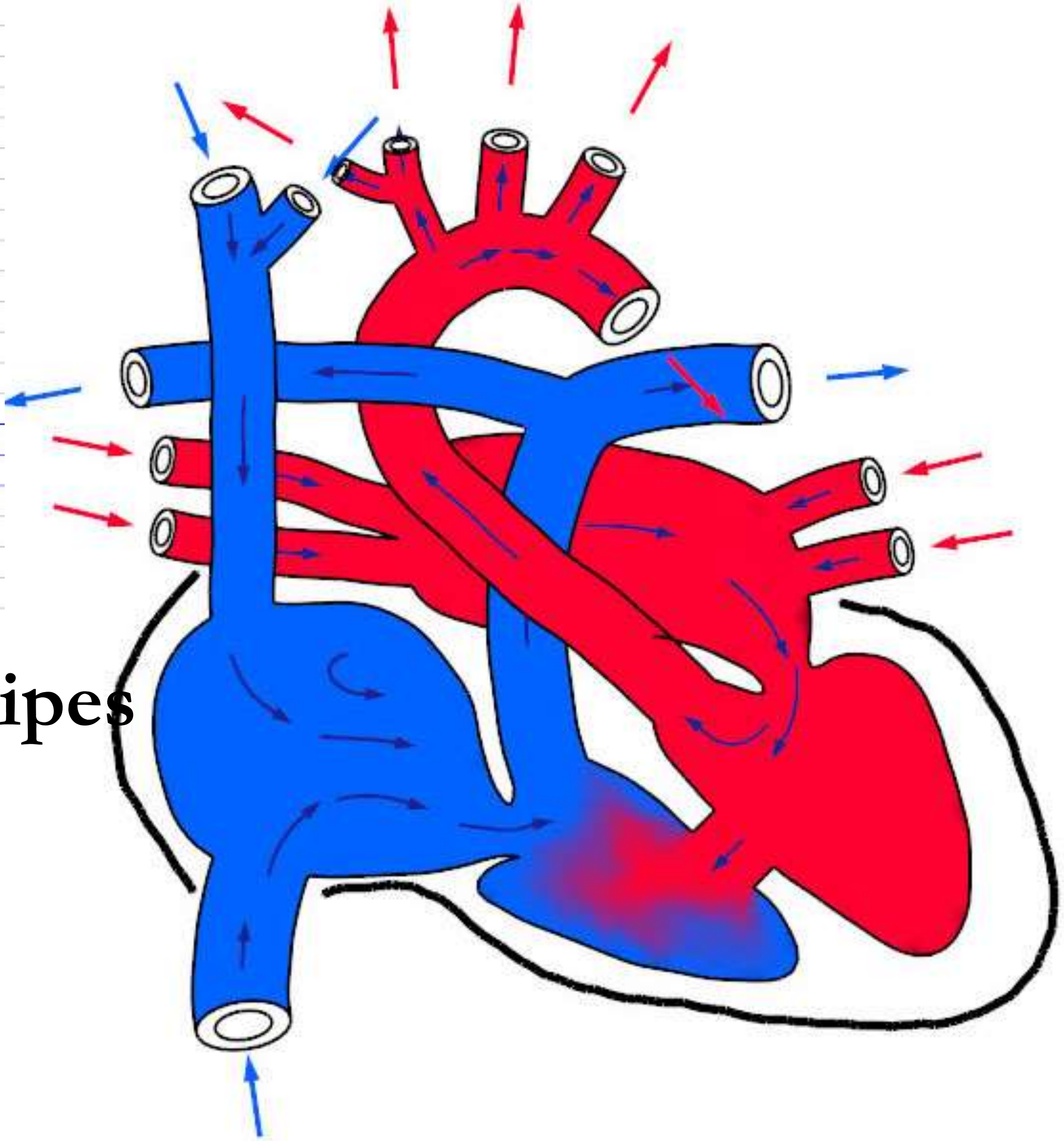






Anisotropic thin pipes

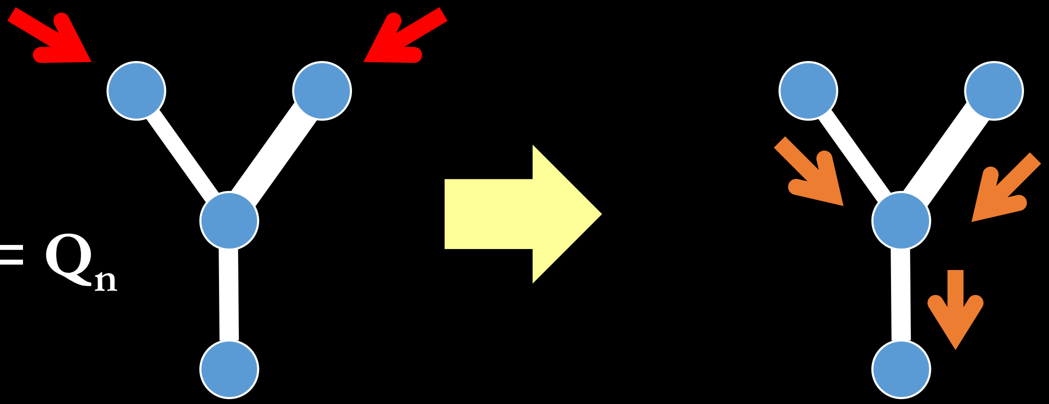
Reduced Graph



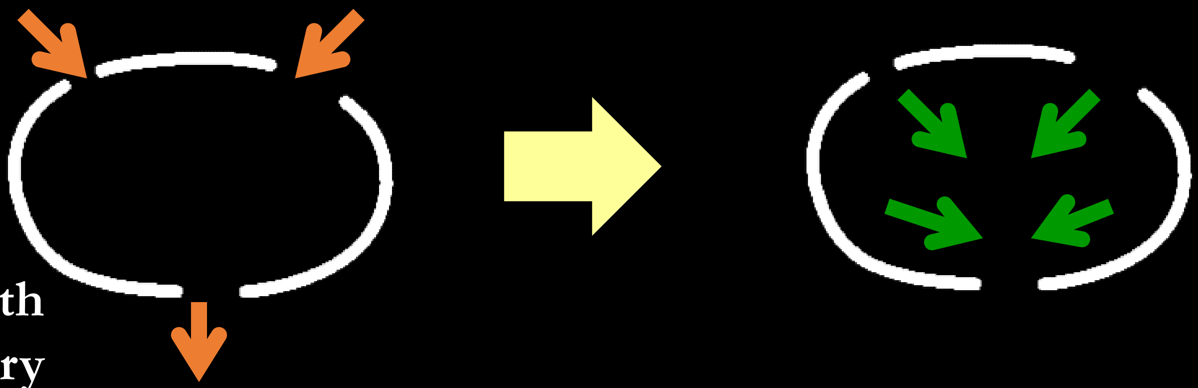
Reduced Geometry

- Hydraulics

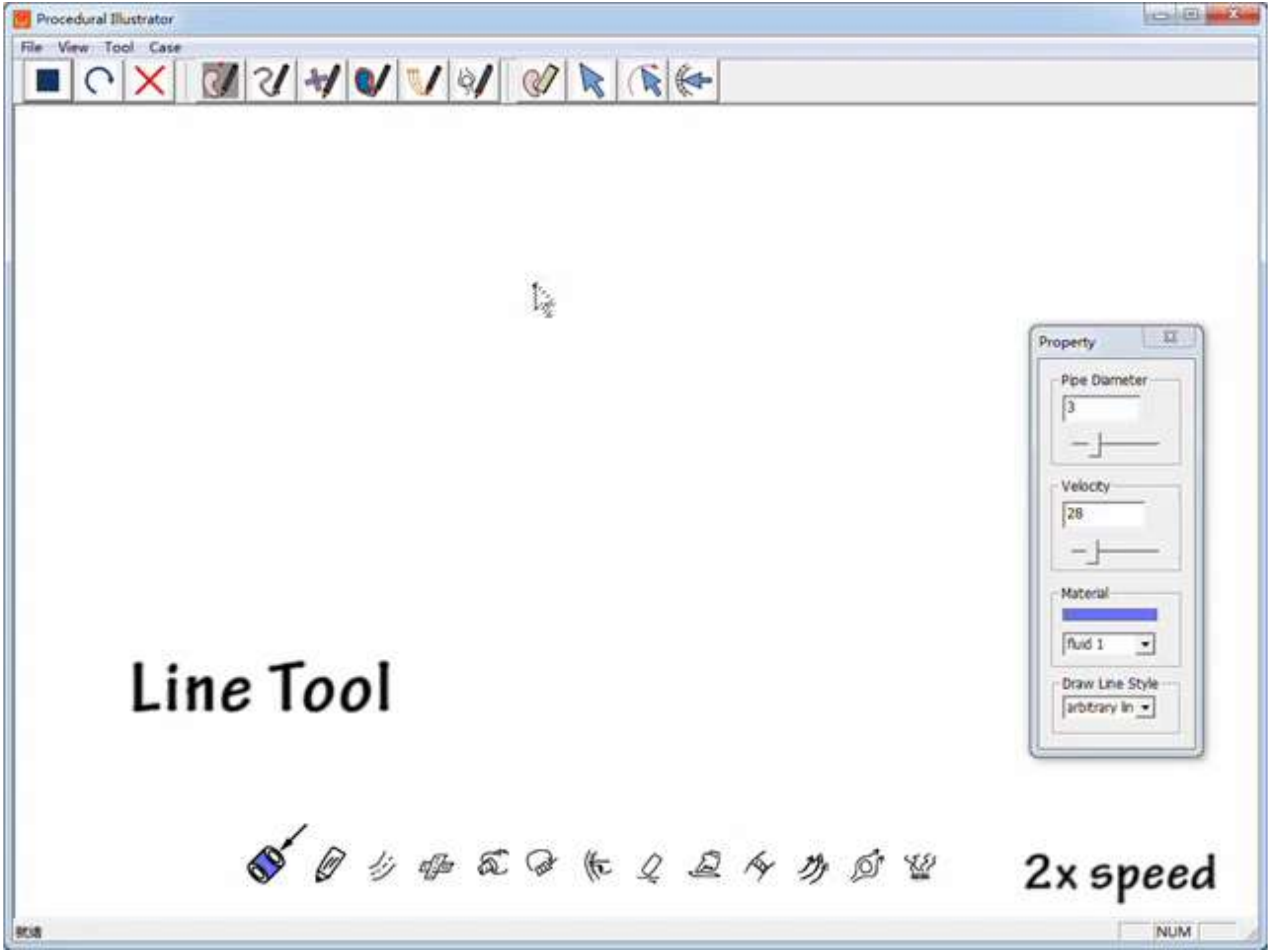
$Q_n = -MQ_e$
$MD_eM^TP_n = Q_n$

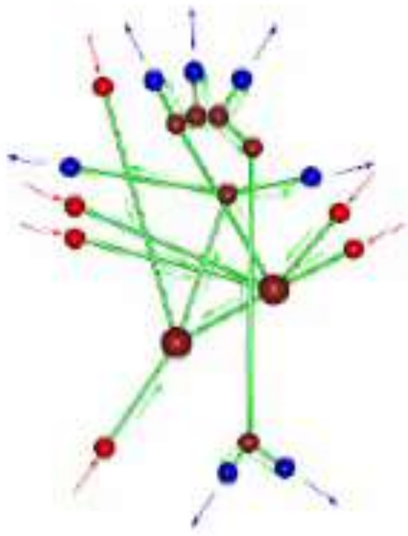


- Hydrodynamics

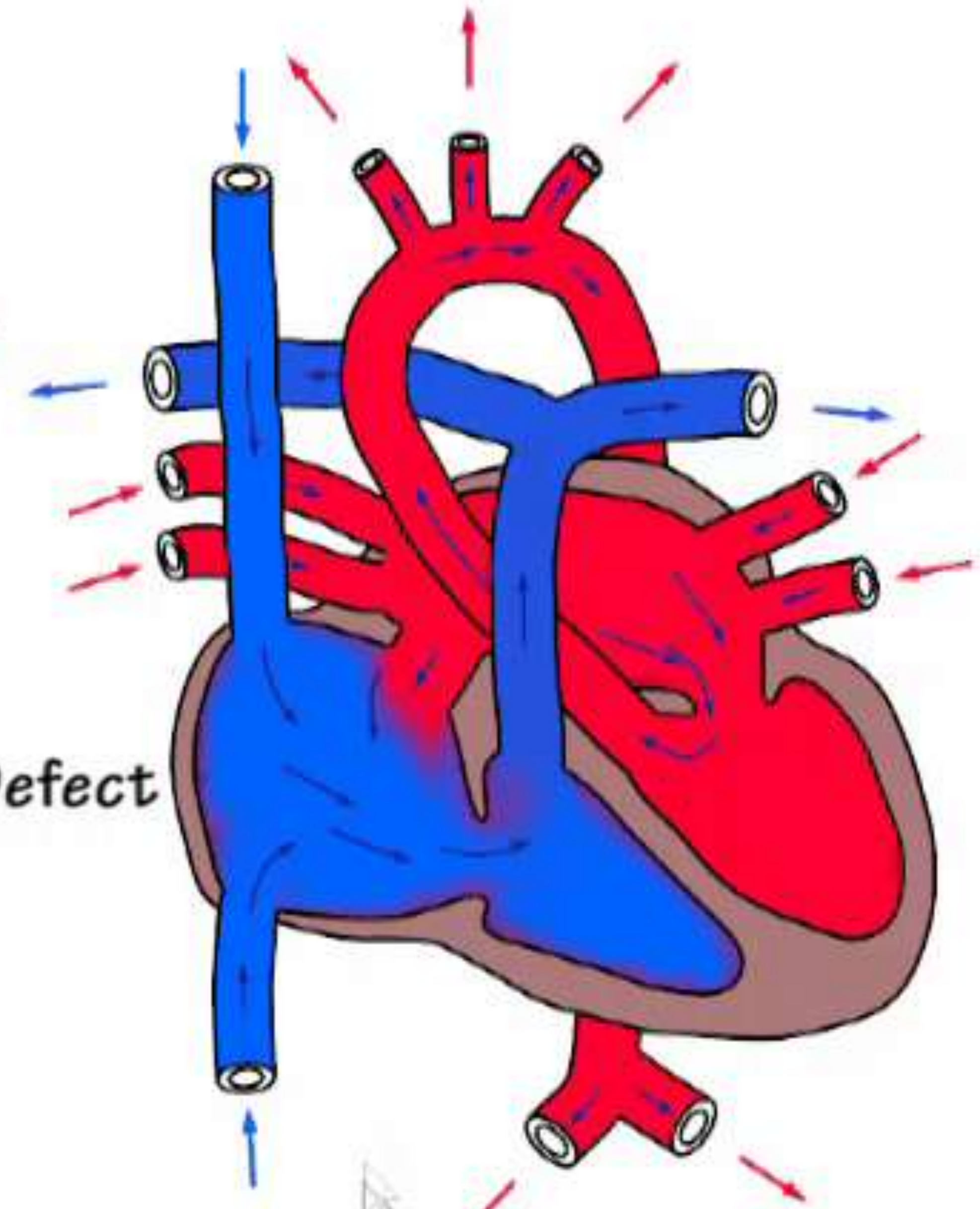


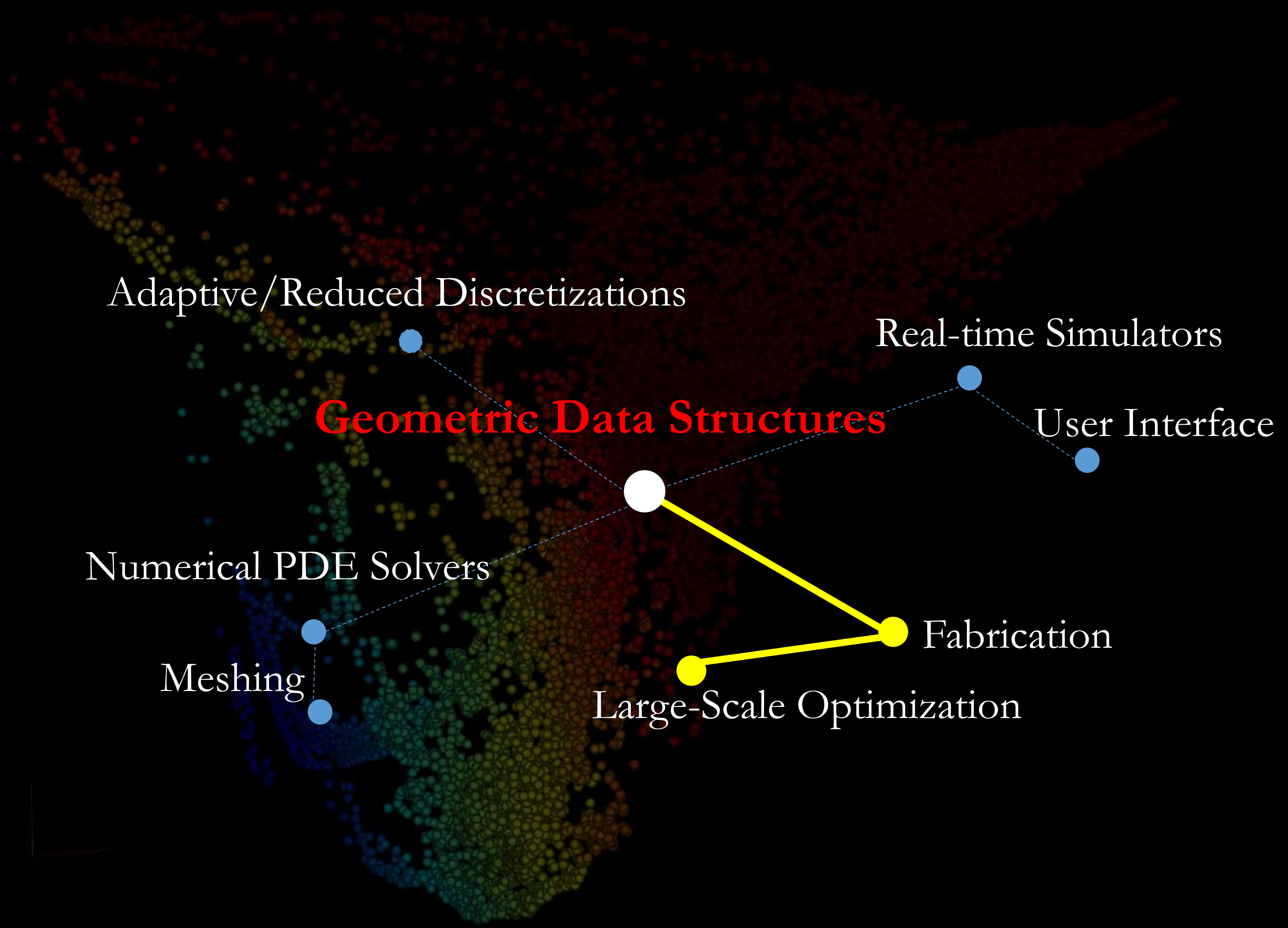
N-S equations with
Dirichlet Boundary


Procedural Illustrator
File View Tool Case
Property
Pipe Diameter
3
Velocity
28
Material
fluid 1
Draw Line Style
arbitrary lin
Line Tool
2x speed
NUM



Atrial Septal Defect (ASD)



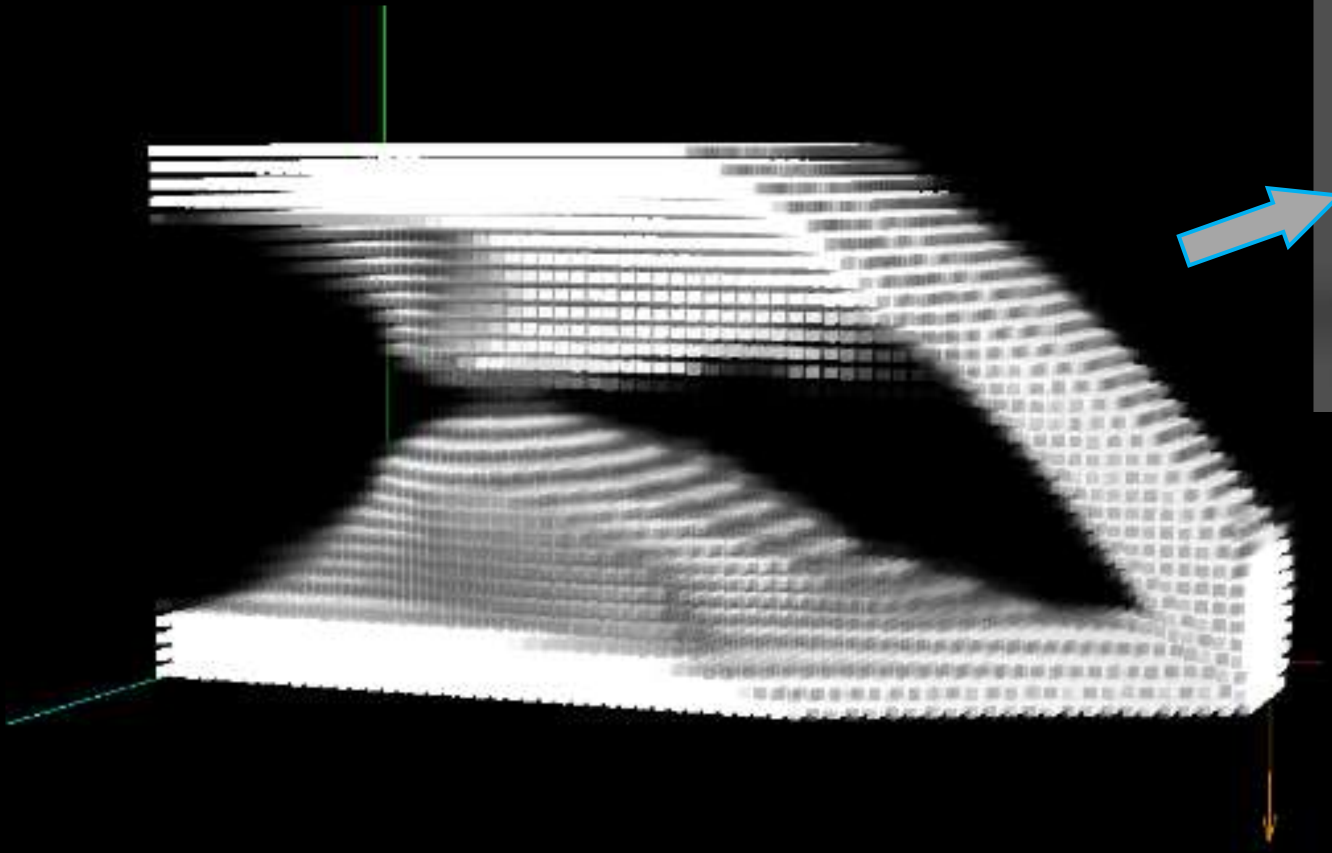

Adaptive/Reduced Discretizations
Real-time Simulators
Geometric Data Structures
User Interface
Numerical PDE Solvers
Fabrication
Meshing
Large-Scale Optimization

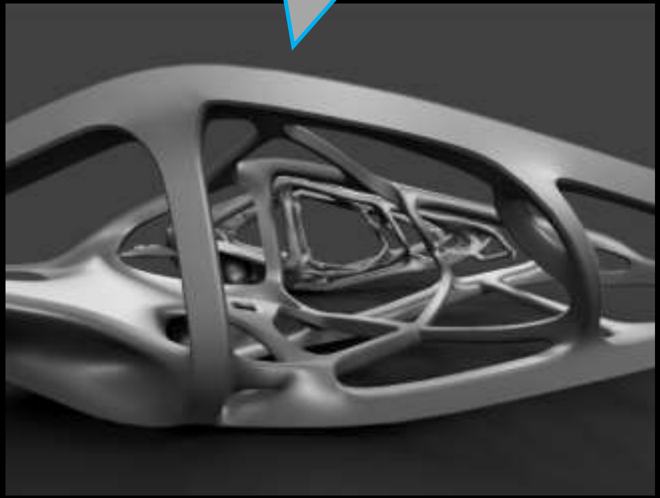
Motivation: Direct Design v.s. Generative Design

Direct Design

Generative Design

Topology Optimization

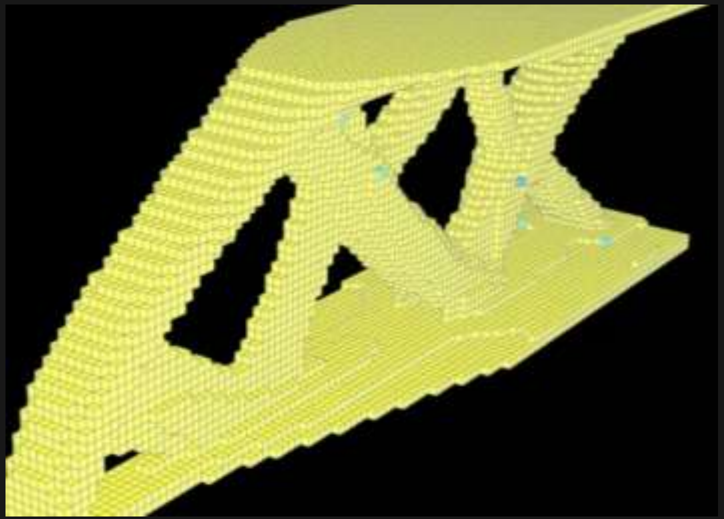




Challenges

Hardware: Object-1000 Plus

- Up to 39.3 x 31.4 x 19.6 in.
- 600dpi (~40 microns)
- 5 trillion voxels

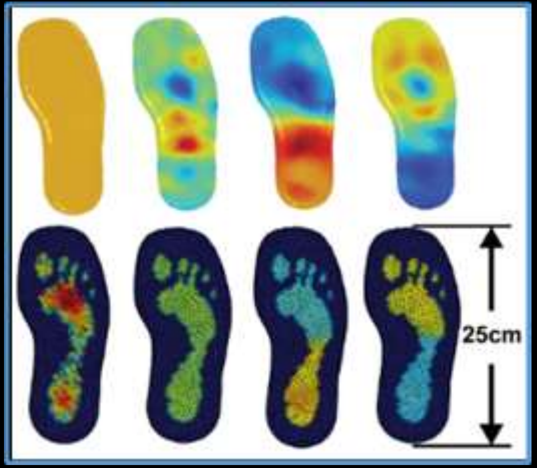


Software: SIMP Topology Optimization

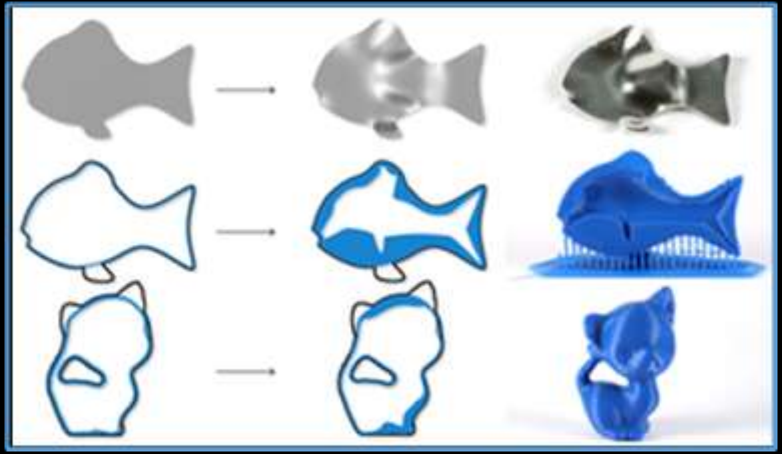
- Up to millions of elements
- Difficult to handle multiple materials

Previous Work: Fabrication-Oriented Optimization

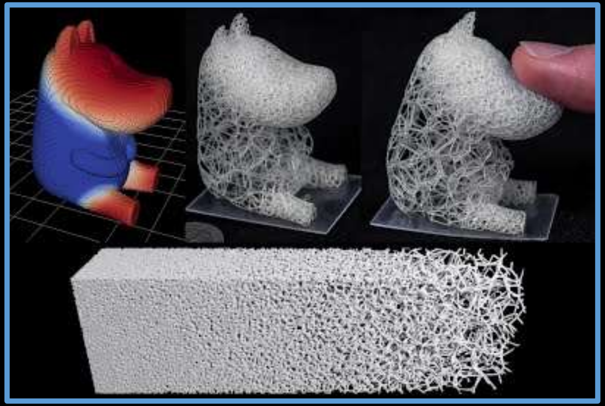
[Lu et.al. 2014]



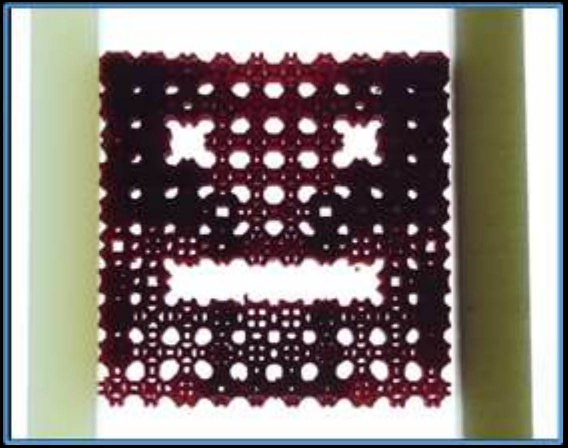
[Xu et.al. 2015]



[Musialski et.al. 2016]



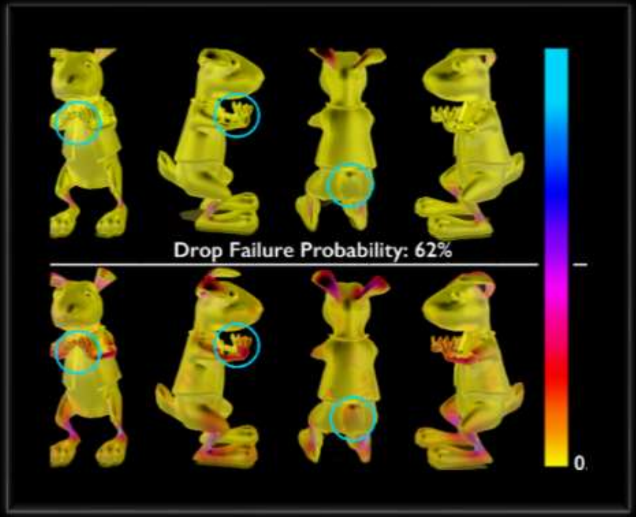
[Matinez et.al. 2016]



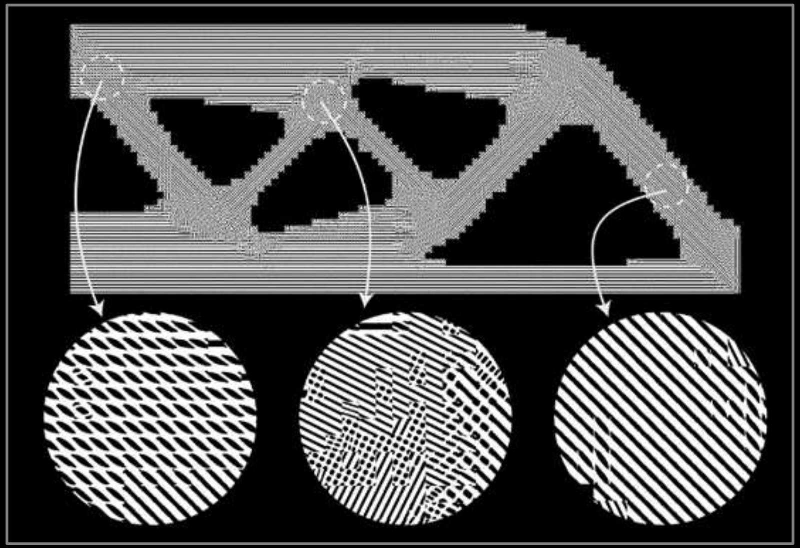
[Panetta et.al. 2015]

[Schumacher et.al. 2015]

Topology Optimization

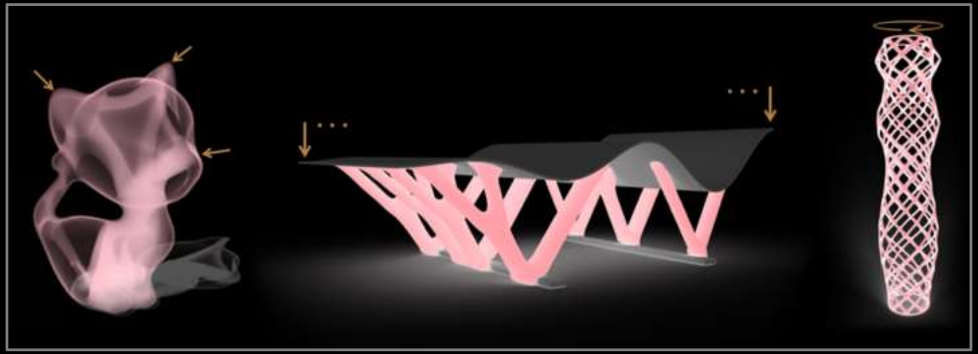


[Langlois et.al. 2016]



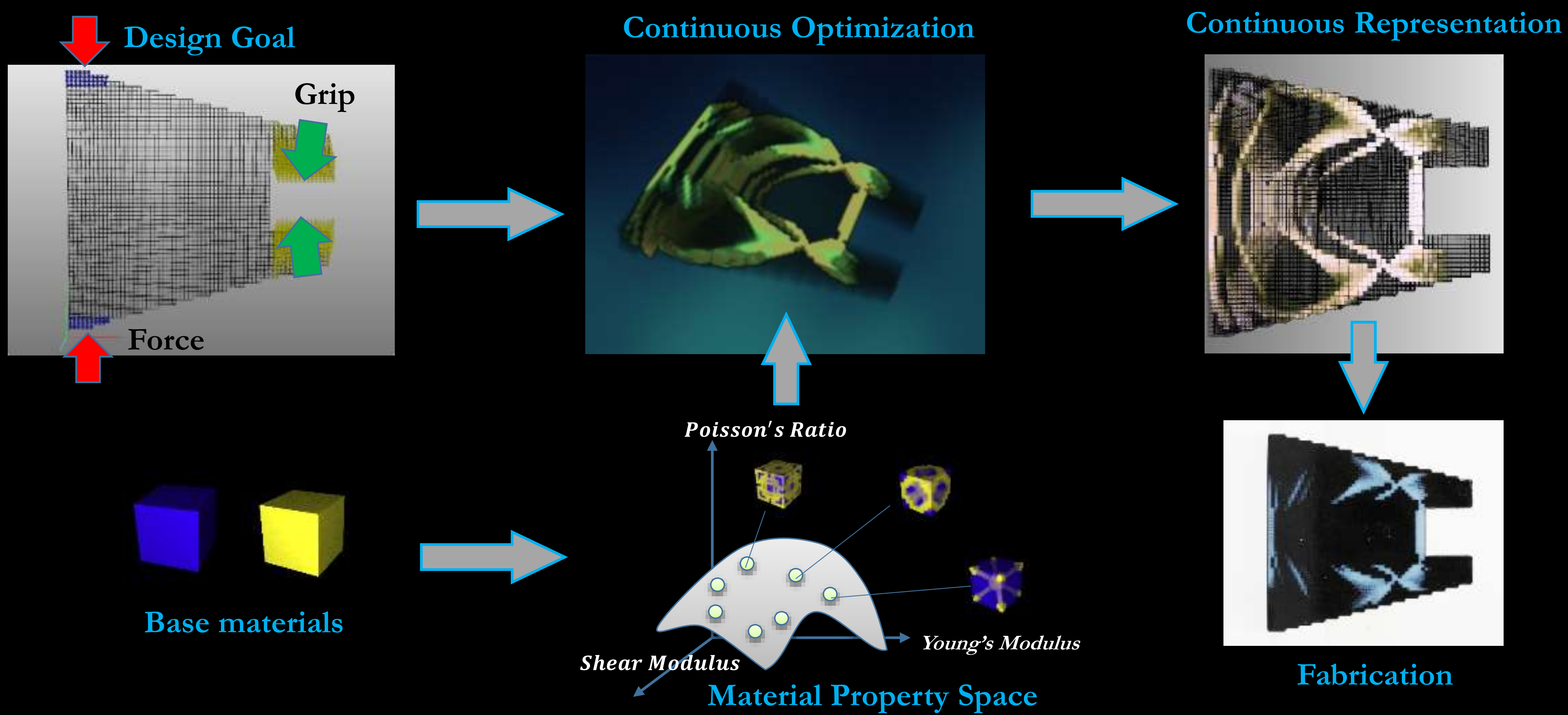
[Liang et.al. 2015]

[Matinez et.al. 2015]

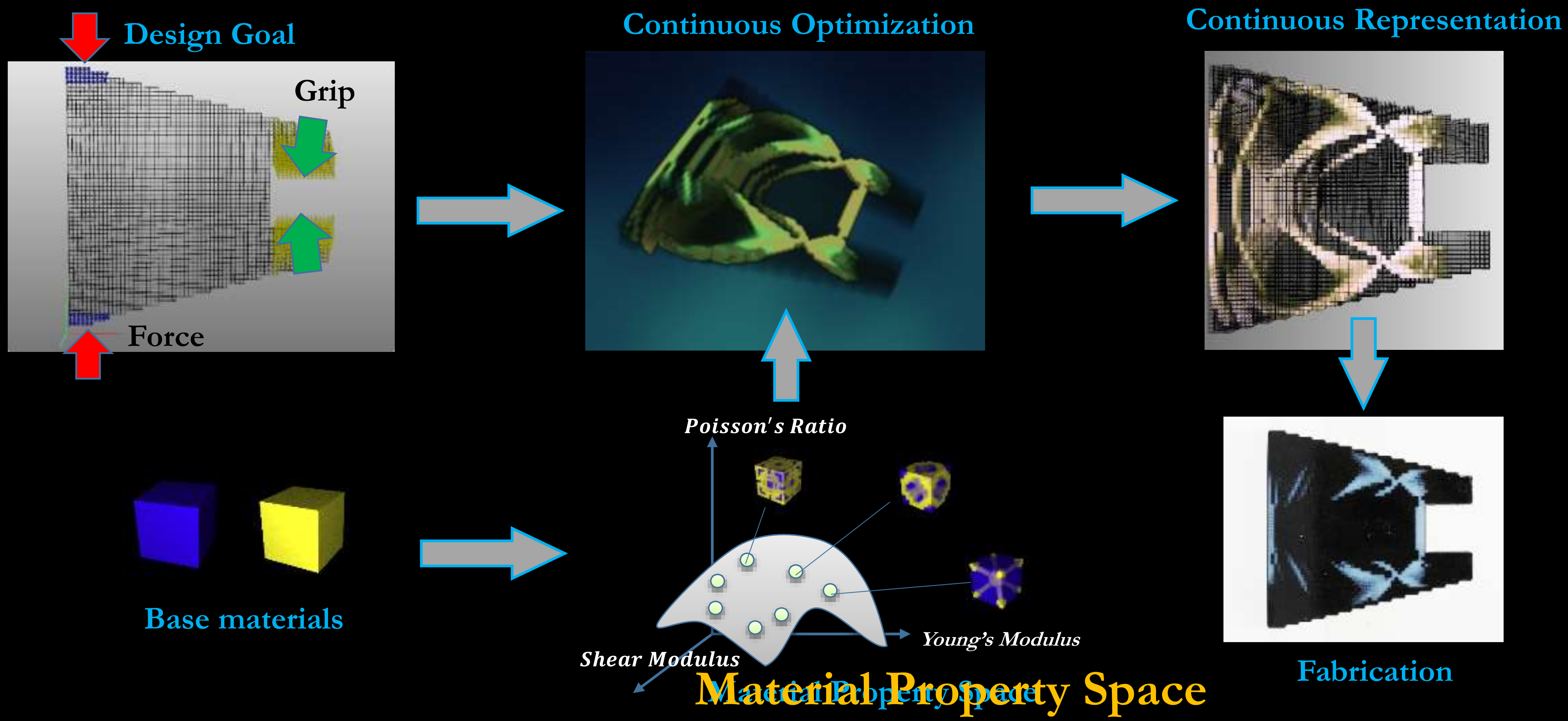


[Wu et.al. 2016]

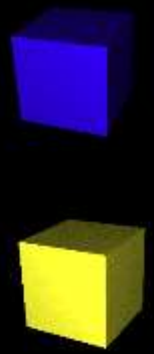
Two-scale Topology Optimization



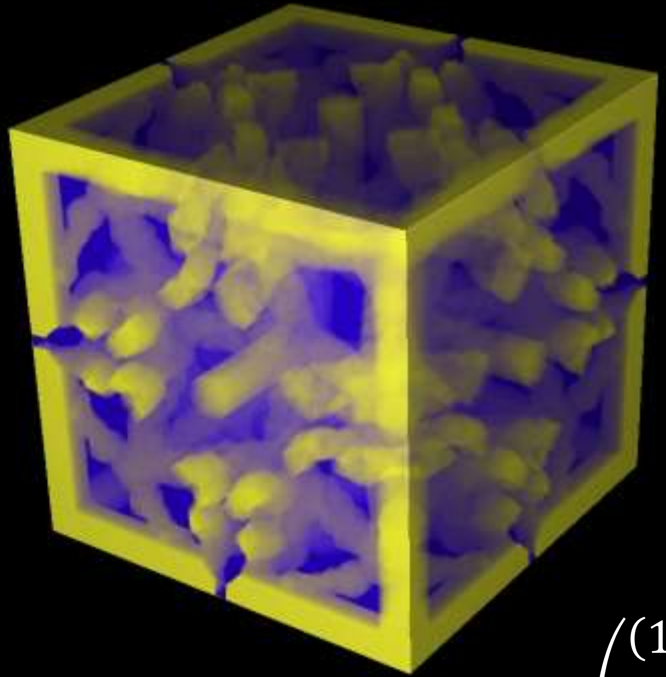
Two-scale Topology Optimization



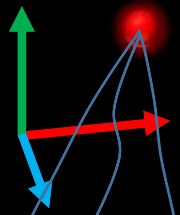
Microstructure



Base materials



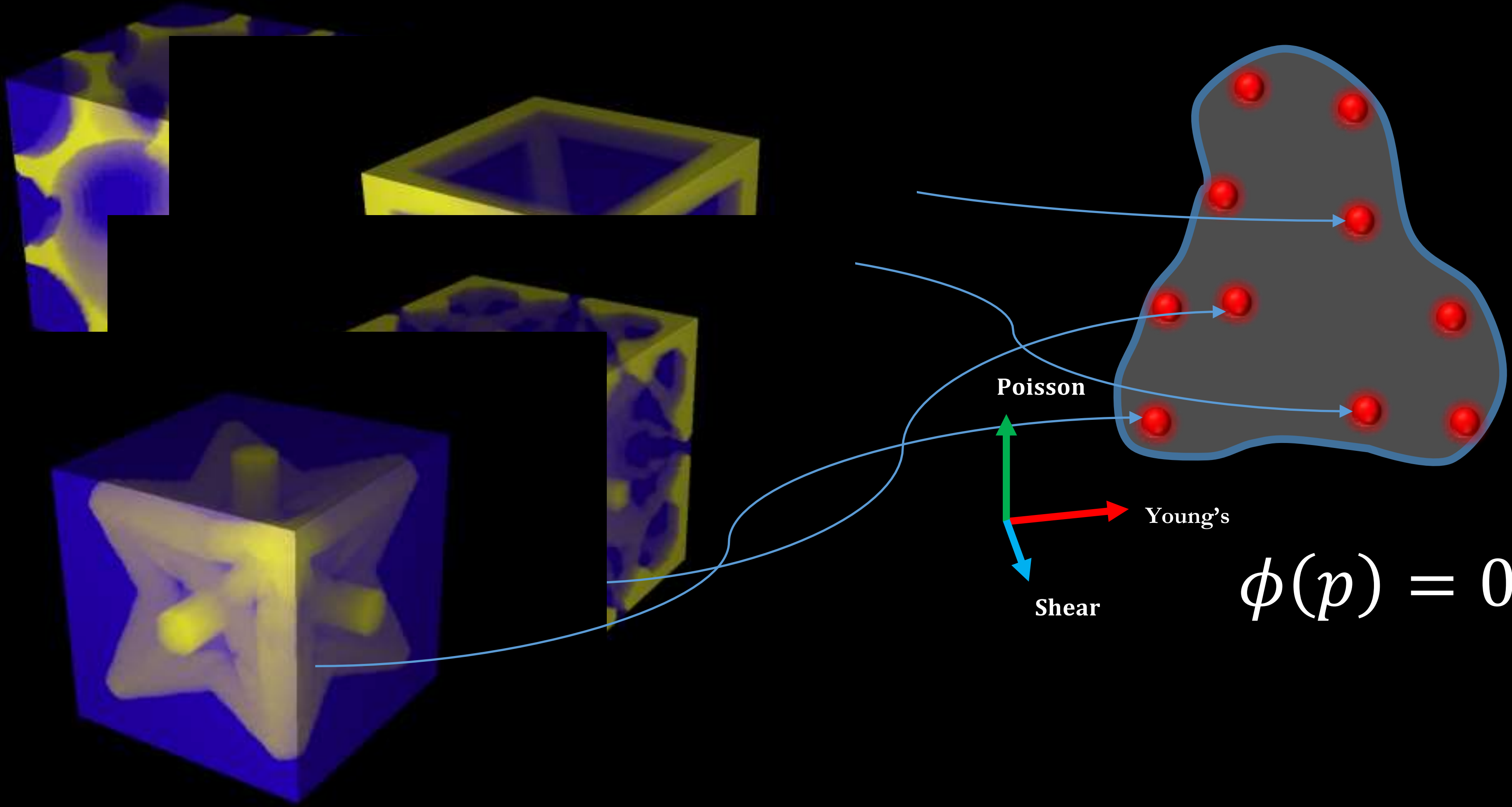
$p = (E, \nu, \mu)^T$



$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\epsilon} = \begin{pmatrix} (1-\nu)\,\hat{E} & \nu\hat{E} & \nu\hat{E} & & & \\ & (1-\nu)\,\hat{E} & \nu\hat{E} & & & \\ & & (1-\nu)\,\hat{E} & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{pmatrix}$$

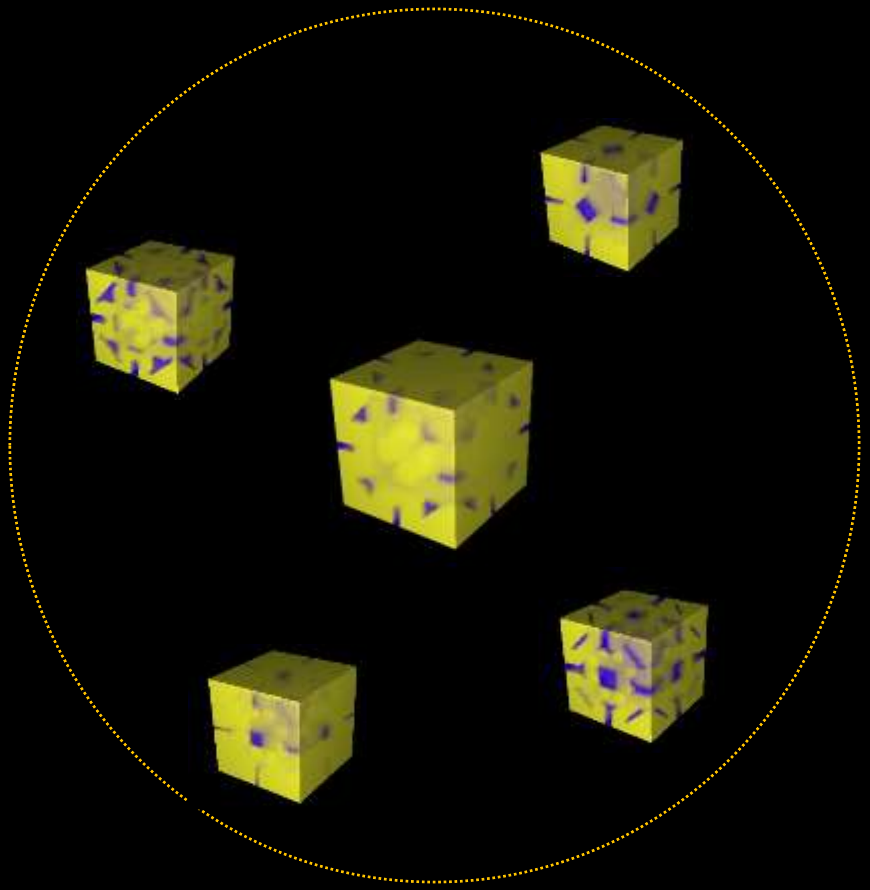
Cubic Material

Continuous Representation: Levelset

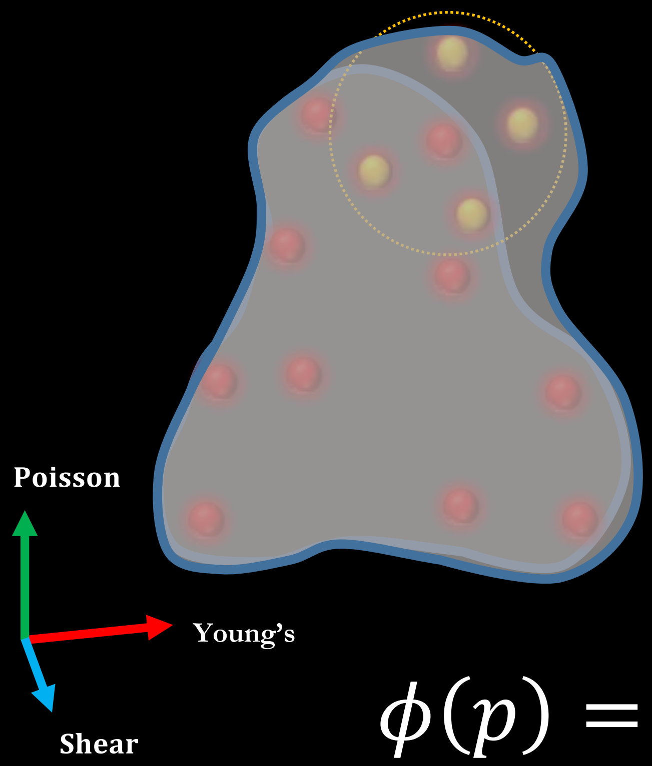


$$\phi(p) = 0$$

Expanding the Achievable Property Domain

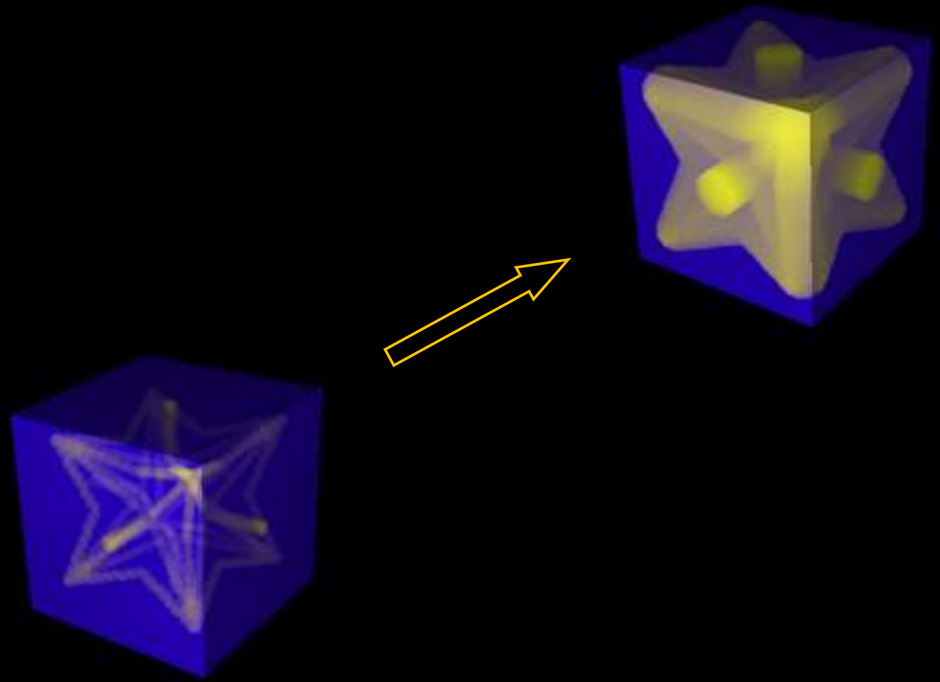


Stochastically-Ordered Sequential Monte Carlo

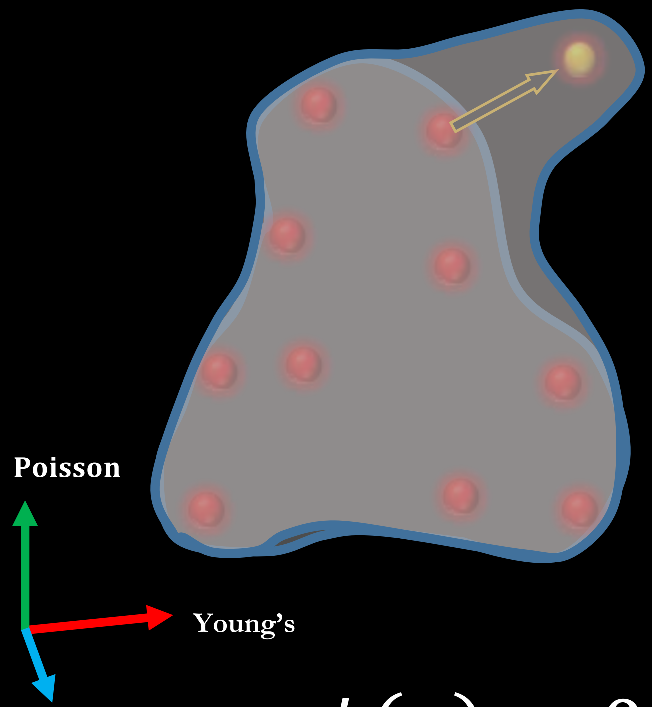


$$\phi(p) = 0$$

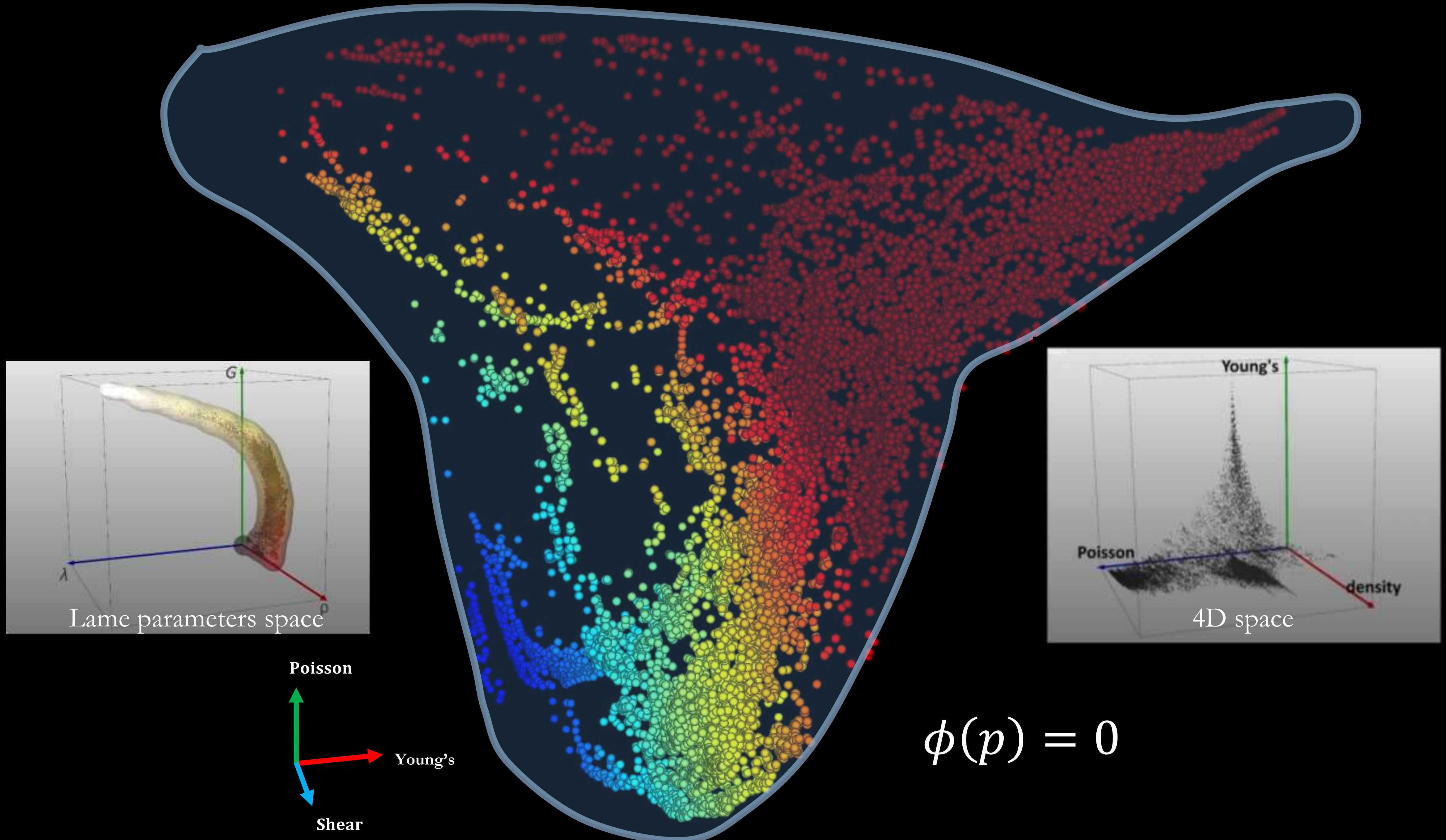
Expanding the Achievable Property Domain



Continuous Microstructure Optimization

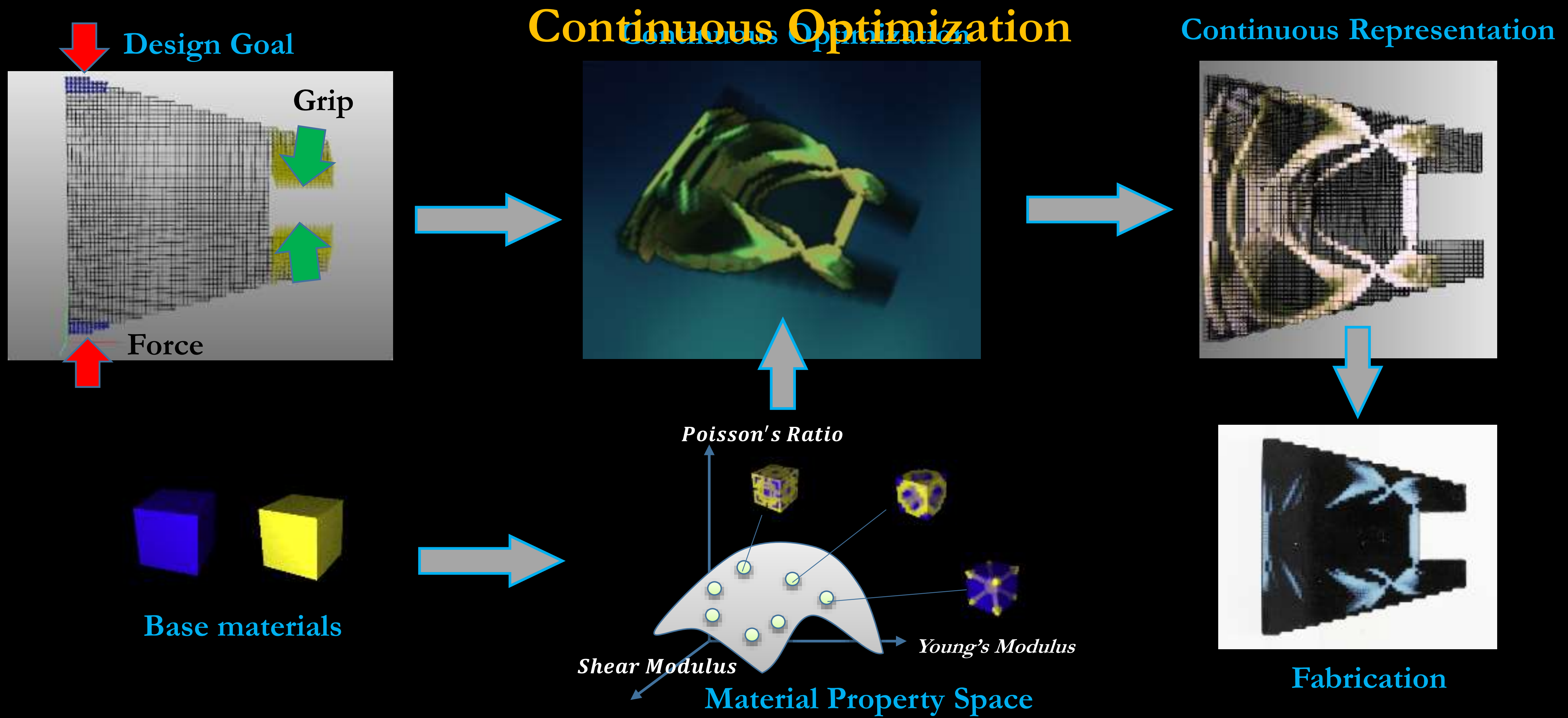


$$\phi(p) = 0$$


G
λ
ρ
Lame parameters space
Poisson
Young's
Shear
Young's
Poisson
density
4D space

$$\phi(p) = 0$$

Two-scale Topology Optimization



Topology Optimization

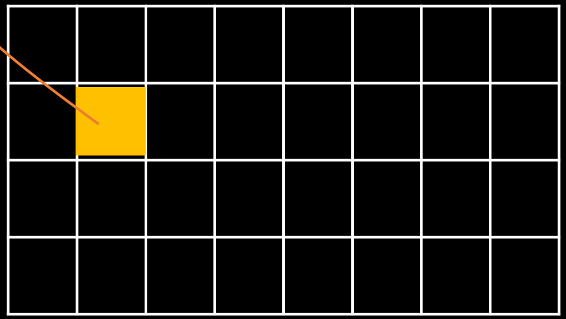
Material property for each cell

$\boldsymbol{p} = [\boldsymbol{\rho_1}, \boldsymbol{E}_1, \boldsymbol{\nu}_1, \boldsymbol{\mu_1}, \boldsymbol{\rho_2}, \boldsymbol{E}_2, \boldsymbol{\nu}_2, \boldsymbol{\mu_2}, \dots]$

$$\min_{\mathrm{p}} : S(\boldsymbol{p}, \boldsymbol{u})$$

$$s.t.: F(\boldsymbol{p}, \boldsymbol{u}) = 0$$

$$\phi(\boldsymbol{p}) \leq 0$$

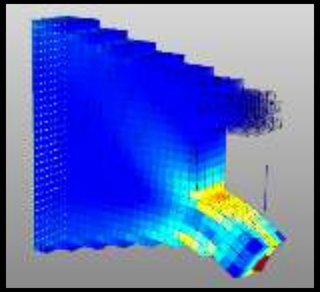


Minimum Compliance/Target Deformation

Linear Elastic FEM:

$F(\boldsymbol{p}, \boldsymbol{u}) = K(\boldsymbol{p})\boldsymbol{u} - \boldsymbol{f} = 0$

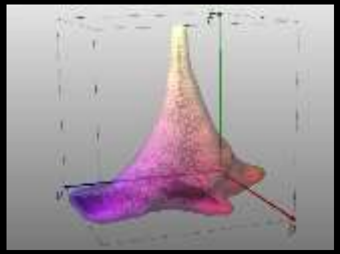
(Adjoint Method)



Levelset Constraints

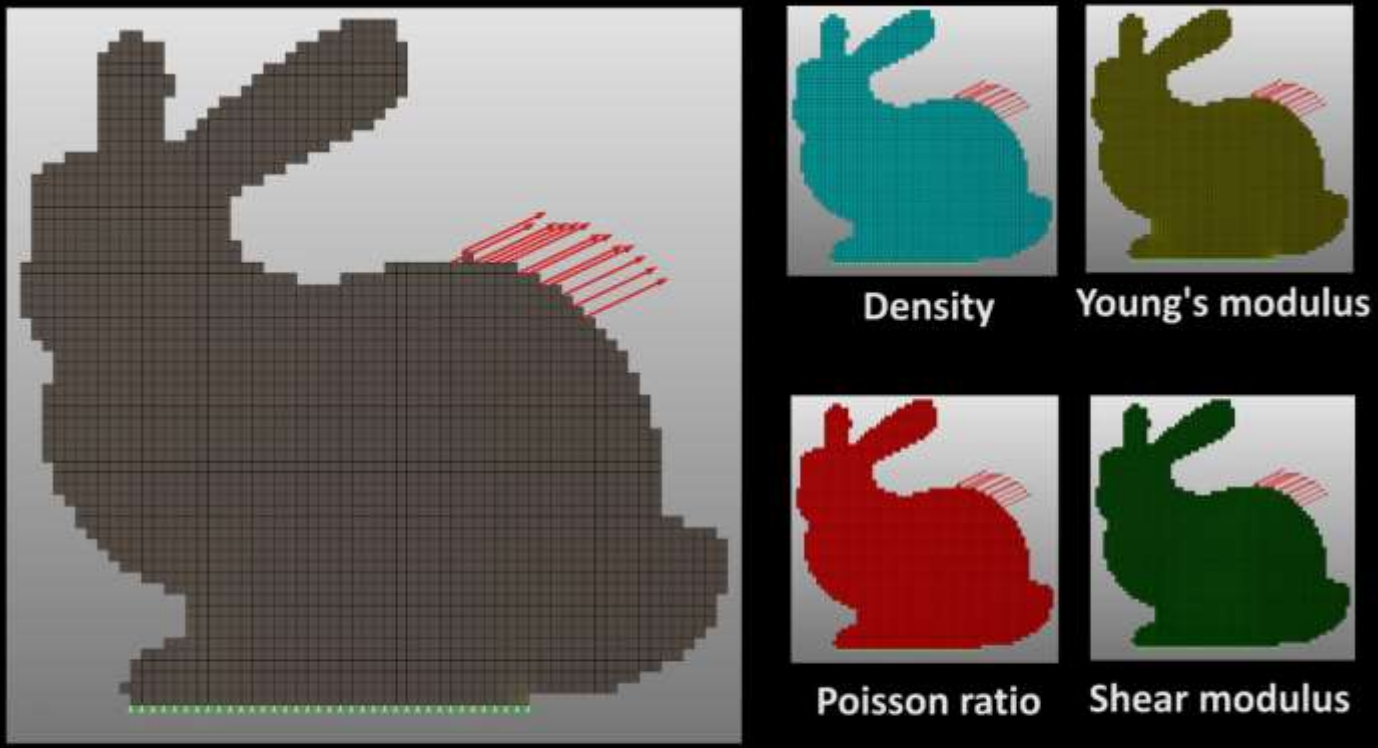
(Interior Point Method,

Finite difference for ϕ_x)



Minimum Compliance

$$S_c(\boldsymbol{p}, \boldsymbol{u}) = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u}$$



Topology optimization iterations: material distribution in 4D space

Density ->

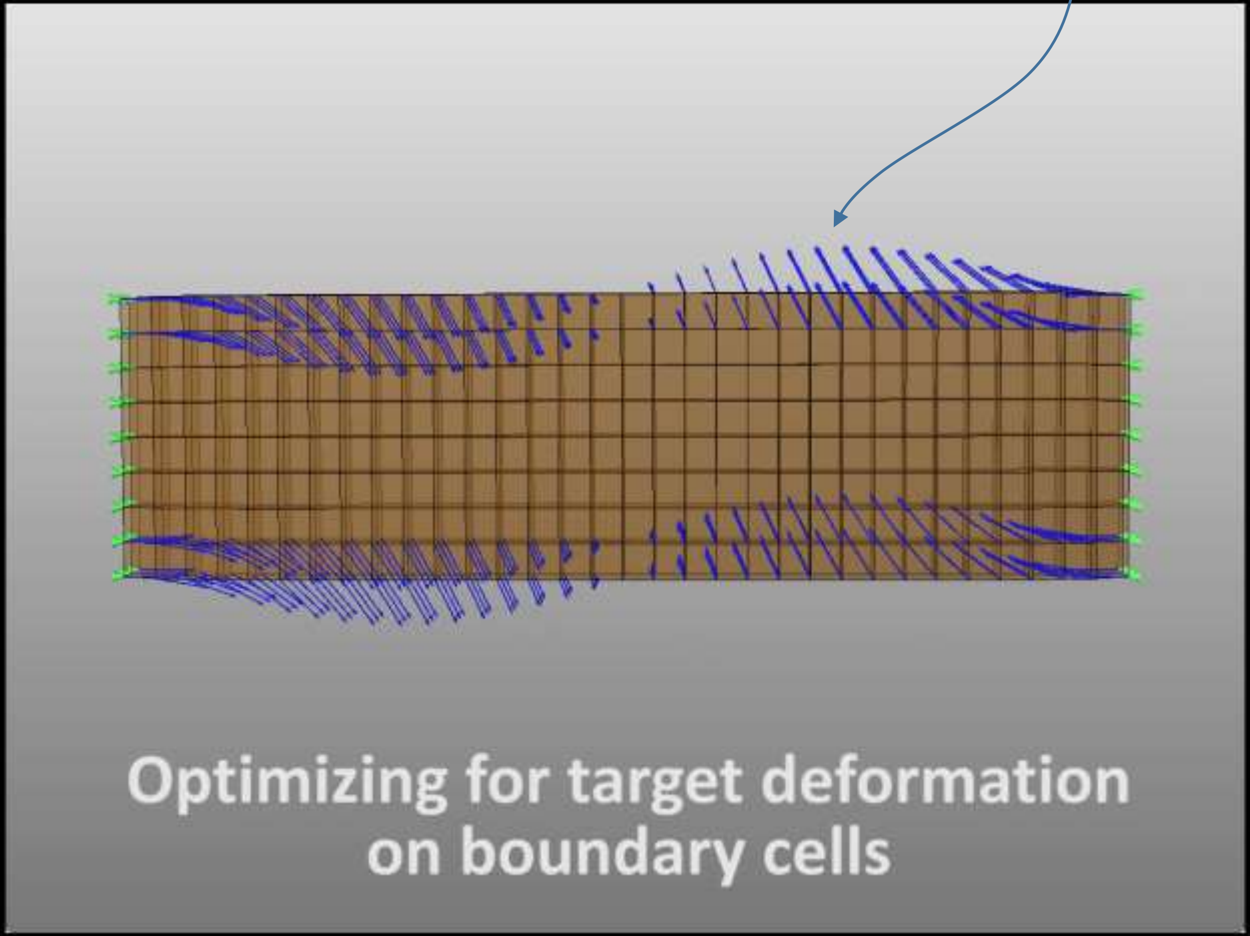
Density, Young's modulus, Poisson's Ratio, …

(0,1] ->

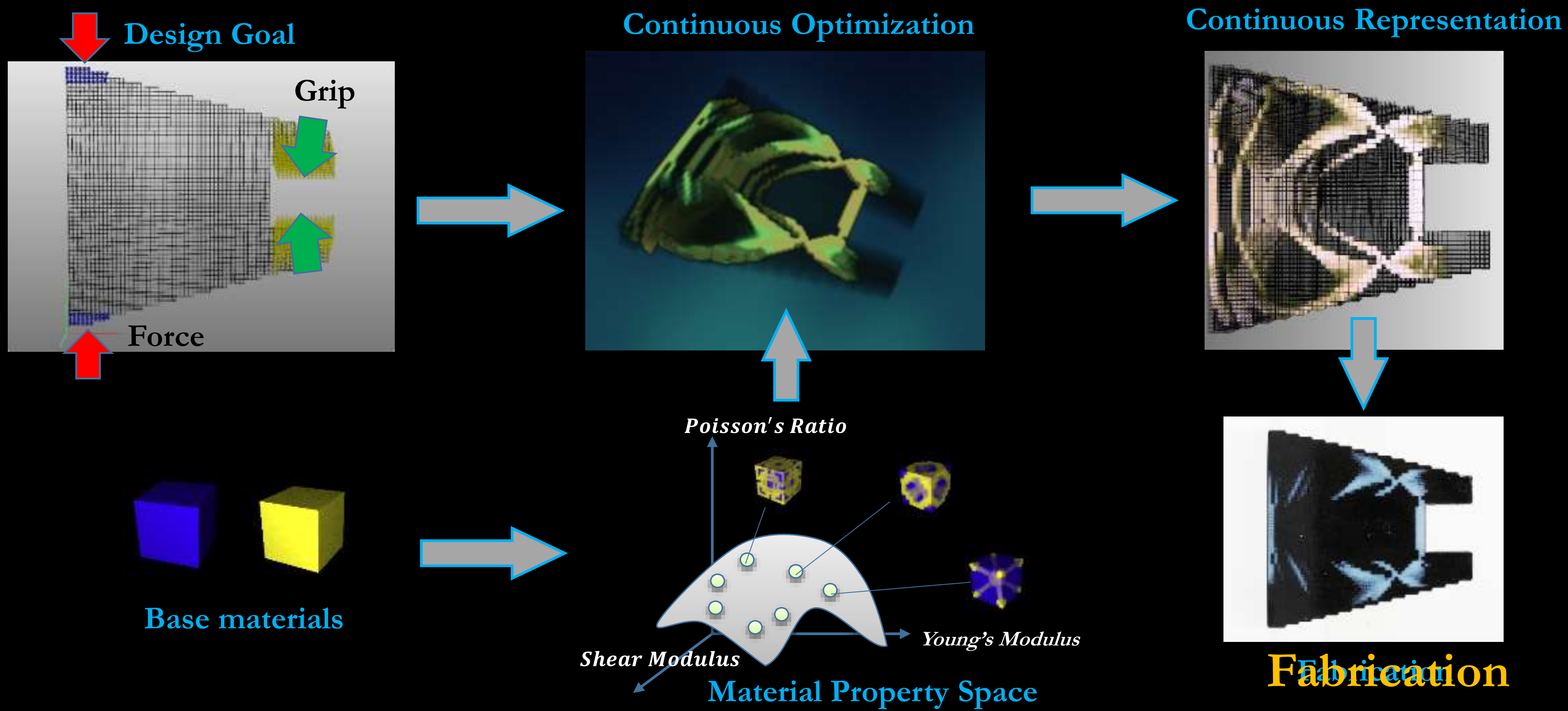
Levelset boundary

Target Deformation

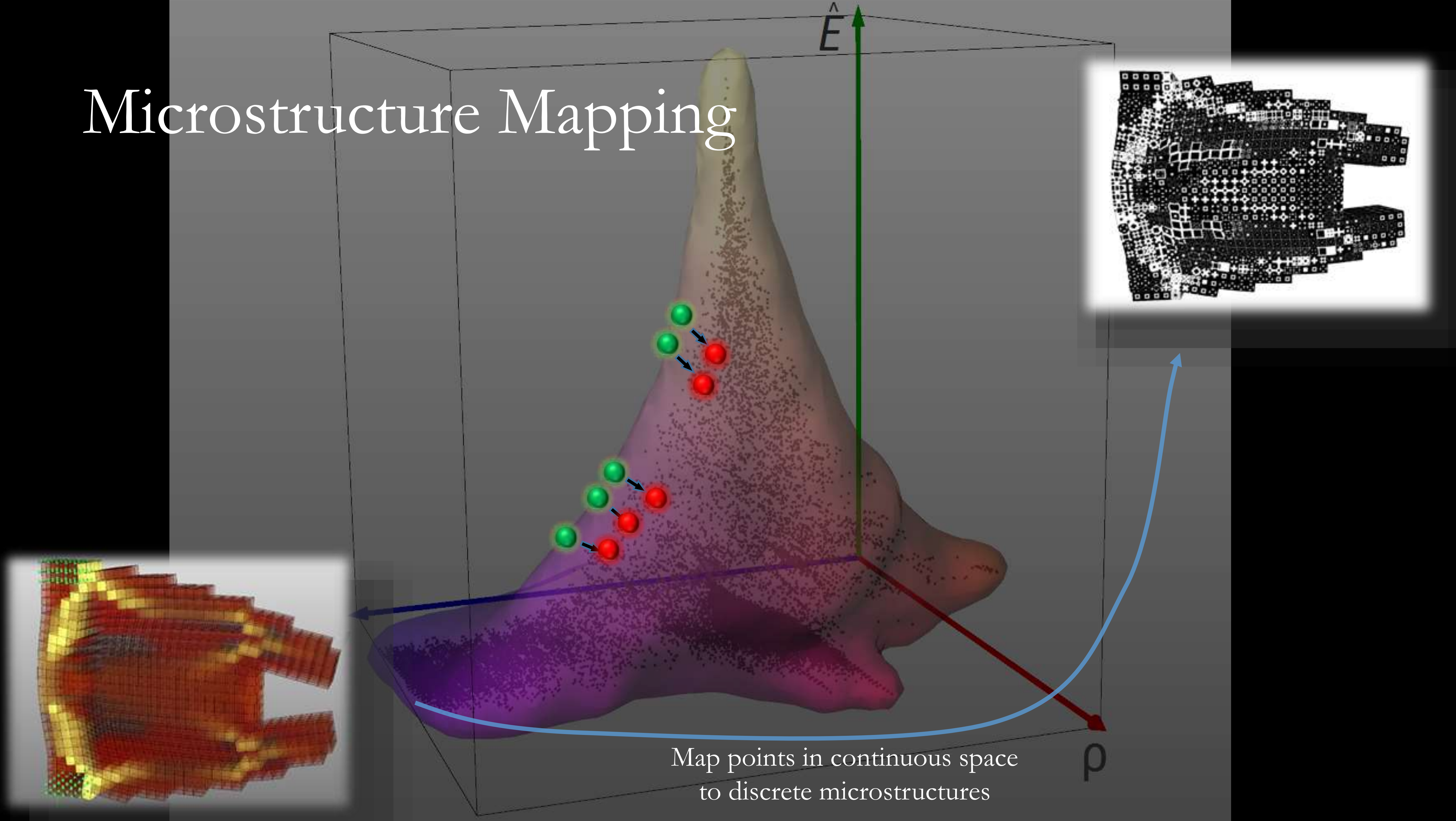
$$S_d(\boldsymbol{p}, \boldsymbol{u}) = (\boldsymbol{u} - \hat{\boldsymbol{u}})^T \boldsymbol{D}(\boldsymbol{u} - \hat{\boldsymbol{u}})$$



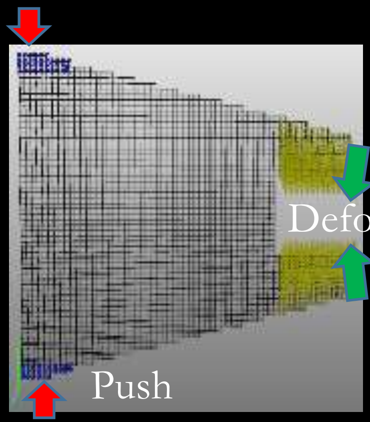
Two-scale Topology Optimization

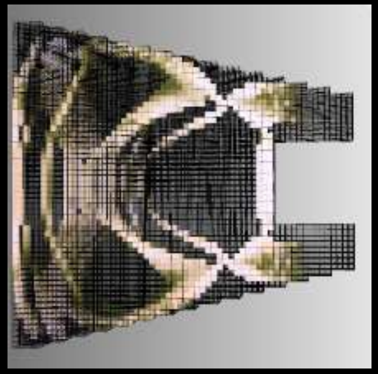


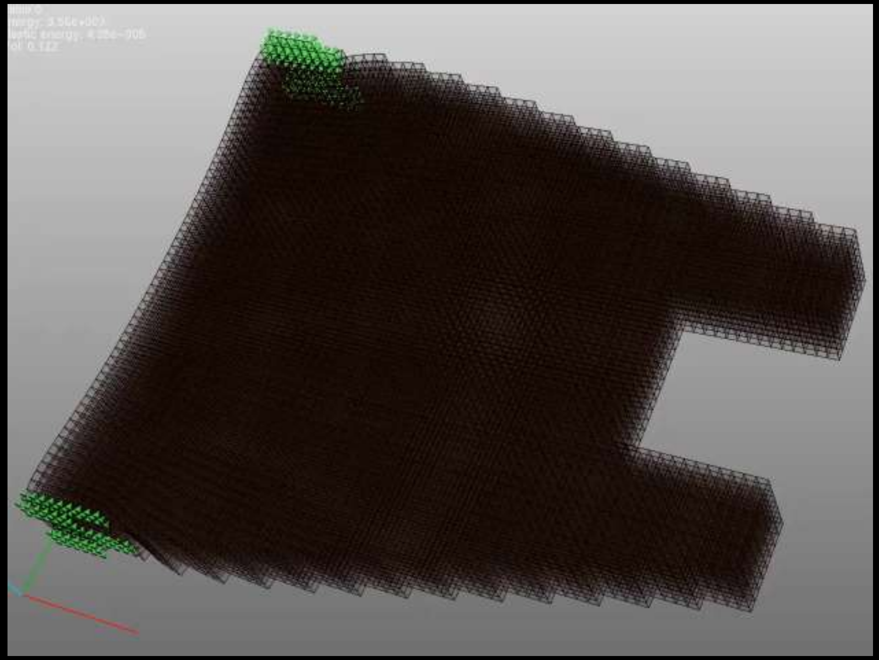
Microstructure Mapping



Example: Soft Gripper



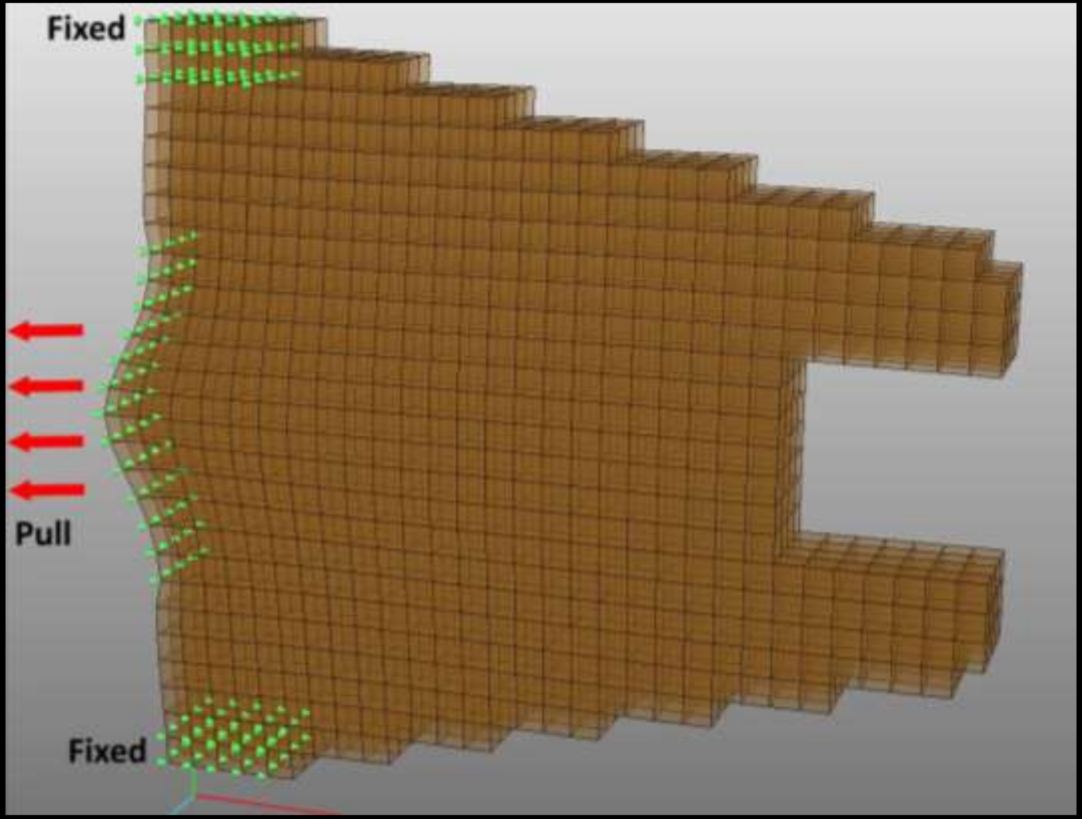




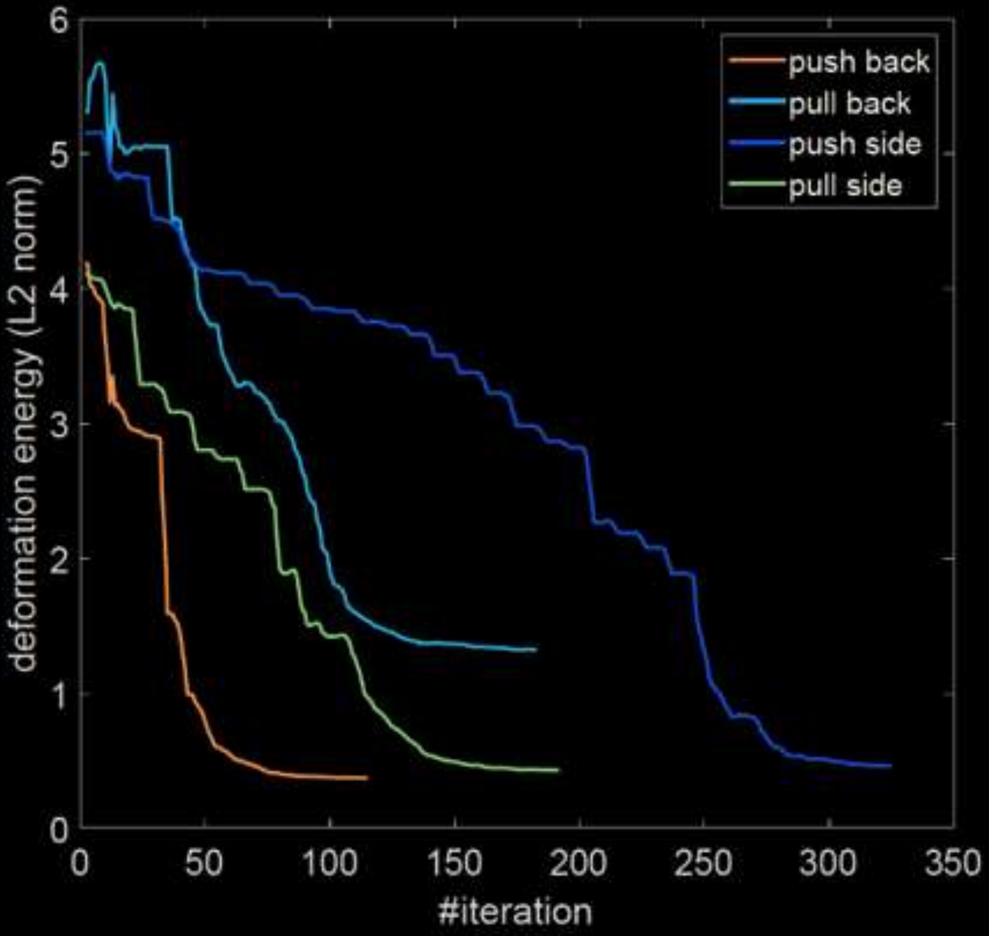
Optimization

Fabrication

Example: Different Gripping Mechanisms

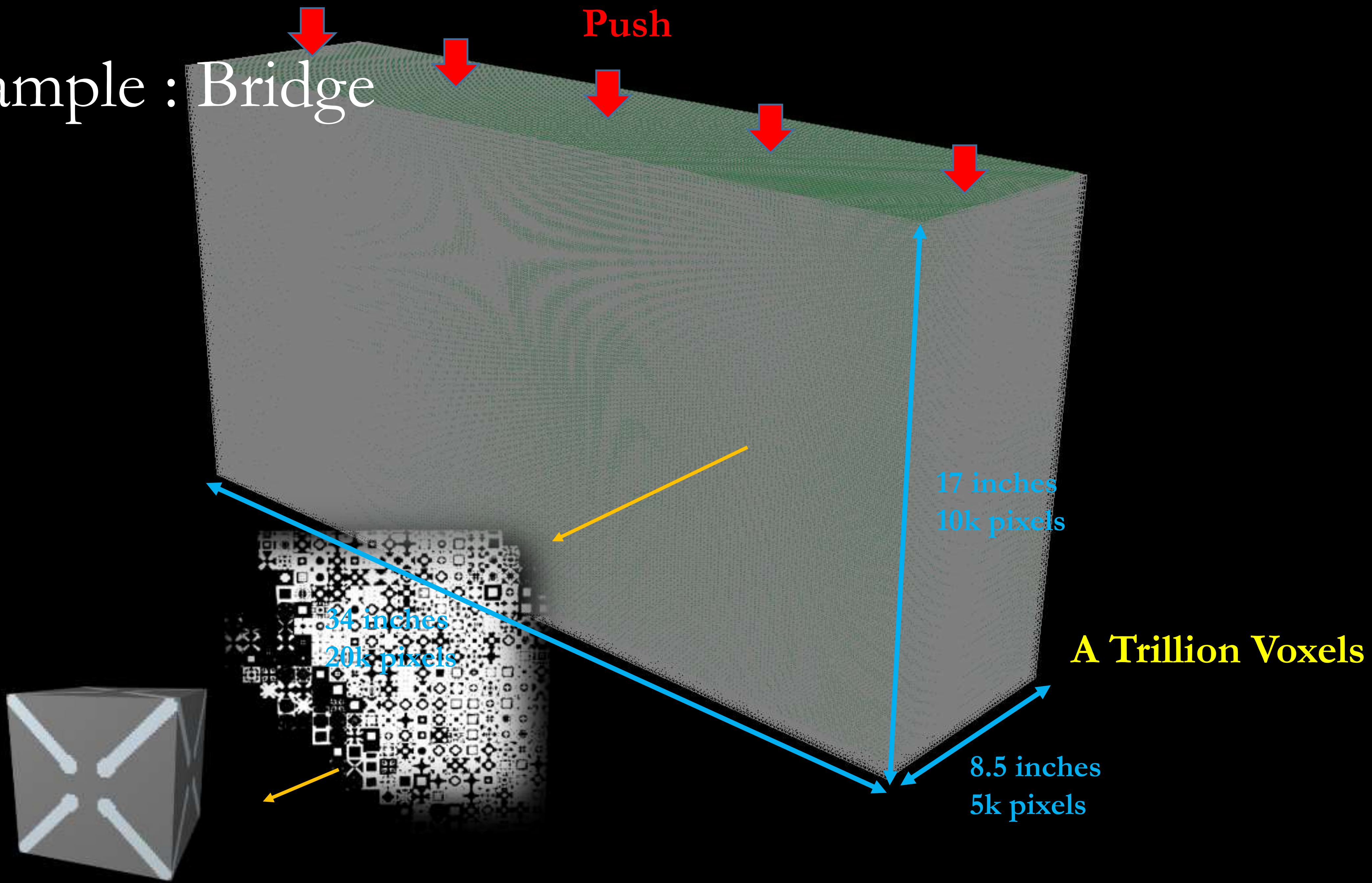


Different gripper structures optimized for the same target deformation

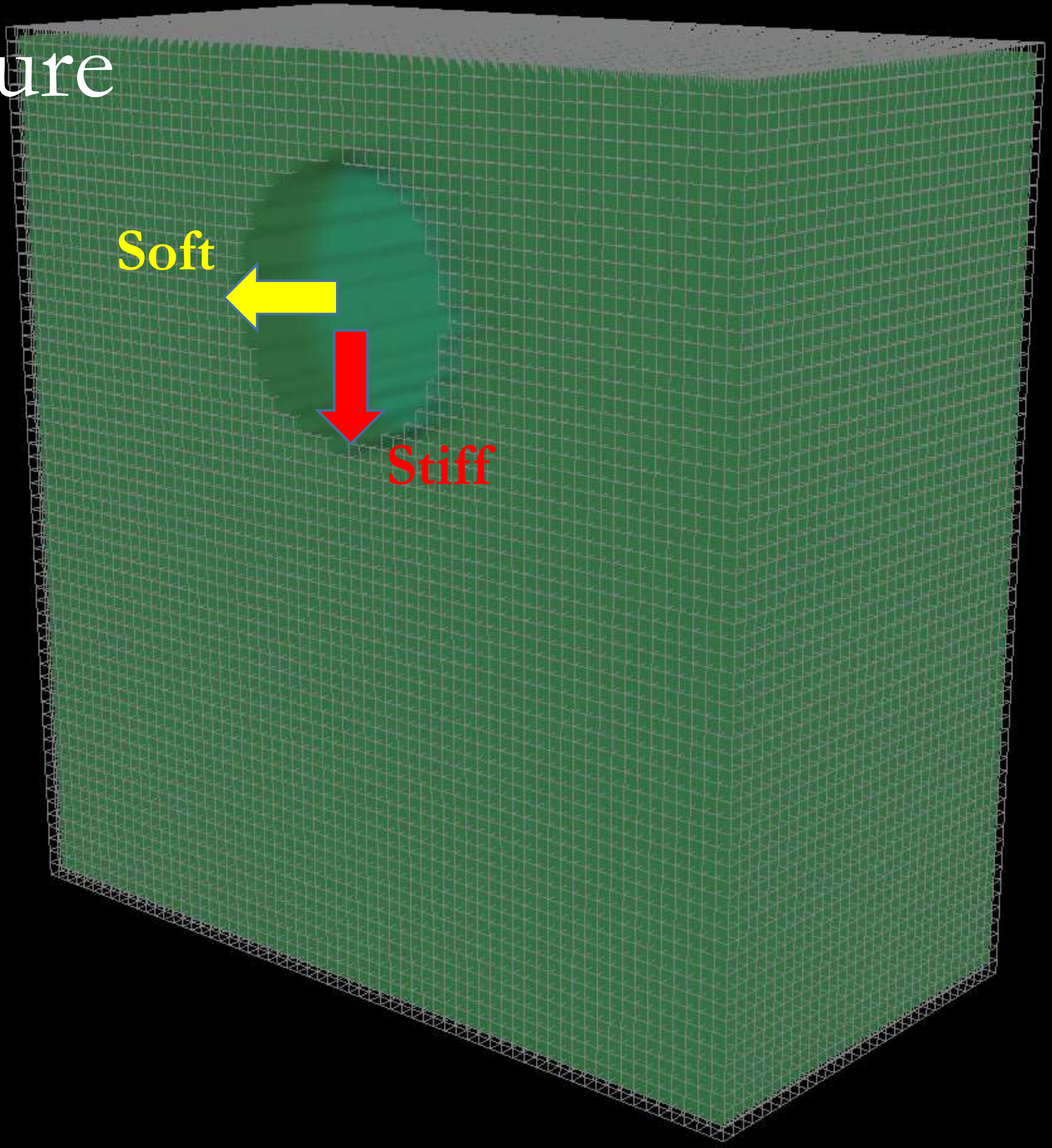


Convergence rate

Example : Bridge



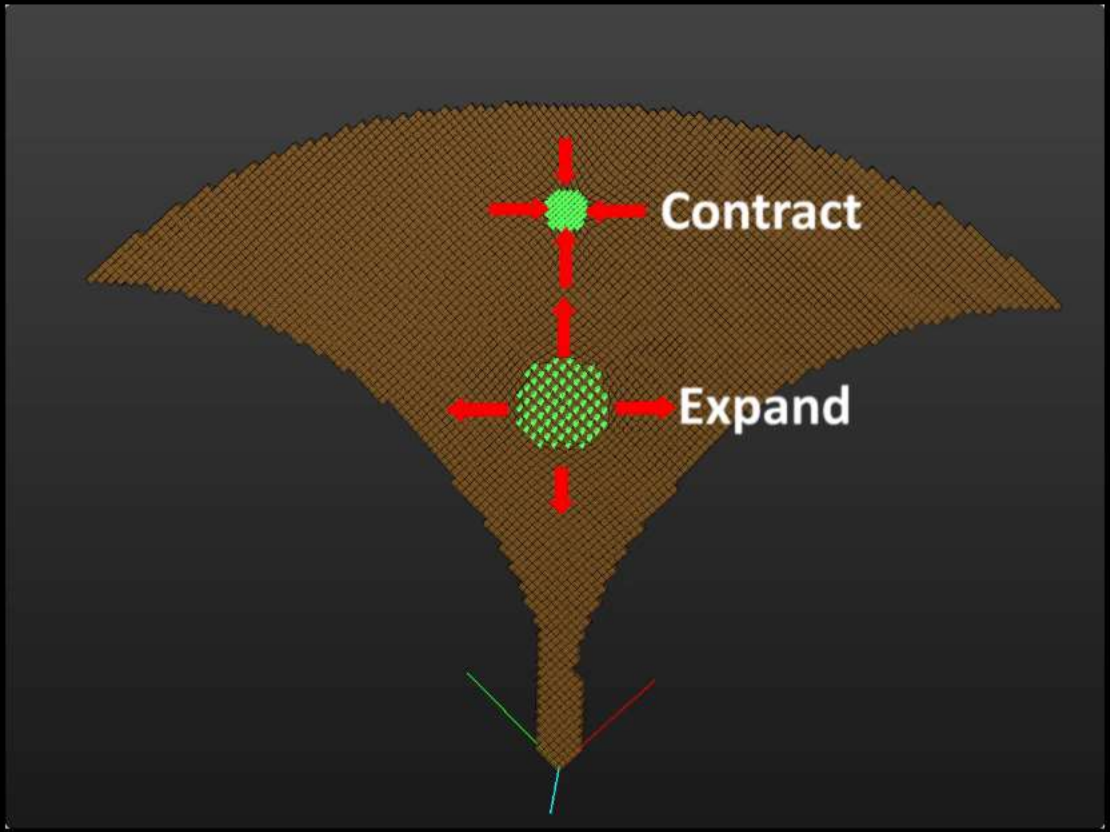
Example : Flexure



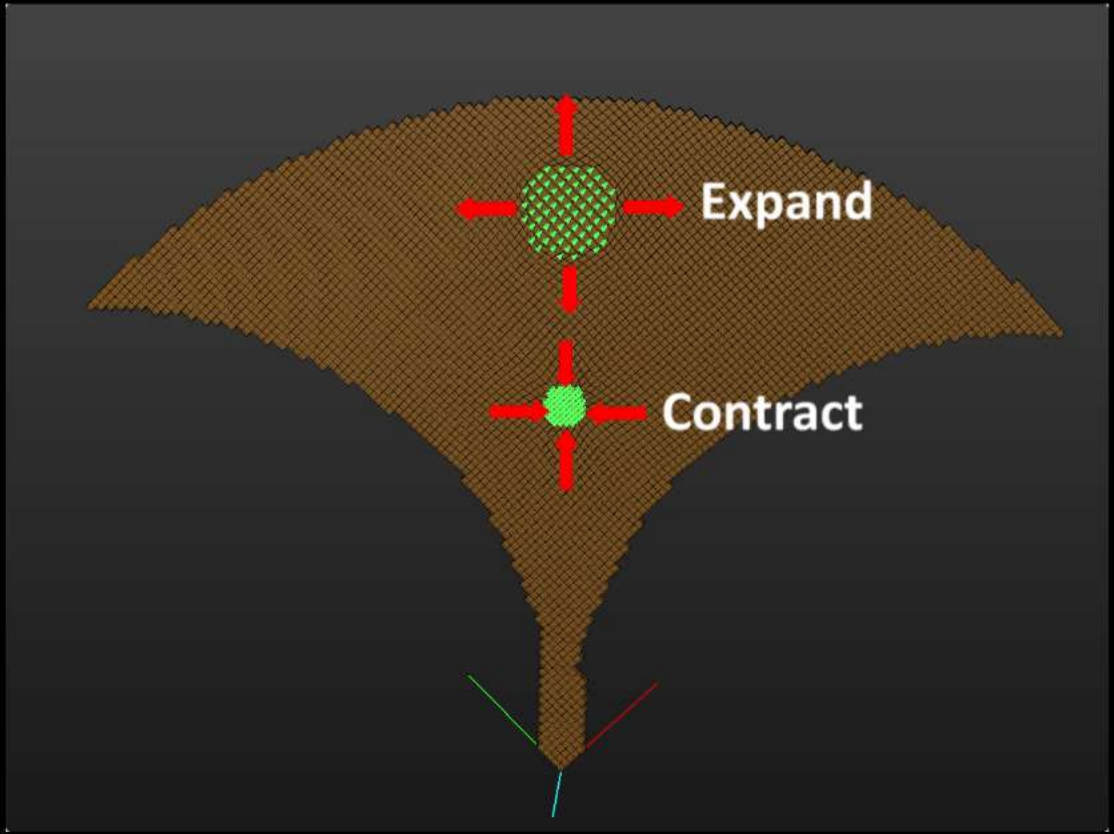
Multiple objectives

Topology Optimization Iterations

Example: Soft Ray



Target 1: flapping down



Target 2: flapping up

Thank you!