Multiscale Methods for Design and Fabrication of Deformable Objects

多尺度方法在设计制造可形变物体方面的应用

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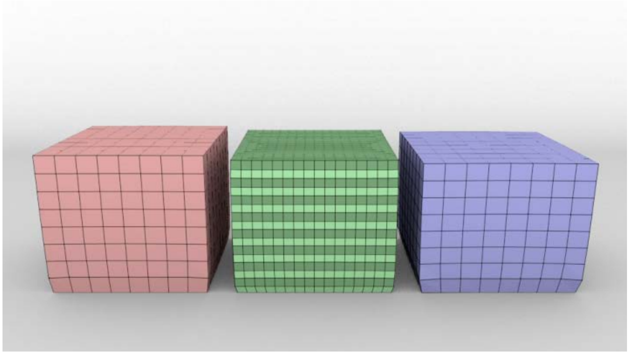
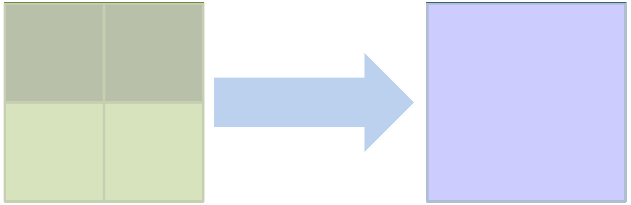
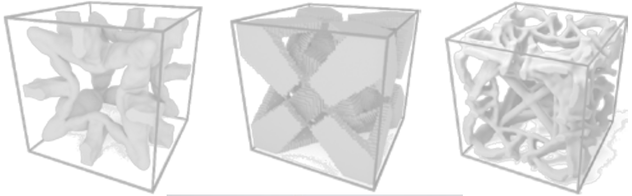
Overview

- **FEM for solid simulation**
- Data-driven coarsening for static simulation
- Topology optimization with microstructures
- Designing dynamic mechanisms





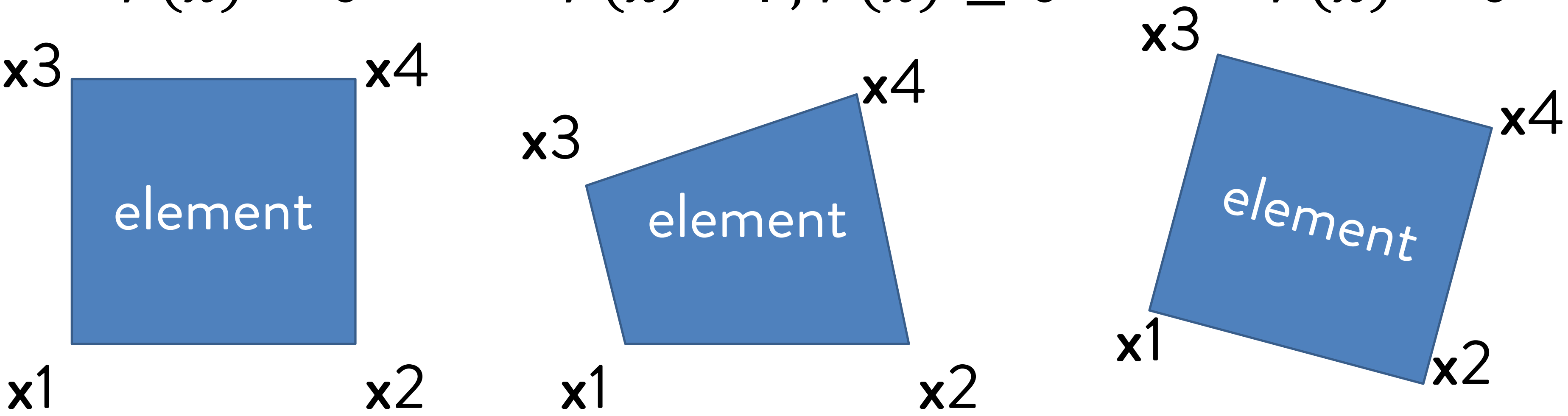


Finite Element Method (FEM)

Degrees of freedom: $\boldsymbol{x}$ coordinates

Elastic energy $V(\boldsymbol{x})$

$V(\boldsymbol{x}) = 0$ $\qquad$ $V(\boldsymbol{x}) = ?, V(\boldsymbol{x}) \geq 0$ $\qquad$ $V(\boldsymbol{x}) = 0$

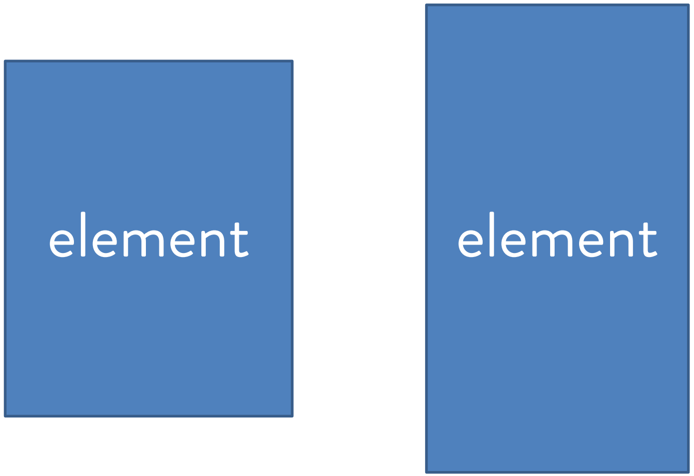


FEM for Hyperelastic Solids

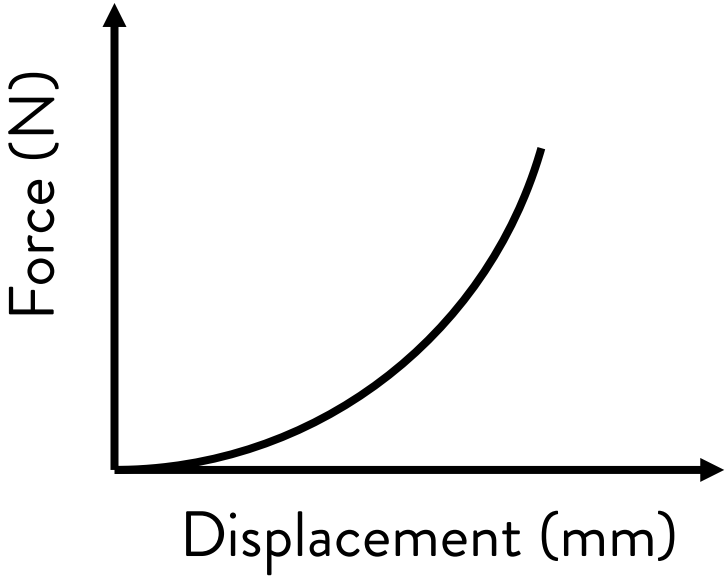
Degrees of freedom: $\boldsymbol{x}$ coordinates

Elastic energy $V(\boldsymbol{x})$

$$V(\boldsymbol{x}_1) < V(\boldsymbol{x}_2)$$



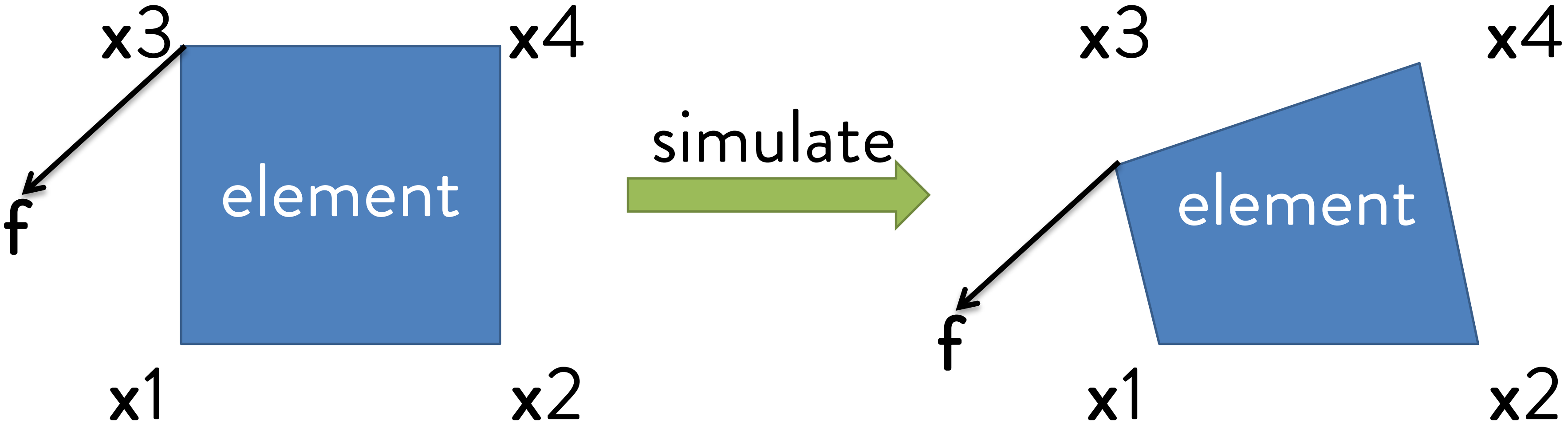
Stress increases with strain



FEM: Simulating Statics

external forces $\boldsymbol{f}$

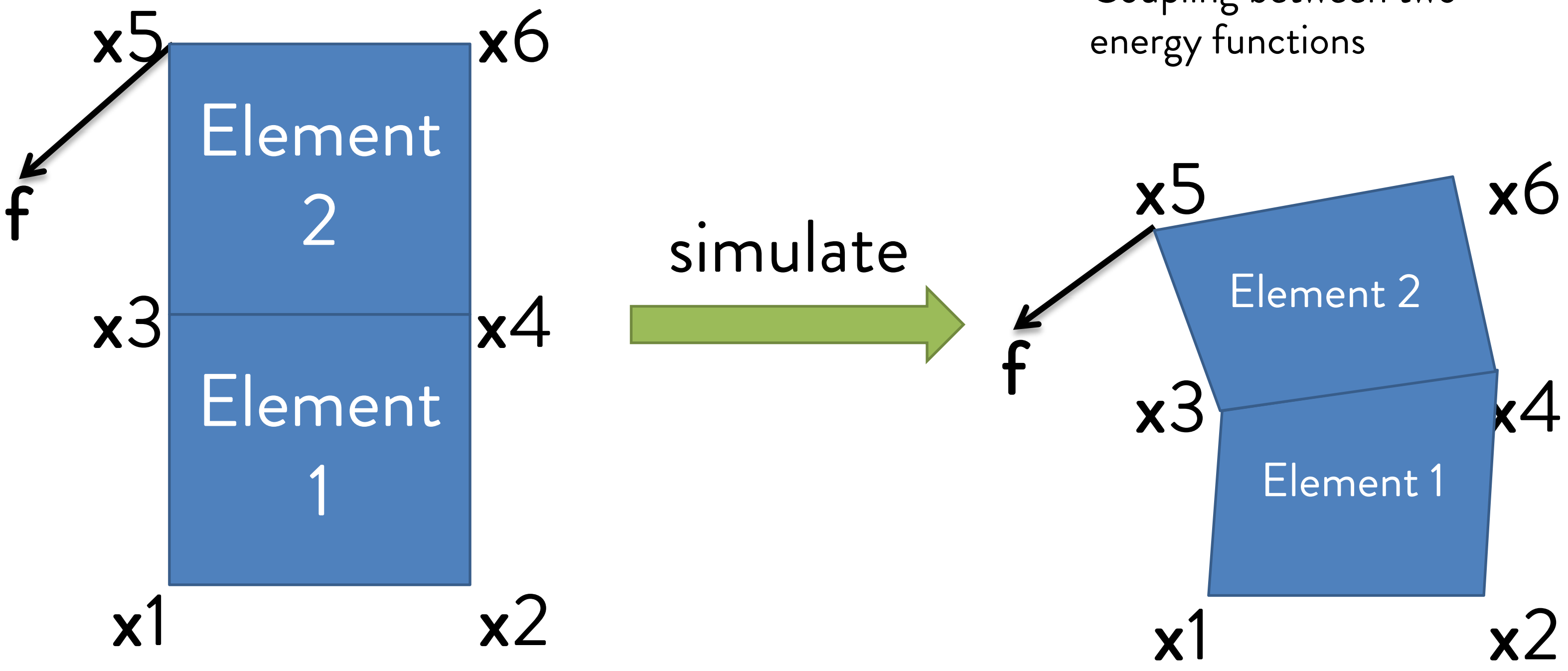
$$\arg\min_{\boldsymbol{x}} V(\boldsymbol{x}) - \boldsymbol{f} \cdot \boldsymbol{x}$$



FEM: Simulating Two Elements

$$\arg\min_{x} \Psi_1(x_1, x_2, \boxed{x_3, x_4}) + \Psi_2(\boxed{x_3, x_4}, x_5, x_6)$$

Coupling between two energy functions



Data-Driven Finite Elements
for Geometry and Material Design

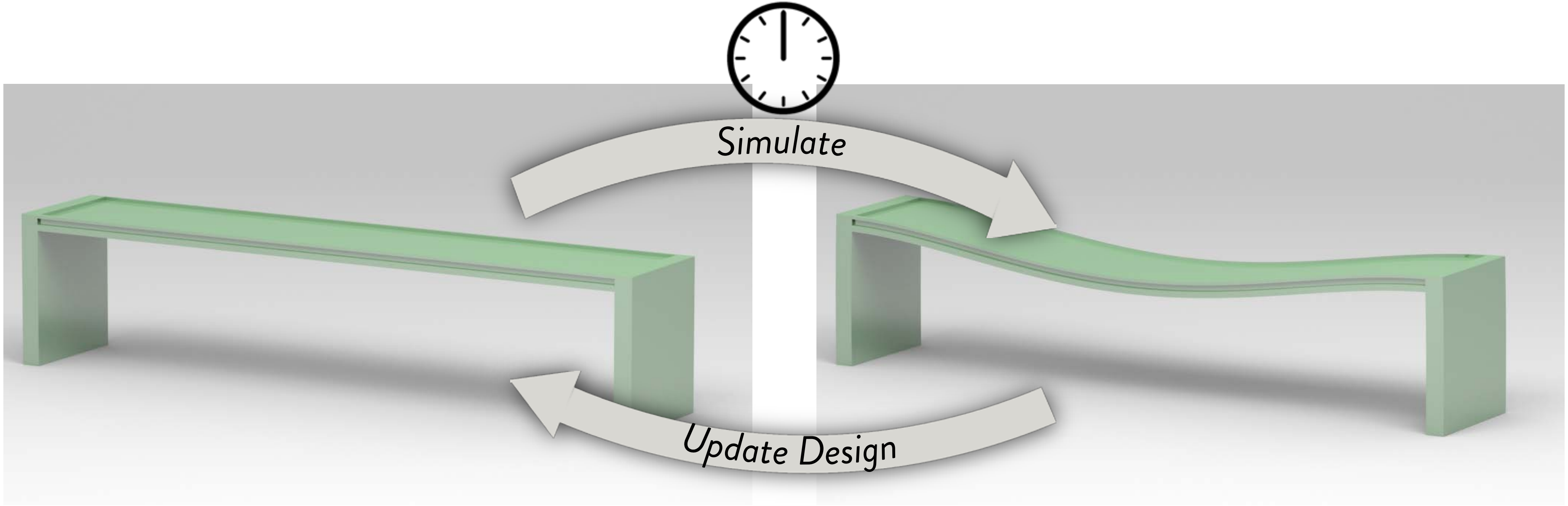
[1]Desai Chen, [12]David I.W. Levin, [123]Shinjiro Sueda, [12]Wojciech Matusik

1 MIT CSAIL

2 Disney Research

3 Cal Poly San Luis Obispo

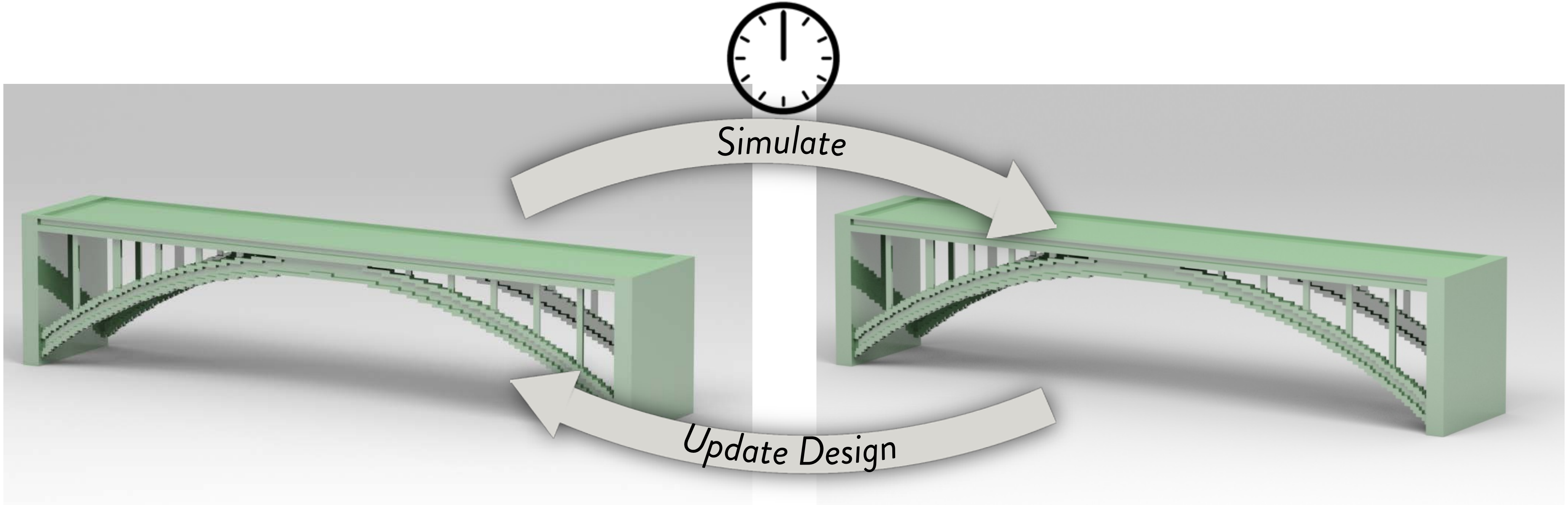
Iterative Design



Computer Aided Design - CAD

Simulation

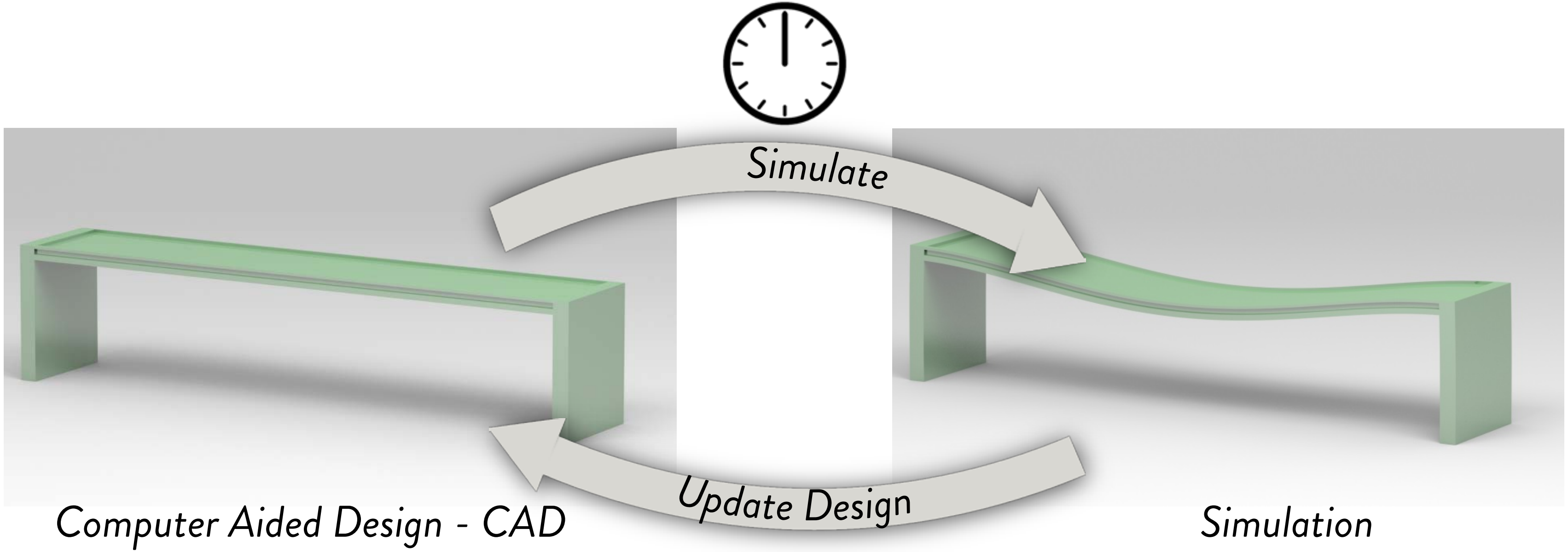
Iterative Design



Computer Aided Design - CAD

Simulation

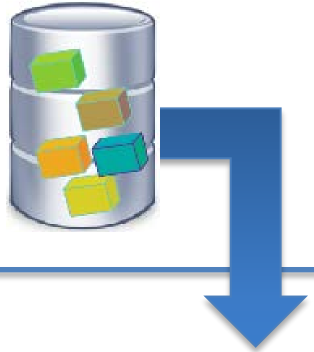
Iterative Design



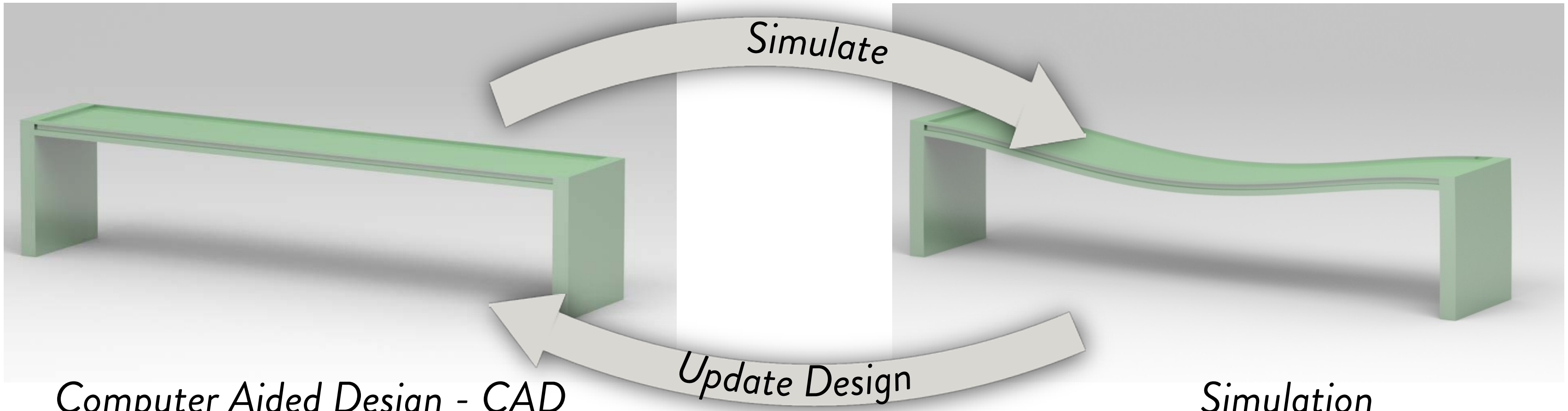
Data Driven Finite Elements

Offline

Metamaterial Database



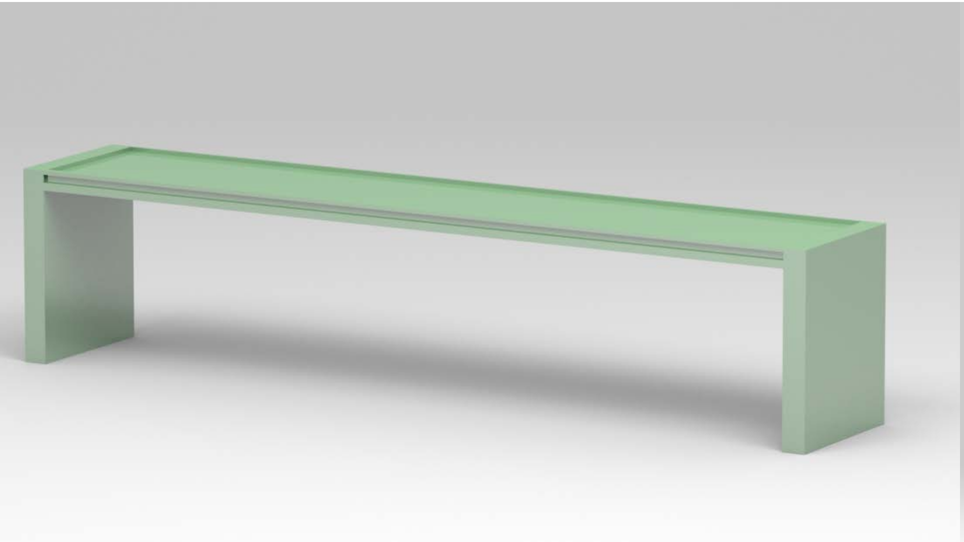
Online



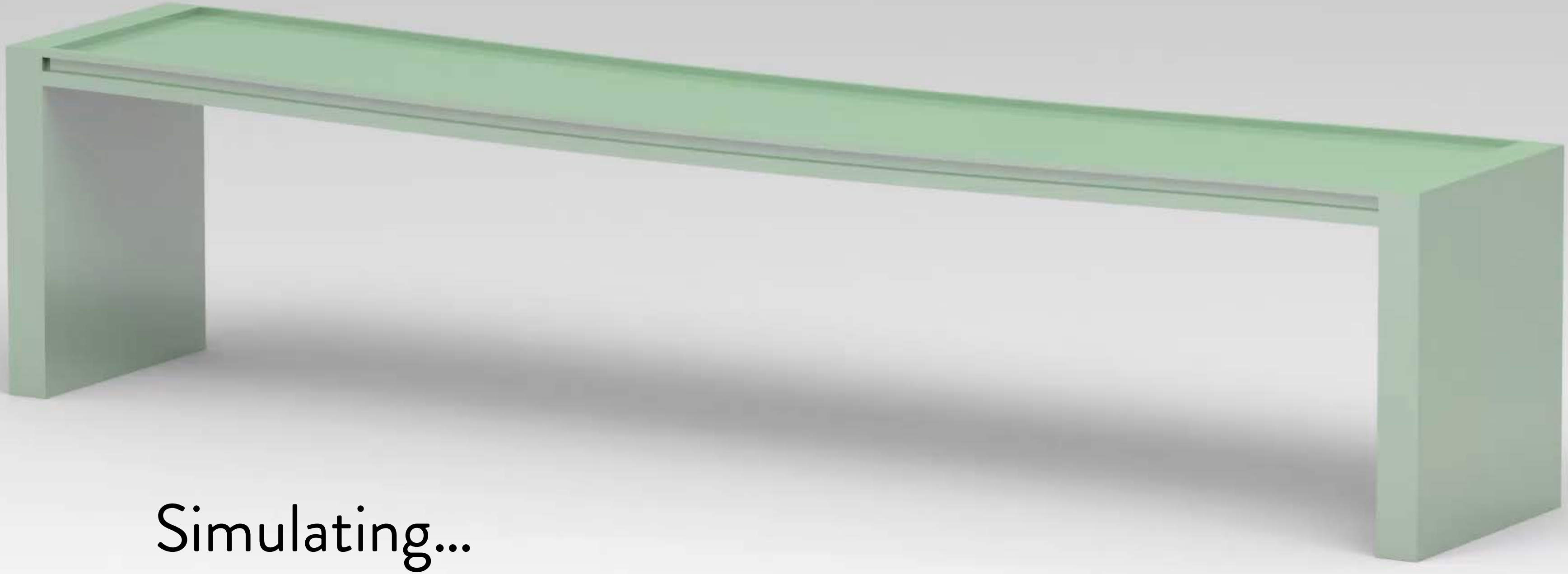
Computer Aided Design - CAD

Simulation

Geometry and Material Design

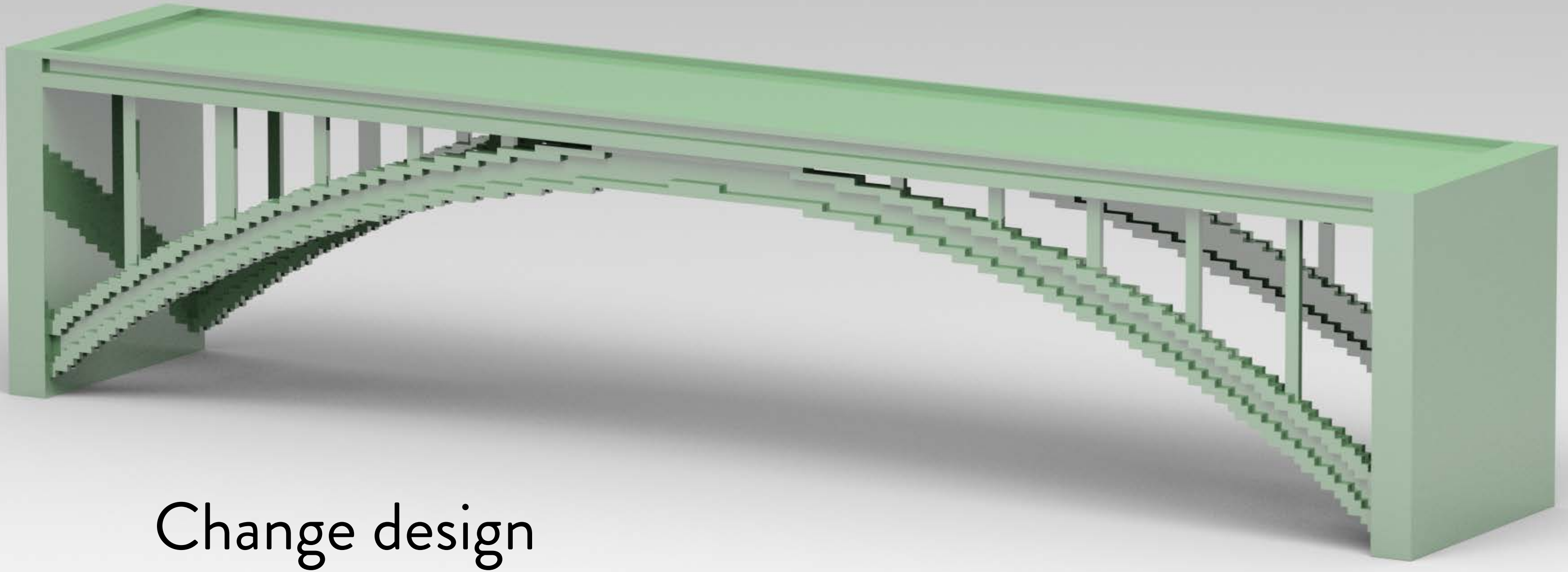


Geometry and Material Design



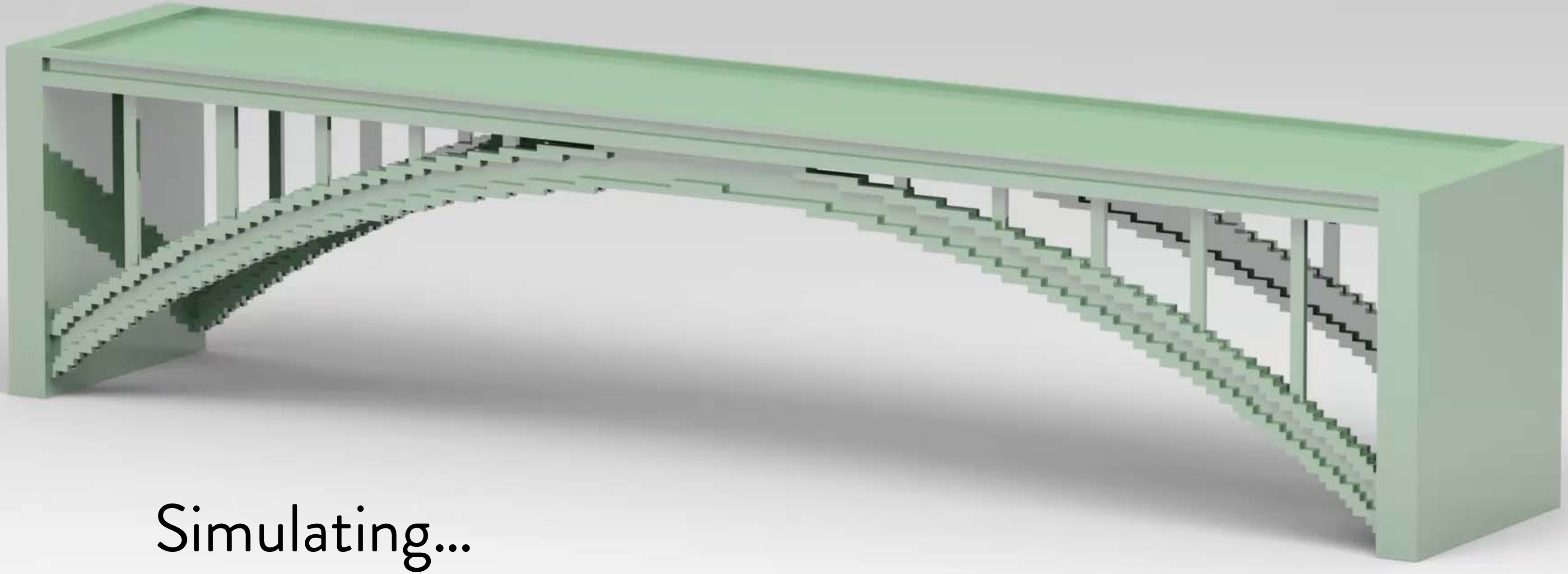
Simulating...

Geometry and Material Design



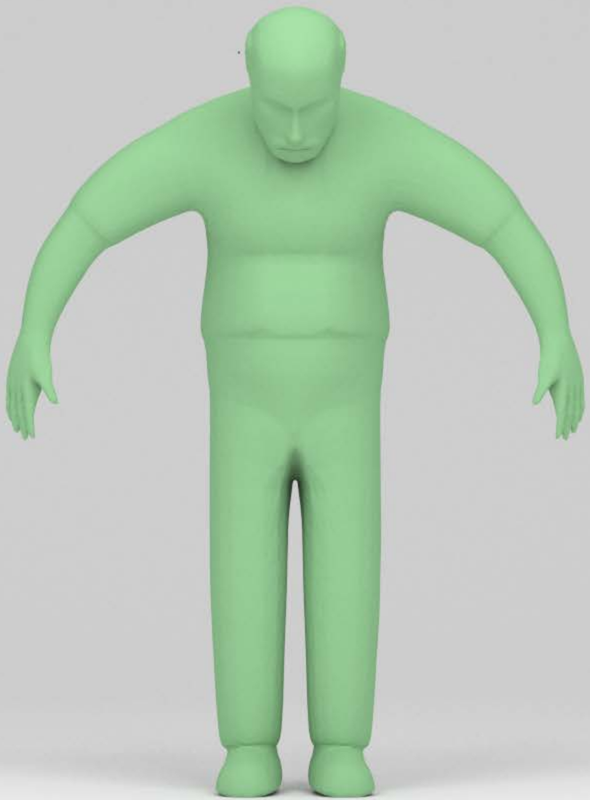
Change design

Geometry and Material Design



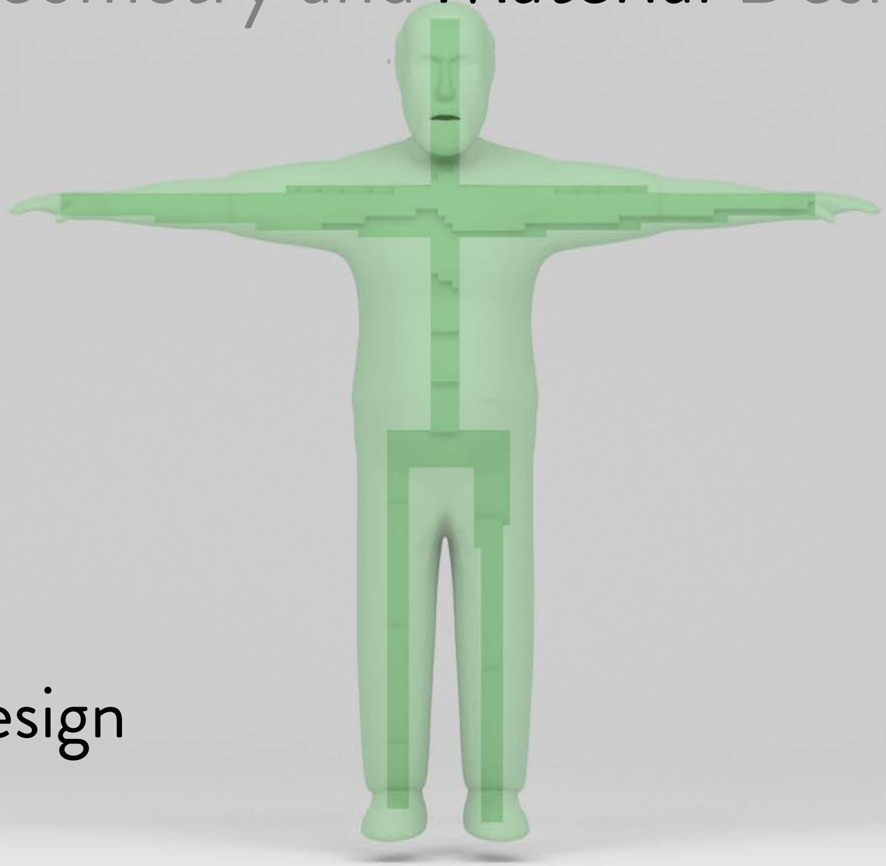
Simulating...

Geometry and Material Design



Simulation

Geometry and Material Design



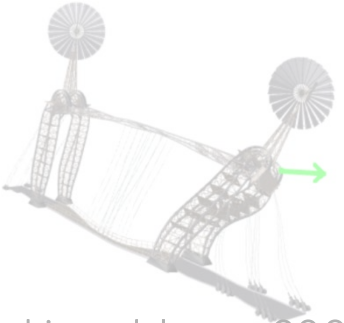
Change design

Geometry and Material Design

Simulation

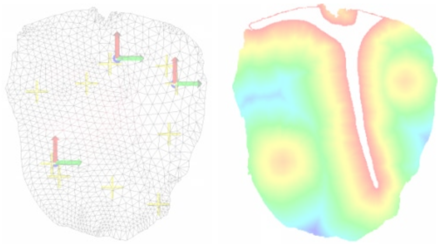
Related Work: Fast FEM with Precomputation

Vibrational Modes



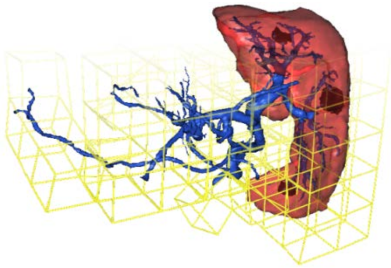
Barbic and James. 2005

Frames



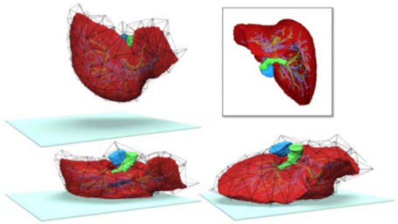
Faure et al. 2011

Shape Function



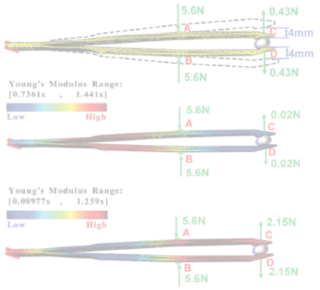
Nesme et al. 2009

Stiffness Tensor

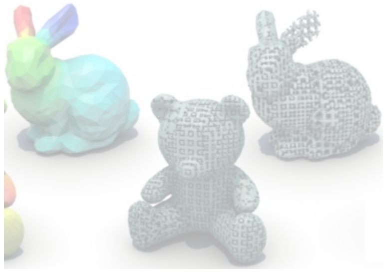


Kharevych et al. 2009

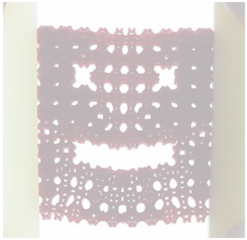
Coarsening Approaches



Xu et al. 2015

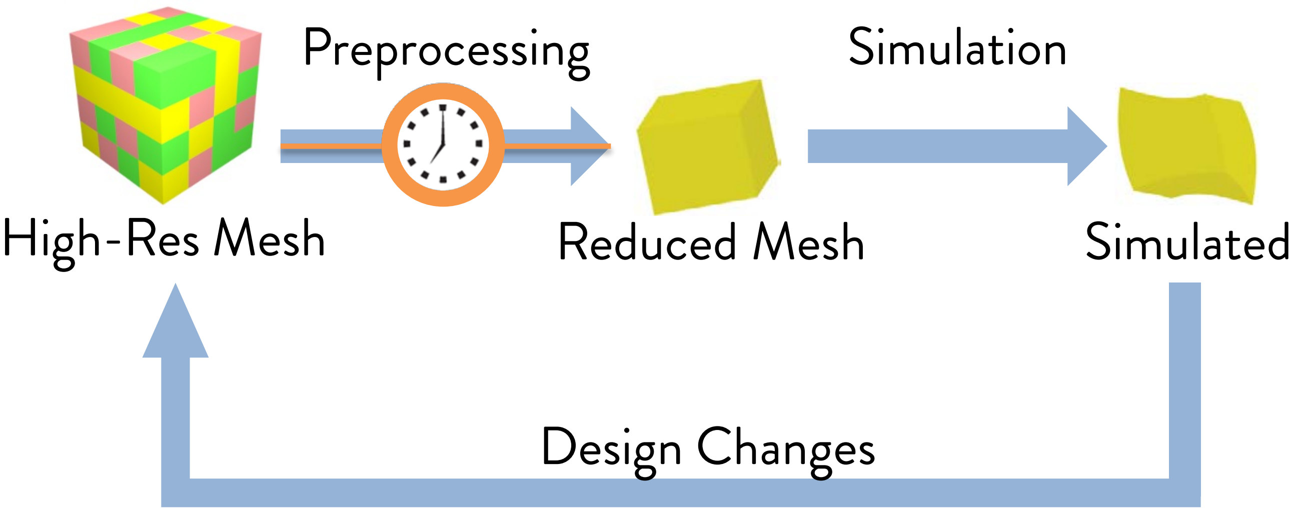


Schumacher et al. 2015

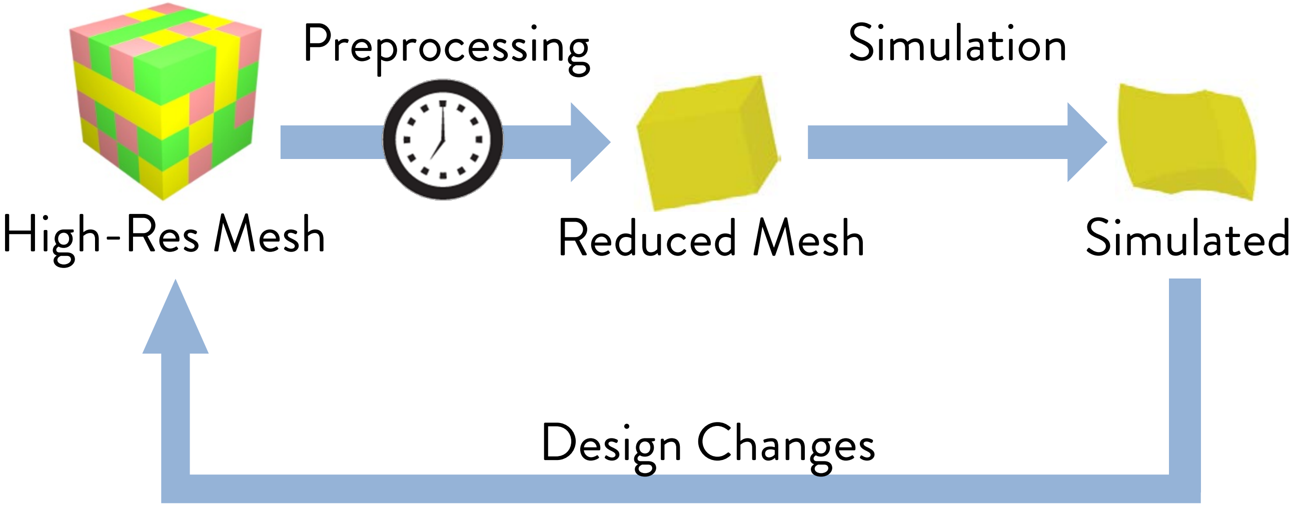


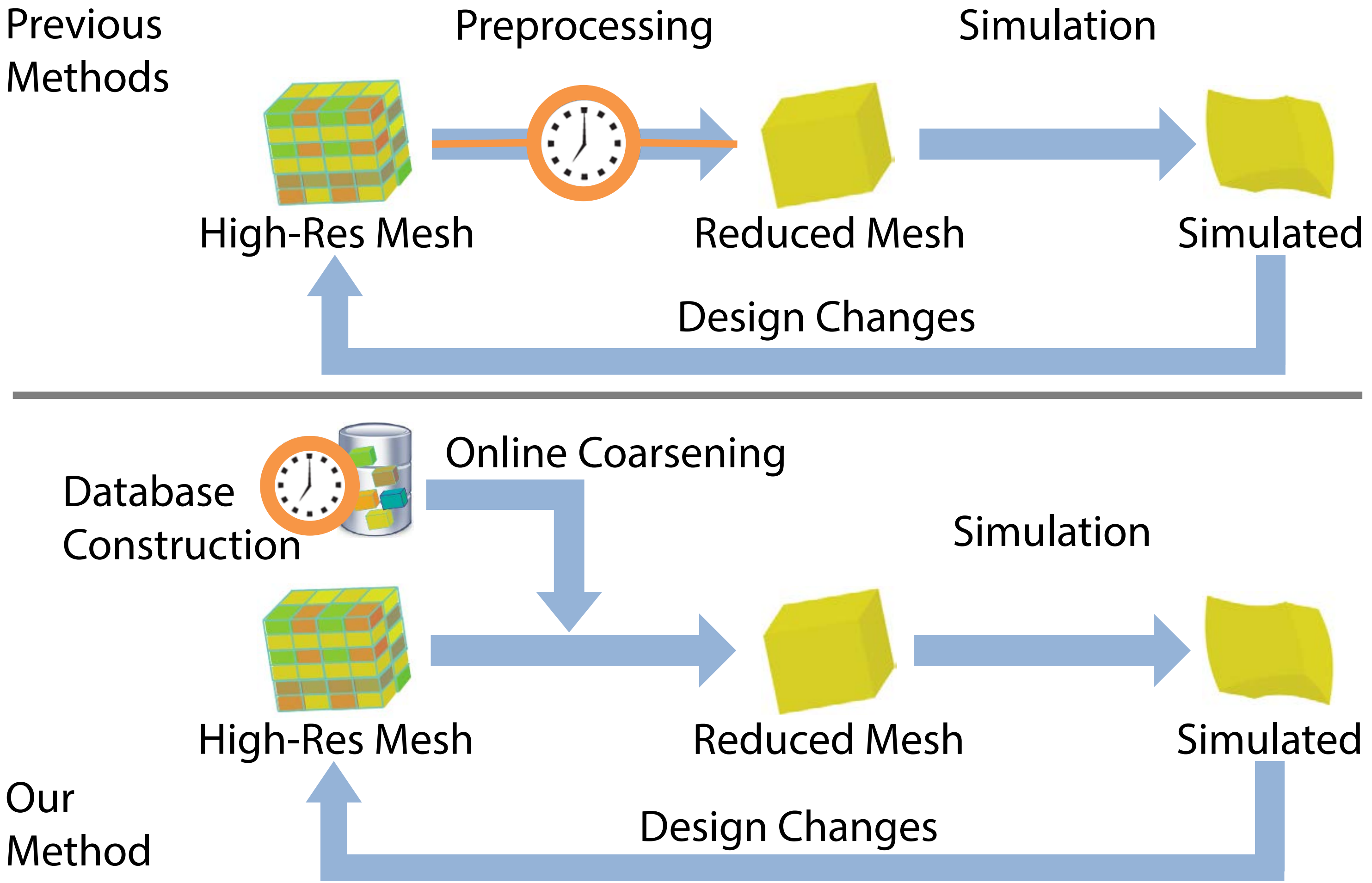
Panetta et al. 2015

Previous Algorithms



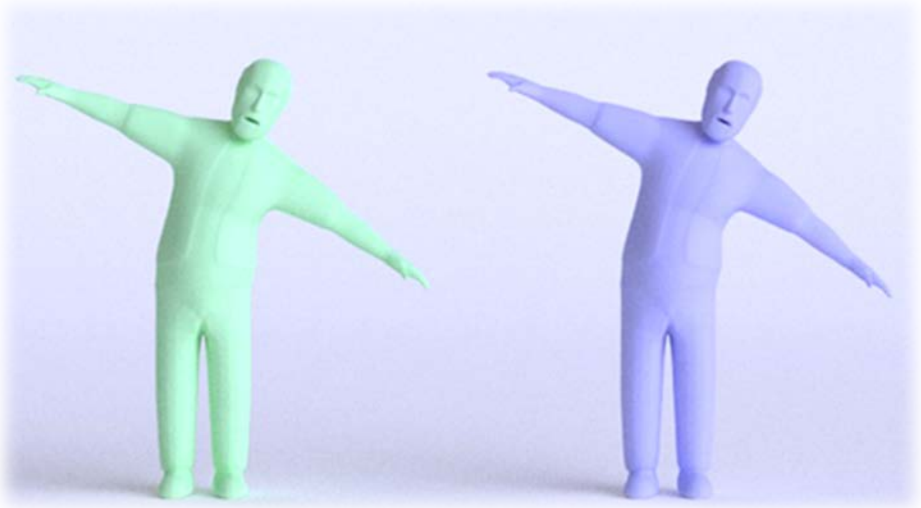
Previous Algorithms



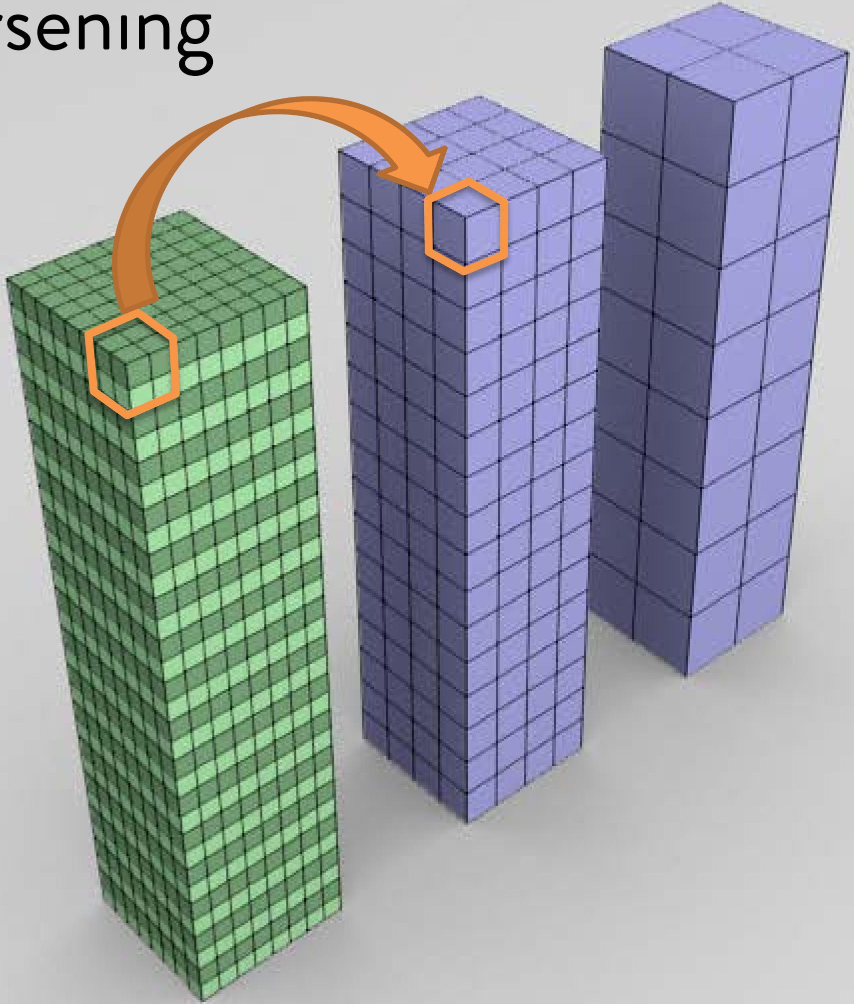

Previous Methods
Preprocessing
Simulation
High-Res Mesh
Reduced Mesh
Simulated
Design Changes
Database Construction
Online Coarsening
Simulation
High-Res Mesh
Reduced Mesh
Simulated
Our Method
Design Changes

Outline

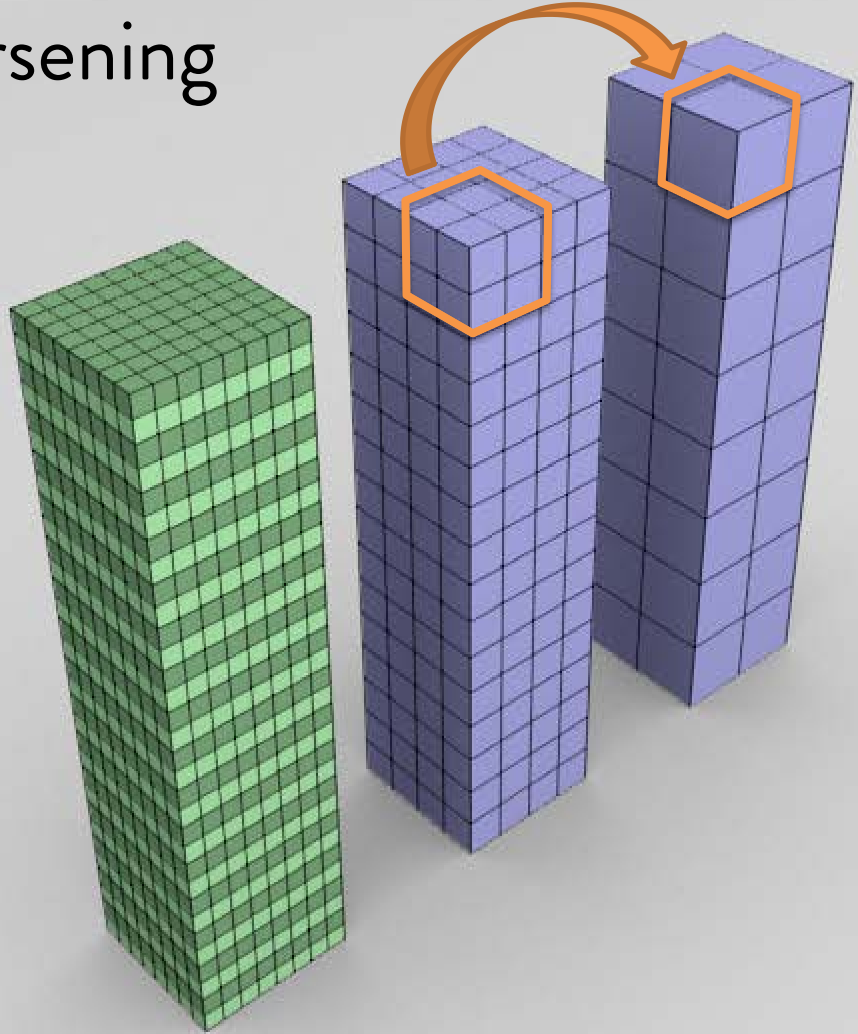
- Introduction
- Coarsening
- Database construction
- Hierarchical coarsening
- Runtime coarsening
- Results



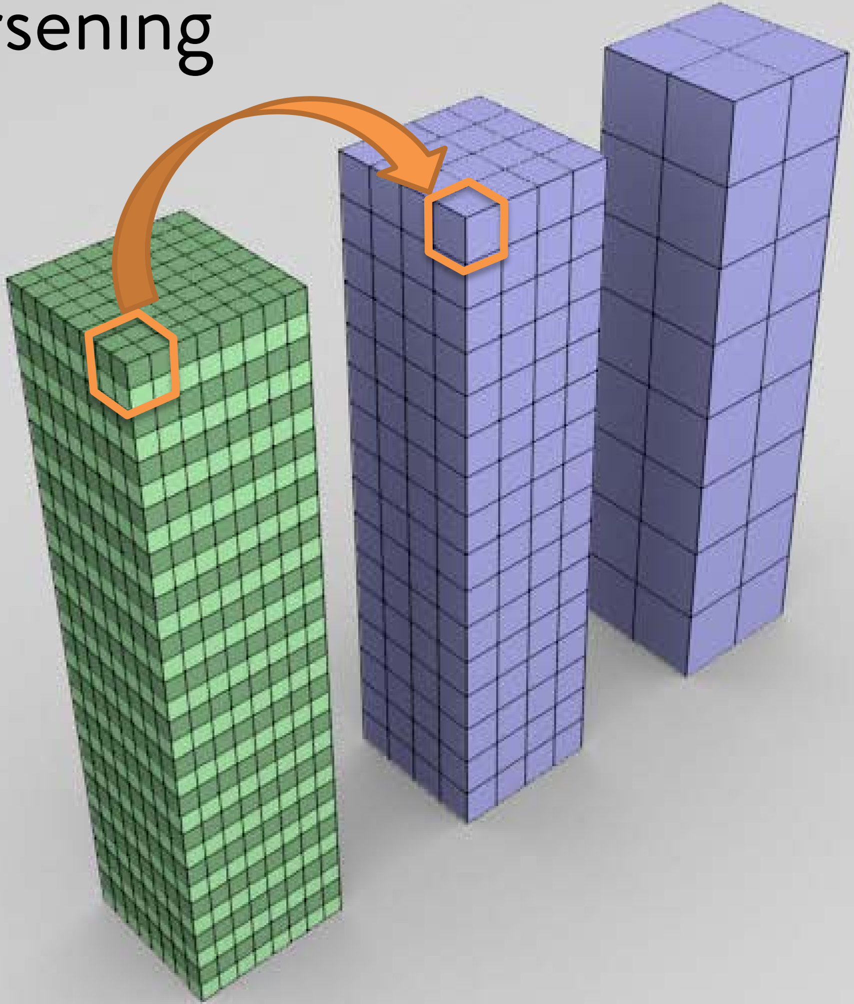
Method: Coarsening



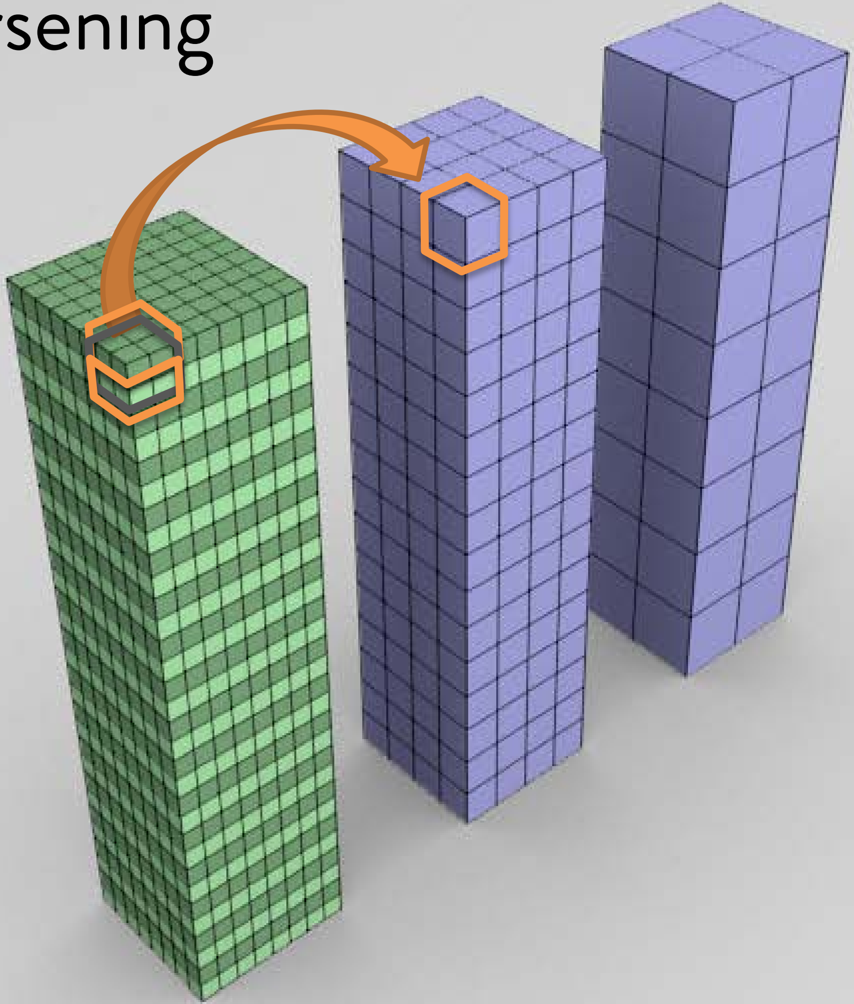
Method: Coarsening



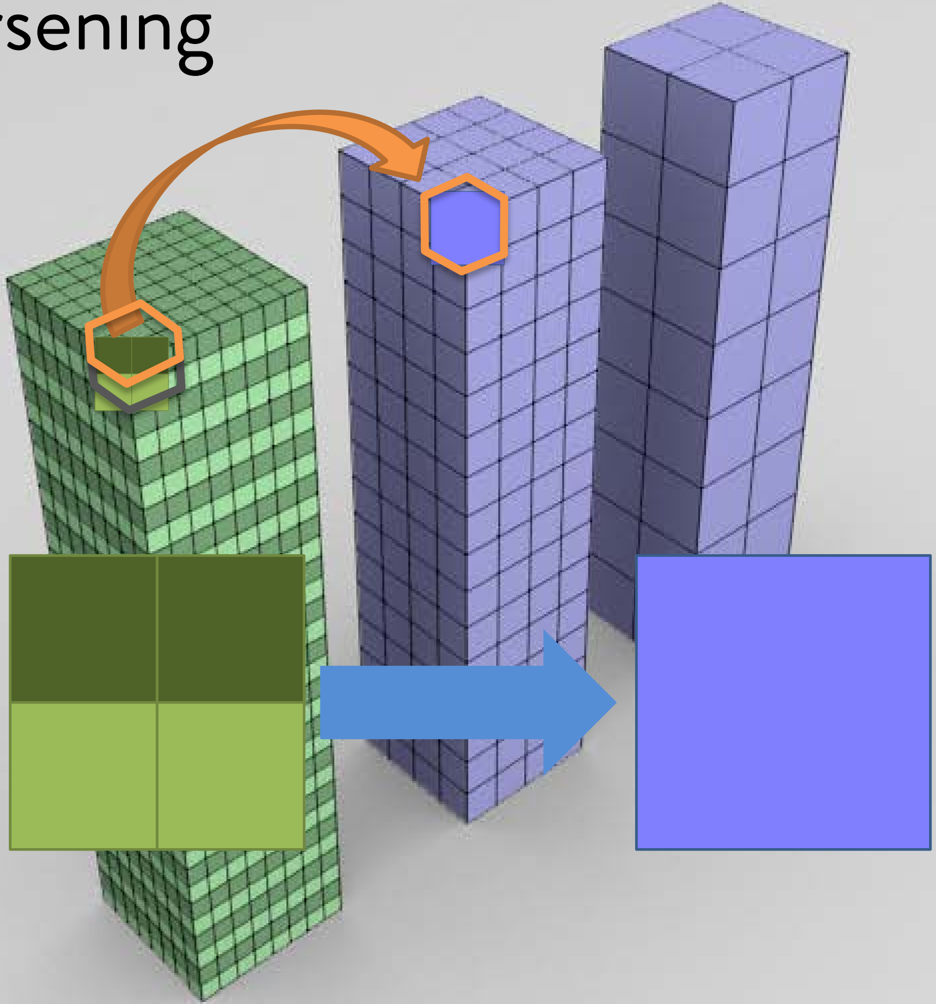
Method: Coarsening



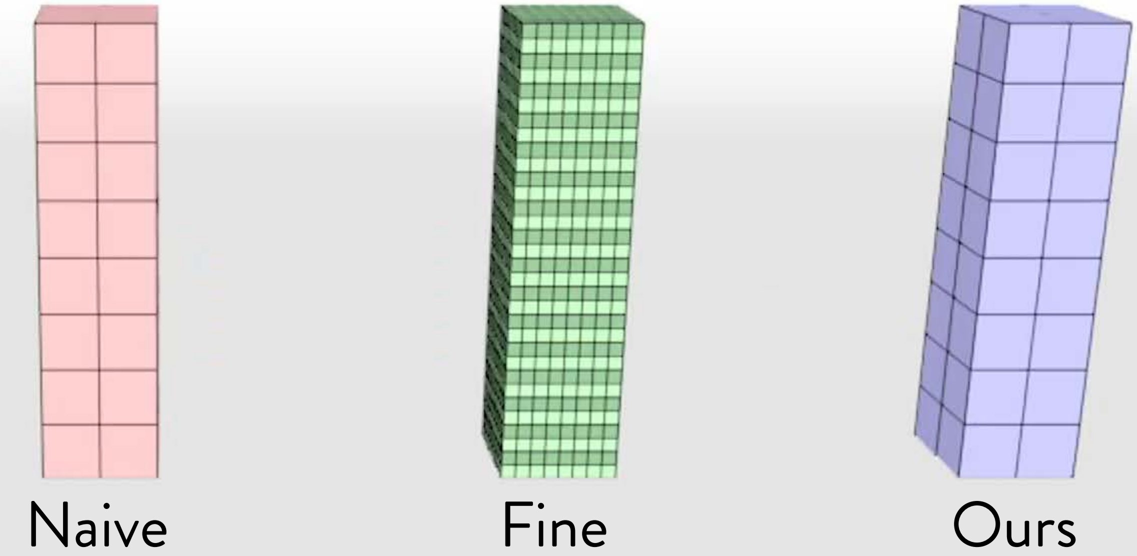
Method: Coarsening



Method: Coarsening



Coarsening



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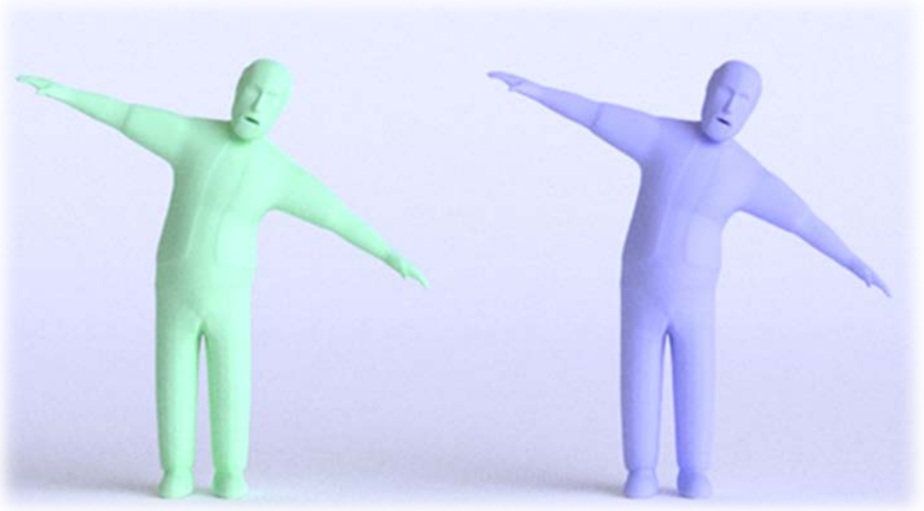
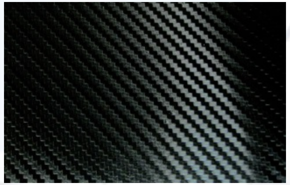


Method: Material Palette

Wood

Steel

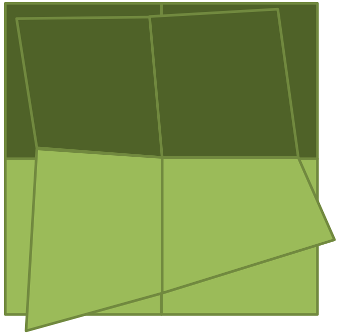
Rubber



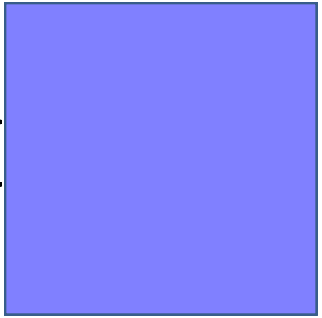
Rubber

Material palette

Method: Coarsening One Block

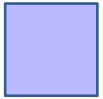


Fine elements

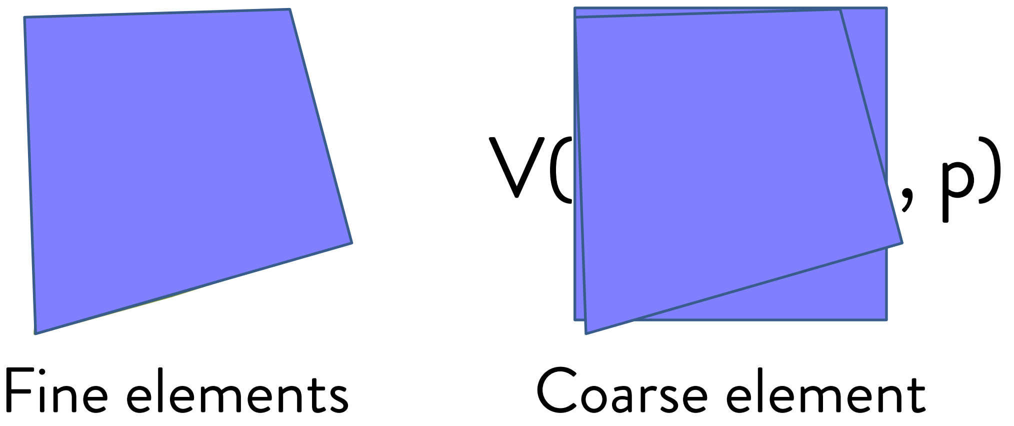
V( , p)

Coarse element

Material parameters

V( , p) – Strain energy function

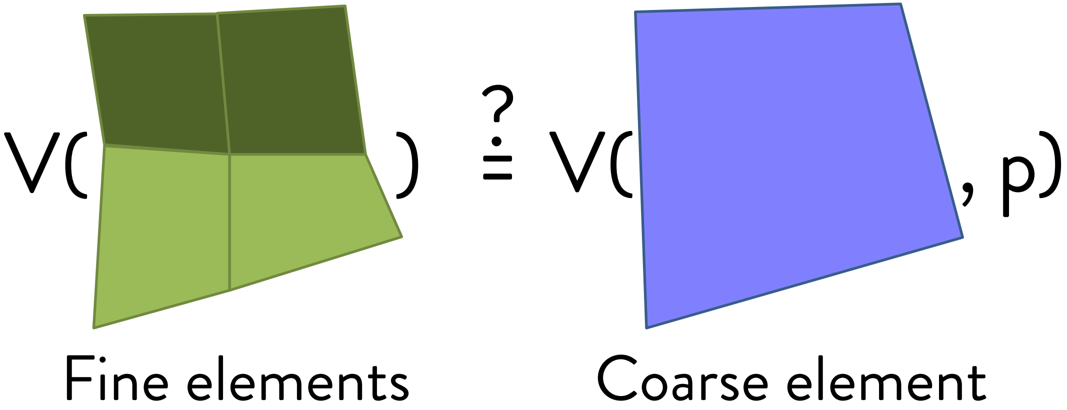
Method: Coarsening One Block



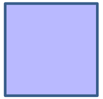
Material parameters

V(, p) – Strain energy function

Method: Coarsening One Block

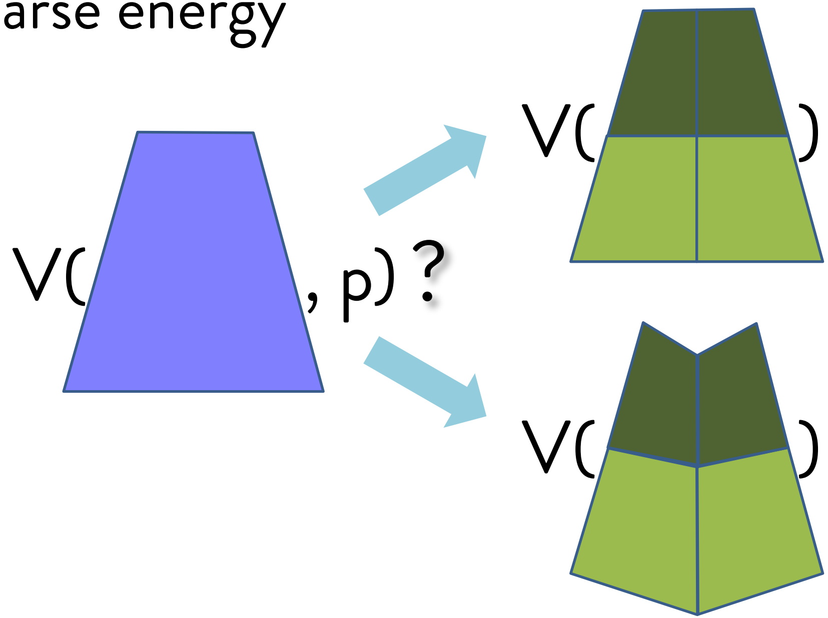


Material parameters

V( , p) – Strain energy function

Method: Coarsening One Block

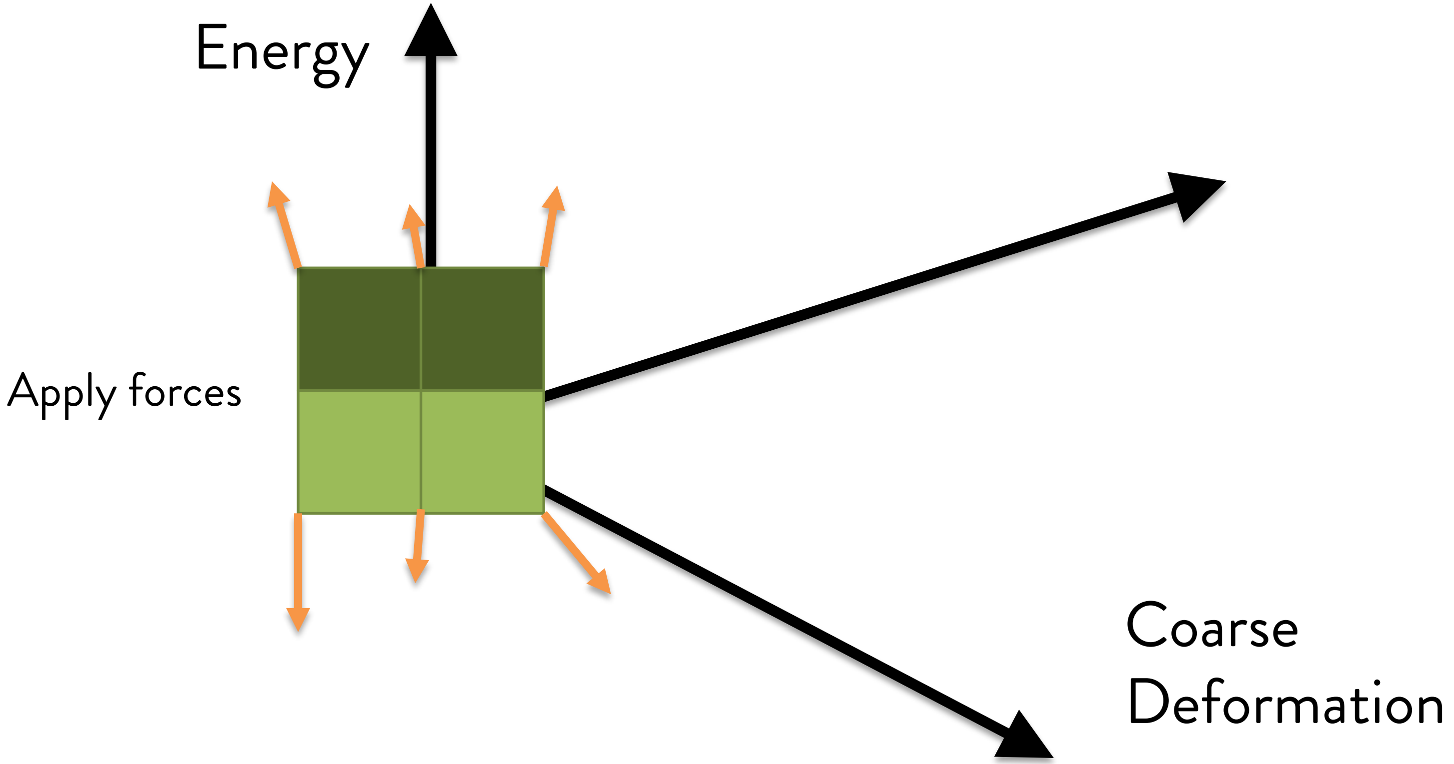
Ambiguity of coarse energy



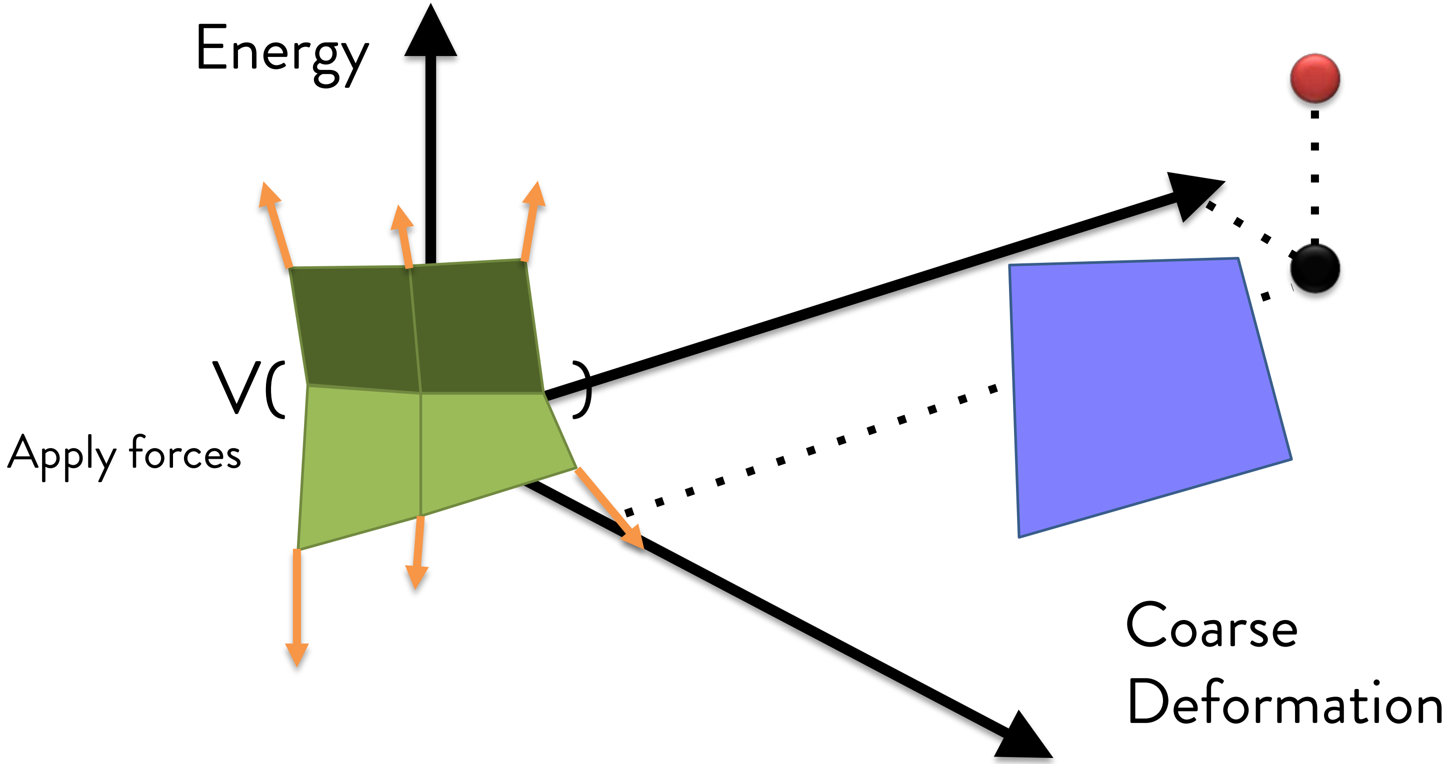
Coarse Deformation

Compatible fine deformations

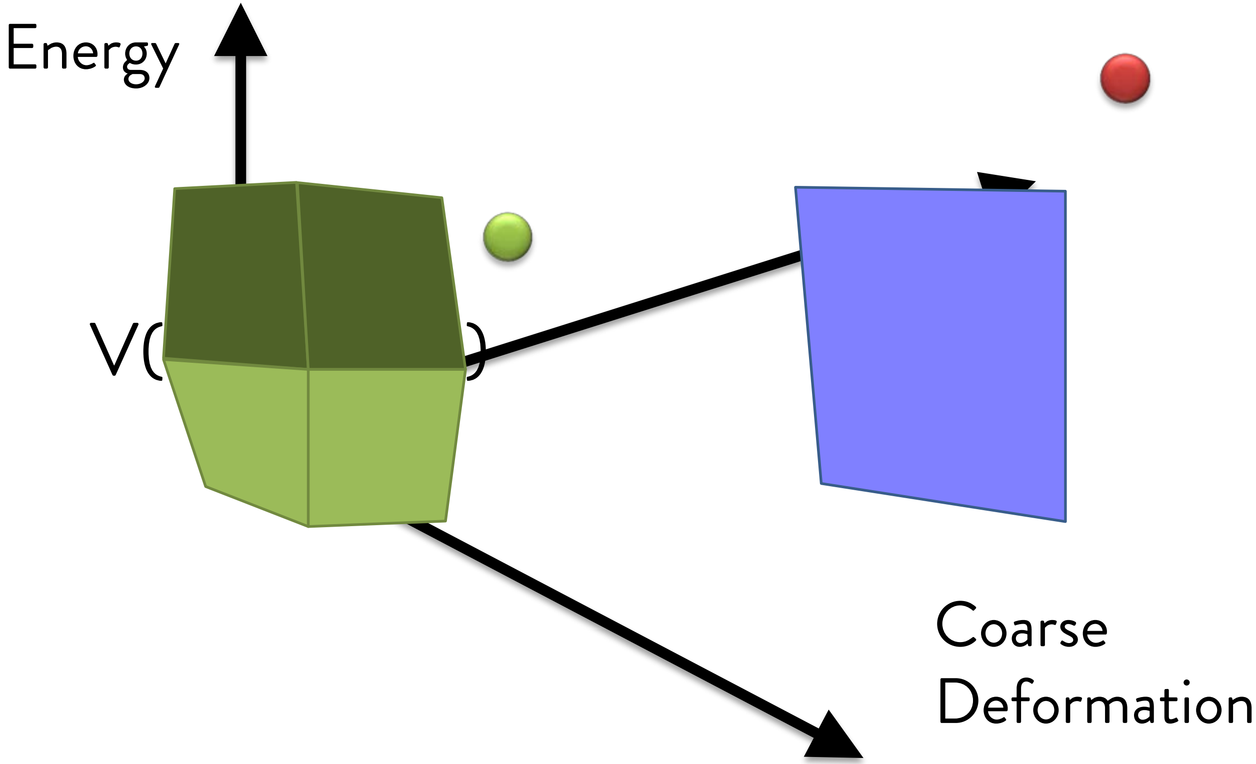
Method: Force-based Sampling



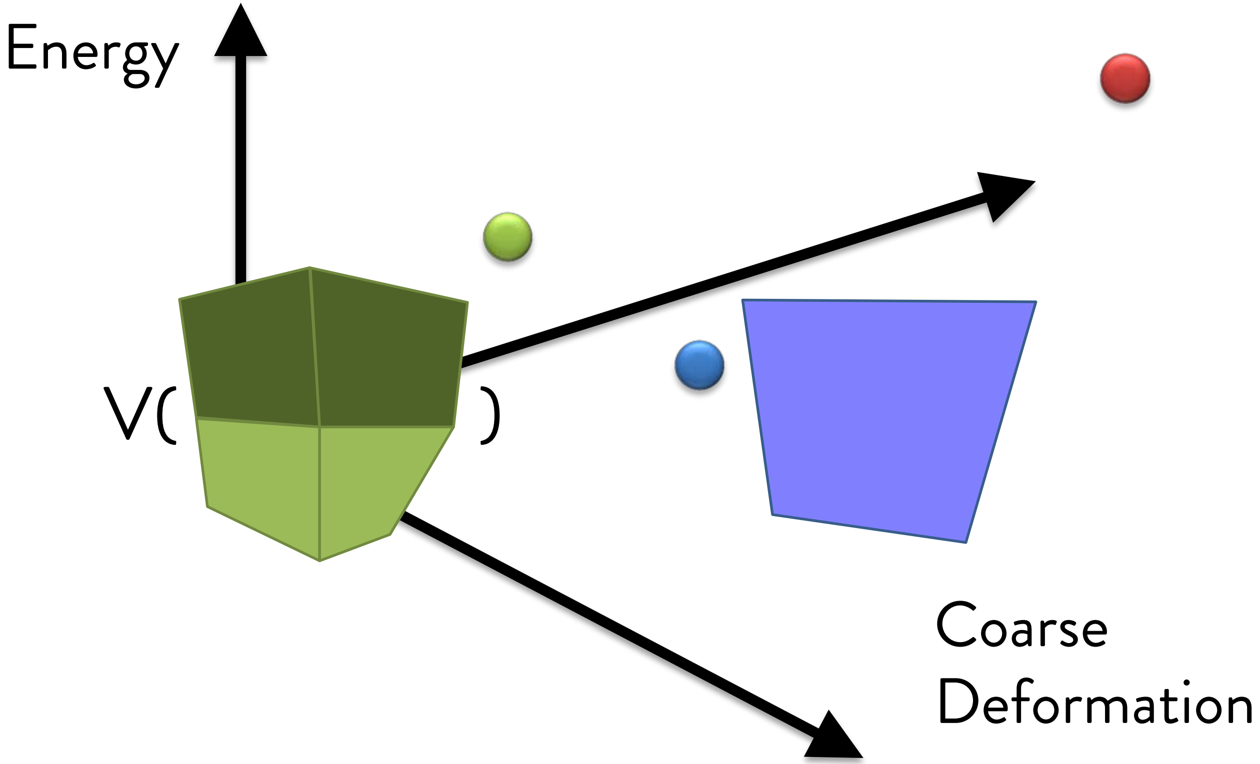
Method: Force-based Sampling



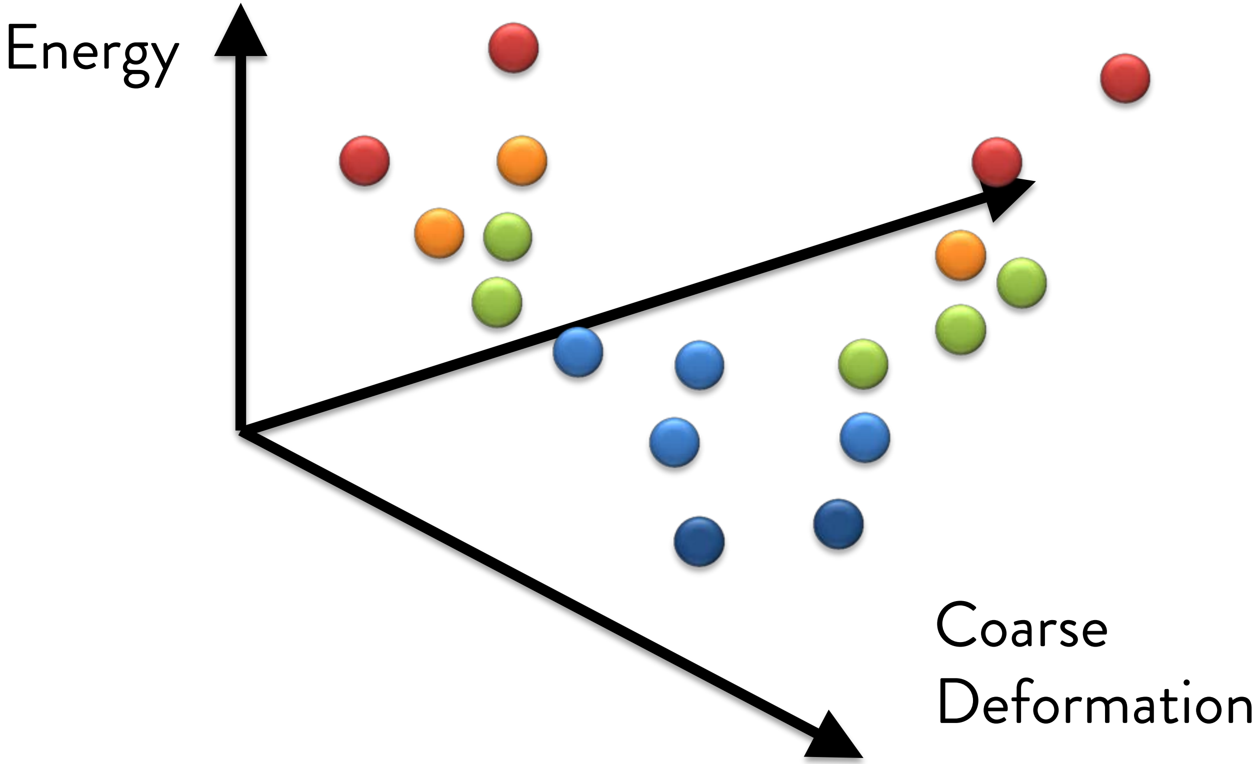
Method: Force-based Sampling



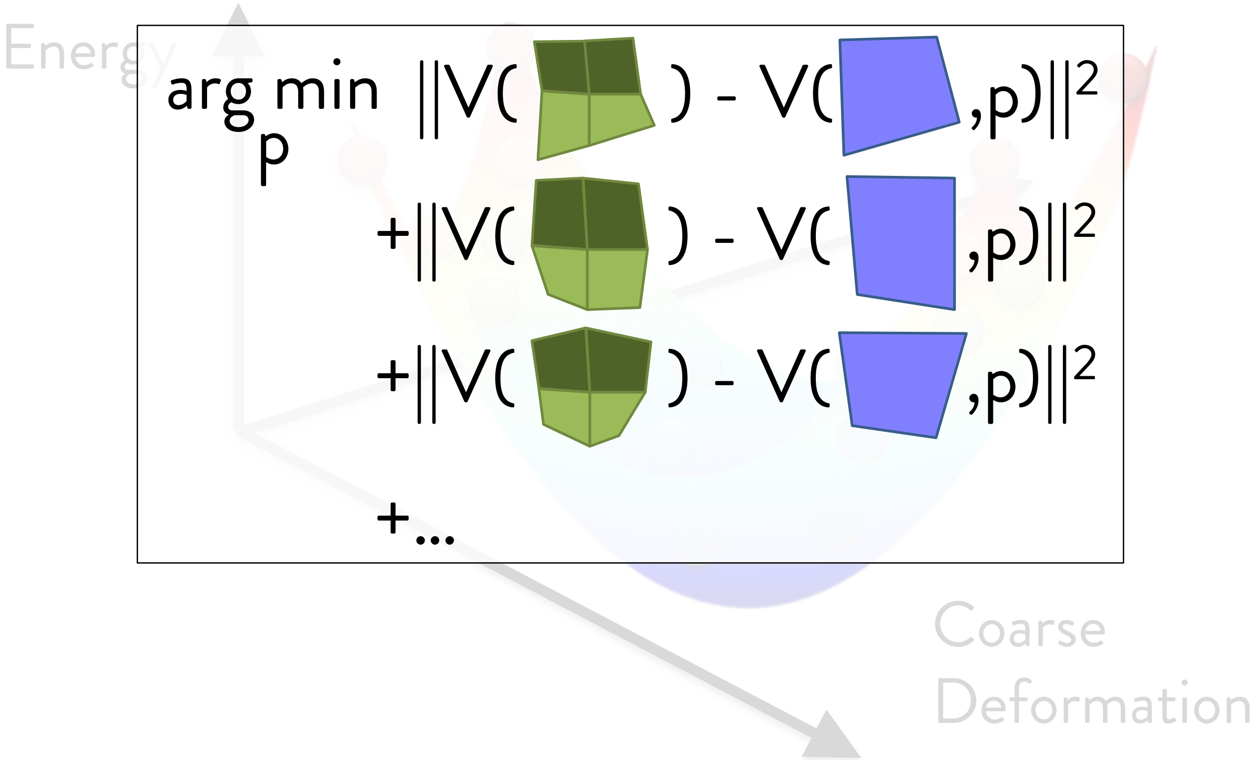
Method: Force-based Sampling



Method: Force-based Sampling

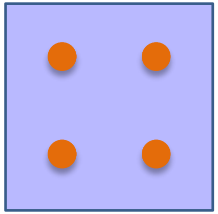


Method: Fitting Strain Energy



Method: Parameterization of Strain Energy

$$p = [p_1, p_2, p_3, p_4]$$

$V($  $, p) = V(F_1, p_1) + V(F_2, p_2) + V(F_3, p_3) + V(F_4, p_4)$

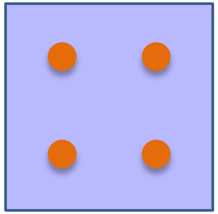
Coarse energy

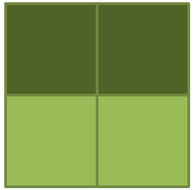
Sum over quadrature points ●

Functions of deformation gradients F

Method: Parameterization of Strain Energy

$$p = [p_1, p_2, p_3, p_4]$$

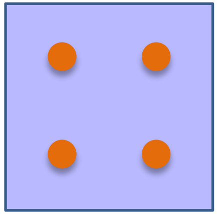
$V($  $, p) = V(F_1, p_1) + V(F_2, p_2) + V(F_3, p_3) + V(F_4, p_4)$

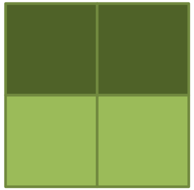


Fine material models

Method: Parameterization of Strain Energy

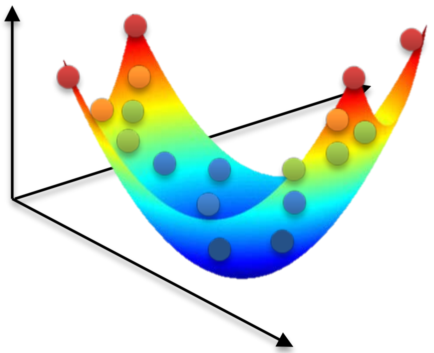
$p = [p_1, p_2, p_3, p_4]$

V( , p) = V( , p_1) + V( , p_2) + V( , p_3) + V( , p_4)

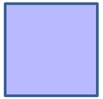


Fine material models

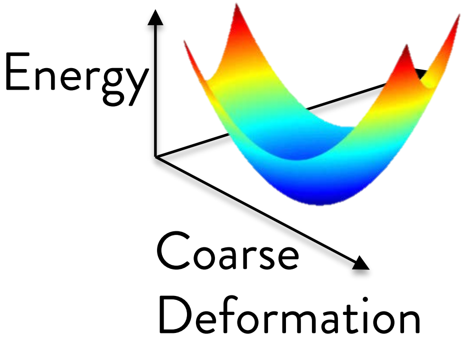
Fit new parameters p to



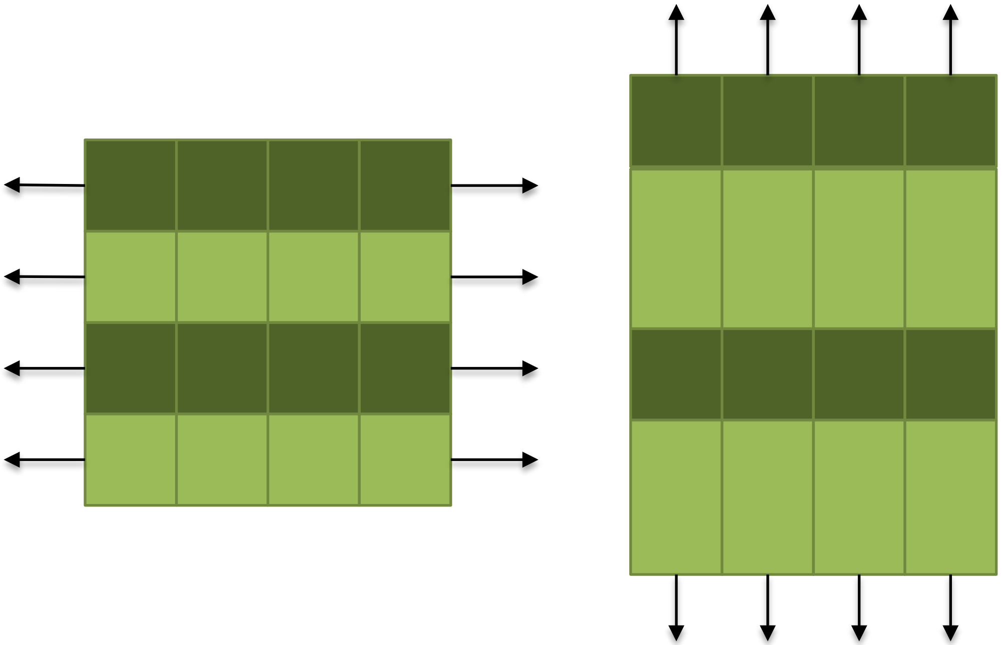
Method: Parameterization of Strain Energy

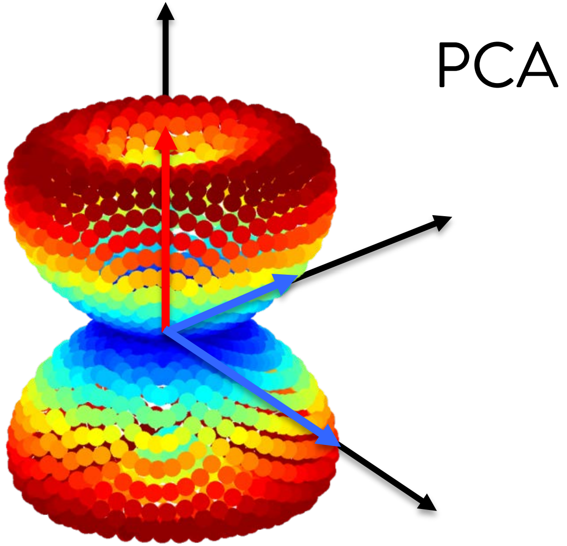
V(, p) – 8 x 3 = 24 dimensional function in 3D

- Invariant to rigid motion
- Polyconvexity for stable simulation
- Extrapolate nicely

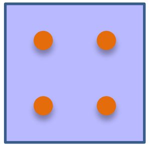


Method: Anisotropy Term



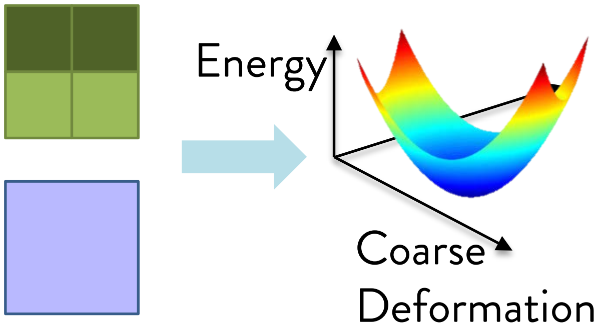


Summary: Fitting Strain Energy

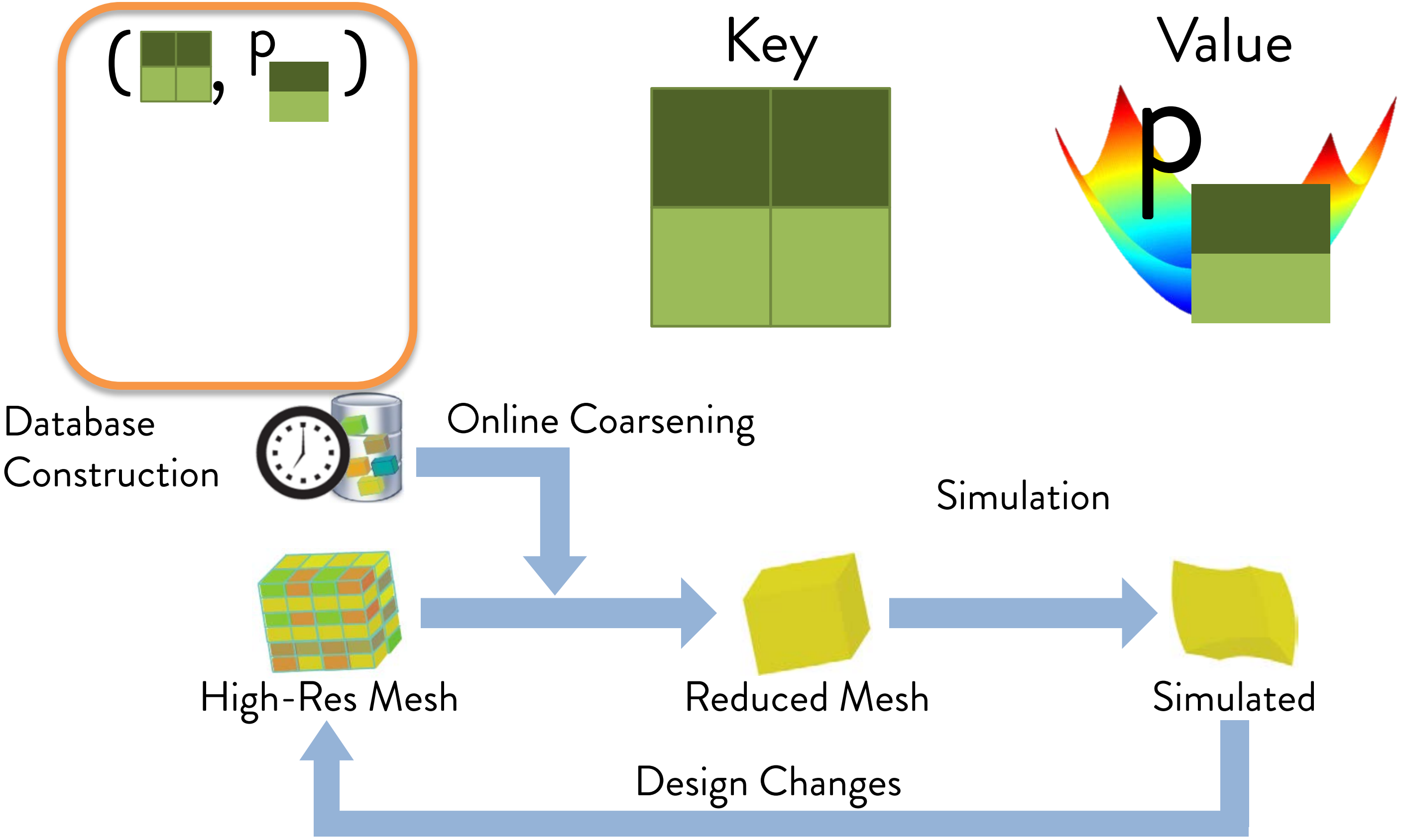
V( , p) = $\sum_i V(F_i, p_i) + C_i(\|F_i v\| - 1)^2$

Coarse material parameters

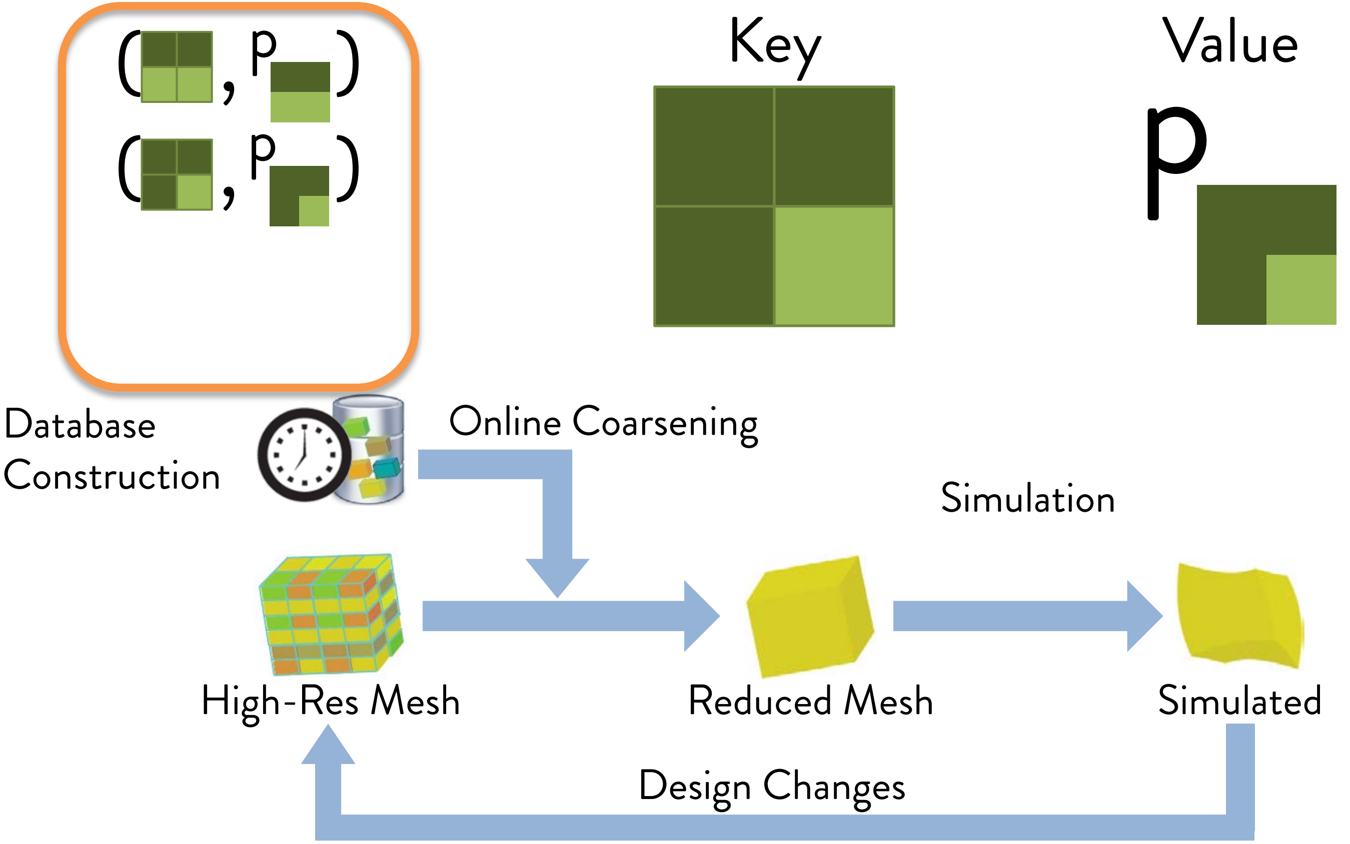
p = [pi, Ci, v]



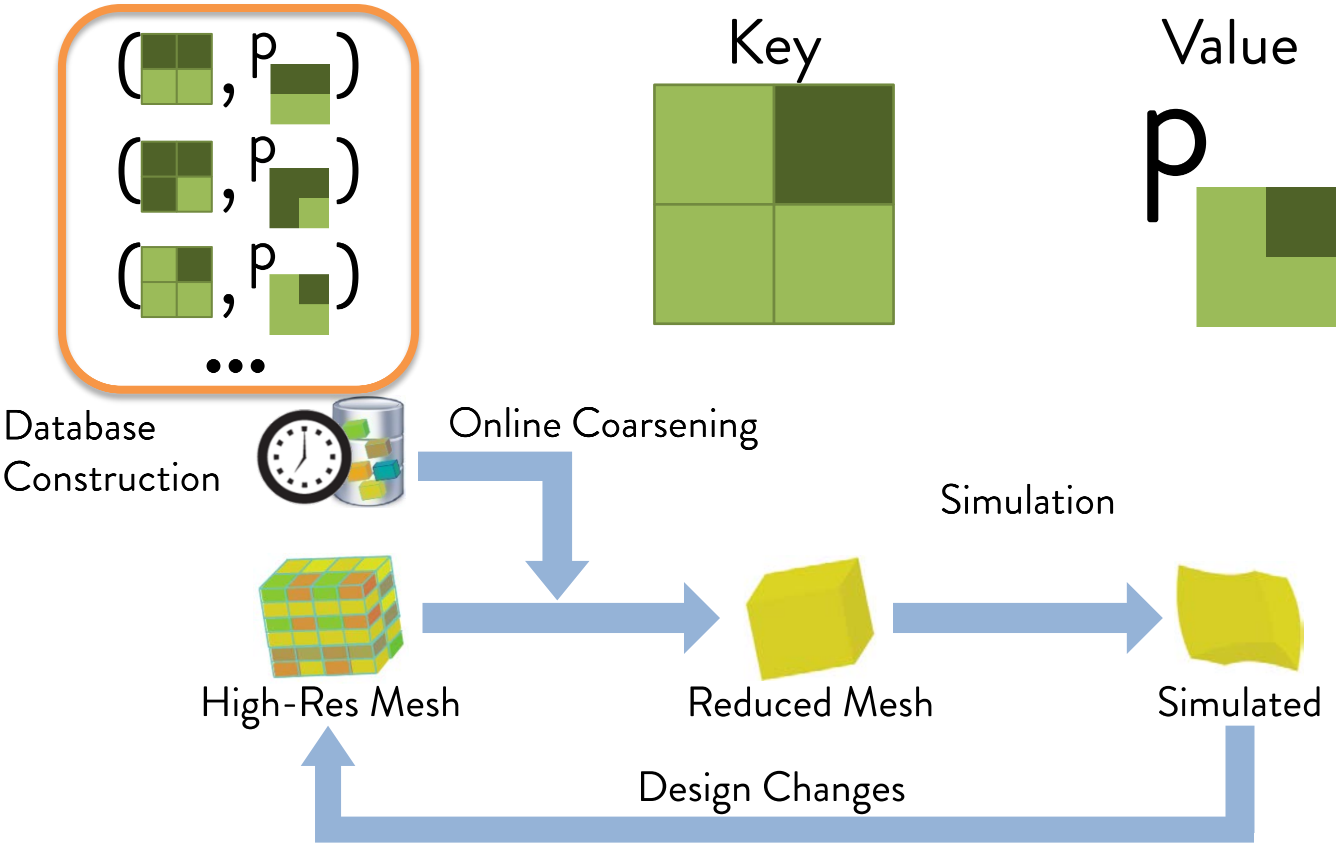
Method: Construct Metamaterial Database



Method: Construct Metamaterial Database

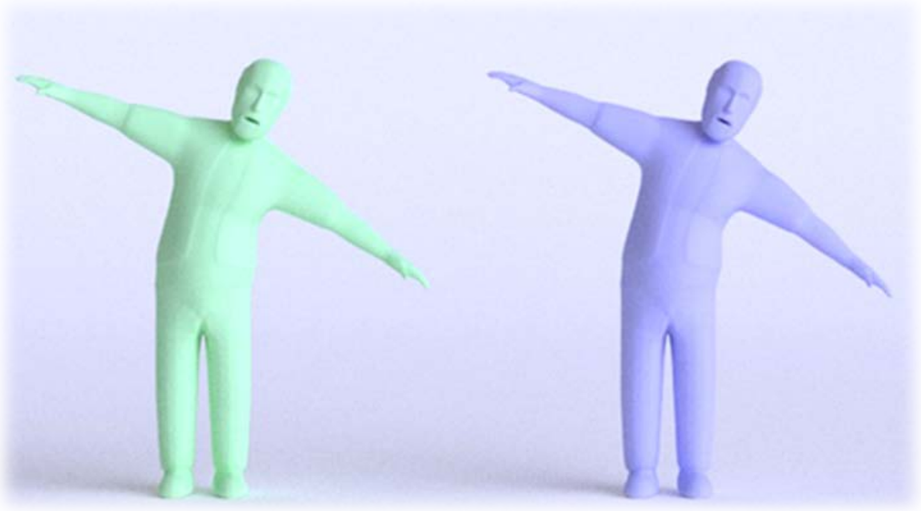


Method: Construct Metamaterial Database



Outline

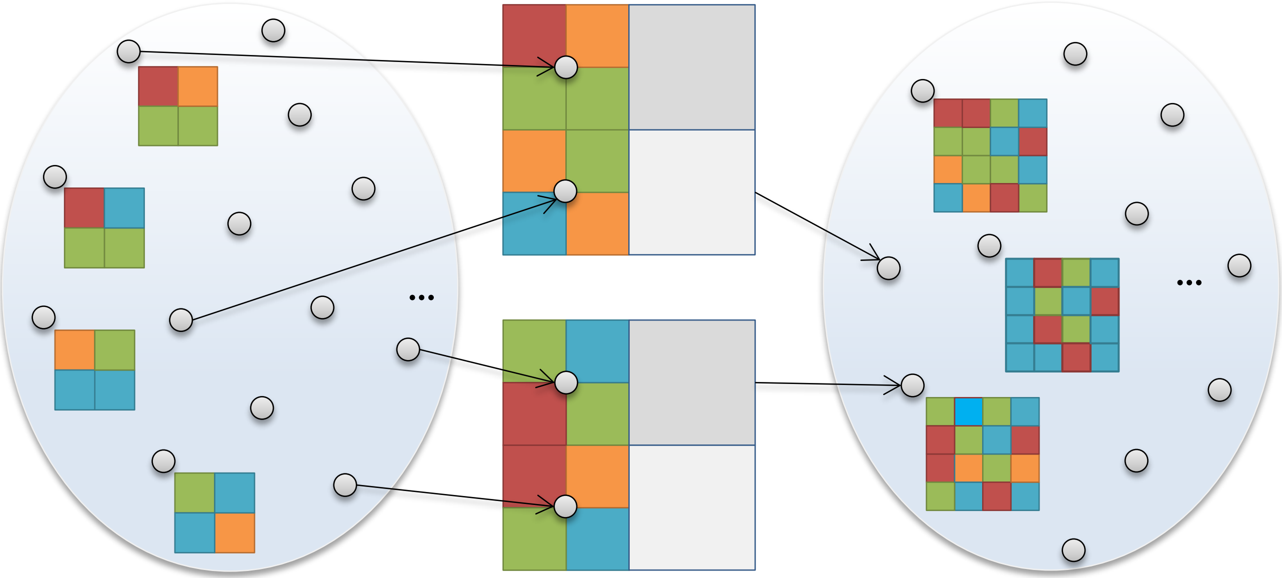
- Introduction
- Coarsening
- Database construction
- Hierarchical database
- Runtime coarsening
- Results



Hierarchical Database

N^{64} combinations

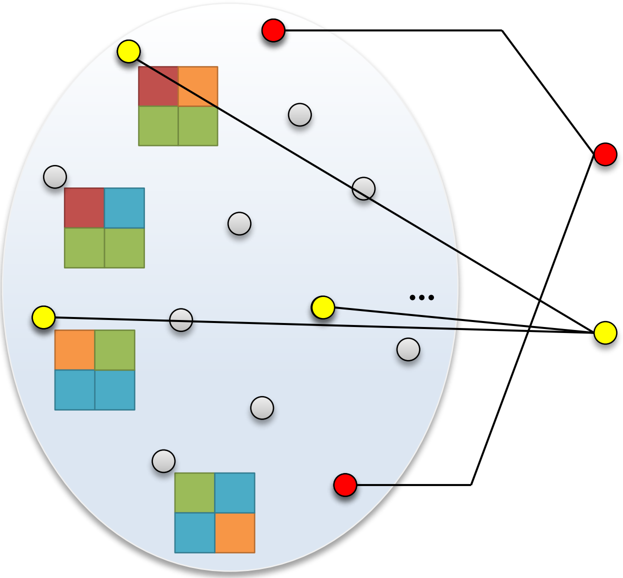
Hierarchical Database



Level 1 database

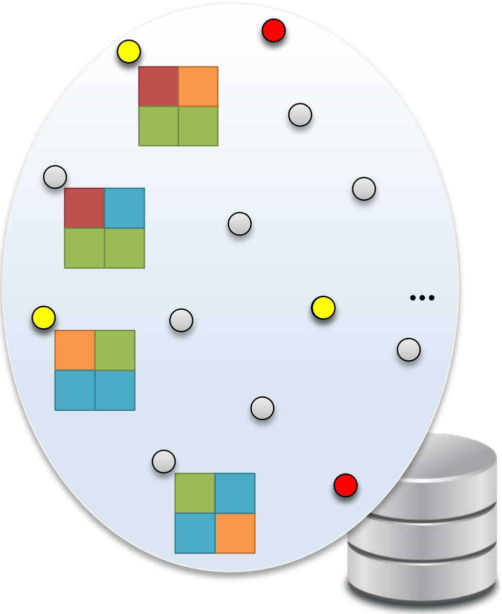
Level 2 database

Hierarchical Database: Furthest Point Sampling



- Choose initial materials
- Repeatedly choose furthest material

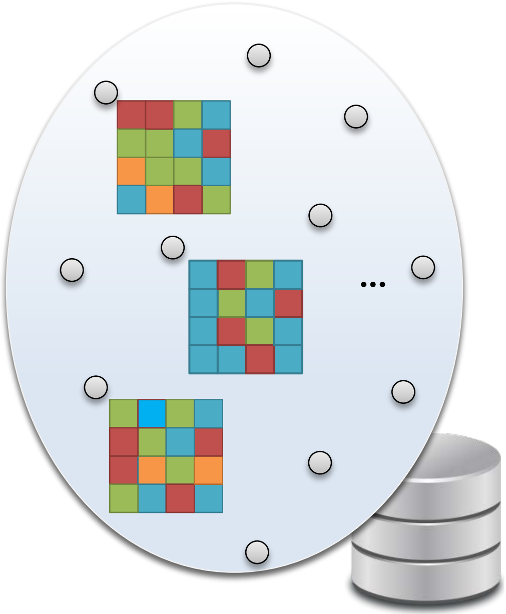
Hierarchical Database: Furthest Point Sampling



- Choose initial materials
- Repeatedly choose furthest material

Compressed database

Hierarchical Database: Furthest Point Sampling



- Choose initial materials
- Repeatedly choose furthest material

Compressed database

Outline

- Introduction
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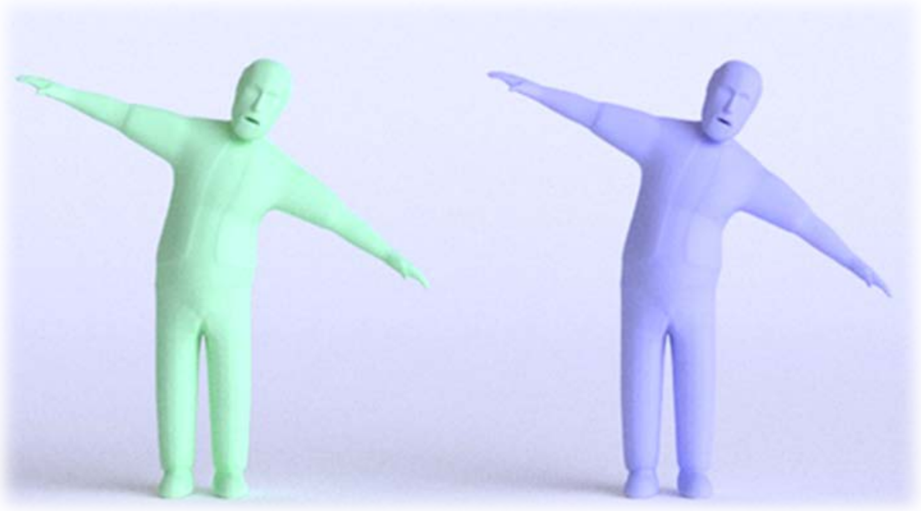
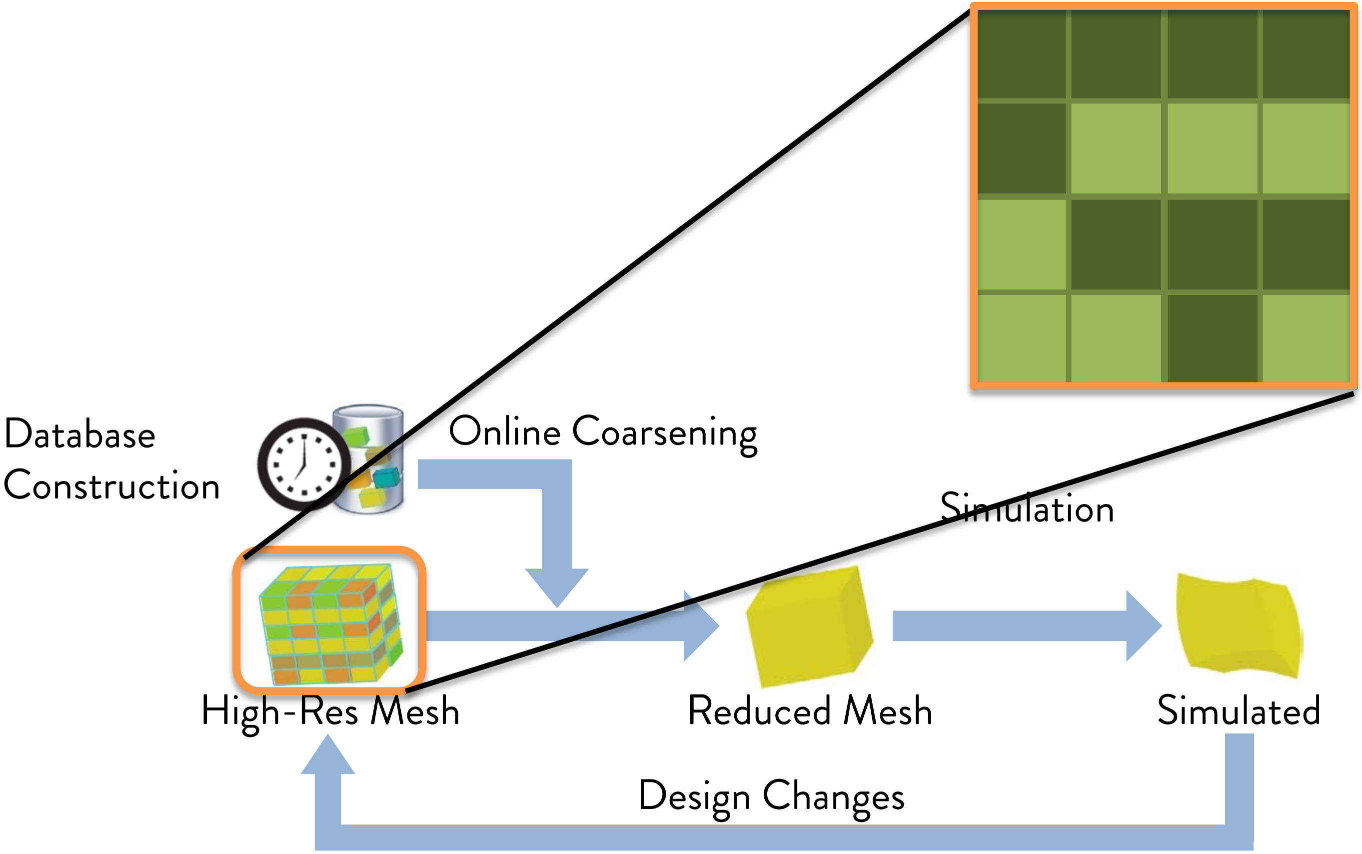
- Results

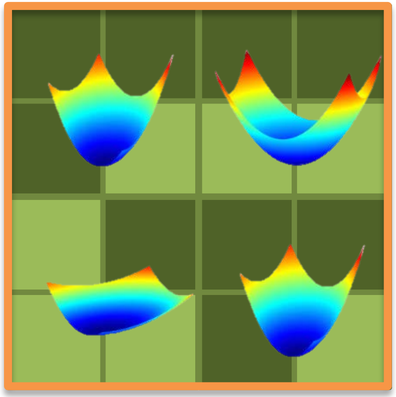


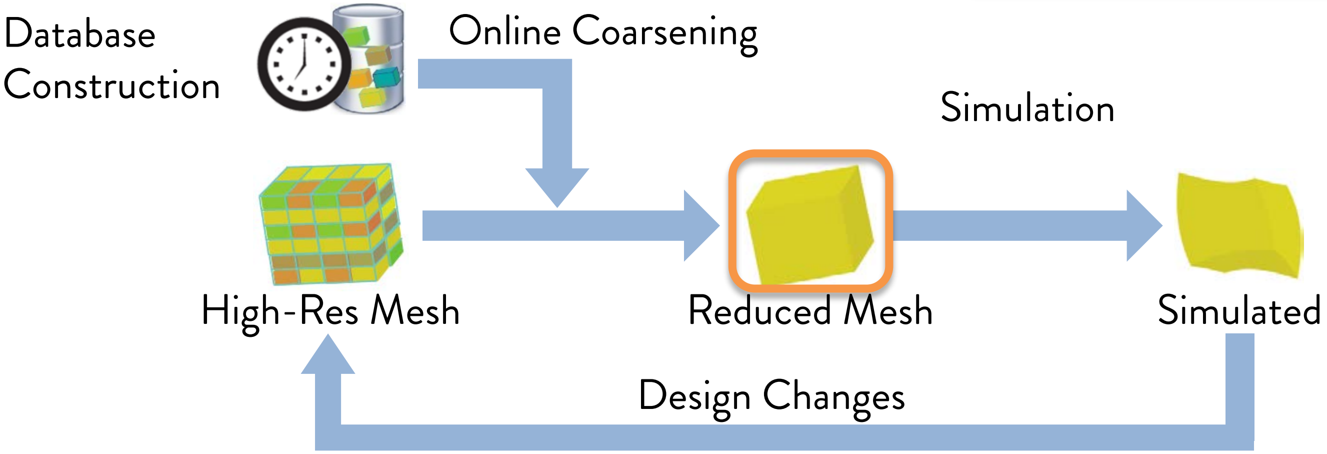
Method: Online Lookup



Method: Online Lookup

Fast DB lookup!





Outline

- Introduction
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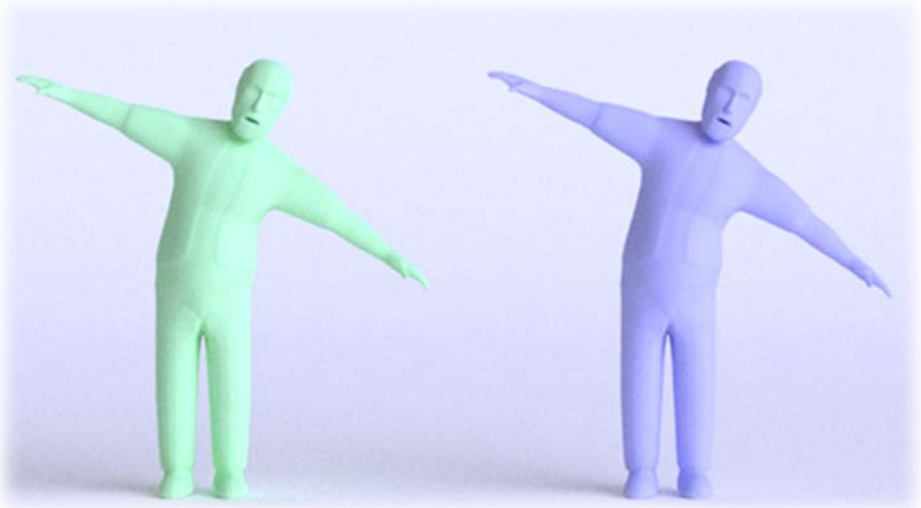
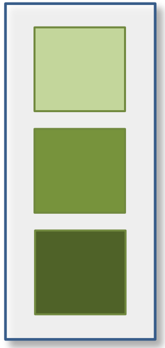
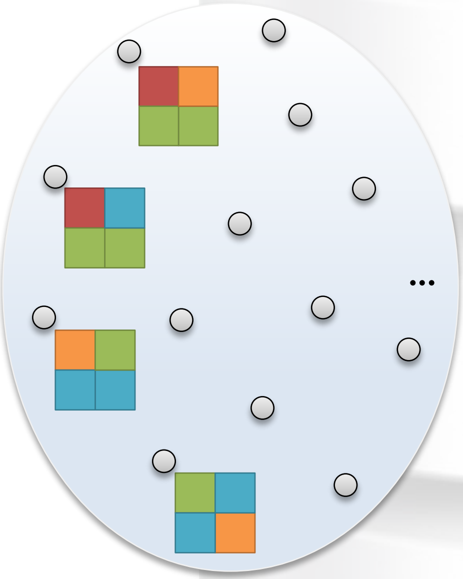
- Hierarchical database
- Runtime coarsening
- **Results**



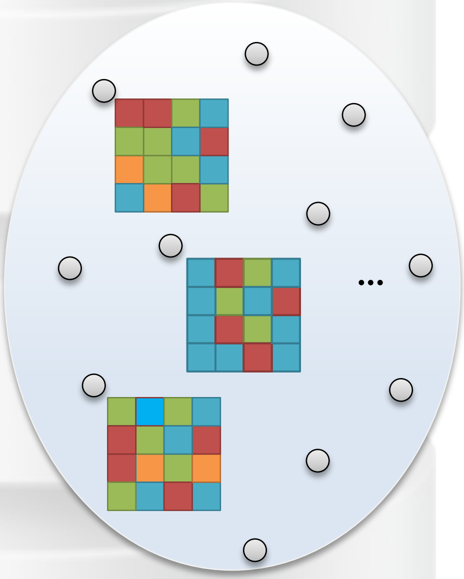
Database



Base materials: 3

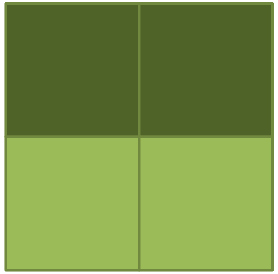


Level 1: 6561 materials
4MB

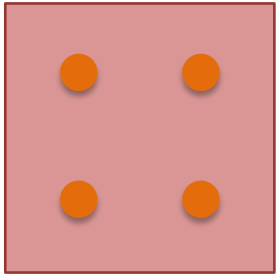


Level 2: 16k materials
20MB

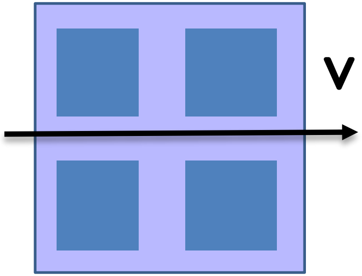
Naïve Vs Coarsened Material



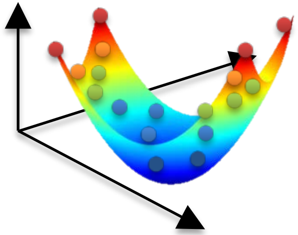
Fine Elements

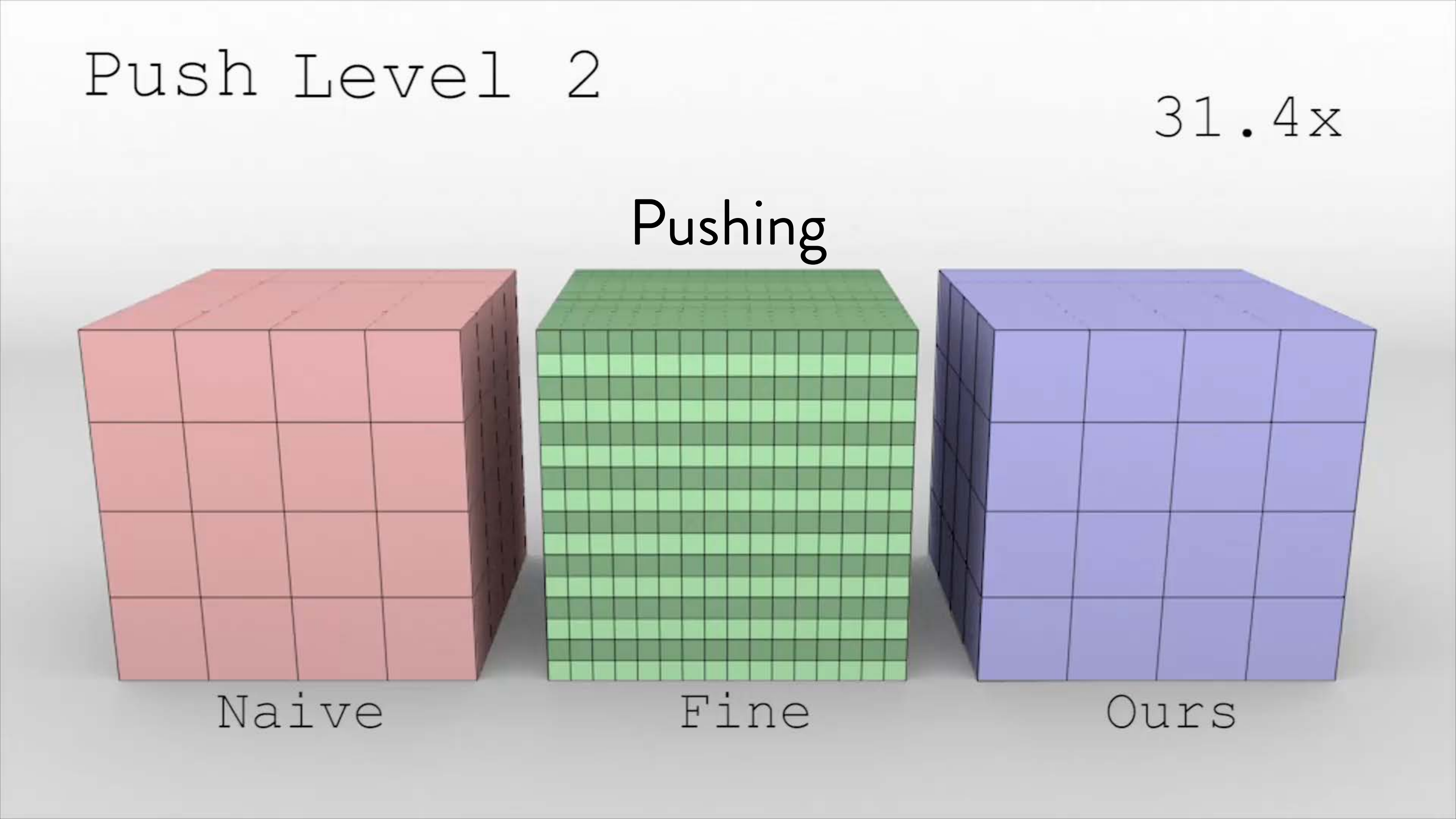


Naïve Material

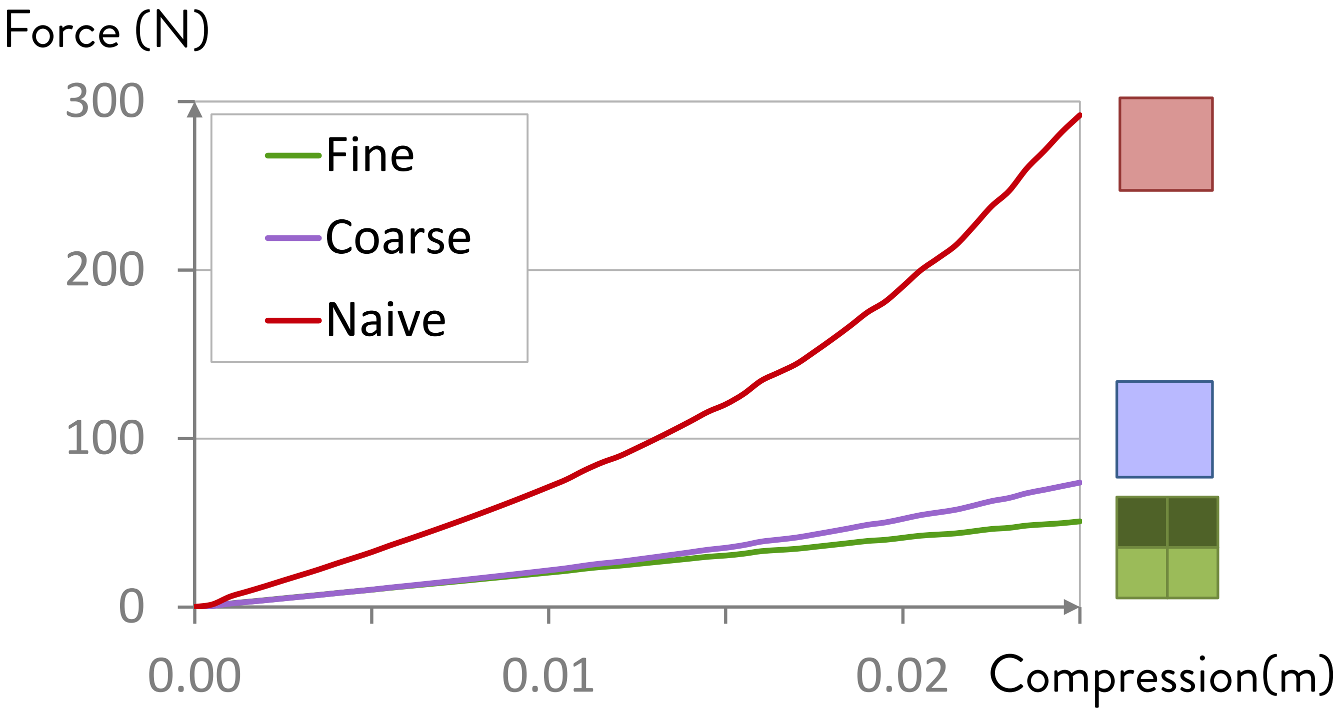


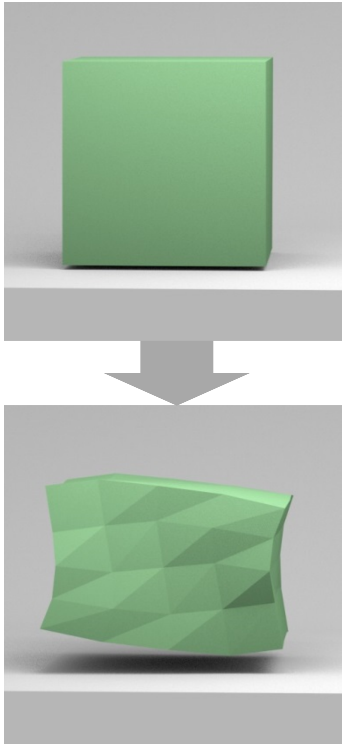
Data-driven Coarsening




Push Level 2
31.4x
Pushing
Naive
Fine
Ours

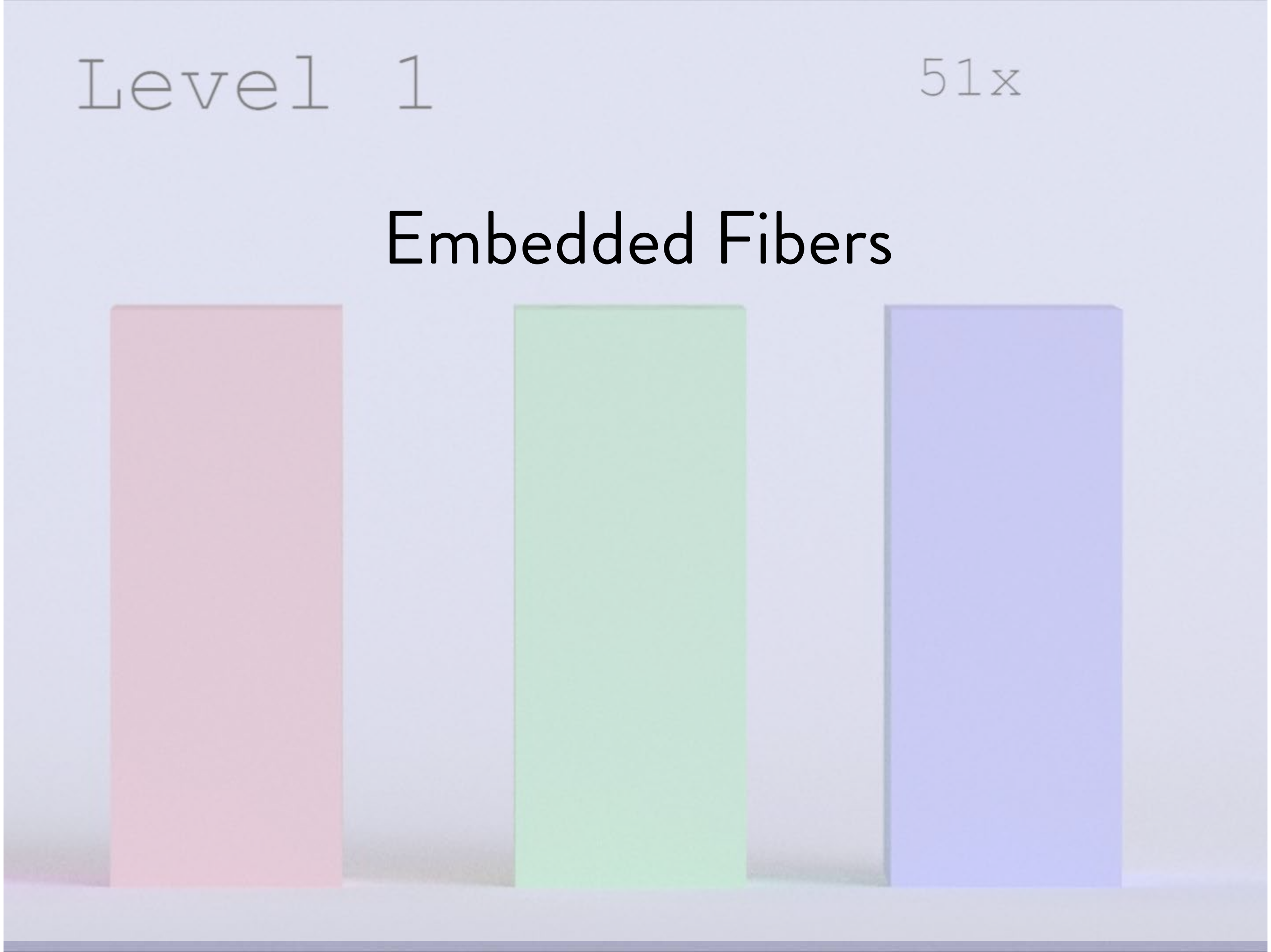
Results: Parameter Fitting Validation



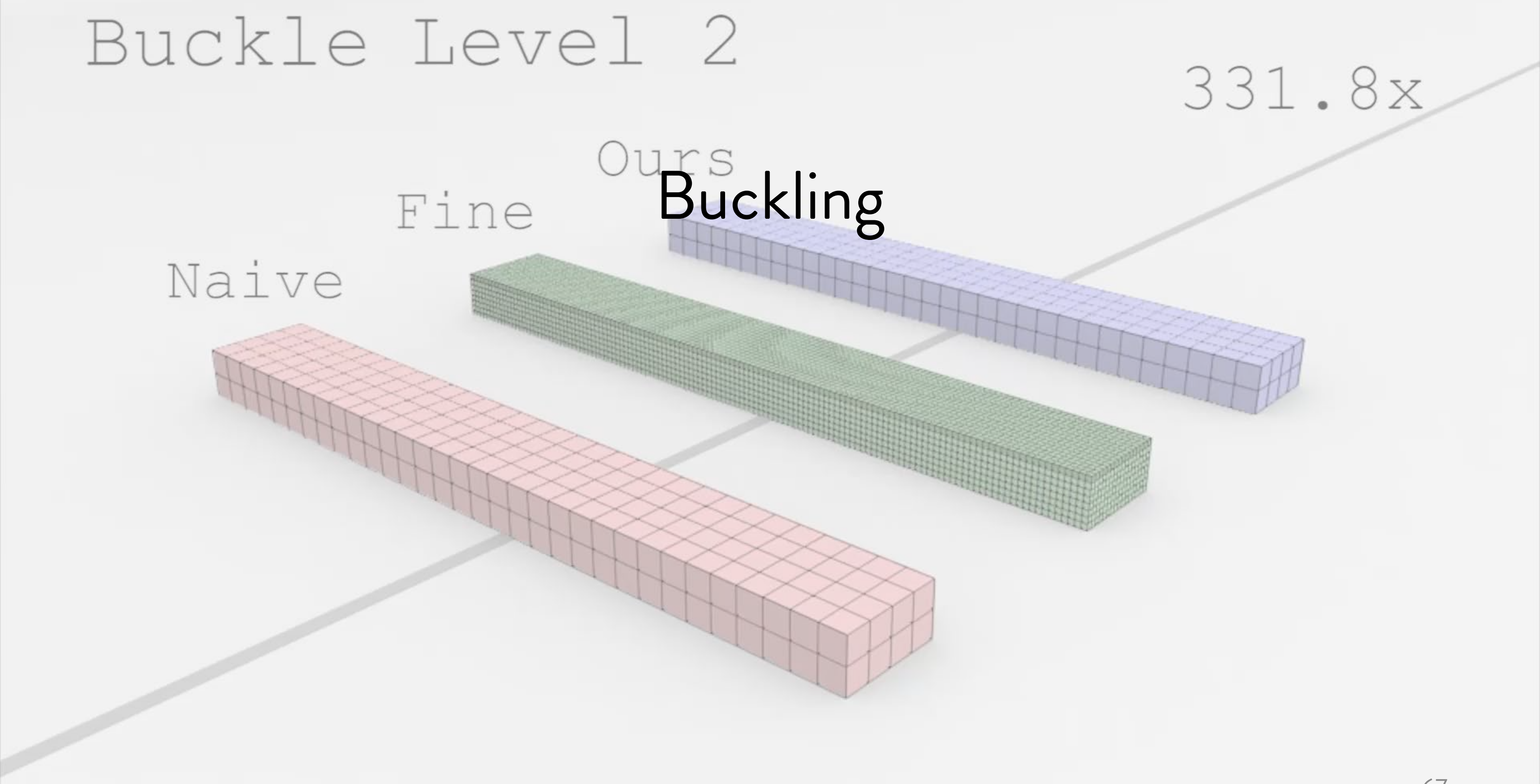


Twisting

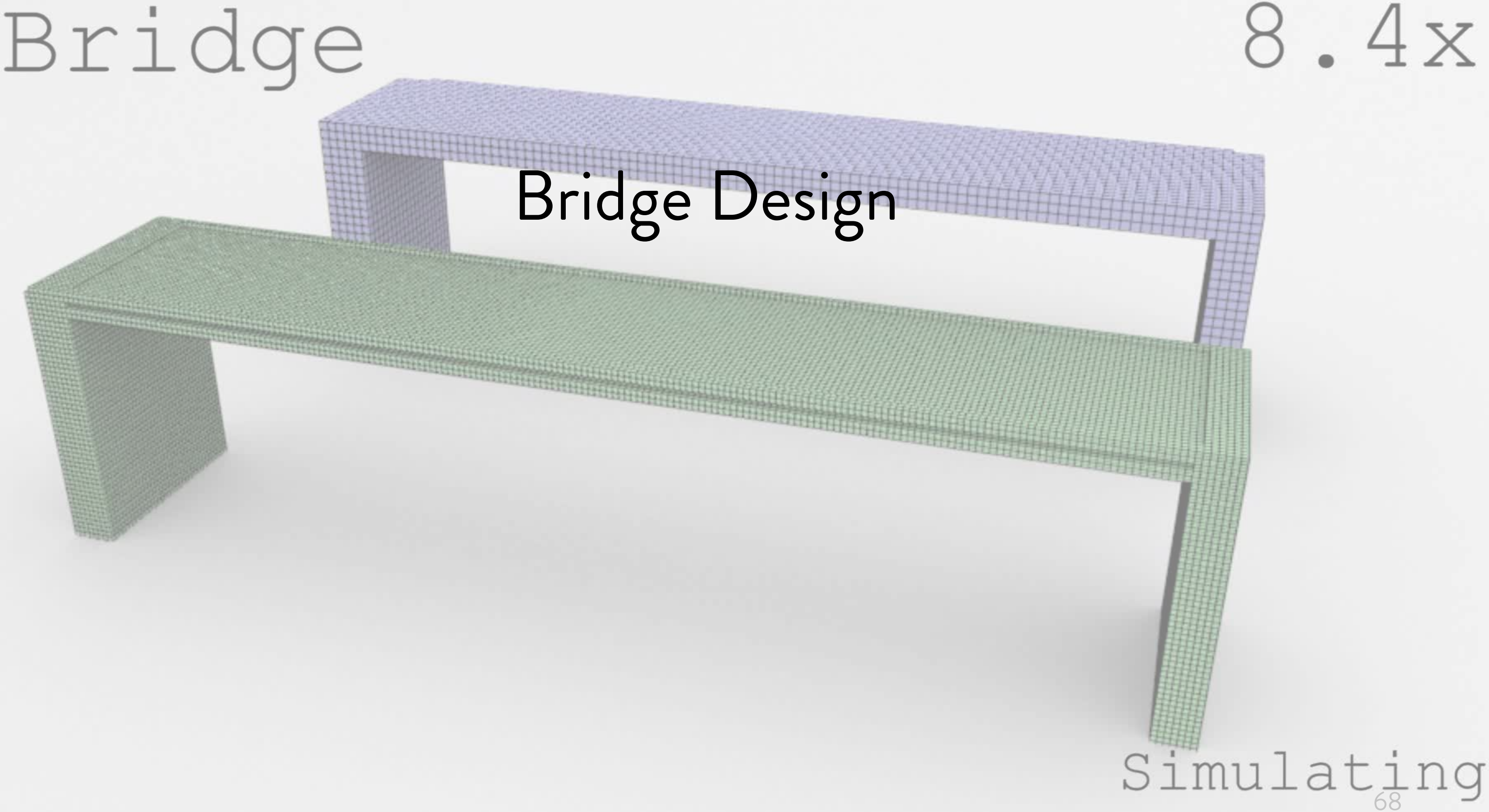
Embedded Fibers



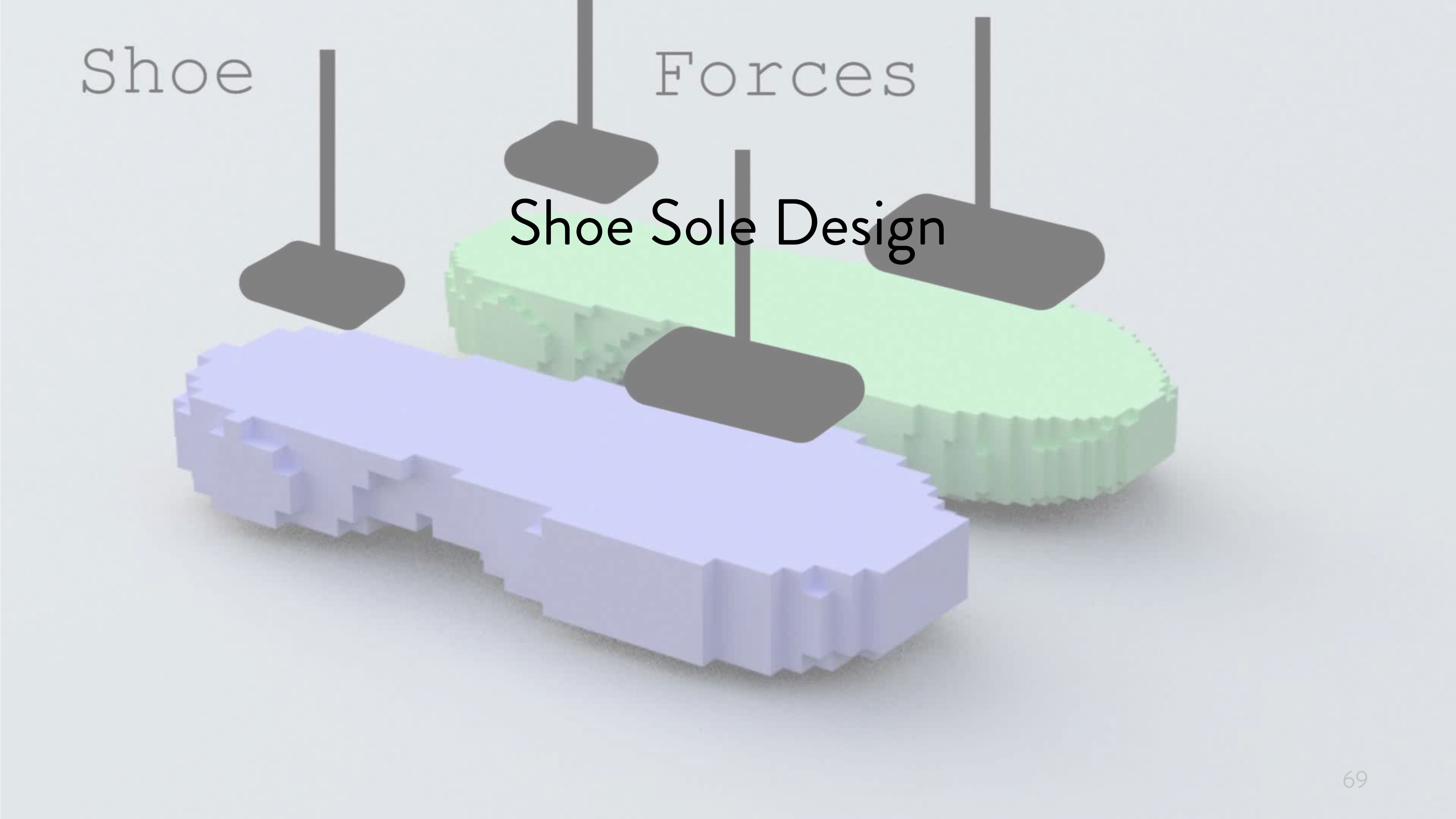
Buckling



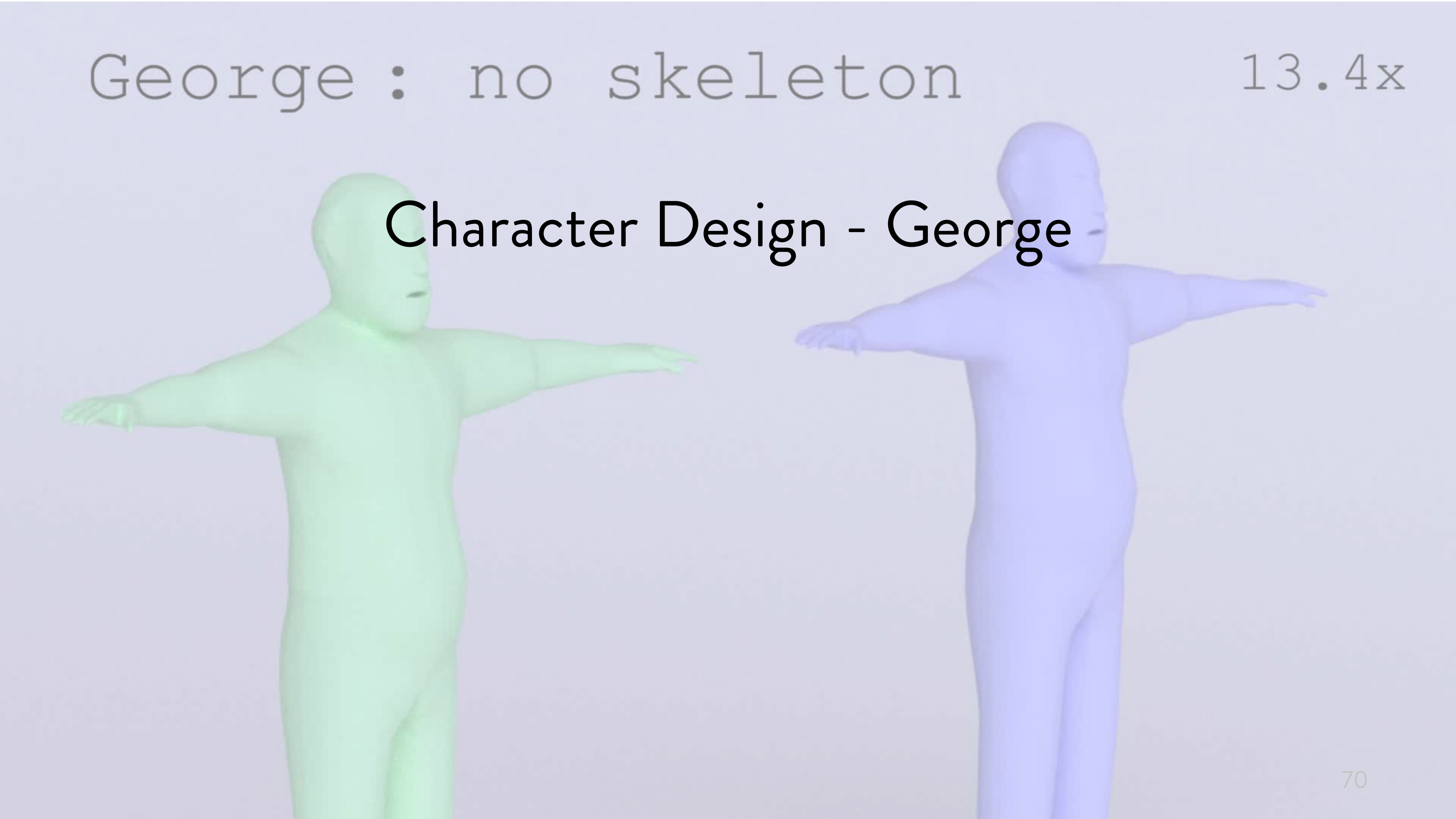
Bridge Design

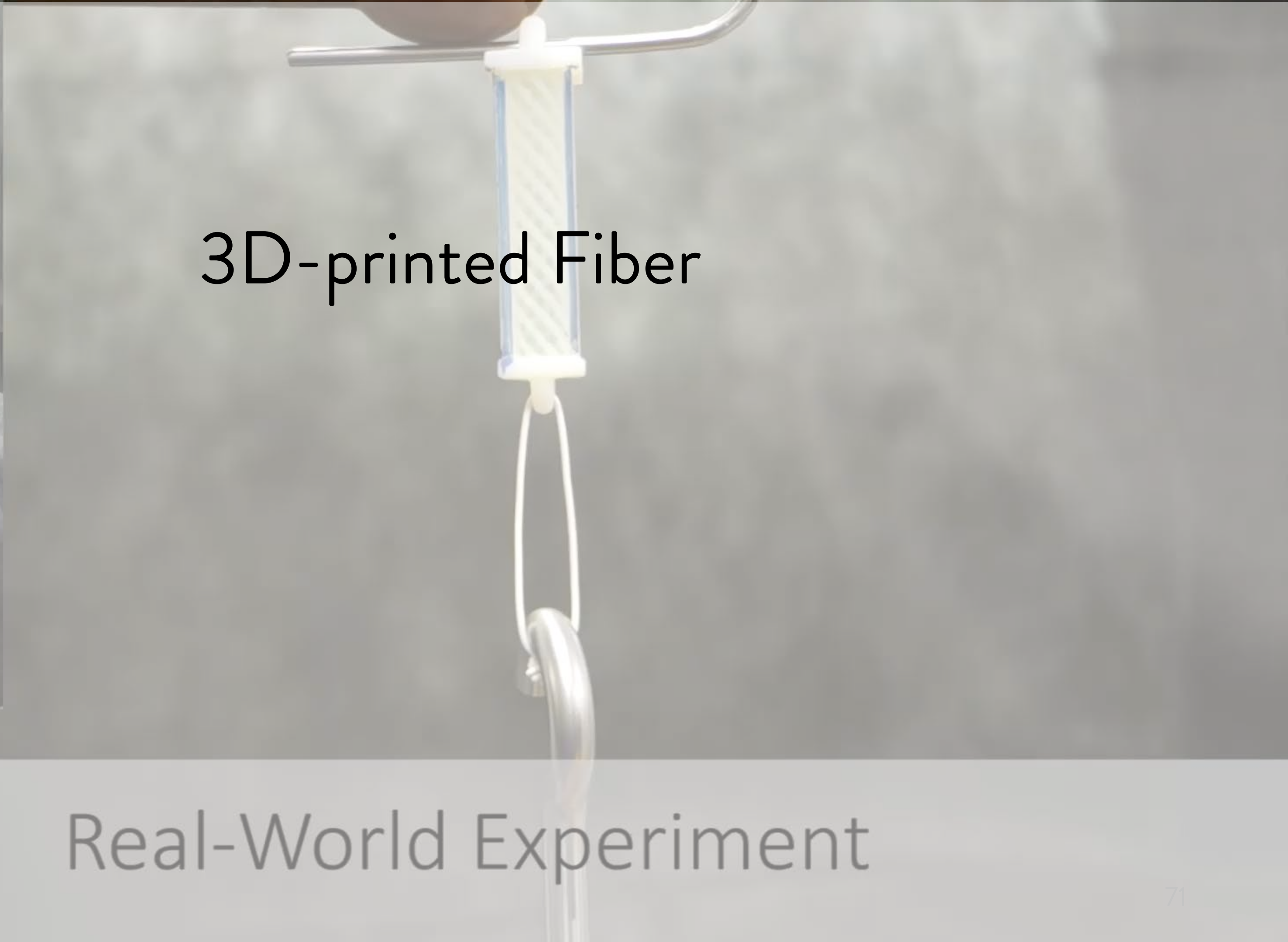


Shoe Sole Design

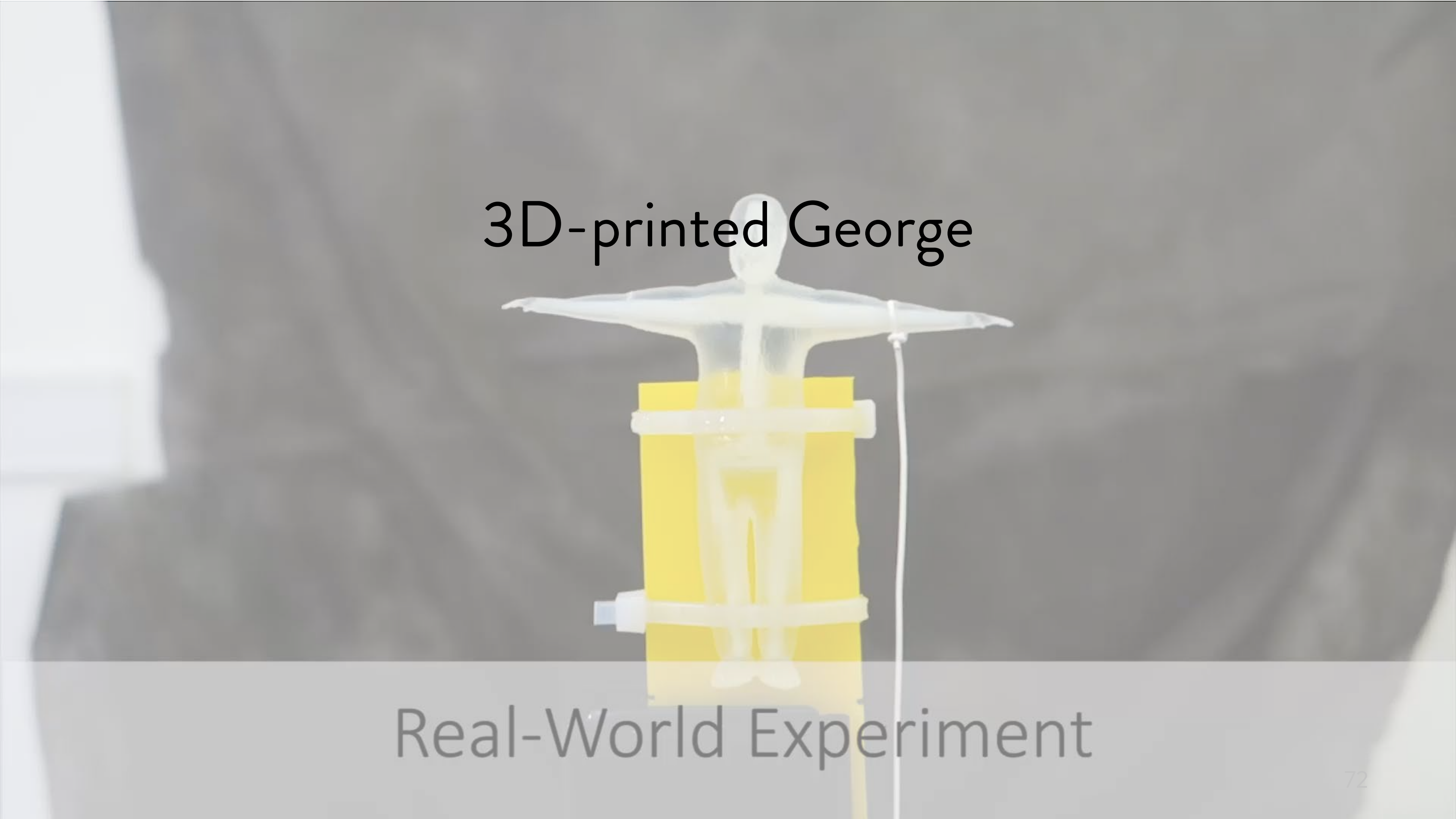


Character Design - George

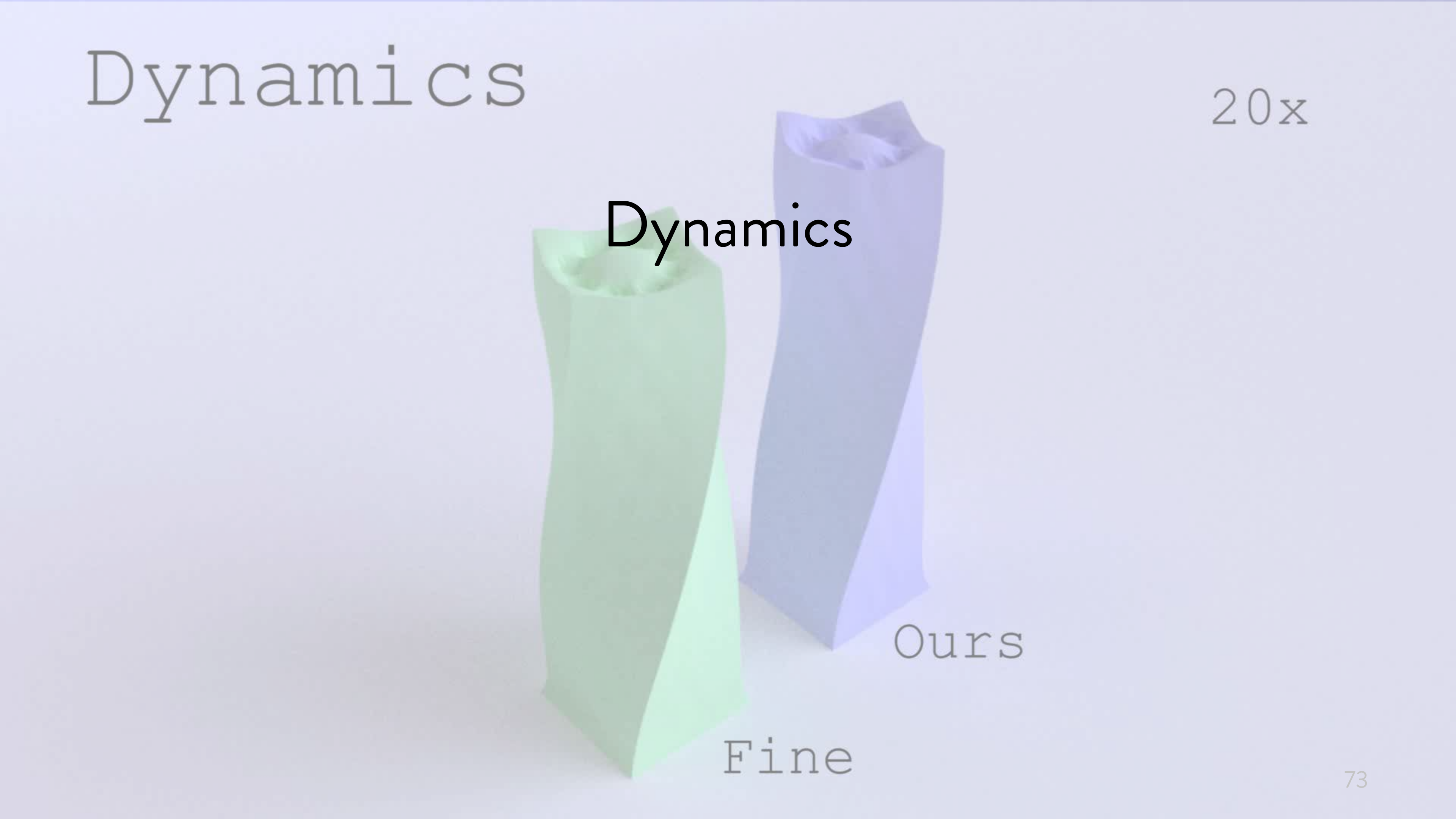




Real-World Experiment


3D-printed George
Real-World Experiment

Dynamics

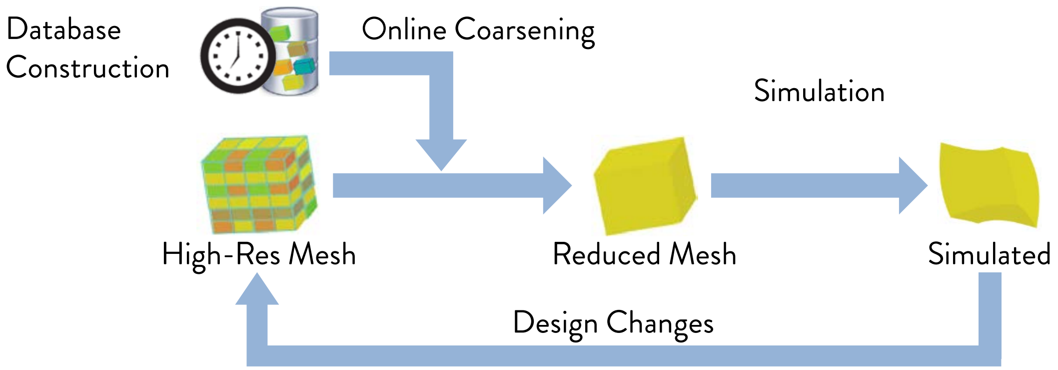


Future Work

- Better energy functions for anisotropic hyperelastic materials
- Continuous material space alleviate combinatorial explosion
- Refine coarse simulation
- Combine with a fast solver such as multigrid

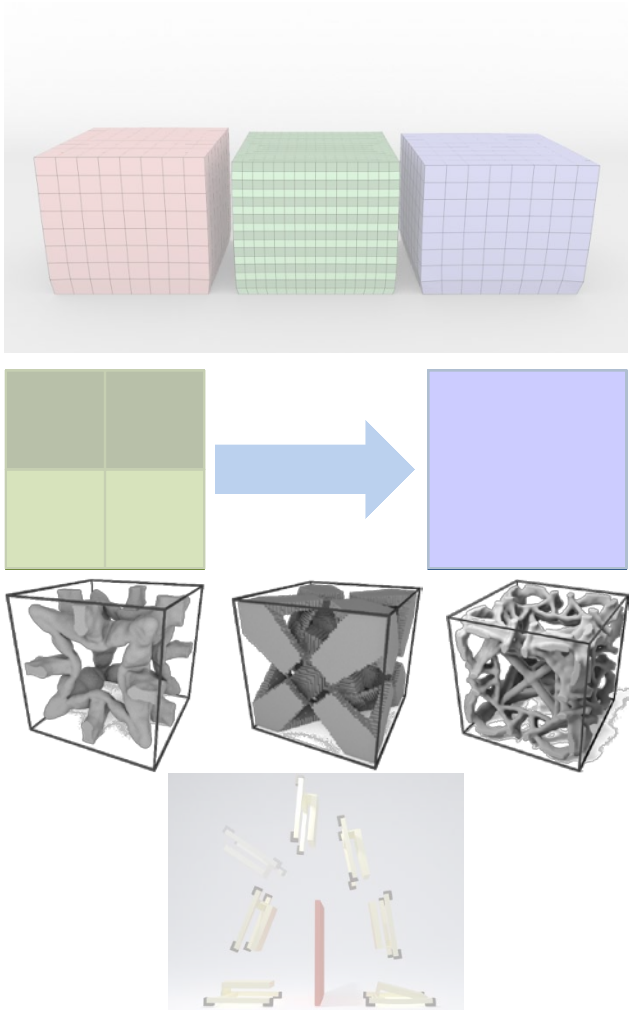
Conclusion

- Data-driven approach to model metamaterials
  - Non-linear hyperelastic materials
- Fast online lookup based on offline computation
- 8-400x speed up



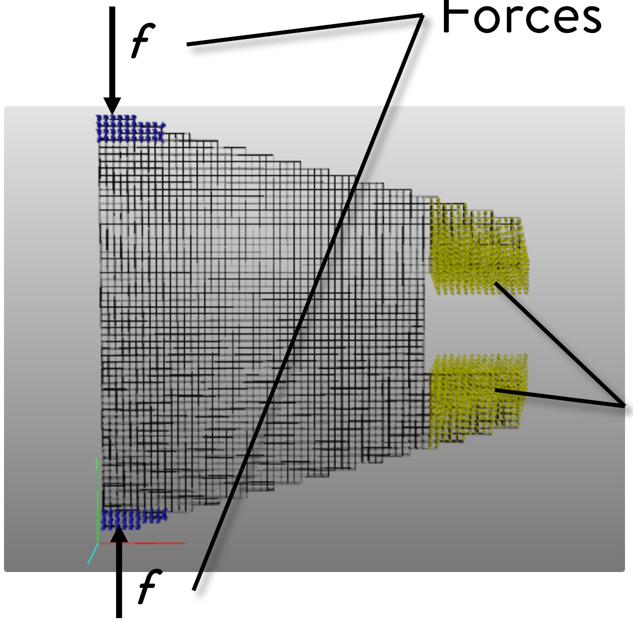
Overview

- FEM for solid simulation
- Data-driven coarsening for static simulation
- **Topology optimization with microstructures**
- Designing dynamic mechanisms



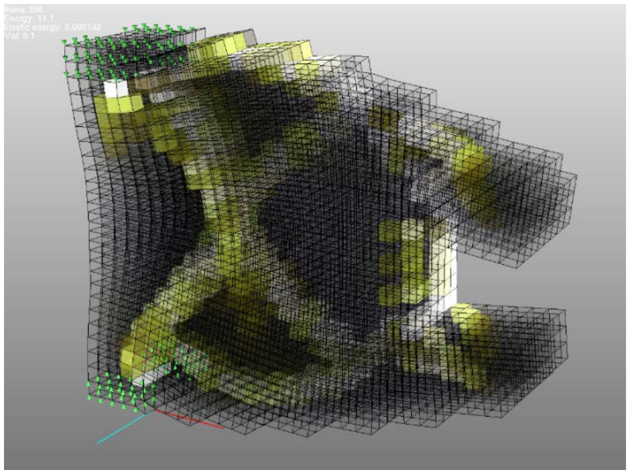
Topology Optimization

Forces



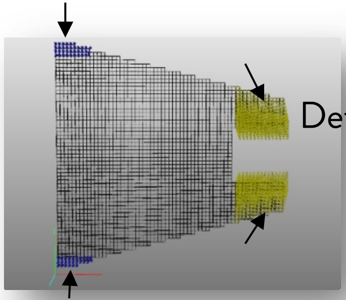
Displacements in region of interest

High level specifications in a design domain



Optimized material distribution

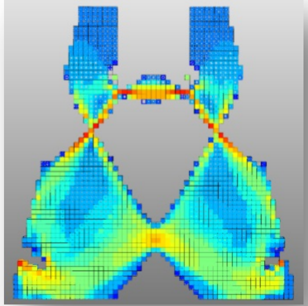
Topology Optimization with Microstructures

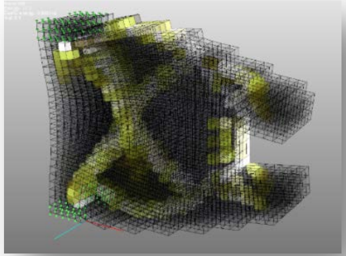


Deform

Design Goal

Topology optimization

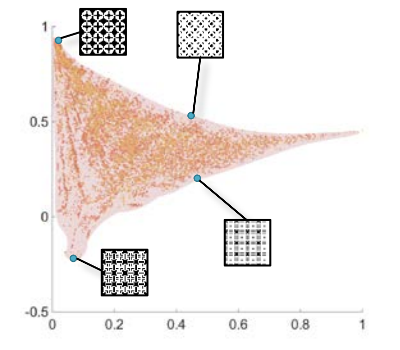




Optimized
material distribution

Rigid material
Soft material

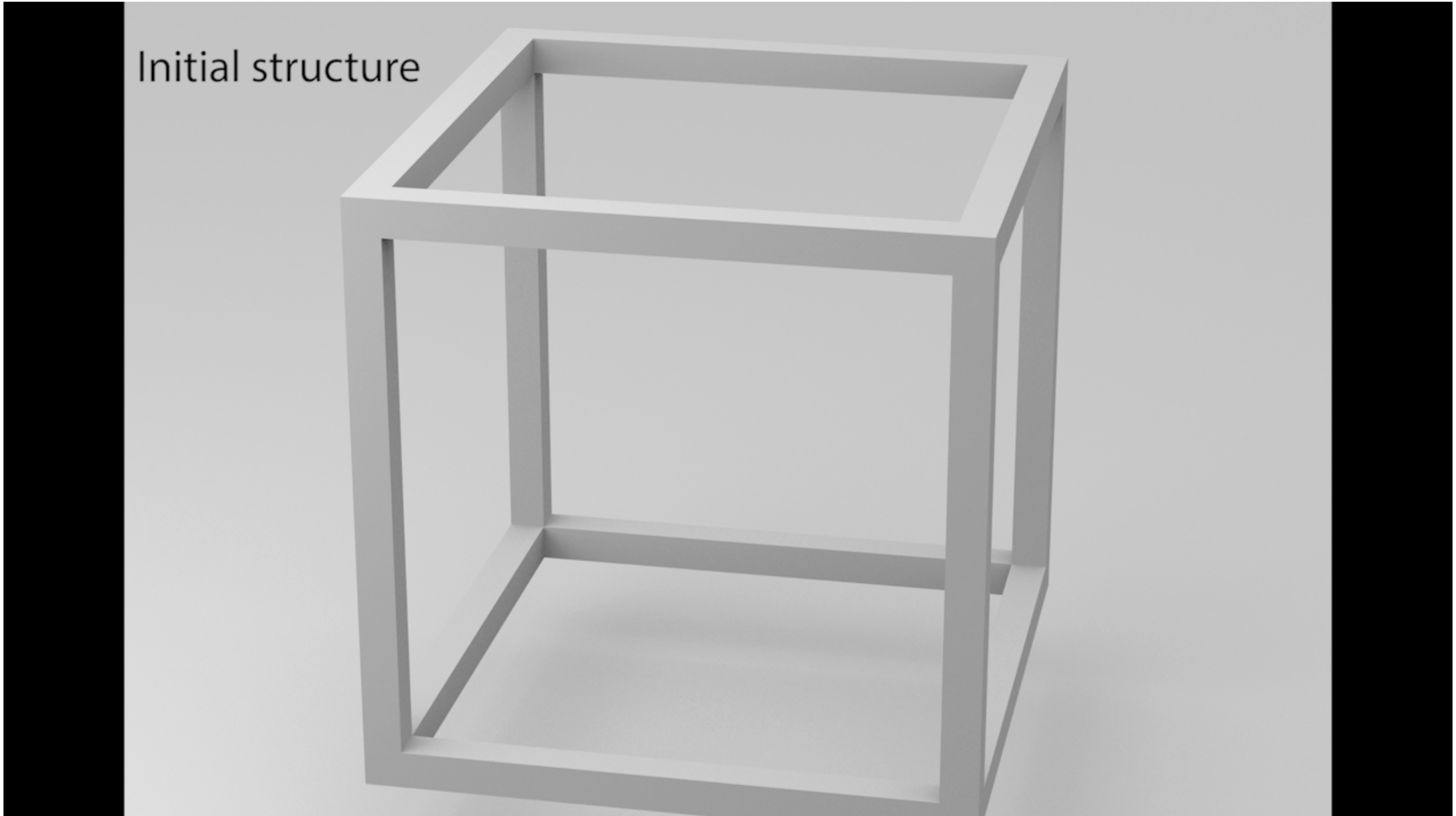
Base materials

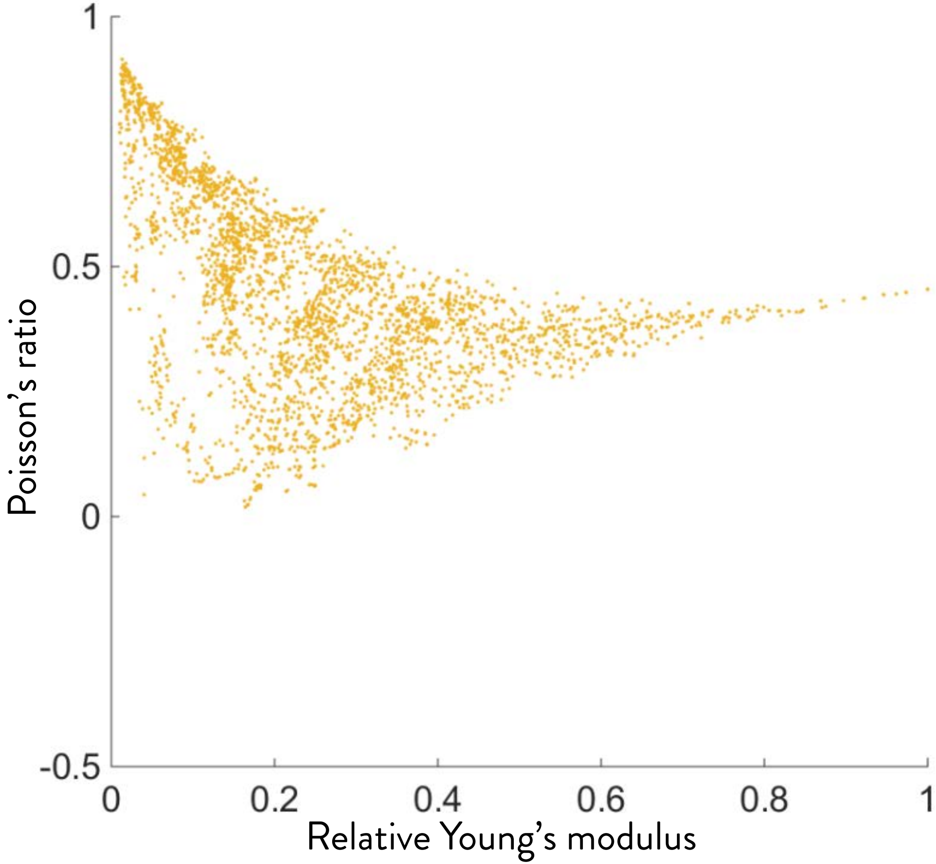


Database of
microstructures

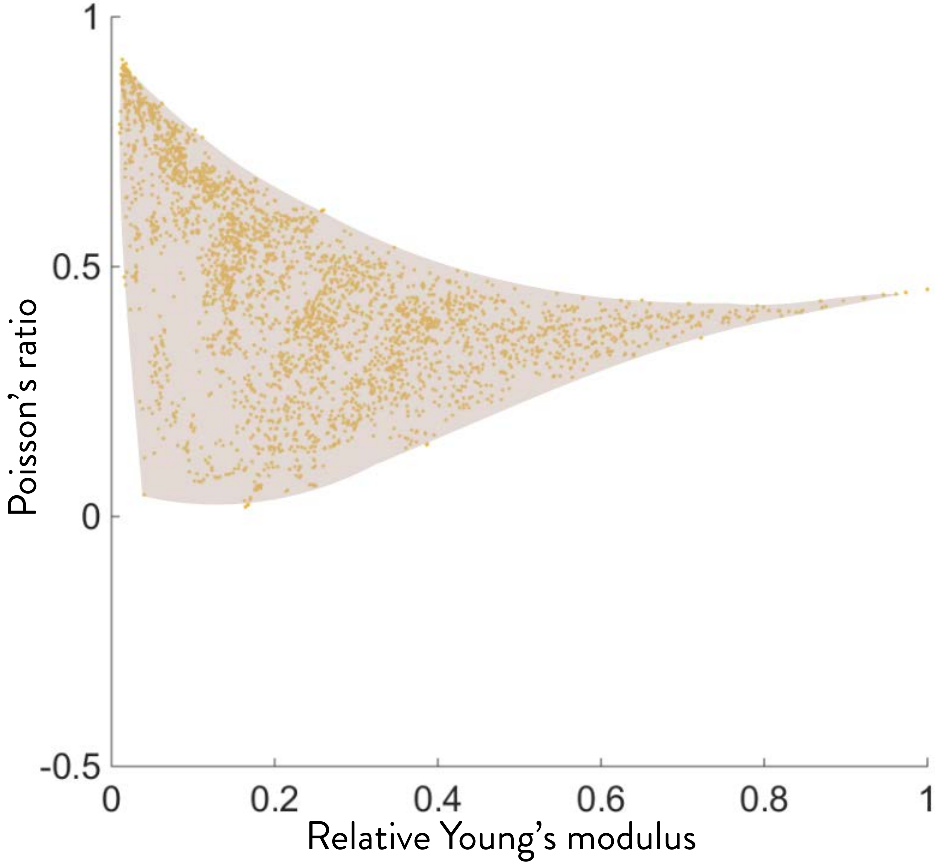
Topology Optimization - Example

Generating Microstructures using Topology Optimization

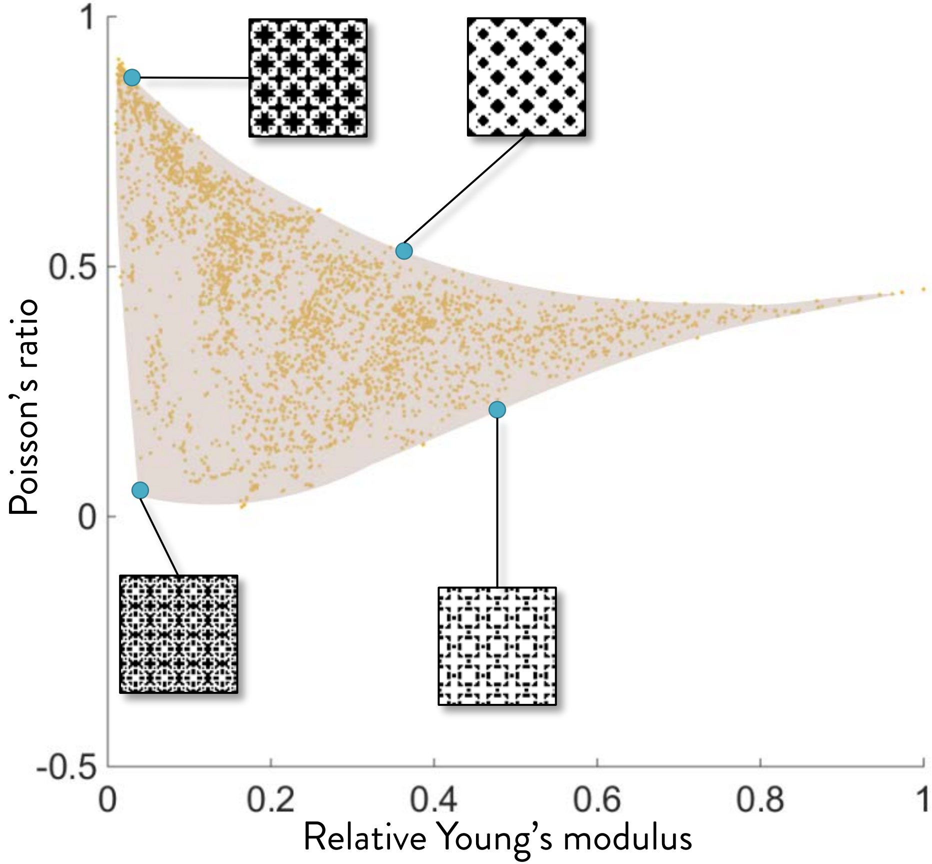




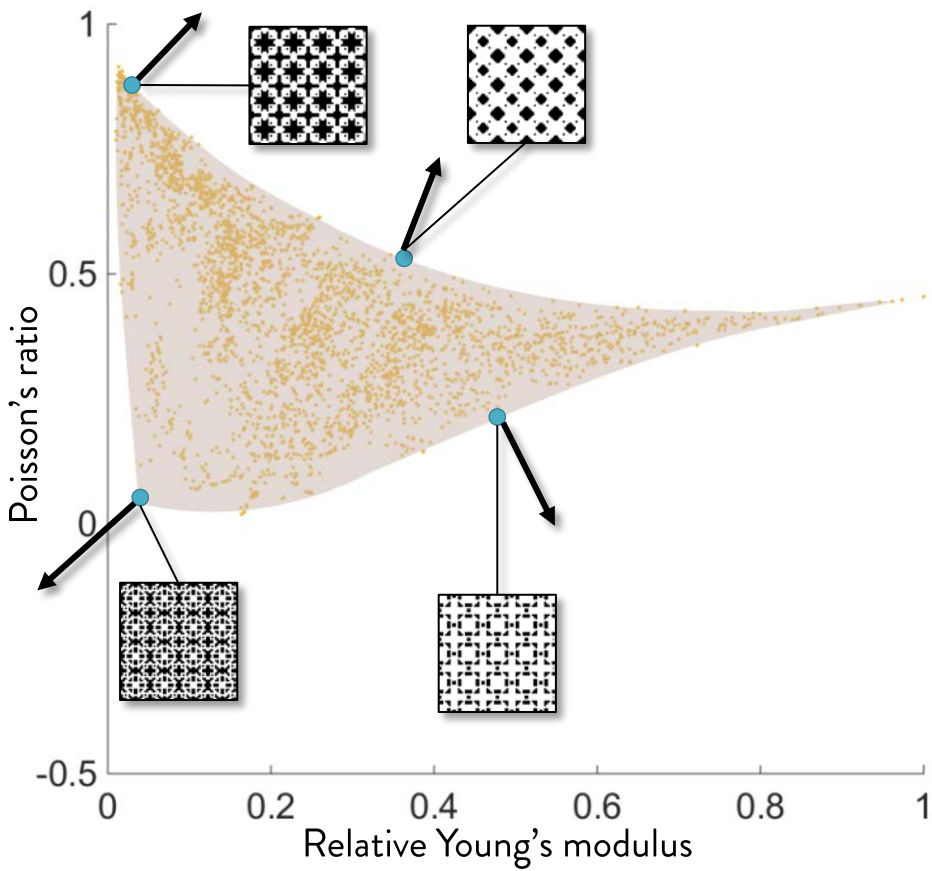
1. Initial samples of microstructures



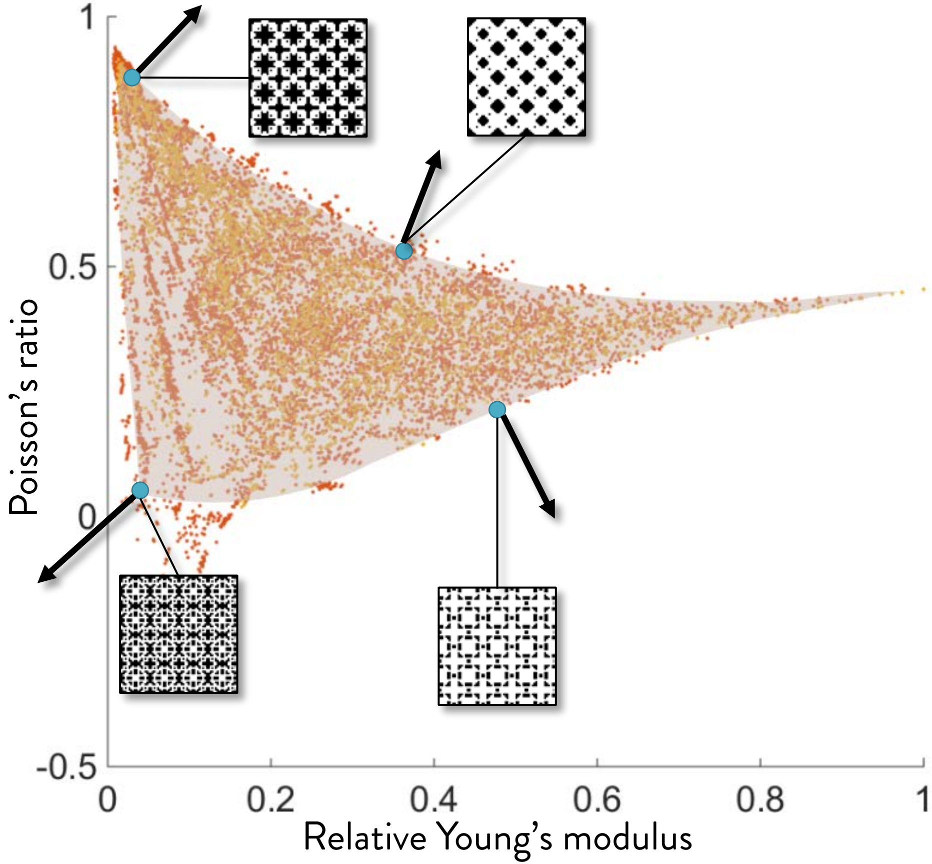
1. Initial samples of microstructures
2. Repeat:
    1. Approximate material gamut with level set



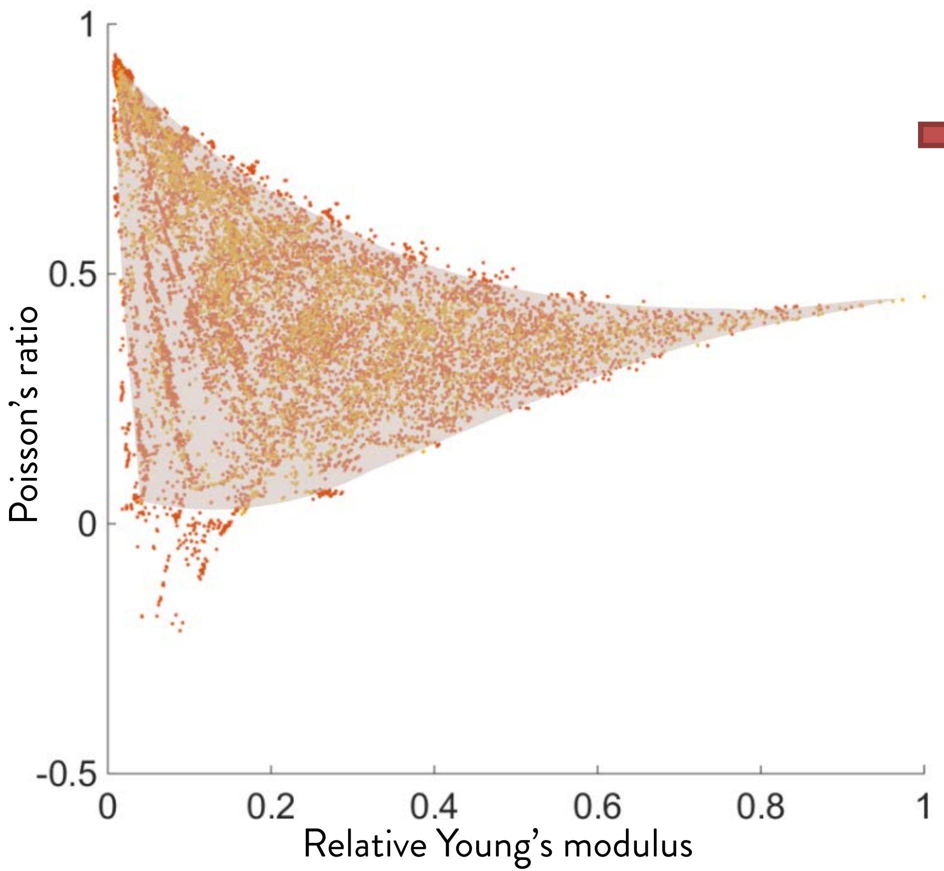
1. Initial samples of microstructures
2. Repeat:
    1. Approximate material gamut with level set
    2. Identify samples near the boundary



1. Initial samples of microstructures
2. Repeat:
    1. Approximate material gamut with level set
    2. Identify samples near the boundary
    3. Compute level set gradient at each boundary sample



1. Initial samples of microstructures
2. Repeat:
    1. Approximate material gamut with level set
    2. Identify samples near the boundary
    3. Compute level set gradient at each boundary sample
    4. Generate new samples along gradient direction



1. Initial samples of microstructures
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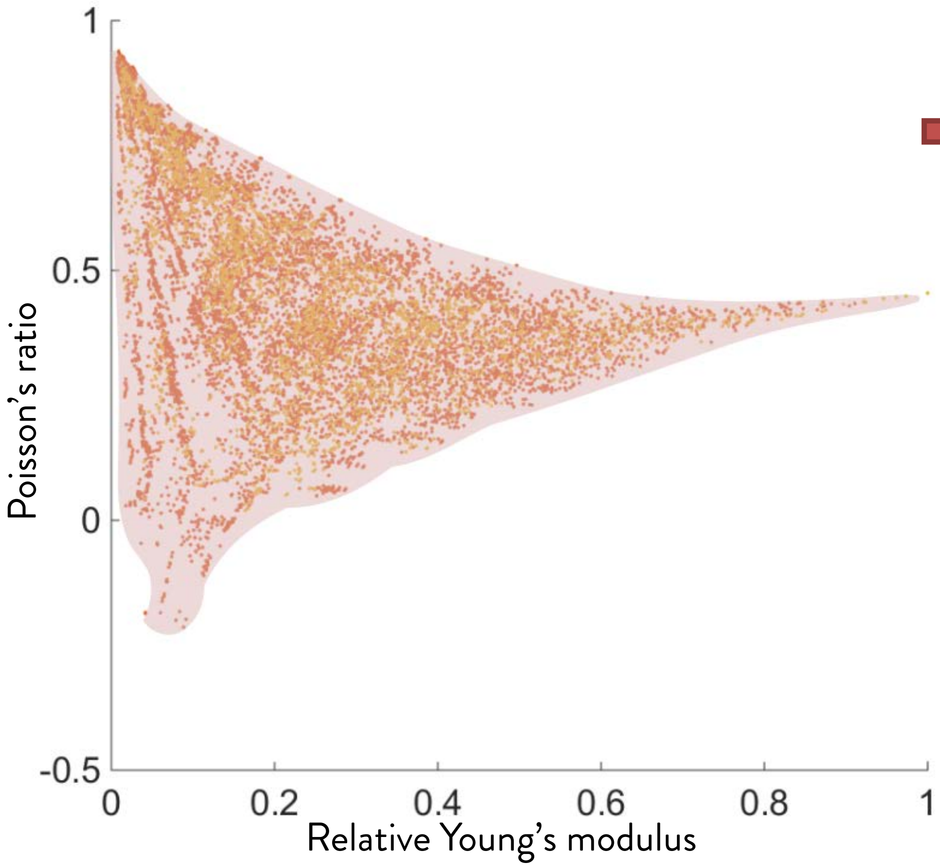

1. Initial samples of microstructures
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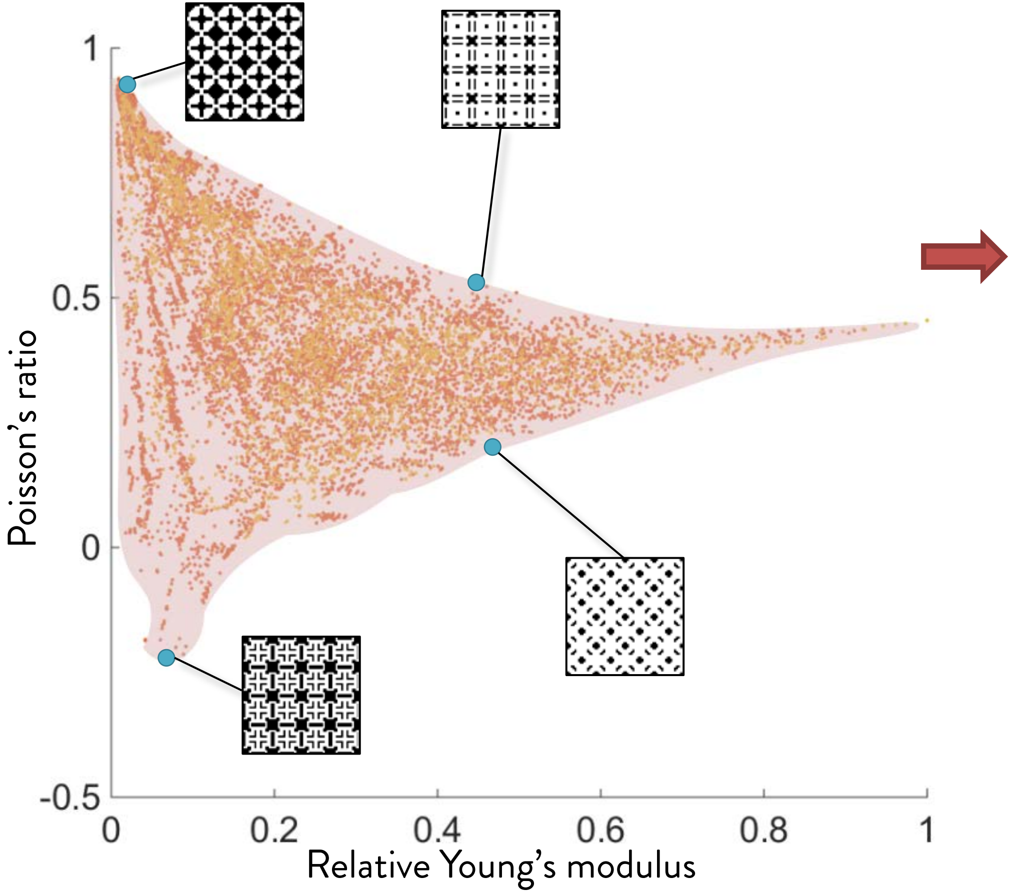
   3. Compute level set gradient at each boundary sample
   4. Generate new samples along gradient direction



1. Initial samples of microstructures
2. Repeat:
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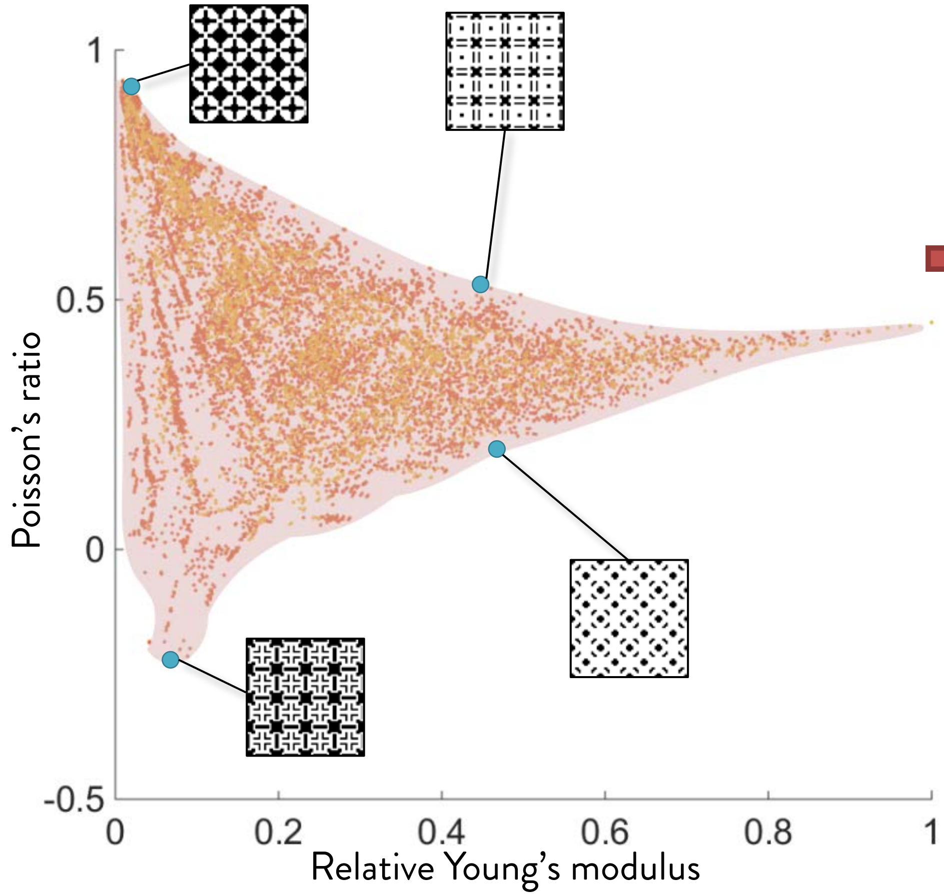

1. Initial samples of microstructures
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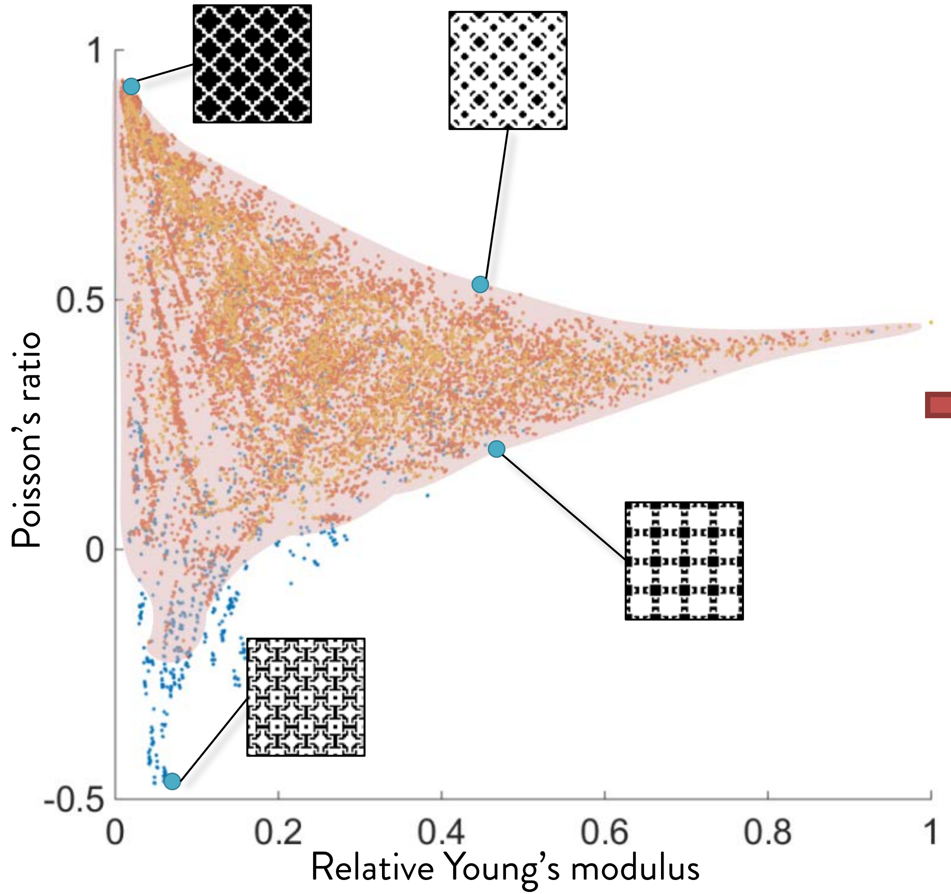

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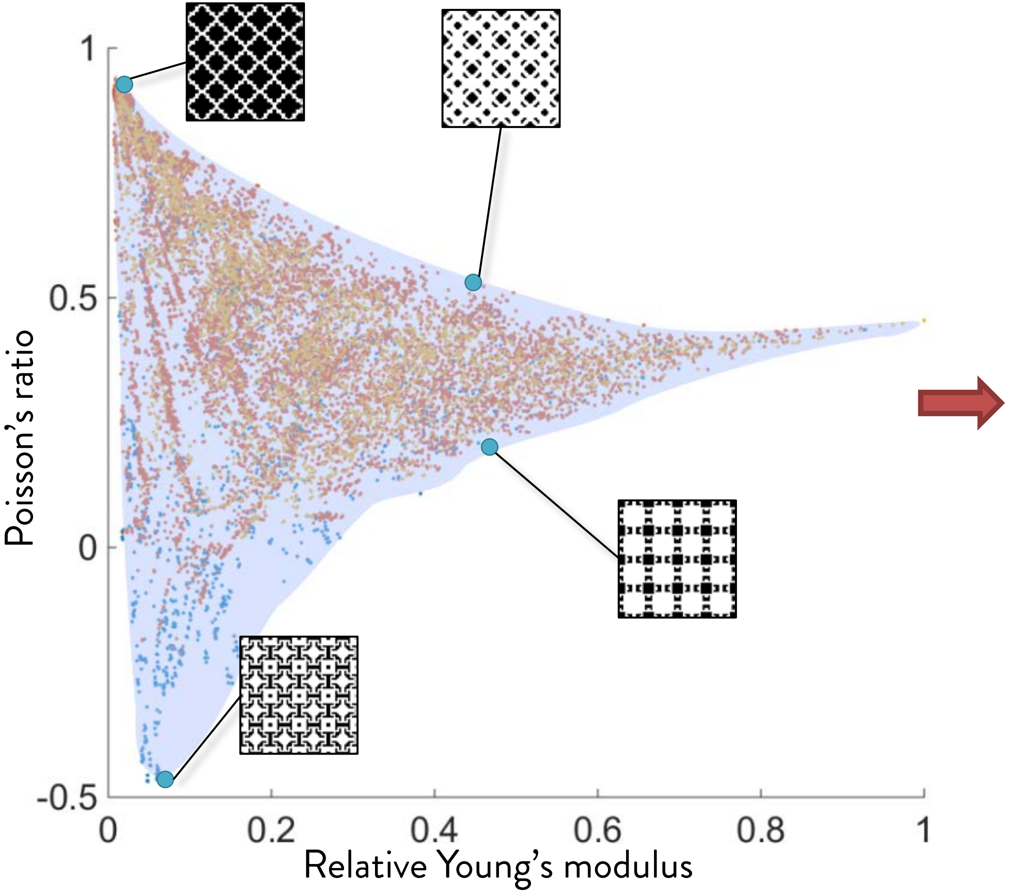
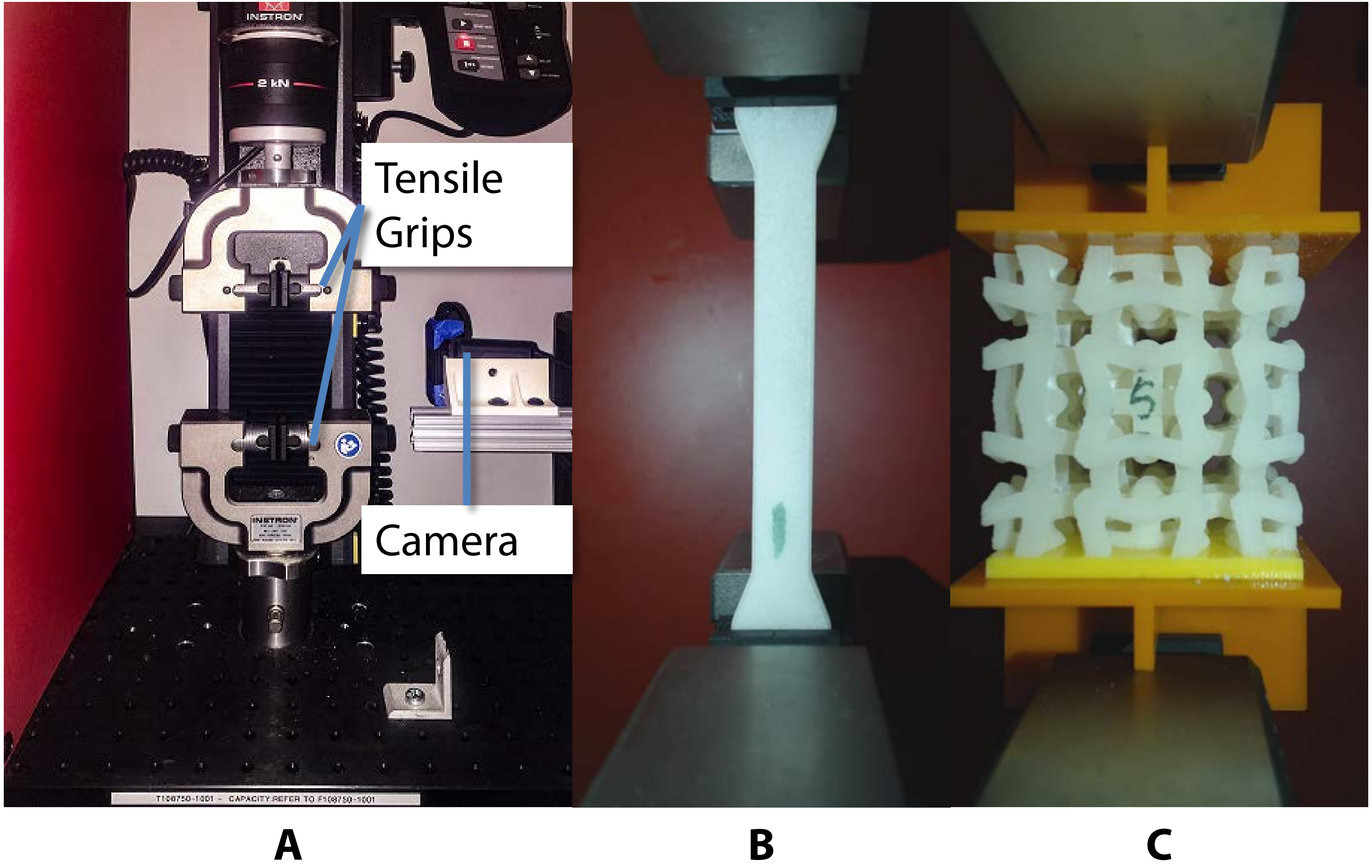
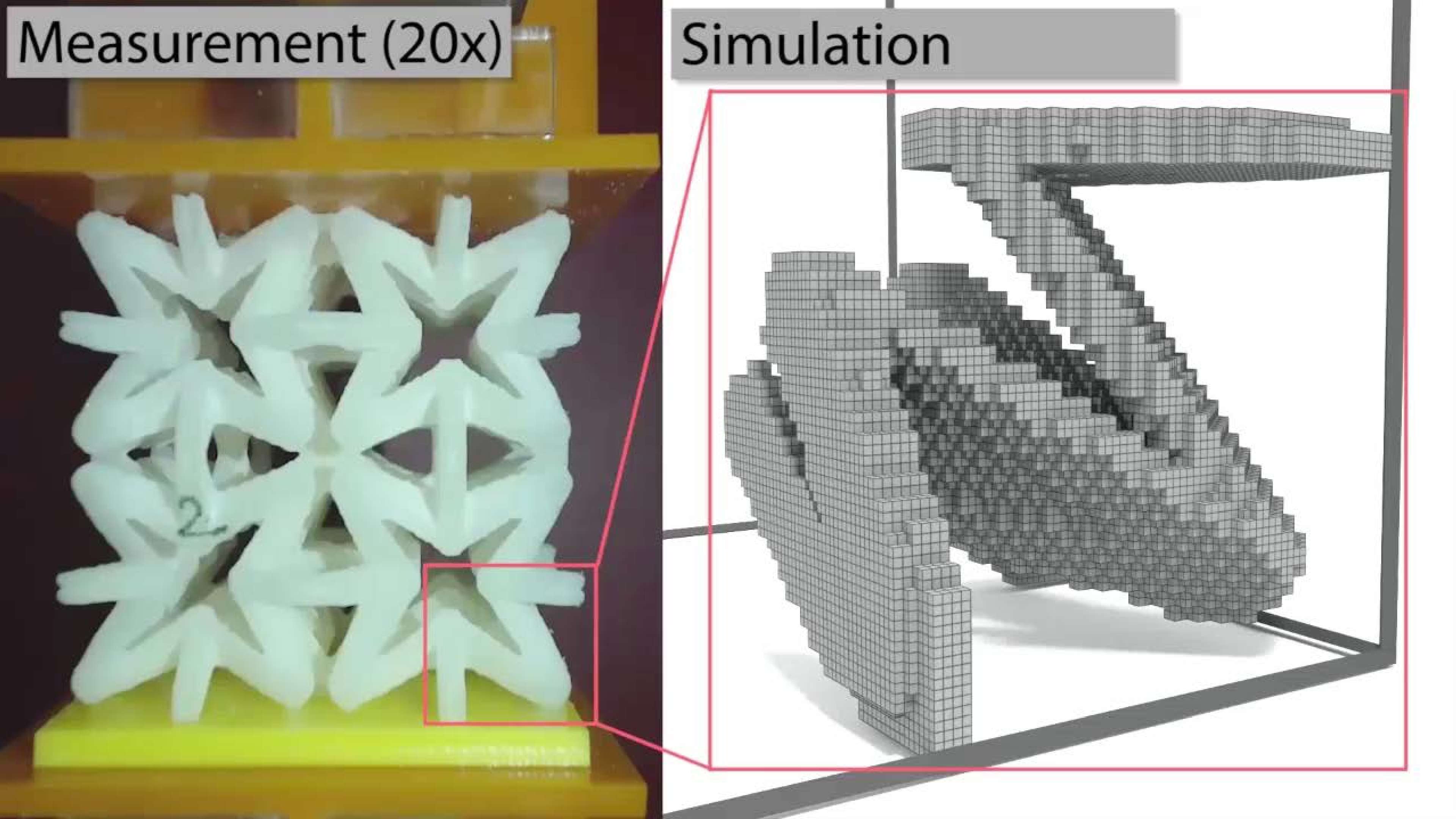


1. Initial samples of microstructures
2. Repeat:
    1. Approximate material gamut with level set
    2. Identify samples near the boundary
    3. Compute level set gradient at each boundary sample
    4. Generate new samples along gradient direction
3. Output level set for topology optimization

Examples with negative poisson's ratio


INSTRON
2 kN
Tensile
Grips
Camera
INSTRON
A
B
C


Measurement (20x)
Simulation

Overview

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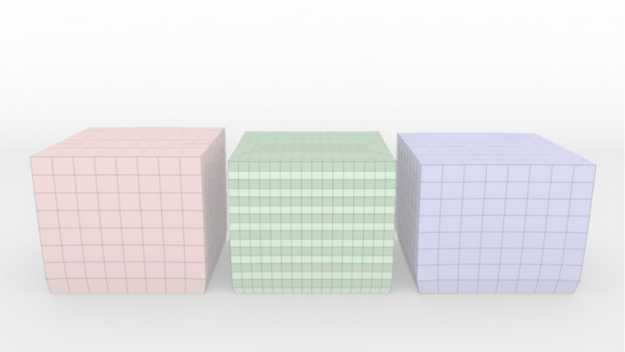
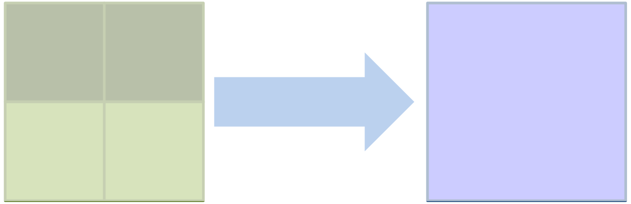
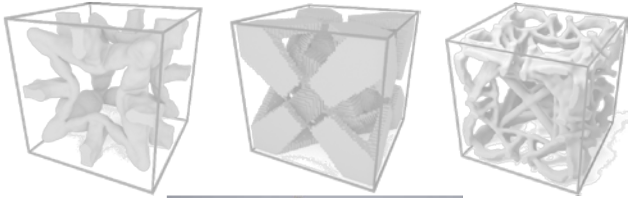
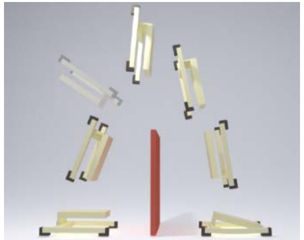








Dynamics-Aware Numerical Coarsening for Fabrication Design

Desai Chen[1,3] David I.W. Levin[2] Wojciech Matusik[1] Danny M. Kaufman[3]

1 MIT CSAIL

2 UNIVERSITY OF TORONTO

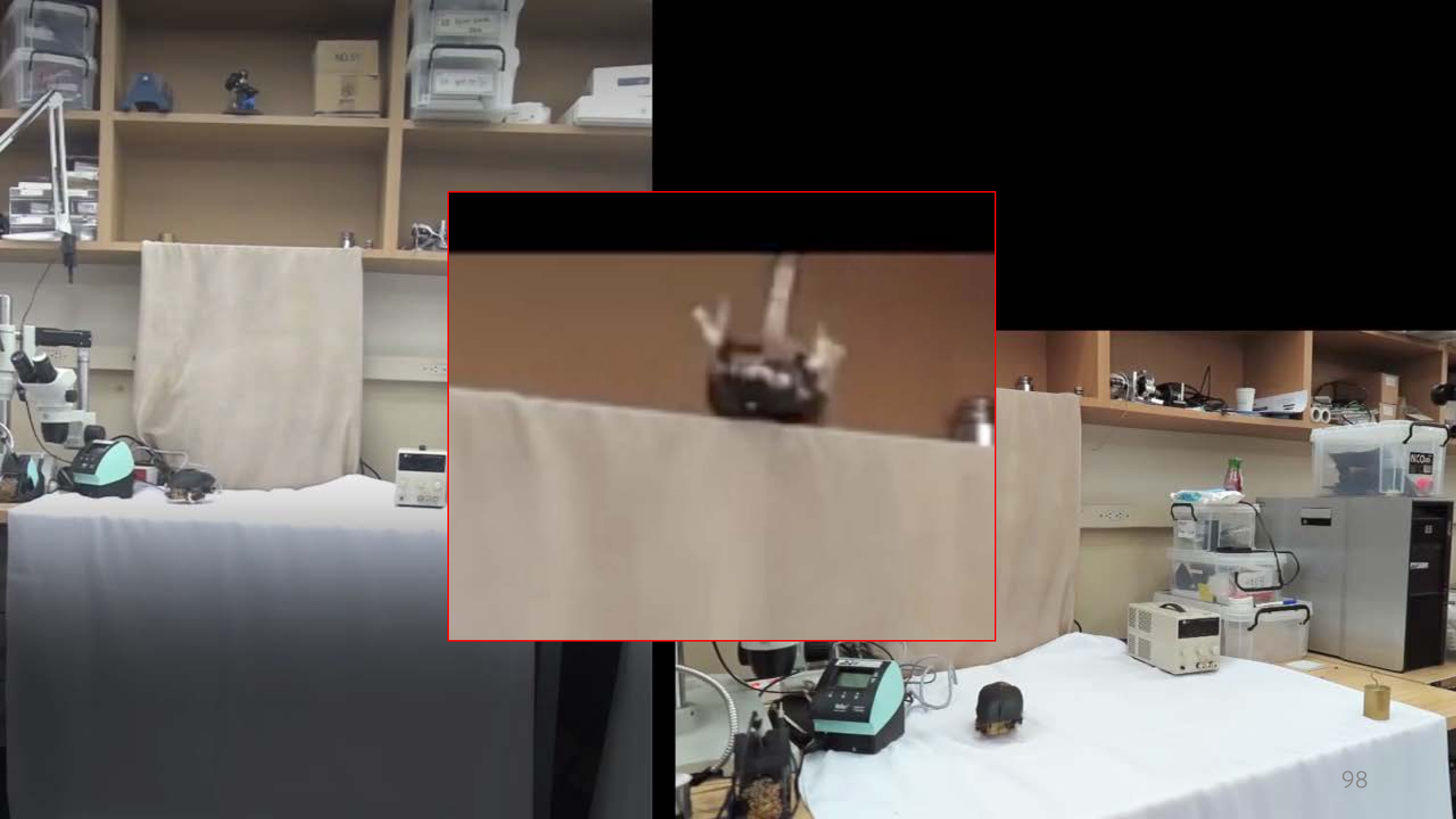
3 Adobe

Introduction – Compliant Dynamic Mechanisms

Jumping stilts

Combustion soft robot
[Bartlett et al., 2015]

Running blades



Introduction – Computational Design

Design

A catapult

Simulation

Design change: make taller

Feedback: Range too short

Introduction – Computational Design

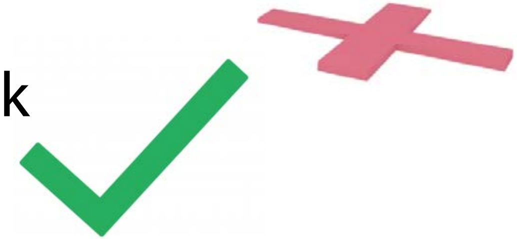
Design

A taller catapult

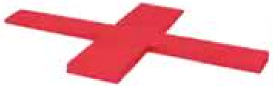
Simulation

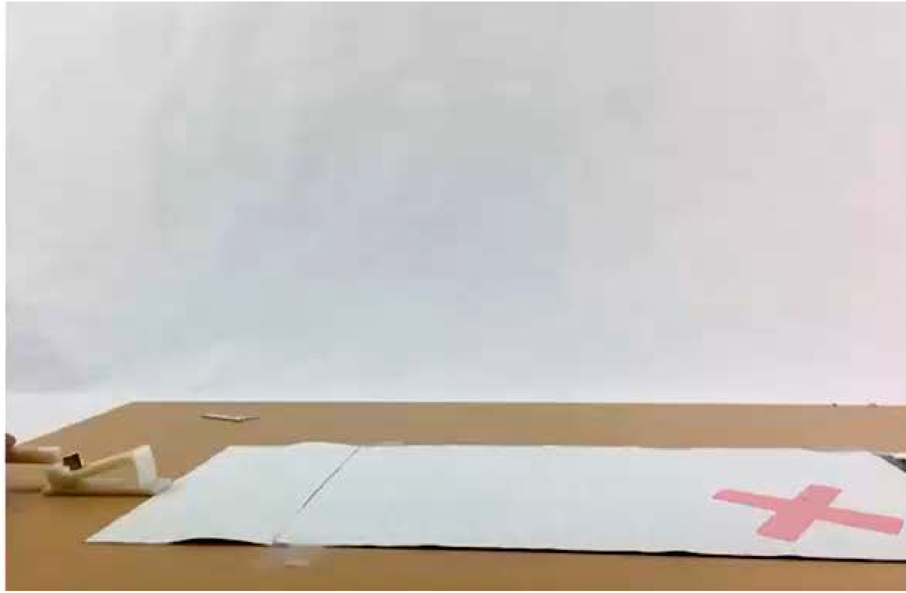
Design change: make taller

Feedback



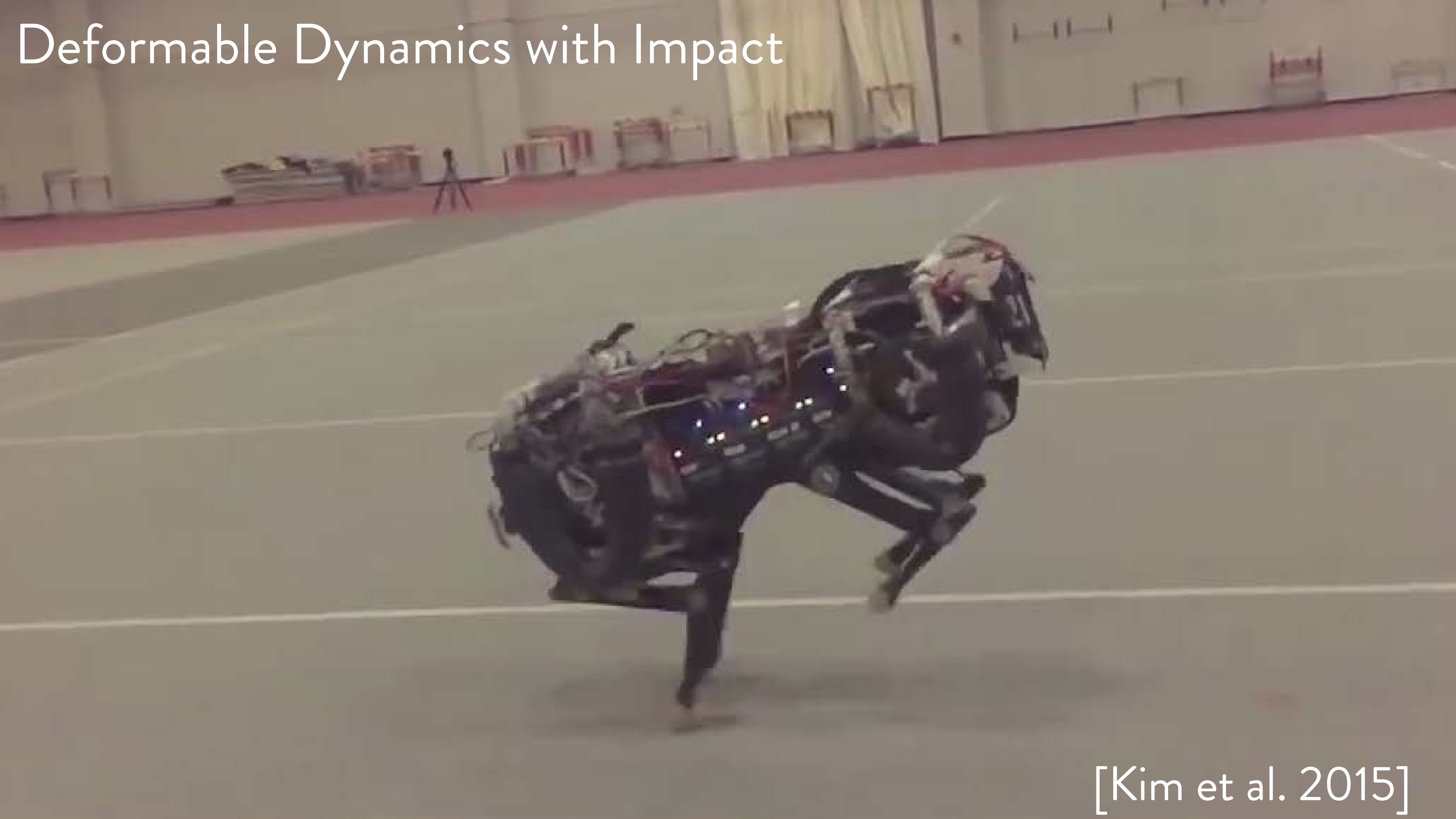
Introduction – Computational Design





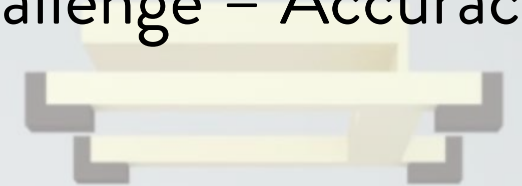
[Chen et al. 2013, Prévost et al. 2013, Skouras et al. 2013, Bächer et al. 2014, Coros et al. 2014, Chen et al. 2014, Musialski et al. 2015]

Deformable Dynamics with Impact

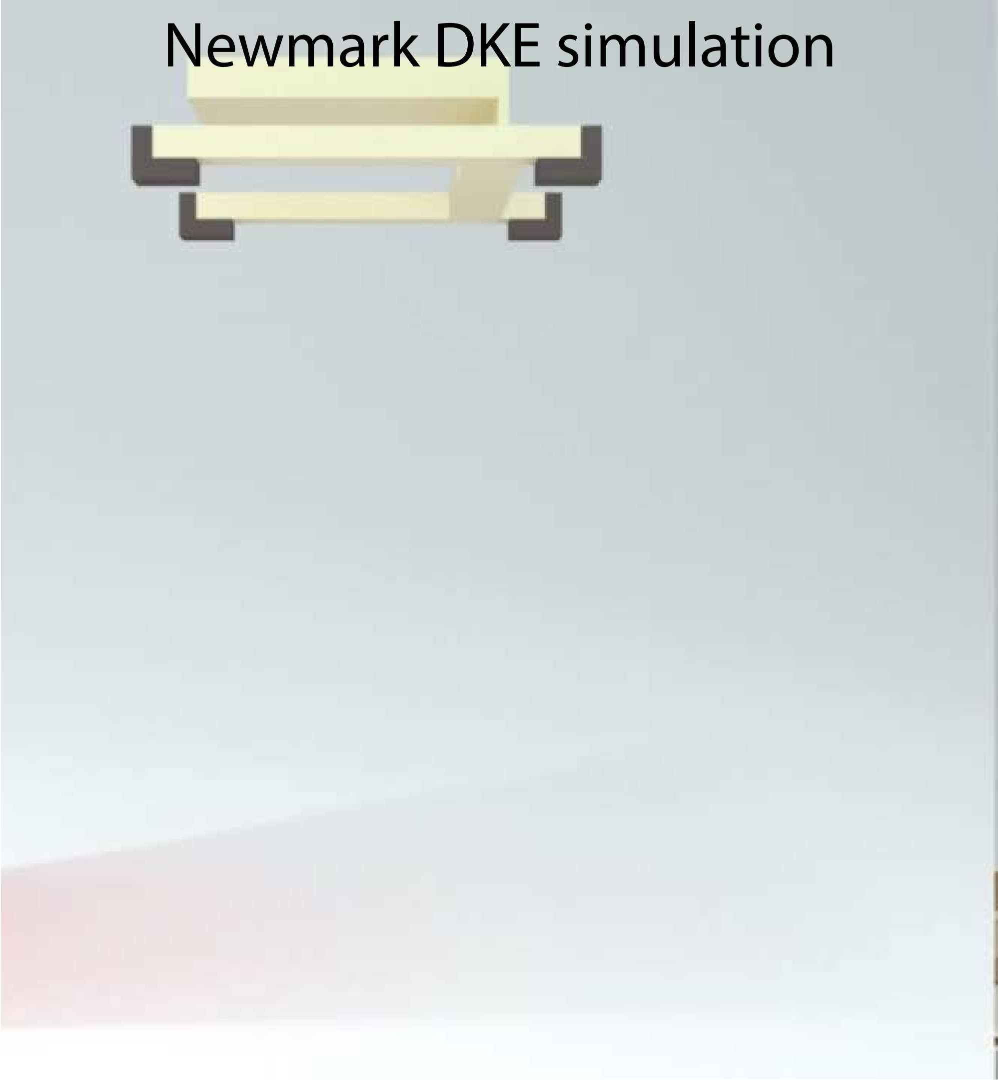


[Kim et al. 2015]

Challenge – Accuracy



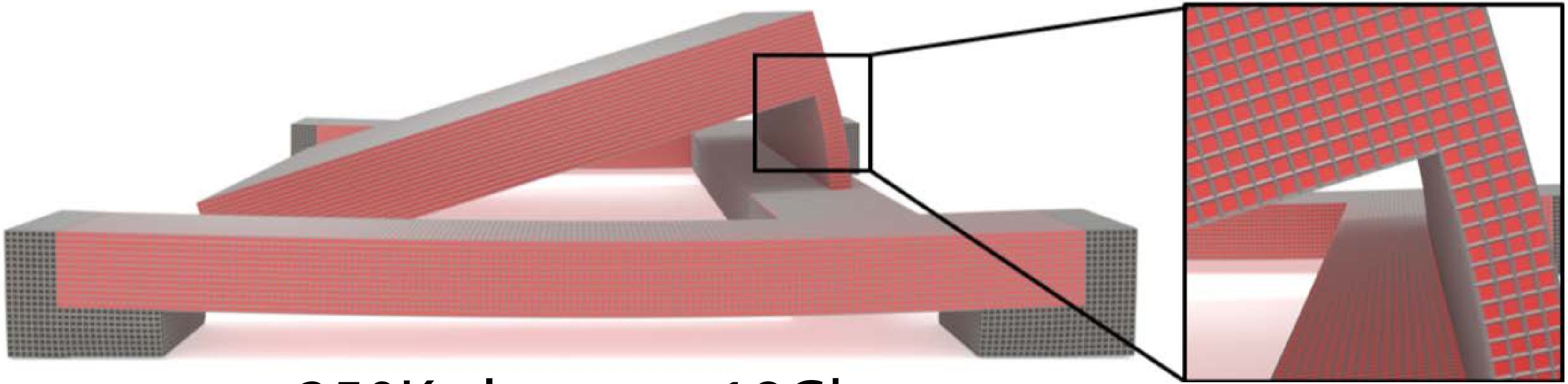
Newmark DKE simulation



Experiment

Challenge – Efficiency

Accurate High-resolution nonlinear FEM

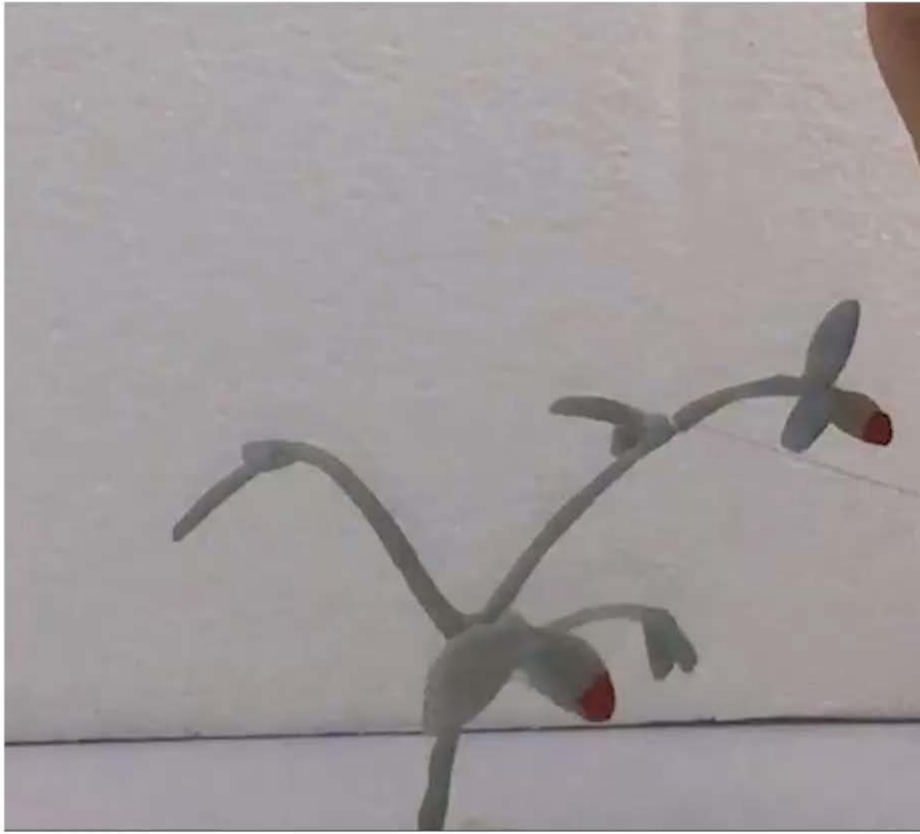


350K elements,18Gb
Simulation time: days

dx = 0.375 mm
dt = 1e-5 sec

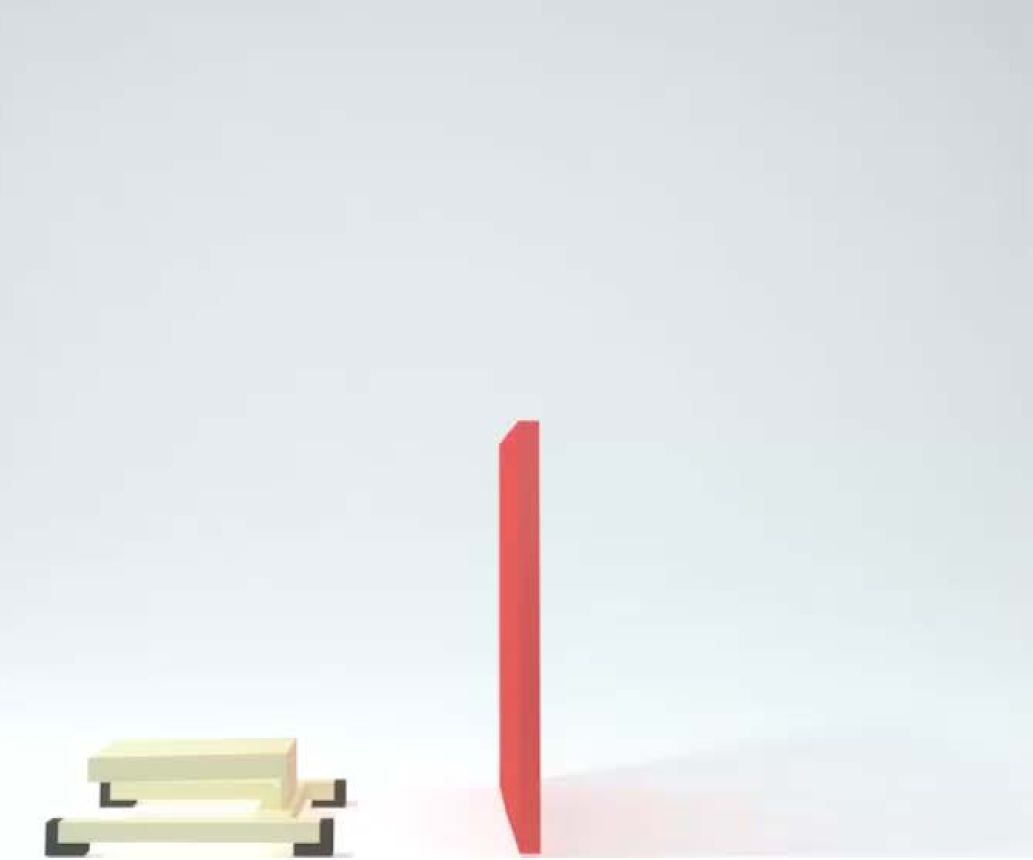
DAC

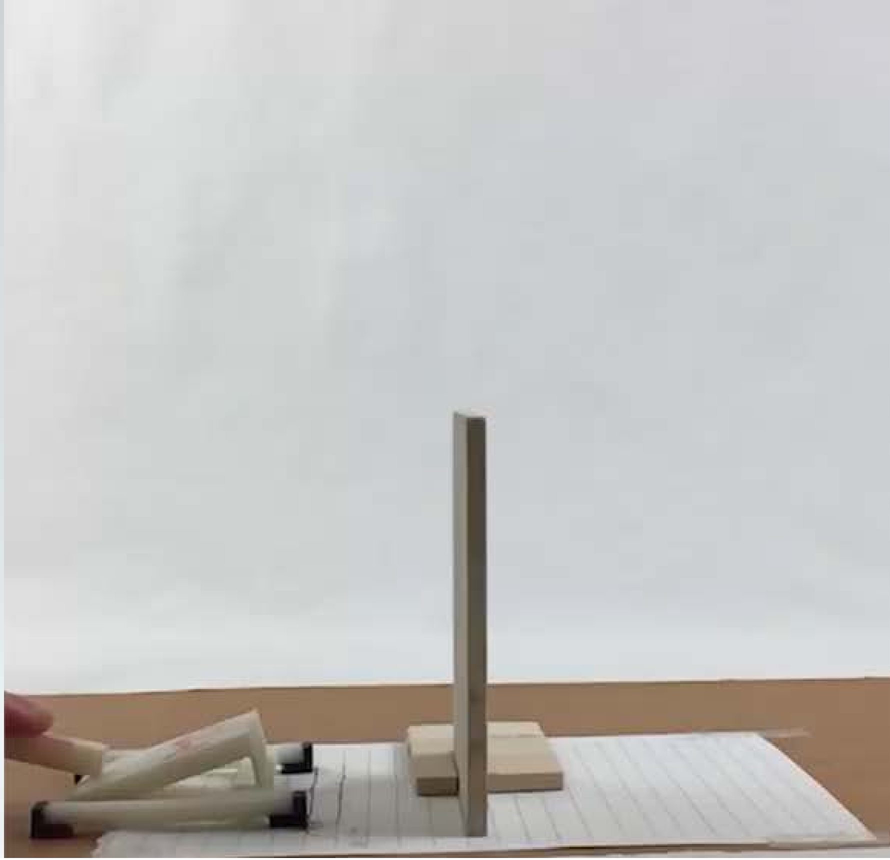
79X speed-up!



BBI

DAC & BBI

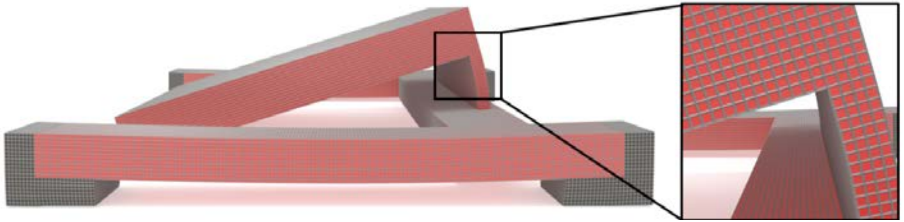




Dynamics-Aware Coarsening (DAC)

Efficient Accuracy and Material Modeling

High-resolution nonlinear FEM



350K elements, 18Gb

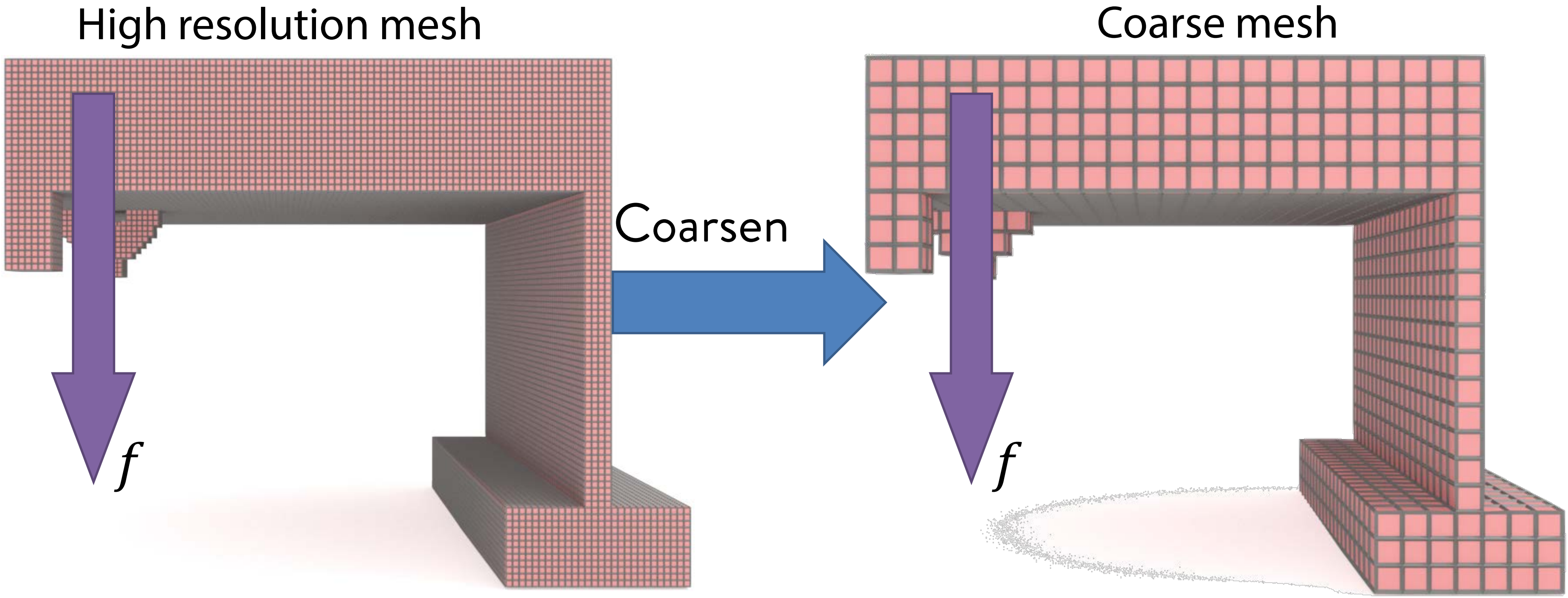
Simulation time: **days**

Unknown material parameters

Young's modulus: 1.9 ± ? GPa

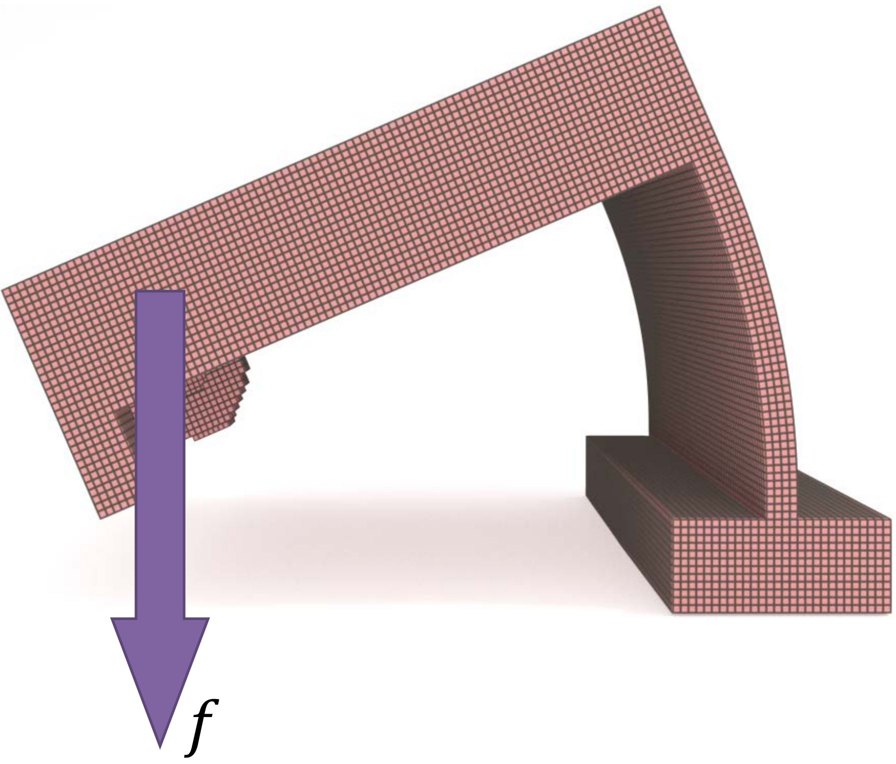
Damping: ?

Method – Dynamics-Aware Coarsening

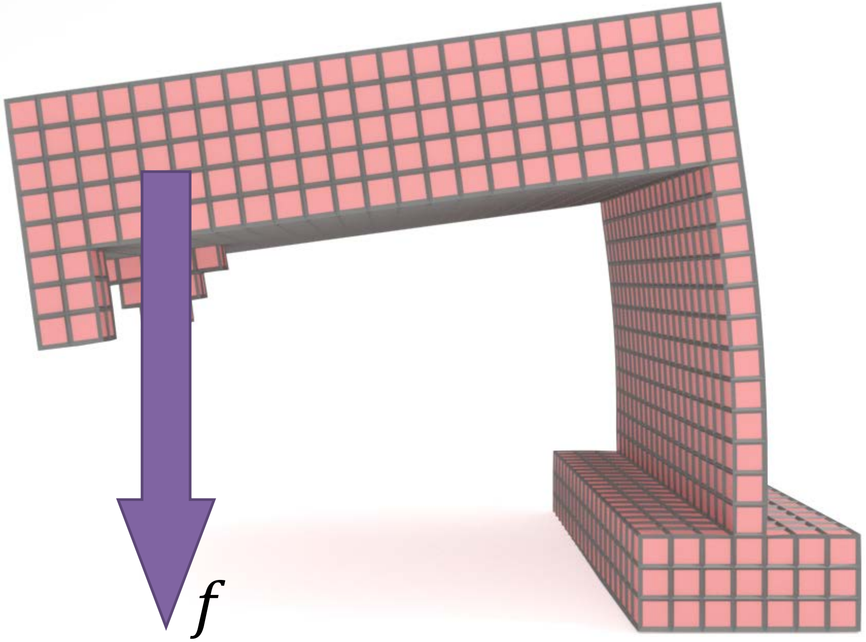


Method – Dynamics-Aware Coarsening

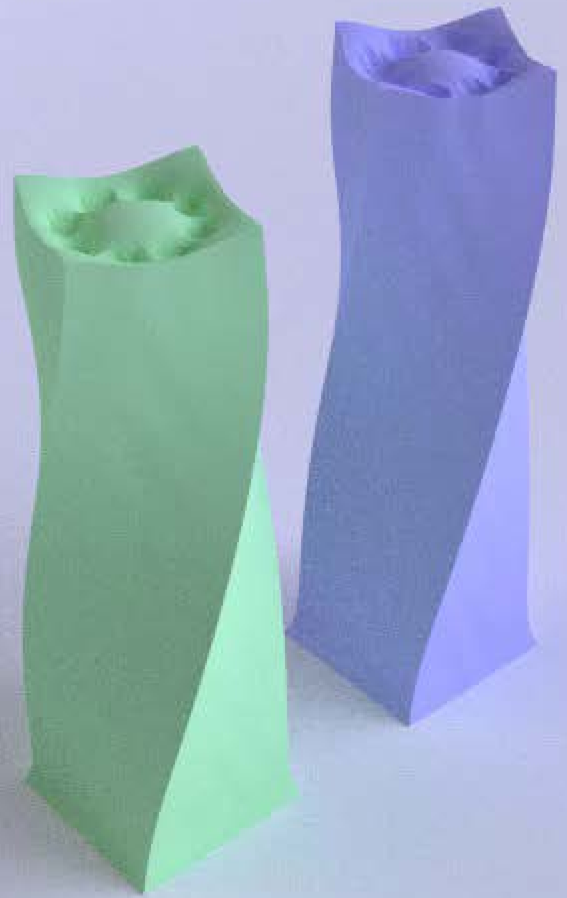
High resolution mesh



Coarse mesh



Method – Energy-based Coarsening

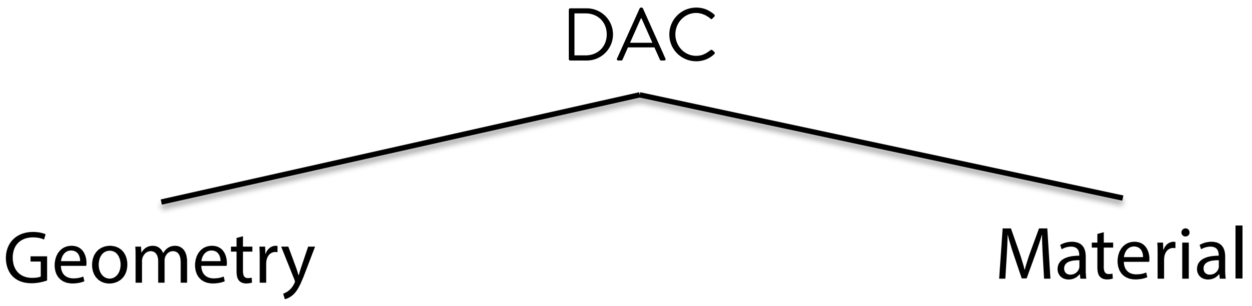


High-res FEA

Energy-based coarsened FEA

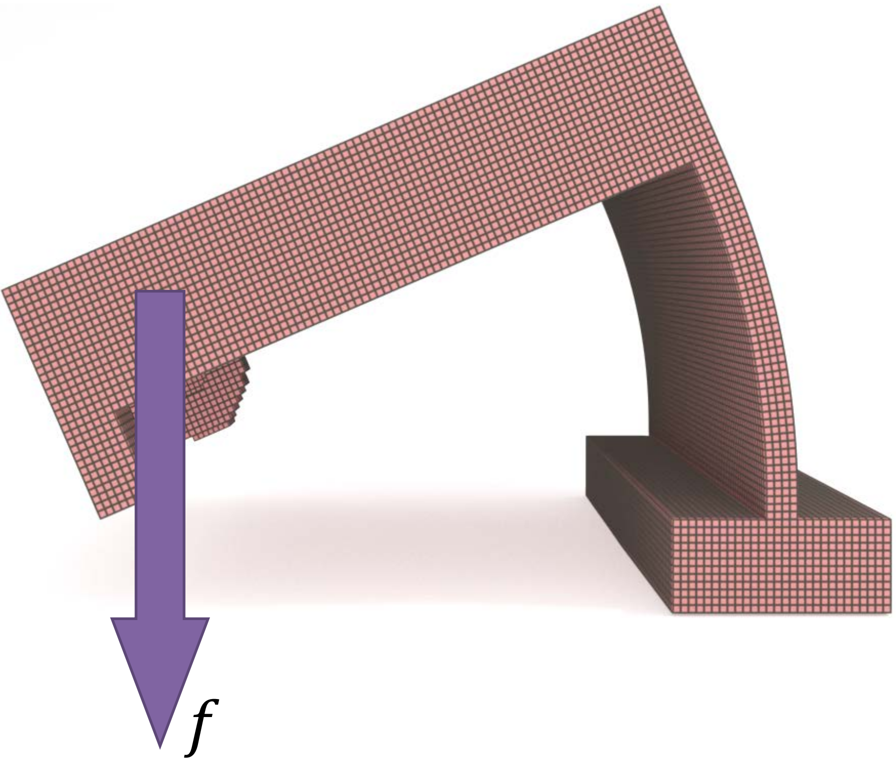
[Chen et. al 2015]

Dynamics-Aware Coarsening

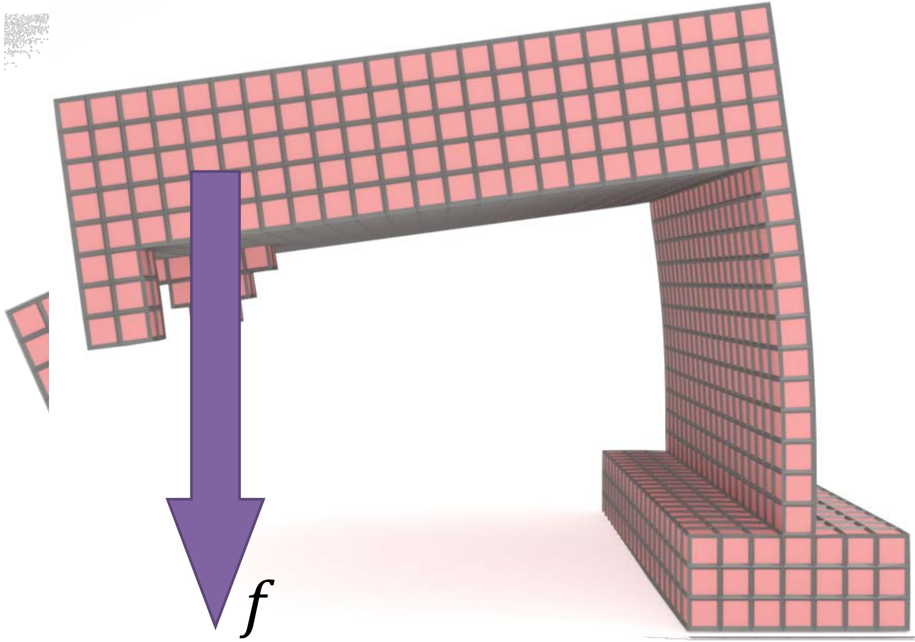


DAC - Capturing Geometry

High resolution finite elements

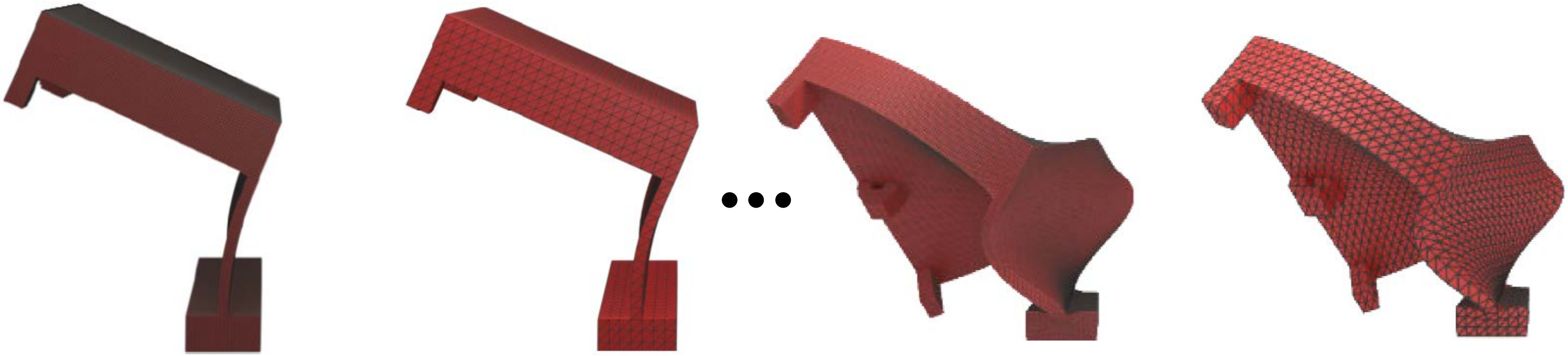


Coarsened finite elements



DAC – Matching Modal Shapes

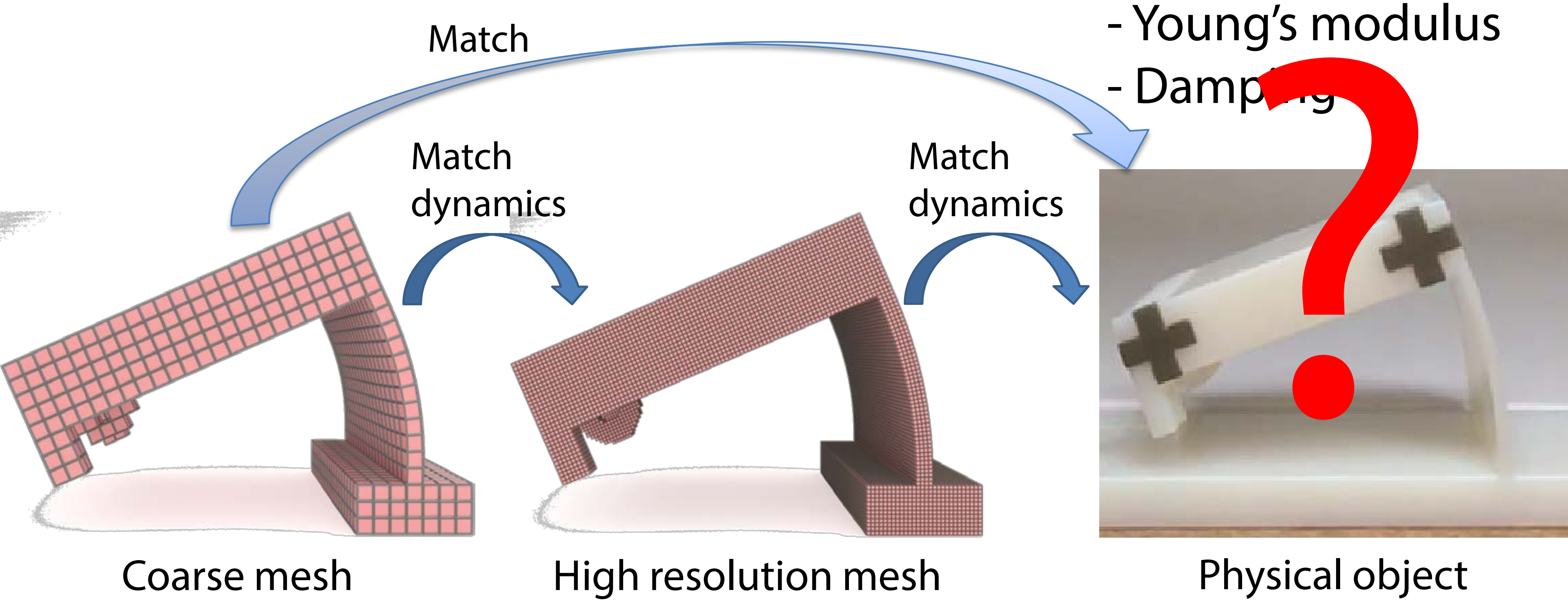
Fine mesh Coarse mesh Fine mesh Coarse mesh



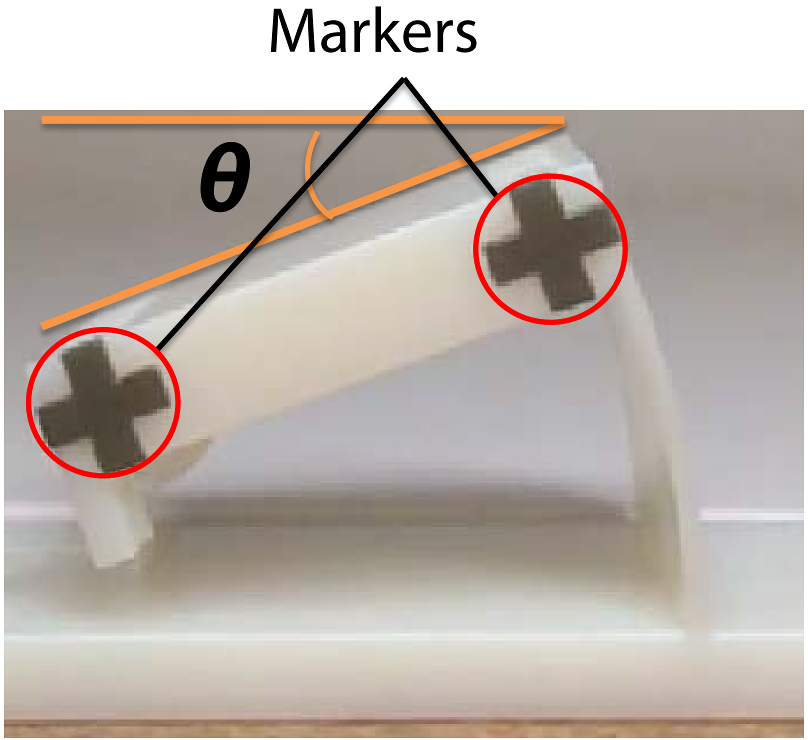
1st mode

5th mode

DAC – Material Modeling



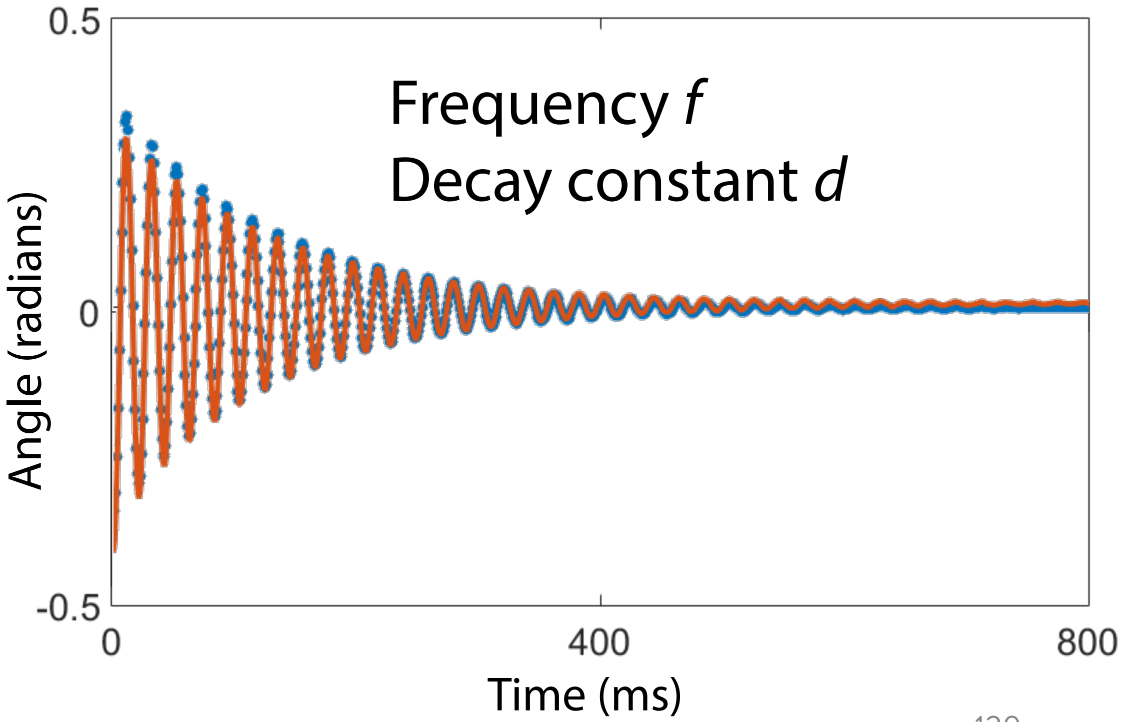
Method – Physical Measurements



Calibration object

Method – Physical Measurements

Calibration object



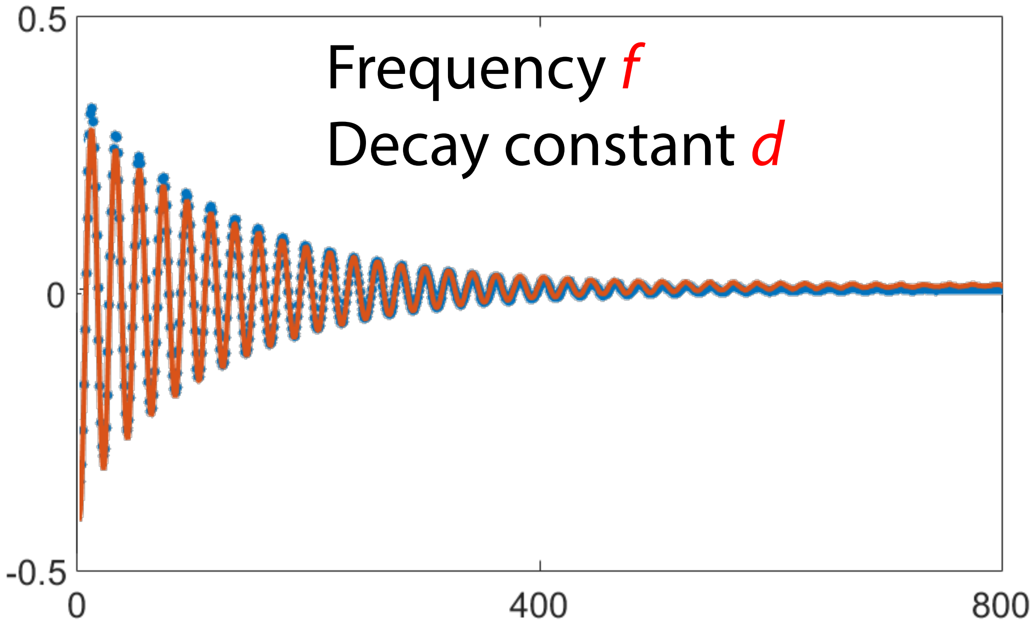
Material parameter from measurement

$$M\ddot{q} = -Eq - bE\dot{q}$$

Young's modulus

Damping coefficient

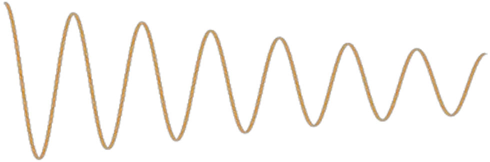
$(E, b) \Leftarrow (f, d) \Leftarrow$



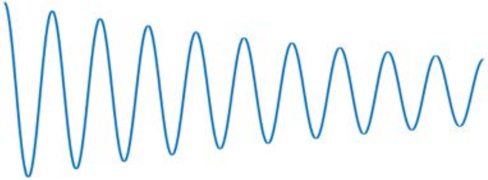
Frequency Scaling

Young's modulus $E = ?$

Measurement

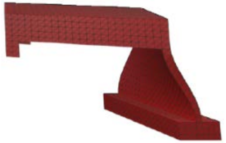


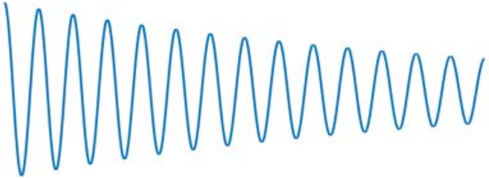
Mode 1



$$\ddot{q}_1 = \lambda_1 q_1 - b\lambda_1 \dot{q}_1$$

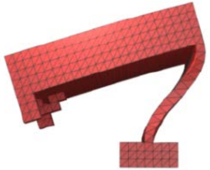
Mode 2

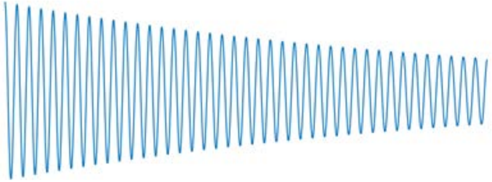




$$\ddot{q}_2 = \lambda_2 q_2 - b\lambda_2 \dot{q}_2$$

Mode 3





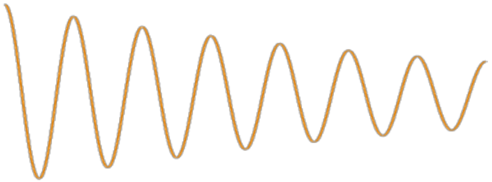
$$\ddot{q}_3 = \lambda_3 q_3 - b\lambda_3 \dot{q}_3$$

⋮

Frequency Scaling

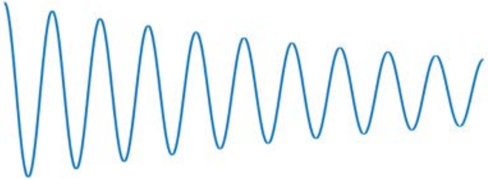
Young's modulus E = ?

Measurement



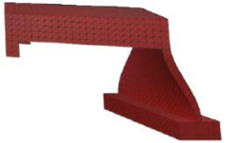
$$E \sim \lambda \sim f^2$$

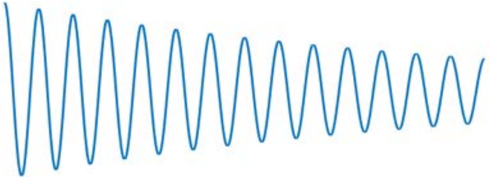
Mode 1



$$\ddot{q}_1 = \lambda_1 q_1 - b\lambda_1 \dot{q}_1$$

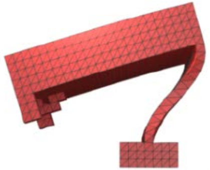
Mode 2

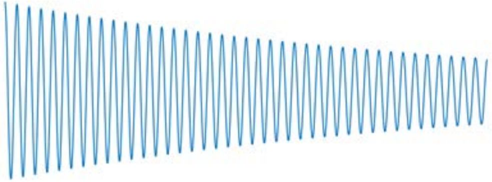




$$\ddot{q}_2 = \lambda_2 q_2 - b\lambda_2 \dot{q}_2$$

Mode 3





$$\ddot{q}_3 = \lambda_3 q_3 - b\lambda_3 \dot{q}_3$$

Frequency Scaling

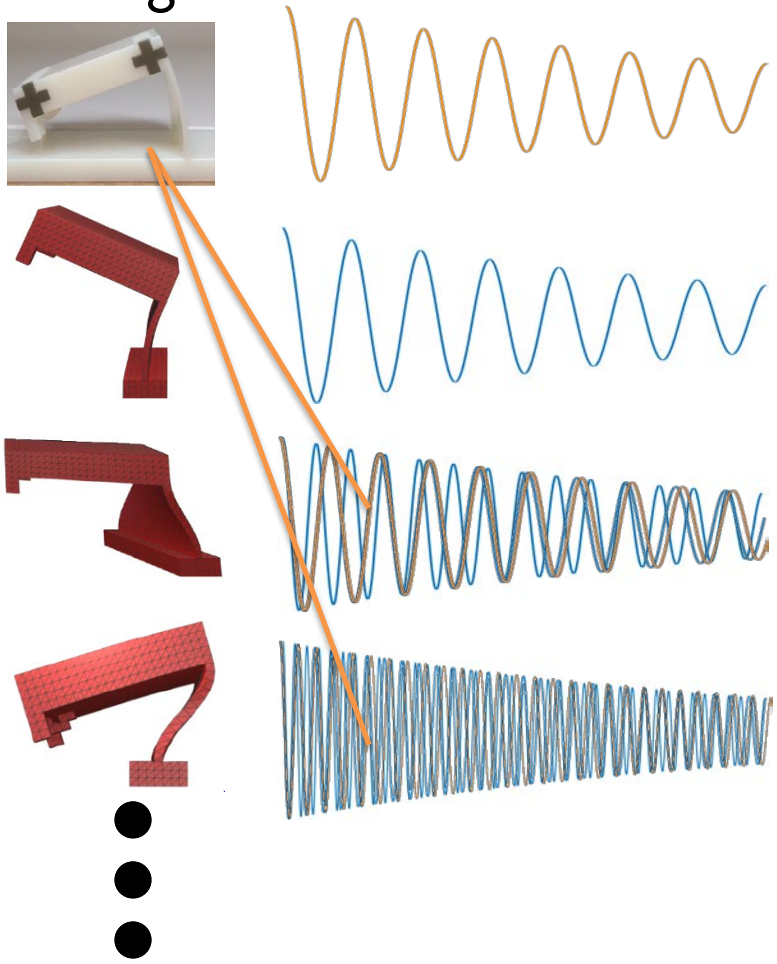
Young's modulus E = ?

Measurement

Mode 1

Mode 2

Mode 3



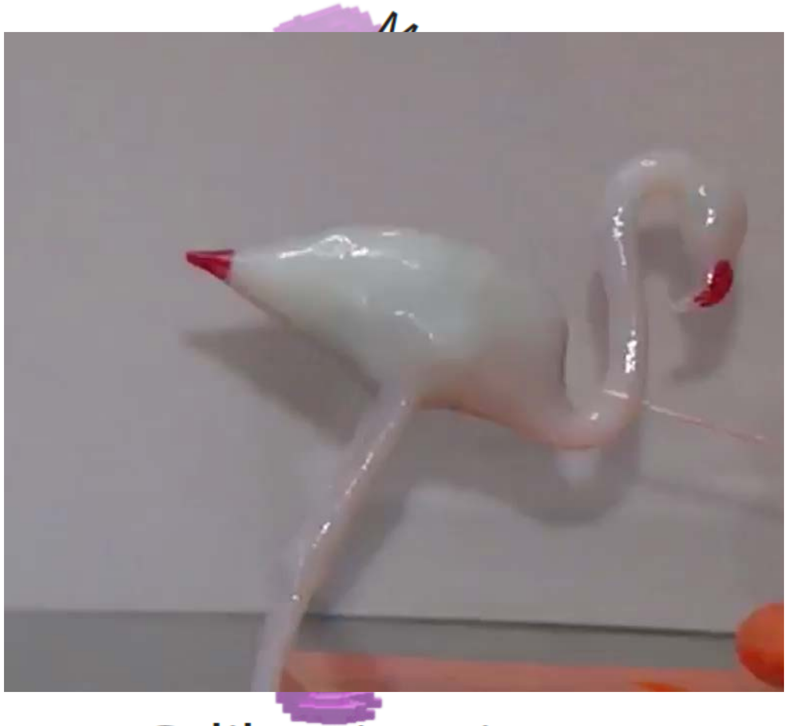
$$E \sim \lambda \sim f^2$$

$$\ddot{q}_1 = \lambda_1 q_1 - b\lambda_1 \dot{q}_1$$

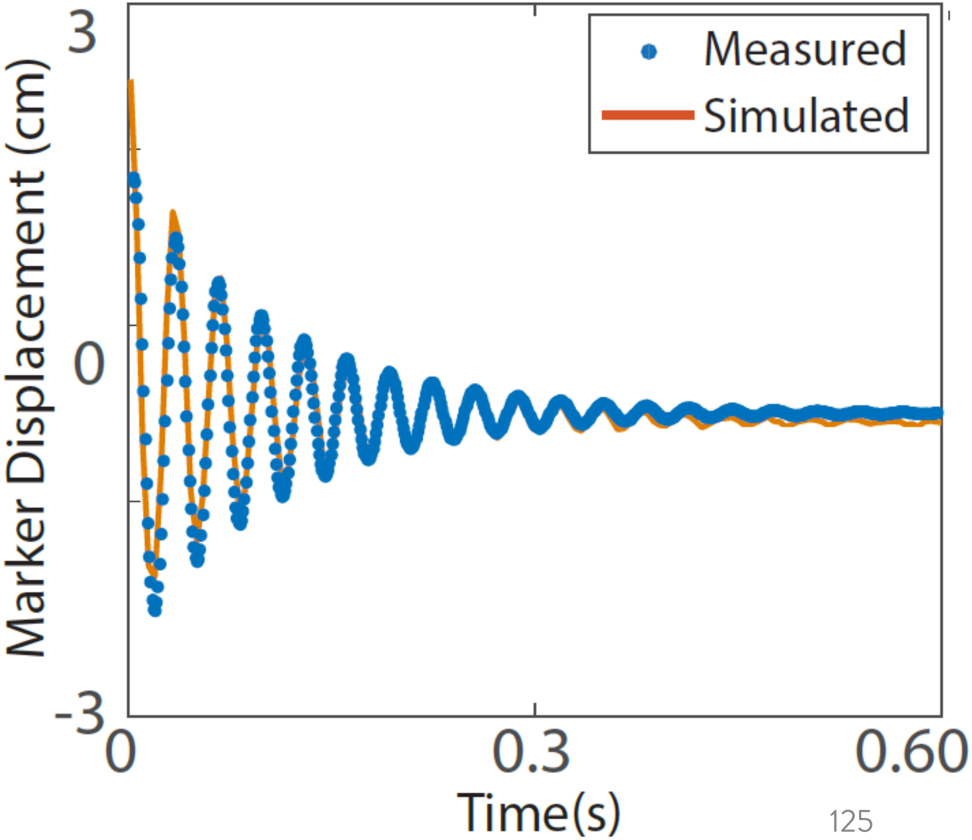
$$\ddot{q}_2 = \lambda_2 q_2 - b\lambda_2 \dot{q}_2$$

$$\ddot{q}_3 = \lambda_3 q_3 - b\lambda_3 \dot{q}_3$$

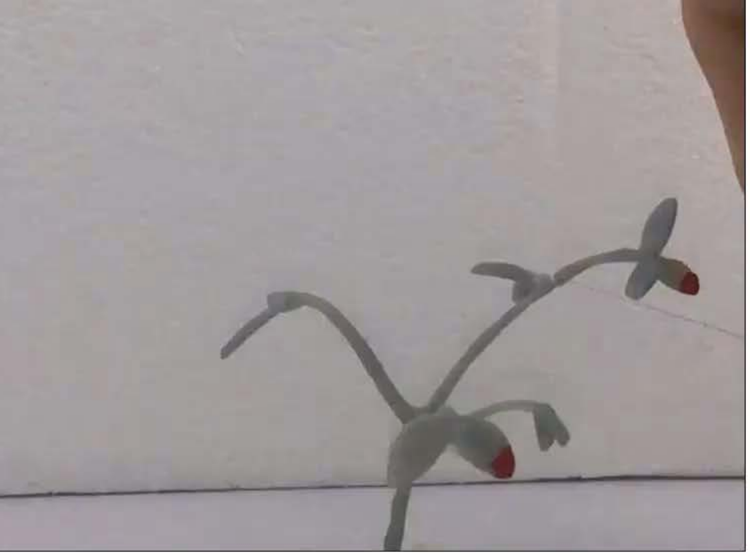
Method – DAC validation

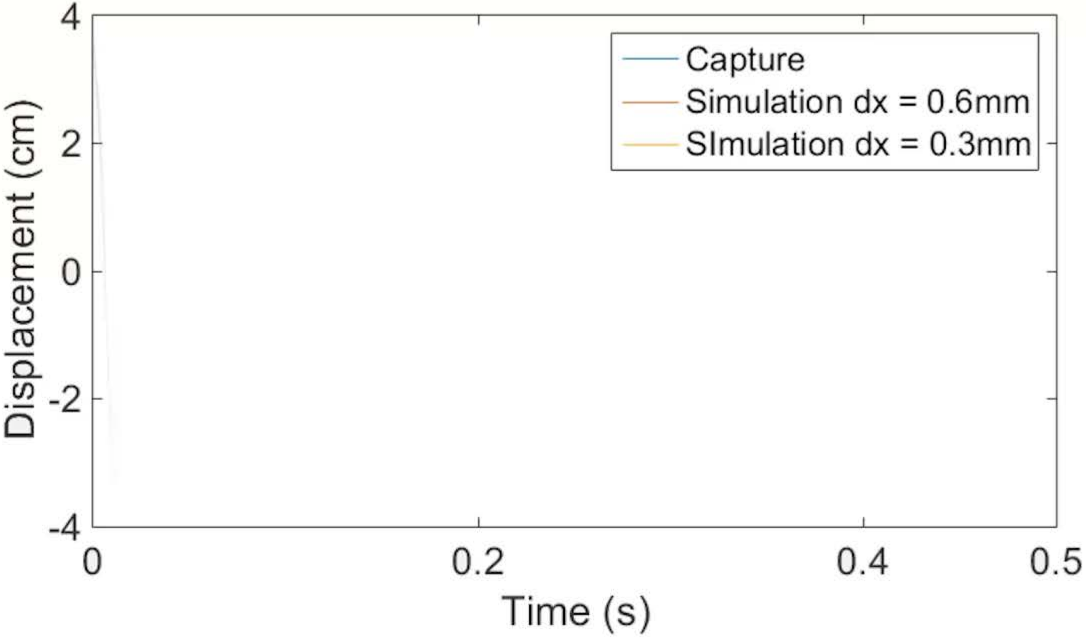


Calibration rig

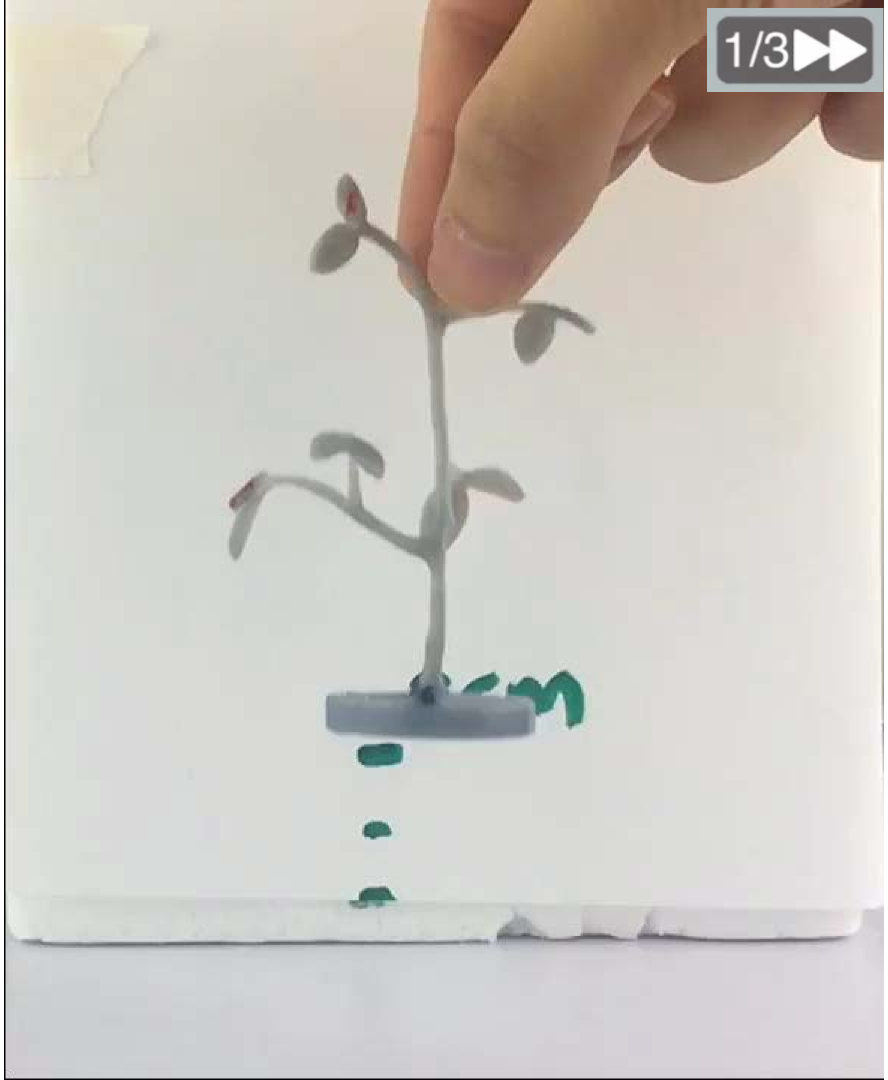


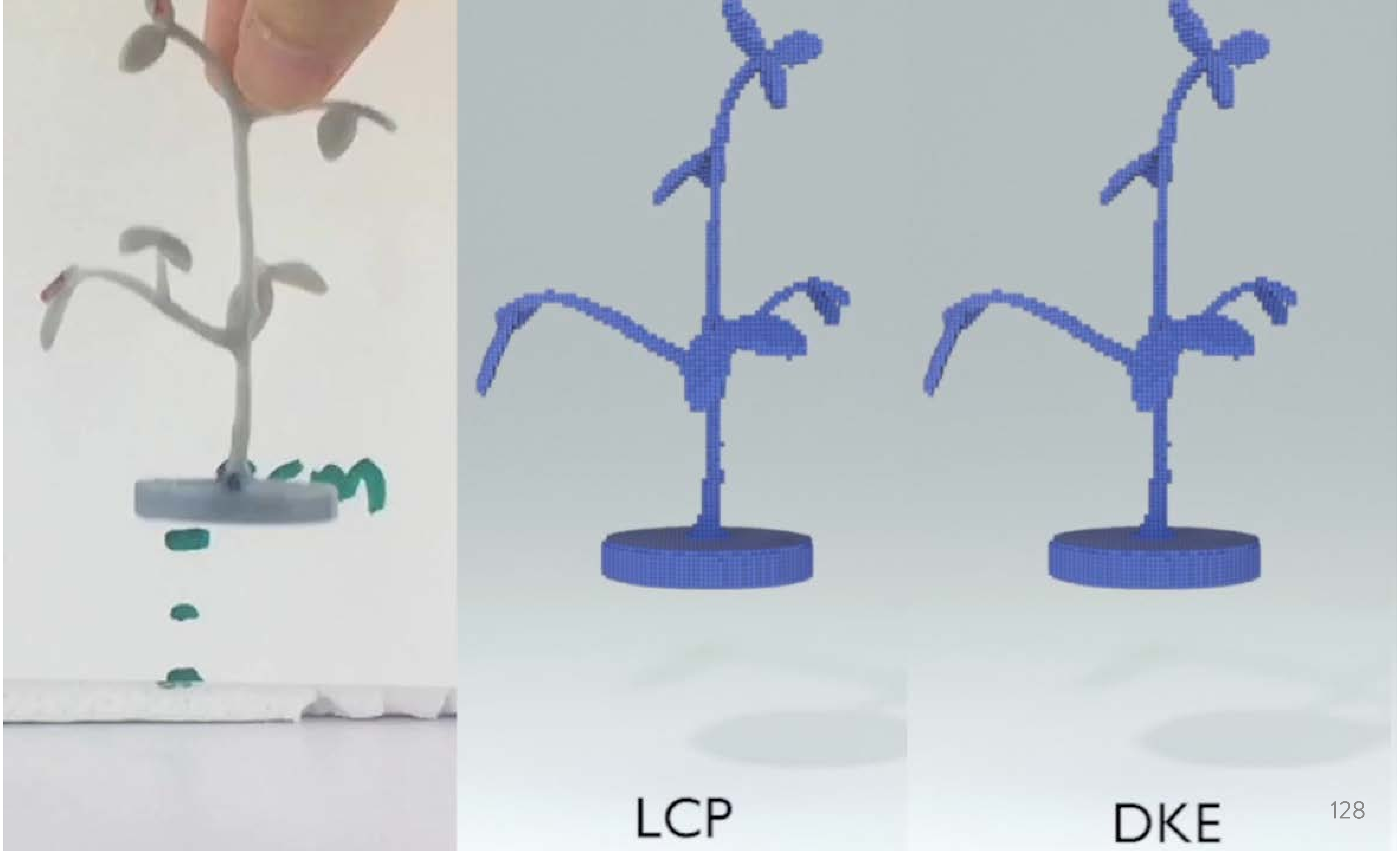
Captured and DAC trajectories

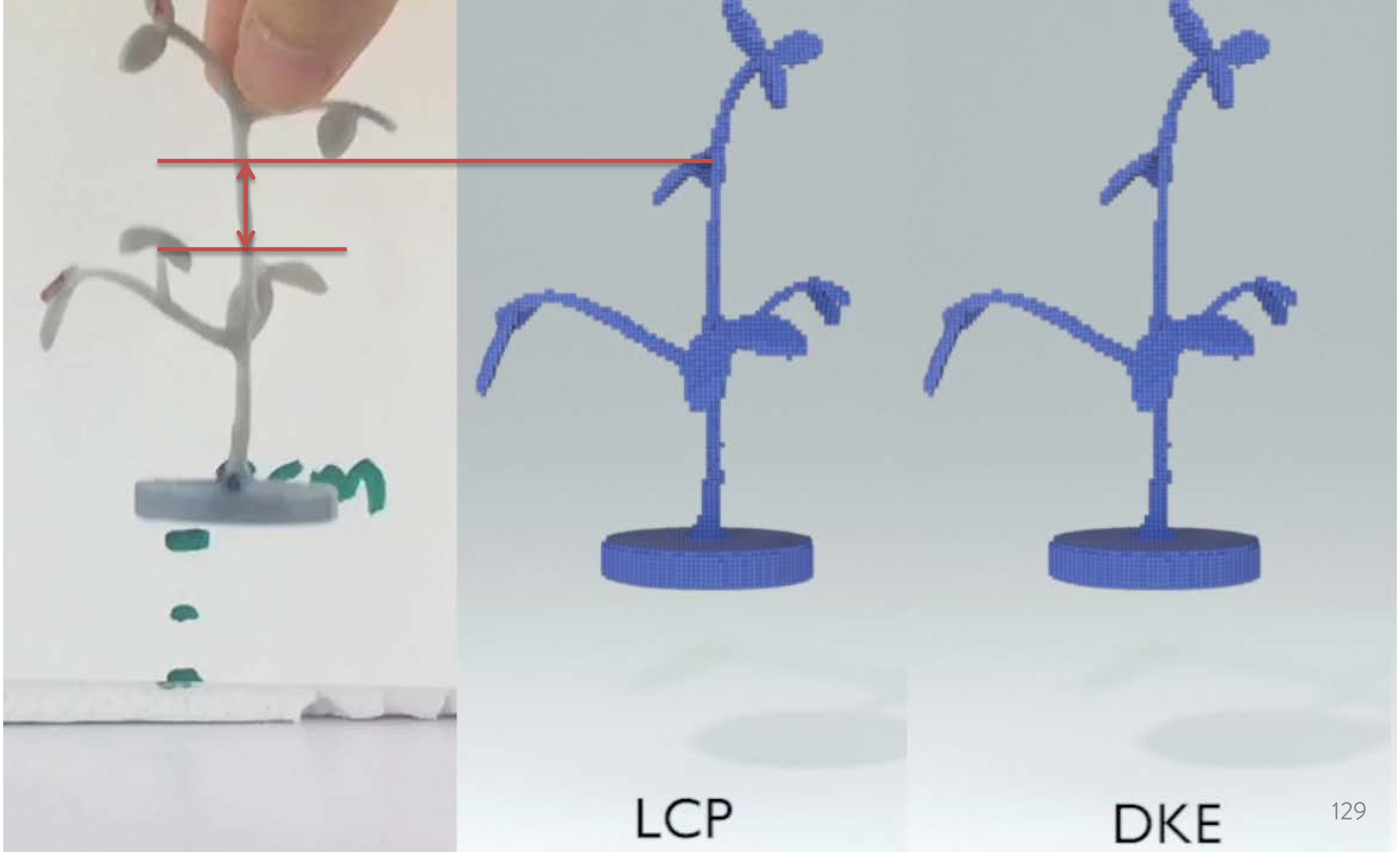




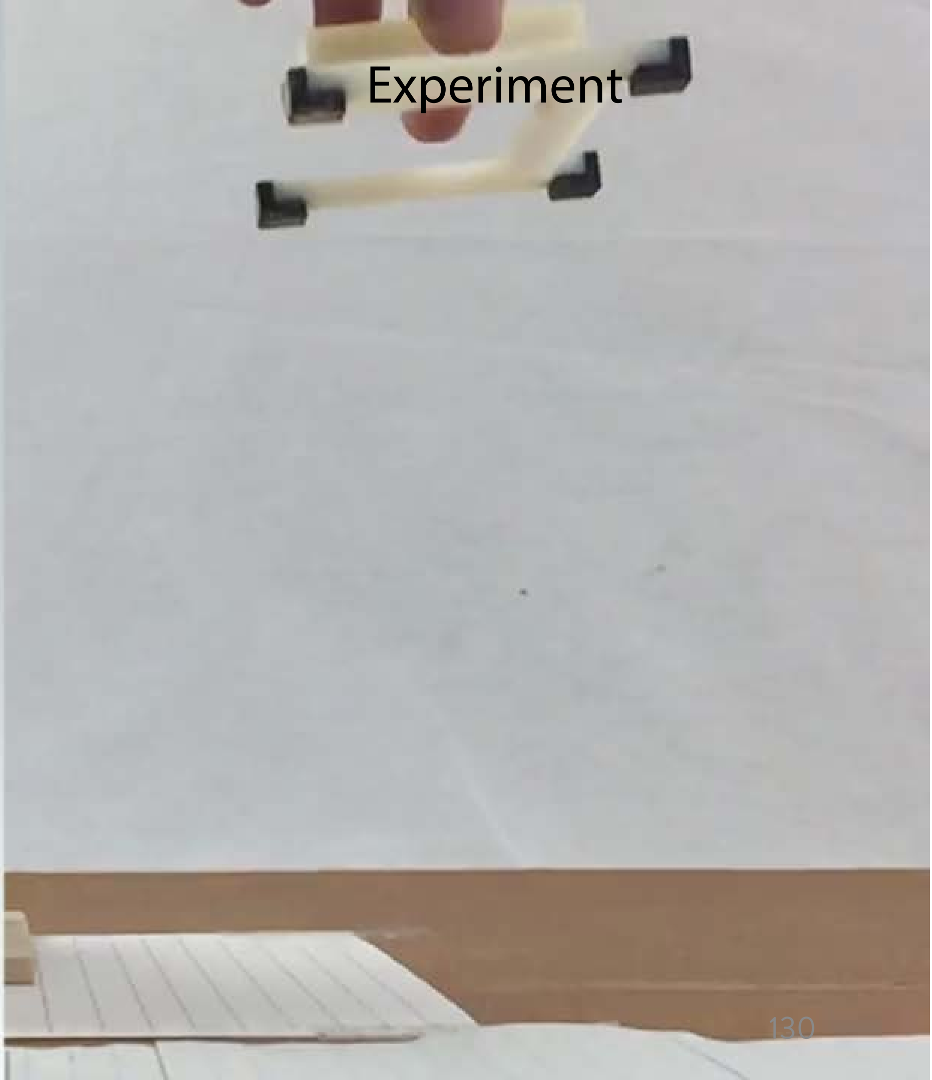
Contact Experiment



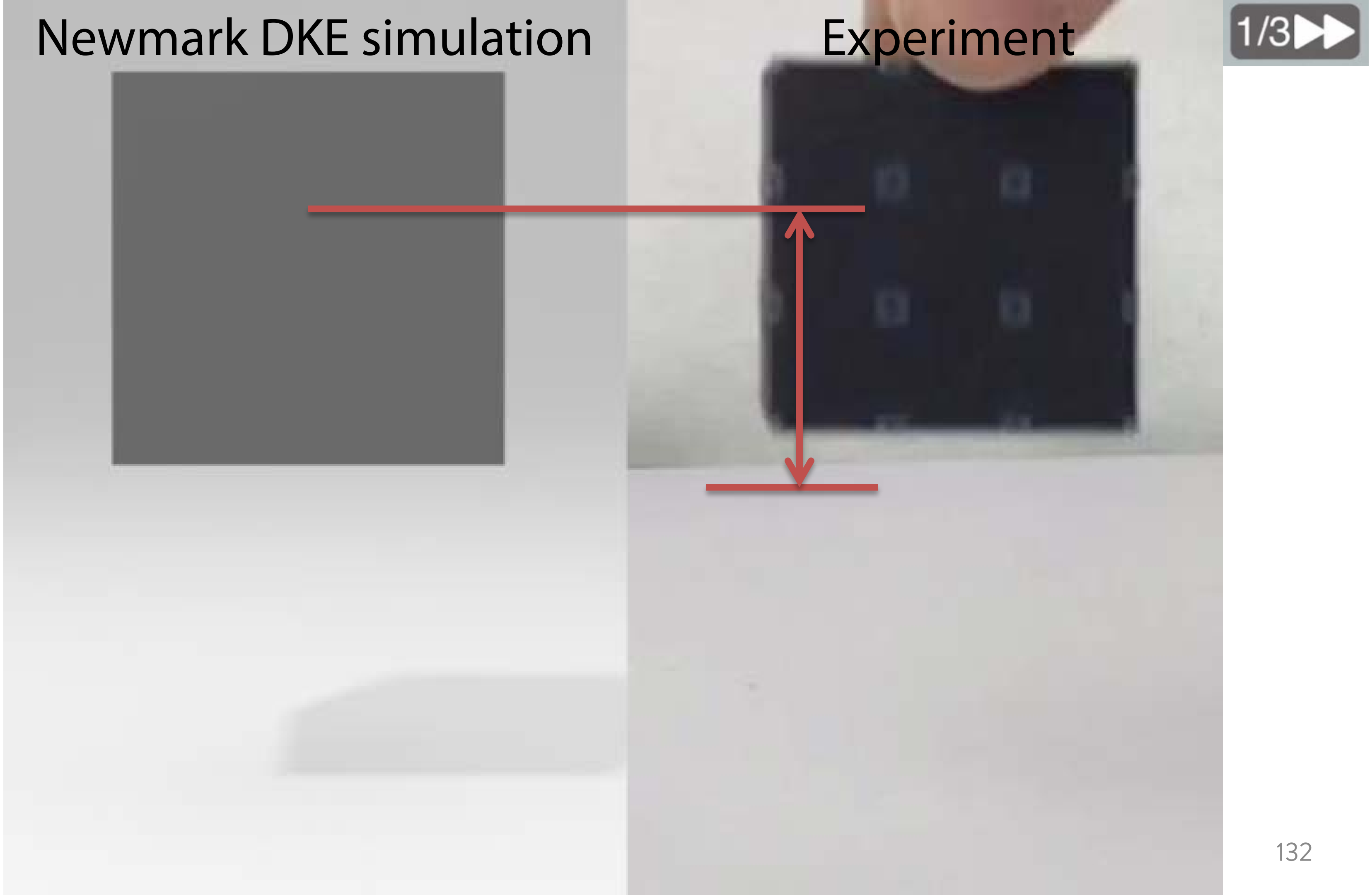

LCP
DKE


LCP
DKE


Newmark DKE Simulation

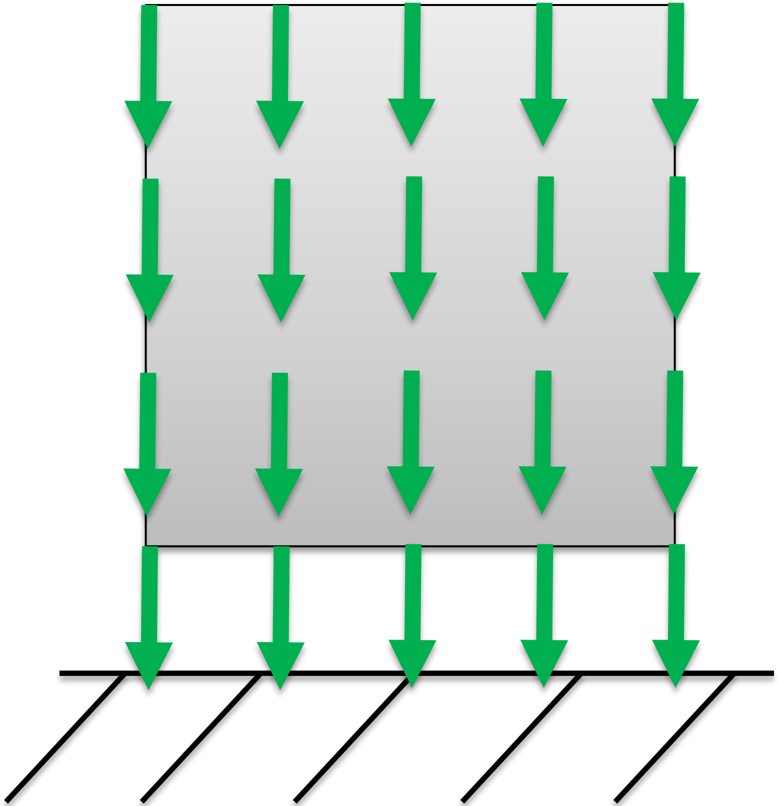

Experiment

Boundary Balacing Impact


Newmark DKE simulation
Experiment
1/3

Inelastic Impact of an Elastic Block

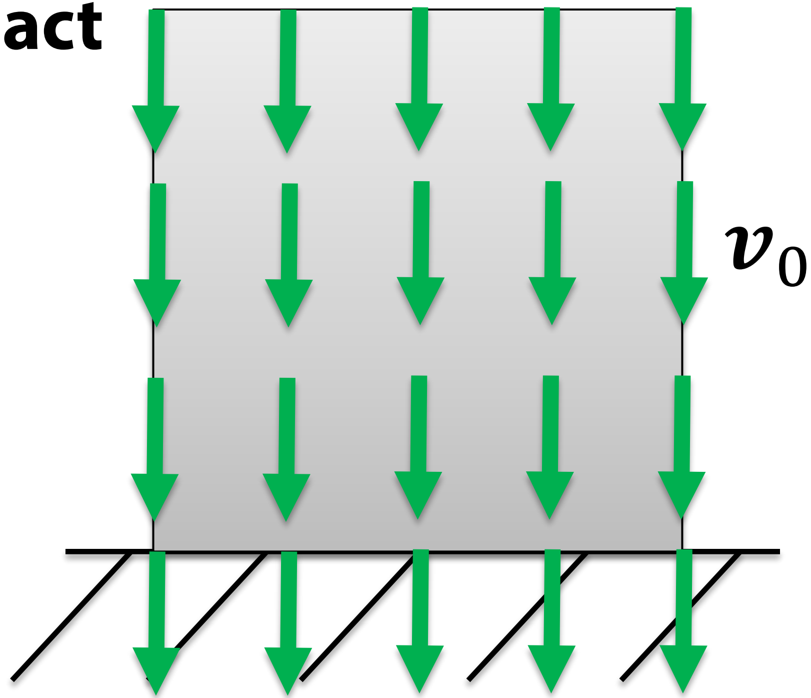
Free fall



Inelastic Impact of an Elastic Block

Free fall

Initial contact

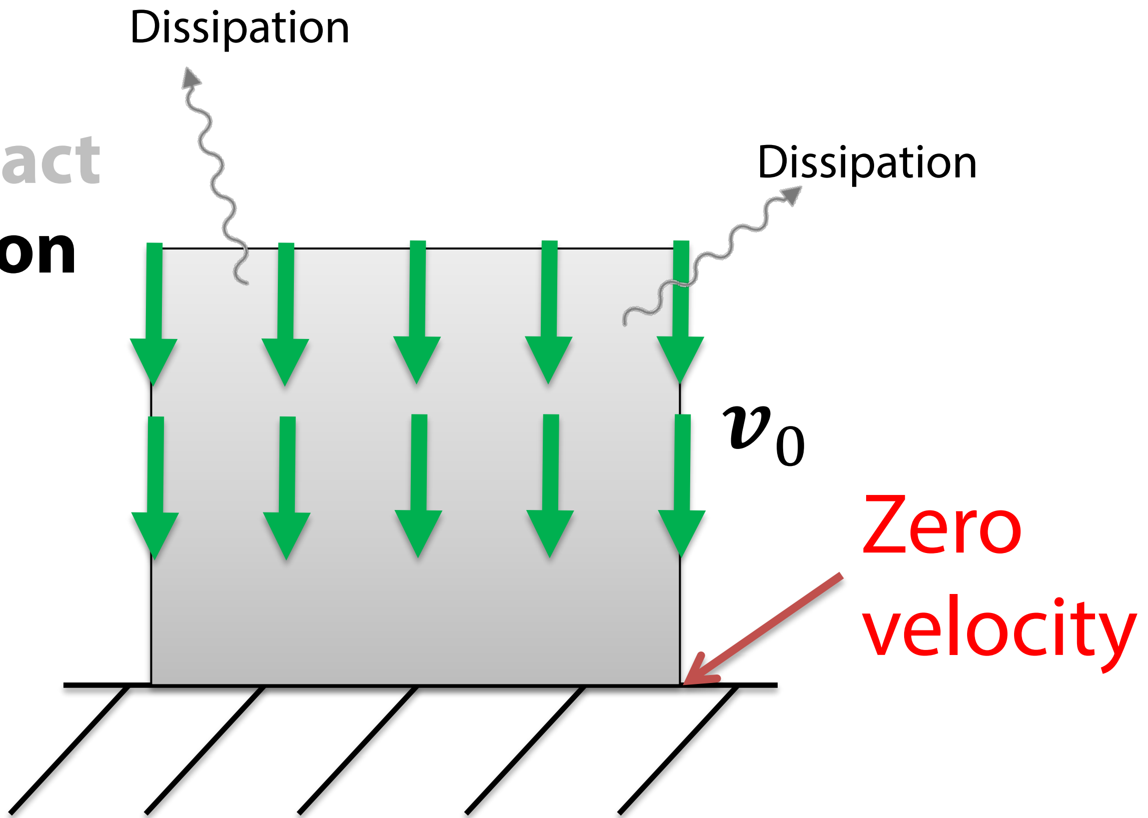


Inelastic Impact of an Elastic Block

Free fall

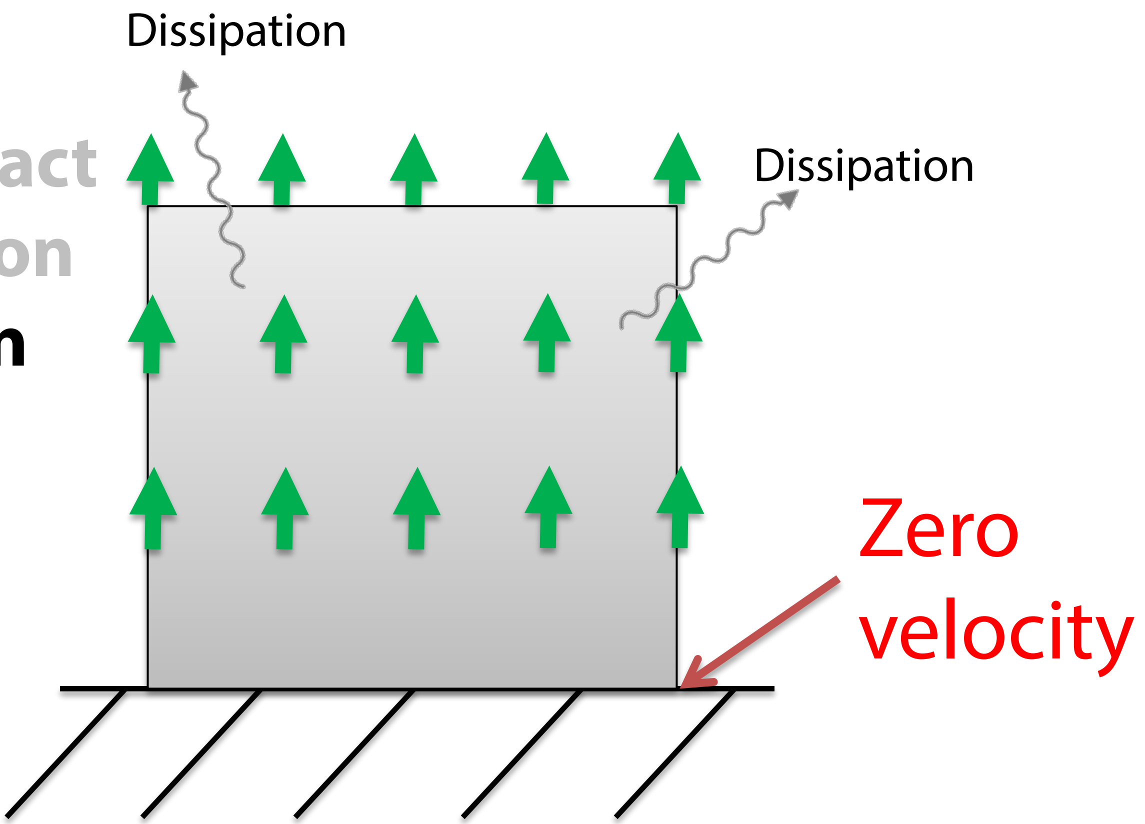
Initial contact

Compression



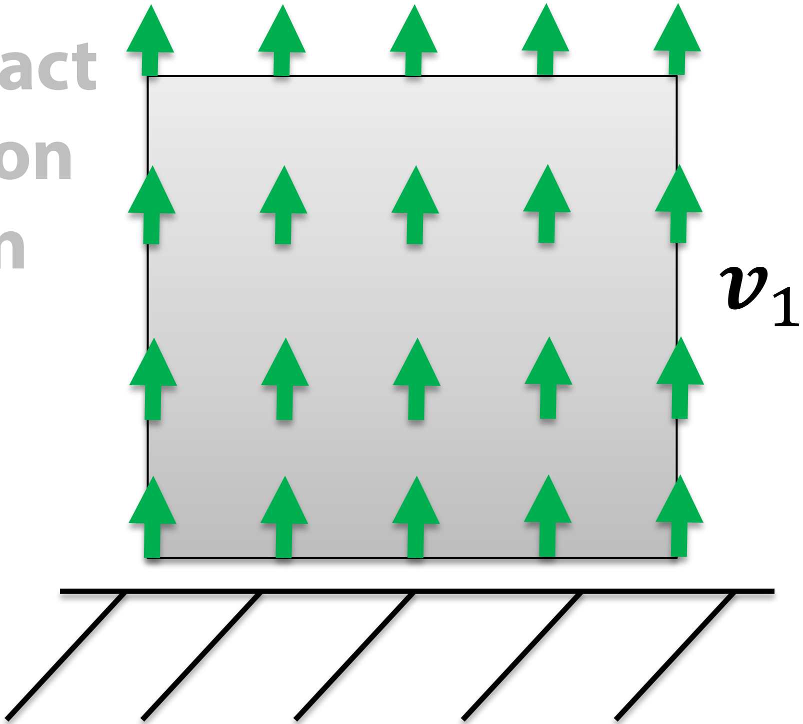
Inelastic Impact of an Elastic Block

Free fall
Initial contact
Compression
Restoration

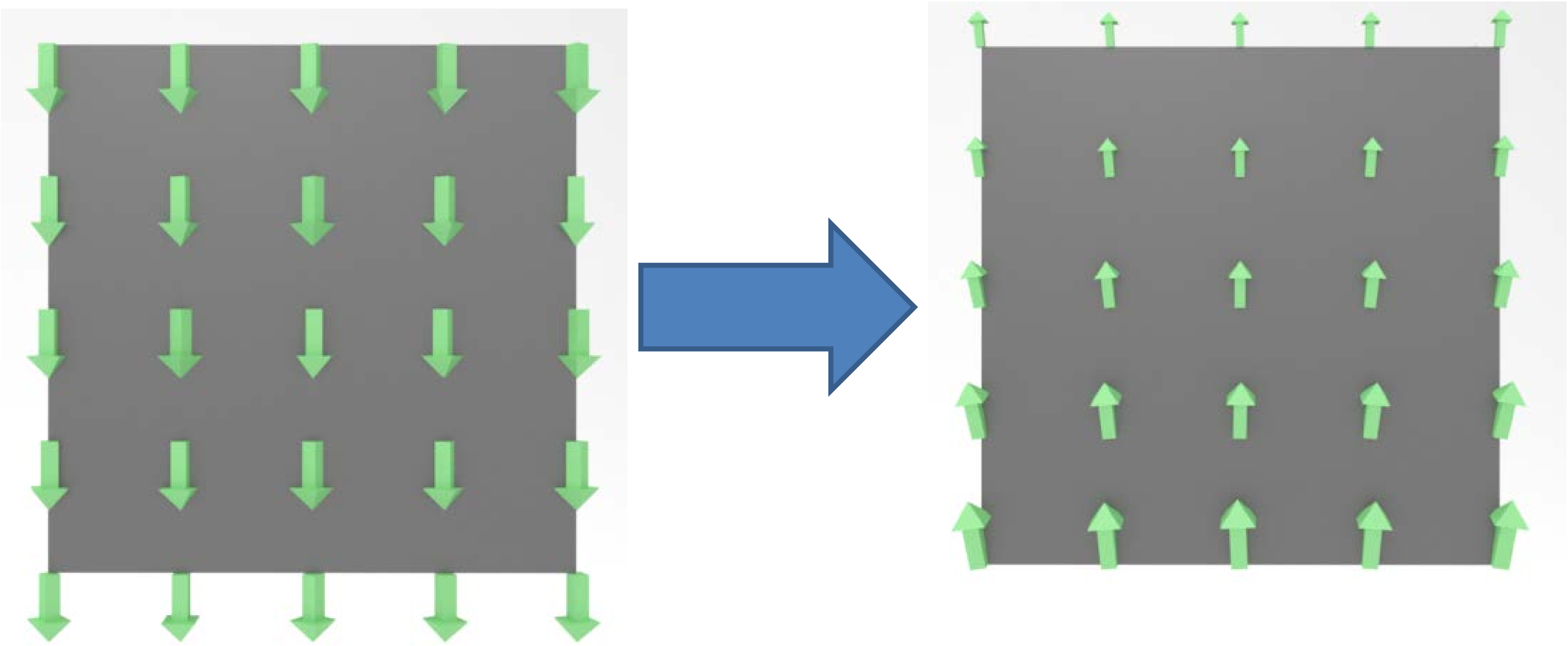


Inelastic Impact of an Elastic Block

Free fall
Initial contact
Compression
Restoration
Rebound

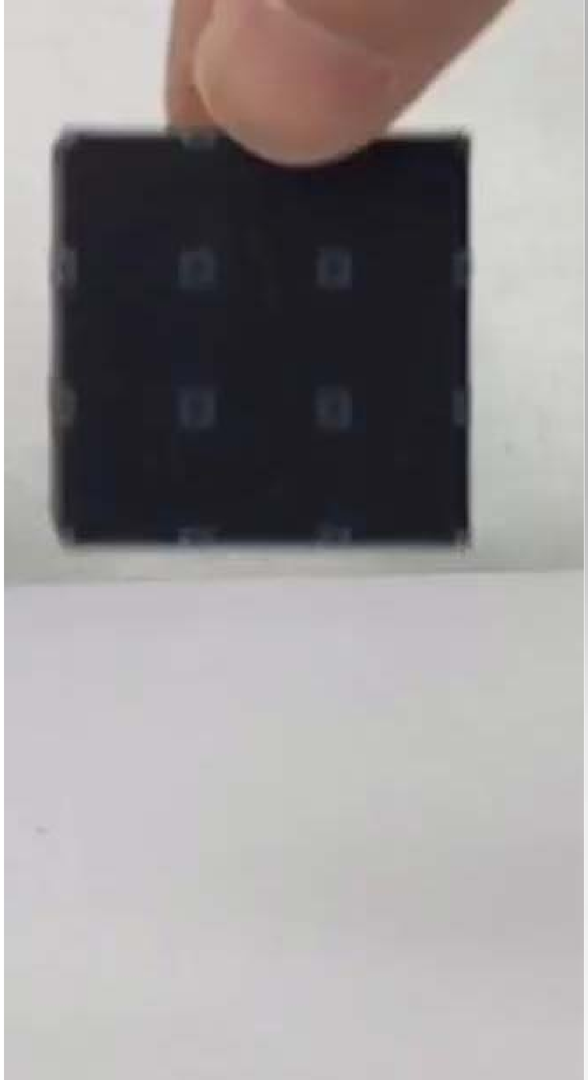


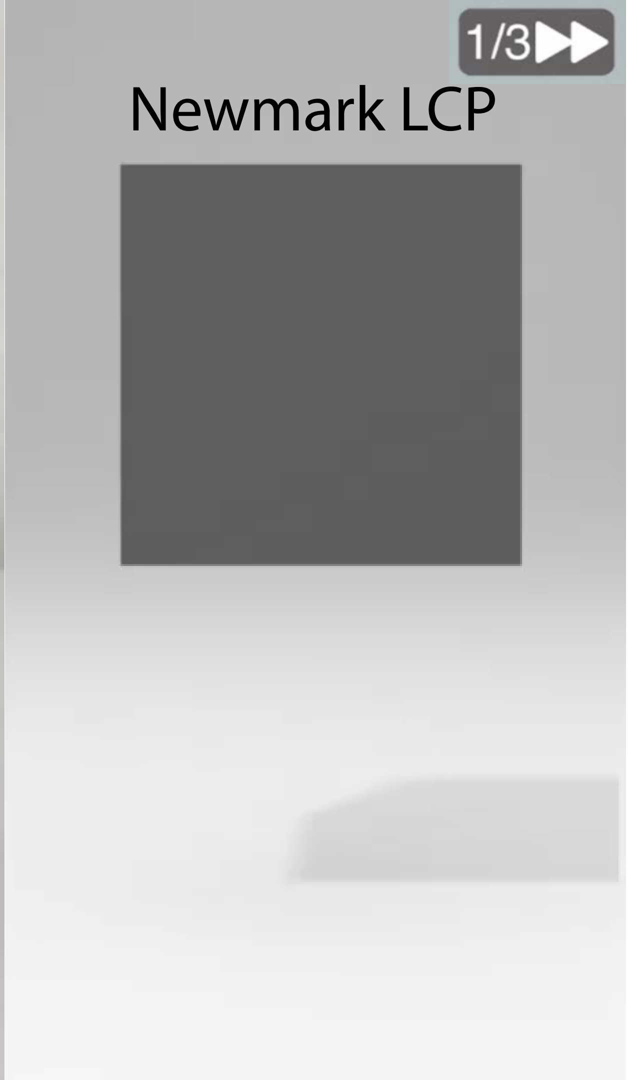
Method – Newmark LCP Contact Model



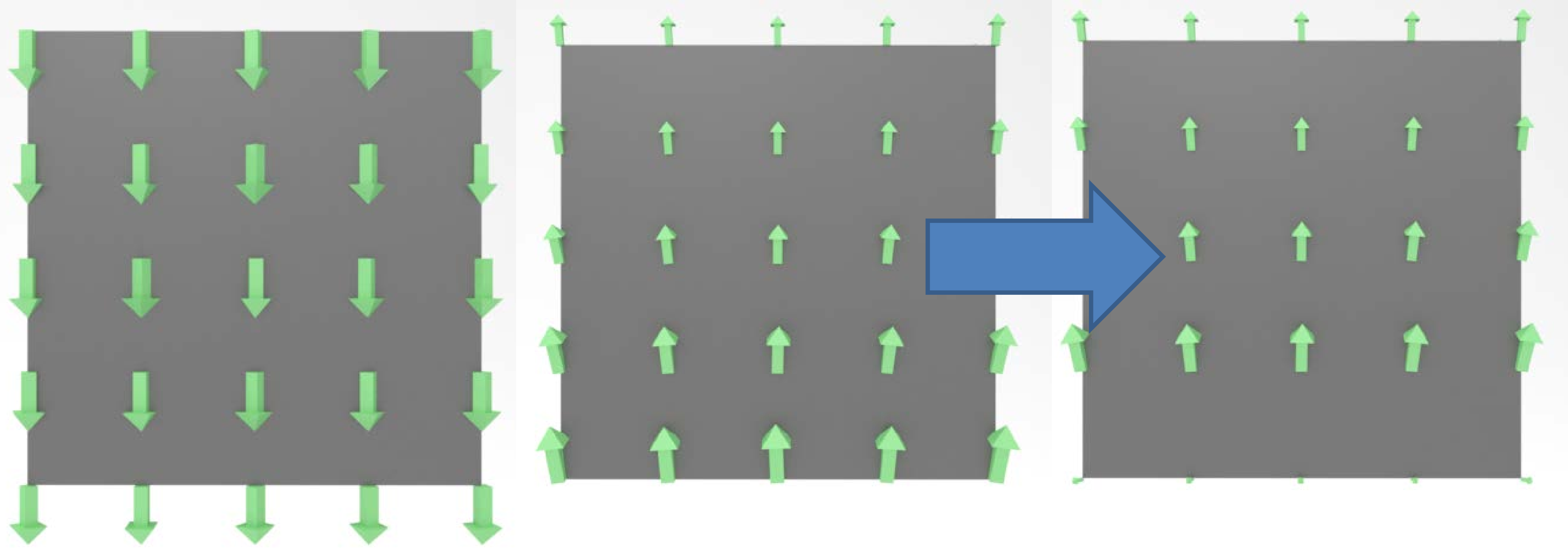
Velocity field before impact

Newmark LCP




1/3
Newmark LCP

Method – DKE Projection

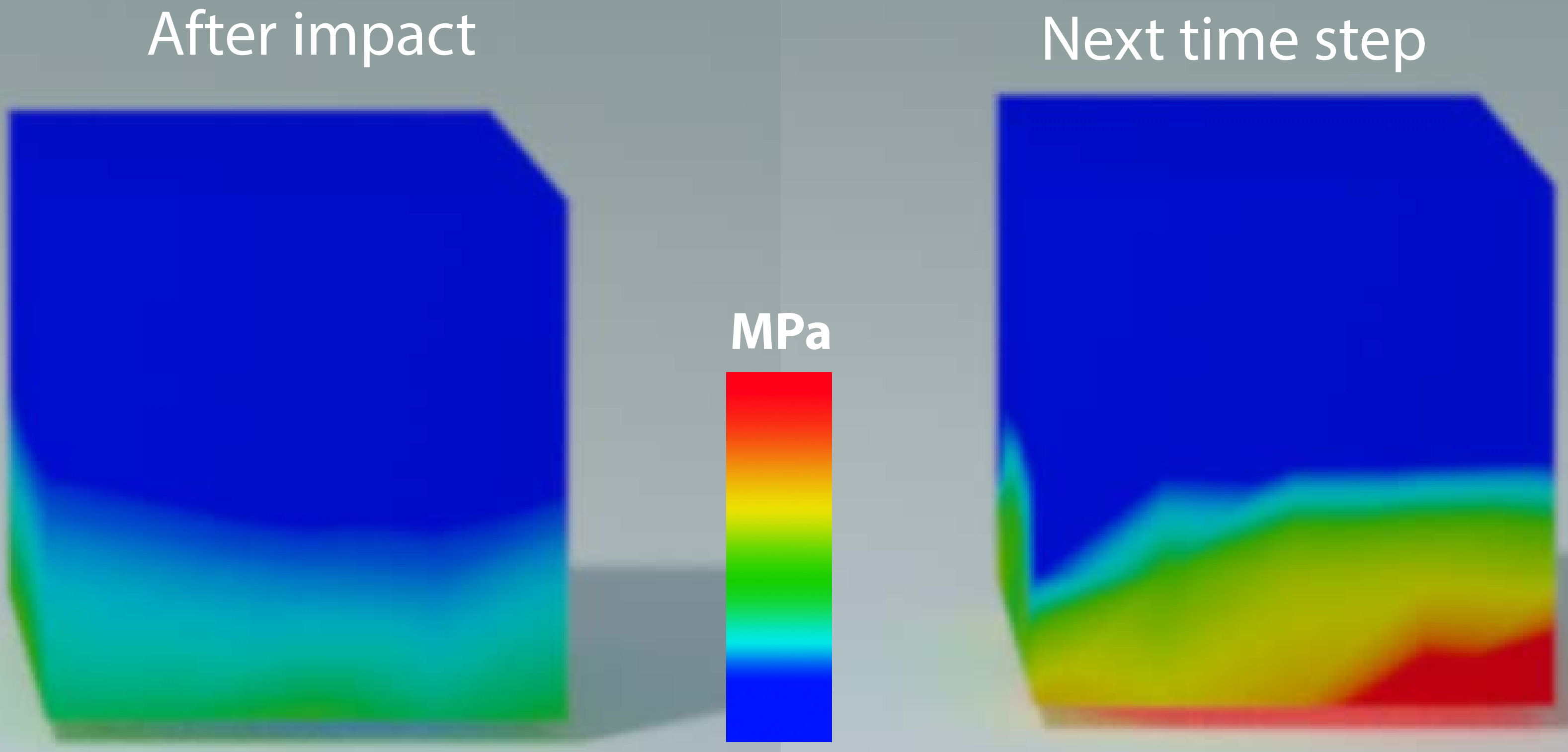


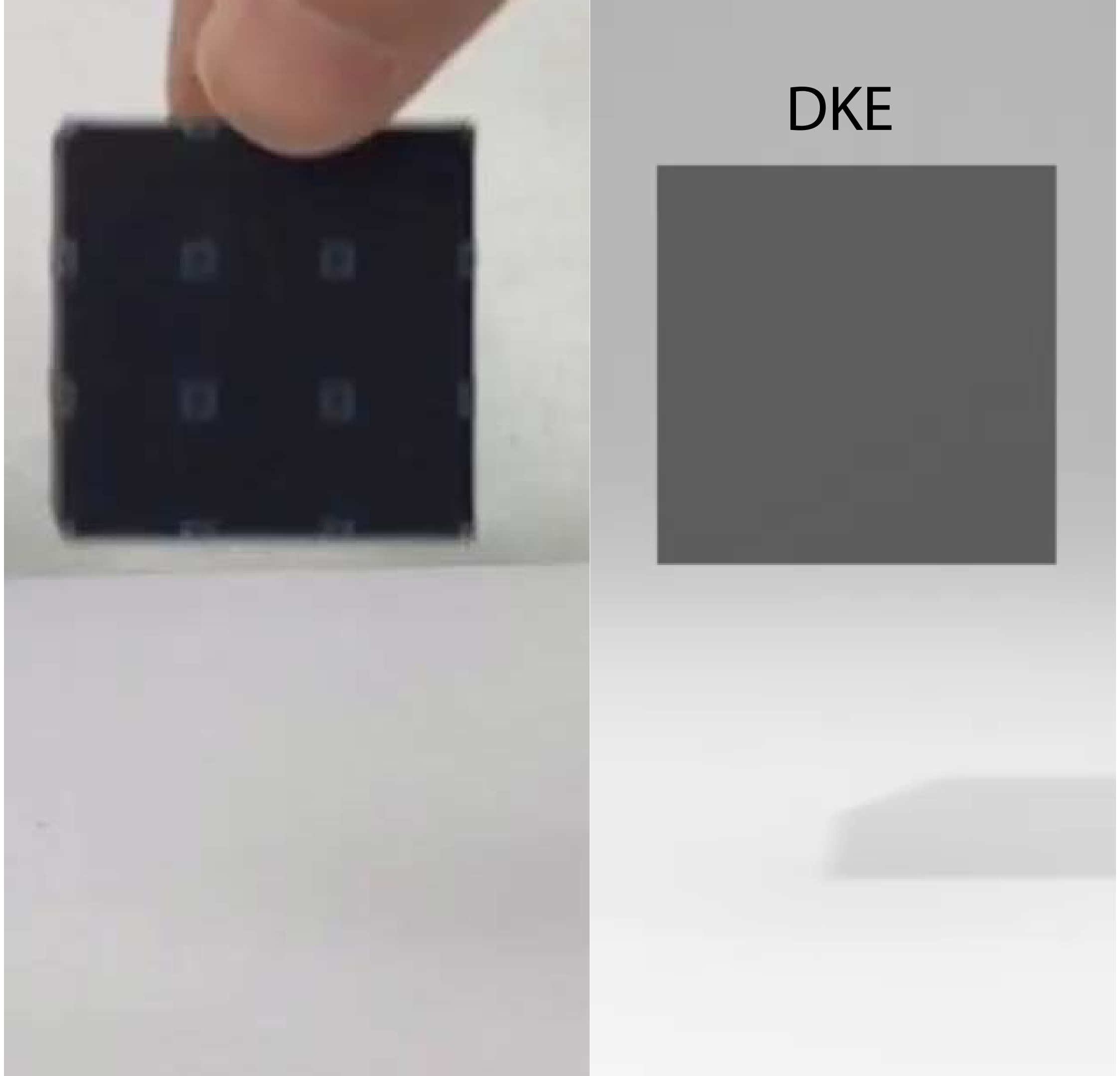
Velocity field before impact

Newmark LCP

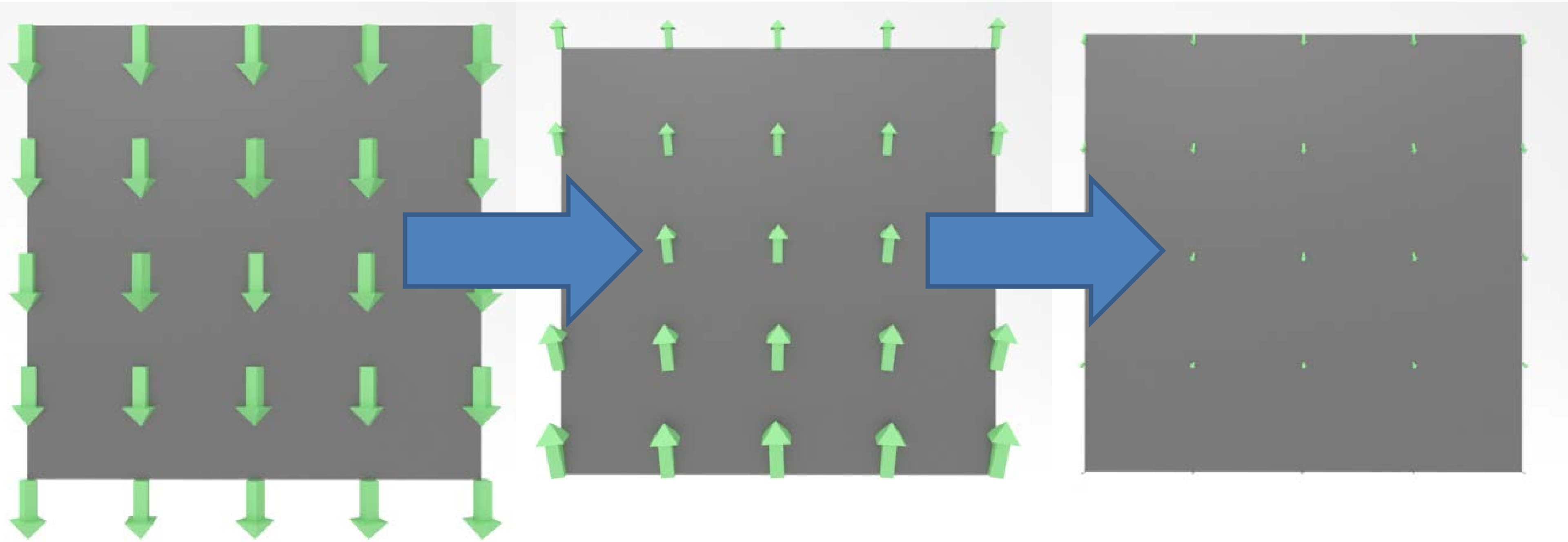
DKE projection

DKE Stress Distribution




DKE

Method – BBI

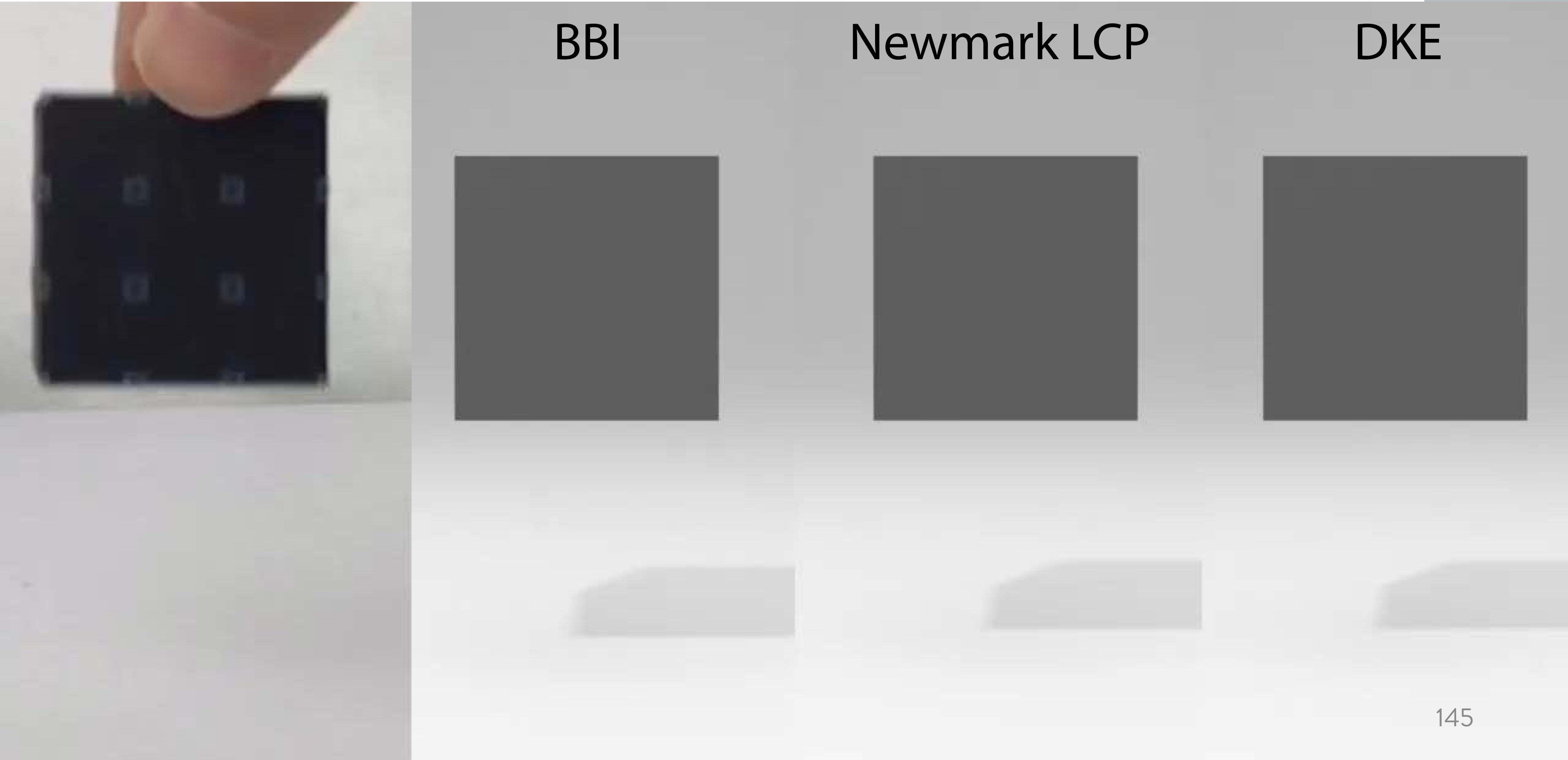


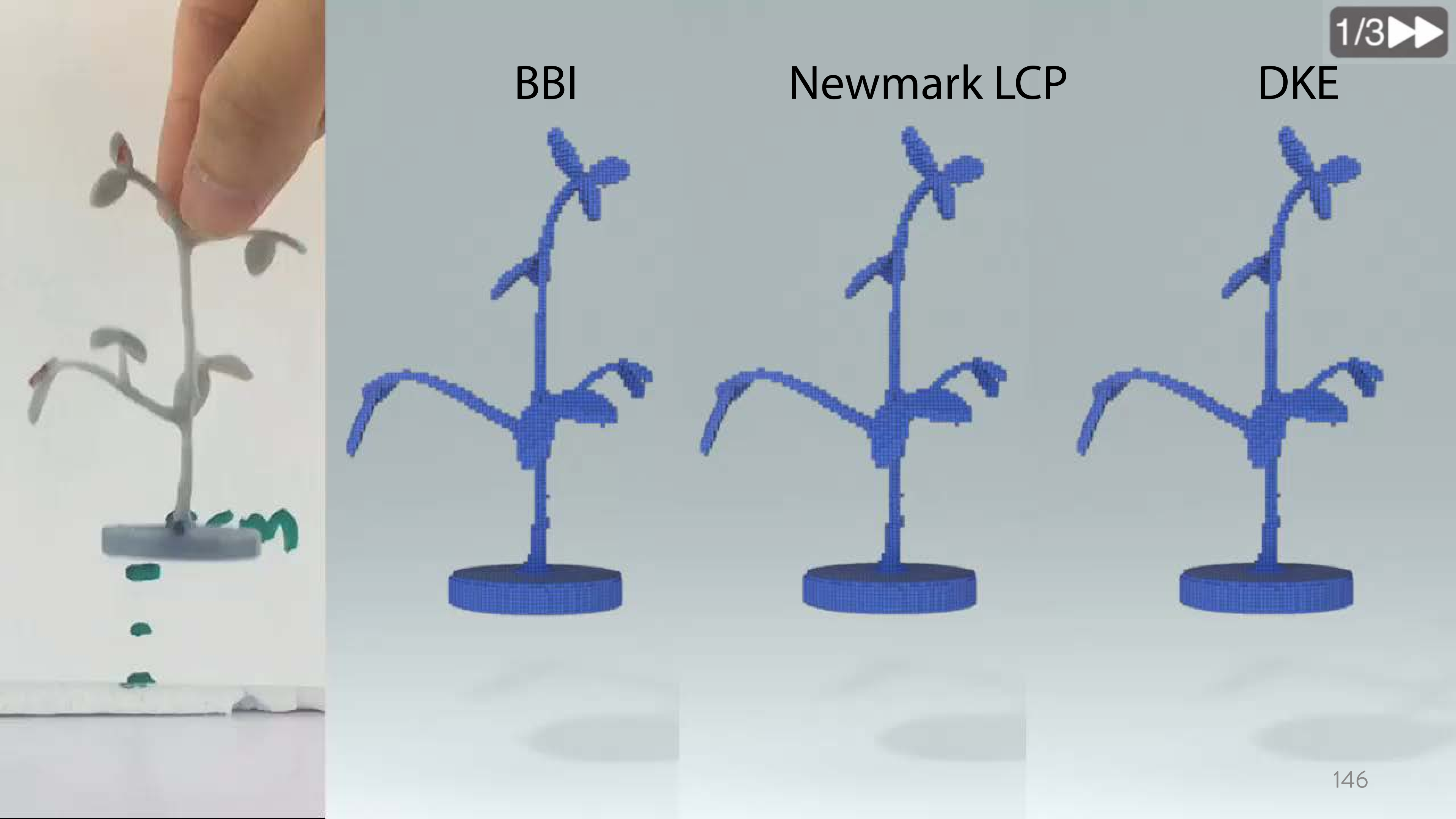
Velocity field before impact

LCP solution

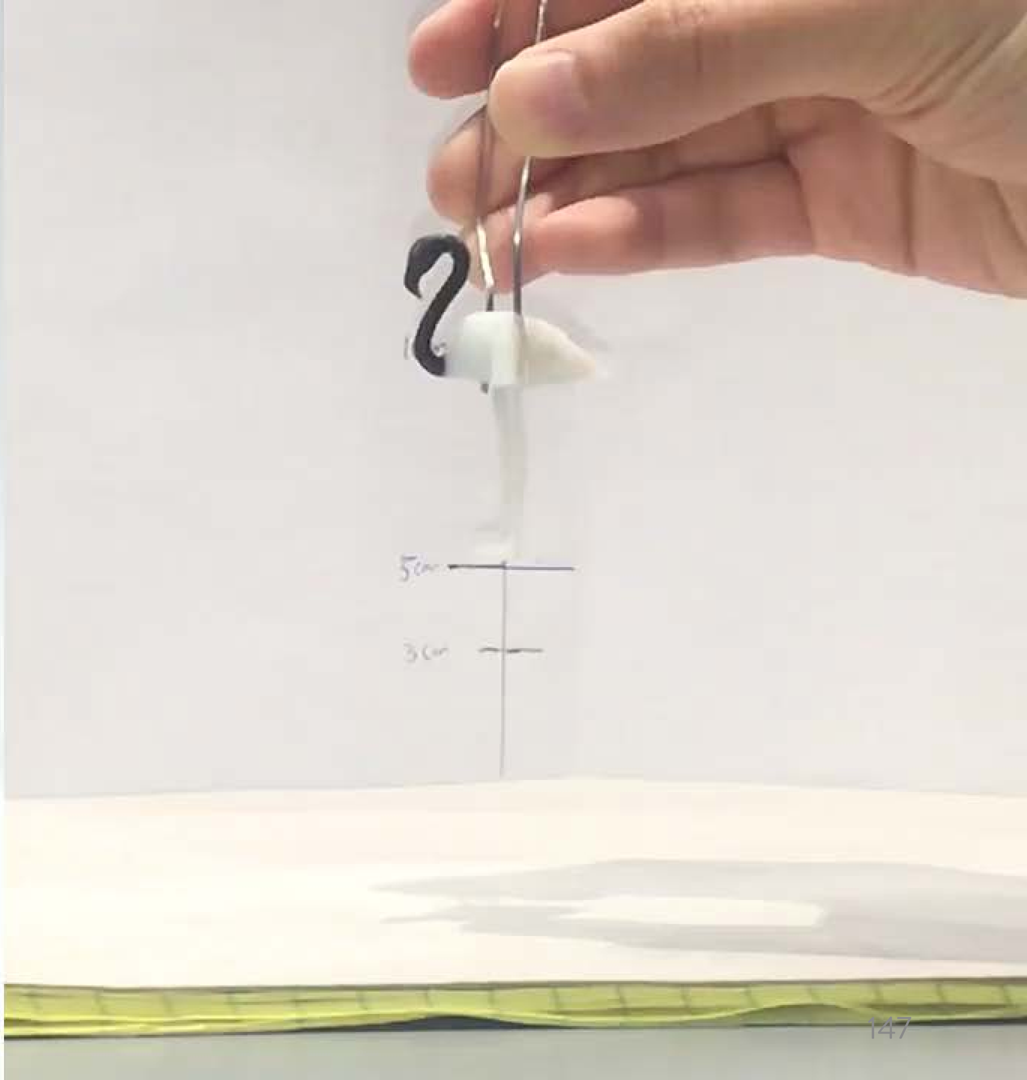
BBI projection

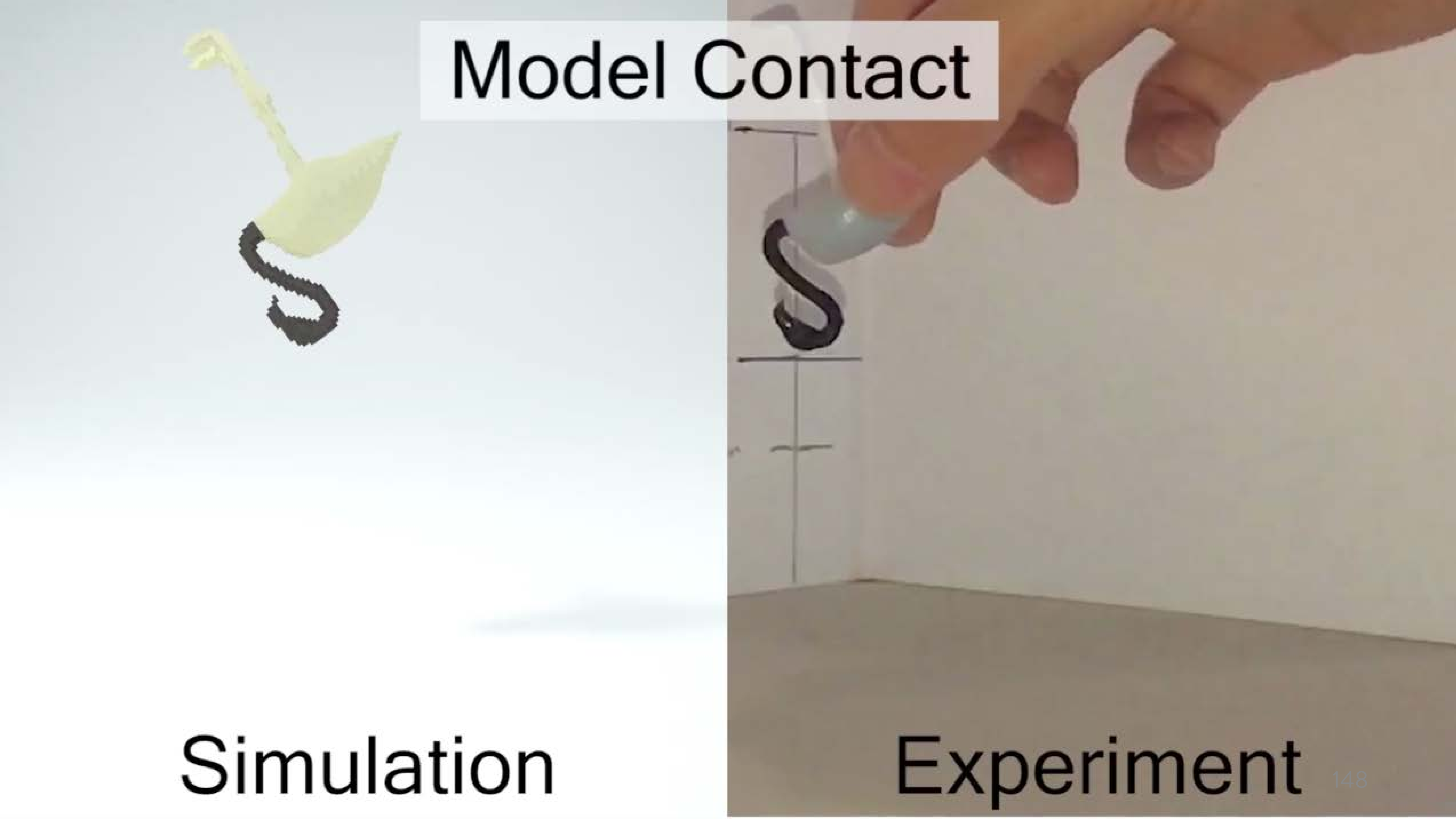
BBI Validation




1/3
BBI
Newmark LCP
DKE

BBI Validation

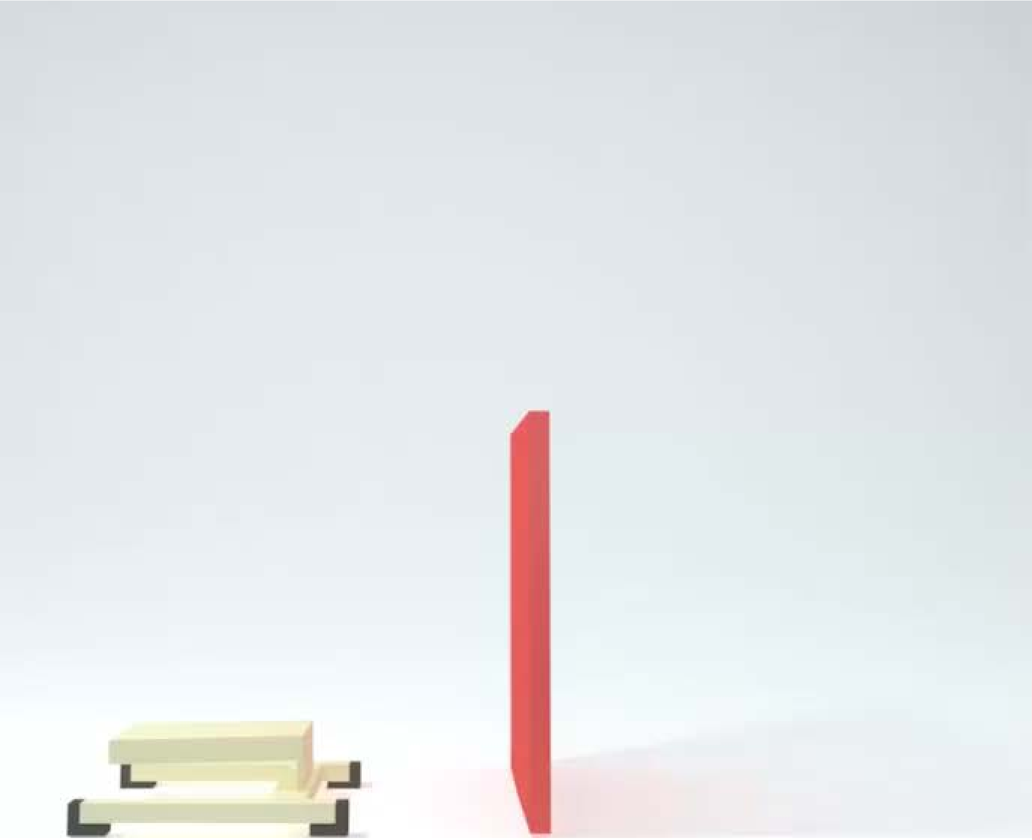



Model Contact
Simulation
Experiment

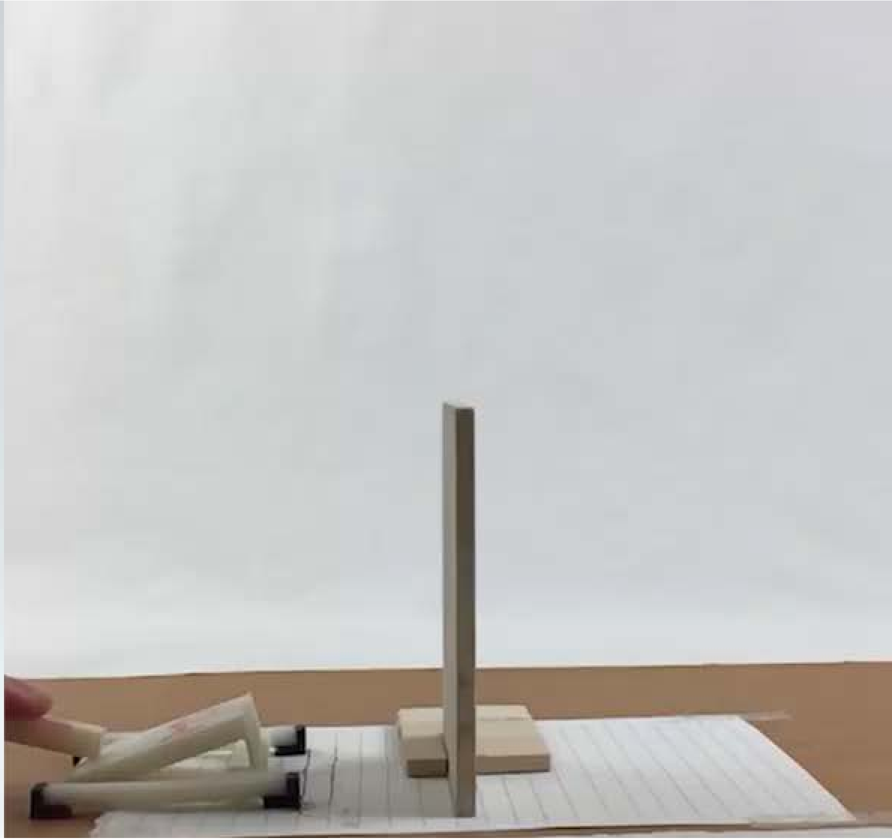
BBI

Design Optimization

Optimizing Dynamic Mechanisms

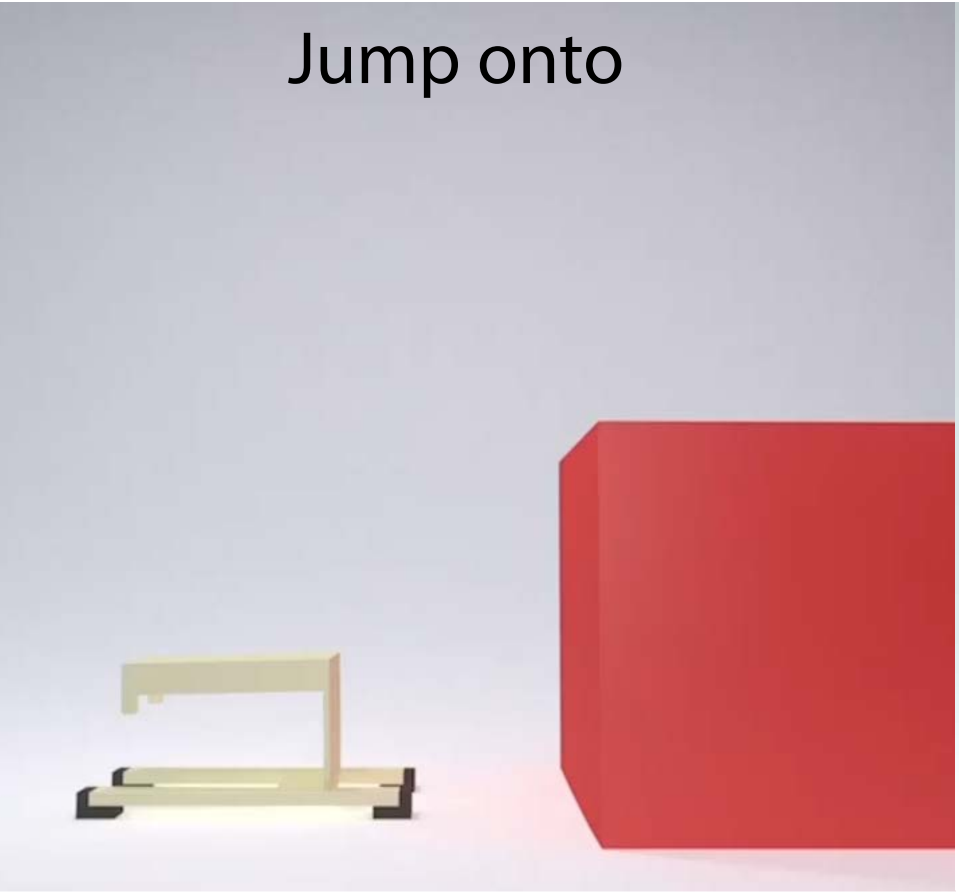


Simulation



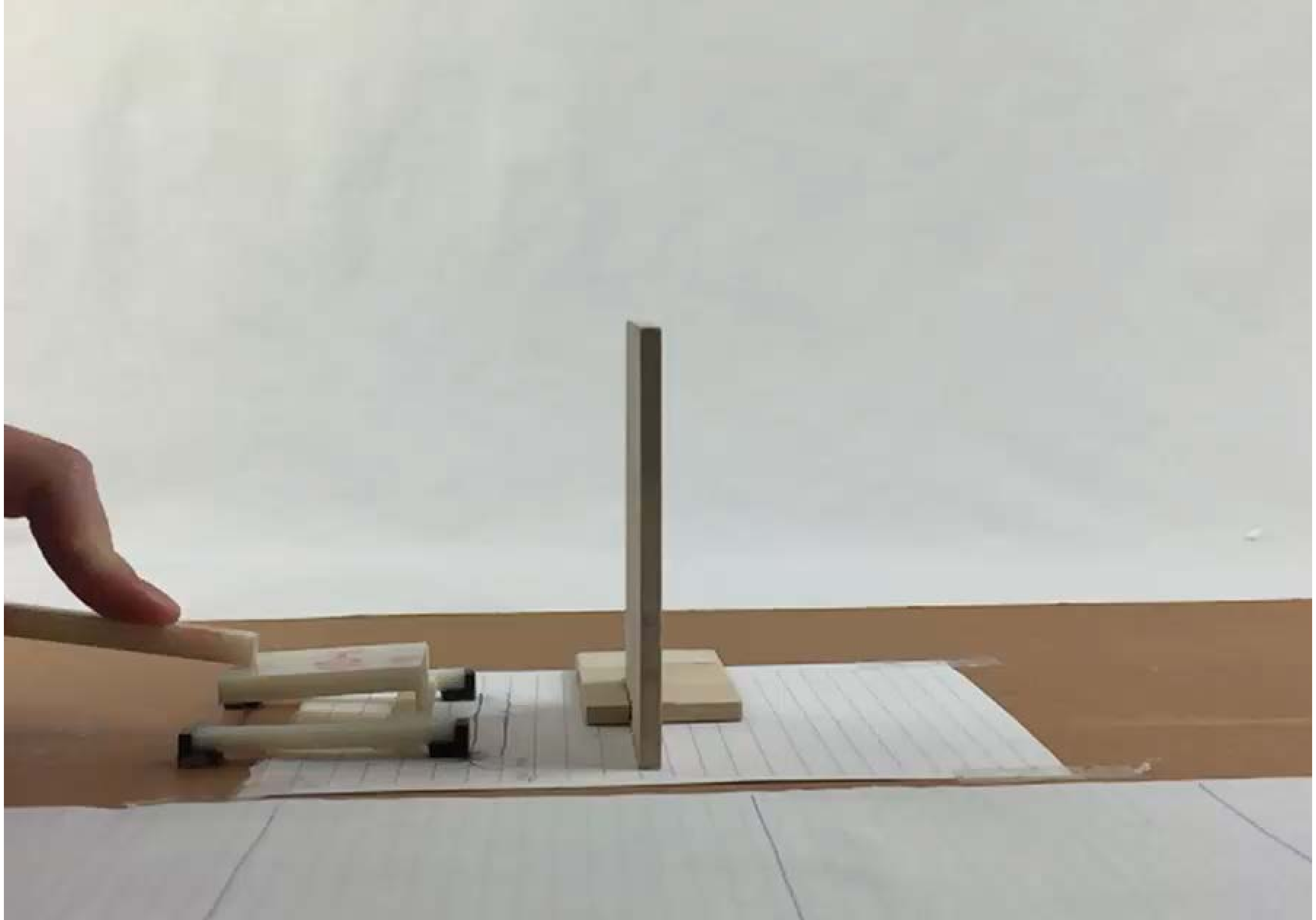
Experiment

Jumper Tasks

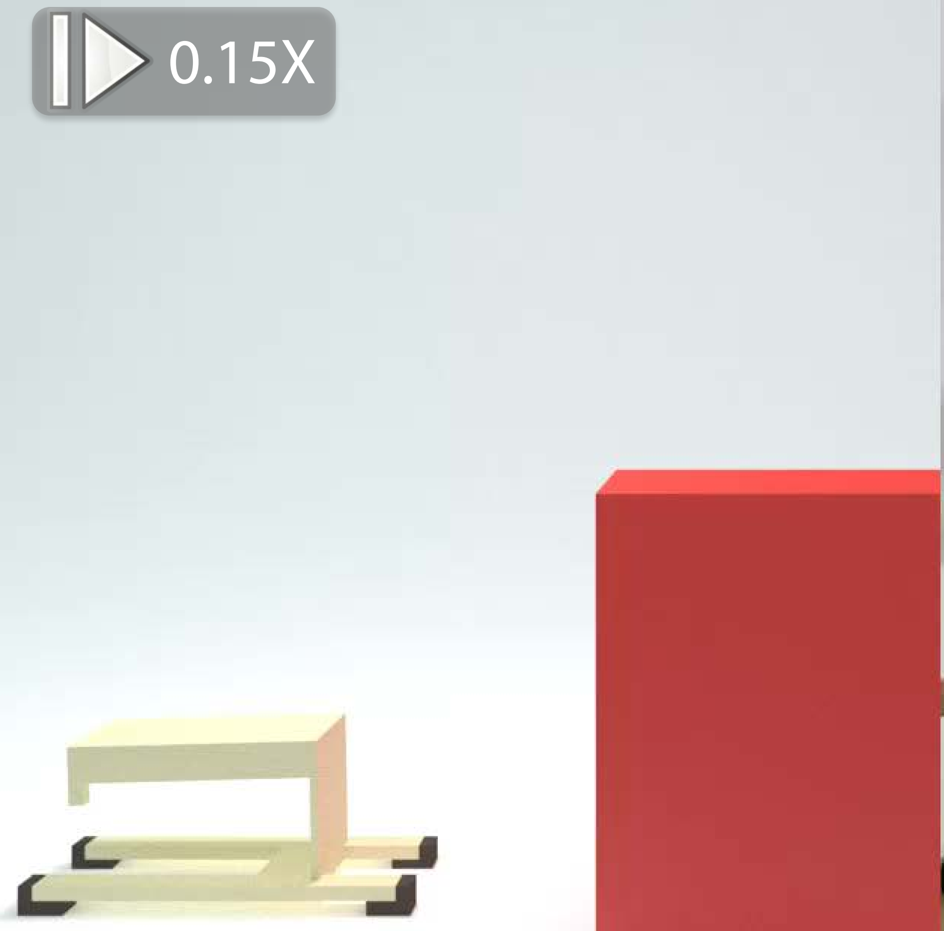


Unsuccessful Starting Jumpers

Unsuccessful Starting Jumpers

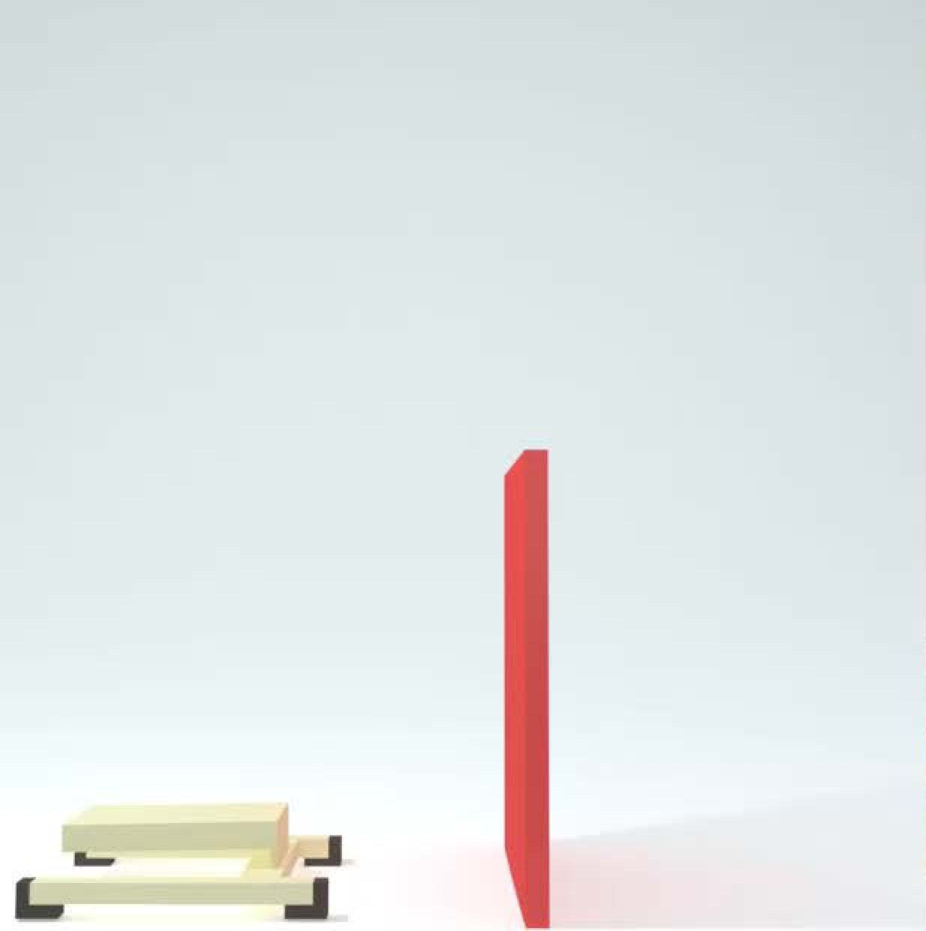


Our Simulation

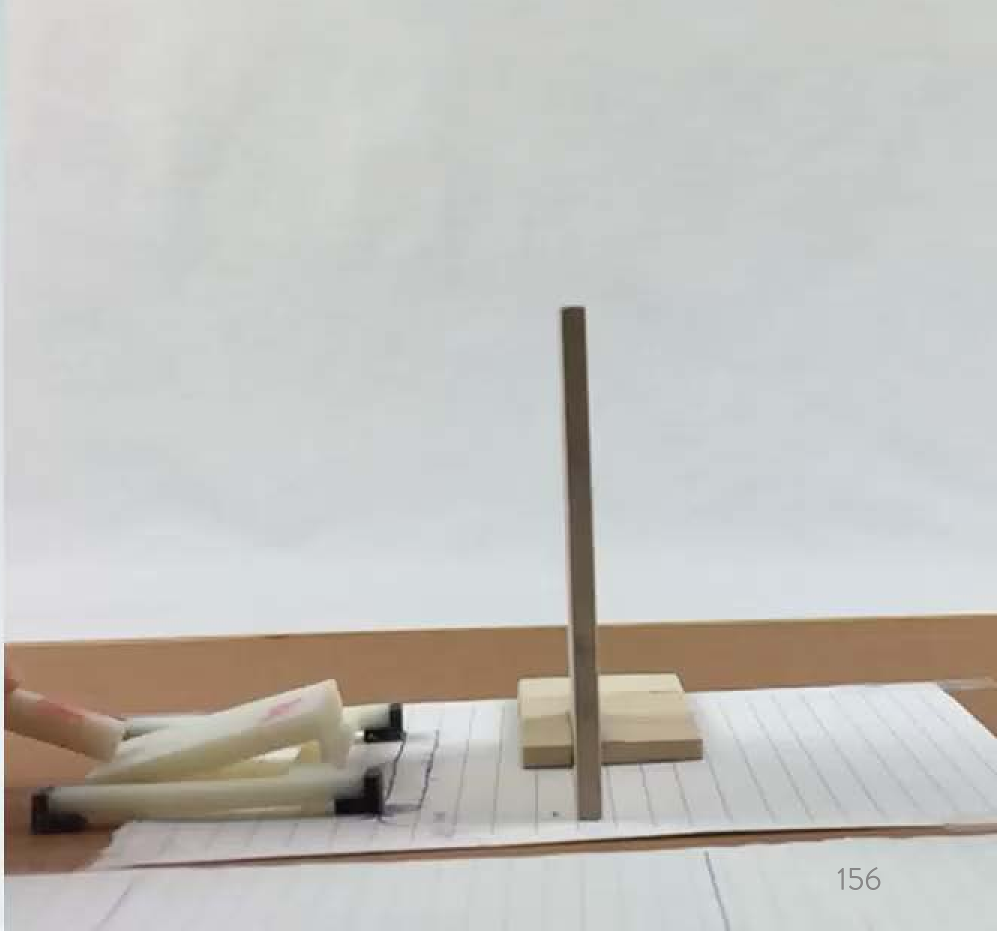


Starting Jumper

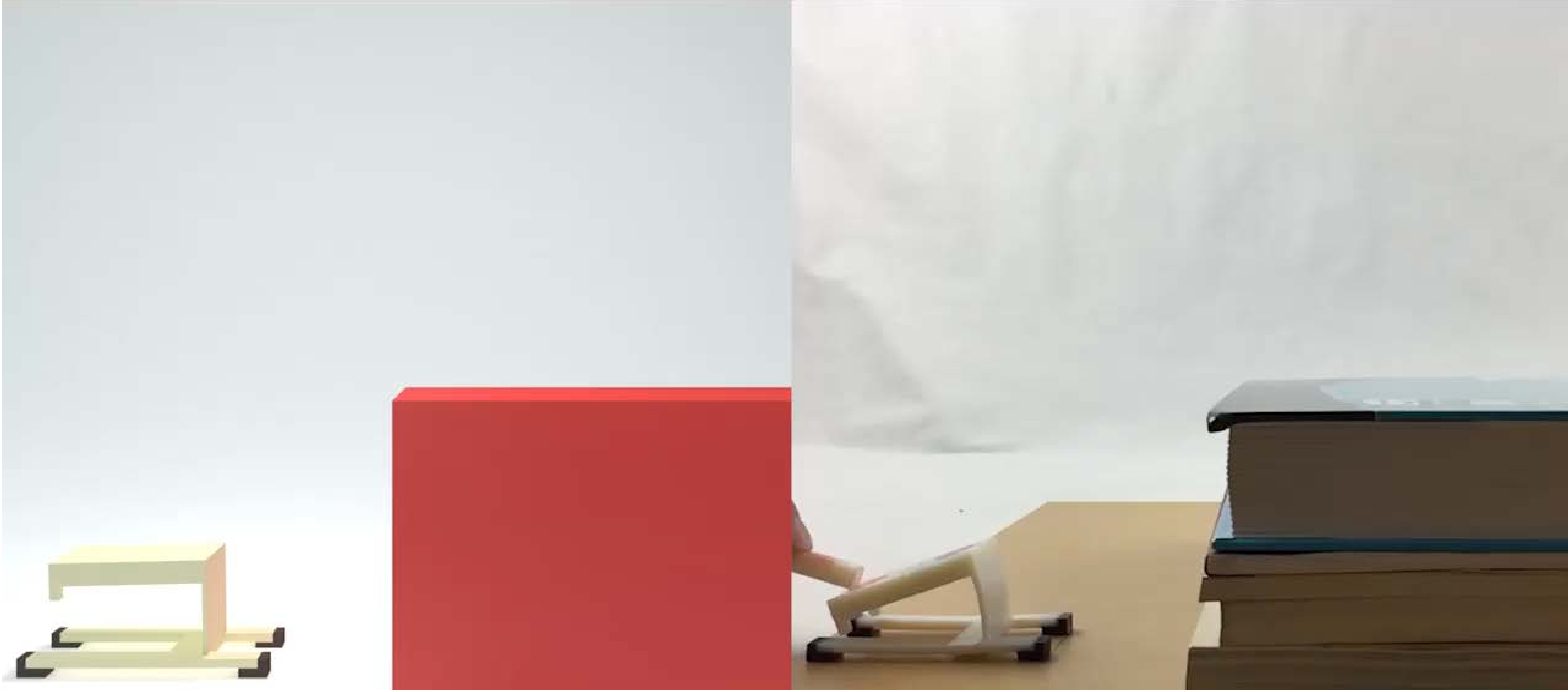
Our Simulation



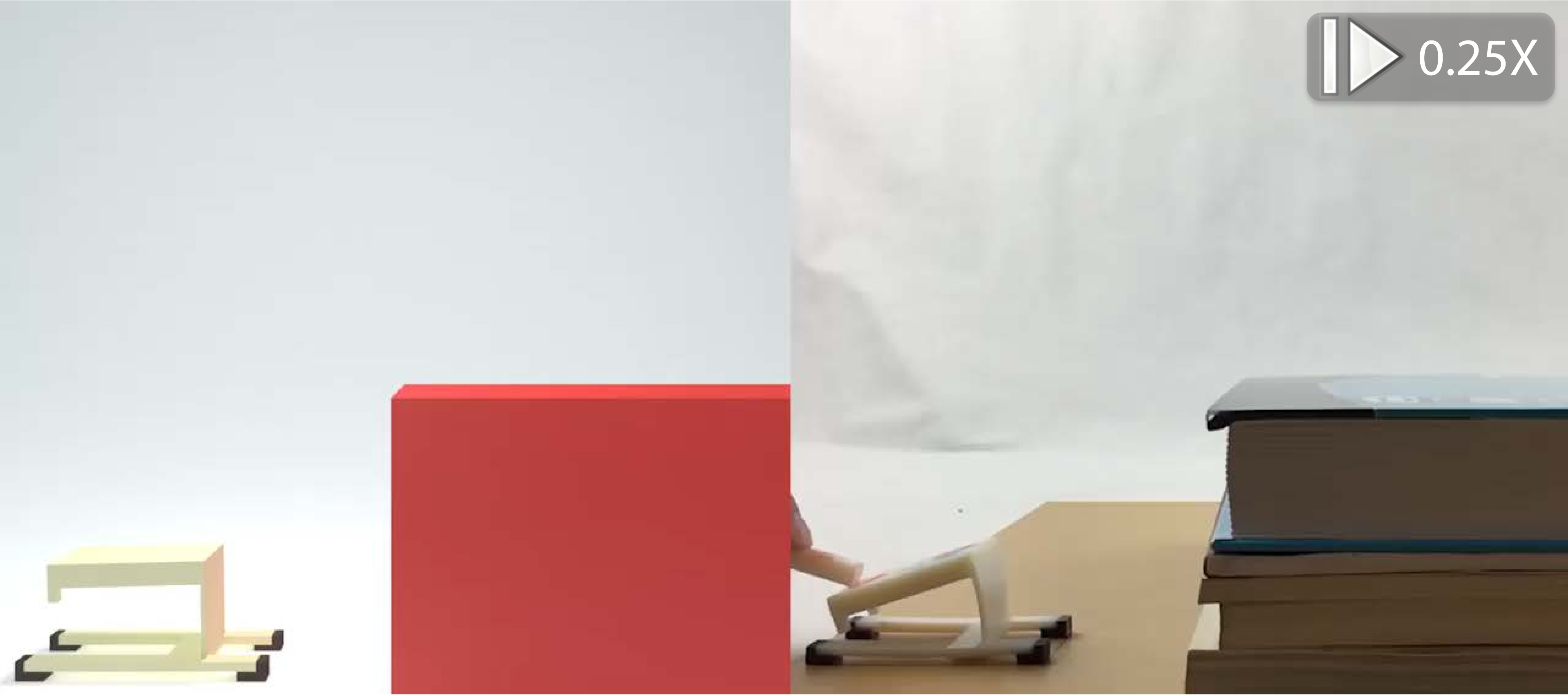
Starting Jumper



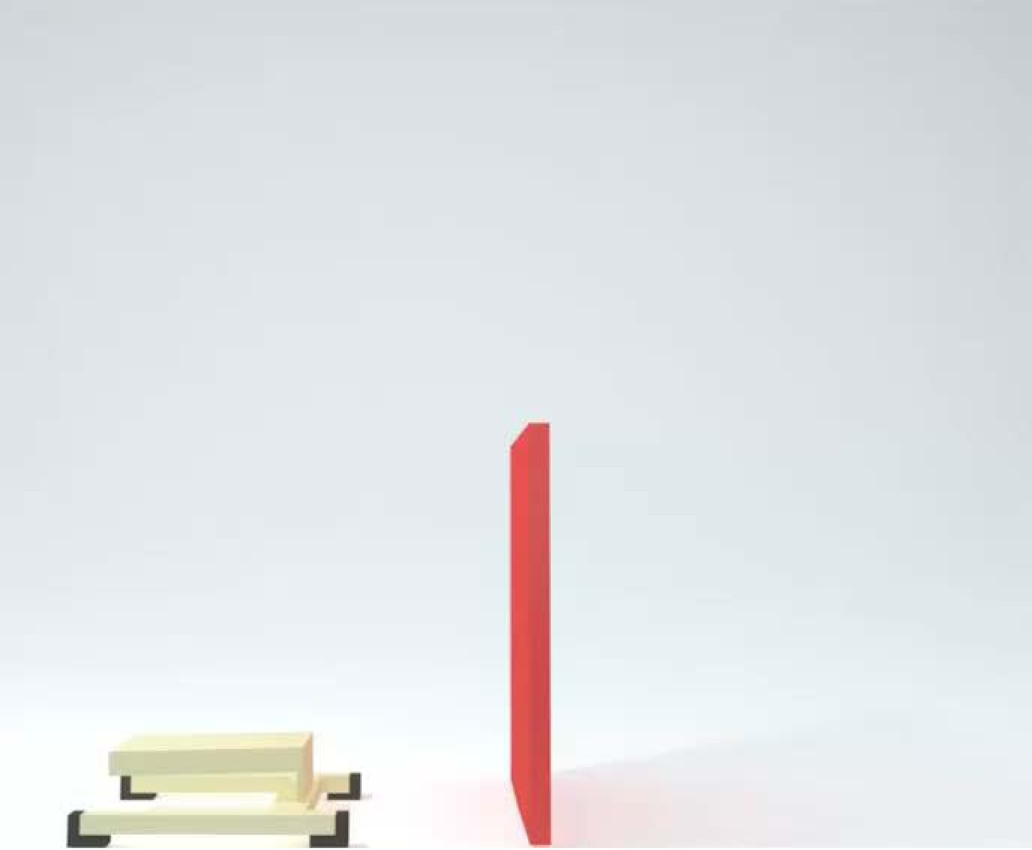
Optimized Designs

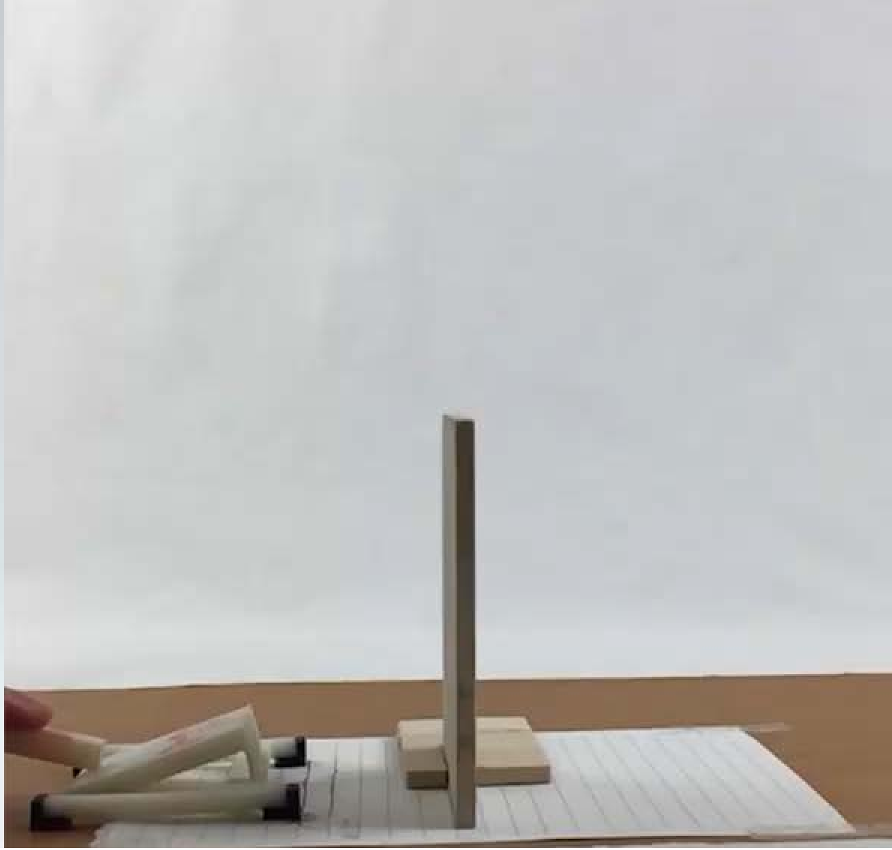


Optimized Designs

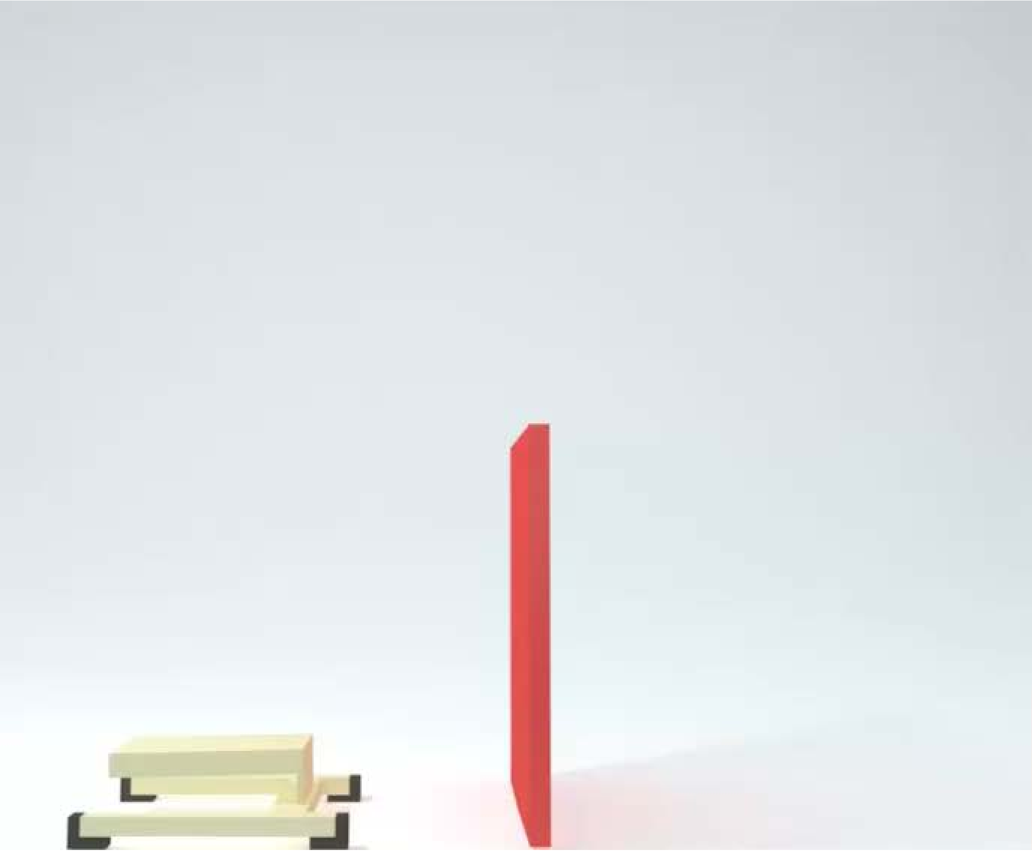


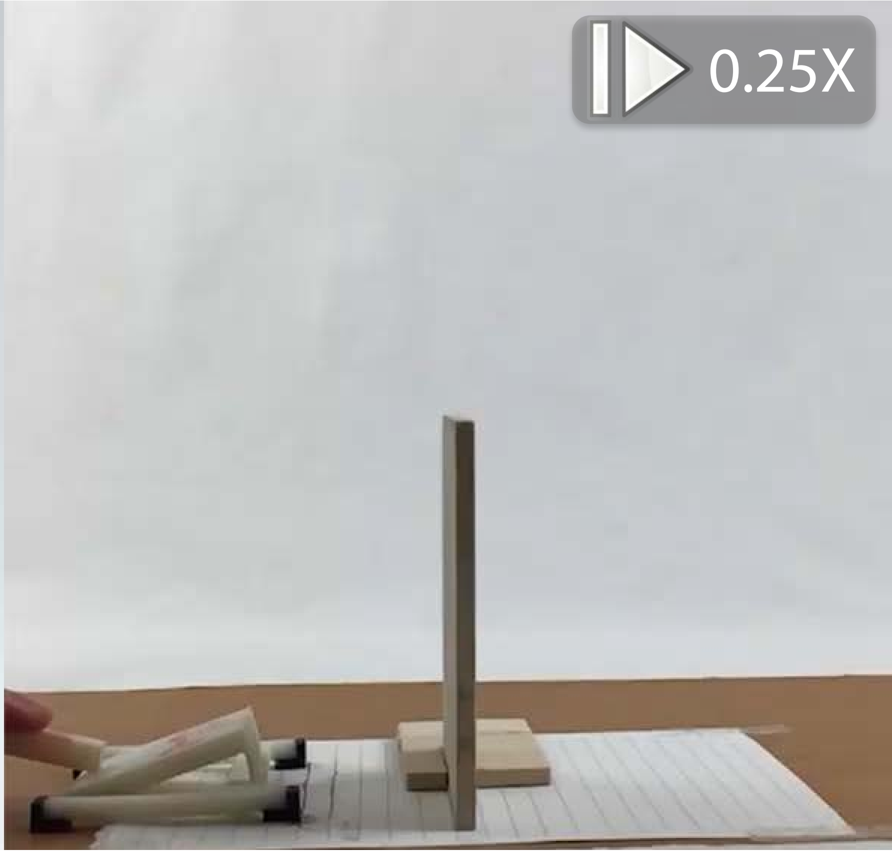
Optimized Designs





Optimized Designs





Conclusion

- Uncertainty
- Materials
- Design optimization

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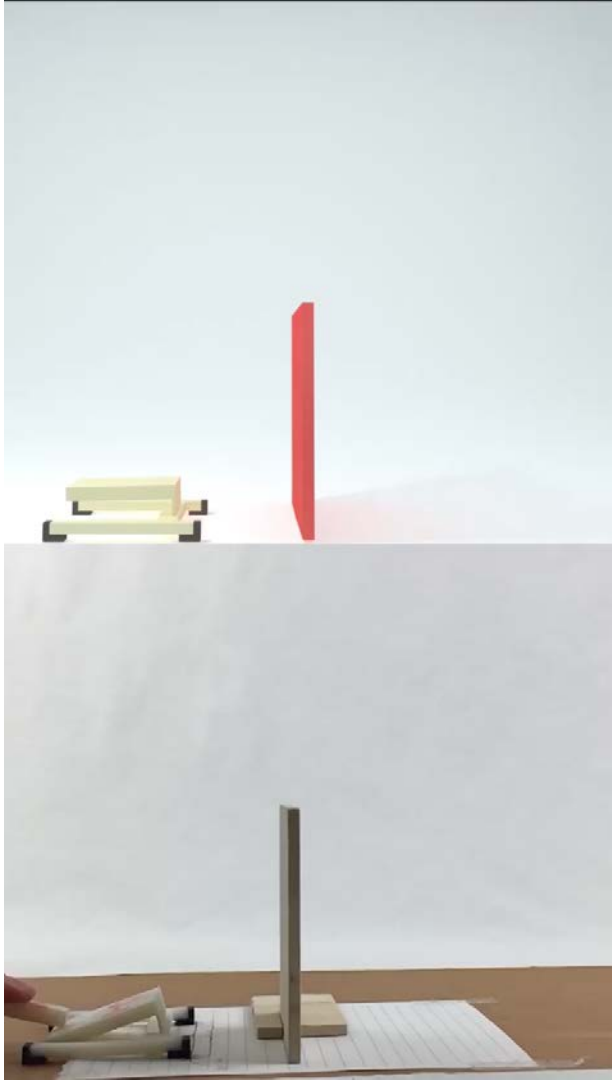

Thank you!