面向等几何分析的计算域参数化

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Outline

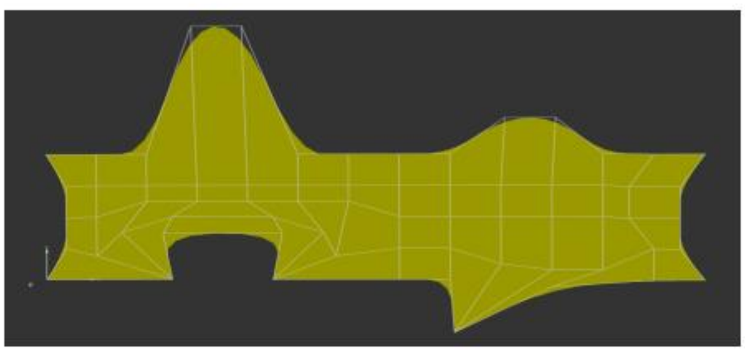
- Introduction
- Optimal parameterization in isogeometric analysis
- Construction of analysis-suitable parameterization
- Conclusion and future work

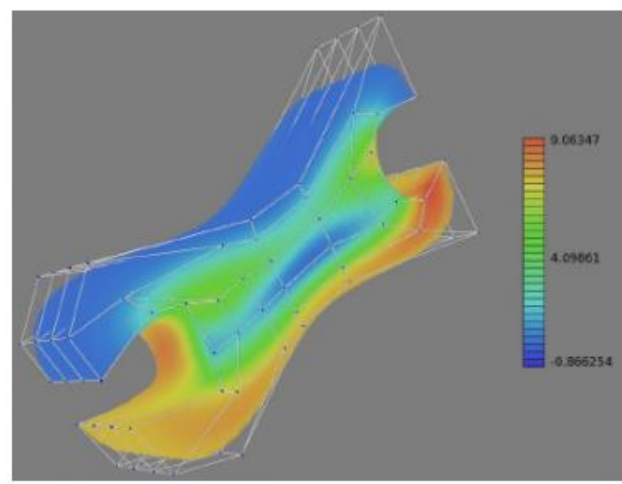
Isogeometric analysis (等几何分析)

- IGA is an isoparametric, exact geometry approach, which is recently providing very promising results as an alternative to FEA.
- proposed by Prof. T. Hughes et al. from University of Texas at Austin in 2005
- motivation:
  - seamless integration of CAD and CAE.
  - avoid geometry approximations of mesh generation in FEA
  - high regularity and refinement of B-spline functions.
- basic idea: use the same standard mathematical representation as in CAD systems (such as NURBS) for both the geometry and the solution field (such as thermal conduction).

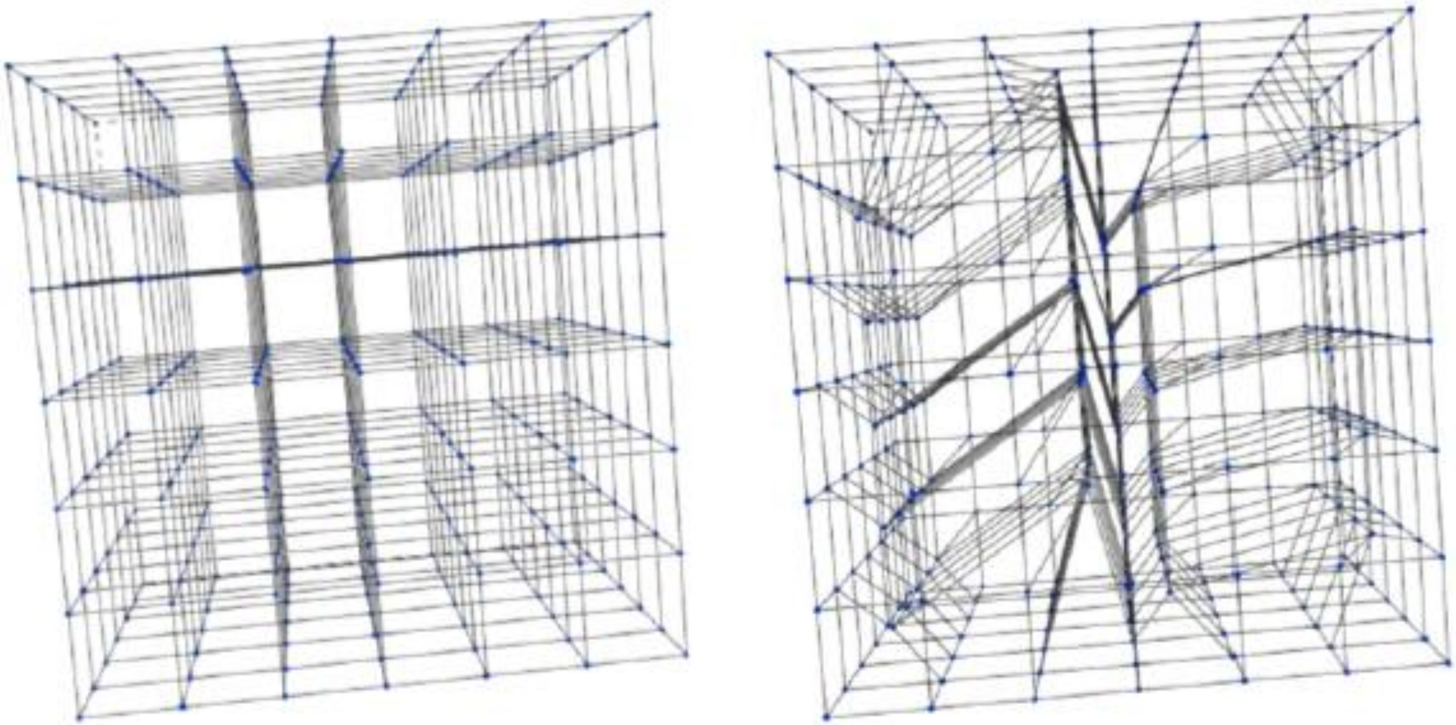
Representation in IGA

- computational domain:
  - 2D: planar B-spline surface
  - 3D: B-spline volume
- solution field :
  - 2D: B-spline surface with 3D control points
  - 3D: B-spline volume with 4D control points



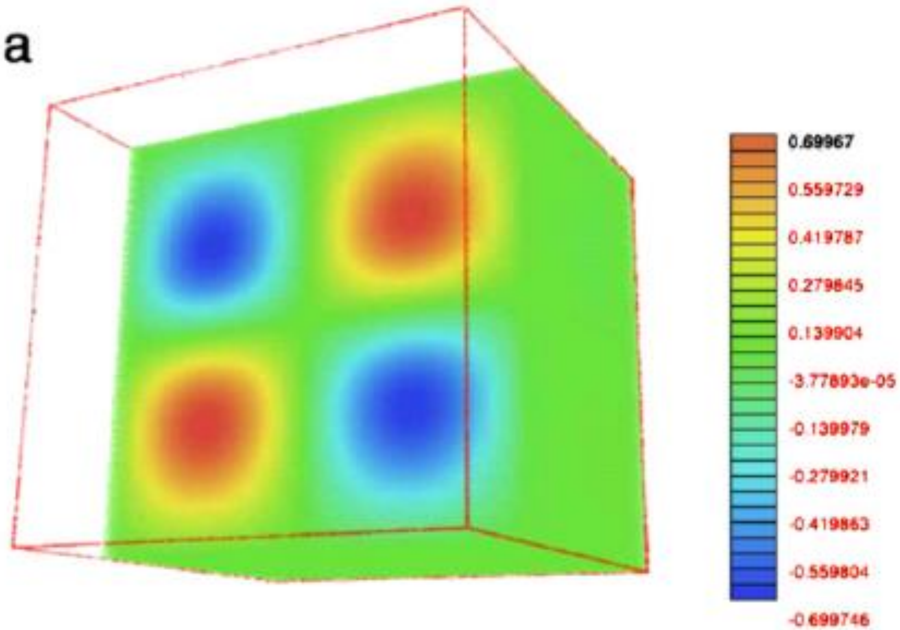


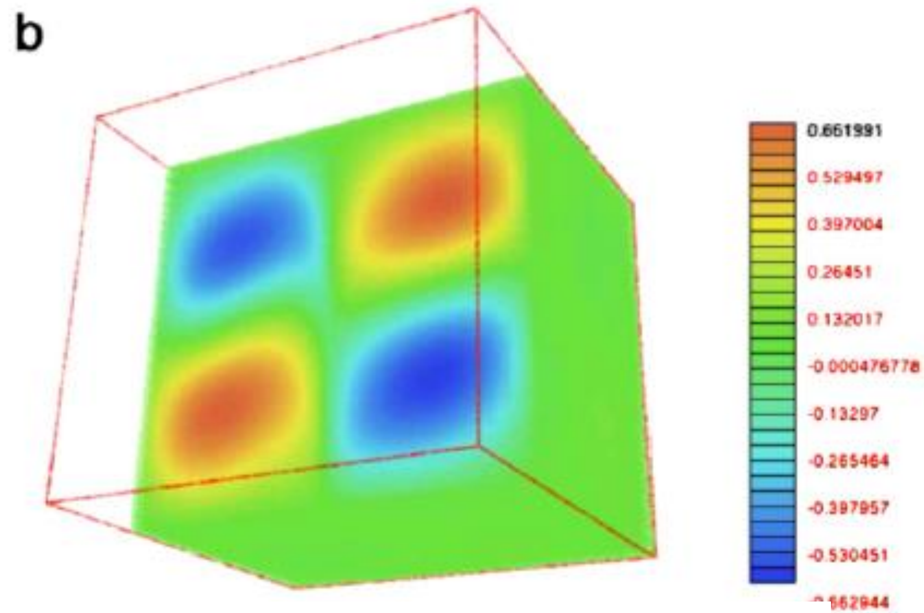
计算域参数化质量对分析结果影响 CAD2013

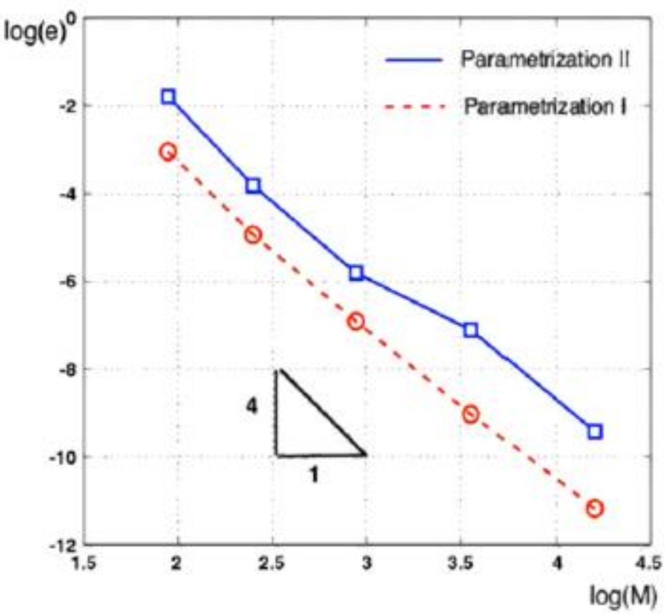


(a) Control point placement I.   (b) Control point placement II.

计算域参数化质量对分析结果影响 CAD2013

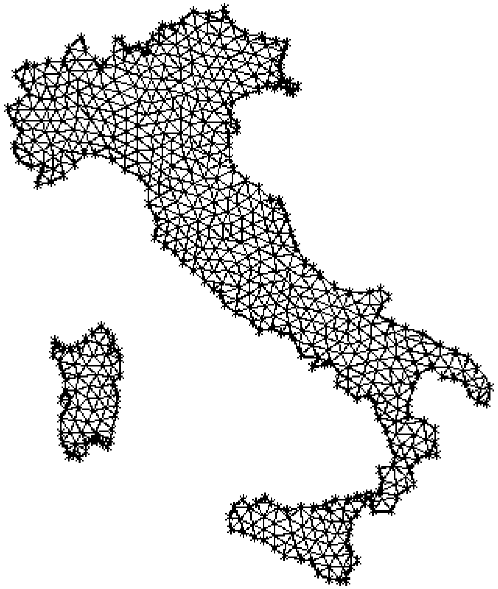






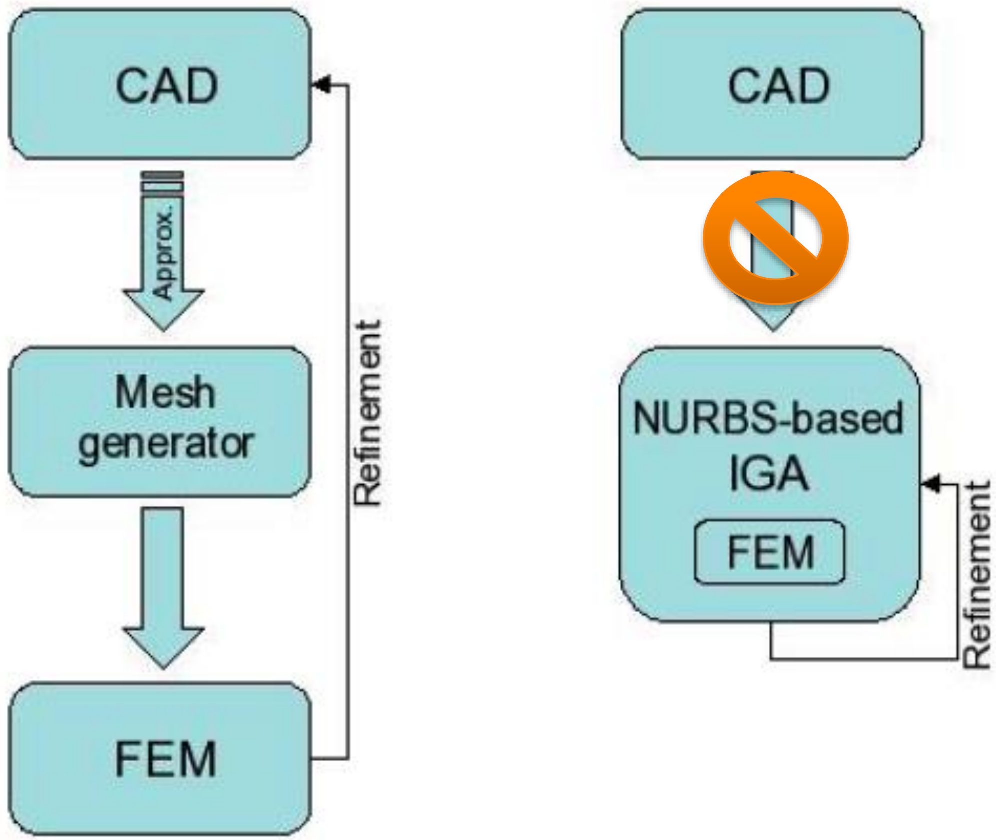
IGA-meshing

- IGA is a spline-version of FEA
- Mesh generation in FEA
- CAD models usually define only the boundary of a solid, but the application of isogeometric analysis requires a volumetric representation
- As it is pointed by Cotrell et al., the most significant challenge facing isogeometric analysis is developing three- dimensional spline parameterizations from boundary information



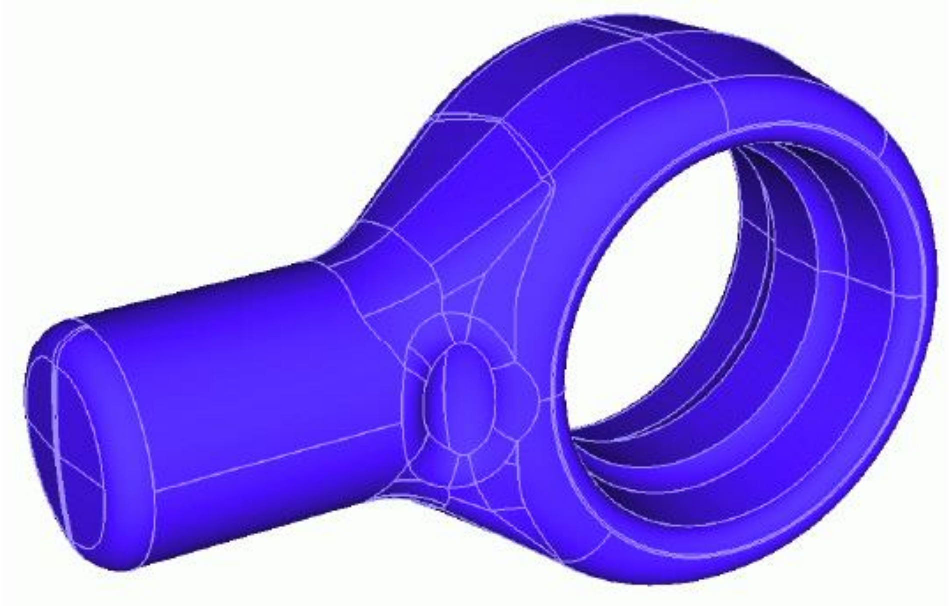
Parameterization of computational domain

- Open problem



Main difficulties

- Trimmed surface
- Complex topology
- Analysis-suitable



Two problems

- Optimal parameterization (r-refinement) from an initial parameterization

- IGA-suitable parameterization of computational domain from given boundary

Related work on parameterization for IGA

- Analysis-aware optimal parameterization

  E. Cohen et al.(CMAME, 2010) , Xu et al.(CMAME,2011), Pilgerstorfer et al (CMAME, 2013)

- Volumetric spline parameterization from boundary triangulation

  T. Martin et al.(CMAME, 2009), Zhang et al.(CMAME, 2012).

- Analysis-suitable planar parameterization from spline boundary

  Xu et al.(CAD, 2013), Gravessen et al.(CMAME, 2014), Xu et al. (CMAME, 2015),

  Nian (CMAME, 2016), Kapl M. et al. (CMAME, 2016) , Buchegger and Jüttler (CAD, 2017)

- Analysis-suitable volume parameterization from spline boundary

  Xu et al.(JCP, 2013), Zhang et al.(CM, 2012), Chan et al (CAD, 2017) ,

  Haberleitner and Jüttler (CAD, 2017)

Related talks in IGA 2017, Pavia, Italy

- Recent Additions to the Isogeometric Segmentation Pipeline. B. Jüttler
- Application of smooth functional surface over unstructured mesh. M. Bercovier
- Analysis-suitable G1 multi-patch domains in IGA. G. Sangalli, M. Kapl and T. Takacs
- Quantification and Control of the Impact of Parametrization on the Performance of Isogeometric Bernstein-Bezier Discretizations.J.A. Evans and L. Engvall
- Trimmed Trivariate Spline Models from Boundary Represented CAD Models and Element Reparameterization. V. Vibeke
- Automatic conversion to IGA-suitable shell models from complicated B-rep data in the industry. T. Tsuduki, K. Sasaya, K. Takada, T. Kikuchi, A. Nagy and D. Benson
- Automatic isogeometric analysis suitable trivariate models generation from standard B-Rep CAD. T. Maquart, T. Elguedj, A. Gravouil and M. Rochette
- Automatic Quadrilateral and Hexahedral Mesh Generation Based on Strebel Differential. N. Lei, X. Zheng and X. Gu
- Robust Hex-Dominant Mesh Generation using Field-Guided Polyhedral Agglomeration. D. Panozzo

Outline

- Parameterization of computational domain
- Optimal parameterization in isogeometric analysis
- IGA-suitable parameterization from boundary
- Conclusion and future work

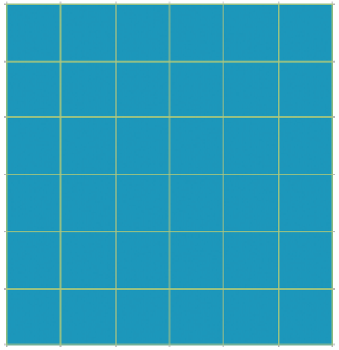
Problem statement

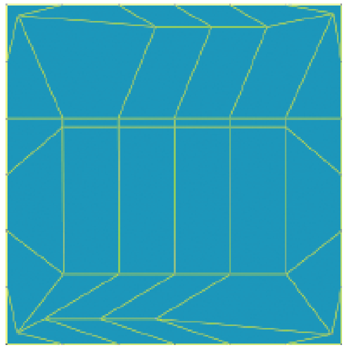
- Refinement in IGA
  - h-refinement： knot insertion
  - p-refinement： degree elevation
  - k-refinement： degree elevation+knot insertion
  - r-refinement： reparameterization of computational domain

r-refinement

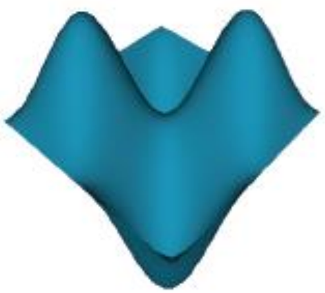
given initial placement of control points of computational domain,reposition the inner control points to achieve more accurate simulation results in isogeometric analysis

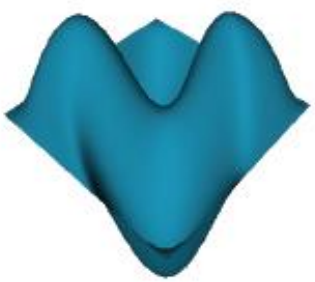
Analysis results

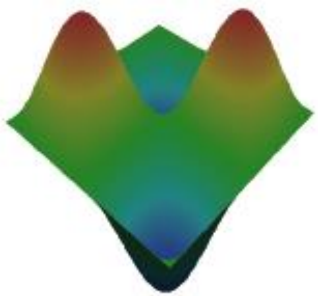




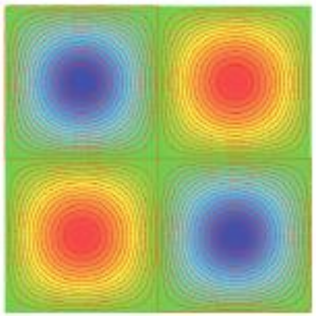
control point placement I   control point placement II

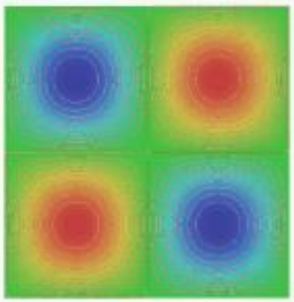




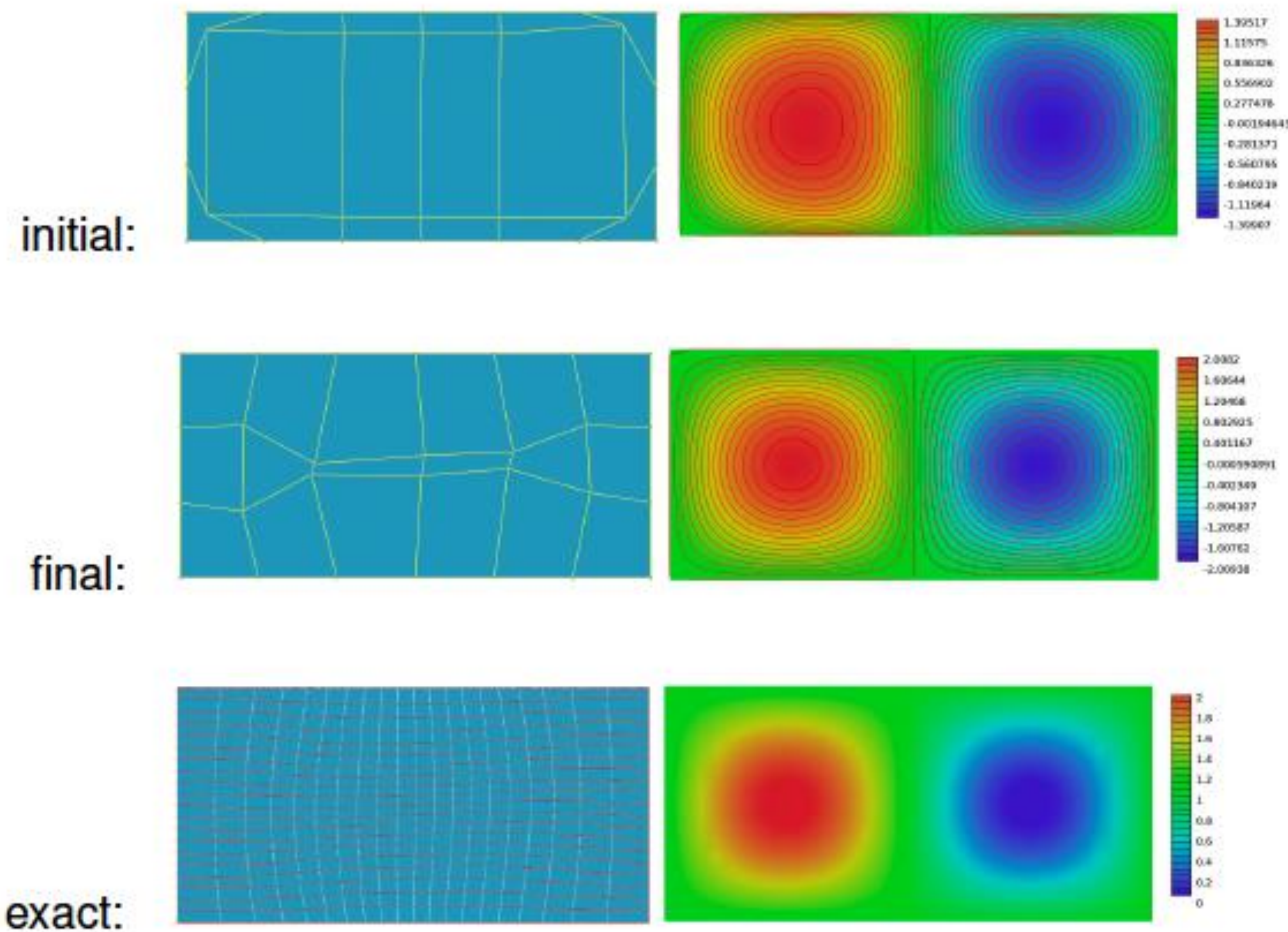


solution I   solution II   exact solution





Case with exact solution



Example with unknown exact solution

- Posterior error estimation
  - model problem:

$$\Delta U = f \quad \text{in } \Omega$$
$$U = U_0 \text{ on } \partial\Omega_D$$

  - U_h is the IGA solution
  - error: $e = U - U_h$

$$||e||^2 \leq C \sum_{K \in \Omega} h_K \int_K (f - \Delta U_h)^2 dK$$

r-refinement

Main idea for r-refinement

reposition inner control points to minimize $\sum_{K\in\Omega} h_K \int_K (f-\Delta U_h)^2 dK$.

$$\Delta U_h = \frac{\partial^2 U_h}{\partial^2 x} + \frac{\partial^2 U_h}{\partial^2 y}$$

$$U_h(x,y) = \mathcal{T}_h(\xi,\eta) = \sum_{i=1}^{n_i}\sum_{j=1}^{n_j} \hat{N}_i^{p_i}(\xi)\,\hat{N}_j^{p_j}(\eta)\,T_{ij}$$

$$\Delta U_h = [(x_\xi^2 - y_\xi^2)(\mathcal{T}_{\eta\eta} - \frac{\partial U_h}{\partial x}x_{\eta\eta} - \frac{\partial U_h}{\partial y}y_{\eta\eta}) - (x_\eta^2 - y_\eta^2)(\mathcal{T}_{\xi\xi} - \frac{\partial U_h}{\partial x}x_{\xi\xi} - \frac{\partial U_h}{\partial y}y_{\xi\xi})]/K$$

Overview

1. Solve IGA problem over given computational domain
2. Compute $\sum_{K \in \Omega} h_K \int_K (f - \Delta U_h)^2 dK$
3. Reposition inner control points by minimizing $\sum_{K \in \Omega} h_K \int_K (f - \Delta U_h)^2 dK$
4. Output final placement of inner control points

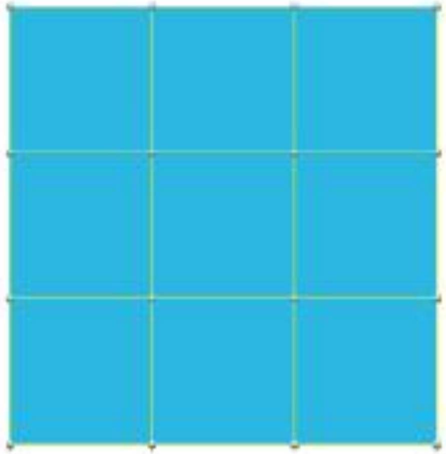
Error assessment

- $e = U - U_h$
- A posteriori error assessment by resolving IGA problem:

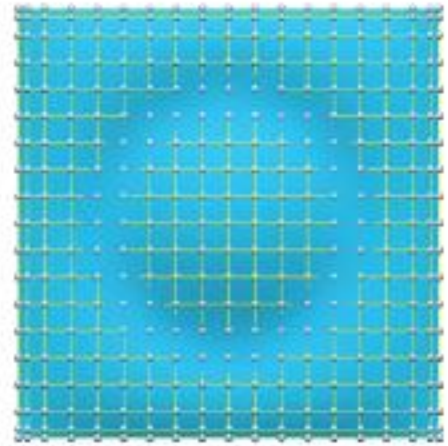
$$\begin{aligned} \Delta e &= f - \Delta U_h && \text{in } \Omega \\ e &= 0 && \text{on } \partial\Omega_D \end{aligned}$$

- Error field e has a B-spline form
- Perform h-refinement to achieve a good approximation
- More accurate but much more expensive
- Used for error assessment in r-refinement method

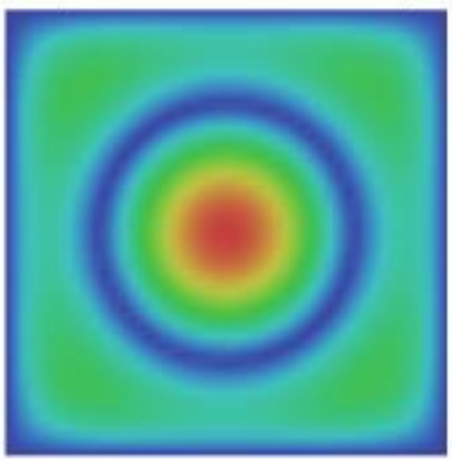
Example of error assessment



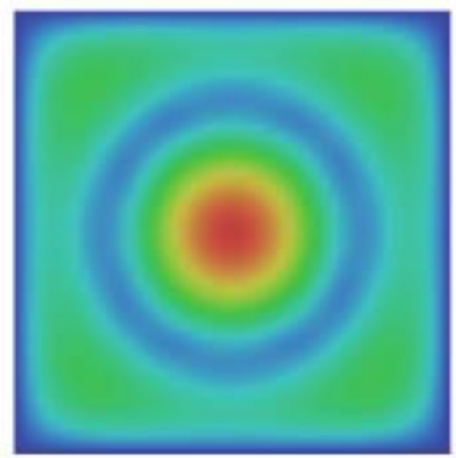
computational domain (CD)



resolved error surface



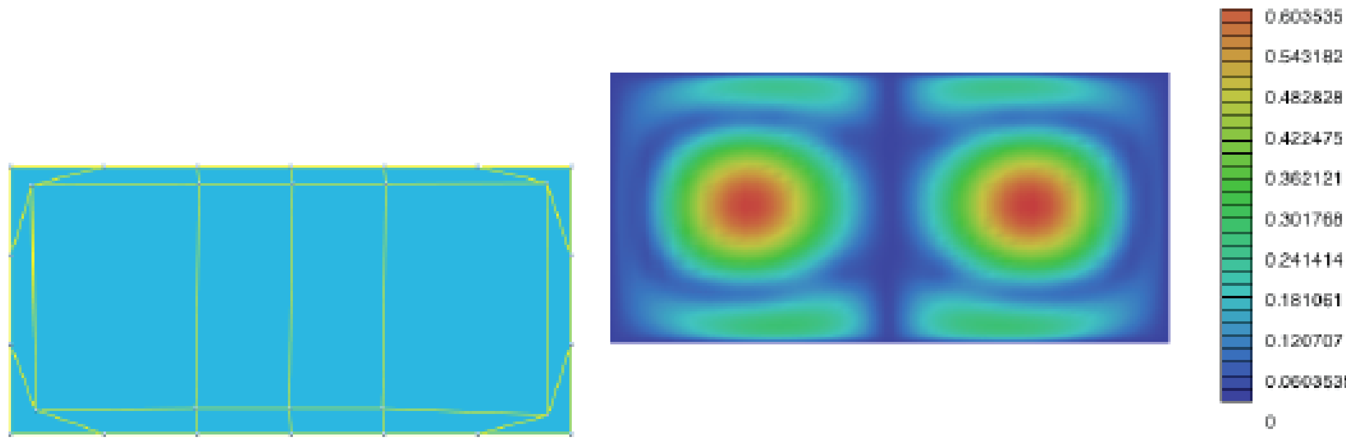
exact error colormap (EC)



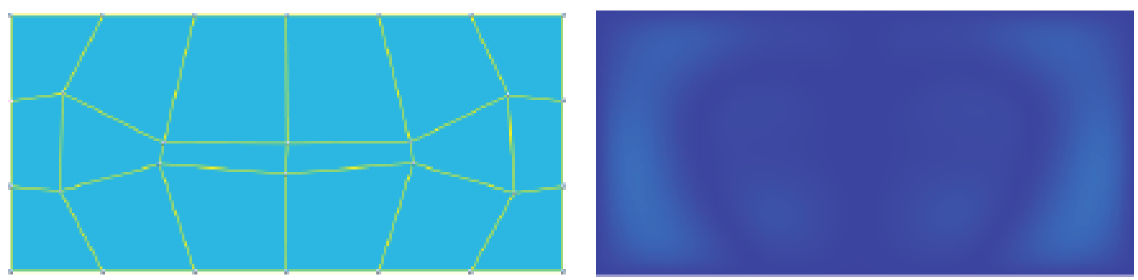
resolved EC

Example with exact solution

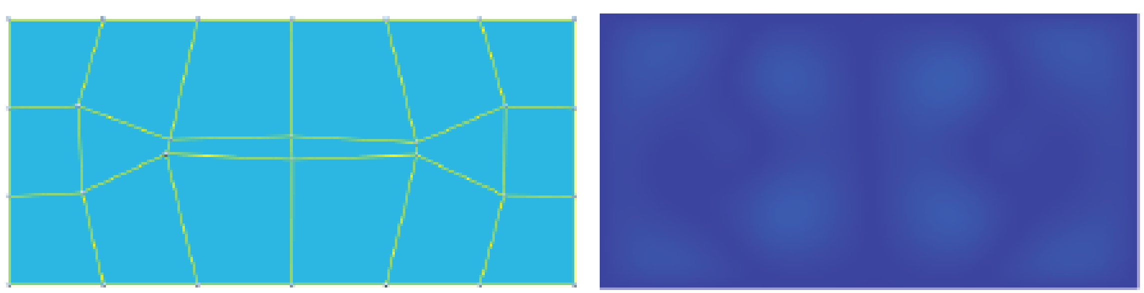
Initial CD and EC:



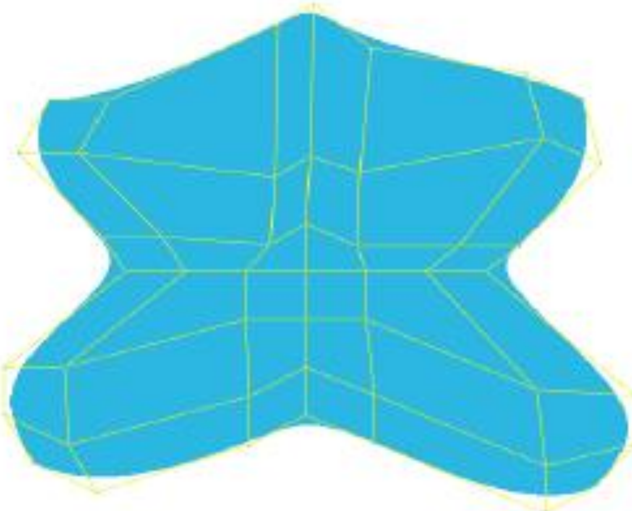
Final CD and EC:



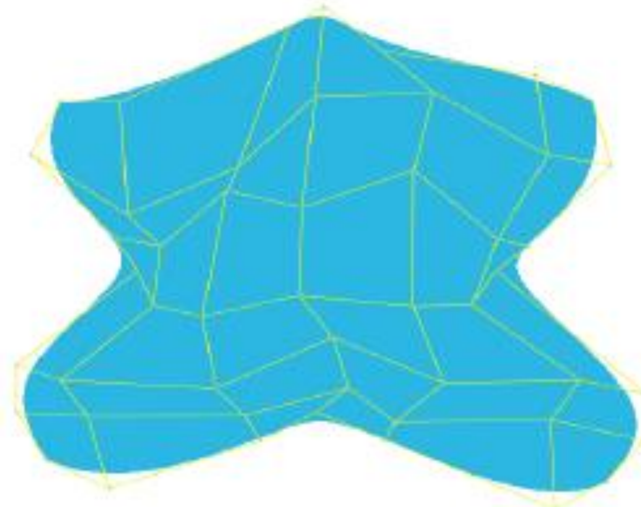
Exact method:



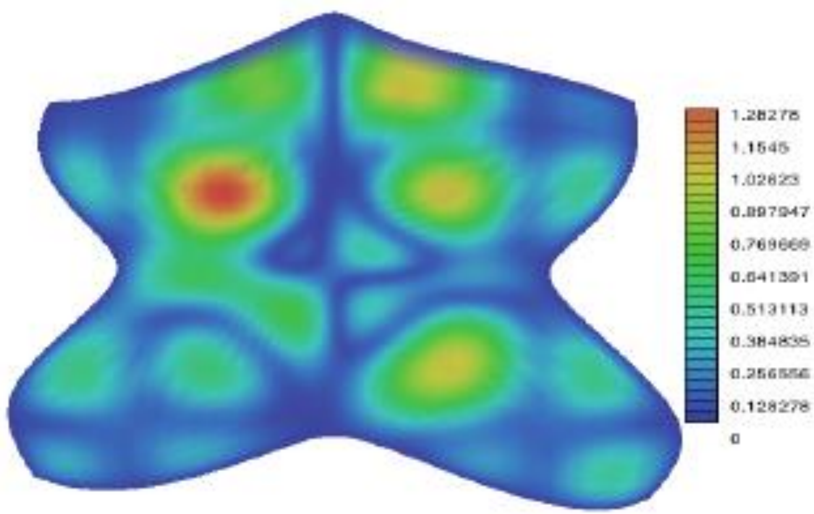
Example with unknown exact solution



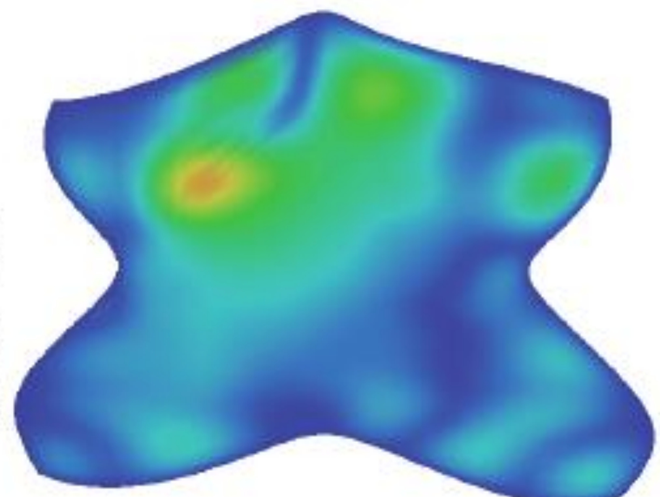
initial CD



CD after r-refinement



initial EC



EC after r-refinement

Local r-refinement

Local r-refinement

given initial placement of control points of computational domain, reposition the control points of patches where the local error indicator exceeds a specified tolerance

$$||e||^2 \leq C \sum_{K \in \Omega} h_K \int_K (f - \Delta U_h)^2 dK \tag{6}$$

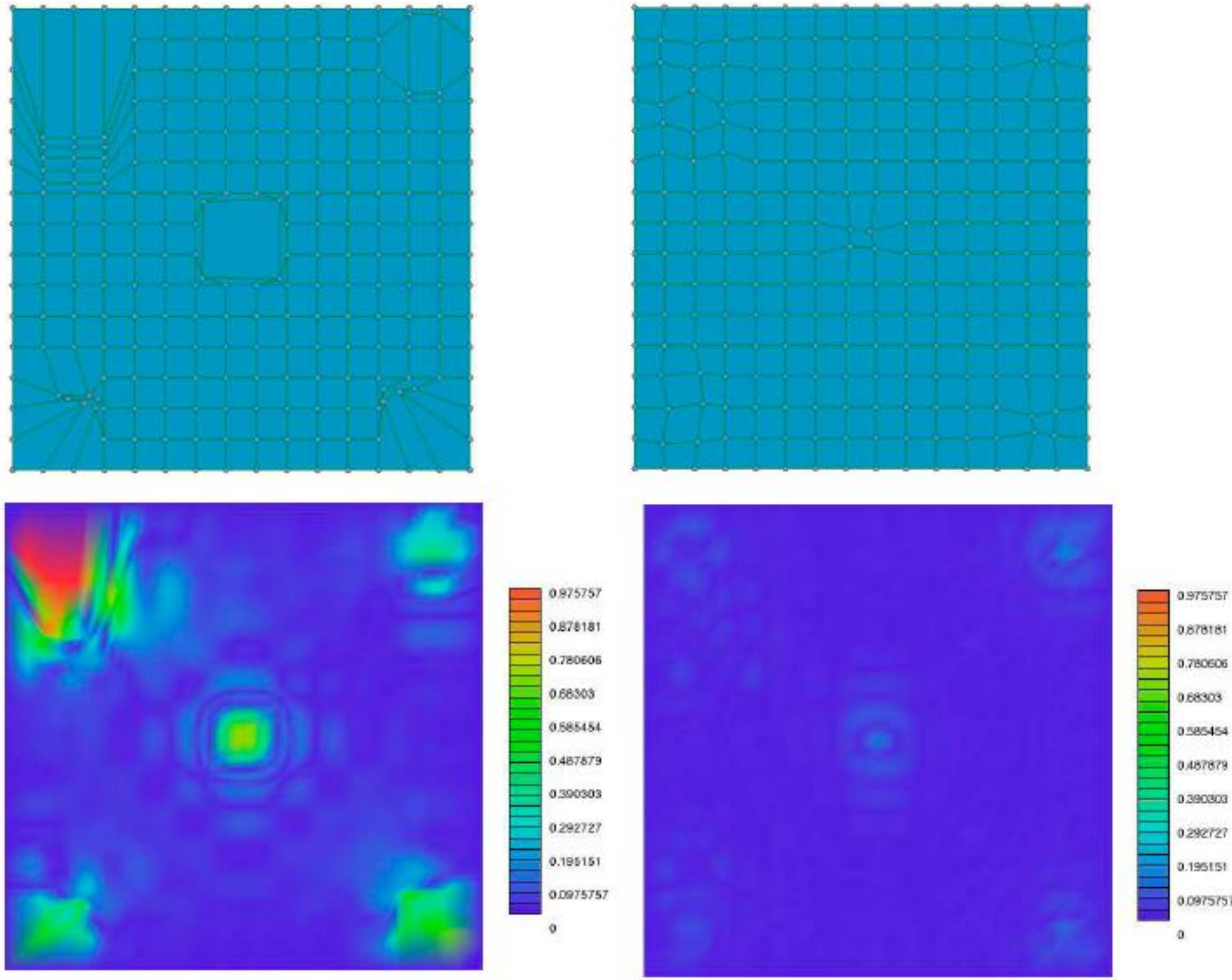
Local error indicator

$e_K = h_K \int_K (f - \Delta U_h)^2 dK$

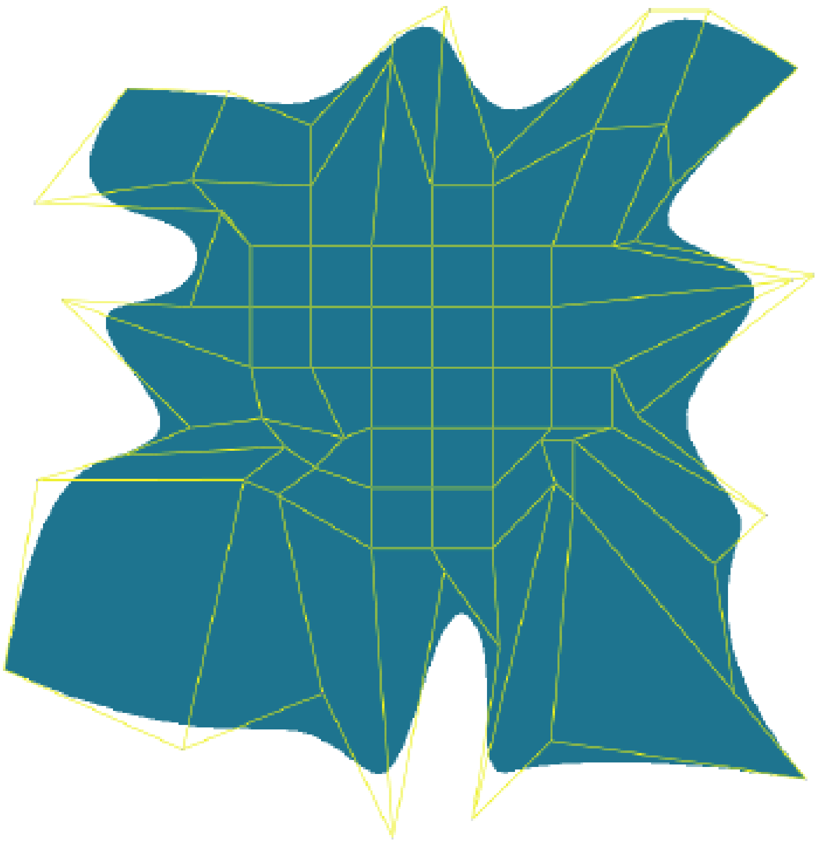
Main steps

- Step 1: Solve isogeometric problem on the given parameterization of computational domain;
- Step 2: Calculate the local error indicator e_K patch by patch;
- Step 3. Compute the specified tolerance $\sigma = \frac{\sum_{K \in \Omega} e_K}{N}$, where N is the number of patches in the computational domain;
- Step 4: Mark the patches K_r where the local error indicator e_{K_r} is greater than σ
- Step 5: Reposition the control points of the marked patches by minimizing $\sum_{K_r \in \Omega} e_{K_r}$, the sum of the error indicator on the marked patches.

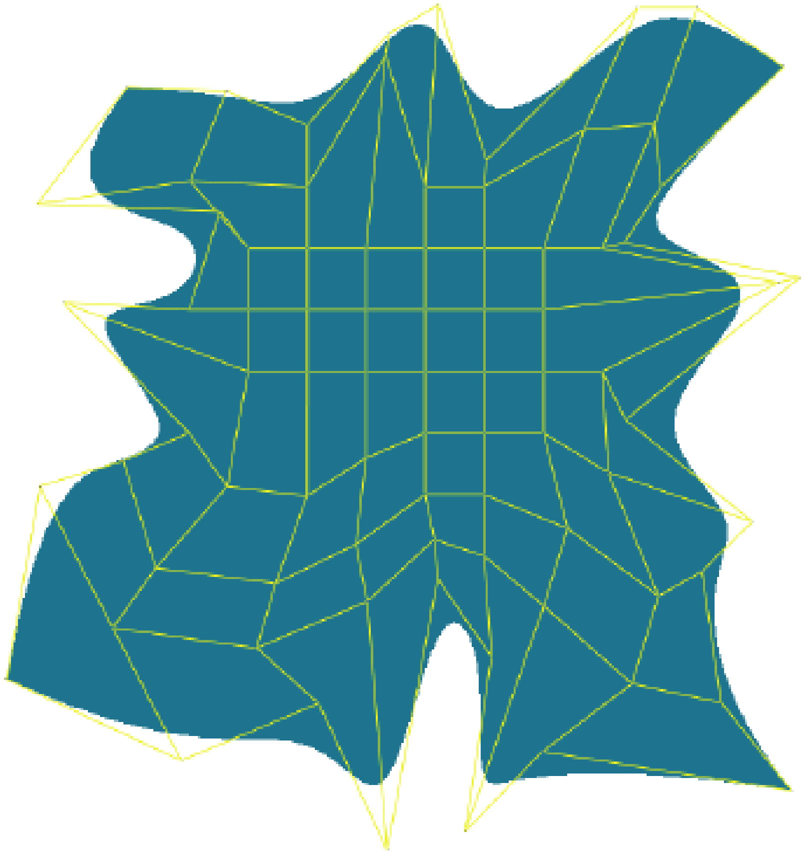
Example with exact solution



Example

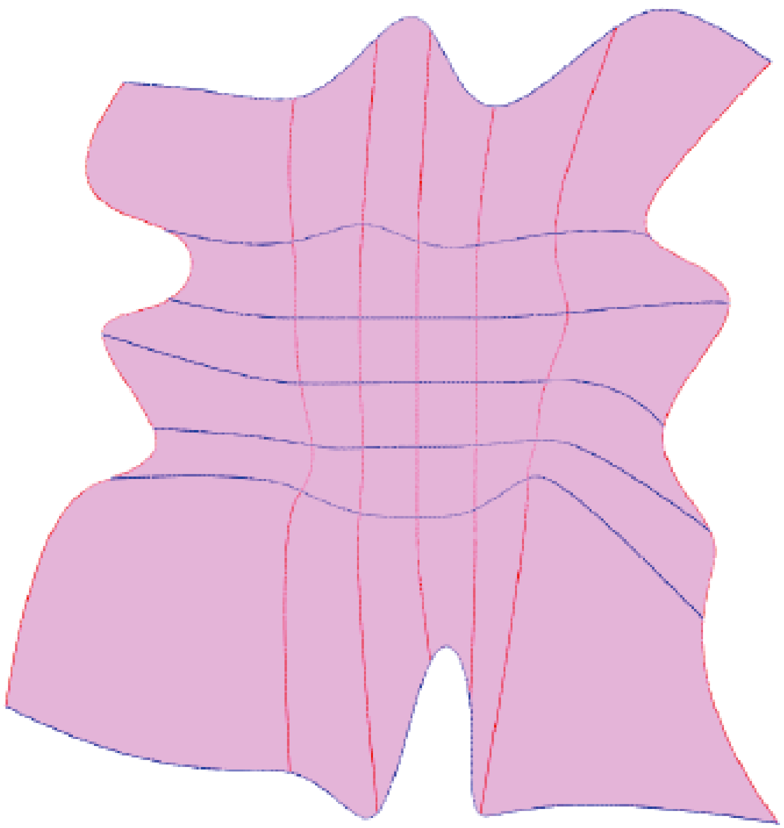


Initial parameterization

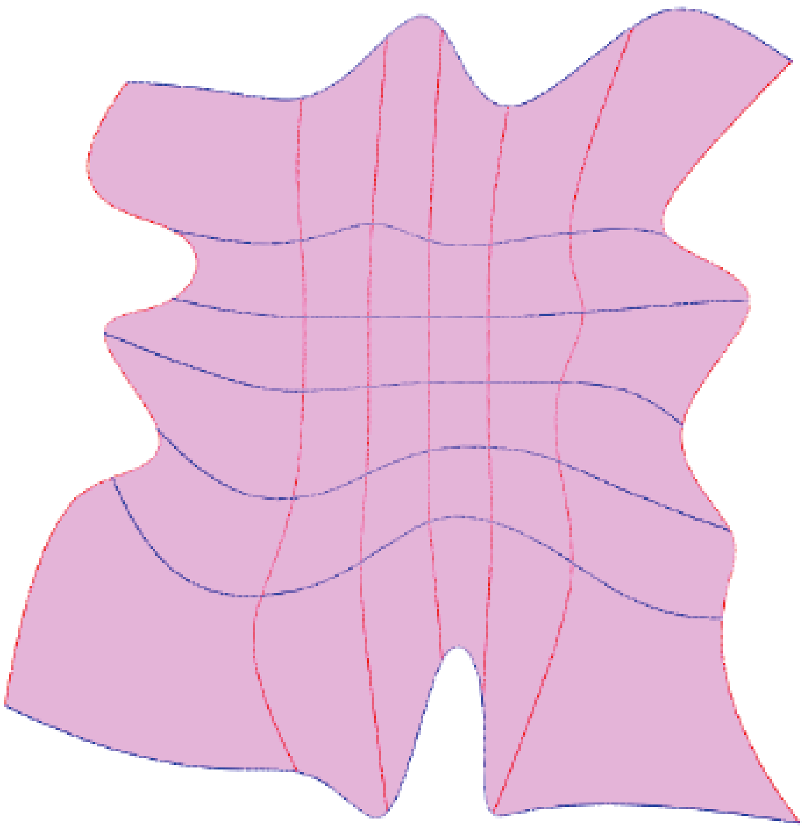


final parameterization

Patch structure

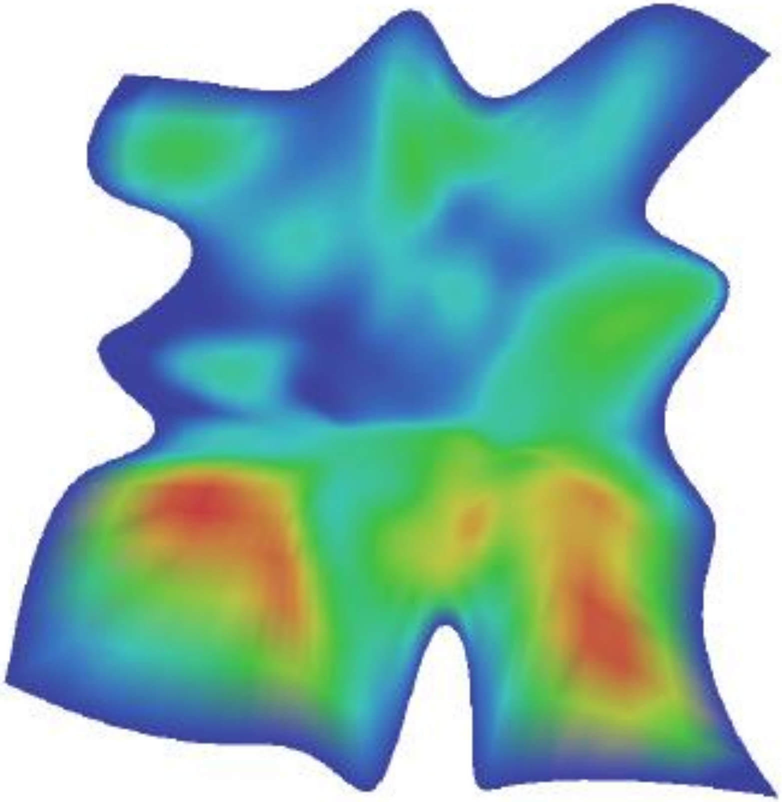


Initial patches

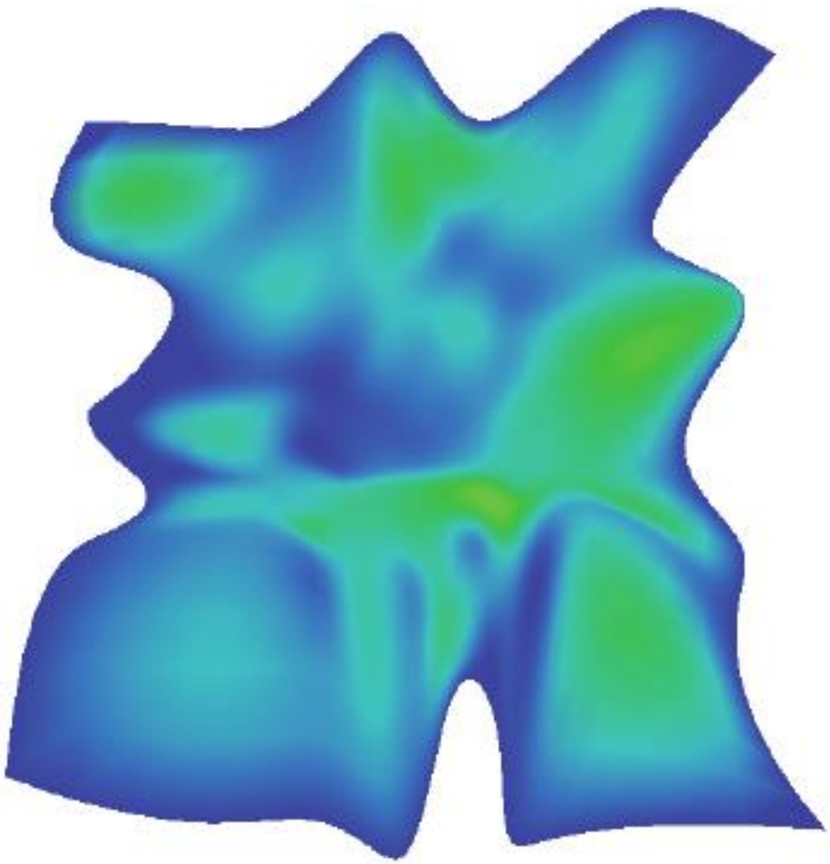


final patches

Error color map

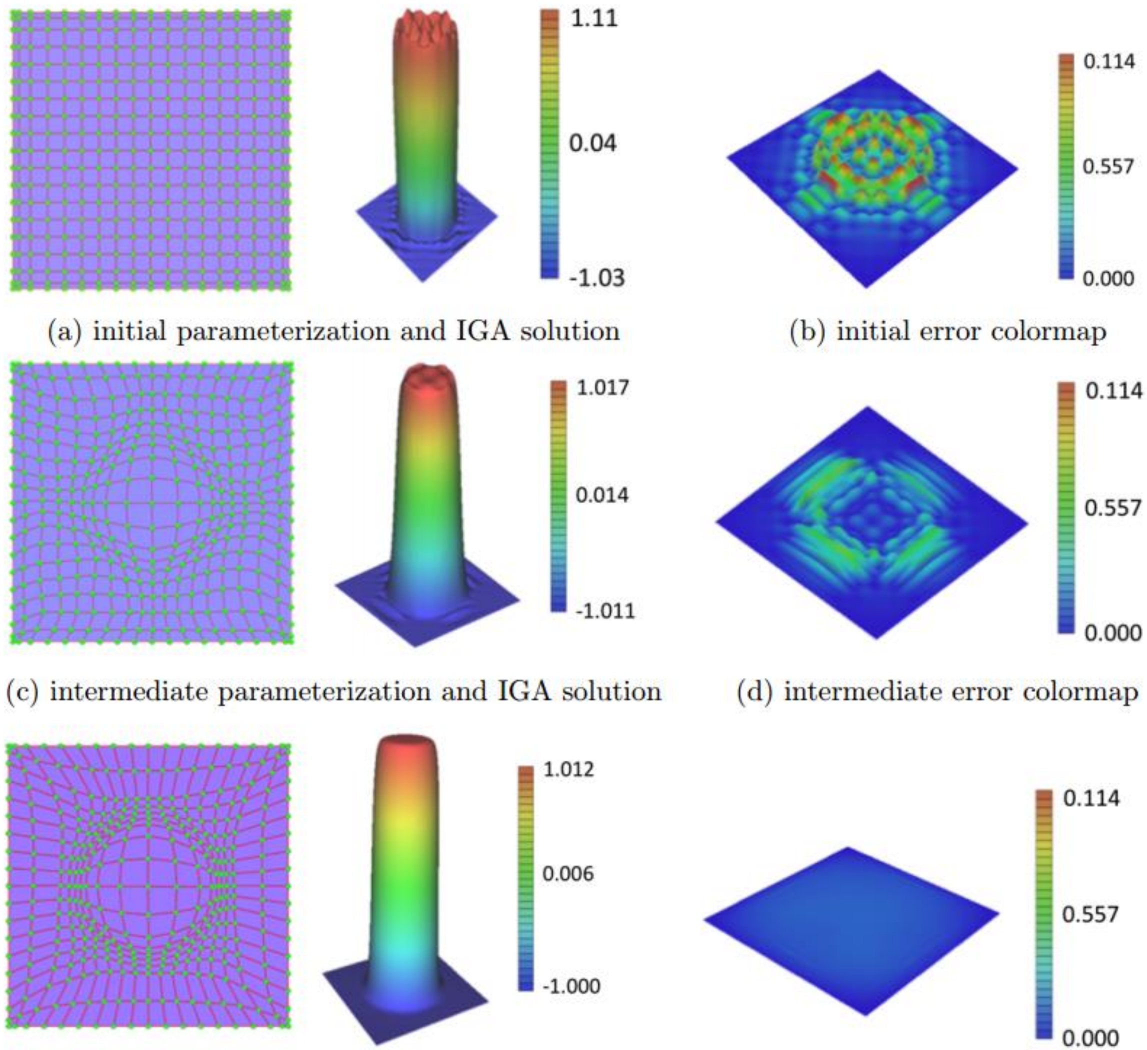


Initial error colormap



final error colormap

r-refinement with monitor function



(a) initial parameterization and IGA solution

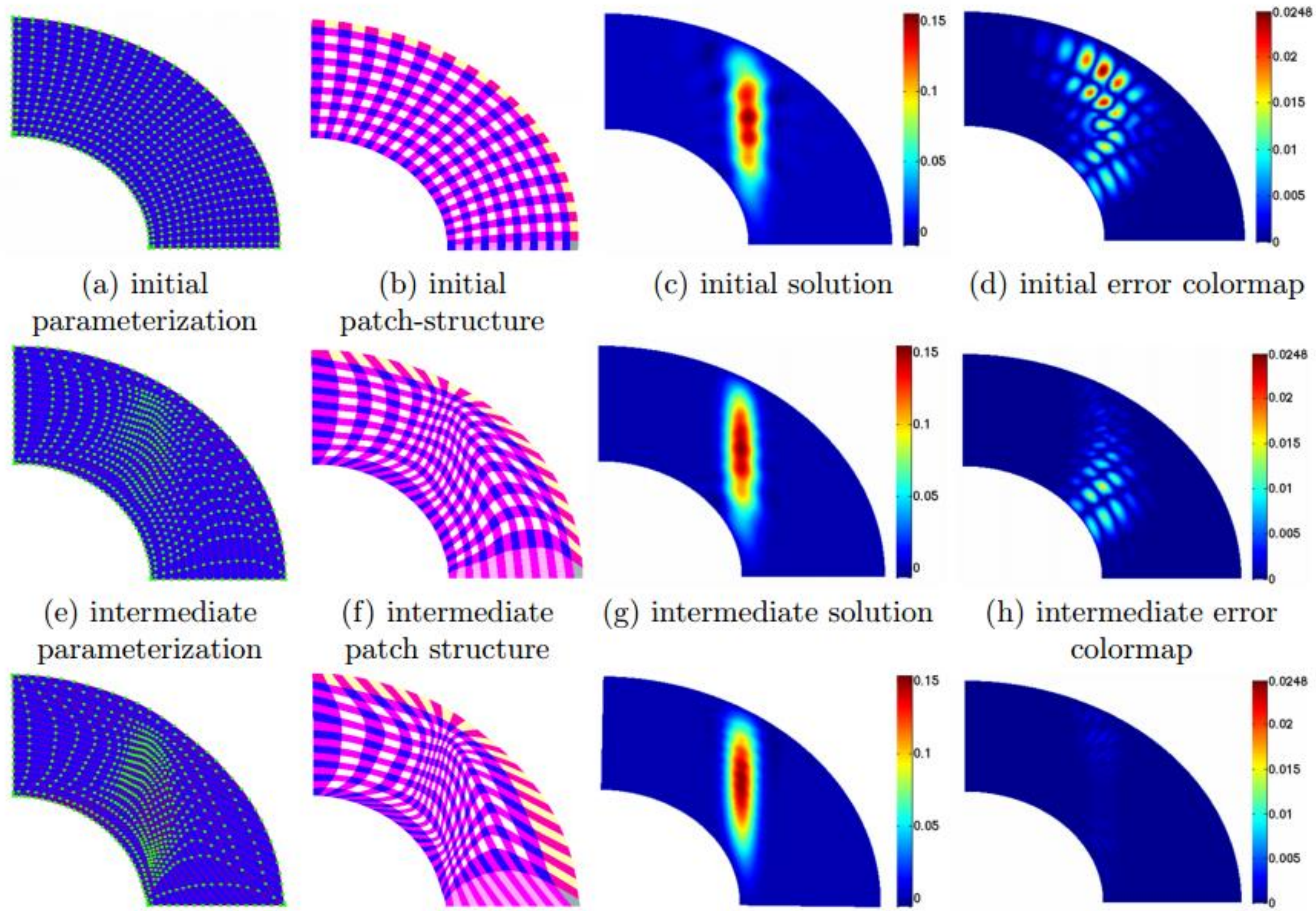
(b) initial error colormap

(c) intermediate parameterization and IGA solution

(d) intermediate error colormap

(e) final parameterization and IGA solution

(f) final error colormap



(a) initial parameterization

(b) initial patch-structure

(c) initial solution

(d) initial error colormap

(e) intermediate parameterization

(f) intermediate patch structure

(g) intermediate solution

(h) intermediate error colormap

(i) final parameterization

(j) final patch-structure

(k) final solution

(l) final error colormap

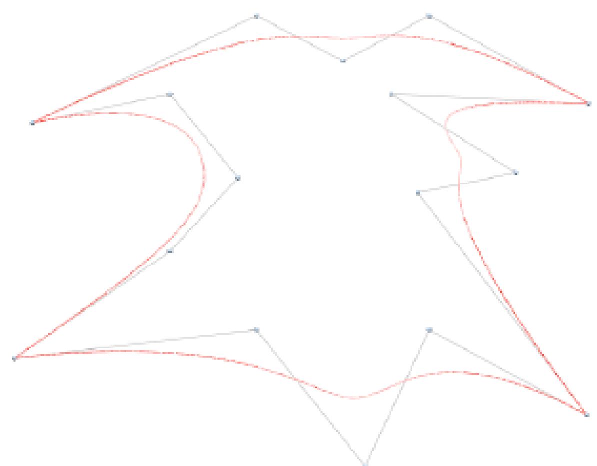
Outline

- Parameterization of computational domain
- Optimal parameterization in isogeometric analysis
- IGA-suitable parameterization from boundary
- Conclusion and future work

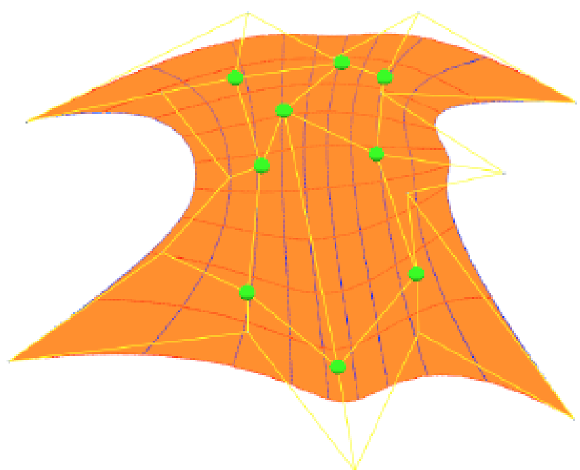
Problem statement

Construction of computational domain from boundary

given boundary control points of computational domain, construct the inner control points to generate analysis-suitable parameterization of computational domain



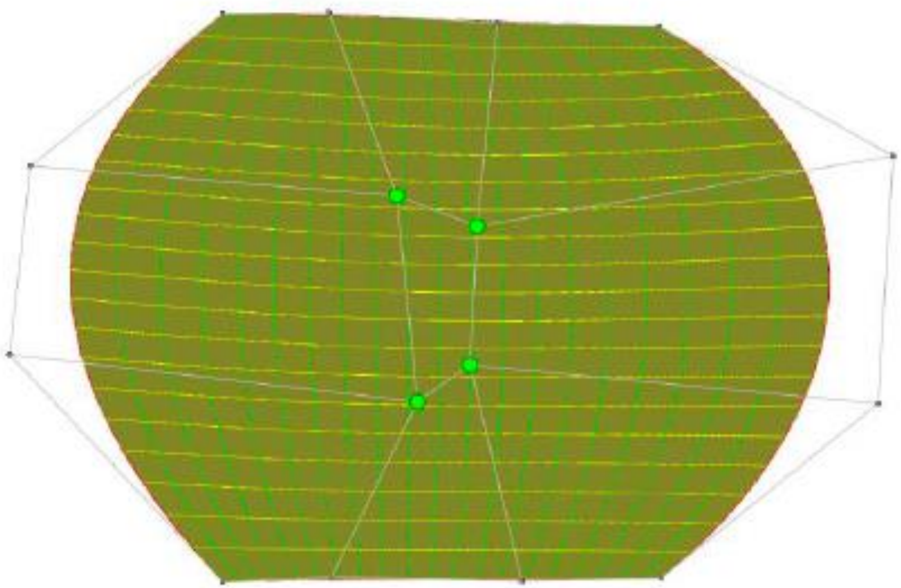
boundary curves



computational domain

Analysis-suitable parameterization

- injective (no self-intersections)
- as uniform as possible
- orthogonal isoparametric curves



CMAME 2013, CAD 2013

Input: six boundary B-spline surfaces

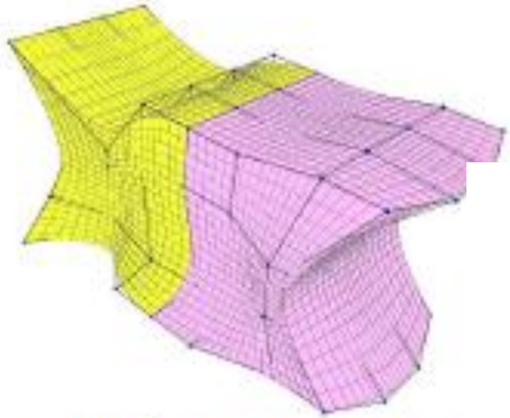
Output: inner control points and the corresponding B-spline volume parameterization

- Construct the initial inner control points by discrete Coons method;
- Construct the constraint condition from boundary B-spline surfaces;
- Solve the following constraint optimization problem by using sequential quadratic programming (SQP for short) method

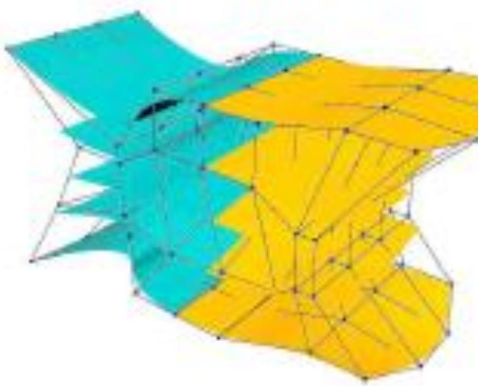
$$
\begin{aligned}
&\mathit{min} \iiint (\| \sigma_\xi \|^2 + \| \sigma_\eta \|^2 + \| \sigma_\zeta \|^2) \\
&+\omega(\| \sigma_{\xi\xi} \|^2 + \| \sigma_{\eta\eta} \|^2 + \| \sigma_{\zeta\zeta} \|^2 \\
&+2 \| \sigma_{\xi\eta} \|^2 +2 \| \sigma_{\xi\zeta} \|^2 +2 \| \sigma_{\eta\zeta} \|^2) d\xi d\eta d\zeta . \\
&s.t. \quad G_{ijk} > 0
\end{aligned}
$$

- Generate the corresponding B-spline volume parameterization $\sigma(\xi, \eta, \zeta)$ as computational domain.

Multi-block case



(a) C^1 B-spline blocks



(b) Isoparametric surfaces and control lattices in C^1 B-spline blocks

$$\frac{\partial}{\partial \xi}\sigma_1(\xi,\eta,\zeta)|_{\xi=\xi_1} = \frac{\partial}{\partial \xi}\sigma_2(\xi,\eta,\zeta)|_{\xi=\xi_1}$$

$$\sum_{\substack{0\le j\le m\\0\le k\le n}} \omega^{1,1}_{i,j,k}\Delta^{1,1}_{i,j,k}N^q_j N^r_k = \sum_{\substack{0\le j\le m\\0\le k\le n}} \omega^{1,2}_{i,j,k}\Delta^{1,2}_{i,j,k}N^q_j N^r_k.$$

$$\omega^{1,1}_{i,j,k}\Delta^{1,1}_{i,j,k} = \omega^{1,2}_{i,j,k}\Delta^{1,2}_{i,j,k}, i = 0, ..., l,$$

Variational harmonic method
(Journal of Computational Physics, 2013)

- Given: computational domain $\mathcal{S}$, parametric domain $\mathcal{P}$,

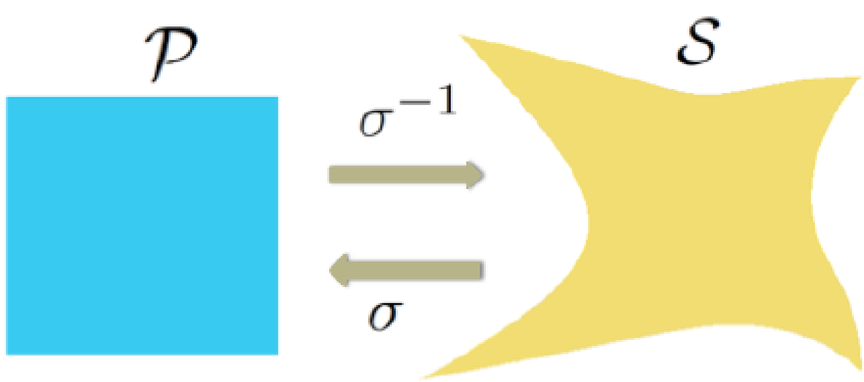
$$S(\xi,\eta) = (x(\xi,\eta), y(\xi,\eta)) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_i^p(\xi) N_j^q(\eta) p_{i,j}$$

- Harmonic mapping: $\sigma : \mathcal{S} \mapsto \mathcal{P}$

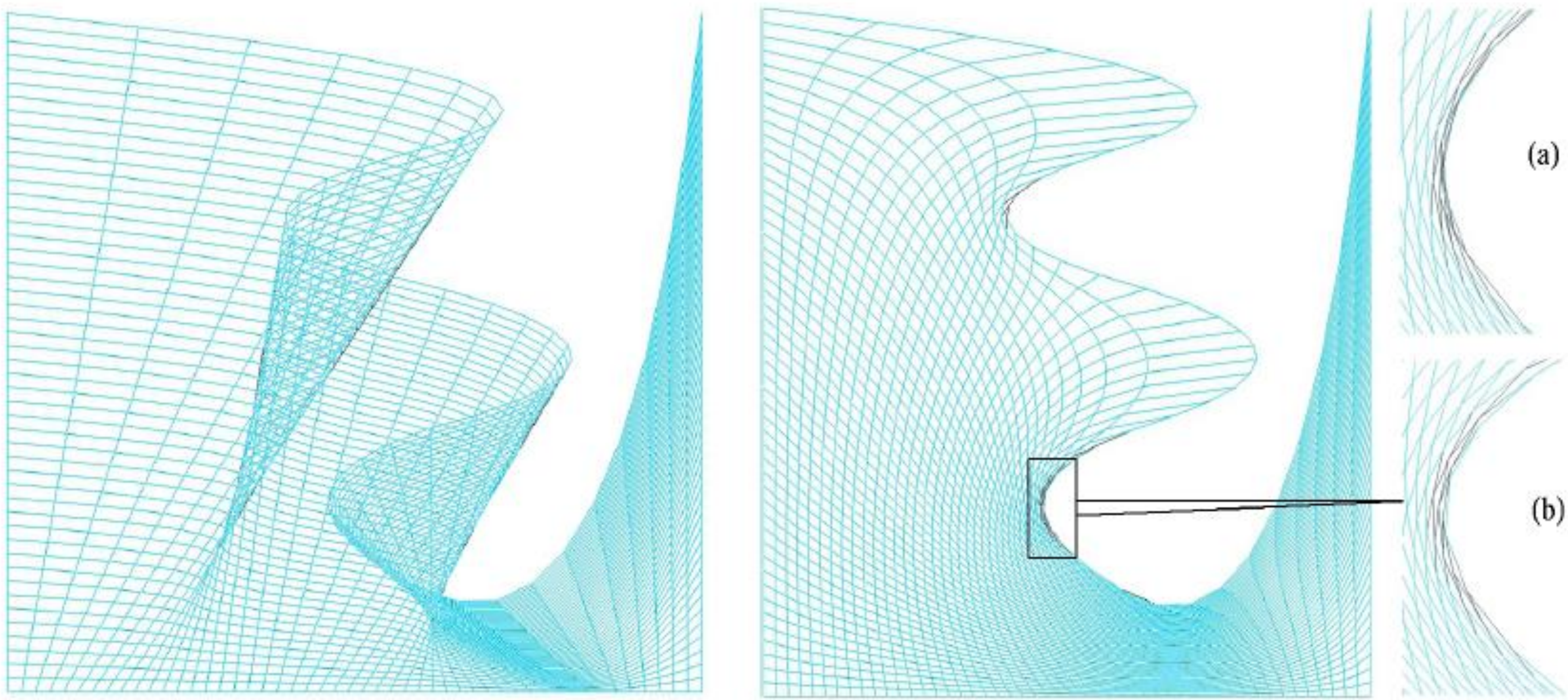
$$\Delta\xi(x,y) = \xi_{xx} + \xi_{yy} = 0$$

$$\Delta\eta(x,y) = \eta_{xx} + \eta_{yy} = 0$$

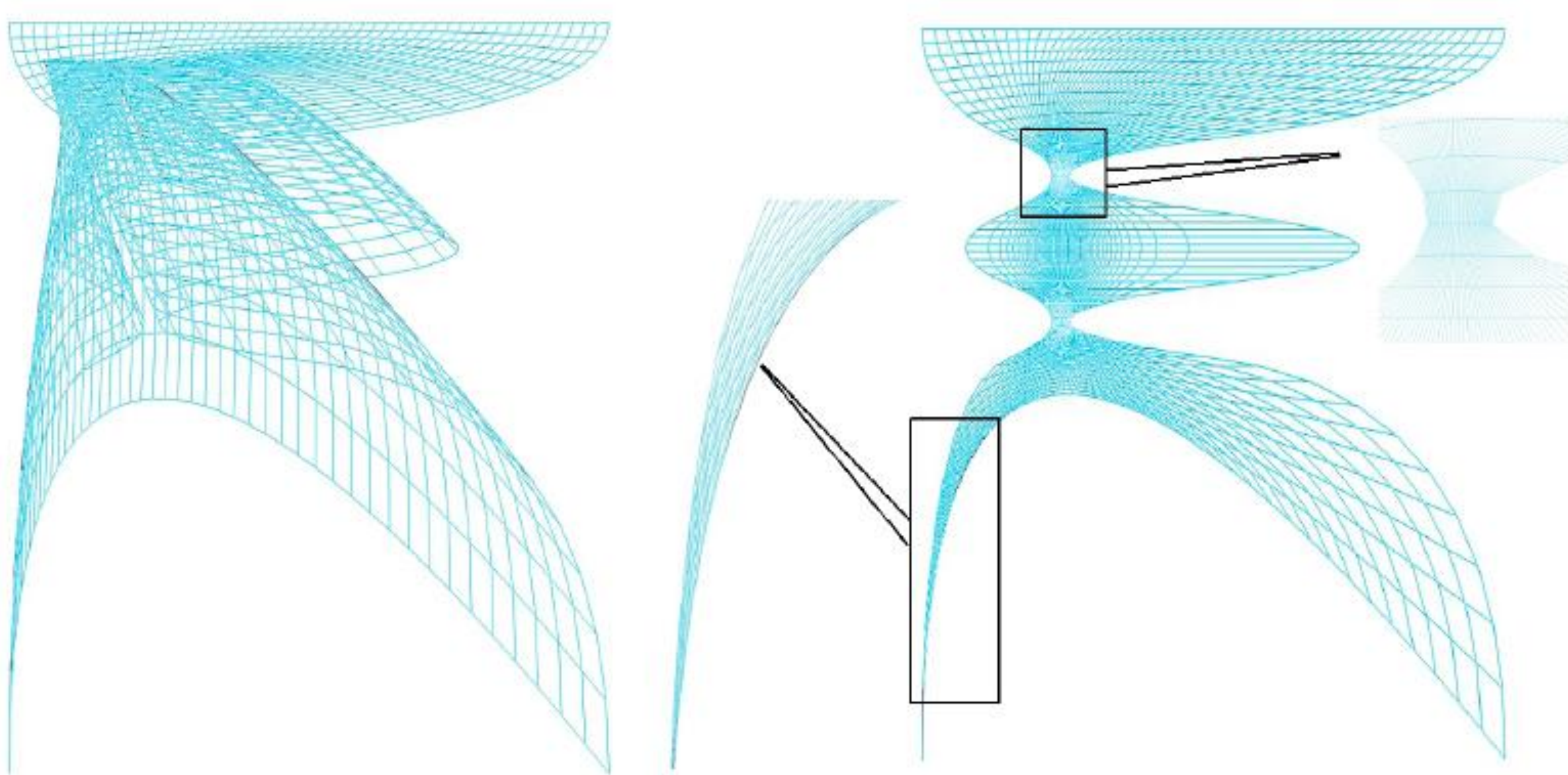
- $\sigma^{-1} : \mathcal{P} \mapsto \mathcal{S}$ is one-to-one



Two examples（1/2）

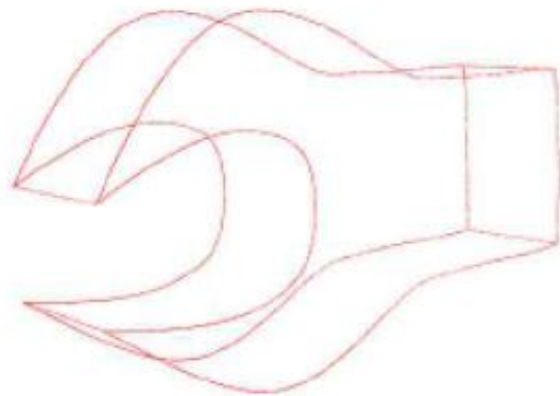


Two examples（2/2）

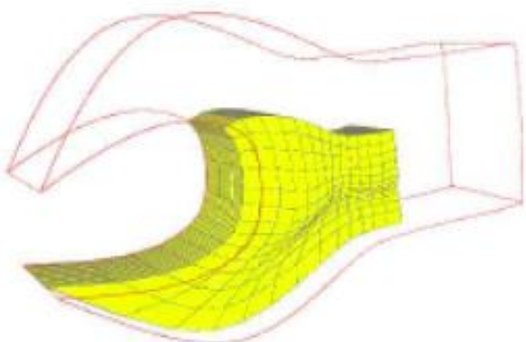


3D example I

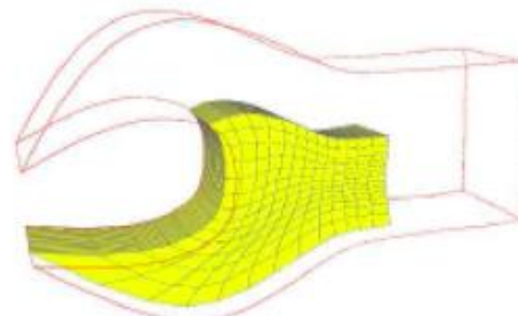
(a) boundary surfaces



(b) boundary curves



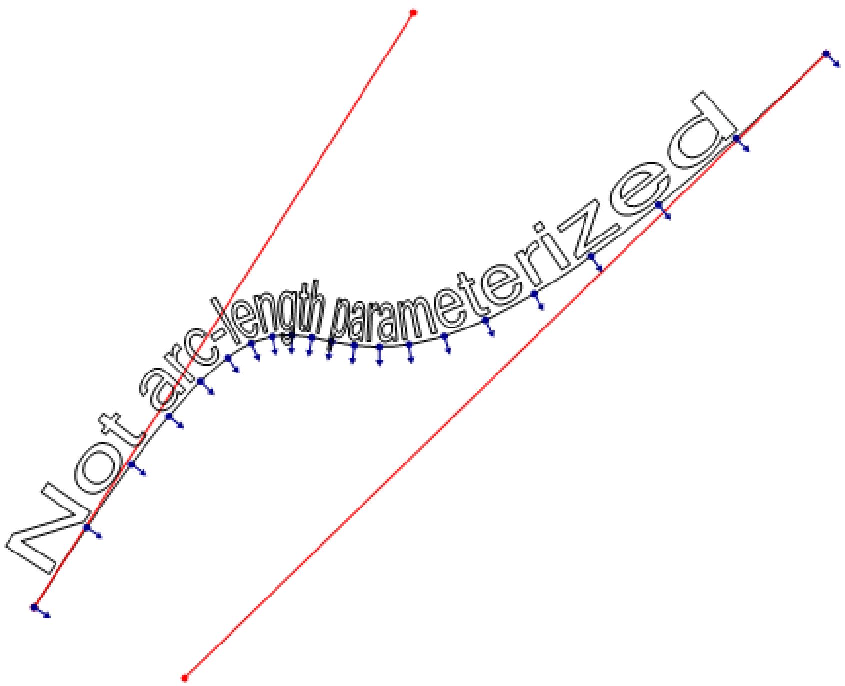
(c) Coons volume

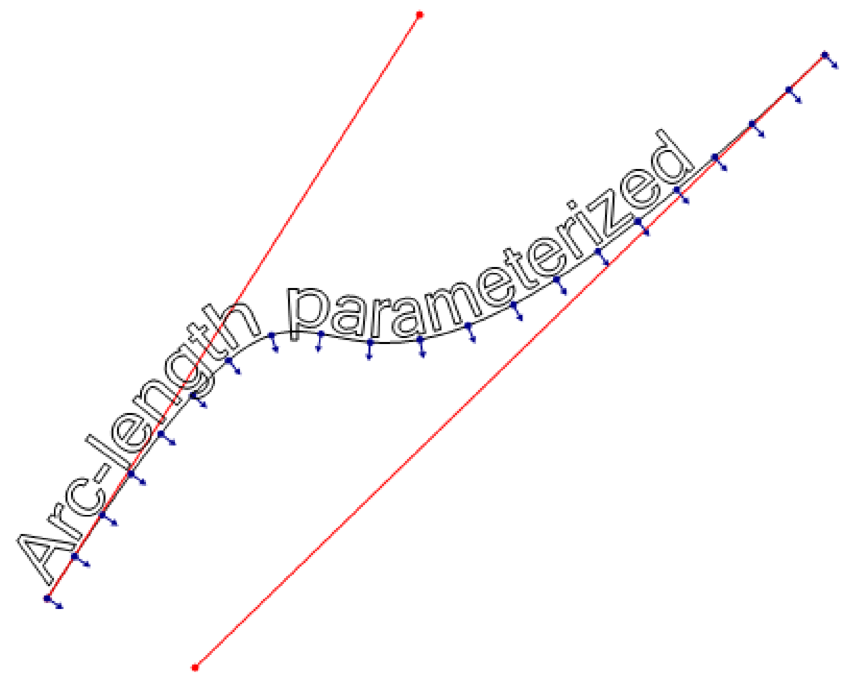


(d) final volume parameterization

Boundary reparameterization for volumetric parameterization (Computational Mechanics, 2014)

Goal: construct optimal Möbius reparameterization of boundary surfaces to achieve high-quality isoparametric structure without changing the boundary geometry





Möbius reparameterization

$$\boldsymbol{R}(u, v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} \lambda_{i,j}\boldsymbol{C}_{i,j} N_i^p \boxed{(u)} N_j^q \boxed{(v)}}{\sum_{i=0}^{n}\sum_{j=0}^{m} \lambda_{i,j} N_i^p \boxed{(u)} N_j^q \boxed{(v)}},$$

$$u = \frac{(1-\alpha)\xi}{\alpha(1-\xi) + (1-\alpha)\xi}$$

$$v = \frac{(1-\beta)\eta}{\beta(1-\eta) + (1-\beta)\eta}$$

New NURBS surface with the same control points but different weights and knot vectors

Optimization method

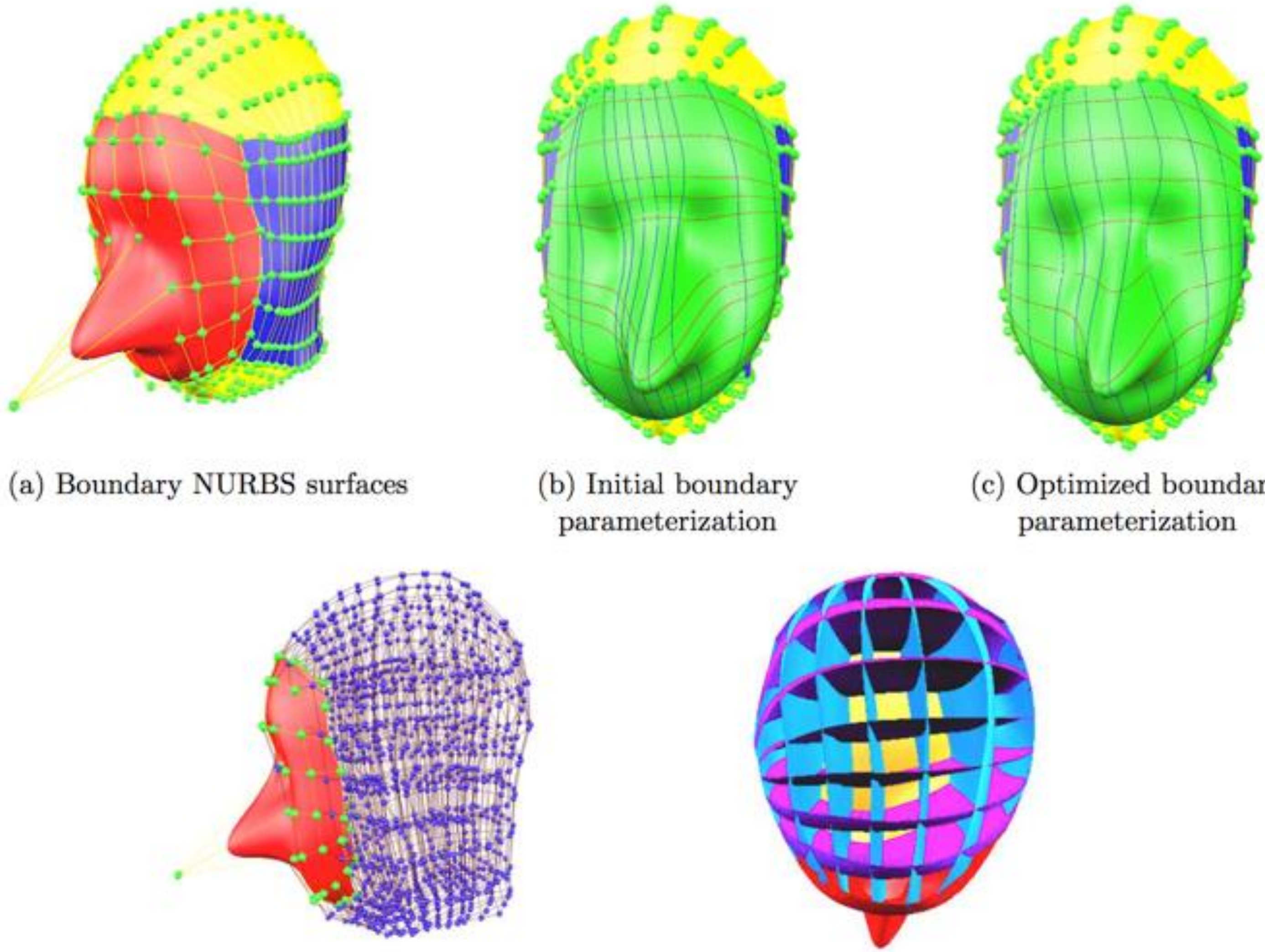
- Find the optimal

$$\alpha, \beta$$

such that the reparameterized NURBS surface minimizes the following objective function

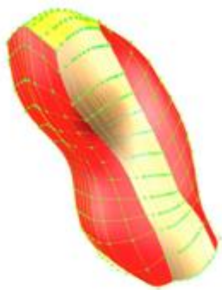
$$\int_{\mathcal{P}} (\mathbf{det}\, \widetilde{\boldsymbol{J}} - J_{avg})^2 + \omega_1(\|\widetilde{\boldsymbol{R}}_{\xi\xi}\|^2 + \|\widetilde{\boldsymbol{R}}_{\eta\eta}\|^2)\, d\xi\, d\eta$$

Reparameterization for VP problem



(a) Boundary NURBS surfaces (b) Initial boundary parameterization (c) Optimized boundary parameterization

(d) Control lattice (e) Final isoparametric structure (top view)



(a) Boundary NURBS surfaces

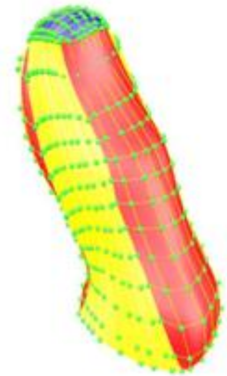
(b) Boundary NURBS curves

(c) Initial boundary parameterization

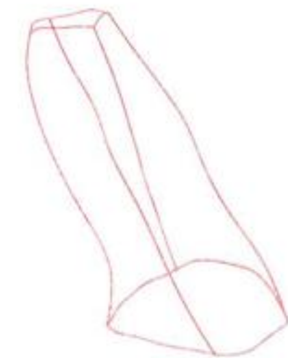
(d) Optimized boundary parameterization

(e) Control lattice

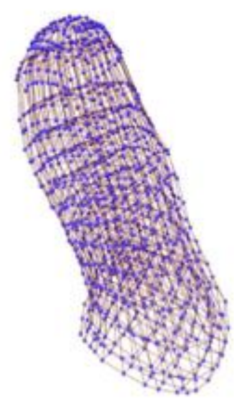
(f) Final isoparametric structure



(a) Boundary NURBS surfaces and control mesh



(b) Boundary NURBS curves

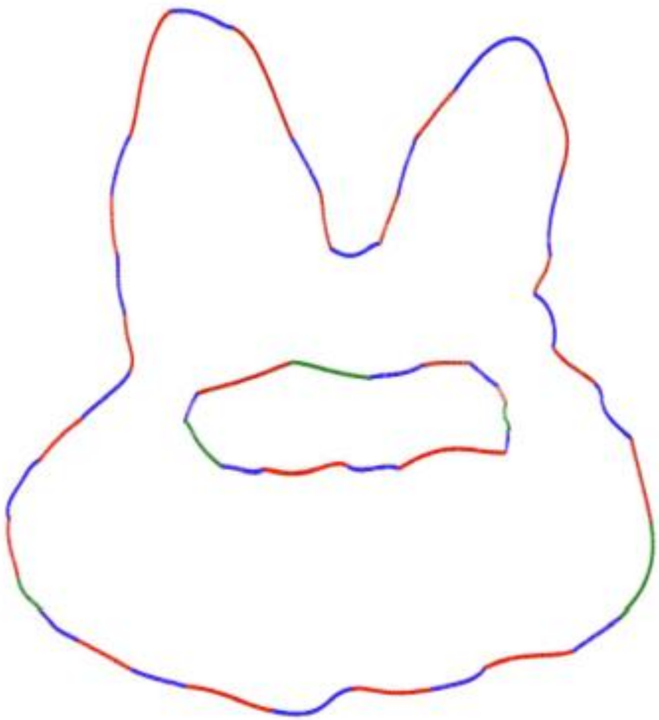


(c) Resulting control lattice

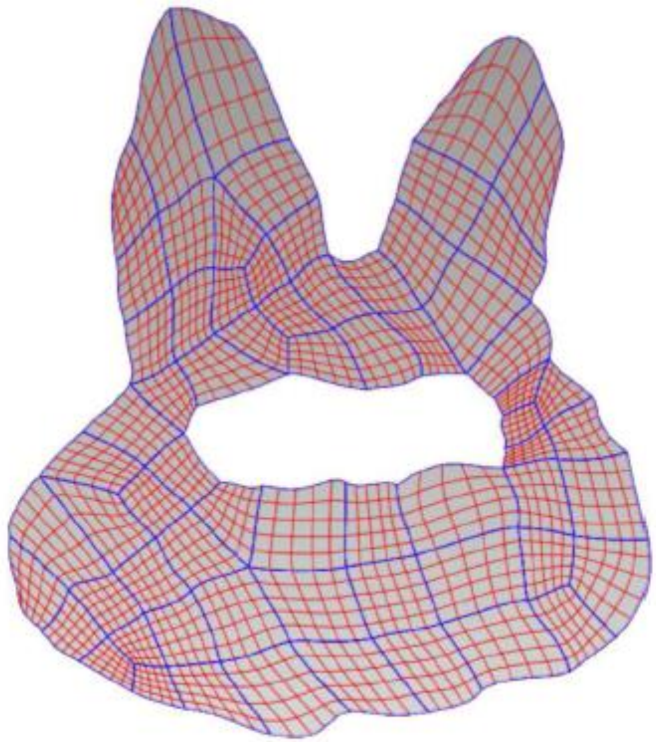
(d) Final isoparametric structure

Planar domain with arbitrary topology

- **Given the boundary spline curves of a planar domain with arbitrary topology, construct the patch structure and control points to obtain IGA-suitable parameterization**



(a) boundary Bézier curves



(e) parameterization result

Desired parameterization method

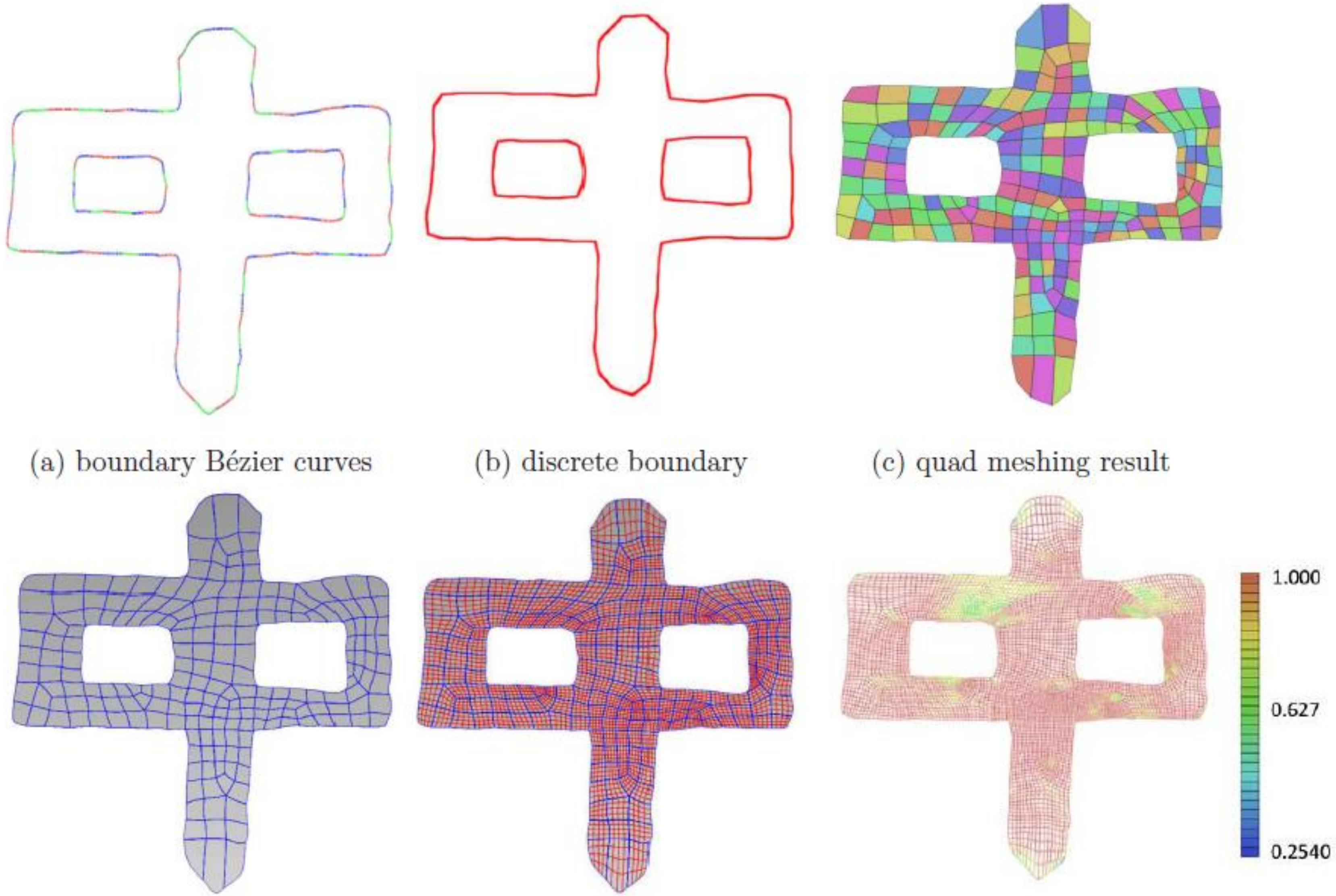
- Boundary-preserving
- Automatic continuity imposition
  ($\neq$ high-order meshing with C^0)
- Automatic construction of segmentation curve
- Injective
- Uniform patch size
- Orthogonal iso-parametric structure

Proposed framework

1. Pre-processing for high-quality parameterization
2. Topology information generation of quadrilateral decomposition
3. Quadrilateral patch partition by global optimization
4. High-quality patch parameterization by local optimization

Reference:
Gang Xu, Ming Li, Bernard Mourrain, Timon Rabczuk, Jinlan Xu, Stephane P.A. Bordas. Constructing IGA-suitable planar parameterization from complex CAD boundary by domain partition and global/local optimization,CMAME, 2018

Framework Overview



(a) boundary Bézier curves (b) discrete boundary (c) quad meshing result

(d) segmentation curves (e) parameterization result (f) Jacobian colormap

Pre-processing of input boundary curves

- Bézier extraction

$$\boxed{\mathbf{N}(\mathbf{t}) = \mathbf{C}\mathbf{B}(\mathbf{t})}$$

$$\boxed{\mathbf{P} = \mathbf{C}\mathbf{Q}}$$

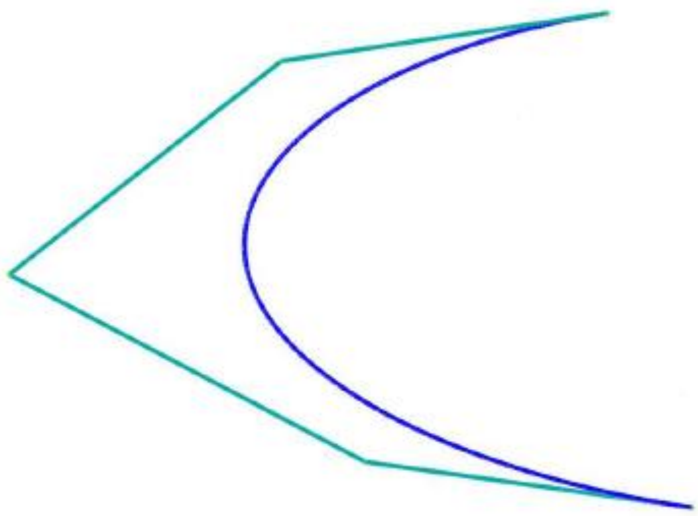
- Bézier subdivision

$$\boxed{\Gamma \geq \log_4 \frac{\sqrt{3}n(n-1)\eta}{8L_{ave}}}$$

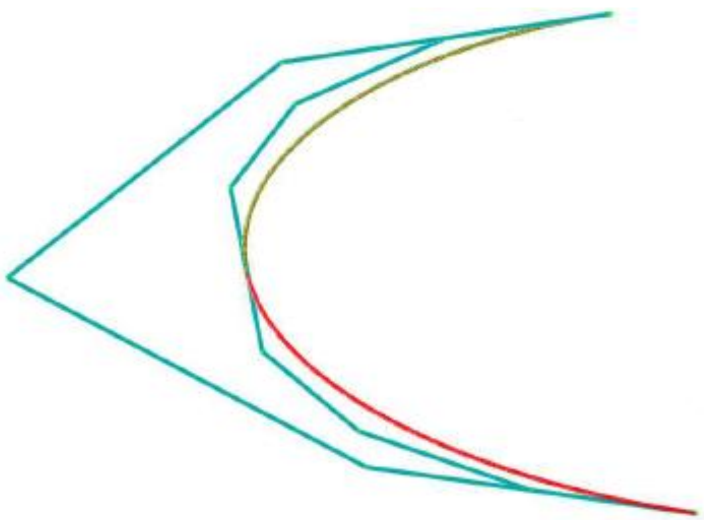
$$\boxed{\eta = \max_{0 \leq i \leq n-2} \{|s^x_{i,k} - 2s^x_{i+1,k} + s^x_{i+2,k}|, |s^y_{i,k} - 2s^y_{i+1,k} + s^y_{i+2,k}|\}}$$

Pre-processing of input boundary curves

- Subdivision of a Bézier curve with concave shape



(a) original Bézier curve



(b) Bézier subdivision

Proposed framework

1. Pre-processing for high-quality parameterization
2. Topology information generation of quadrilateral decomposition
3. Quadrilateral patch partition by global optimization
4. High-quality patch parameterization by local optimization

Ref:
Gang Xu, Ming Li, Bernard Mourrain, Timon Rabczuk, Jinlan Xu, Stephane P.A. Bordas. Constructing IGA-suitable planar parameterization from complex CAD boundary by domain partition and global/local optimization,CMAME, 2018

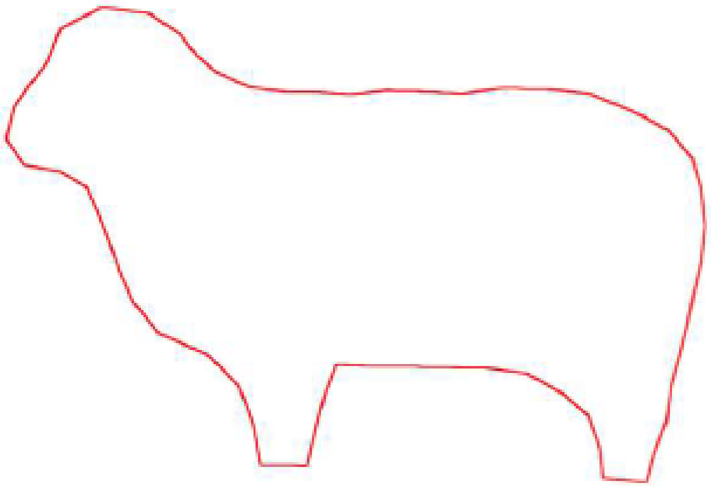
Global optimization method

- Propose a global optimization method to construct the four-sided curved partition of the computational domain

Topology generation of quadrilateral decomposition → Laplacian smoothing method → Construction of segmentation curves

Topology generation of quadrilateral decomposition

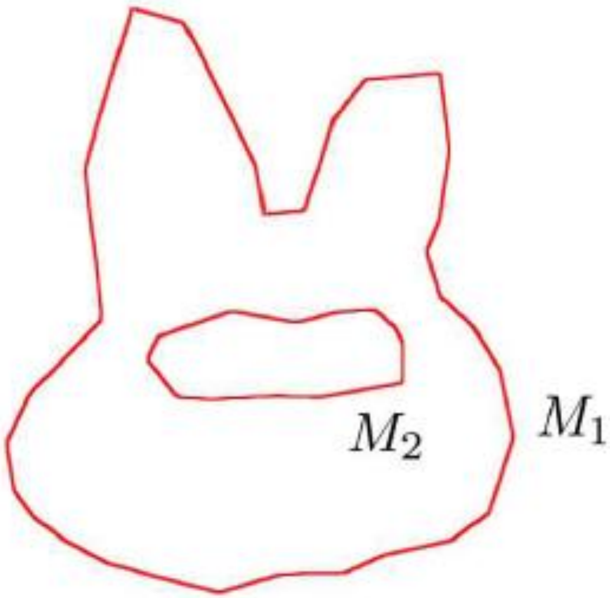
- Step. 1: Construct the discrete boundary by connecting the endpoints of the extracted Bézier curves.



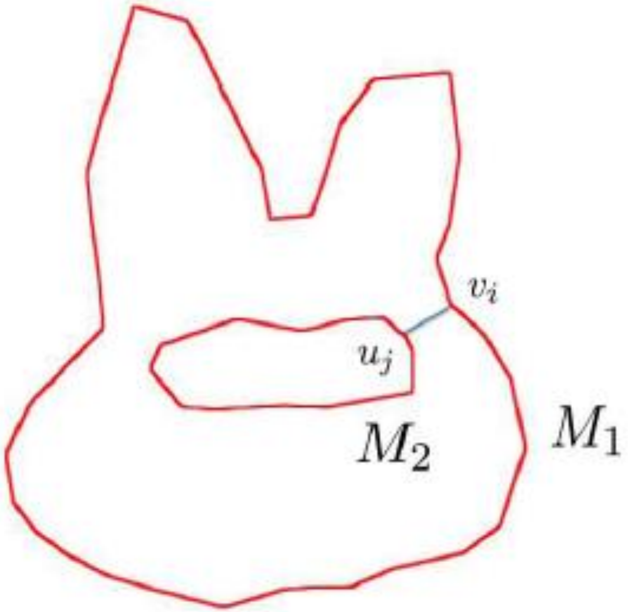
(a)input discrete boundary

Step. 2

- Multiply-connected region → simply-connected region.



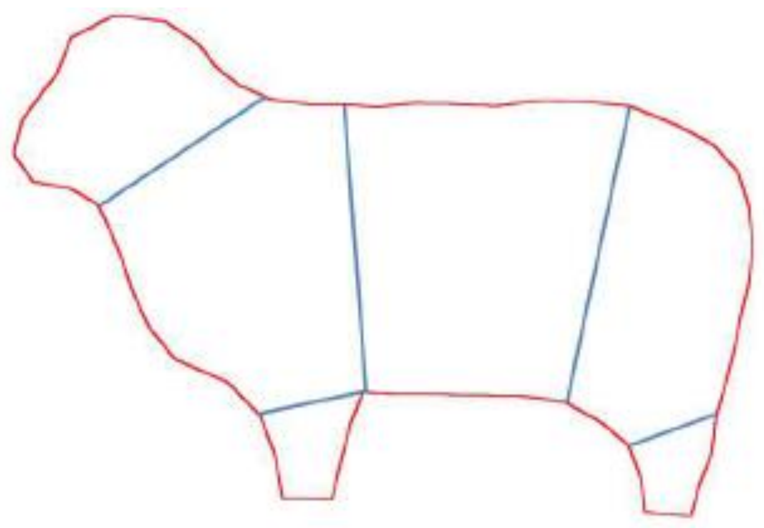
(a)multiply-connected domain



(b) simply-connected domain

Step. 3

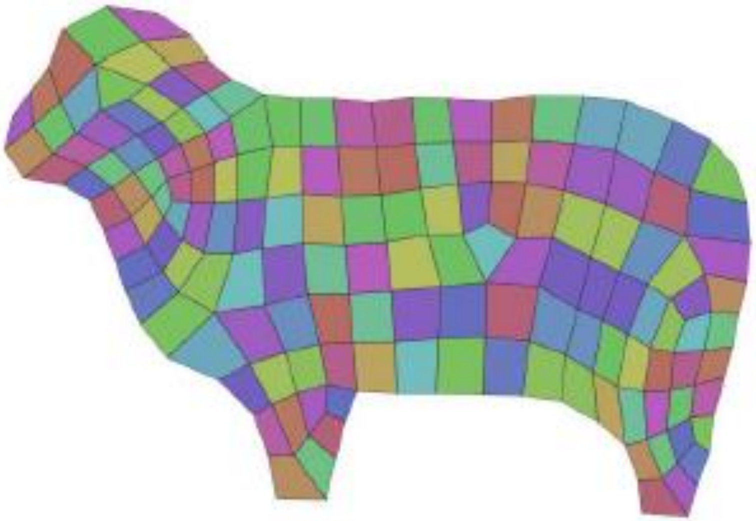
- Approximate convex decomposition of the simply-connected regions



(b) quasi-convex polygon decomposition

Step. 4

- For each quasi-convex polygon obtained in Step.3, generate the quadrangulation topology information
- Only introduce irregular vertices with valence 3 or 5, which guarantees the solution existence for G1 planar parameterization around the irregular vertex



(c) quad-meshing result by our method with 147 elements and 14 irregular vertices

Reference:
K.Takayama,D.Panozzo,O. Sorkine-Hornung Pattern-based quadrangulation for N-sided patches. CGF, 2015

Laplacian smoothing

- We adapt an iterative Laplacian smoothing method to improve the quality of the quad mesh.

$$x_i^k = \frac{\sum_{j=1}^{N_i} x_j^{k-1}}{N_i}, \qquad y_i^k = \frac{\sum_{j=1}^{N_i} y_j^{k-1}}{N_i}$$

Termination rules:

$$\frac{[\sum_{i=1}^{m}[(x_i^k - x_i^{k-1})^2 + (y_i^k - y_i^{k-1})^2]]^{1/2}}{[\sum_{i=1}^{m}[(x_i^{k-1})^2 + (y_i^{k-1})^2]]^{1/2}} < \delta$$

Construction of segmentation curves

- The segmentation curves should interpolate two vertices on the quad mesh Q(V,E).
- Global optimization method to construct the optimal shape of segmentation curves.

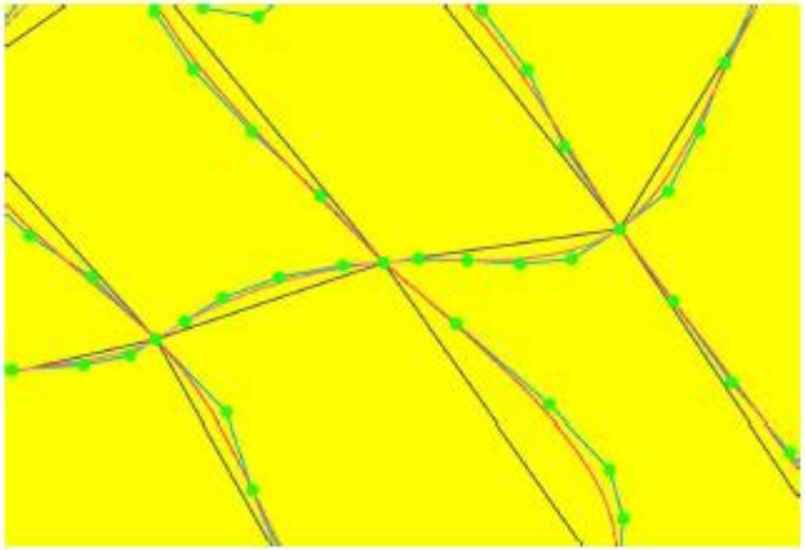


Fig.5(a) segmentation curves(red) and quad edges(black)

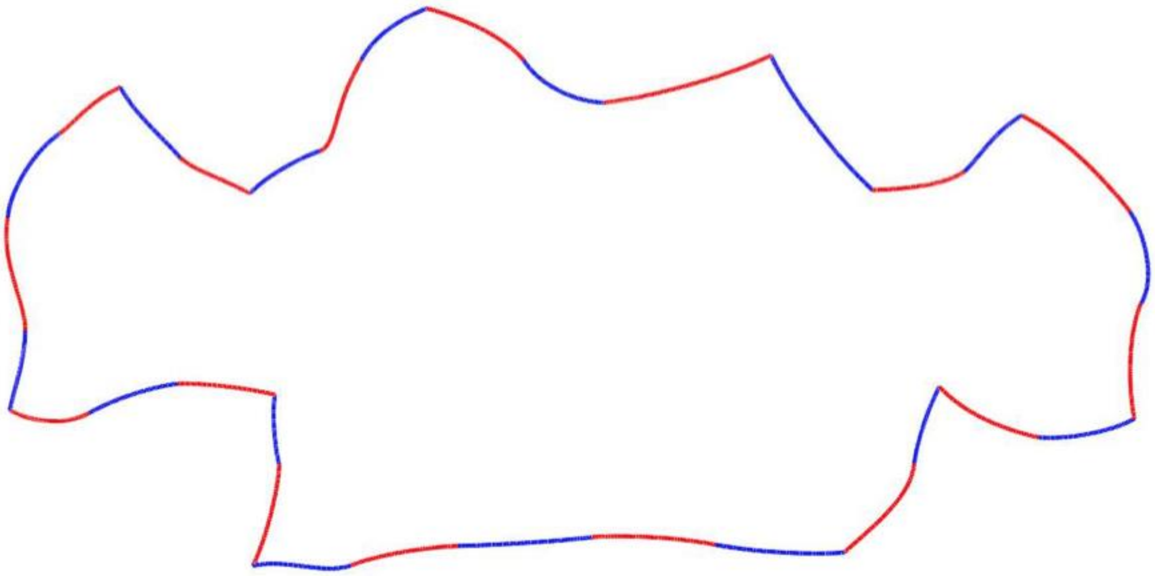
Desired parameterization method

- Boundary-preserving
- Automatic continuity imposition

  ($\neq$ high-order meshing with C^0)
- Automatic construction of segmentation curve
- Injective
- Uniform patch size
- Orthogonal iso-parametric structure

Uniform patch size

Computing the area of planar region bounded by B-spline curves?



Computing the area of planar region with Bézier boundary

- For the planar region bounded by N pieces of Bézier curves

$$\boldsymbol{S}_k(t) = (S_k^x(t), S_k^y(t)) = \sum_{i=1}^{n} (s_{i,k}^x, s_{i,k}^y) B_i^n(t)$$

Then the area A(Ω) of the planar region is

$$A(\Omega) = \frac{1}{4n} \sum_{k=1}^{N} \sum_{j=0}^{2n-1} (c_j^k - d_j^k)$$

$$c_j^k = \sum_{r=max(0,j-n)}^{min(j,n-1)} \frac{\binom{n}{r}\binom{n-1}{j-r}}{\binom{2n-1}{j}} s_{r,k}^x (s_{j-r+1,k}^y - s_{j-r,k}^y)$$

$$d_j^k = \sum_{r=max(0,j-n)}^{min(j,n-1)} \frac{\binom{n}{r}\binom{n-1}{j-r}}{\binom{2n-1}{j}} s_{r,k}^y (s_{j-r+1,k}^x - s_{j-r,k}^x)$$

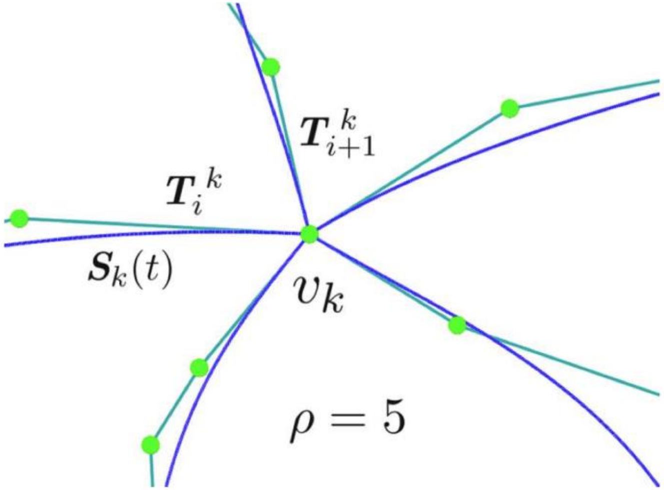
Global optimization method

- Objective functions:

$$F_{\text{uniform}} = \frac{1}{L}\sum_{i=0}^{L}(A_i - A_{ave})^2$$

$$F_{\text{shape}} = \sum_{k=0}^{N}\int_0^1 \sigma_1 \|\boldsymbol{S}_k^{'}(t)\|^2 + \sigma_2 \|\boldsymbol{S}_k^{''}(t)\|^2 dt$$

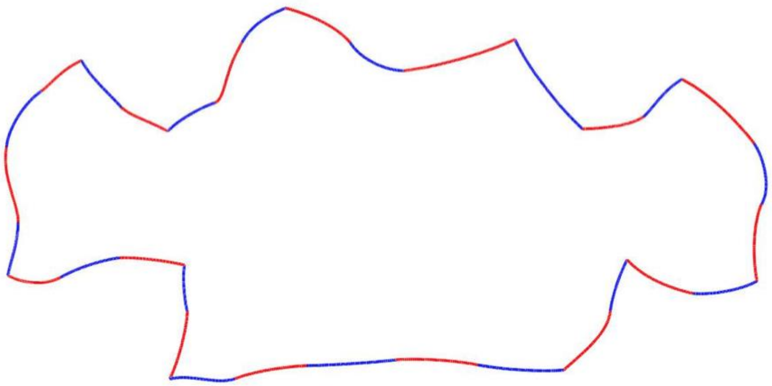
$$F_{\text{tangent}} = \sum_{k=0}^{N}\sum_{i=1}^{\rho}\left(\frac{\boldsymbol{T}_i^{k}\cdot\boldsymbol{T}_{i+1}^{k}}{\|\boldsymbol{T}_i^{k}\|\|\boldsymbol{T}_{i+1}^{k}\|} - \cos\frac{2\pi}{\rho}\right)^2$$



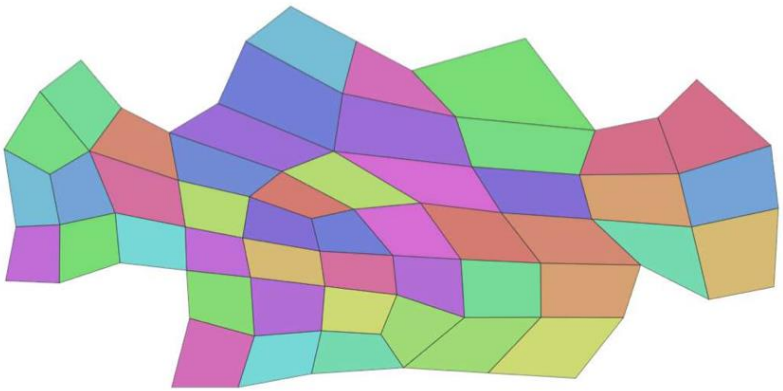
$$F = \omega_1 F_{\text{uniform}} + \omega_2 F_{\text{shape}} + \omega_3 F_{\text{tangent}}$$

$$\underset{s_{i,k}}{\arg\min}\quad F$$

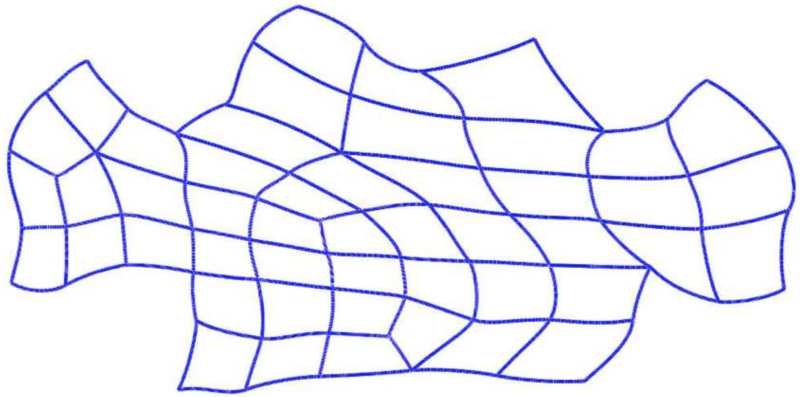
An example



(a) boundary Bézier curves



(e) quad meshing result II



(f) segmentation curves II

Proposed framework

1. Pre-processing for high-quality parameterization
2. Topology information generation of quadrilateral decomposition
3. Quadrilateral patch partition by global optimization
4. High-quality patch parameterization by local optimization

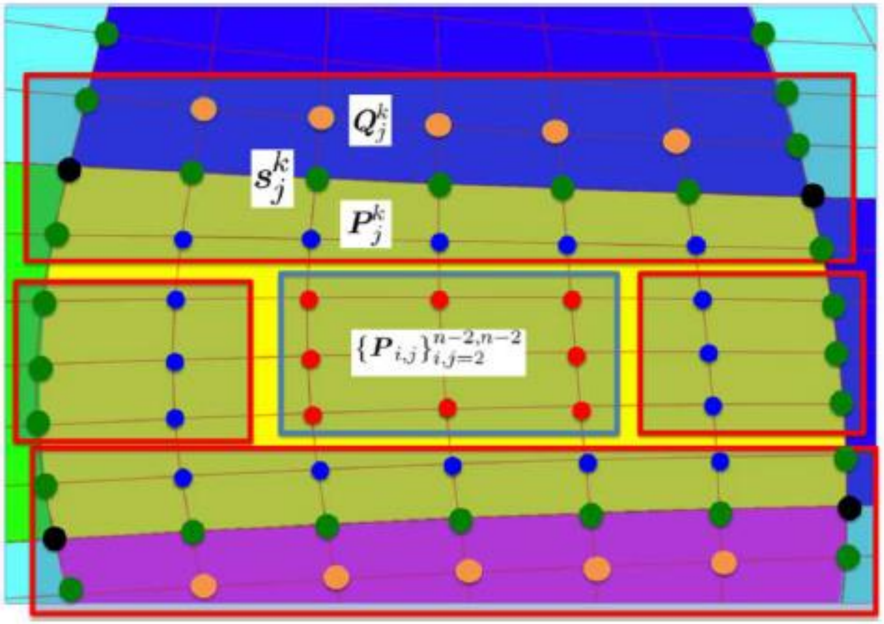
Ref:
Gang Xu, Ming Li, Bernard Mourrain, Timon Rabczuk, Jinlan Xu, Stephane P.A. Bordas. Constructing IGA-suitable planar parameterization from complex CAD boundary by domain partition and global/local optimization,CMAME, 2018

High-quality patch parameterization

Step 1: Construction of boundary second-layer control points with orthogonality optimization and continuity constraints.

Step 2: Local C1 linear-energy-minimizing method for constructing inner control points.

Step 3: Find out the invalid patches on the parameterization, then recover patch validity.



Step. 1: Construction of boundary control points

Initial construction

→

Imposition of C^1-continuity at regular region

→

Imposition of G^1-continuity around the irregular vertices

Initial construction

- Firstly, we will describe the initial construction by orthogonality optimization.

$$\boxed{\boldsymbol{P}^0_{n-1,j} = \boldsymbol{P}_{n,j} + \frac{(\boldsymbol{P}_{0,j} - \boldsymbol{P}_{n,j})}{n}}$$

$$\boxed{\underset{\boldsymbol{P}_{n-1,j}}{\arg\min} \int_0^1 (<\boldsymbol{r}_{1,u}(1,v), \boldsymbol{r}_{1,v}(1,v)>)^2 dv}$$

$$\boxed{\begin{aligned} \boldsymbol{r}_{1,u}(1,v) &= n\sum_{j=0}^{n} B_l^n(v)\Delta^{1,0}\boldsymbol{P}_{n-1,l}, \\ \boldsymbol{r}_{1,v}(1,v) &= n\sum_{j=0}^{n-1} B_l^{n-1}(v)\Delta^{0,1}\boldsymbol{P}_{n,l}, \end{aligned}}$$

Imposition of C¹-continuity by Lagrange Multiplier method

Minimize the change of related control points along the segmentation curves such that they satisfy the C¹-constraints

$$\boxed{\boldsymbol{s}_j^k - \boldsymbol{P}_j^k = \boldsymbol{Q}_j^k - \boldsymbol{s}_j^k,}$$

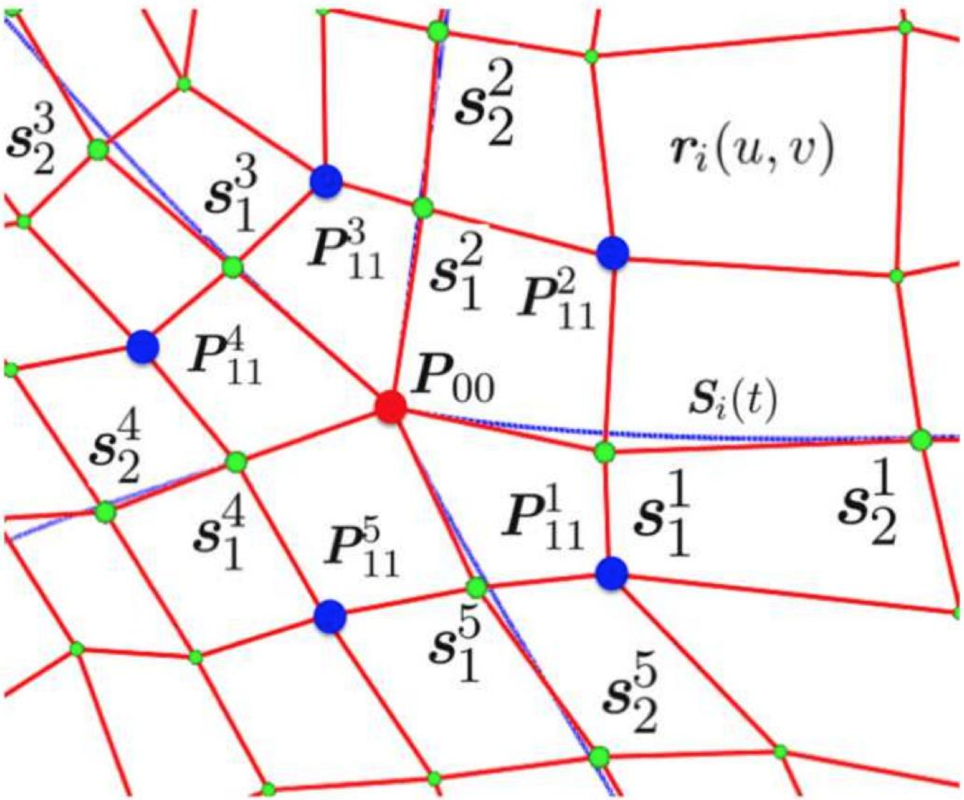
$$\boxed{\mathrm{Min} \sum_{k=1}^{N} \sum_{j=0}^{n} (\|\boldsymbol{P}_j^k - \bar{\boldsymbol{P}}_j^k\|^2 + \|\boldsymbol{Q}_j^k - \bar{\boldsymbol{Q}}_j^k\|^2)}$$

the Lagrange function:

$$\boxed{L = \sum_{i=0}^{N} \sum_{j=0}^{n} (\|\boldsymbol{P}_j^k - \bar{\boldsymbol{P}}_j^k\|^2 + \|\boldsymbol{Q}_j^k - \bar{\boldsymbol{Q}}_j^k\|^2) + \sum_{i=0}^{N} \sum_{j=0}^{n} \lambda_{k,j} (2\boldsymbol{s}_j^k - \boldsymbol{P}_j^k - \boldsymbol{Q}_j^k)}$$

Imposition of G^1-continuity around irregular vertex

- Some special treatments should be done achieve G^1-continuity at the irregular vertices.



G^1-continuity

Imposition of G¹-continuity (Mourrain et al， CAGD 2016)

- The G¹-continuity constraints around the irregular vertex can be described as follows:

$$(\boldsymbol{s}_1^i - \boldsymbol{P}_{00}) = \alpha_i(\boldsymbol{s}_1^{i+1} - \boldsymbol{P}_{00}) + \beta_i(\boldsymbol{s}_1^{i-1} - \boldsymbol{P}_{00}),$$

$$\boldsymbol{0} = n\alpha_i(\boldsymbol{P}_{11}^i - \boldsymbol{s}_1^i) + n\beta_i(\boldsymbol{P}_{11}^{i-1} - \boldsymbol{s}_1^i) - (n-1)(\boldsymbol{s}_2^i - \boldsymbol{s}_1^i) + (\boldsymbol{s}_1^i - \boldsymbol{P}_{00})$$

$$\begin{pmatrix} \alpha_1 & 0 & \dots & 0 & \beta_1 \\ \beta_2 & \alpha_2 & \dots & 0 & 0 \\ 0 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \alpha_{M-1} & 0 \\ 0 & 0 & \dots & \beta_M & \alpha_M \end{pmatrix} \begin{pmatrix} \boldsymbol{P}_{11}^1 \\ \boldsymbol{P}_{11}^2 \\ \boldsymbol{P}_{11}^3 \\ \vdots \\ \boldsymbol{P}_{11}^{M-1} \\ \boldsymbol{P}_{11}^M \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \boldsymbol{H}_3 \\ \vdots \\ \boldsymbol{H}_{M-1} \\ \boldsymbol{H}_M \end{pmatrix}$$

There exists unique solution for M=3 and M=5

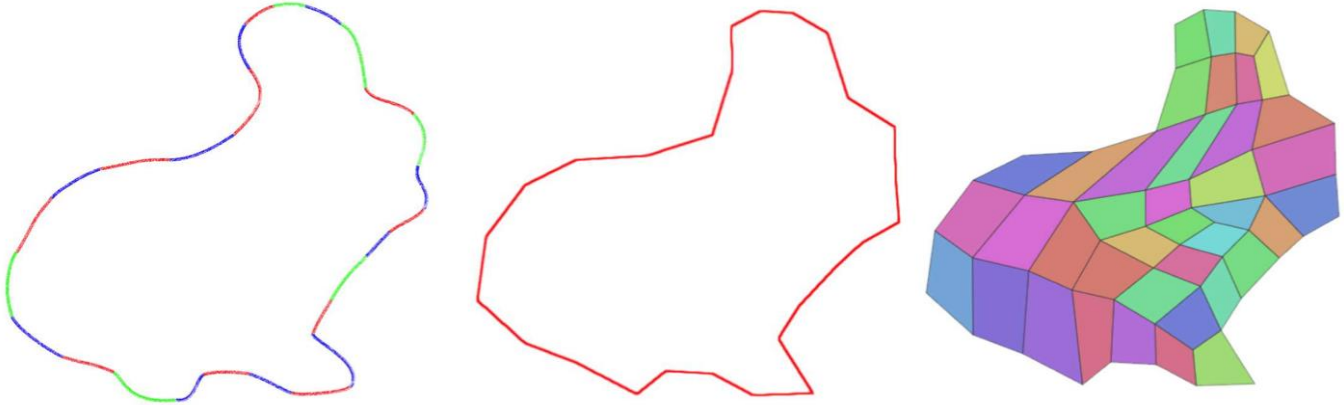
High-quality patch parameterization

Step 1: Construction of boundary second-layer control points with orthogonality optimization and continuity constraints.

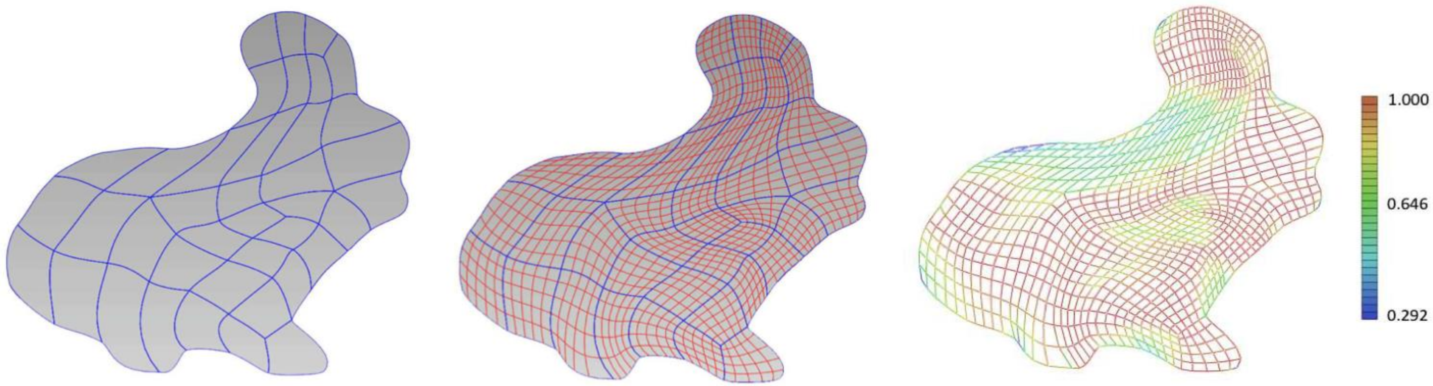
Step 2: Local C1 linear-energy-minimizing method for constructing inner control points.

Step 3: Find out the invalid patches on the parameterization, then recover patch validity.

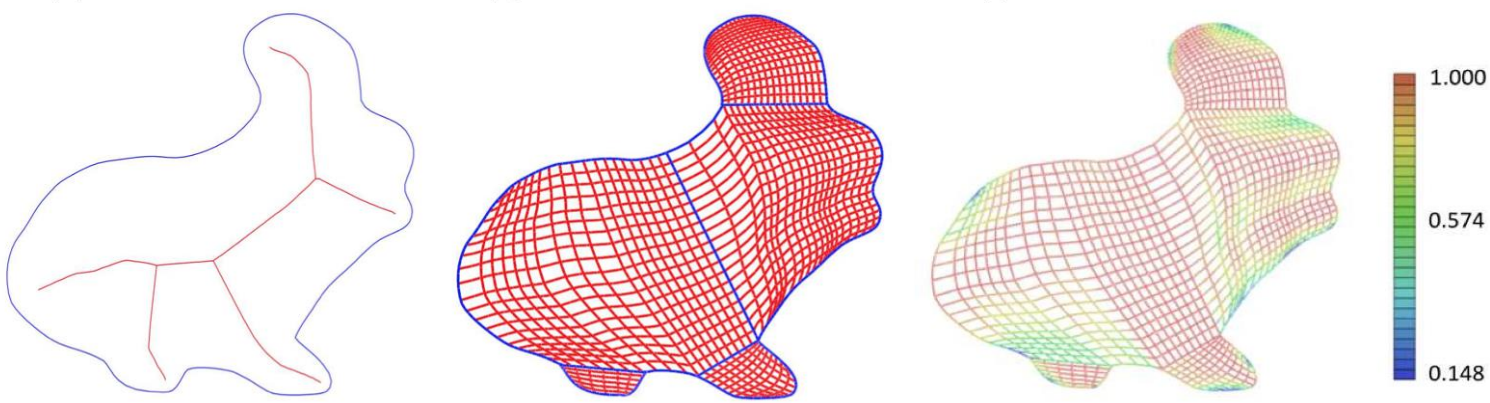
Example I



(a) boundary Bézier curves (b) discrete boundary (c) quad meshing result

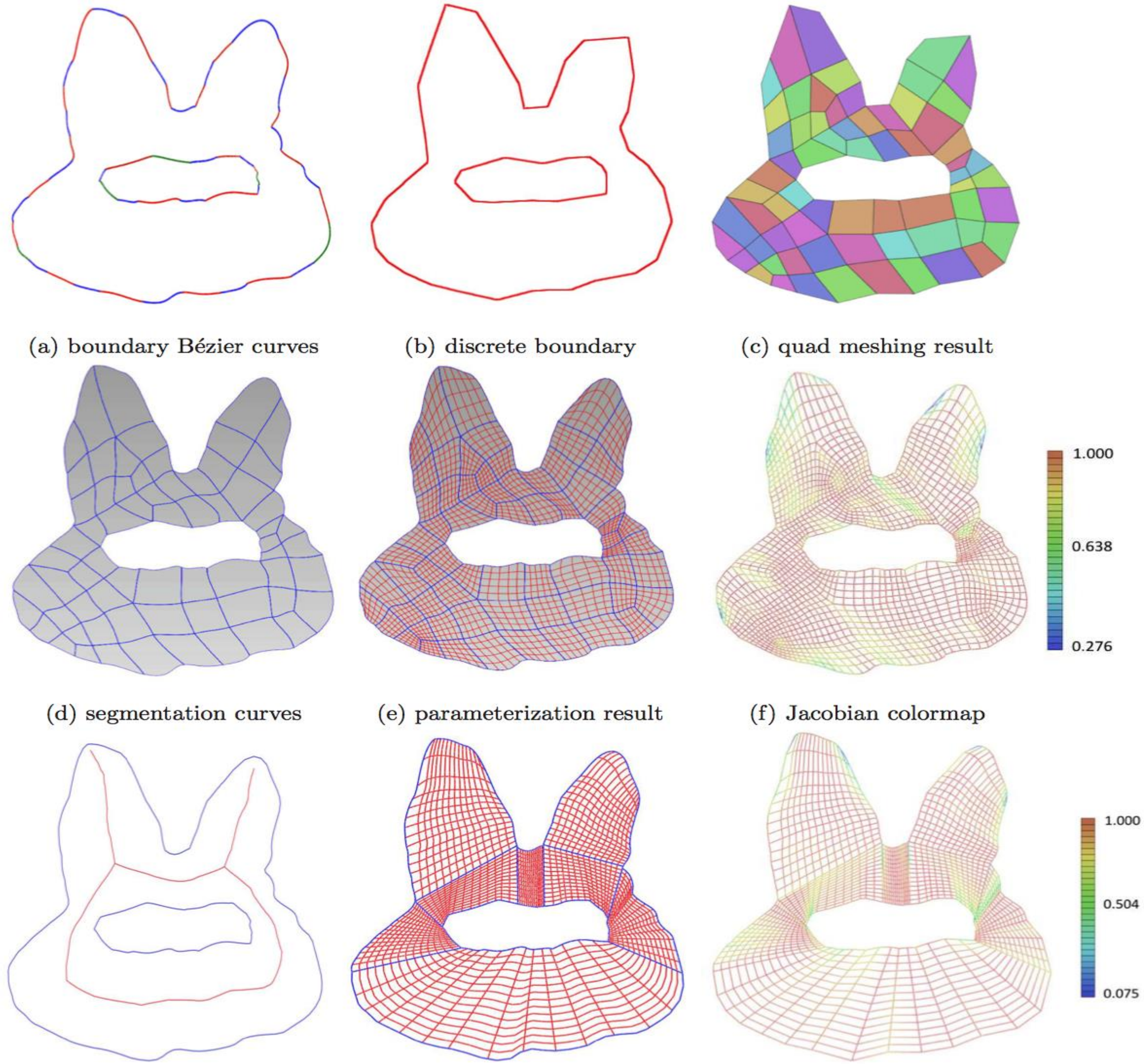


(d) segmentation curves (e) parameterization result (f) Jacobian colormap



(g) extracted skeleton [42] (h) skeleton-based (i) Jacobian colormap of (h)

Example II

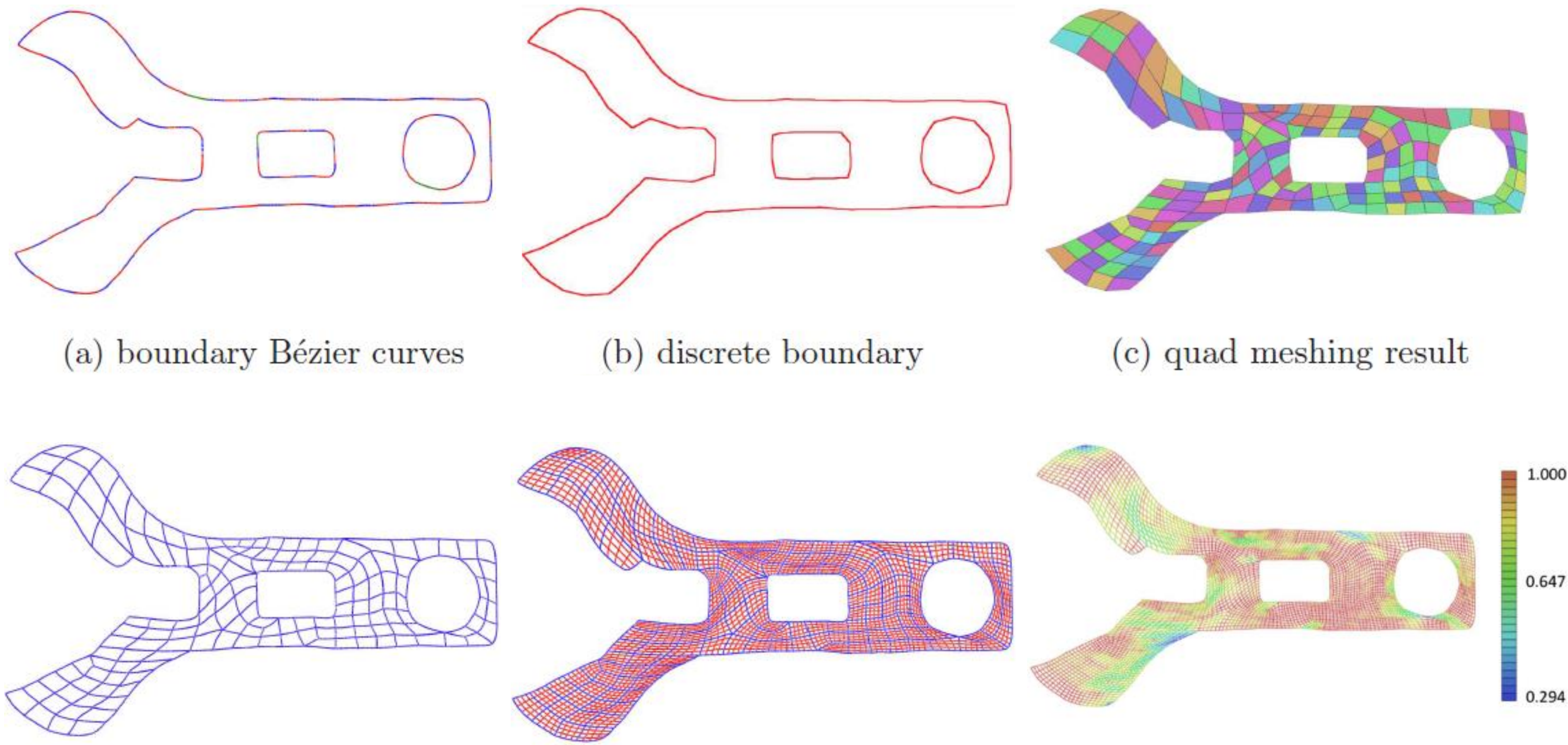


(a) boundary Bézier curves (b) discrete boundary (c) quad meshing result

(d) segmentation curves (e) parameterization result (f) Jacobian colormap

(g) extracted skeleton [42] (h) skeleton-based parameterization [42] (i) Jacobian colormap of (h)

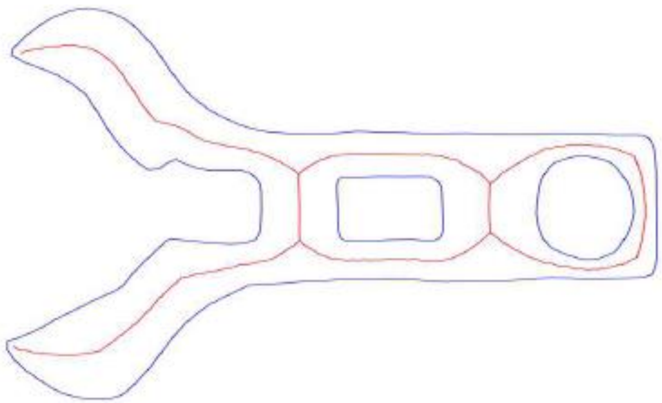
Example III



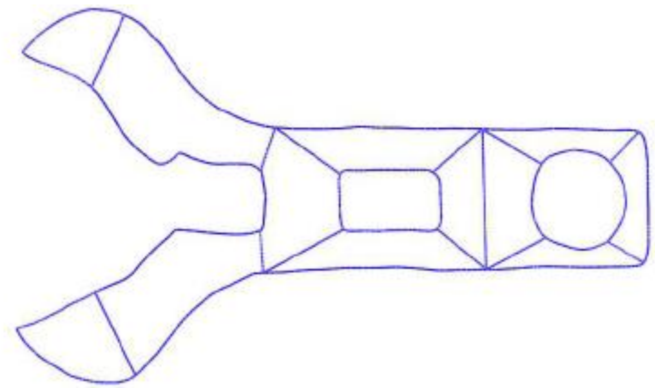
(a) boundary Bézier curves (b) discrete boundary (c) quad meshing result

(d) segmentation curves (e) parameterization result (f) Jacobian colormap

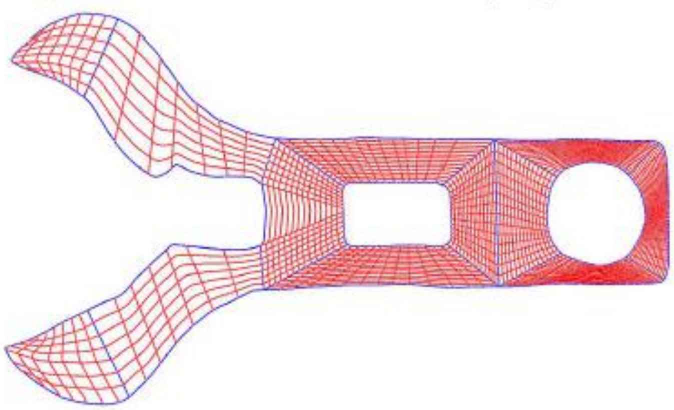
Example III with the skeleton-based decomposition



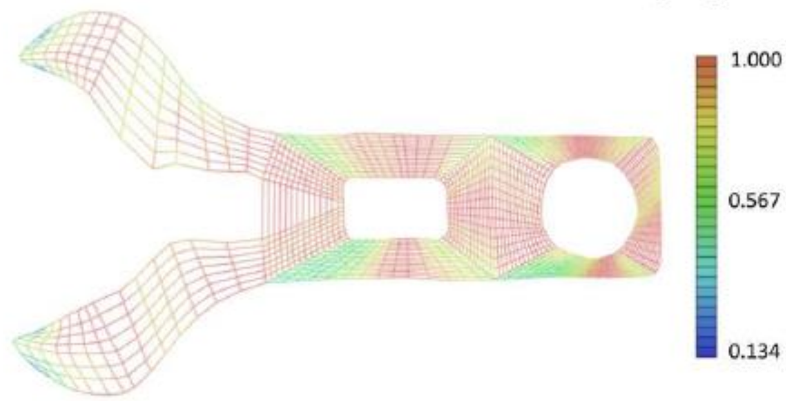
(g) extracted skeleton [42]



(h) skeleton-based domain partition[42]



(i) skeleton-based parameterization [42]



(j) Jacobian colormap of (i)

Fig. 11. Example V.

Quality comparison

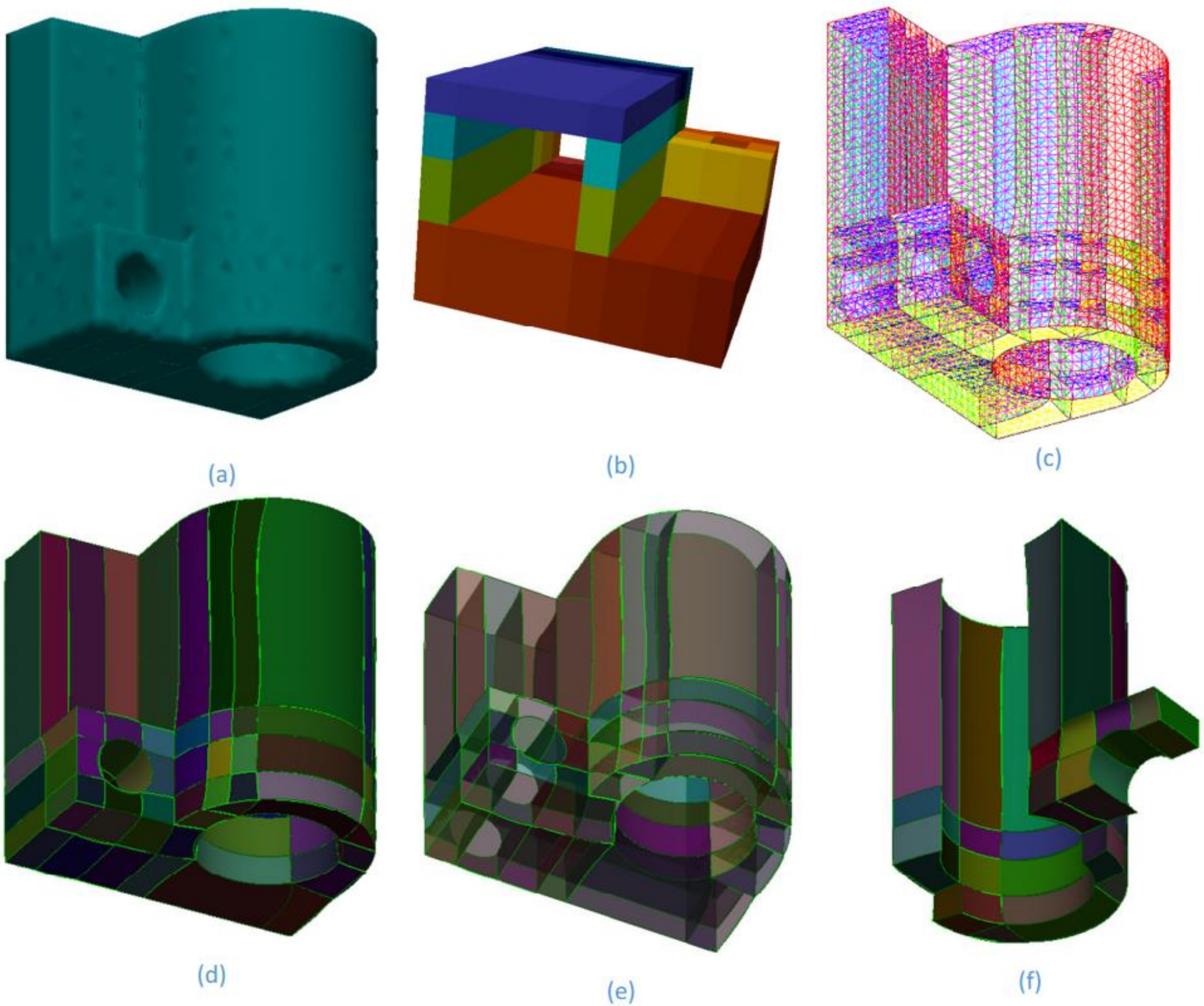
Table 2: Quantitative data for planar parameterization in Fig. 9, Fig. 10 and Fig. 11. p: degree of planar parameterization; # SD: number of subdomains by domain decomposition;# Patch: number of Bézier patches; # Con.: number of control points.

Example	Method	p	# SD	#Patch	# Con.	Scaled Jacobian		Conditional number	
						Average	Min	Average	Max
Fig. 9	Our method	6	39	39	1467	0.8843	0.292	2.76	8.06
	Xu et al.[42]	6	5	35	1309	0.5172	0.148	5.36	16.31
Fig. 10	Our method	5	66	66	1768	0.9194	0.276	2.42	10.18
	Xu et al.[42]	5	8	56	1507	0.7801	0.075	4.35	18.23
Fig. 11	Our method	5	155	155	3720	0.9017	0.294	2.57	7.86
	Xu et al.[42]	5	12	132	3282	0.7894	0.134	4.23	15.64

Parameters and computing time

Example	F_{shape} in (14)		F in (16)			$E(\boldsymbol{r})$ in (28)		# T_1	# T_2	# T
	σ_1	σ_2	ω_1	ω_2	ω_3	τ_1	τ_2			
Fig. 1	2.0	1.0	2.0	1.0	50.0	2.0	1.5	73.22	177.86	251.08
Fig. 8(c)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	22.68	26.64	49.32
Fig. 8(g)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	22.96	27.18	50.14
Fig. 8(k)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	27.68	36.32	63.90
Fig. 9	1.0	2.0	1.0	2.0	50.0	1.0	2.0	32.74	53.84	86.58
Fig. 10	2.0	1.0	1.0	2.0	50.0	2.0	1.0	41.02	61.36	102.38
Fig. 11	2.0	1.0	2.0	2.0	50.0	2.0	1.0	75.70	209.68	285.38

Volumetric parameterization with polycube structure

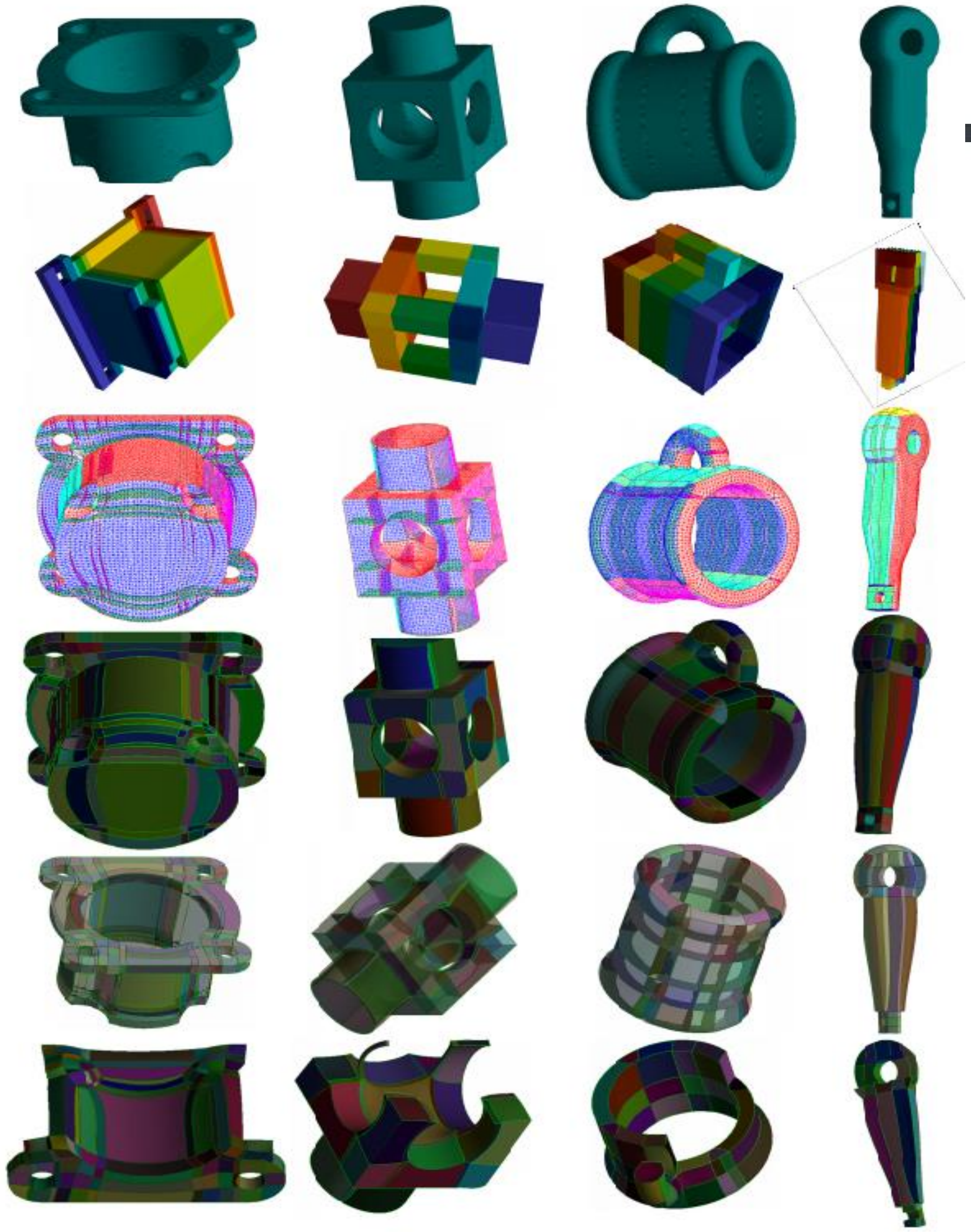


(a) (b) (c) (d) (e) (f)

Framework

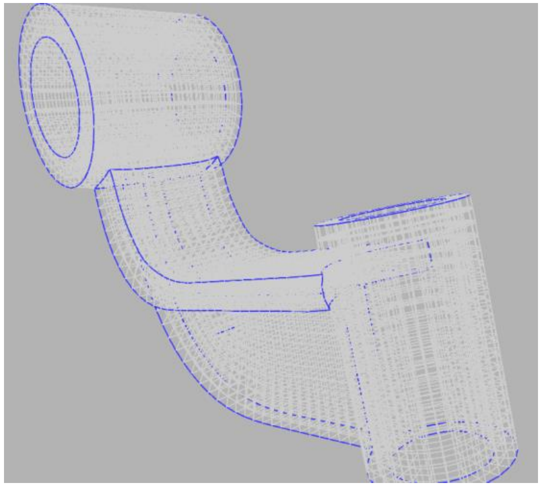
- Generate ℓ_1-based polycube structure from the given triangle mesh
- Directed graph simplification of polycube structure
- Construction of cutting surfaces
- Volume interpolation from the cutting spline surfaces

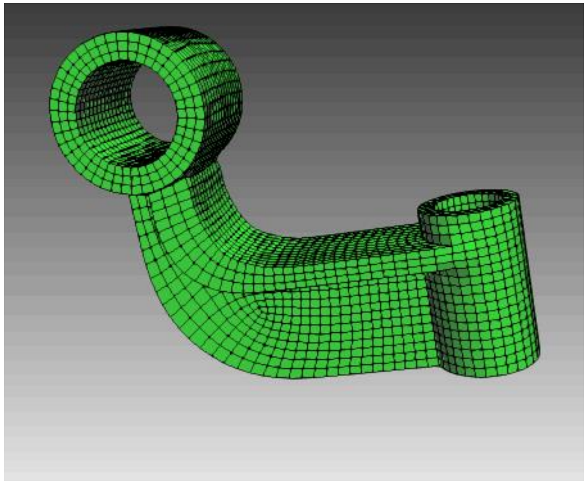
Examples



G^1 volumetric interpolation

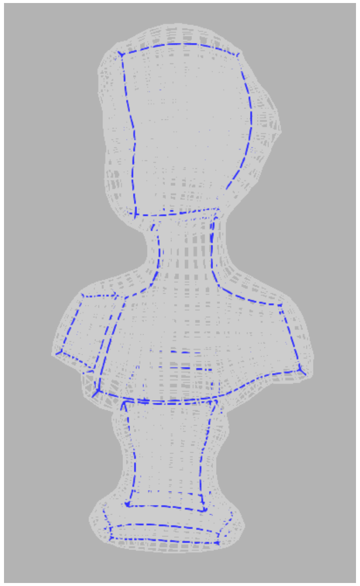
Constructing unstructured trivariate Bézier representation with geometric continuity constraints at extraordinary vertices/edges to interpolate a specified unstructured hexahedral mesh



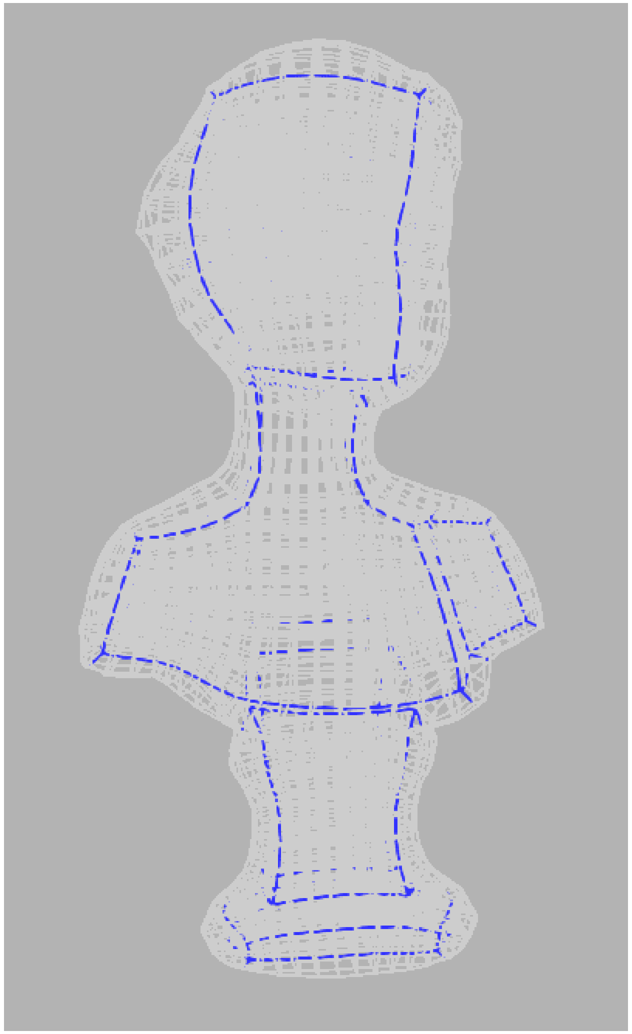


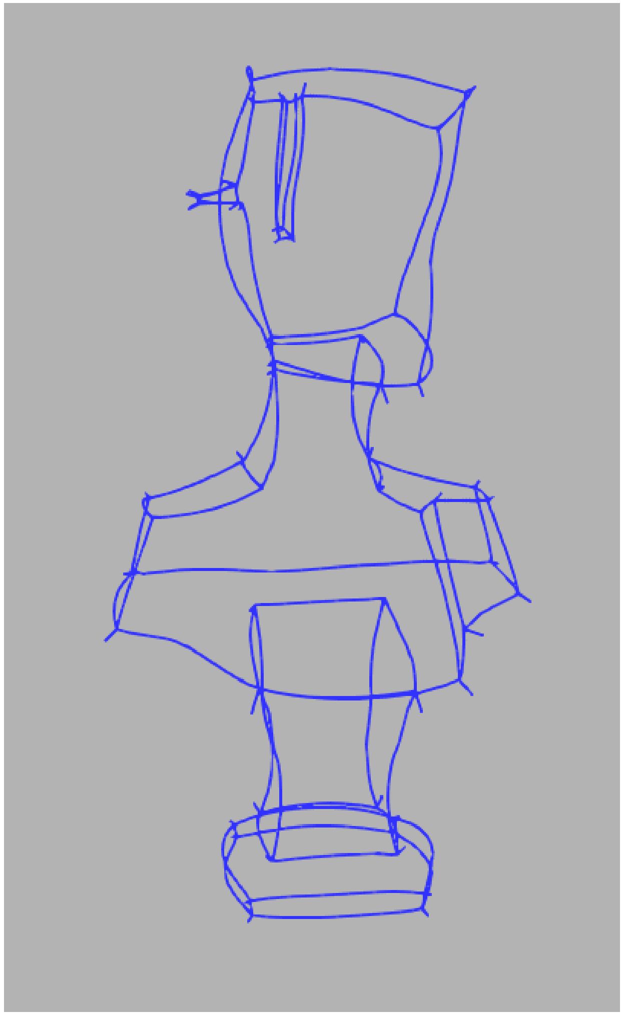
Main difficulty ---- definition of G^1 continuity

- G^1-continuity at regular parts
- How to define the G^1-continuity along the irregular edges?
- How to define the G^1-continuity around irregular vertices?

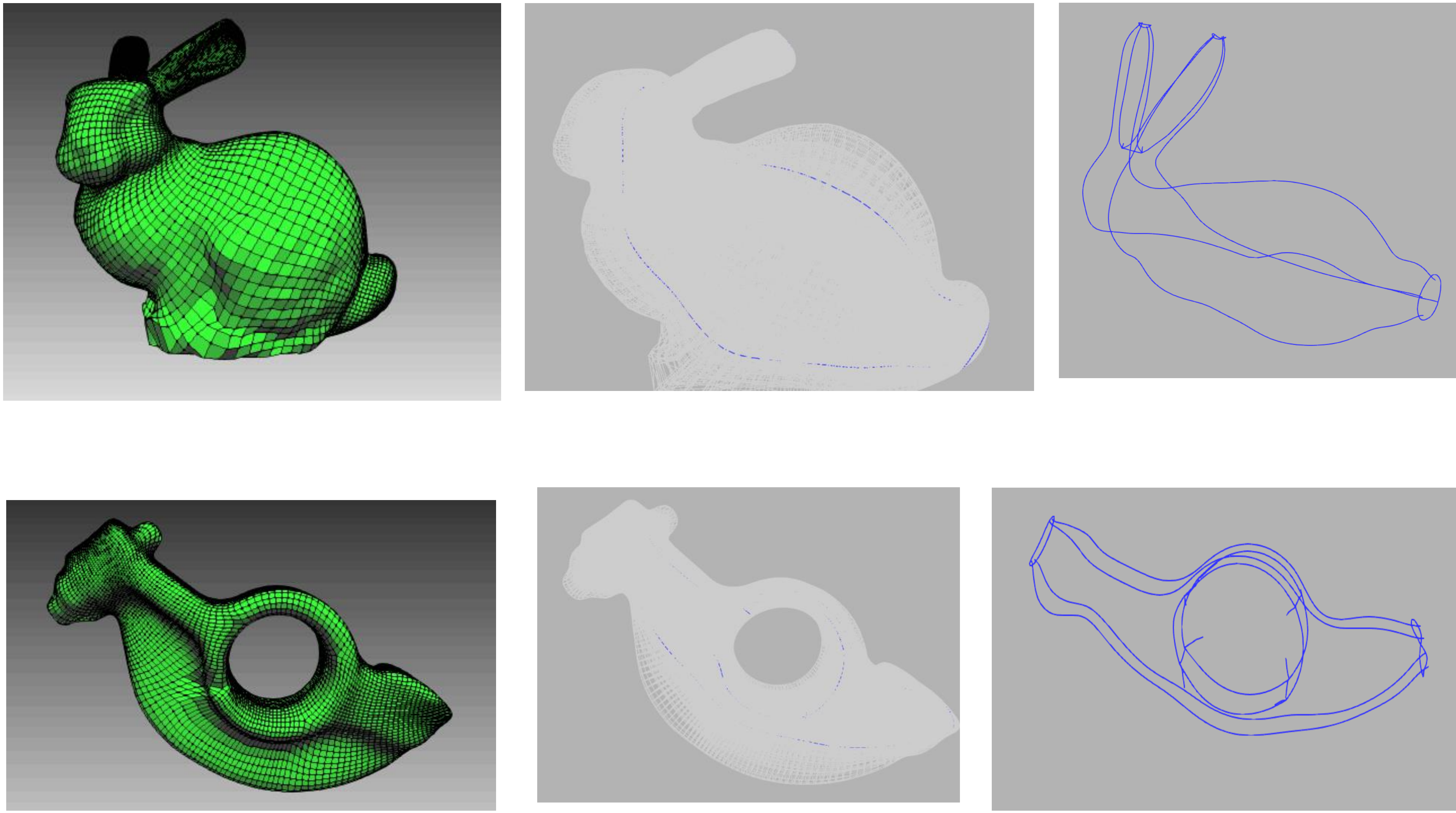


Extraction of irregular structure for hex-mesh

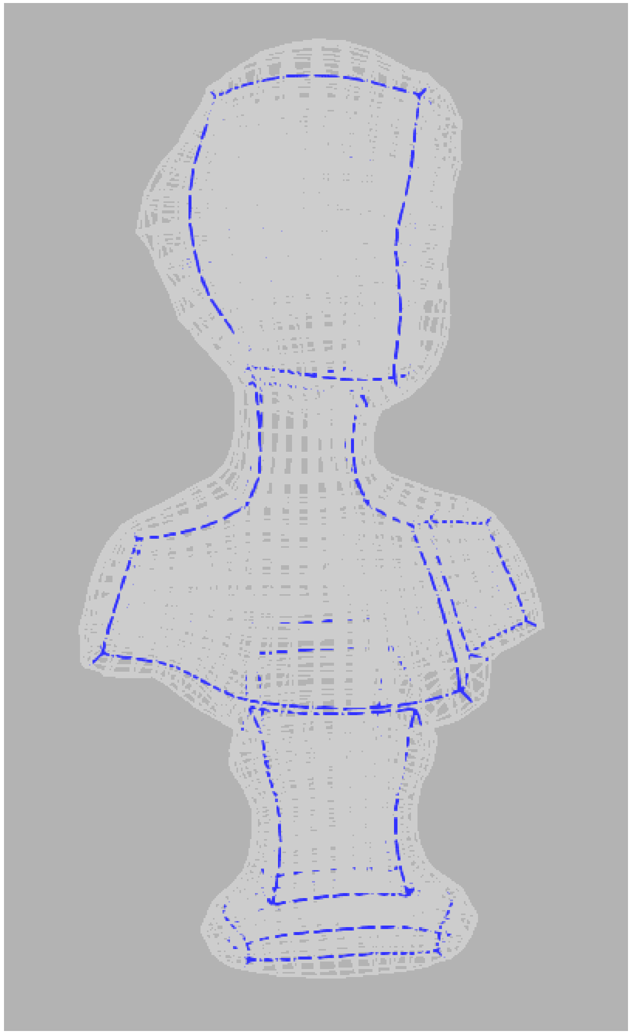


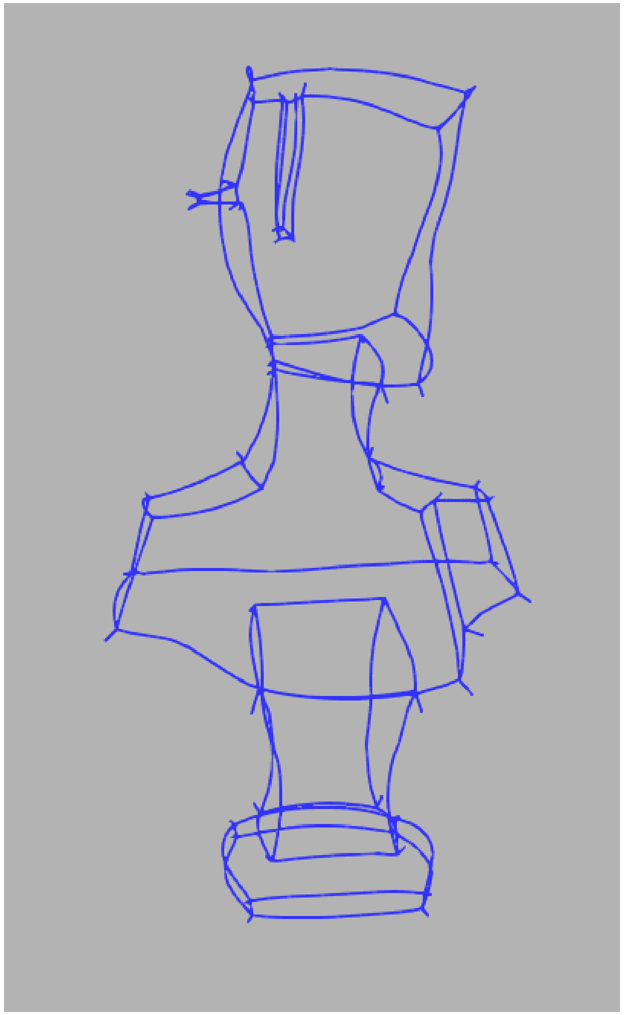


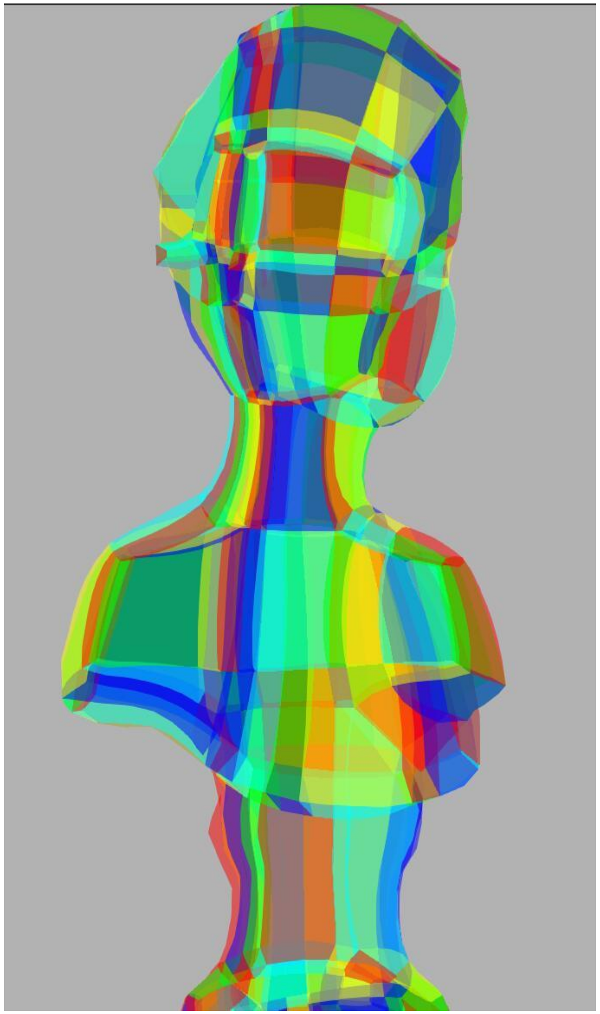
Extraction of irregular structure for hex-mesh



Extraction of irregular structure for hex-mesh

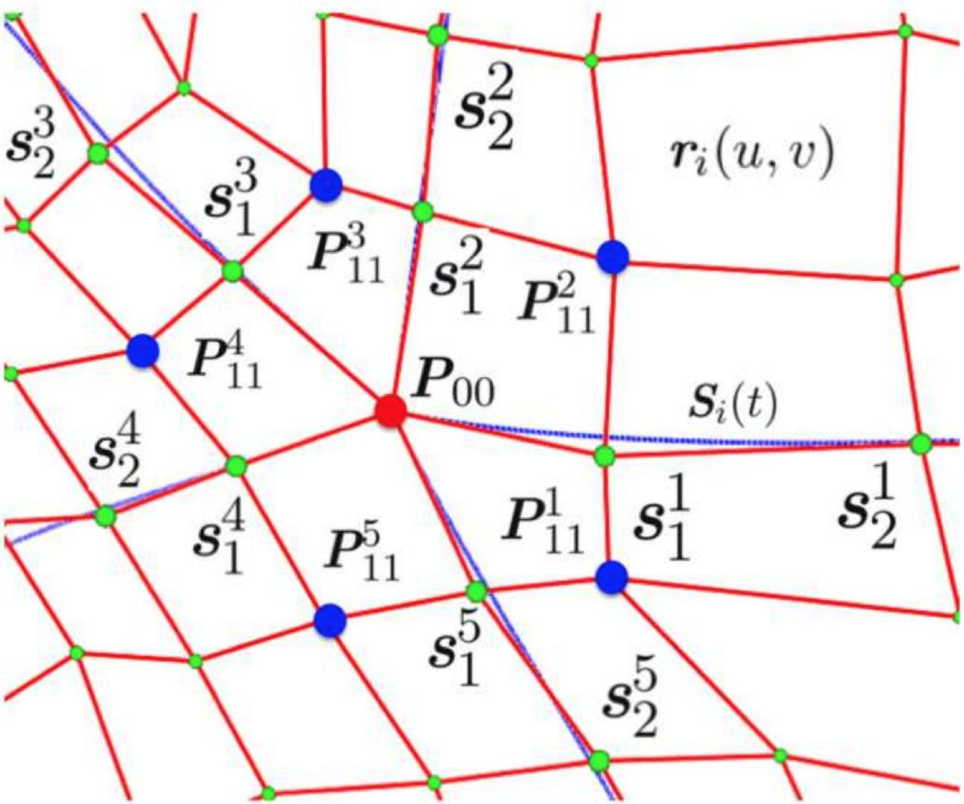




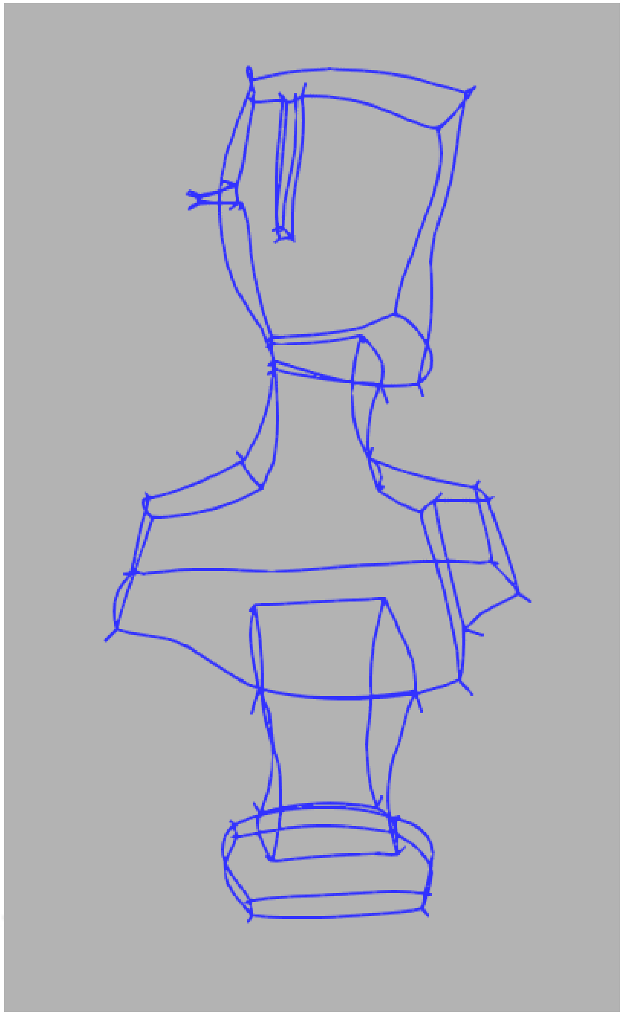


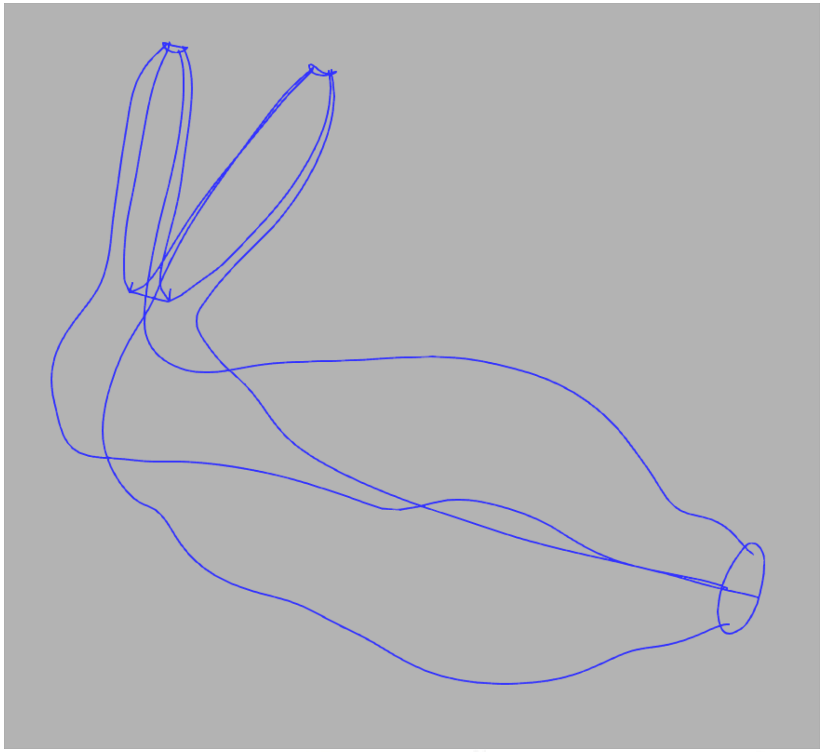
G^1-continuity along the irregular edges

- Extension of the G^1-continuity around the irregular vertex in planar parameterization.



G^1-continuity around irregular vertex





Outline

- Parameterization of computational domain
- Optimal parameterization in isogeometric analysis
- IGA-suitable parameterization from boundary
- Conclusion and future work

Conclusion

- Parameterization of computational domain
- Optimal parameterization
- IGA-suitable parameterization of the computational domain with complex CAD boundary.
- Future work
  - Time-varying parameterization for IGA-dynamic problems
  - Automatic imposition of continuity constraints
  - Volume parameterization for CAD models with trimmed patches

Thanks for your attention !