

Tetrahedral Meshing in the Wild

Yixin Hu¹, Qingnan Zhou², Xifeng Gao¹,
Alec Jacobson³, Denis Zorin¹, Daniele Panozzo¹

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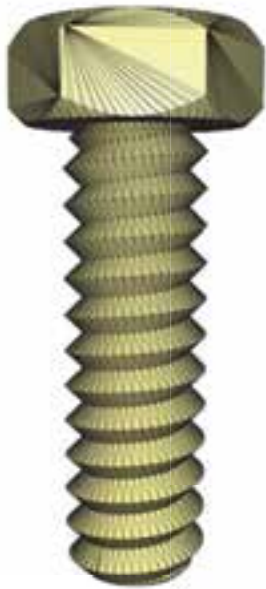
Adobe

3

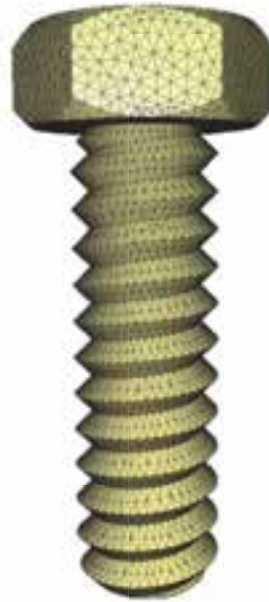


Computer Science
UNIVERSITY OF TORONTO

Tetrahedral Meshing



Surface Representation
(From Thingi10k)



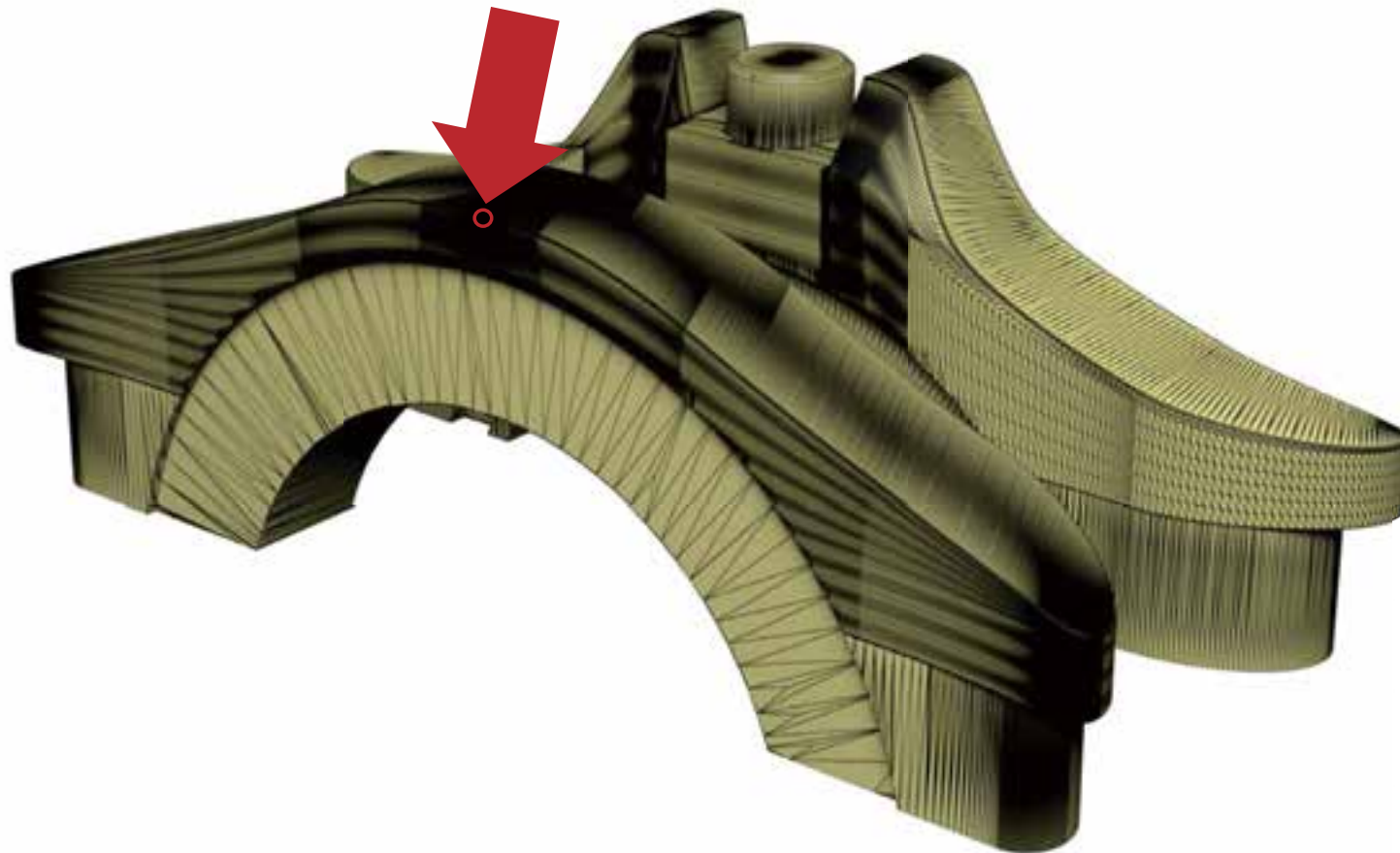
Volumetric Representation
(Generated by TetWild)



Physical Simulation



Why is it a Hard Problem?



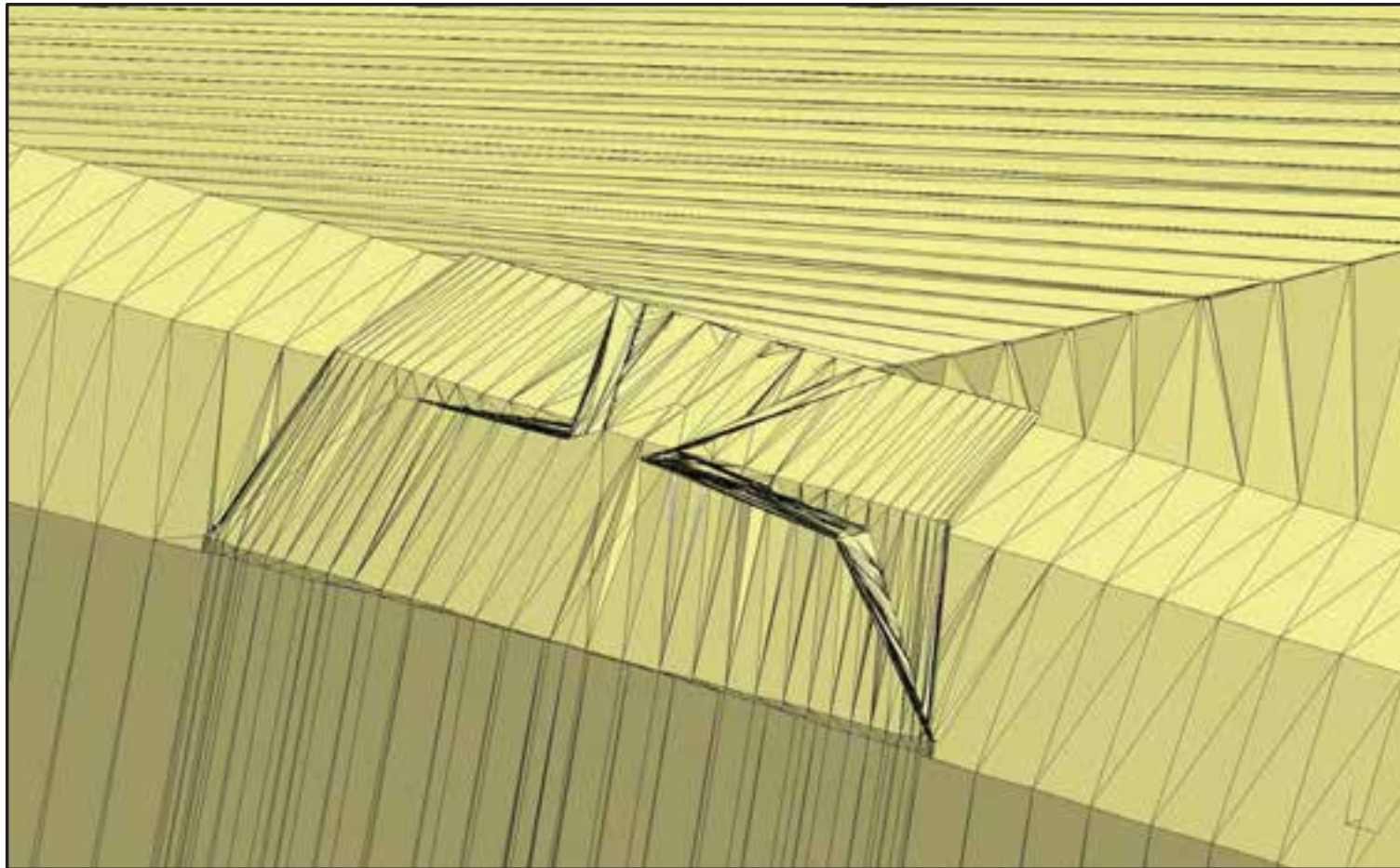
Yixin Hu, 2



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Why is it a Hard Problem?



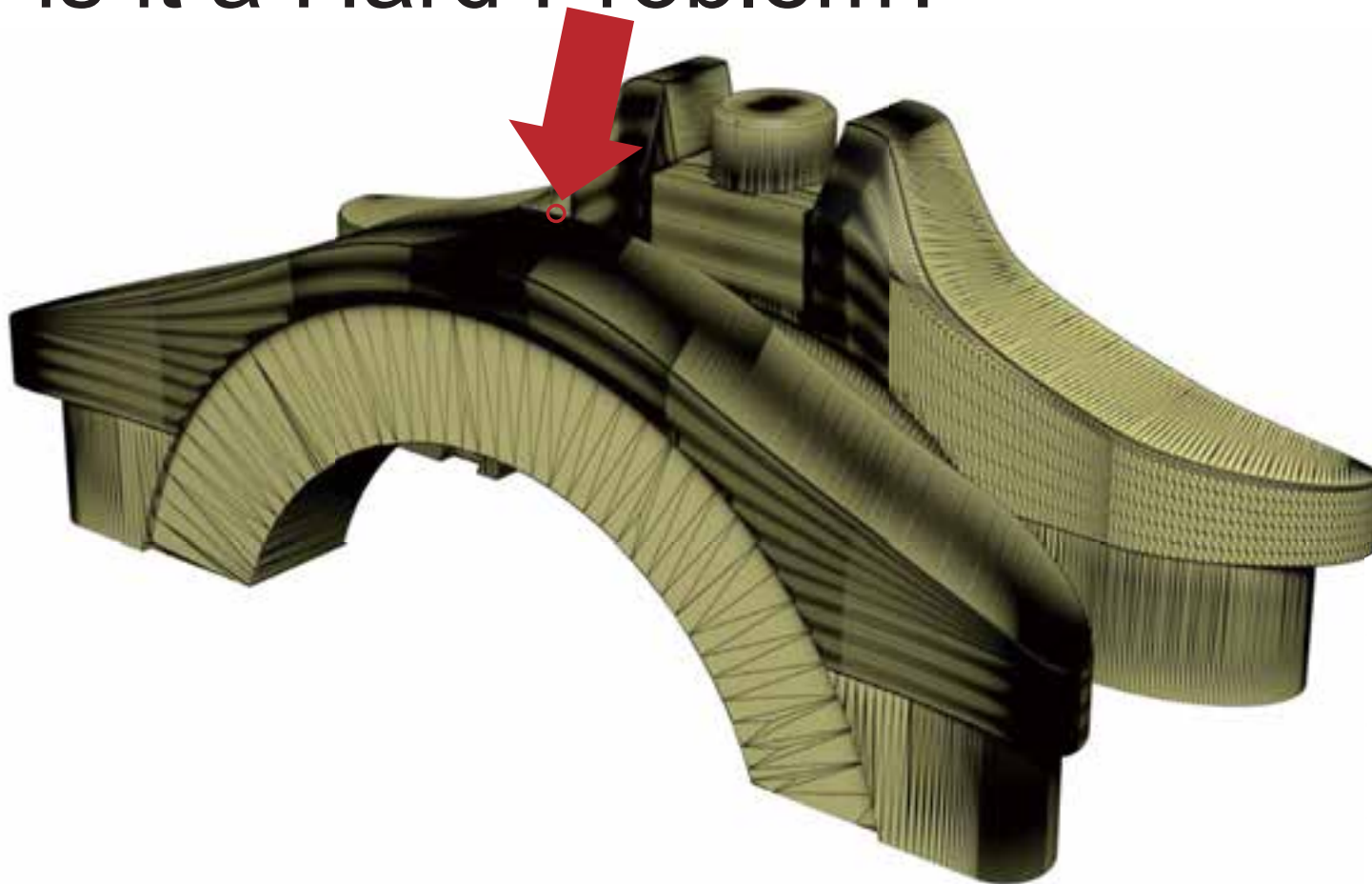
Yixin Hu, 3



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Why is it a Hard Problem?



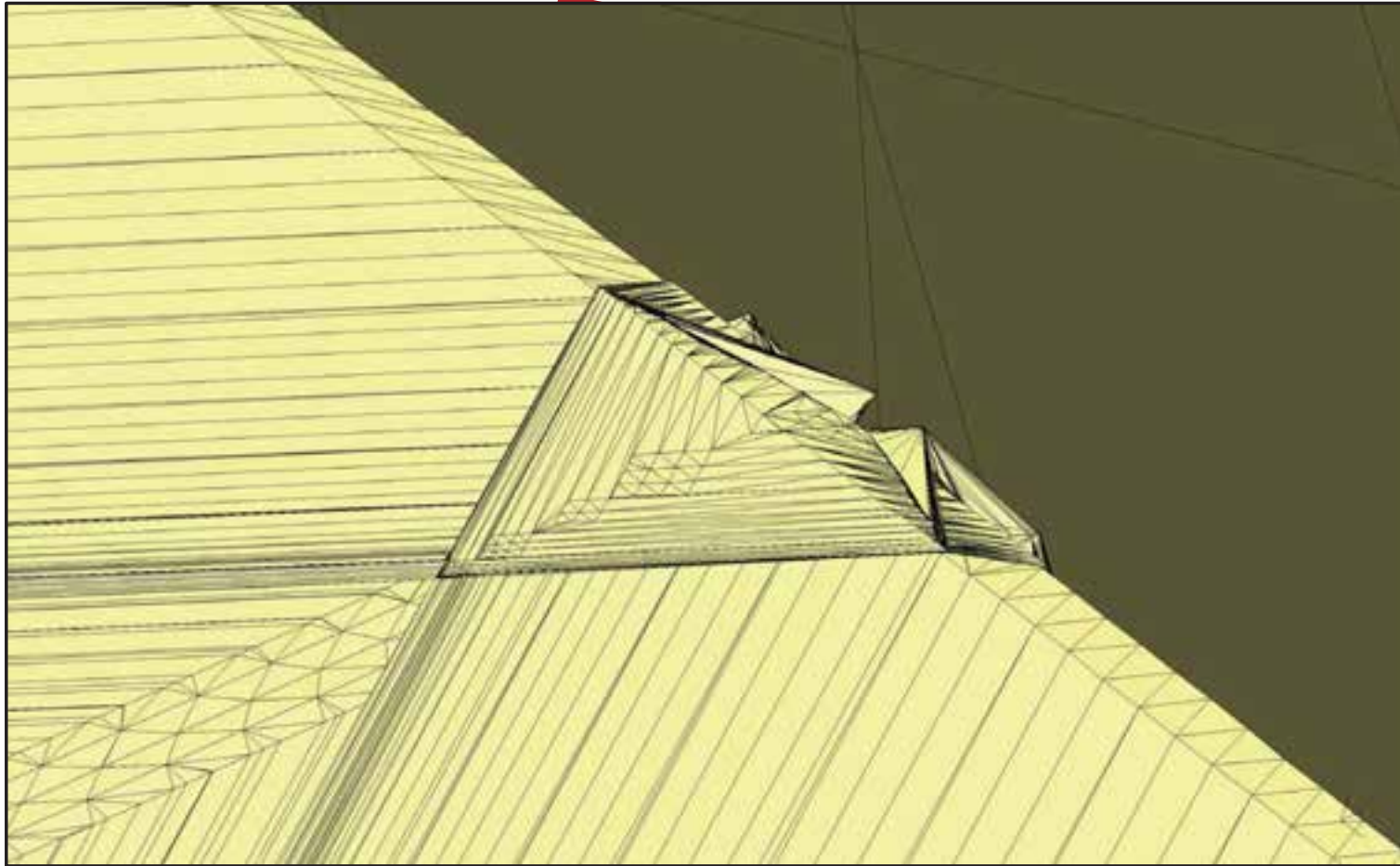
Yixin Hu, 4



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Why is it a Hard Problem?



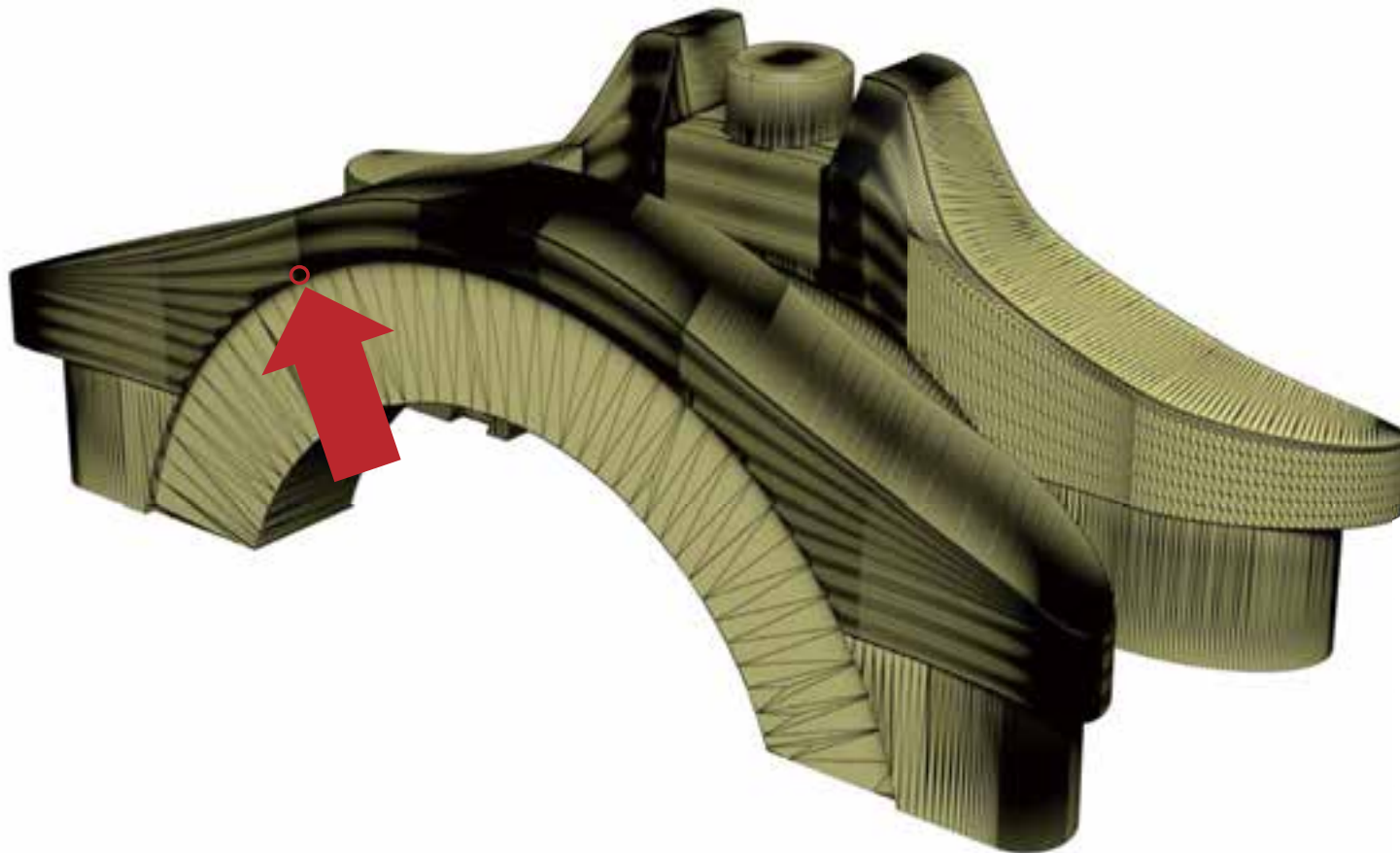
Yixin Hu, 5



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Why is it a Hard Problem?



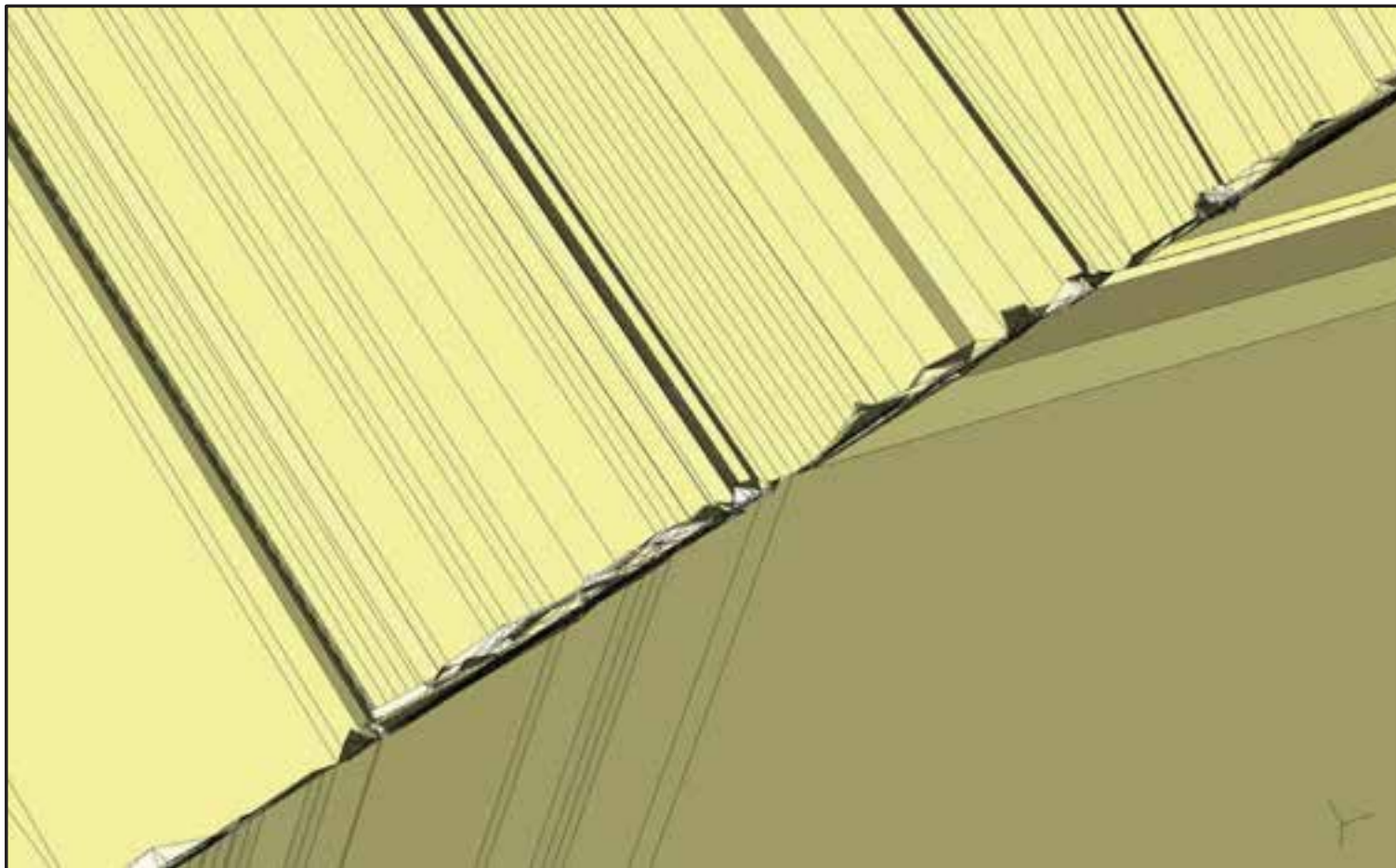
Yixin Hu, 6



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Why is it a Hard Problem?



Yixin Hu, 7



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Why is it a Hard Problem?



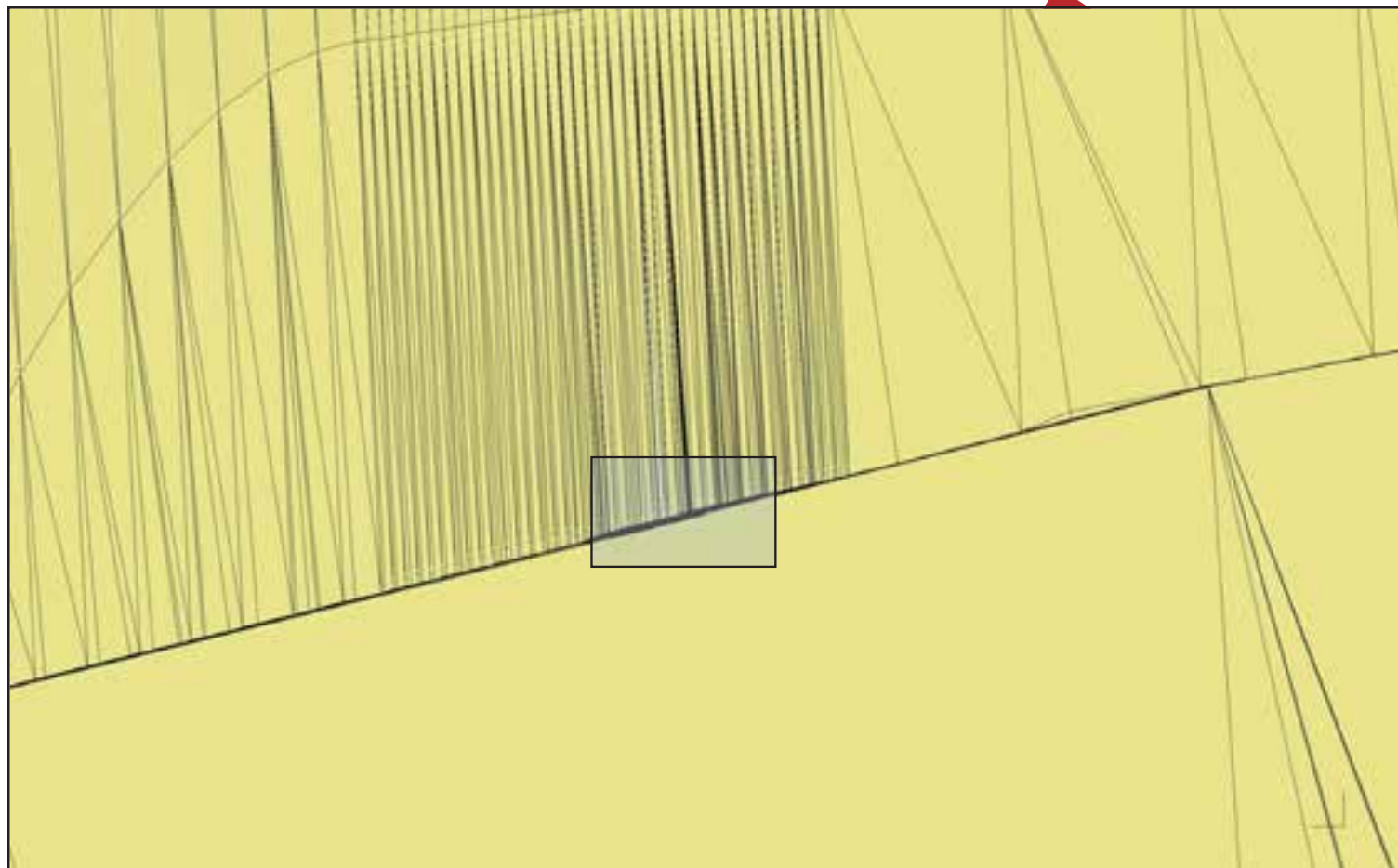
Yixin Hu, 8



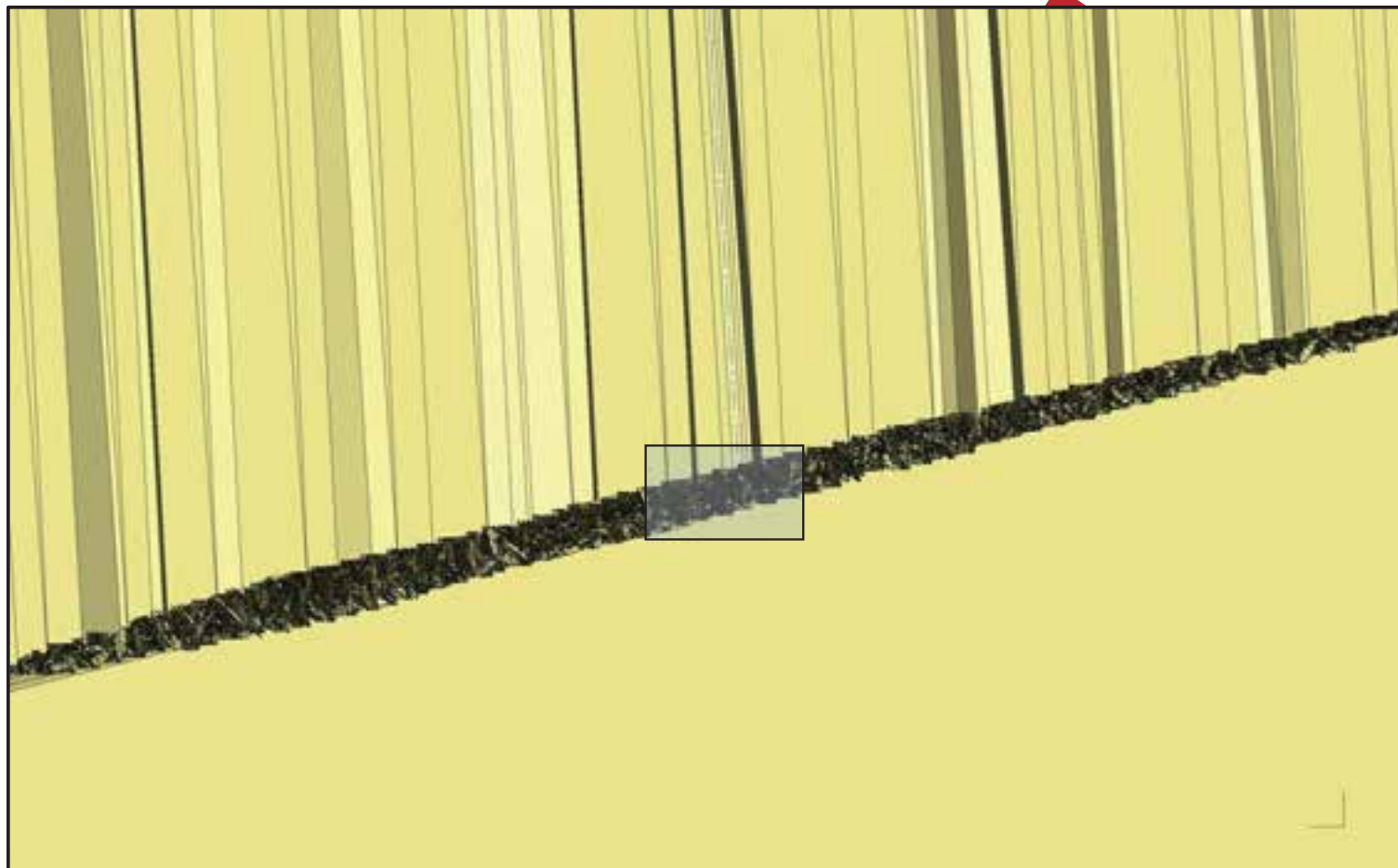
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Why is it a Hard Problem?



Why is it a Hard Problem?



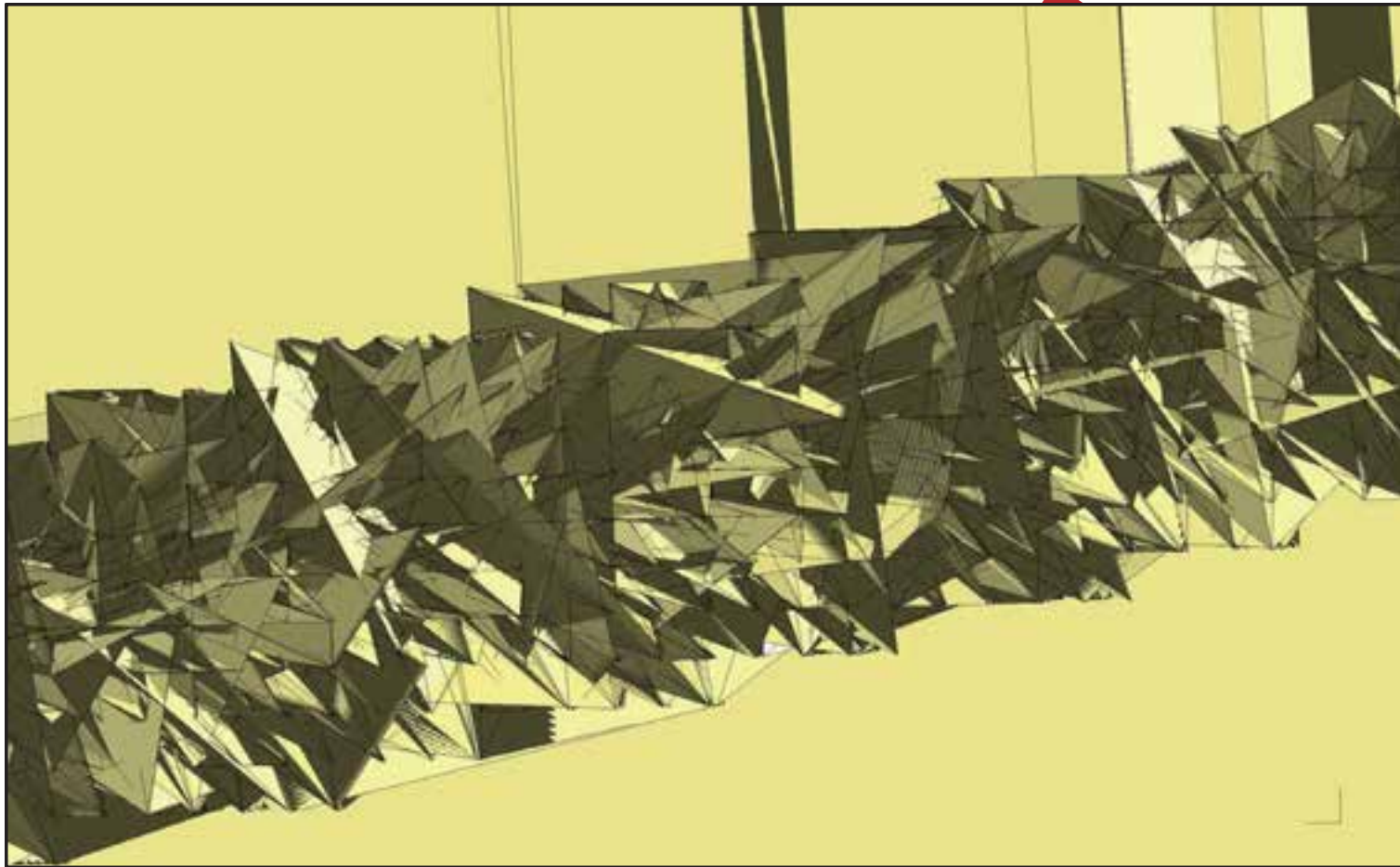
Yixin Hu, 10



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Why is it a Hard Problem?



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New Problem

[cs.GR] 2 Jul 2016

Thing10K: A Dataset of 10,000 3D-Printing Models

Qingnan Zhou
New York University

Alec Jacobson
Columbia University

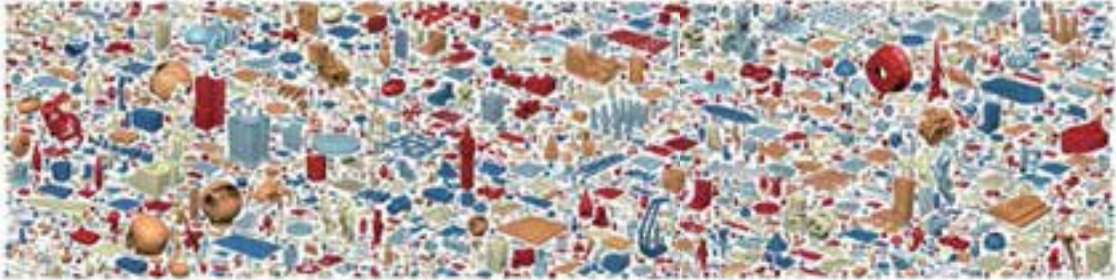


Figure 1: The Thing10K dataset contains 10,000 models from featured "things" on thingiverse.com, a popular online repository.

Abstract

Empirically validating new 3D-printing related algorithms and implementations requires testing data representative of inputs encountered in the wild. An ideal benchmarking dataset should not only draw from the same distribution of shapes people print in terms of class (e.g., toys, mechanisms, jewelry), representation type (e.g., triangle soup meshes) and complexity (e.g., number of facets), but should also capture problems and artifacts endemic to 3D print-

the demand for state-of-the-art processing techniques and automation within 3D printing pipelines.

However, testing remains inadequate. Existing datasets contain only sanitized models (e.g., [Aim@Shape 2004; Levoy et al. 2005; Myles et al. 2014]) or draw from populations containing raw models not specifically intended for printing (rather, e.g., for shape classification [Shilane et al. 2004; Chang et al. 2015] or scene understanding [Nathan Silberman & Fergus 2012; Choi et al. 2016]).





Tetrahedral Meshing on Messy Surface?

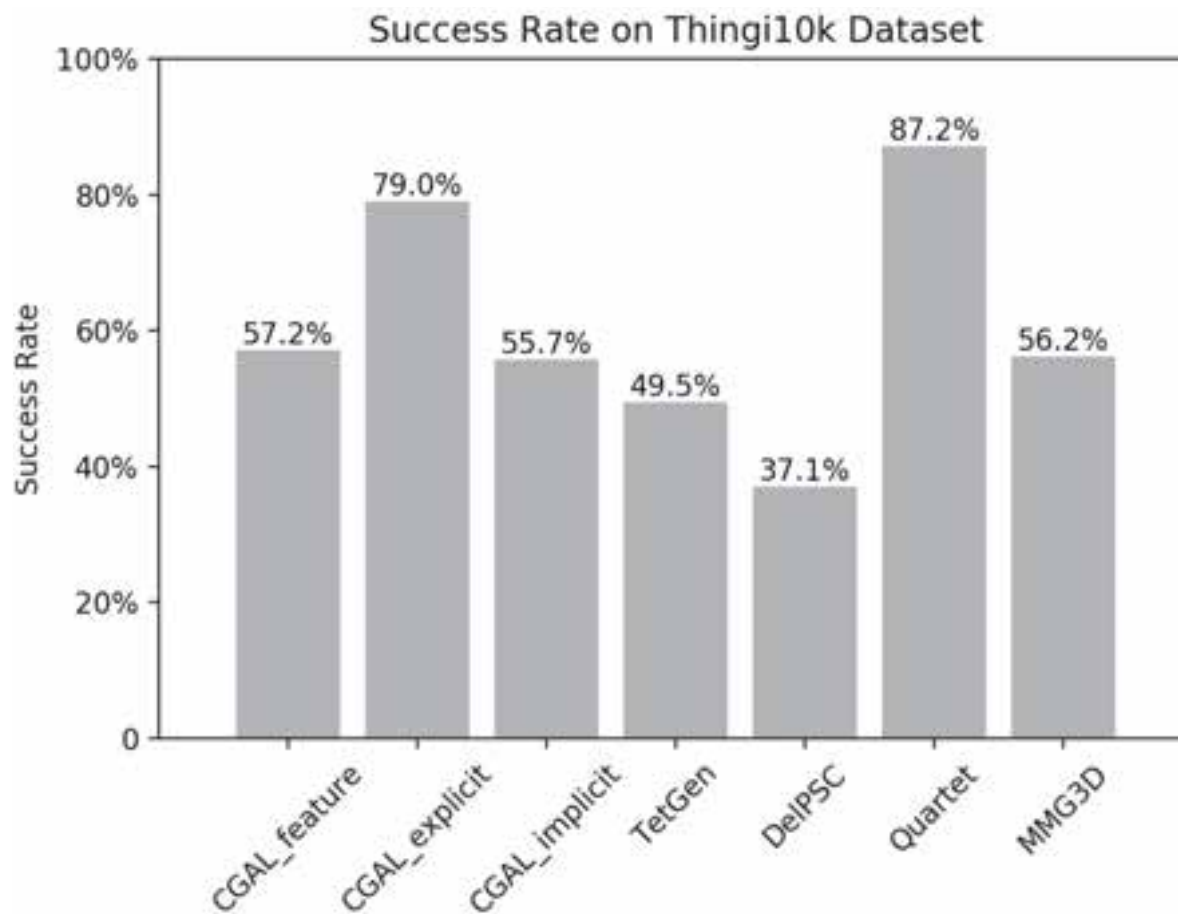
Yixin Hu, 13



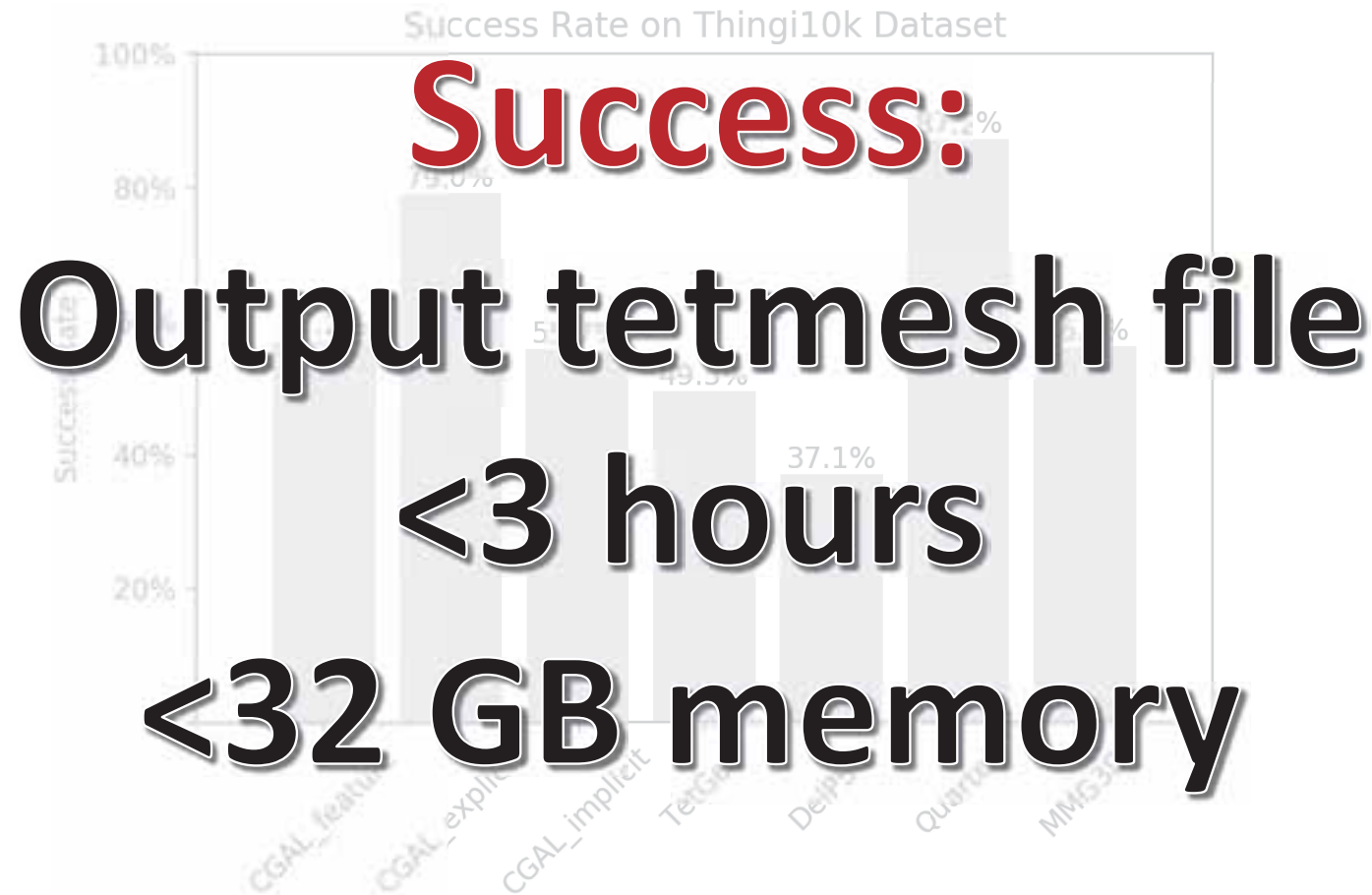
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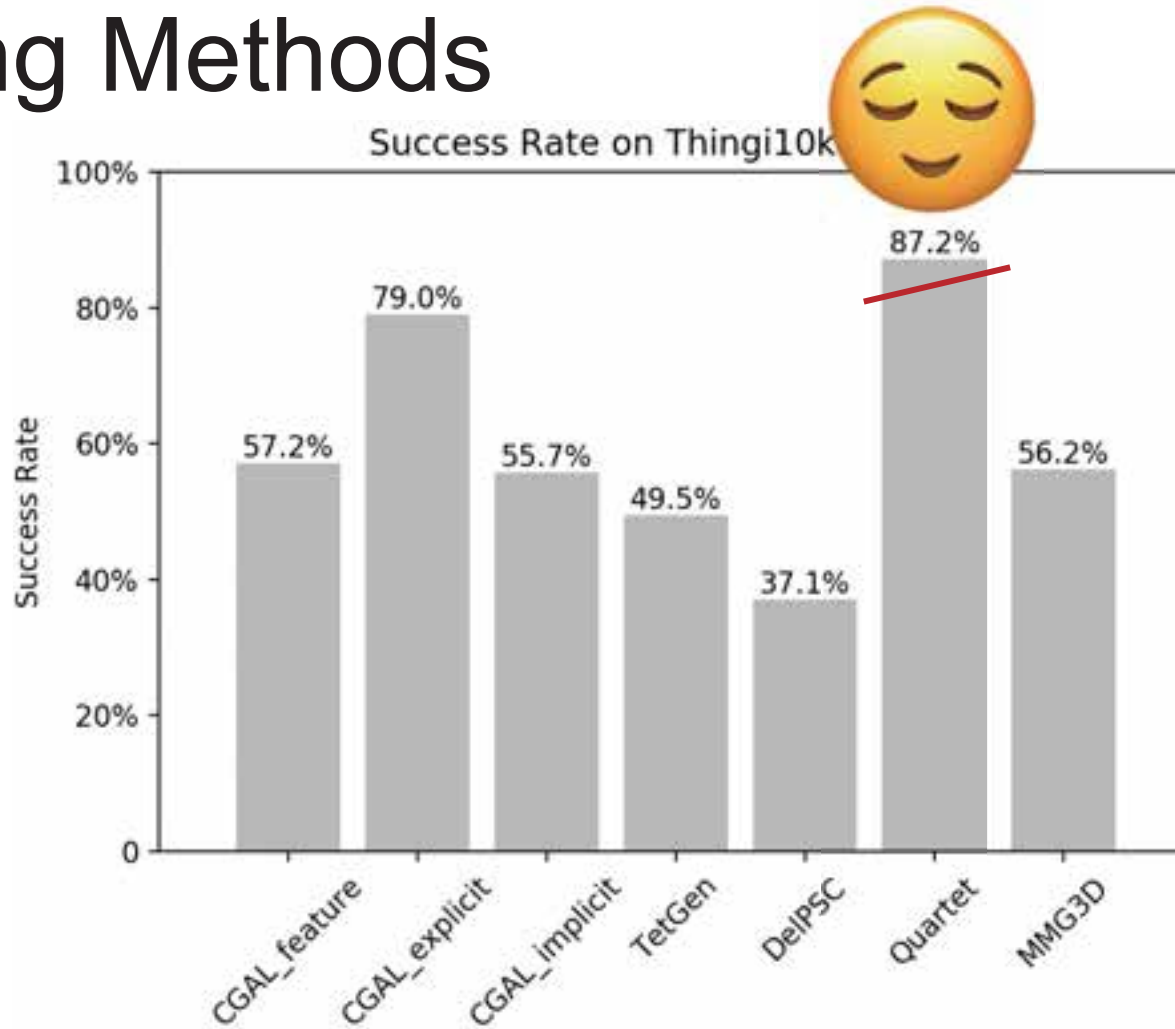
Existing Methods



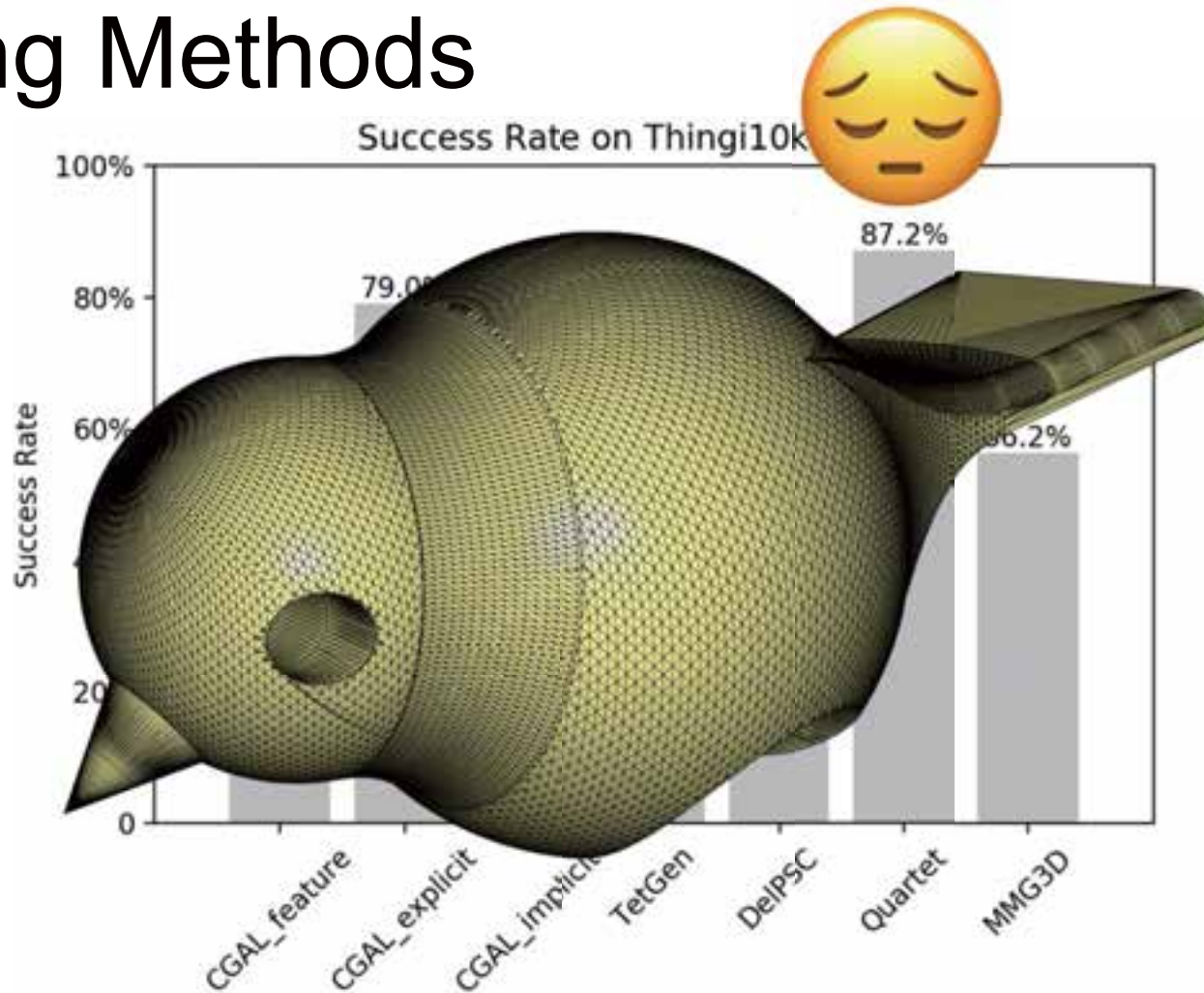
Existing Methods



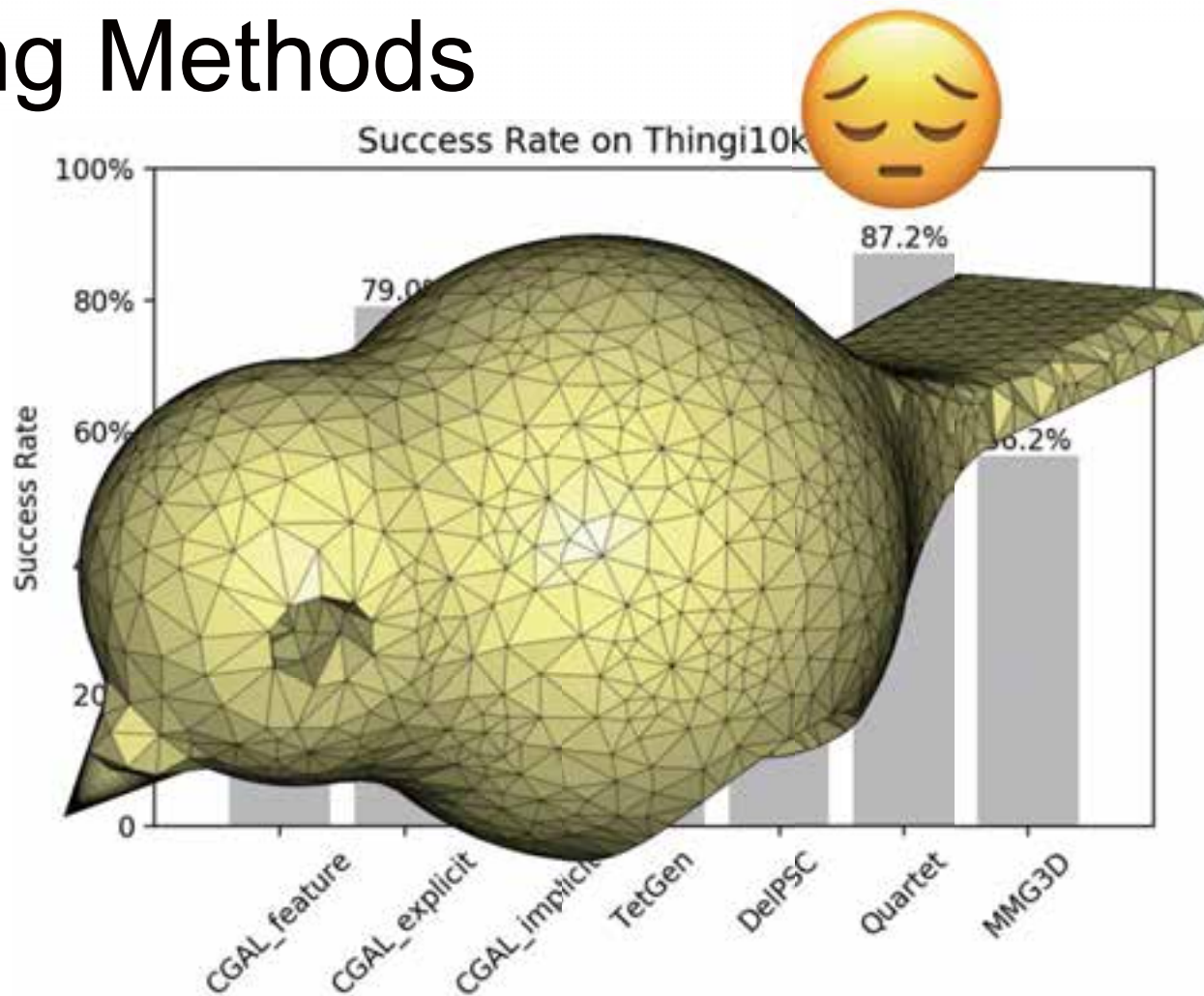
Existing Methods



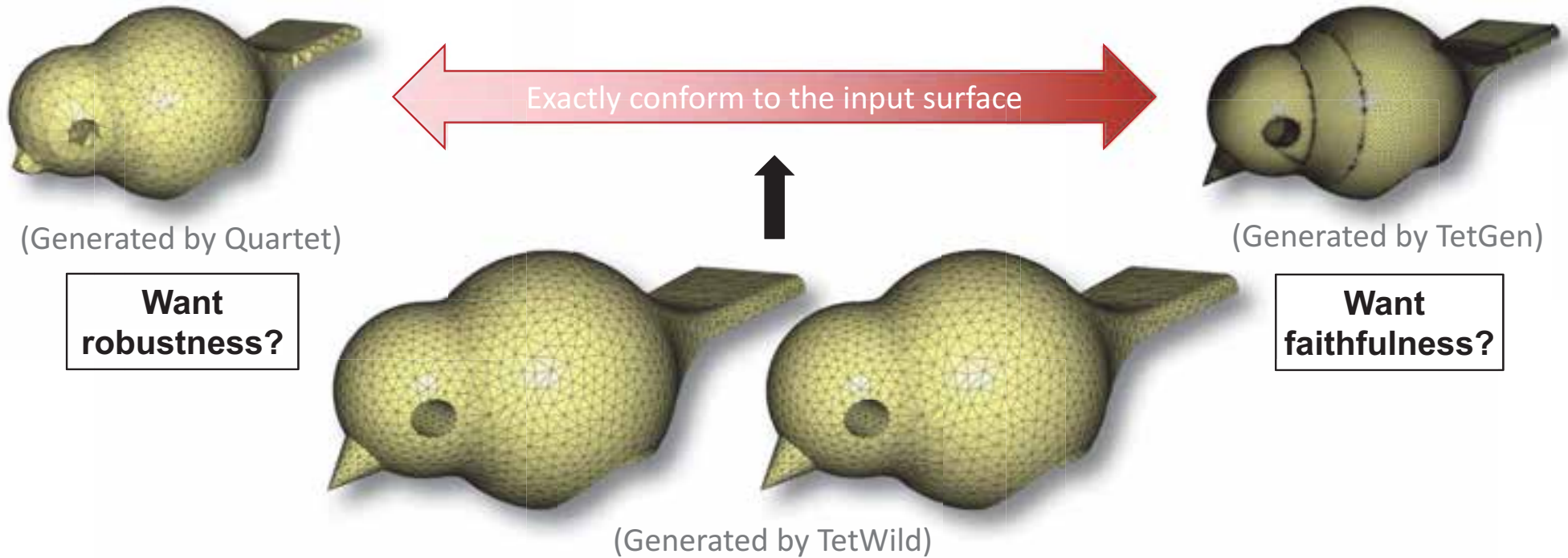
Existing Methods



Existing Methods



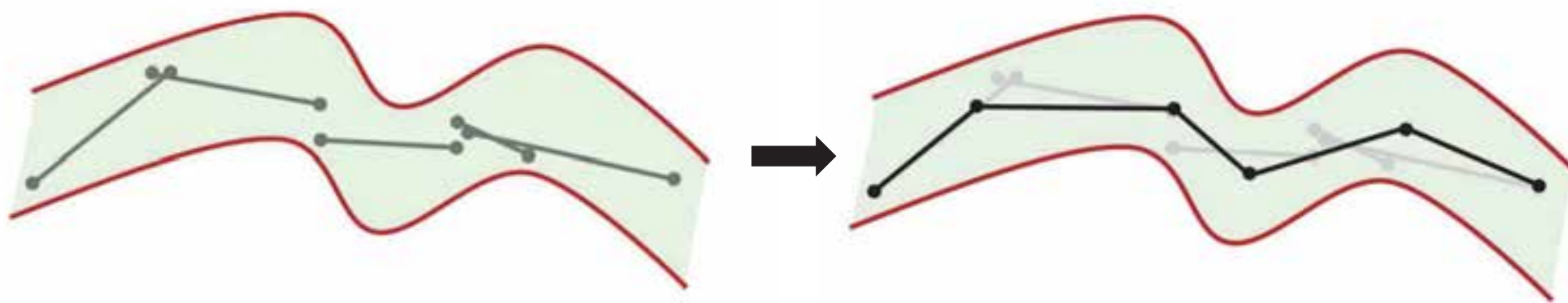
New Method



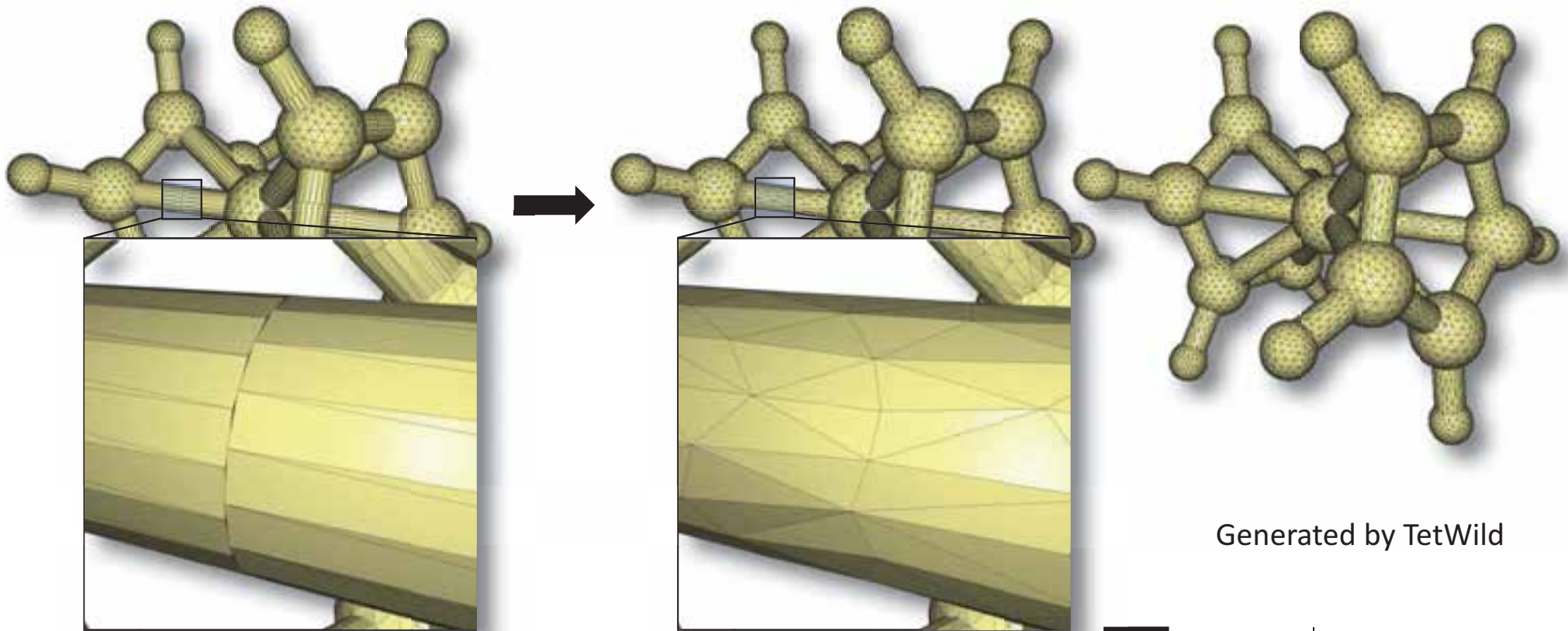
Our method: 1. Closed under rationals. 2. Approximate input surface within an envelope.

Why Envelope?

- Optimize boundary inside an envelope



Input with Gaps



Generated by TetWild

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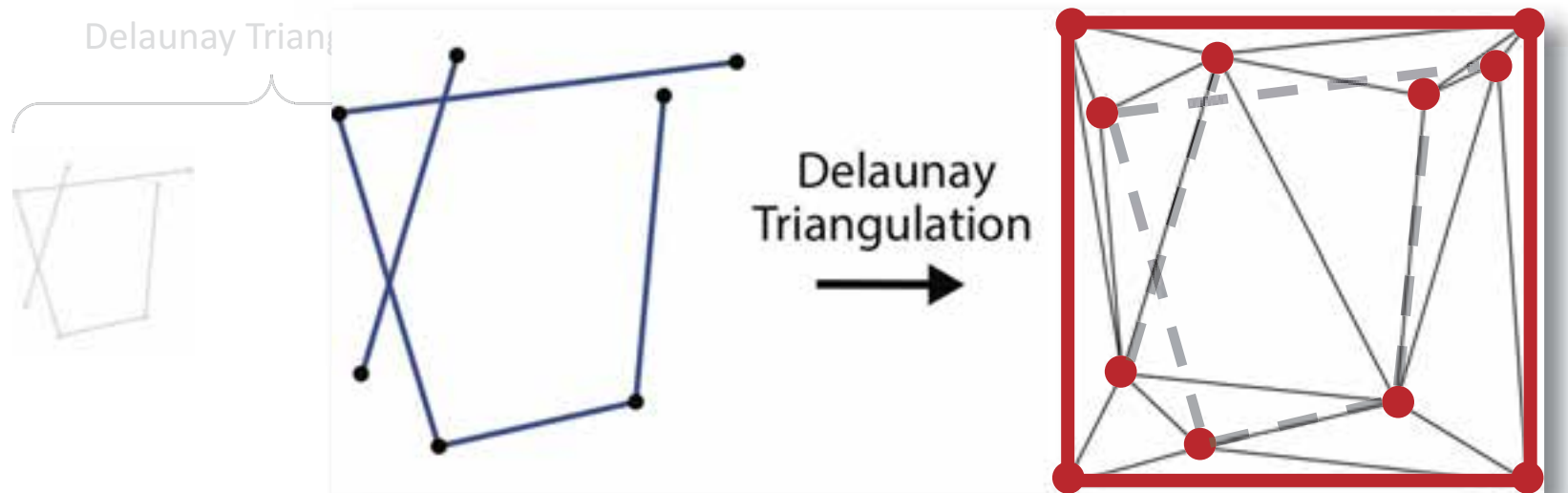
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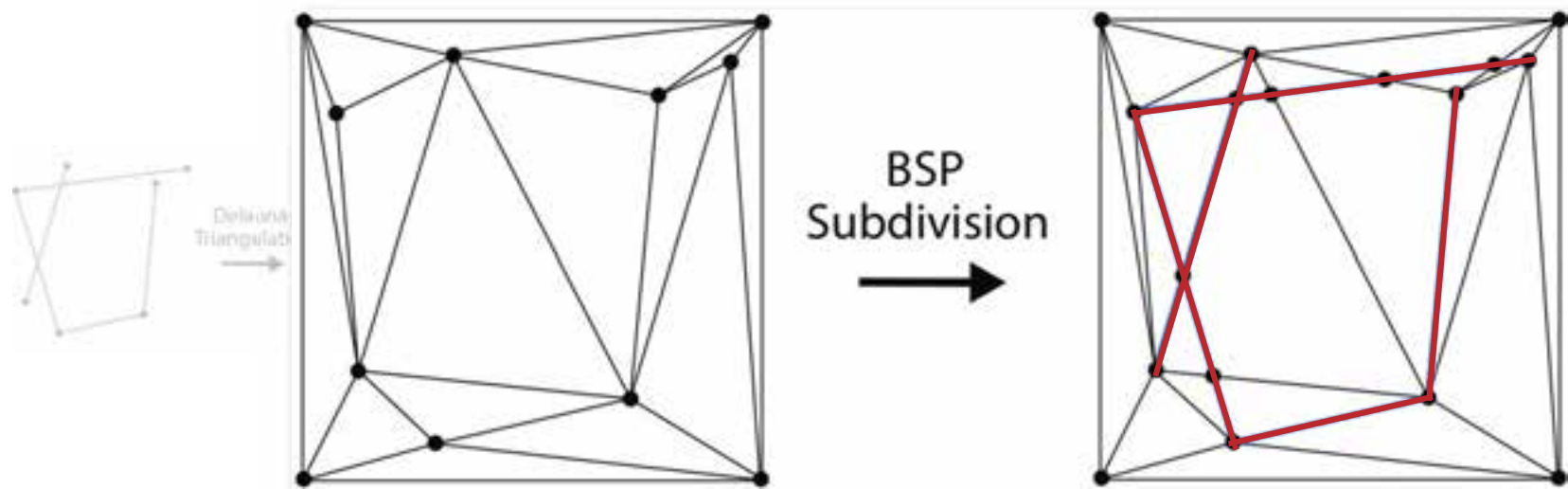
TetWild

- **Input:** Triangle soup that forms the surface of a 3D shape.
 - No assumption about the input.
- **Output:** Approximated constrained tetrahedral mesh. (Envelope)
 - Able to clean up imperfect regions on the input and produce high-quality output.

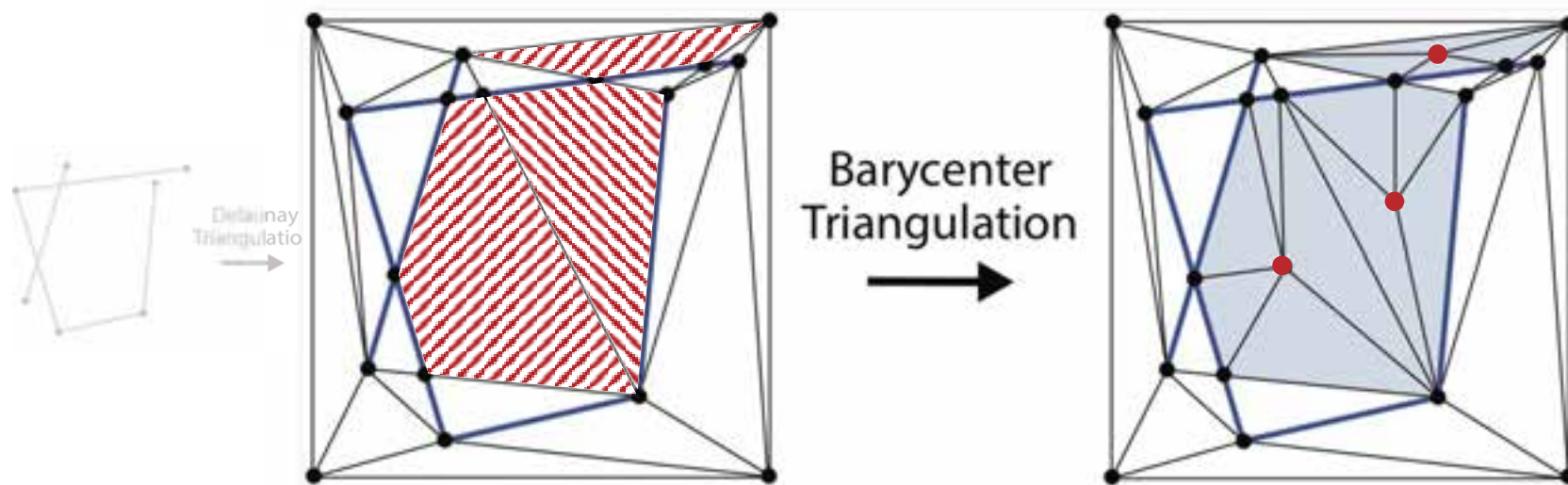
Pipeline



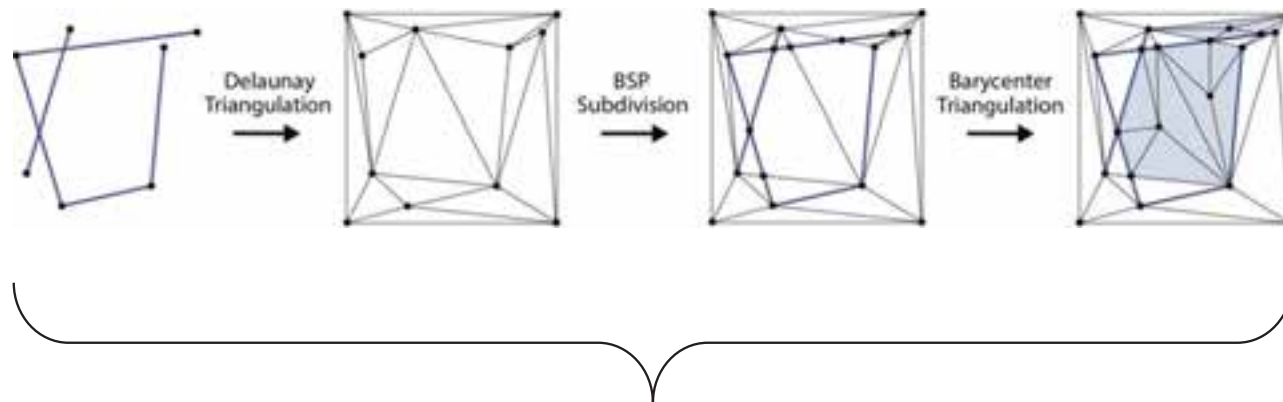
Pipeline



Pipeline



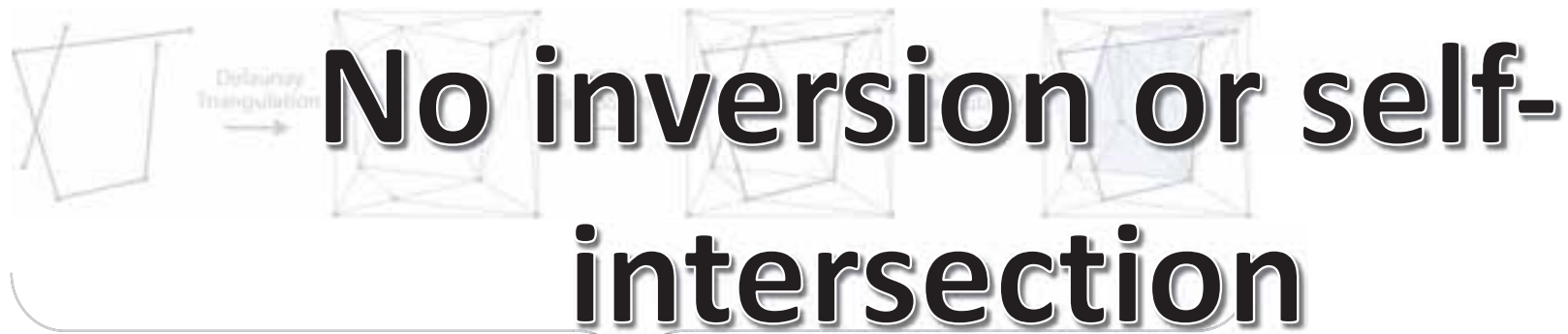
Pipeline



Stage I: Valid Mesh Generation
(Rational)

Pipeline

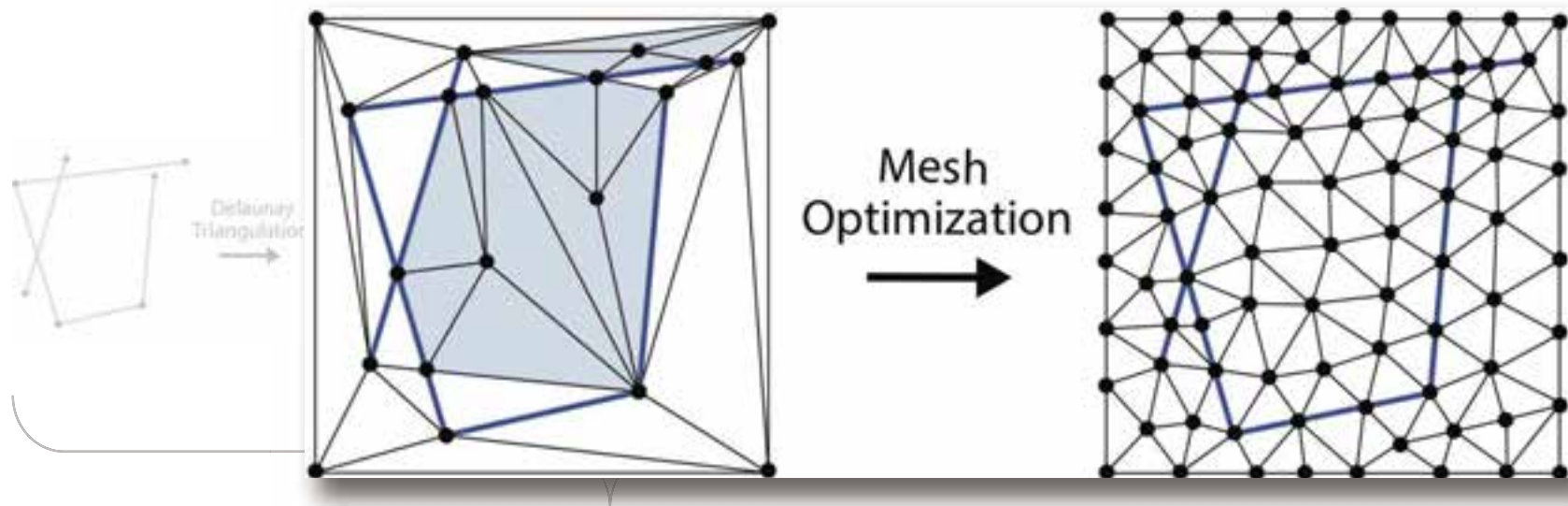
Valid:



Input boundary preserved

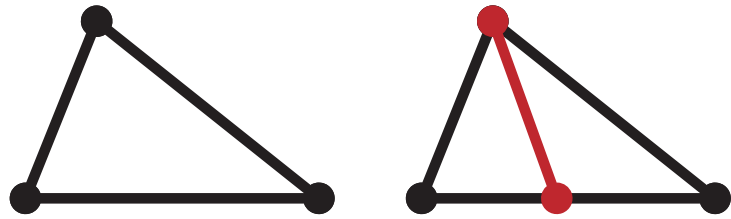


Pipeline

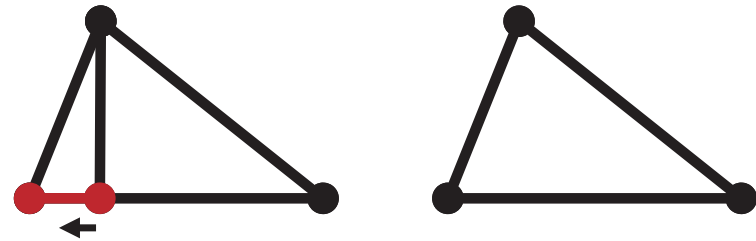


Stage I: Valid Mesh Generation
(Rational)

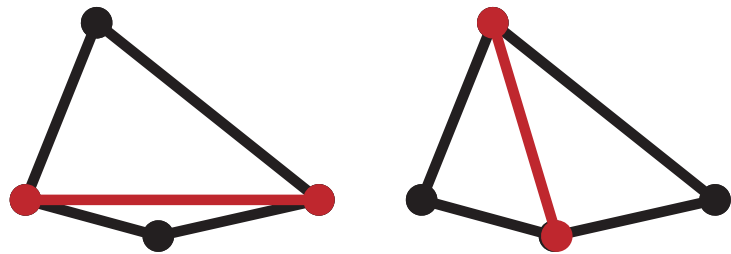
Mesh Optimization



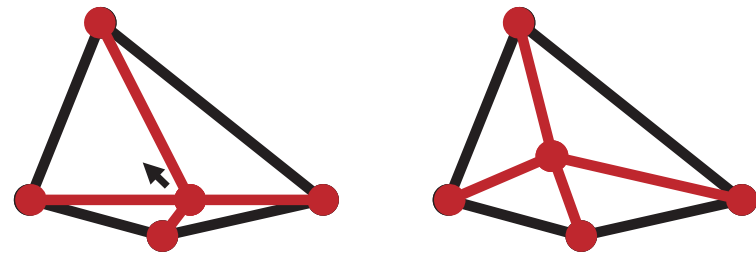
Edge Splitting



Edge Collapsing



Face Swapping

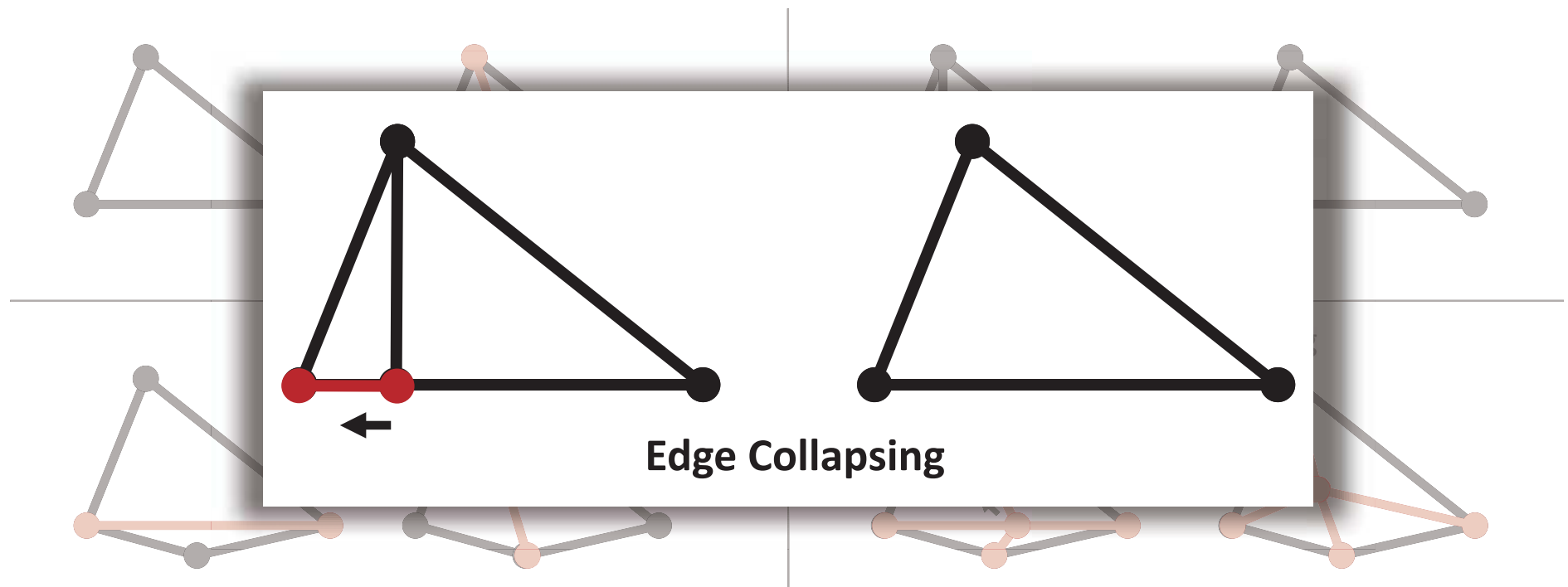


Vertex Smoothing

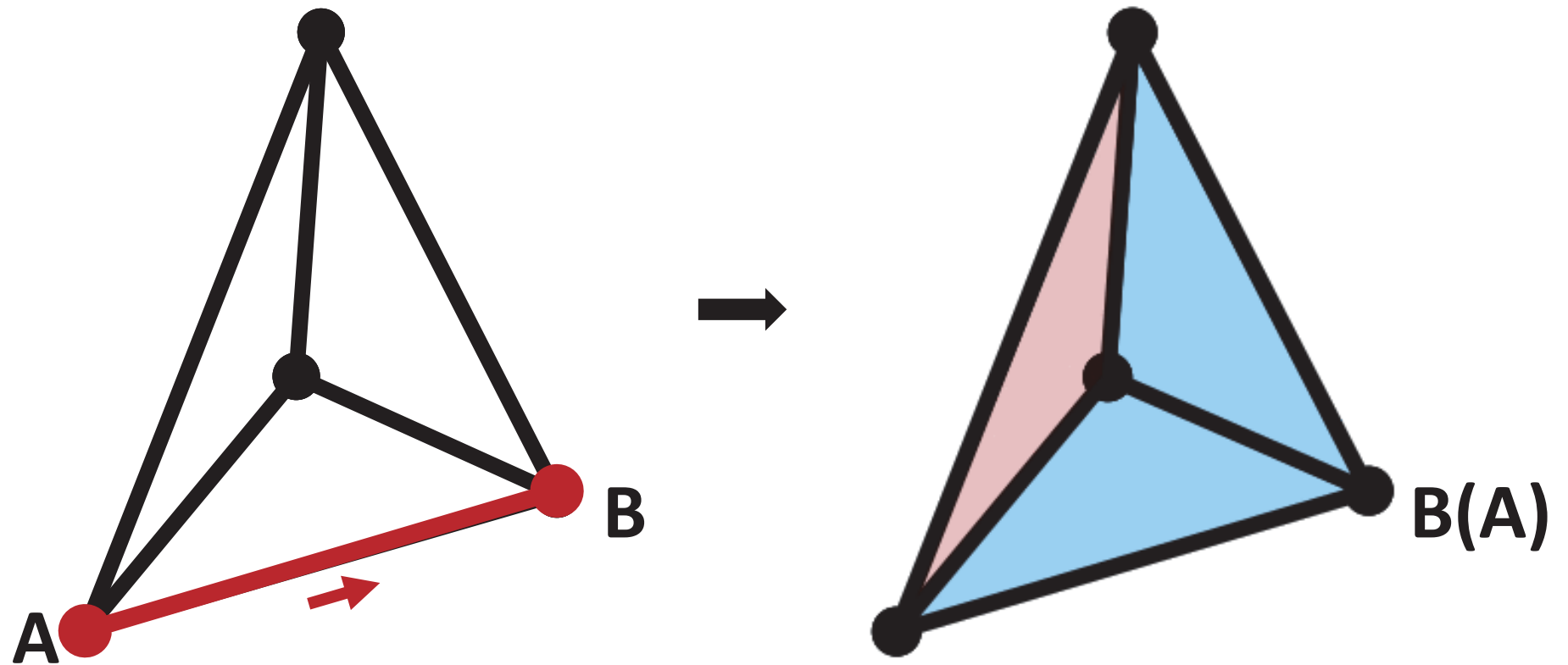


Mesh Optimization

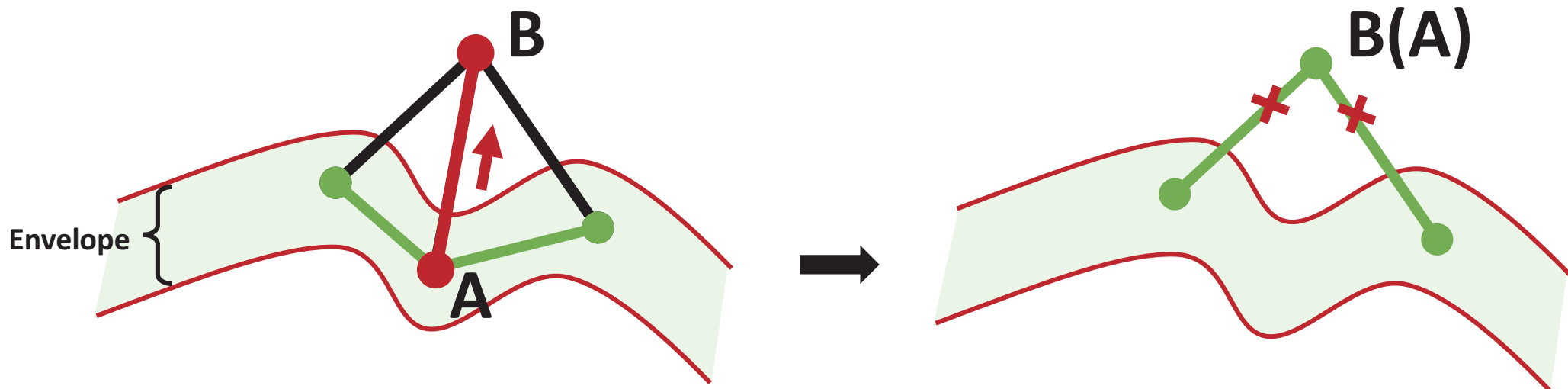
Conformal AMIPS 3D Energy



Inversion Check



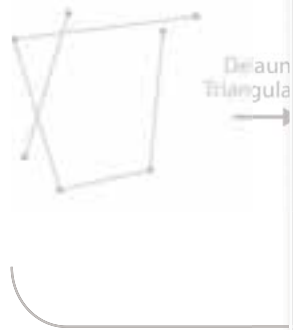
Envelope Check



Envelope Check



Pipeline



Filtering

Robust Inside-Outside Segmentation using Generalized Winding Numbers

Alec Jacobson¹ Ladislav Kavan^{2,1} Olga Sorkine-Hornung¹
¹ETH Zurich ²University of Pennsylvania



Figure 1: The Big SigCut input mesh has 3442 pairs of intersecting triangles (bright red), 1020 edges on open boundaries (dark red), 344 non-manifold edges (purple) and 67 connected components (randomly colored). On top of those problems, a SIGGRAPH logo shaped hole is carved from her side.

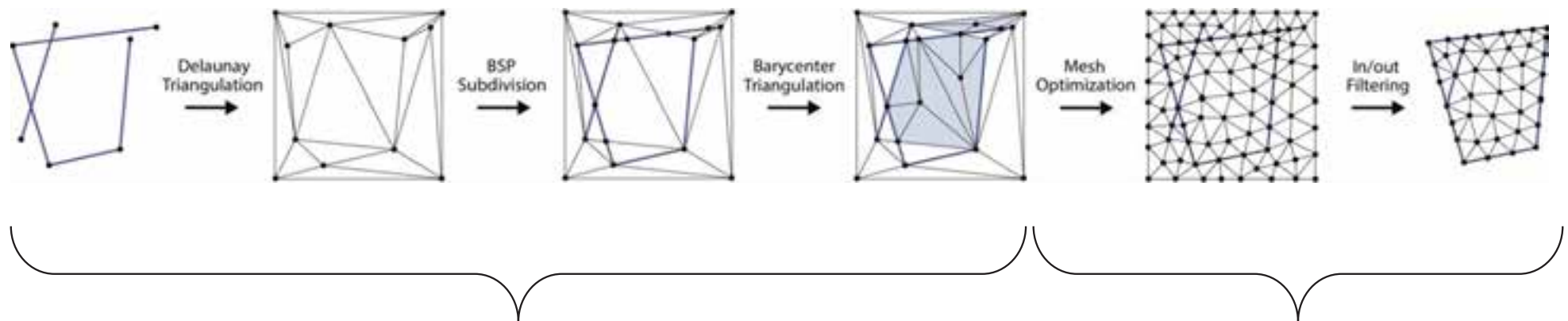
Abstract

Solid shapes in computer graphics are often represented with boundary descriptions, e.g. triangle meshes, but animation, physically-based simulation, and geometry processing are more realistic and accurate when explicit volume representations are available. Tetrahedral meshes which exactly contain (interpolate) the input boundary description are desirable but difficult to construct for a large class of input meshes. Character meshes and CAD models are often composed of many connected components with numerous self-

physically-based simulation of a hippopotamus would look quite different (and unrealistic) if handled as a thin shell, rather than a solid. Since many operations in animation, simulation and geometry processing require an explicit representation of an object's volume, for example for finite element analysis and solving PDEs, a conforming¹ tetrahedral meshing of the surface is highly desired, as it enables volumetric computation with direct access to and assignment of boundary surface values. However, a wide range of "real-life" models, although they appear to describe the boundary of a solid object, are in fact unsuitable with current tools, due to the presence of



Pipeline

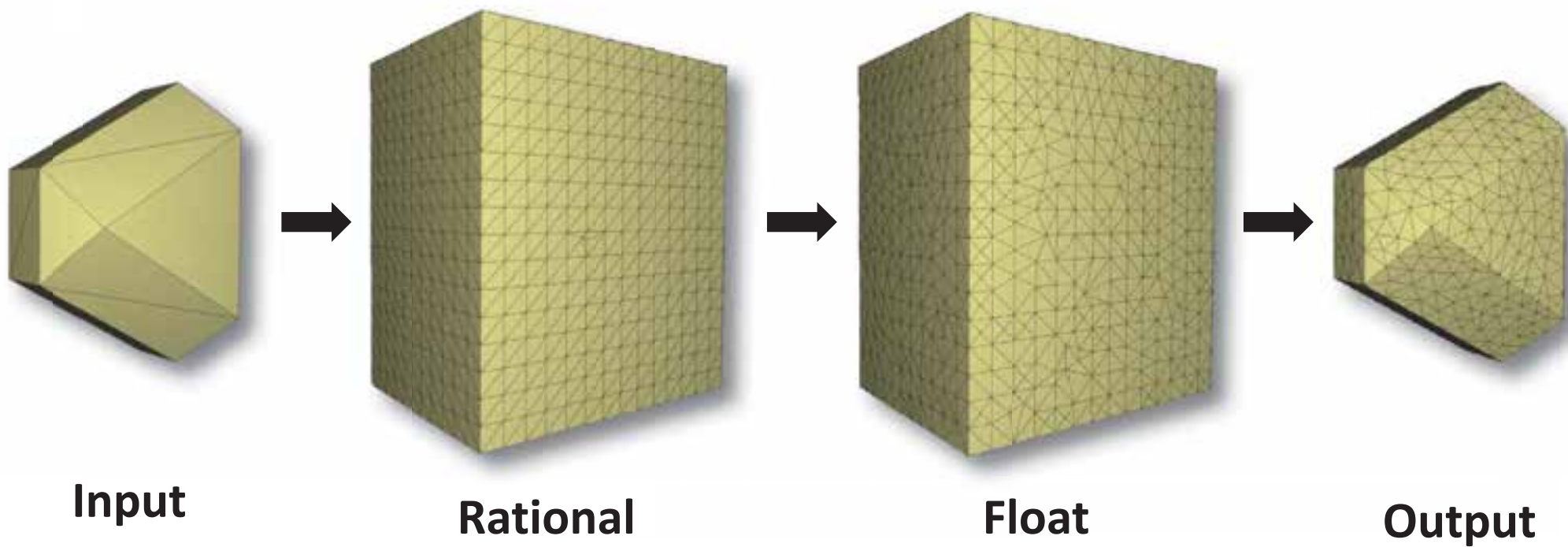


Stage I: Valid Mesh Generation
(Rational)

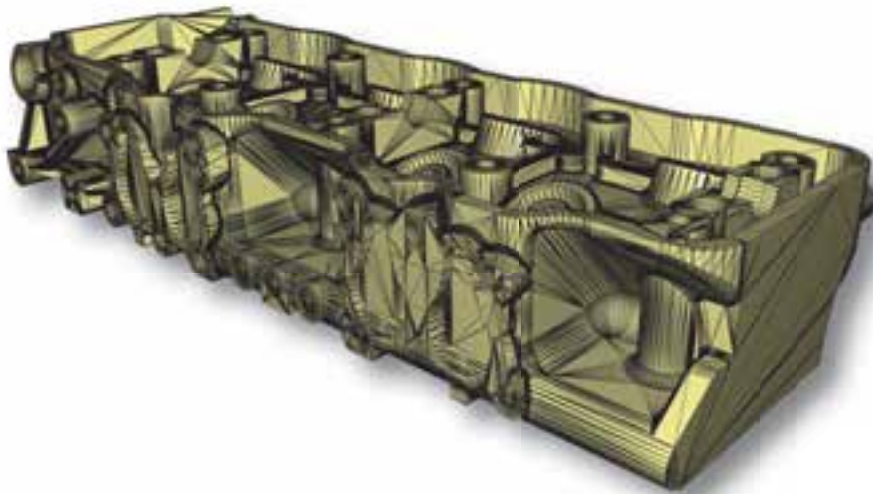
Stage II: Mesh Improvement
(Mixed \rightarrow Double)



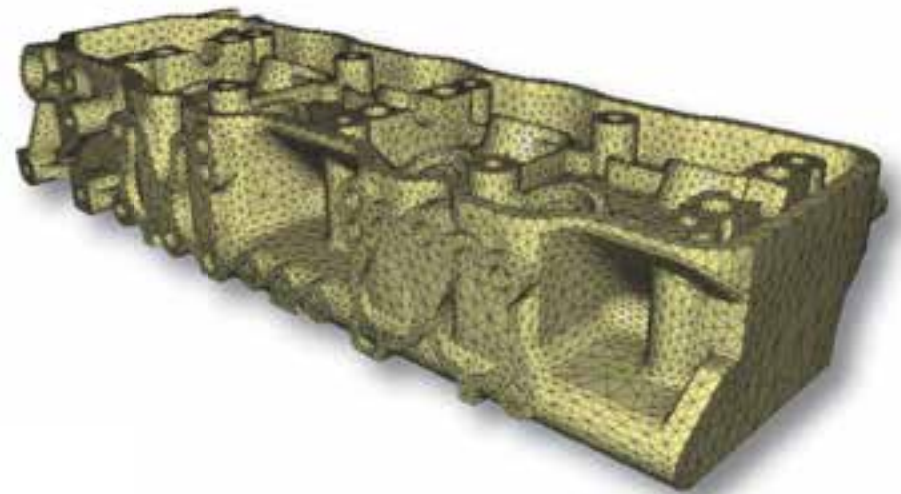
Pipeline



Parameters: Ideal Edge Length



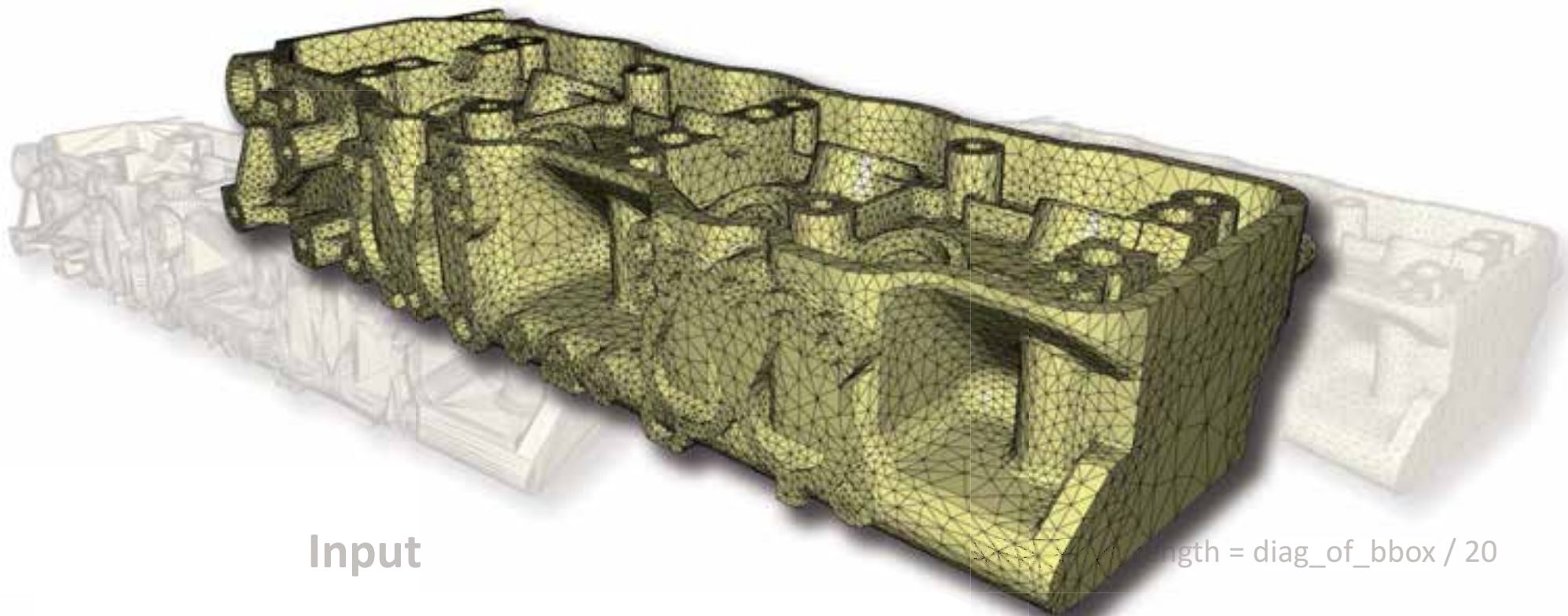
Input



$$\text{Ideal_edge_length} = \text{diag_of_bbox} / 20$$



Parameters: Ideal Edge Length

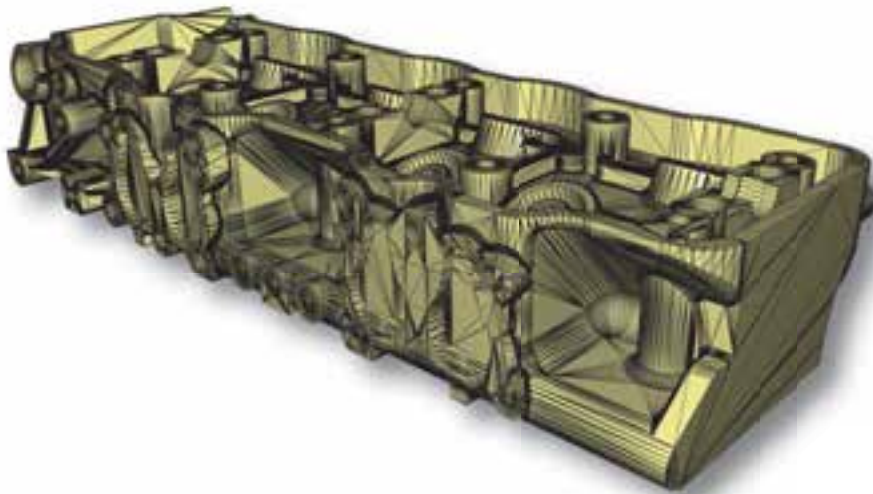


Input

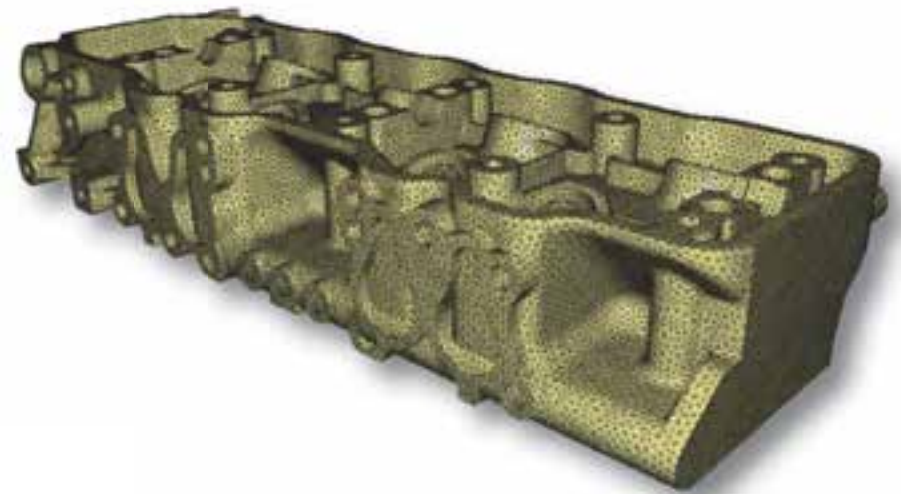
length = diag_of_bbox / 20



Parameters: Ideal Edge Length



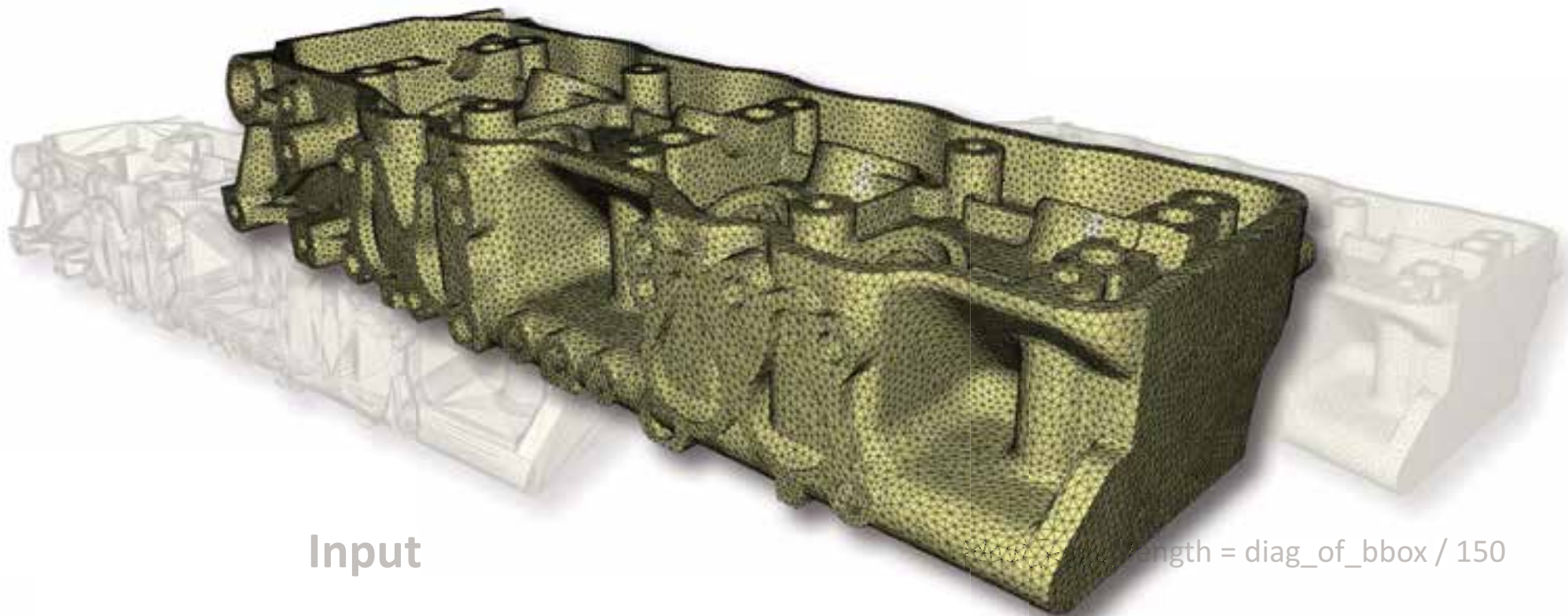
Input



$$\text{Ideal_edge_length} = \text{diag_of_bbox} / 150$$



Parameters: Ideal Edge Length

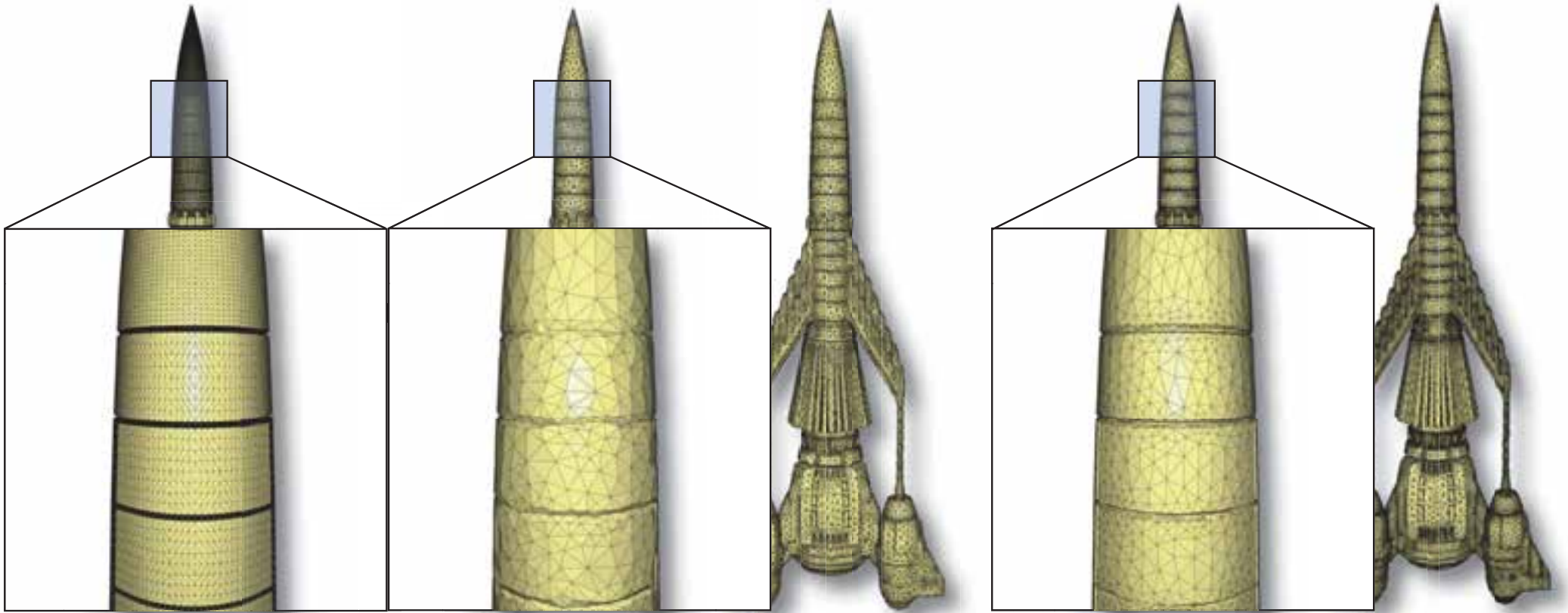


Input

Ideal Edge Length = $\text{diag_of_bbox} / 150$



Parameters: Envelope Size



Input

Envelope size = $\text{diag_of_bbox} / 1000$

Envelope size = $\text{diag_of_bbox} / 3000$



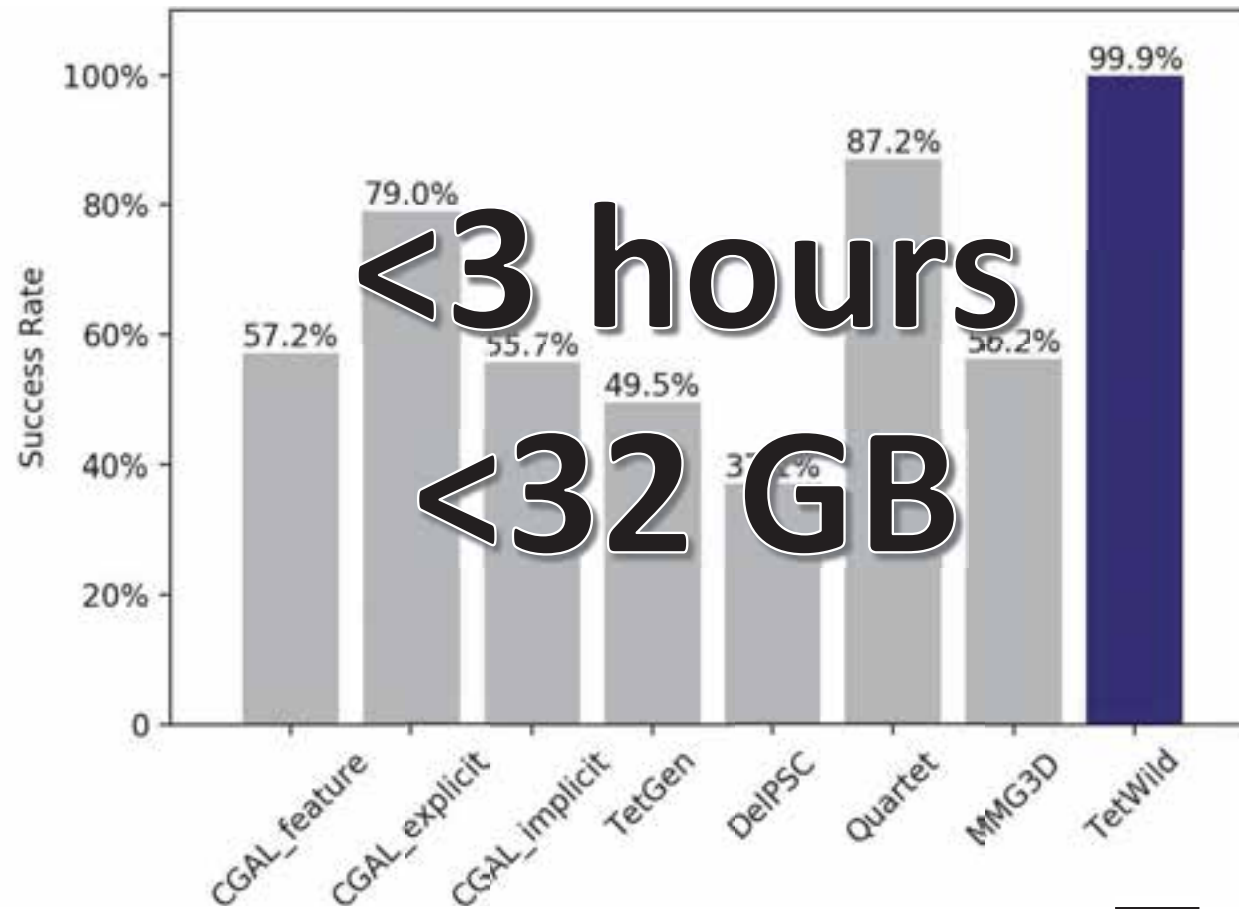
Parameters: Envelope Size



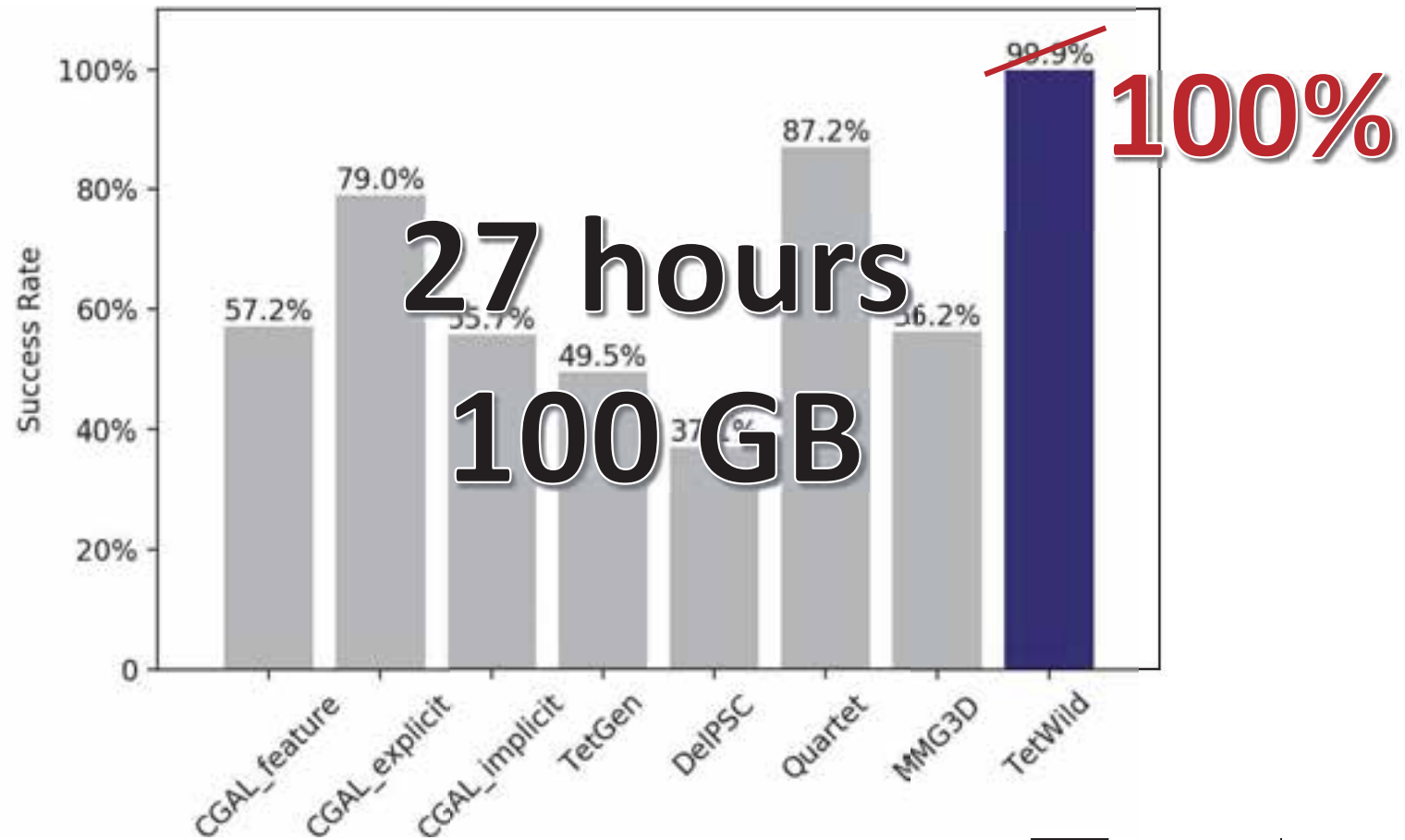
Envelope size = $\text{diag_of_bbox} / 1000$



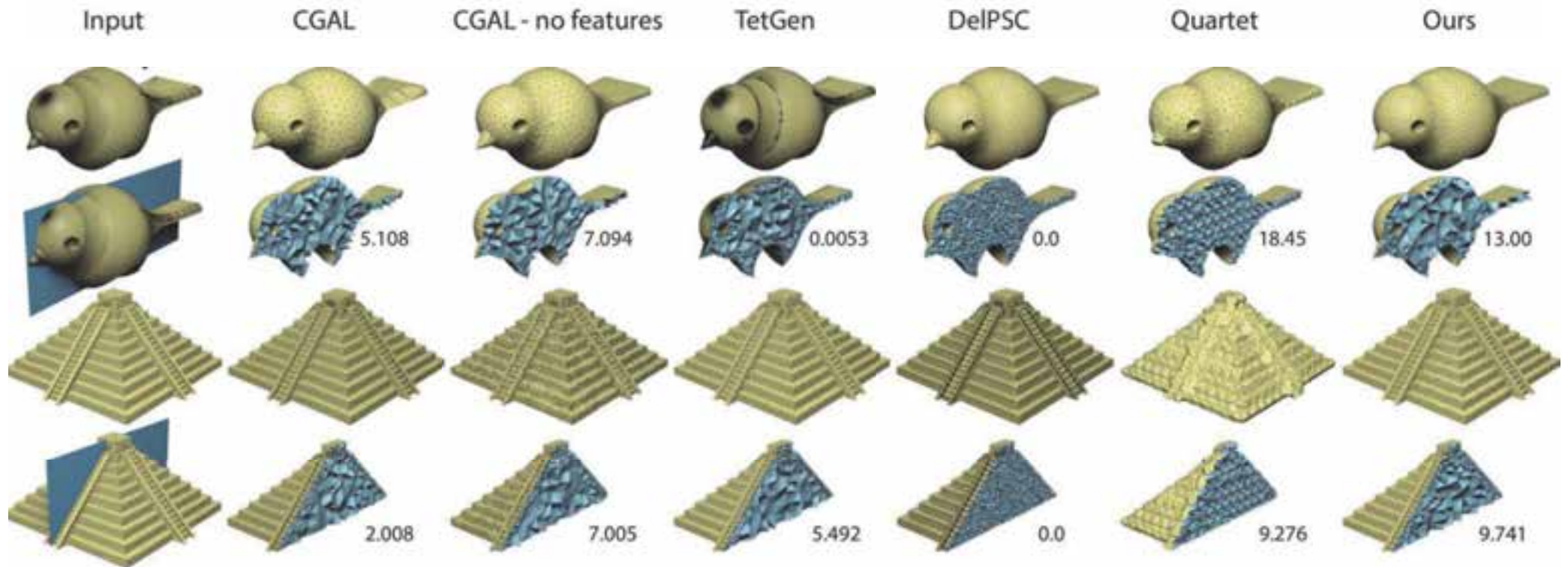
Comparison: Success Rate on 10k Models



Comparison: Success Rate on 10k Models



Comparison



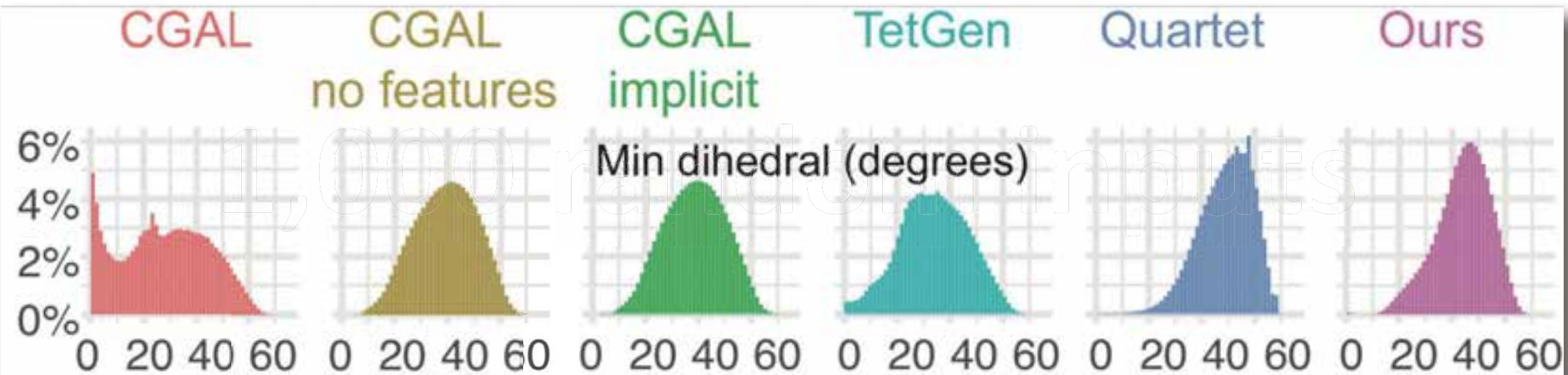
Comparison

measures

1,000 random inputs

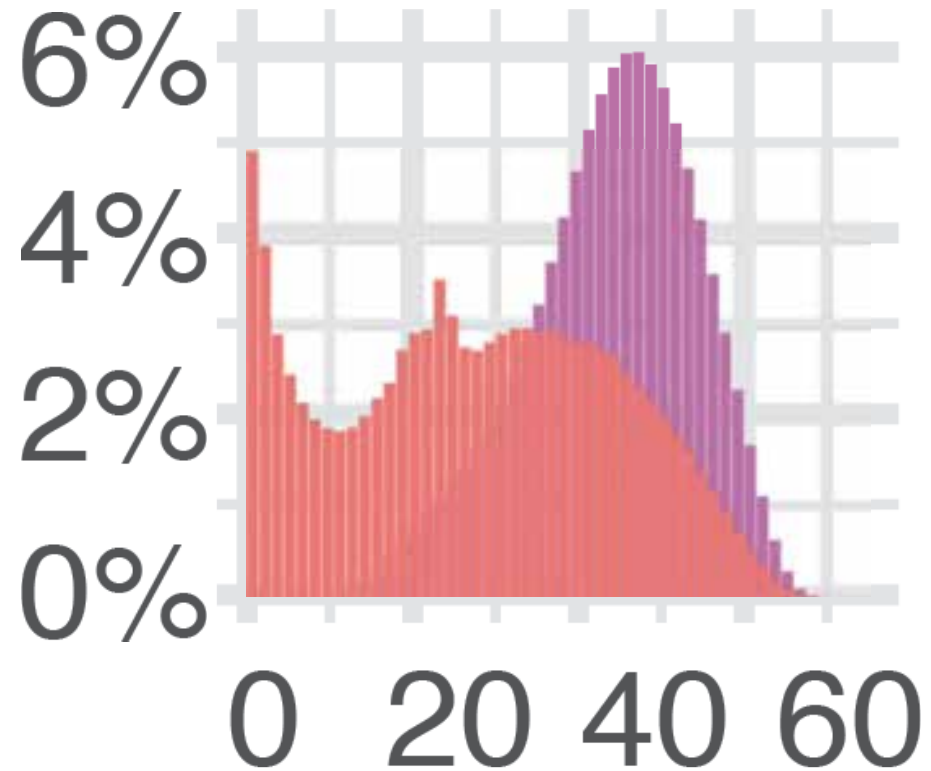


Comparison: Quality in Different Measures



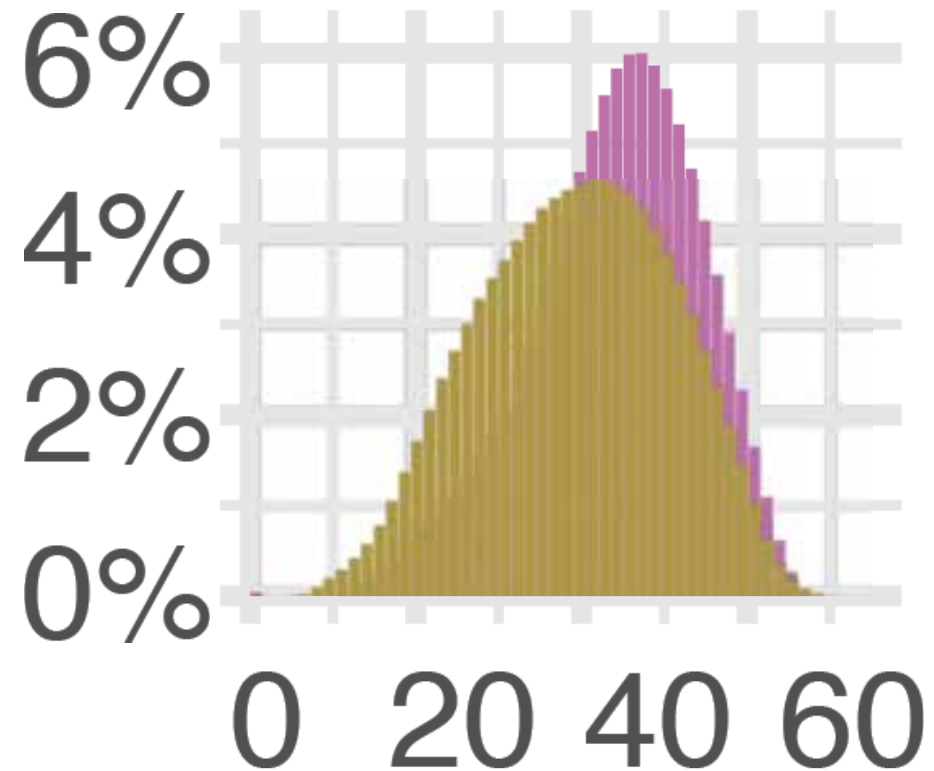
Comparison: Quality in Different Measures

**Min dihedral angle:
TetWild vs. CGAL**



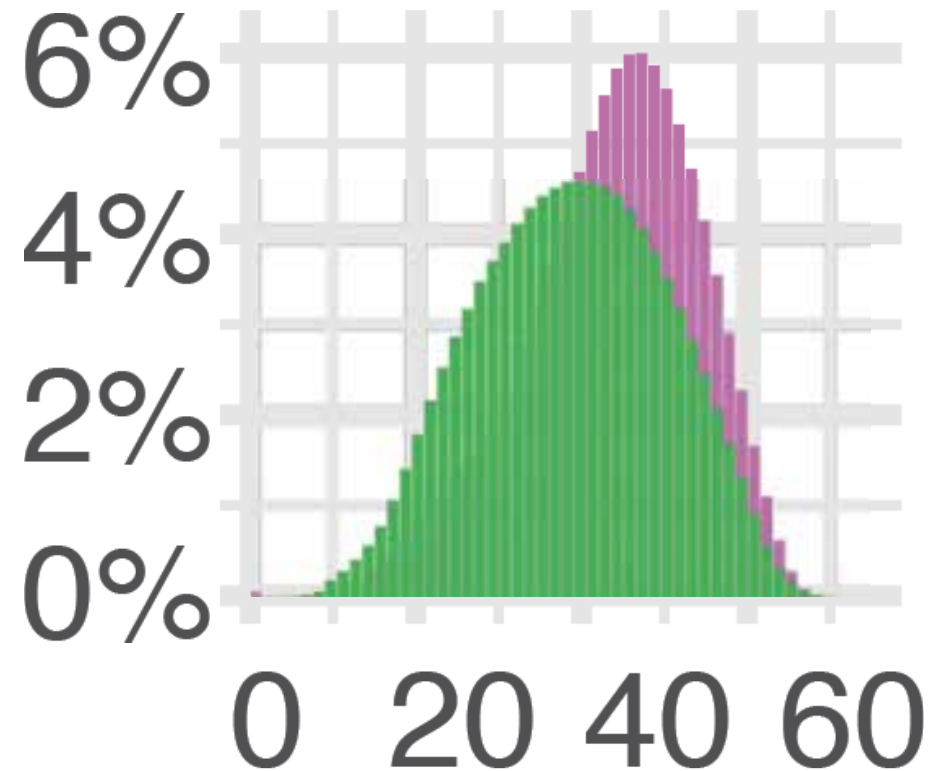
Comparison: Quality in Different Measures

**Min dihedral angle:
TetWild vs.
CGAL no feature**



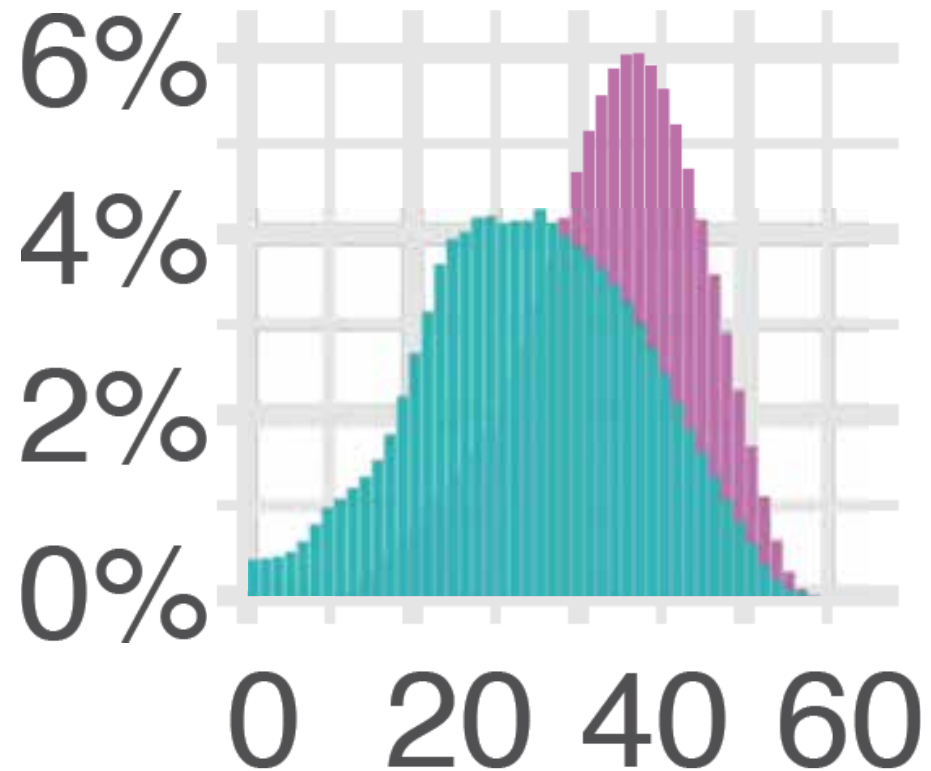
Comparison: Quality in Different Measures

**Min dihedral angle:
TetWild vs.
CGAL implicit**



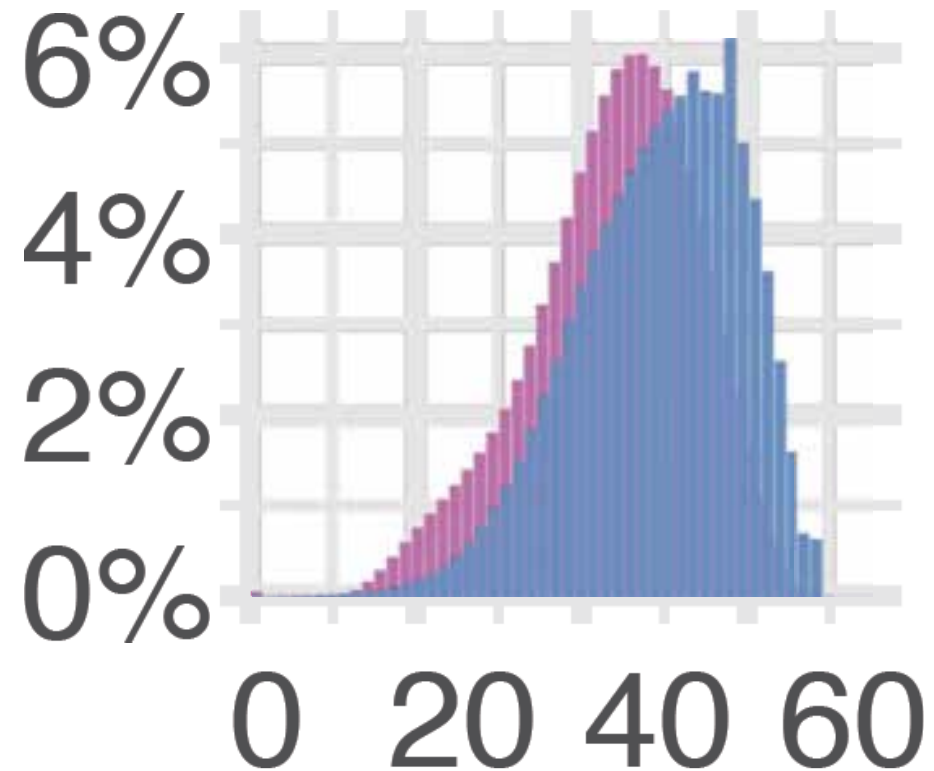
Comparison: Quality in Different Measures

**Min dihedral angle:
TetWild vs. TetGen**

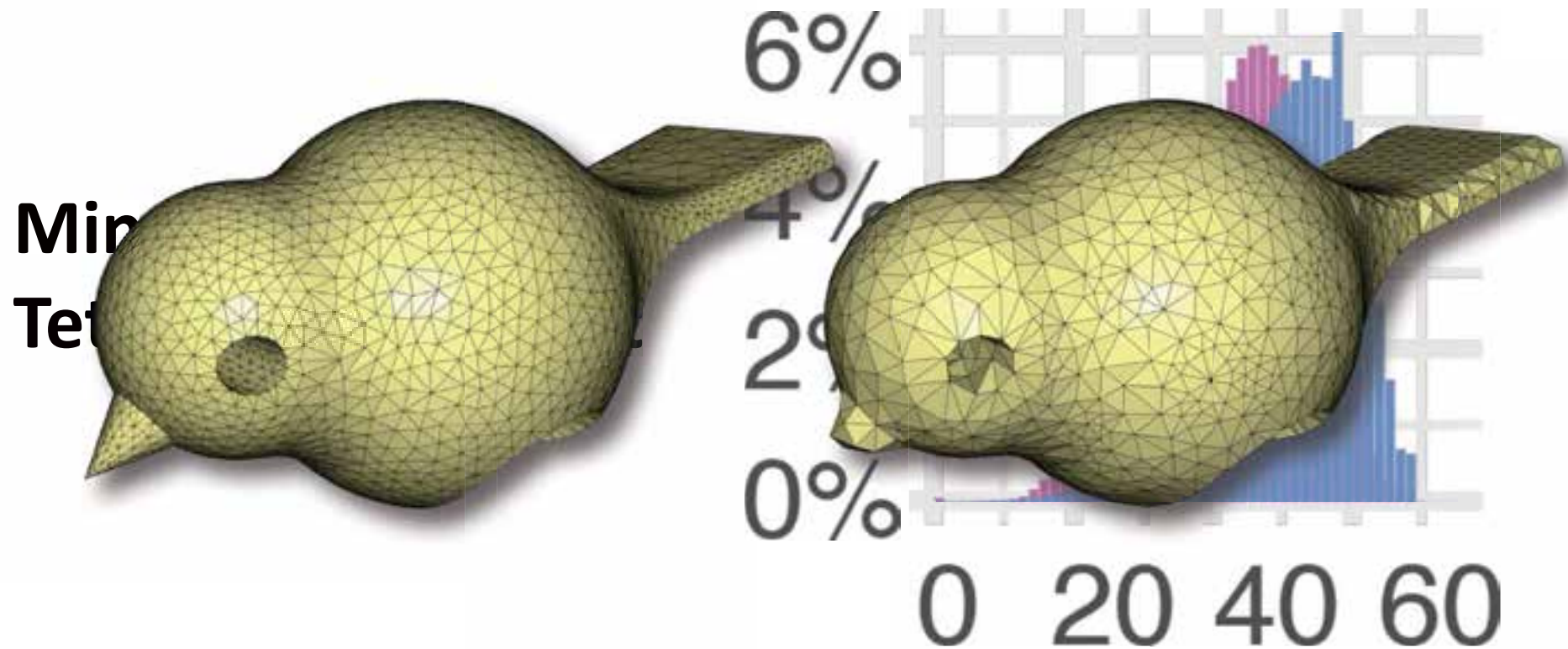


Comparison: Quality in Different Measures

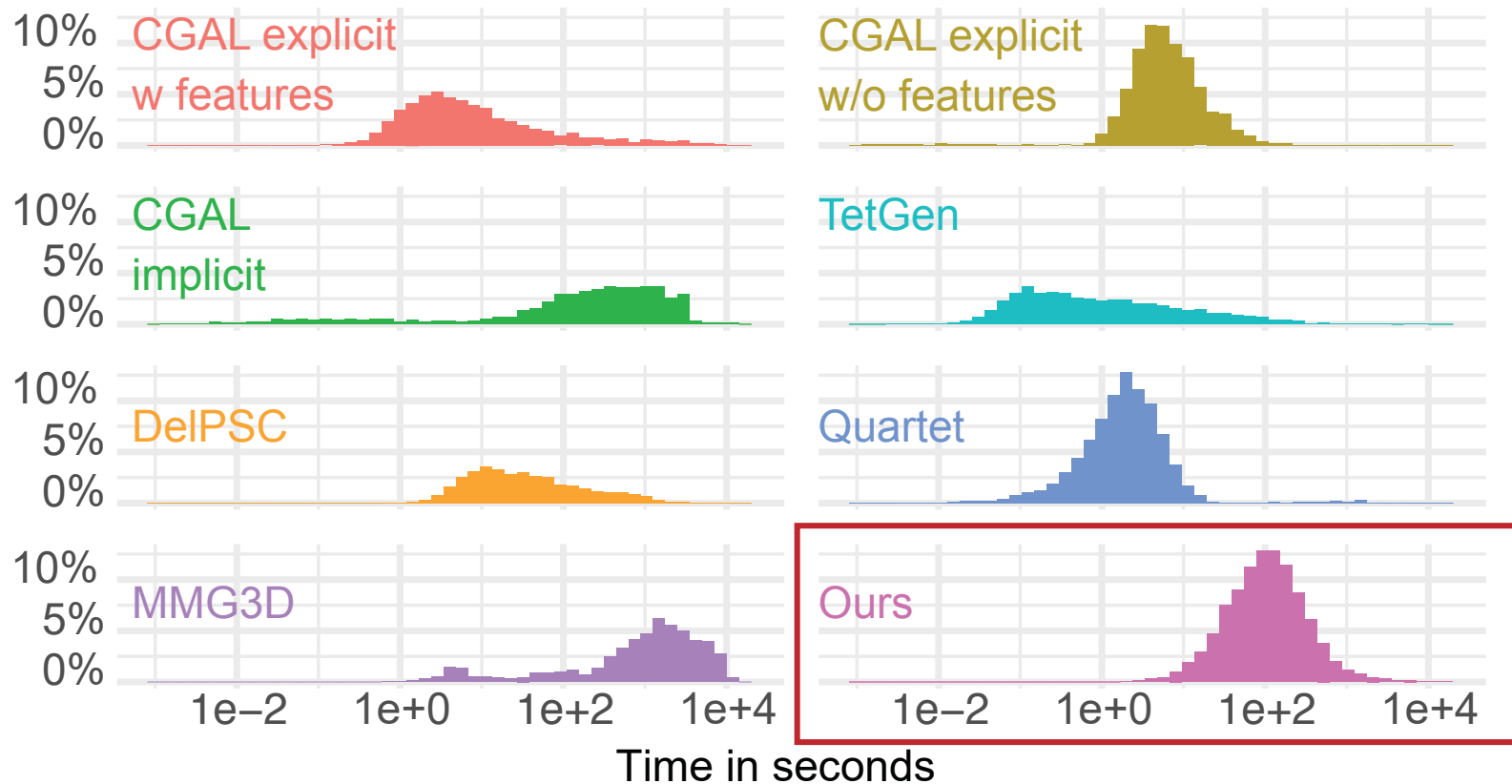
**Min dihedral angle:
TetWild vs. Quartet**



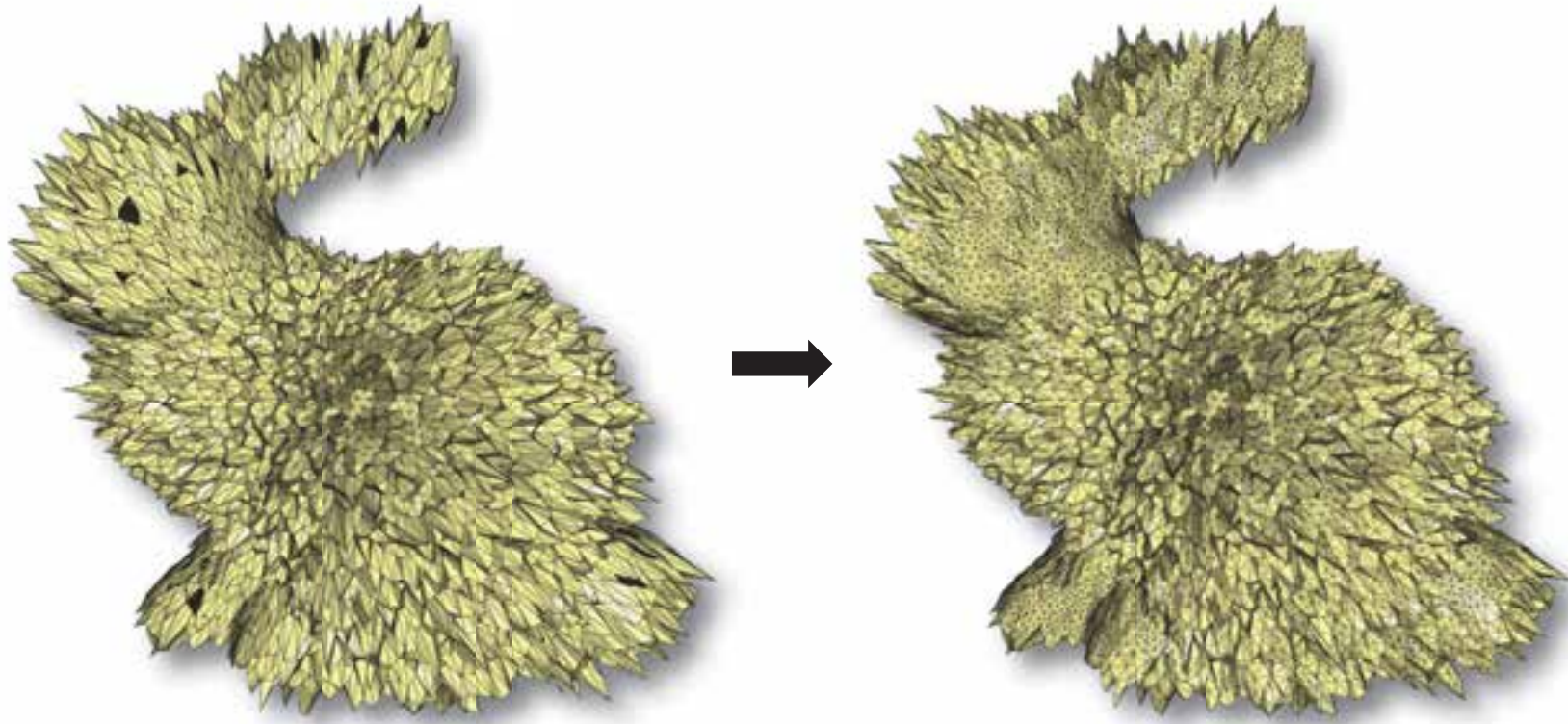
Comparison: Quality in Different Measures



Comparison: Running Time on 10k Models



Noise Stress-Test



Input, noise = 0.05

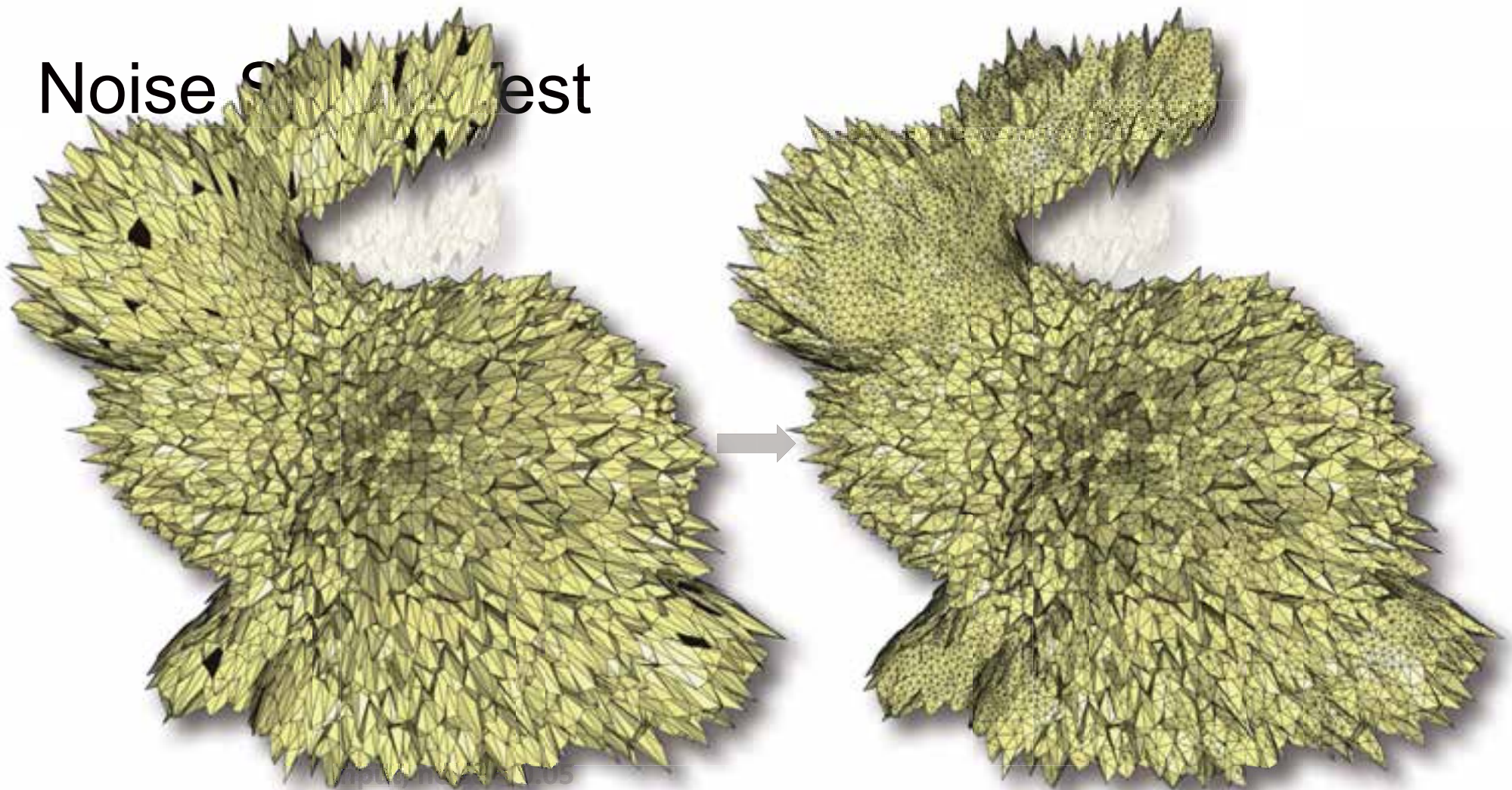
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Noise Sensitivity Test



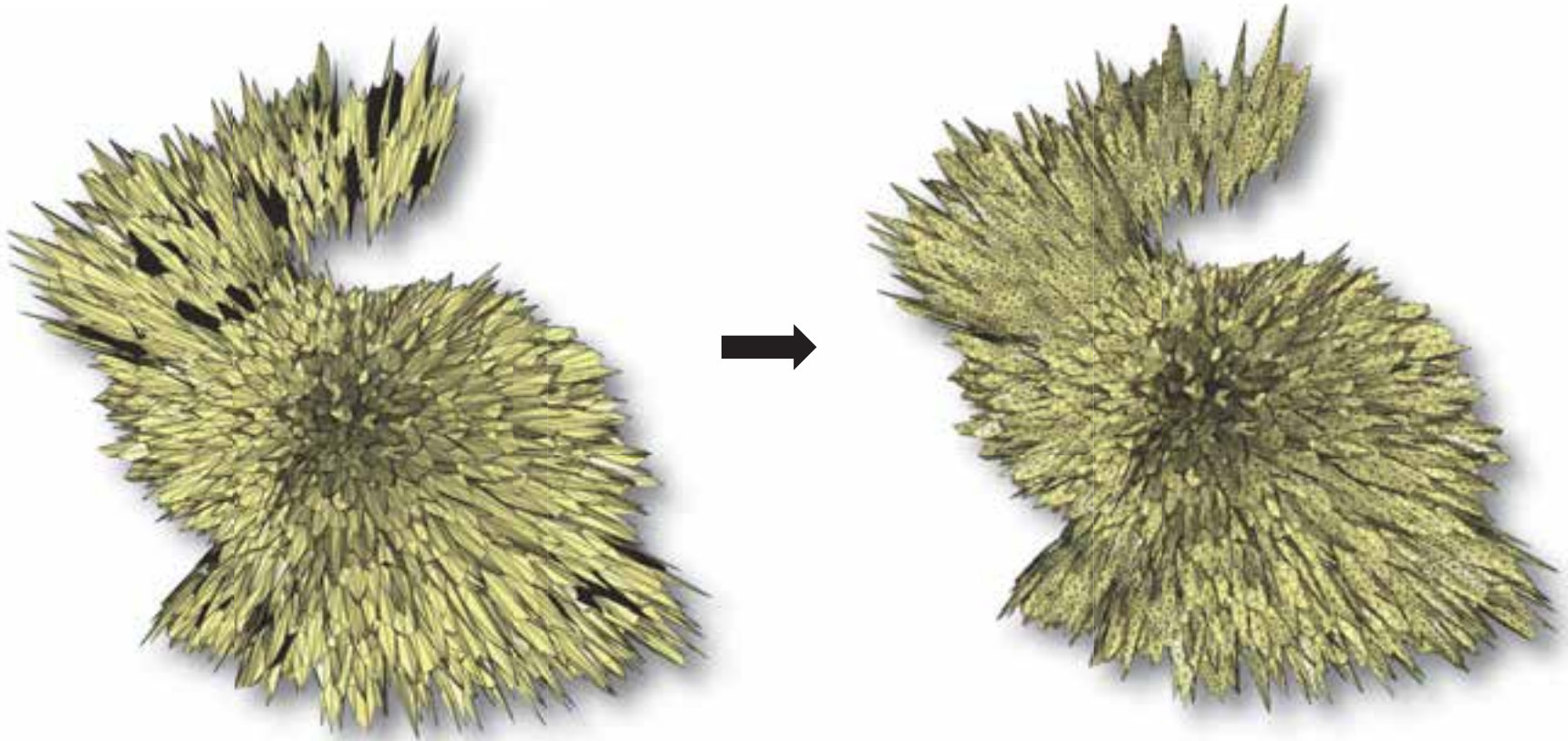
Yixin Hu, 56



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Noise Stress-Test



Input, noise = 0.1

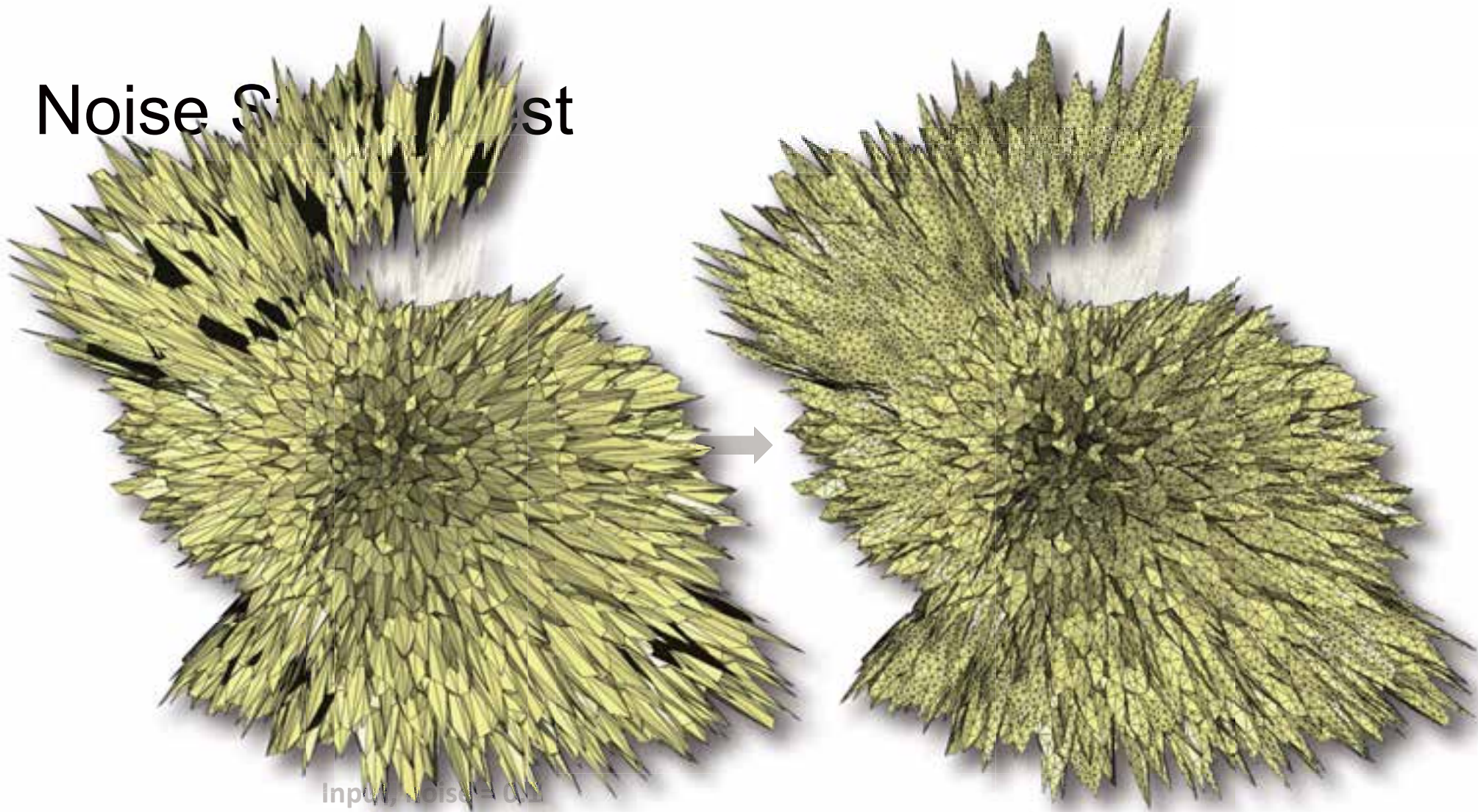
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Noise S...st



Input, noise = 0.1

Yixin Hu, 58



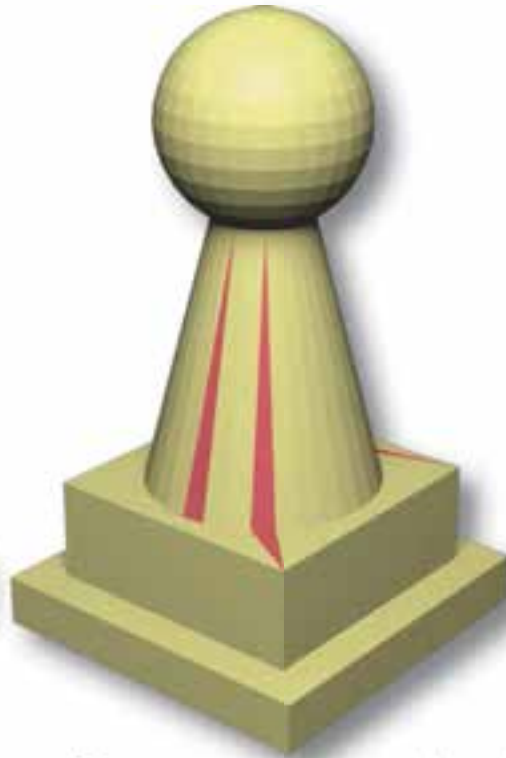
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Surface Repair



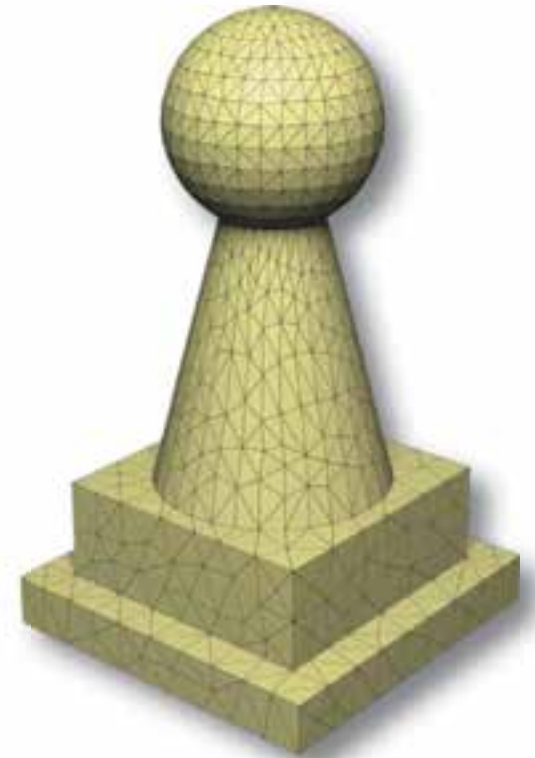
Input



Self-intersection(red)



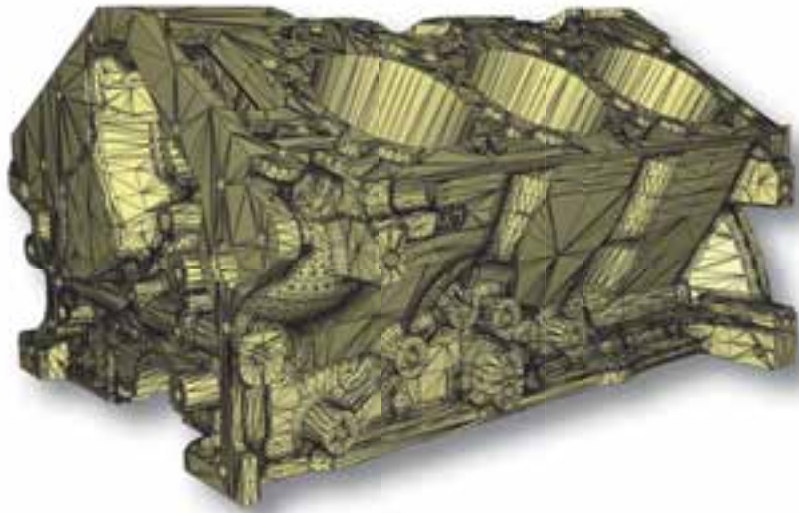
MeshFix



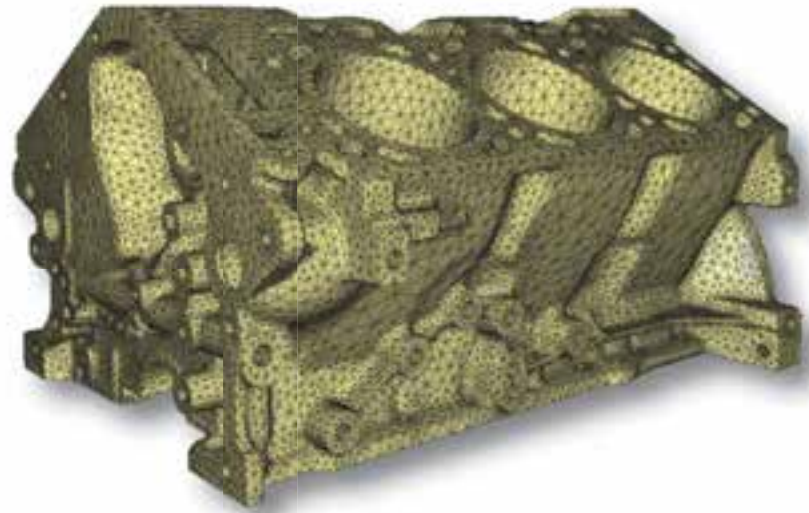
TetWild



CAD models



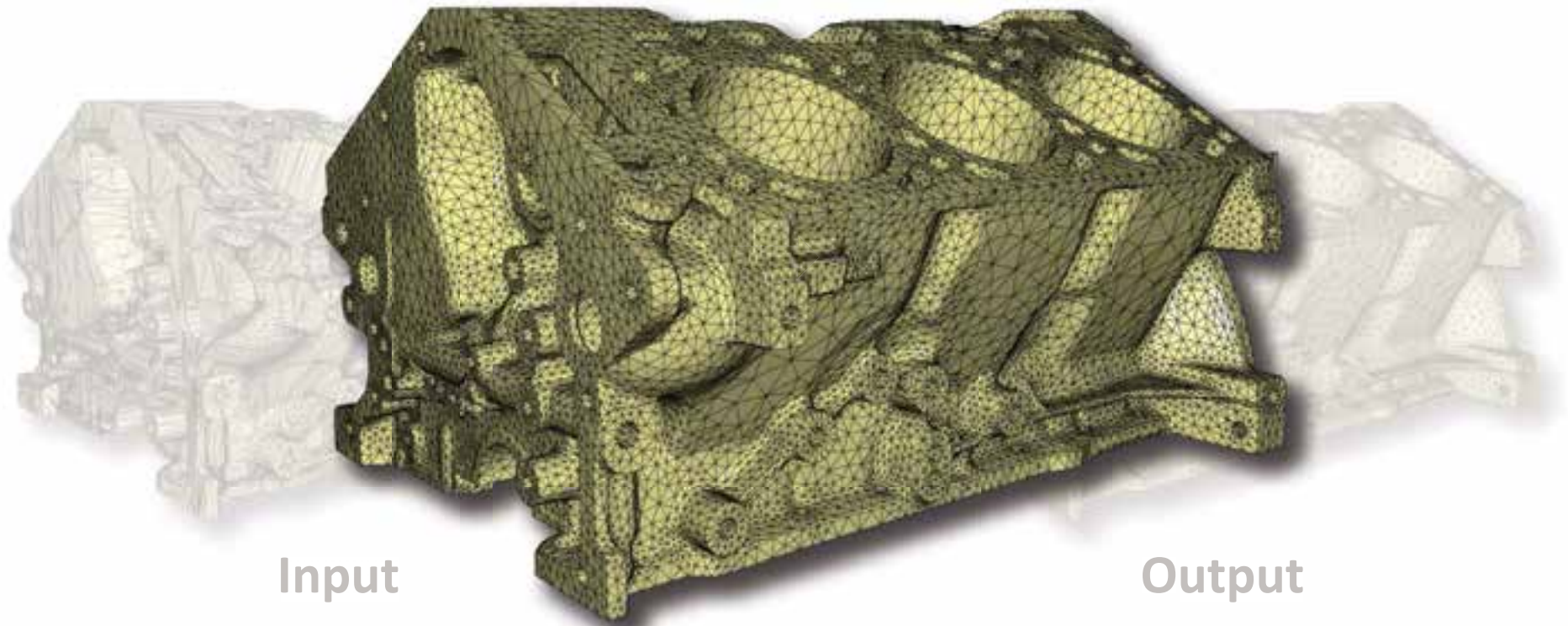
Input



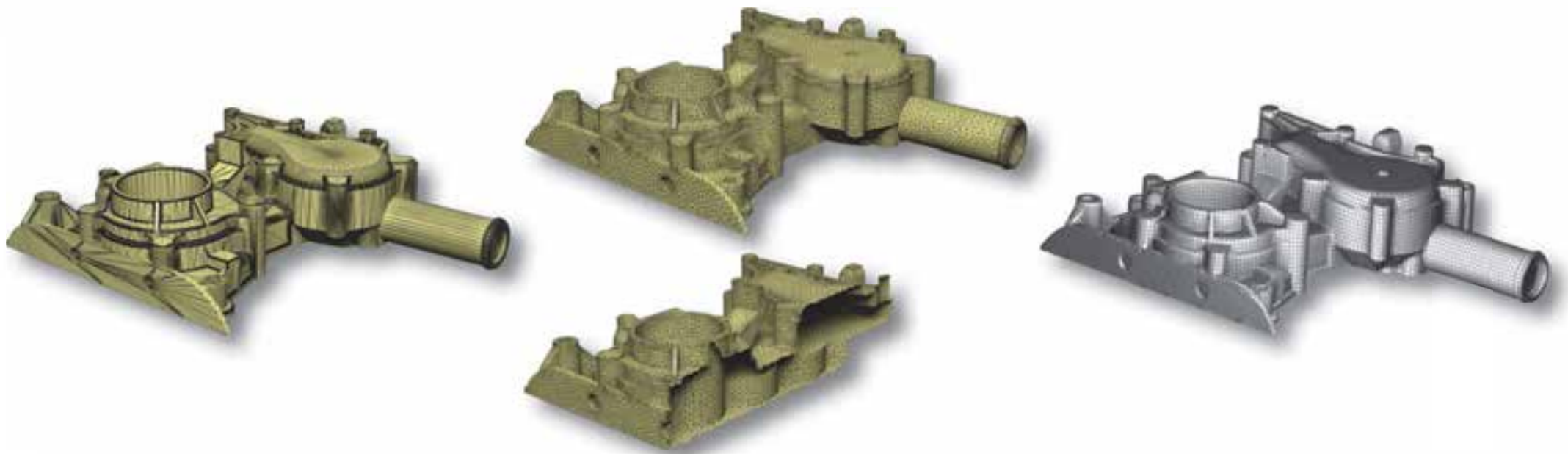
Output



CAD models



Structured Meshing: Quadmesh



Input

Surface of Tetmesh

Quadmesh



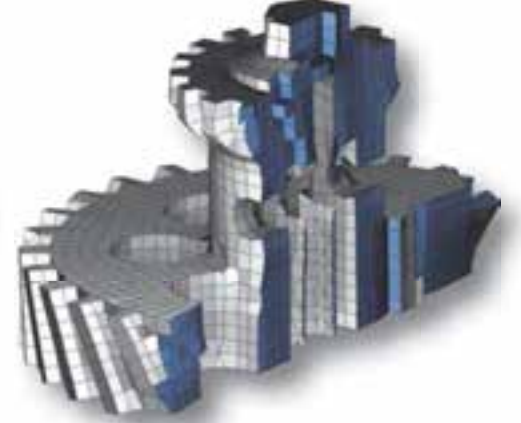
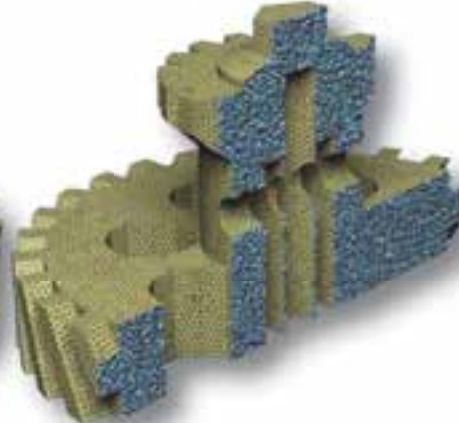
Structured Meshing: Hexmesh



Input



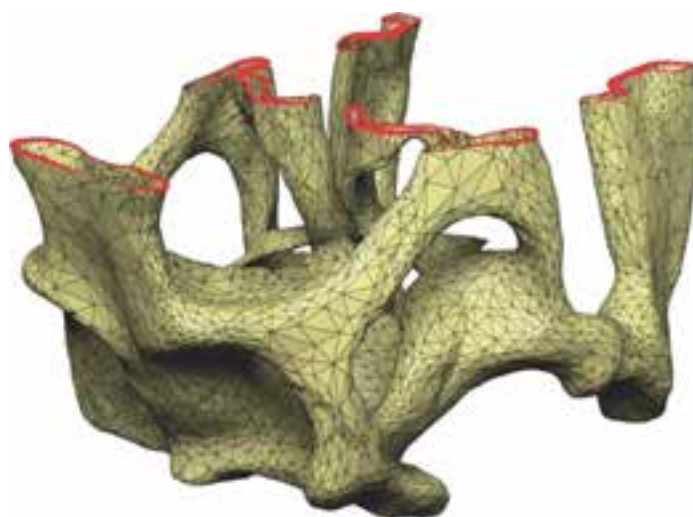
Tetmesh



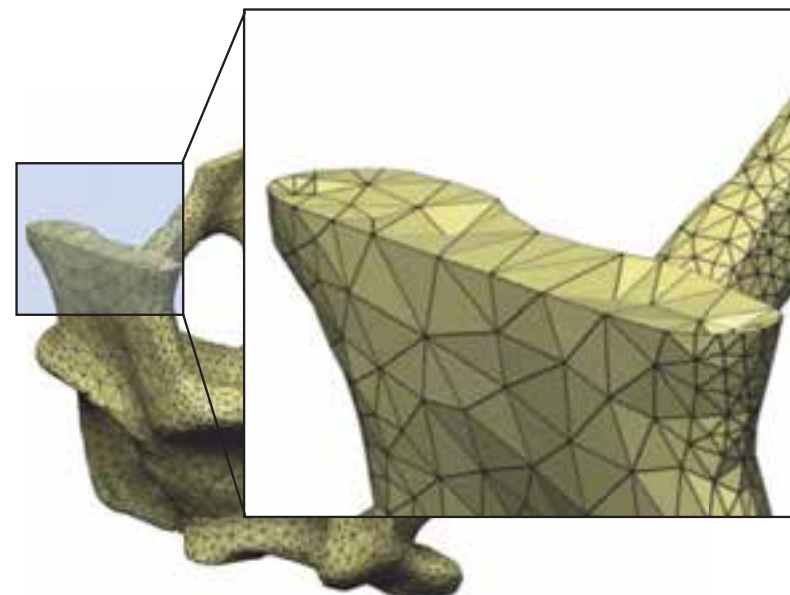
**Hex-dominant
Mesh**



Input with Open Boundary



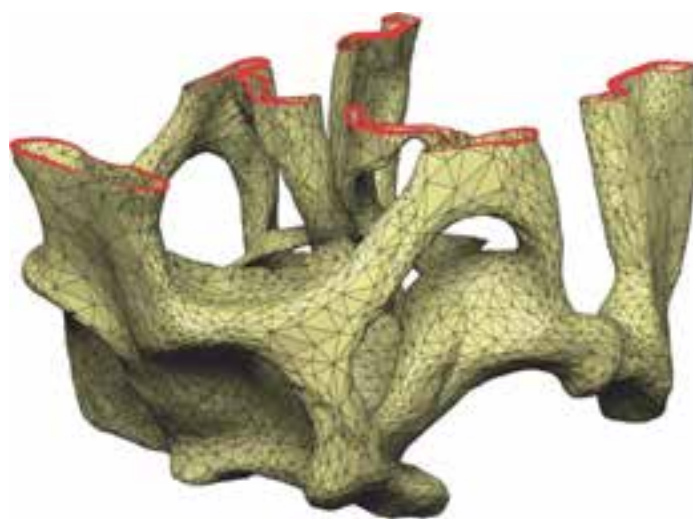
Input



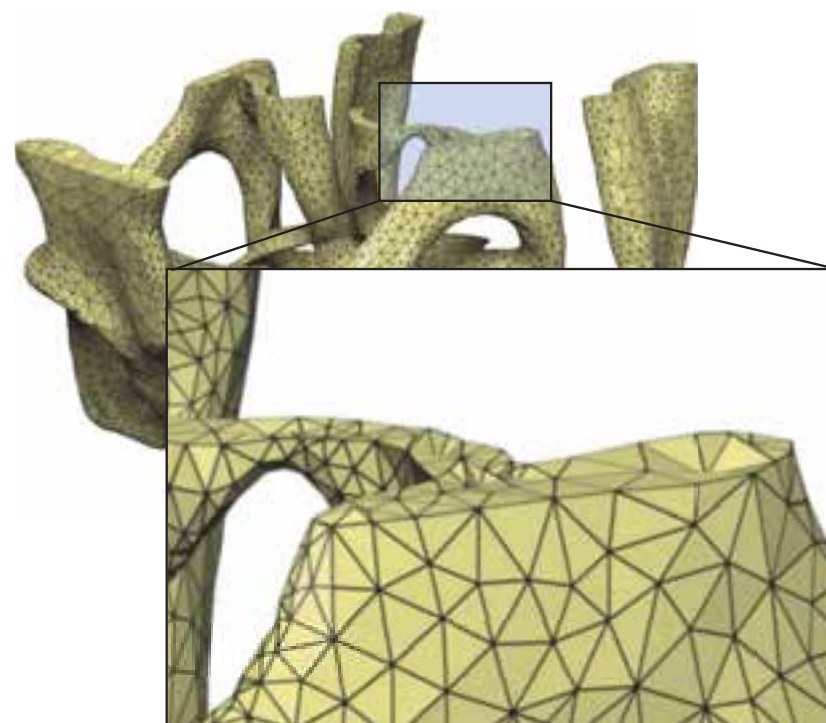
Output w/ smoothing



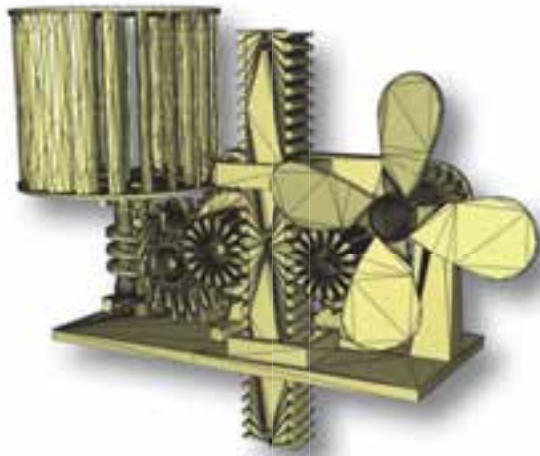
Input with Open Boundary



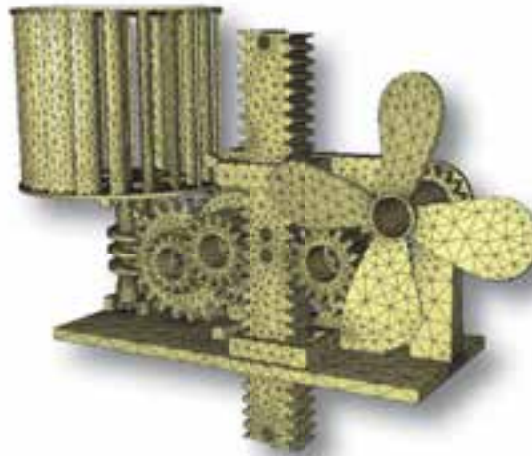
Input



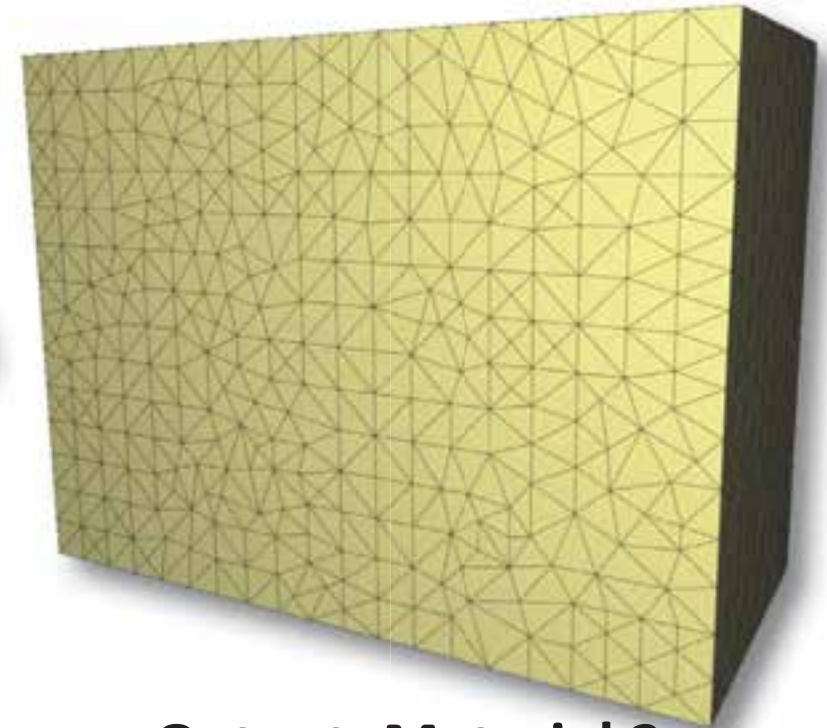
Meshing for Multi-material Solids



Input



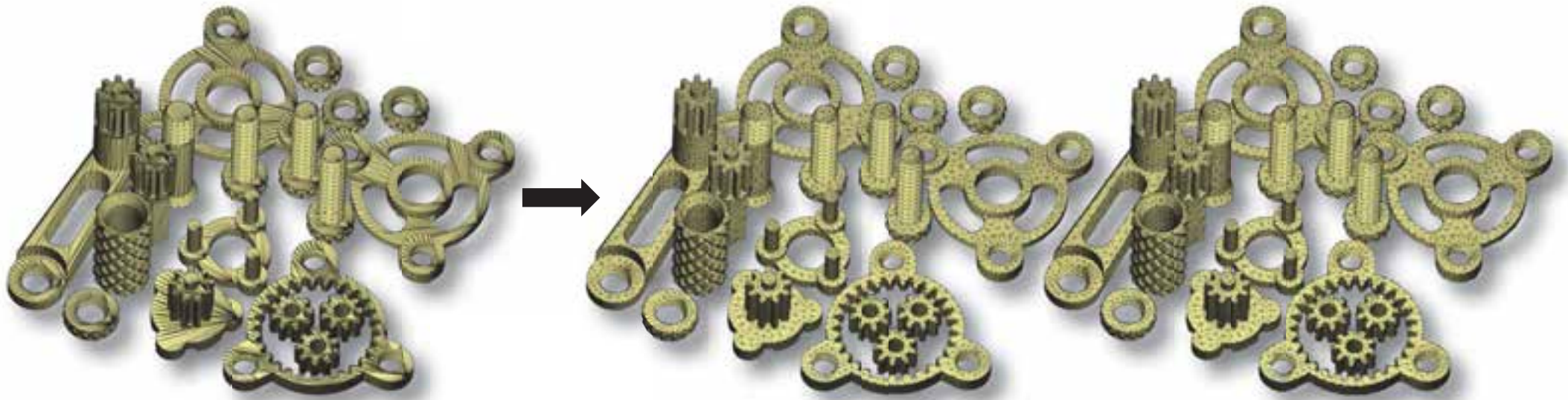
Output: Material 1



Output: Material 2



Multi-component

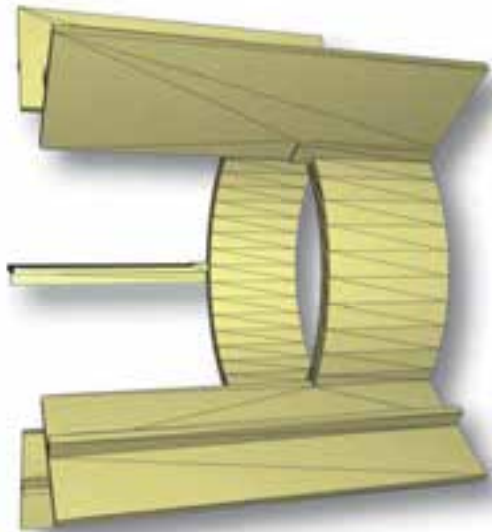


Input

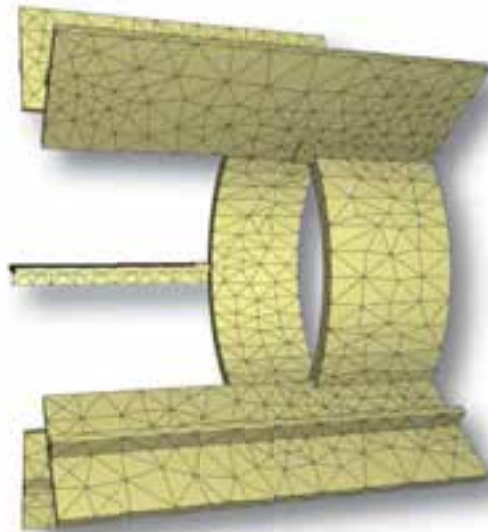
Output



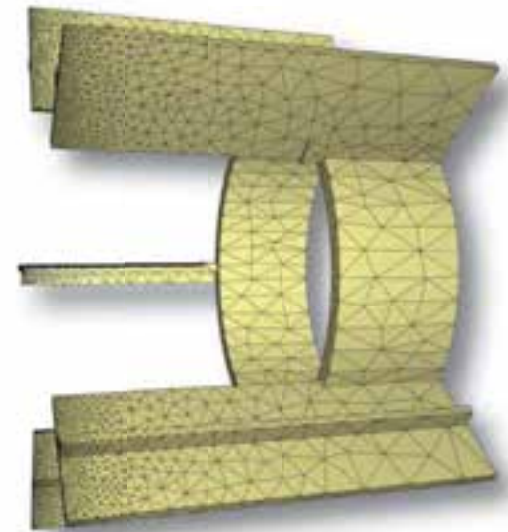
Spatially Varying Sizing Field



Input



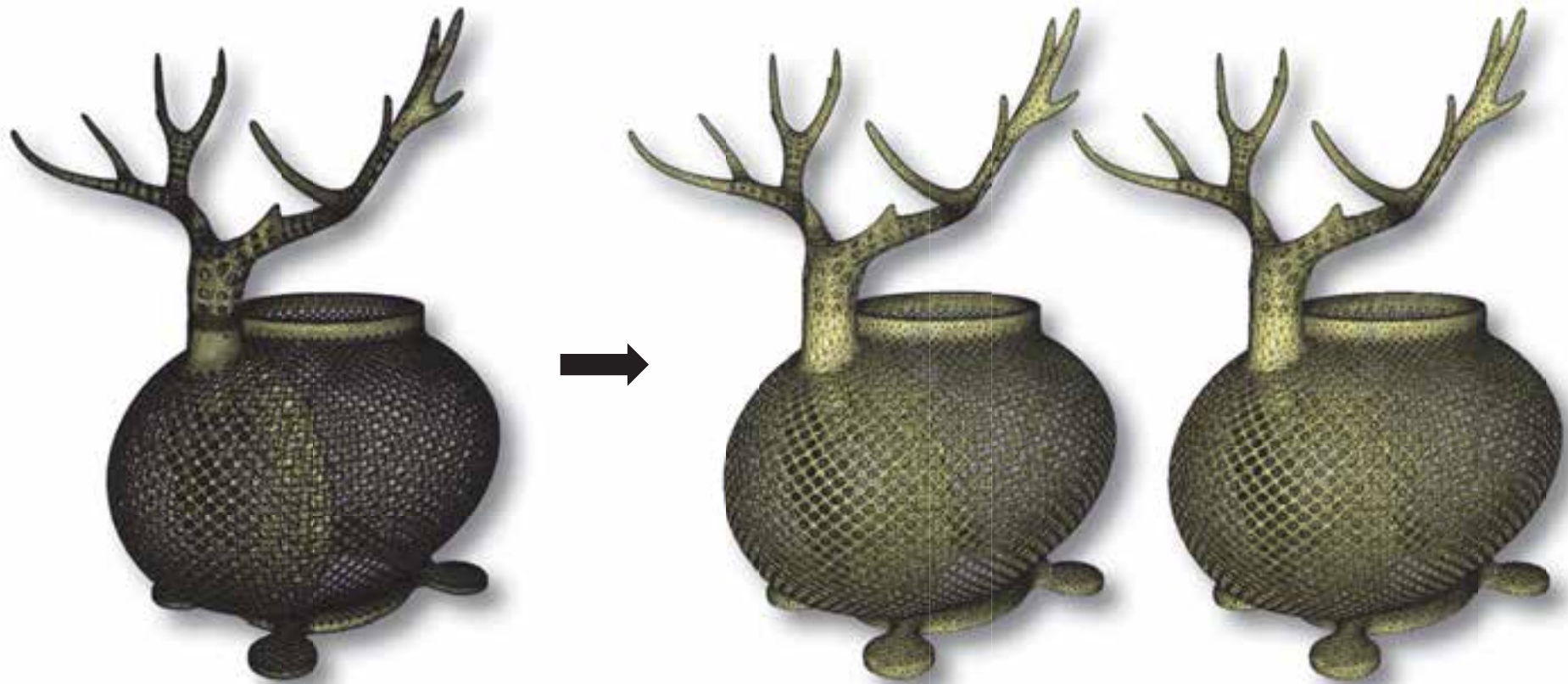
Without Sizing Field



With Sizing Field



More Examples: Thin Structure



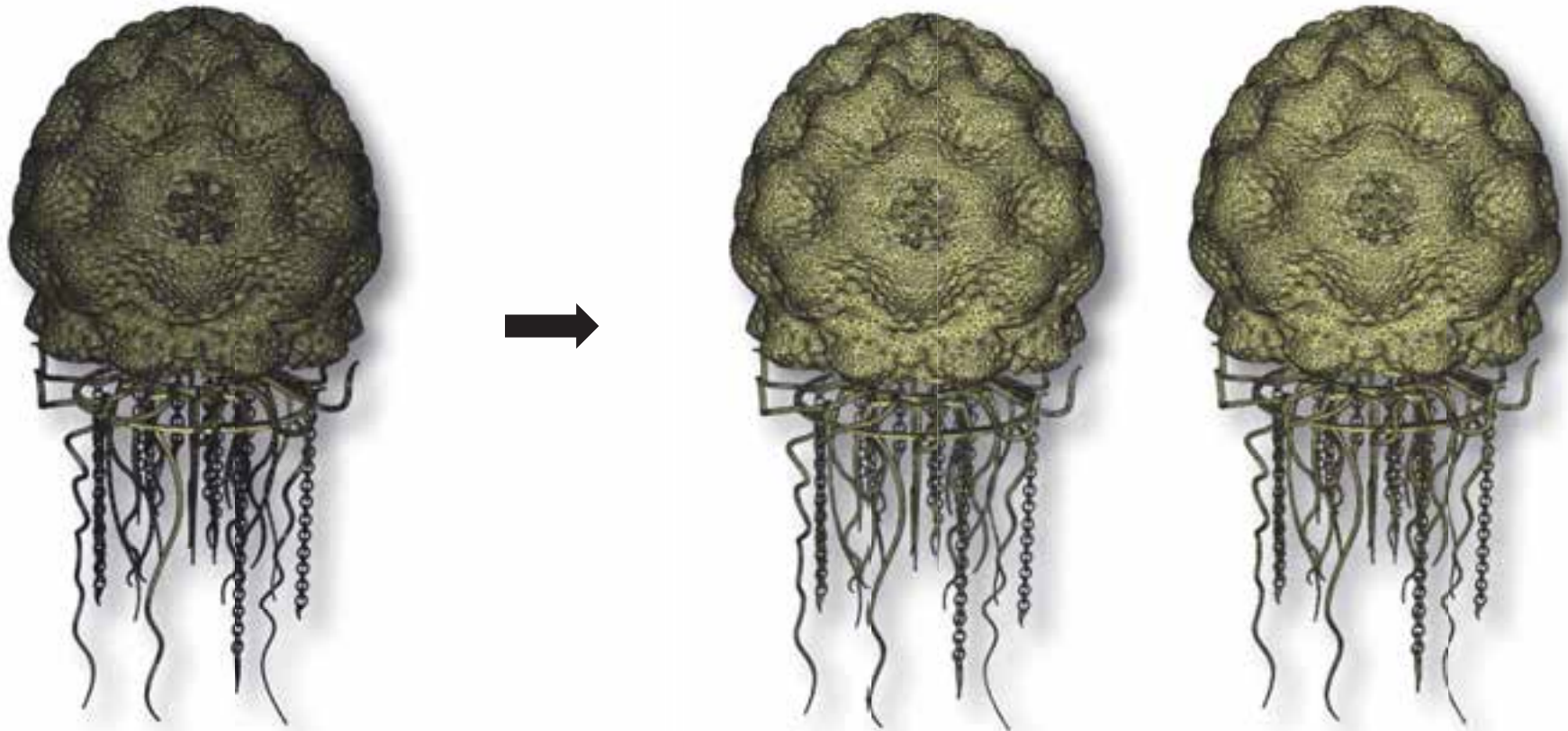
Yixin Hu, 69



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More Examples: Thin Structure

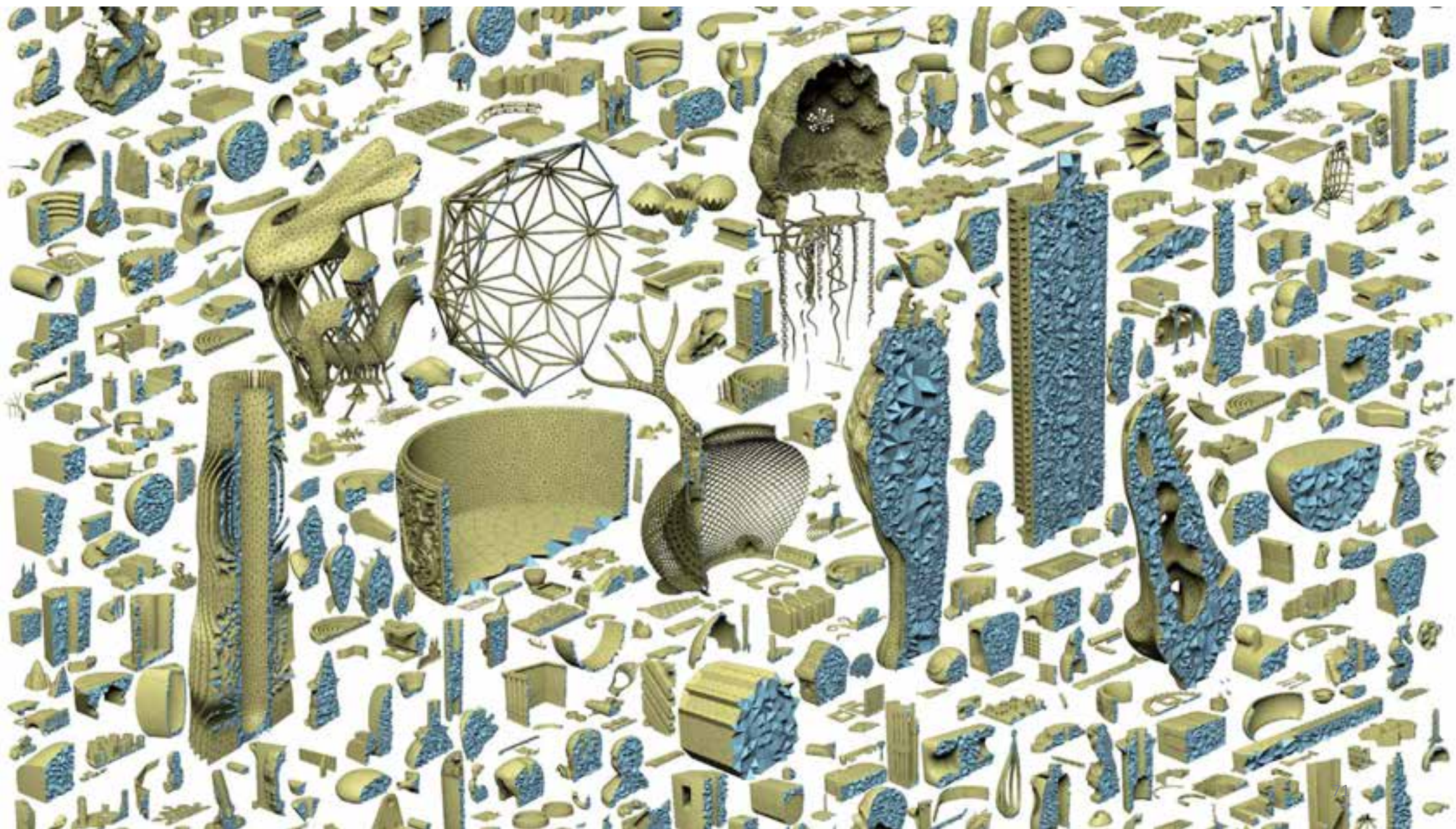


Yixin Hu, 70



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MATHEMATICAL SCIENCES**

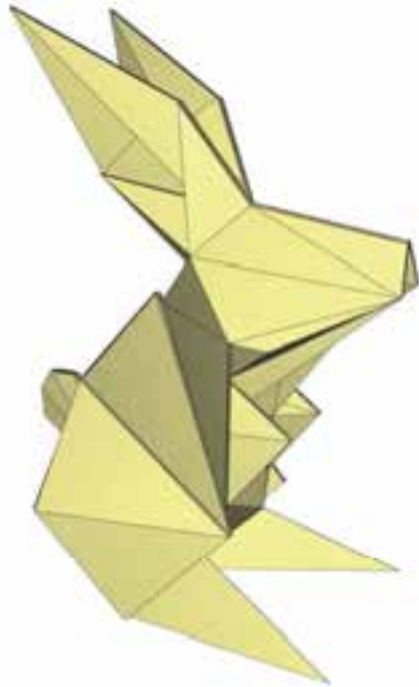


Limitations

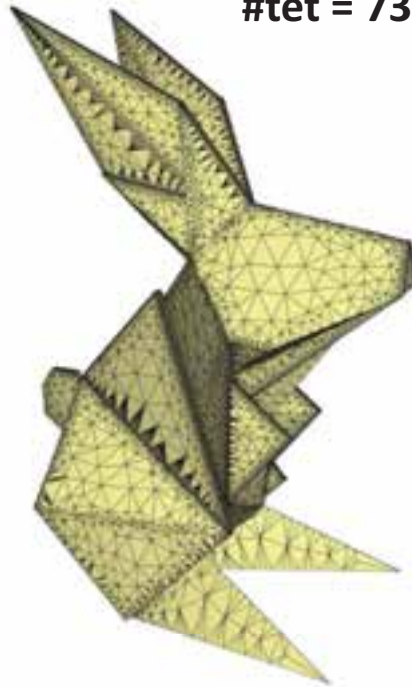
- Our algorithm handles sharp features in a soft way.
 - There always a balance between how well the features are preserved and how much time it takes.
- For repairing surface purpose, our method is limited to surface without large open regions.
- Rational-based computation and envelop check are time-consuming.



CGAL with Feature vs. Ours

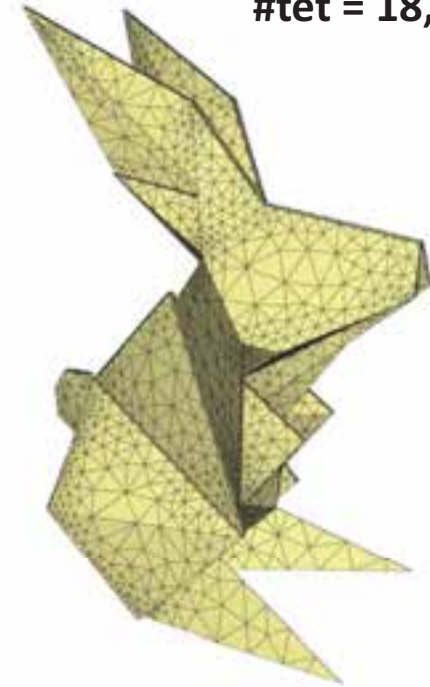


Input



#tet = 73,378

CGAL with feature

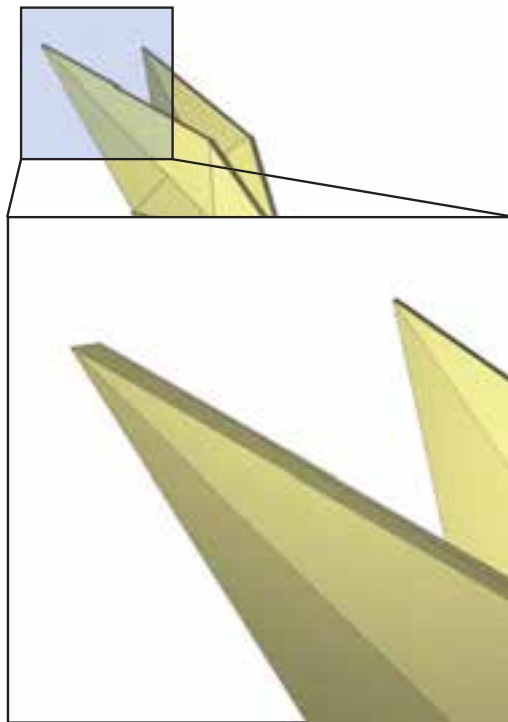


#tet = 18,882

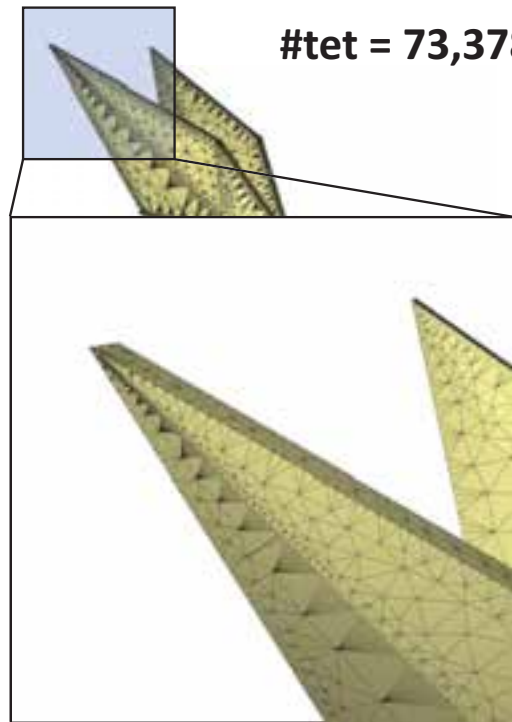
TetWild



CGAL with Feature vs. Ours

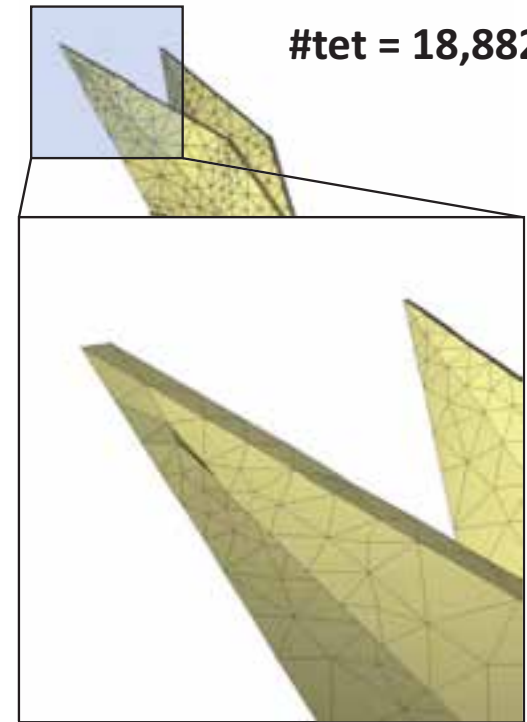


Input



#tet = 73,378

CGAL with feature



#tet = 18,882

TetWild



Limitations

- Our algorithm handles sharp features in a soft way.
 - There always a balance between how well the features are preserved and how much time it takes.
- For repairing surface purpose, our method is limited to surface without large open regions.
- Rational-based computation and envelop check are time-consuming.



Conclusion

- **Robust:**
 - No assumption about the input.
 - Verified on 10,000 models and achieved 100% success rate.
- **Automatic:**
 - No complex user-controlled parameters.
 - Pre-set general parameters tested on 10k dataset.
- **High-quality output.**



Open-source Implementation

- GitHub Repository: <https://github.com/Yixin-Hu/TetWild>



Acknowledgements

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Thank you!

- Email: yixin.hu@nyu.edu
- GitHub Repository: <https://github.com/Yixin-Hu/TetWild>

