

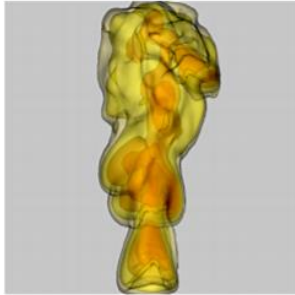
Space-time Tomography for Continuously Deforming Objects

Guangming Zang, Ramzi Idoughi, Ran Tao, Gilles Lubineau, Peter Wonka, Wolfgang Heidrich,
King Abdullah University of Science And Technology

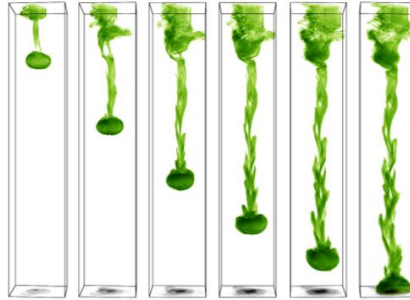




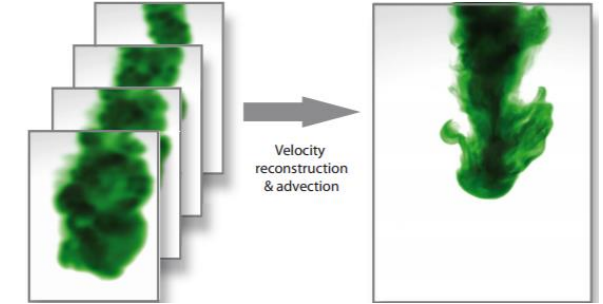
Tomography in graphics



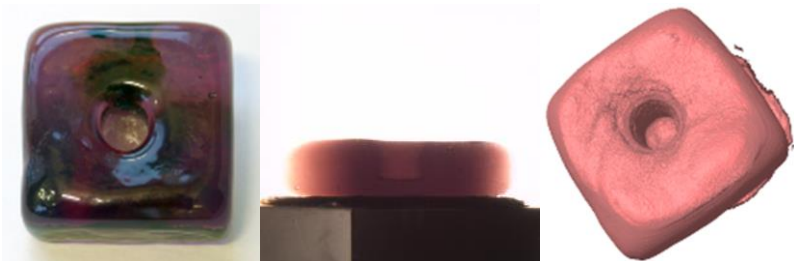
Gas Flows capture
[Atcheson et al. 2008]



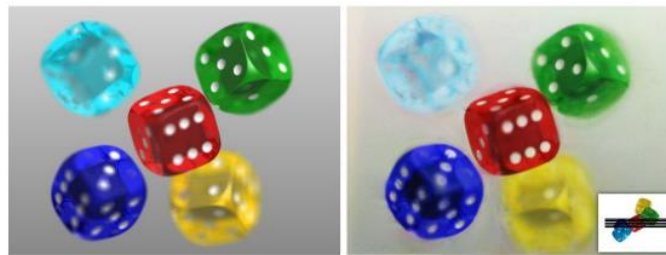
Fluid scanning
[Gregson et al. 2012]



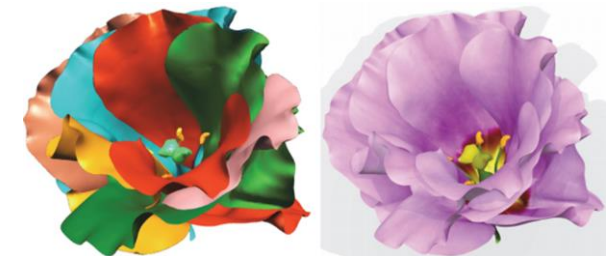
Velocity reconstruction
[Gregson et al. 2014]



Transparent reconstruction
[Trifonov et al. 2006]



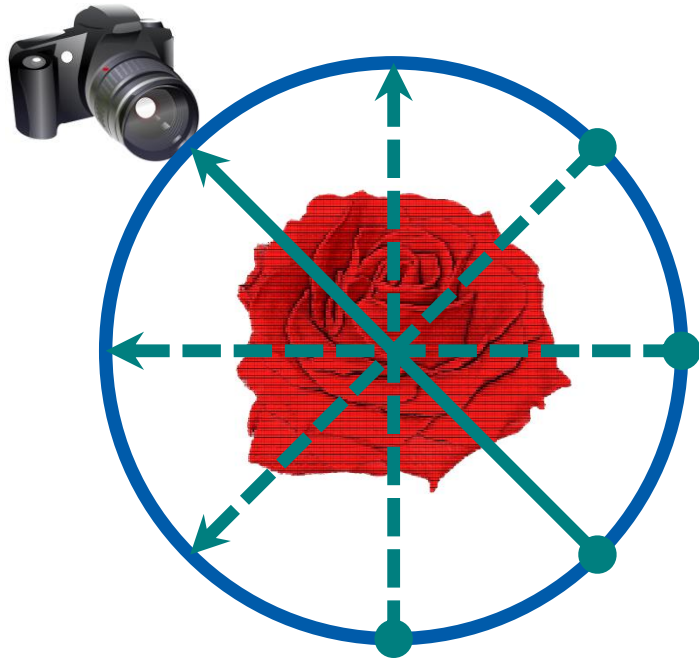
Multi-layer 3D display
[Wetzstein et al. 2011]



Flower modeling
[Ijiri et al. 2014]



Motivation



Unknowns: $\sim 500^3$ voxels

Equations: 2M pixels \times Number of projections



Large number of projections + priors



Motivation

Time for one projection: ~ 5 s

Is it possible to scan continuously deforming objects ?





Motivation

Classical reconstruction using 5520 projections (duration: 9h32m)



Before scan



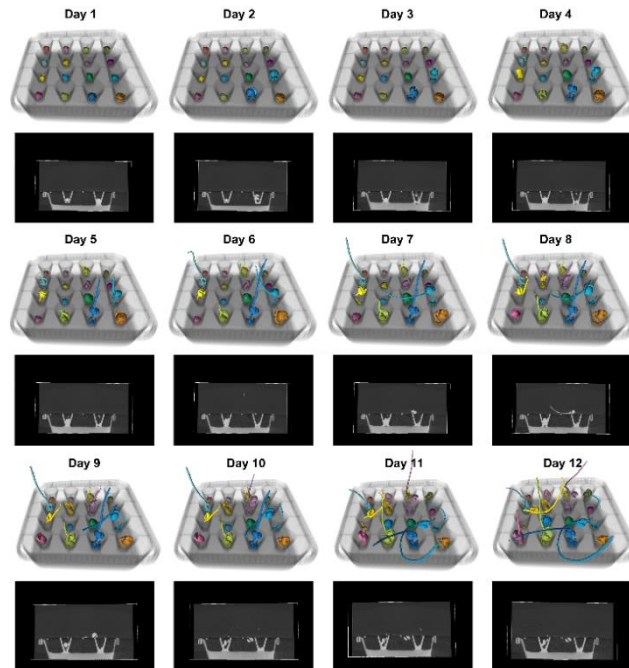
Wilting rose



After scan

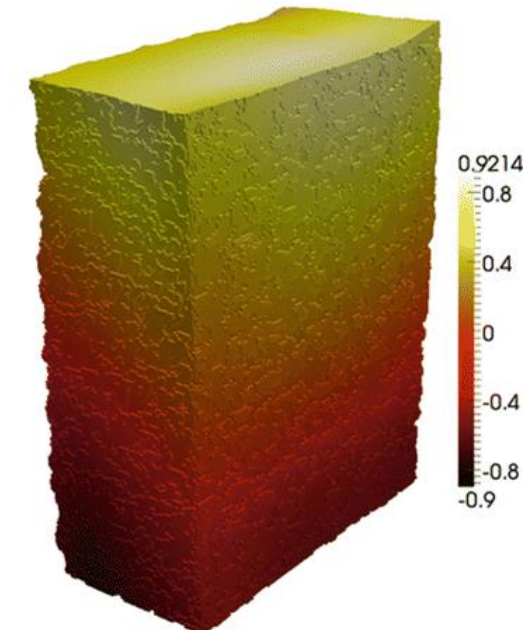
Related work

Discrete deformations / Key framing



Growing plant animation

[Kato et al. 2017]



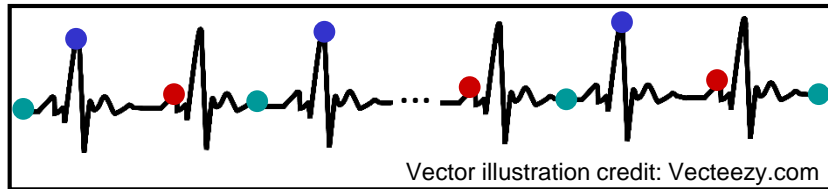
Digital Volume Correlation to retrieve
the deformation field

[Hild et al. 2014]

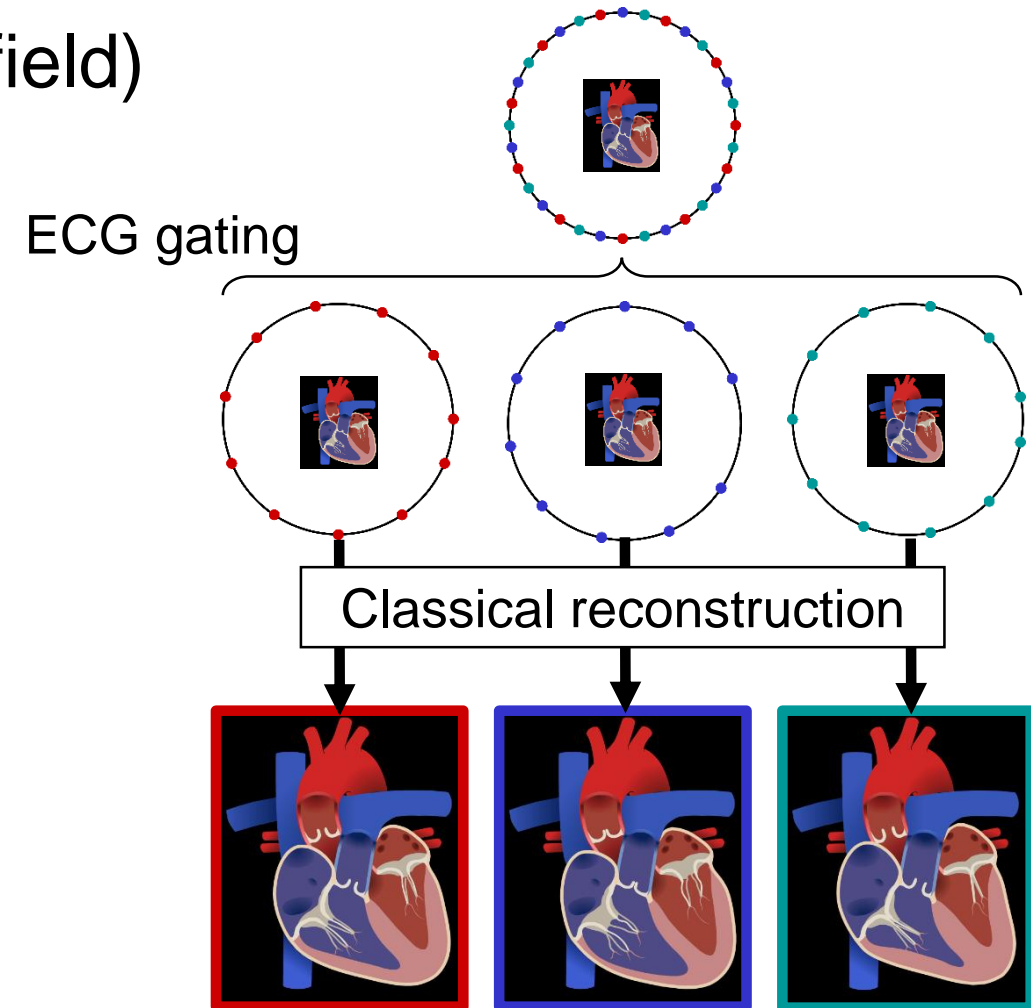


Related work

Periodic deformations (Medical field)



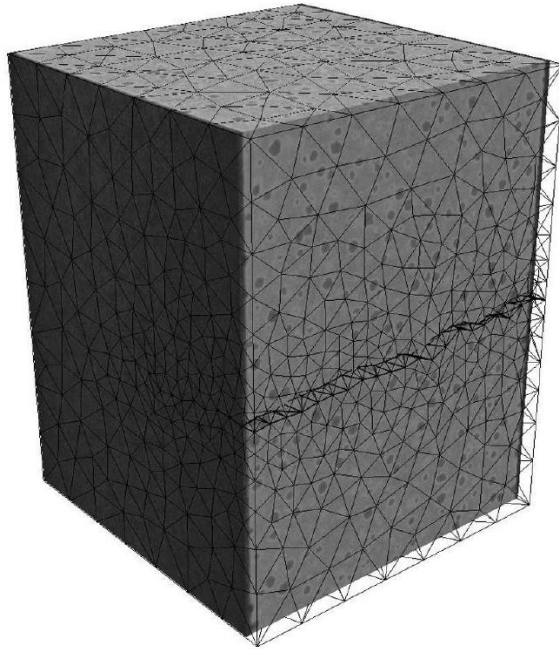
Electrocardiography (ECG)



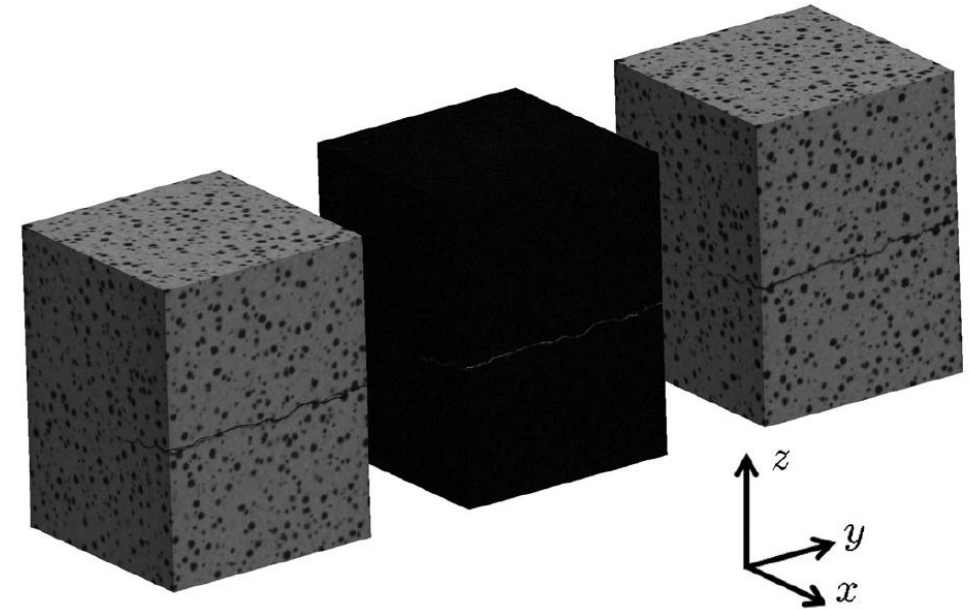
credit: austincc.edu

Related work

General deformations (parametric approach)



Use of basis-functions approach
(e.g. finite element framework)



The volume is advected from a reference, using an estimated displacement.



Objective

Space-time tomography reconstruction



Before scan



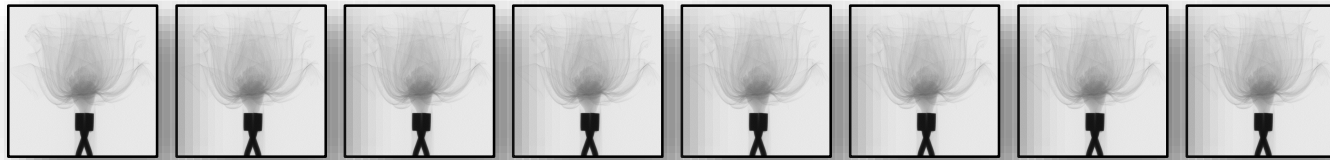
Wilting rose



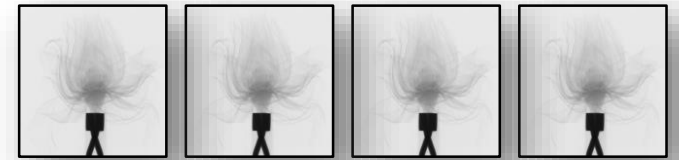
After scan



Space-time tomography



...



Assumption: relatively slow deformations (less than 6 voxels/minute)



Space-time tomography



Frame 01



Frame 02



Frame 46



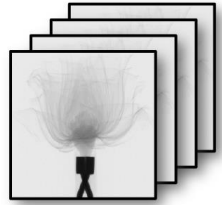
Frame 91



Frame 92



Space-time tomography



Frame 01



Frame 02



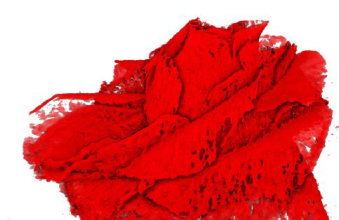
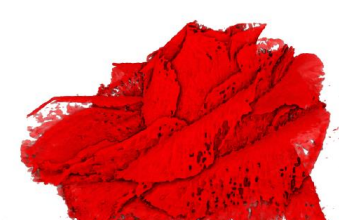
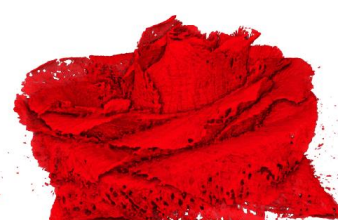
Frame 46



Frame 91

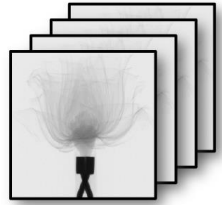


Frame 92

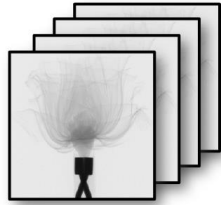




Space-time tomography



Frame 01



Frame 02



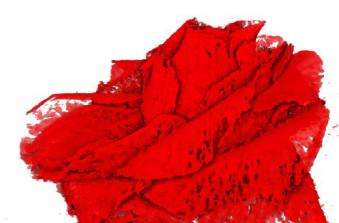
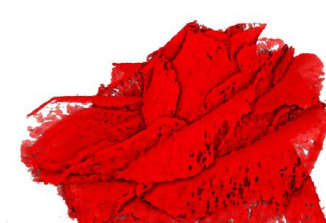
Frame 46



Frame 91



Frame 92

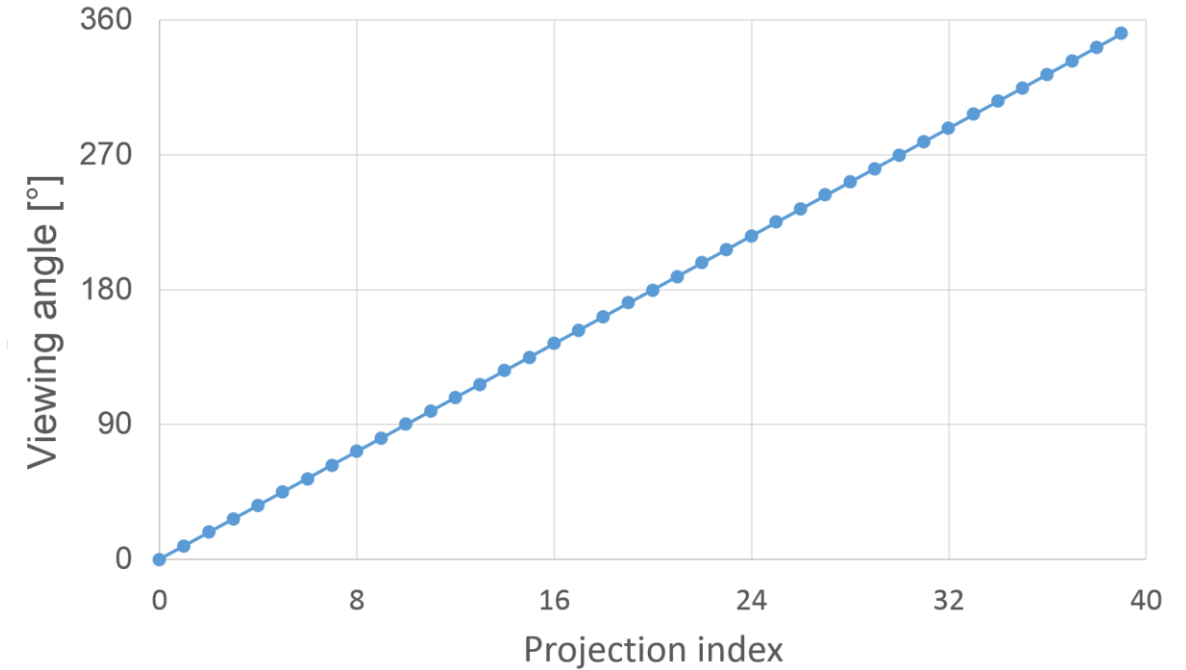
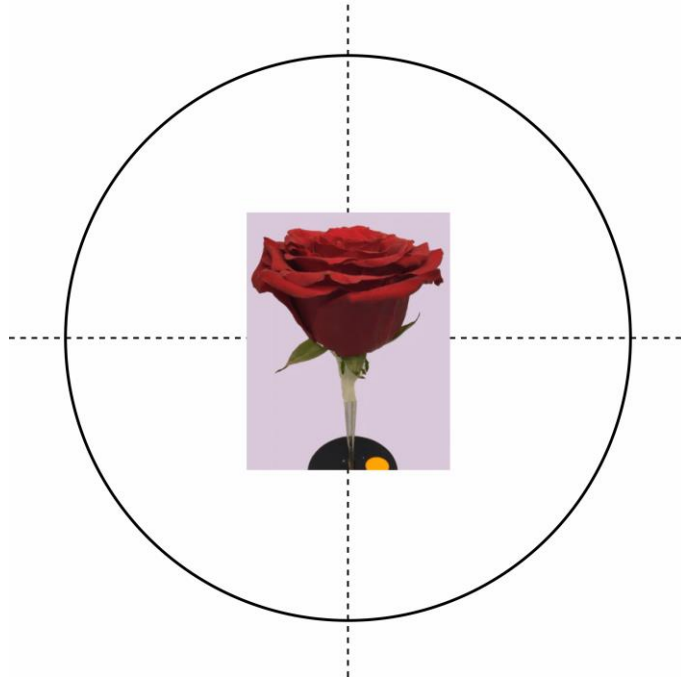




Overview

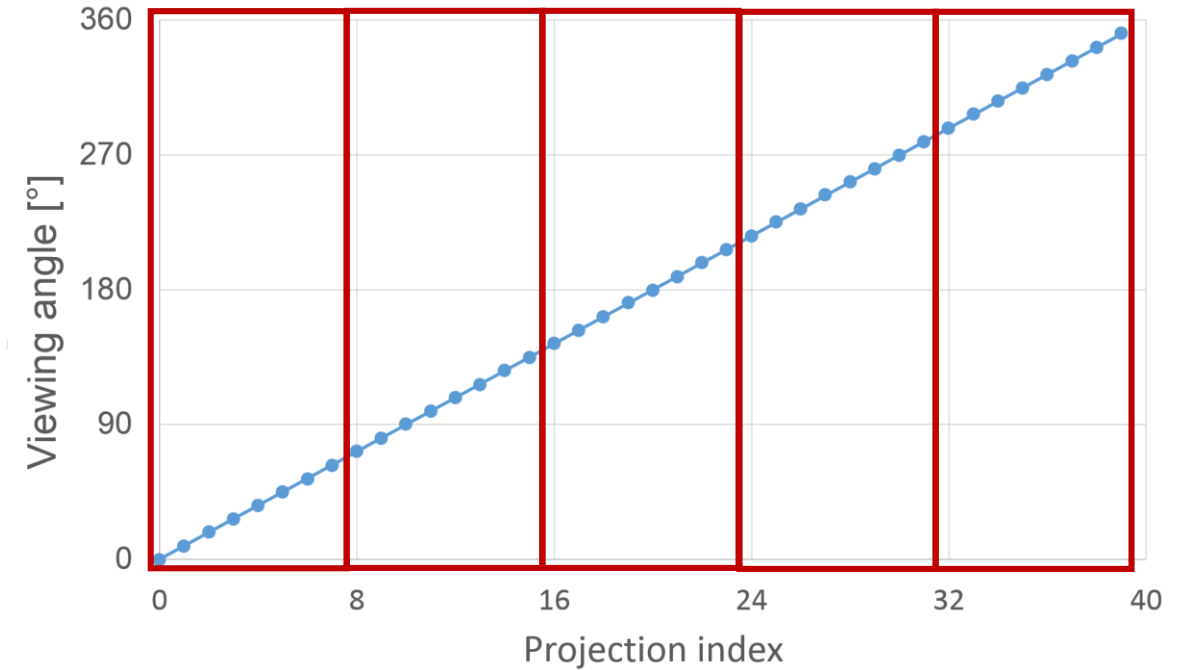
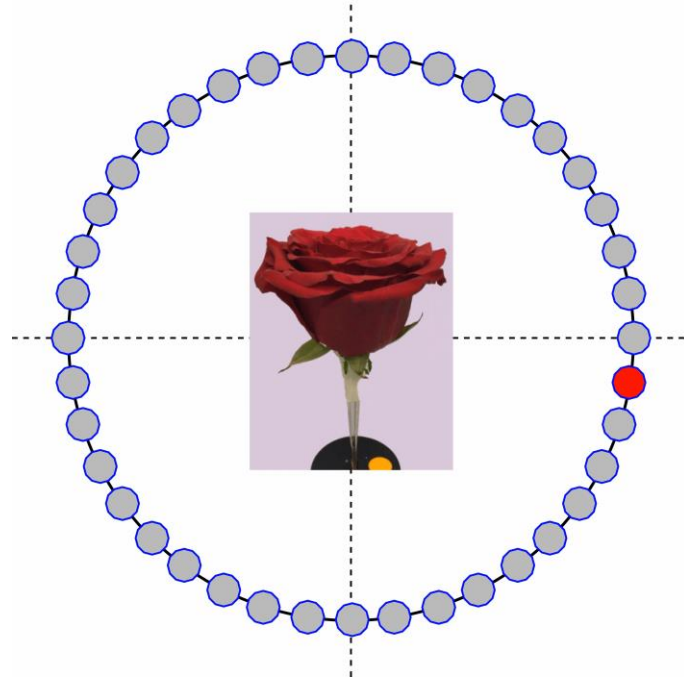
- Data acquisition
- 4D tomographic reconstruction
- Results

Data acquisition



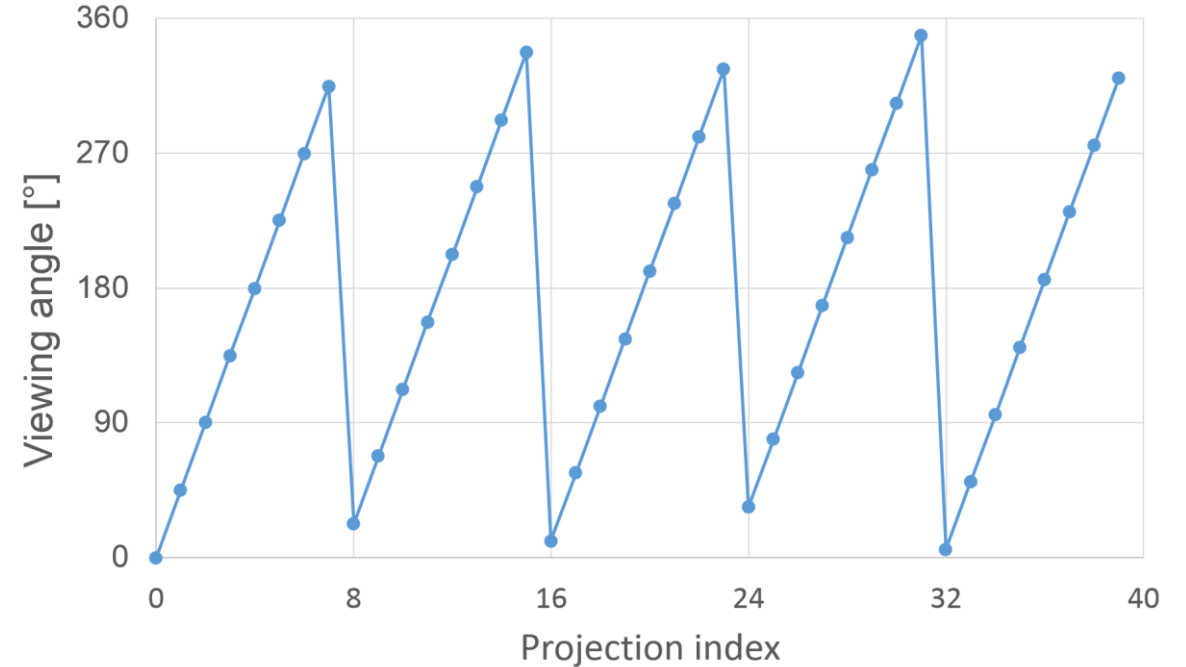
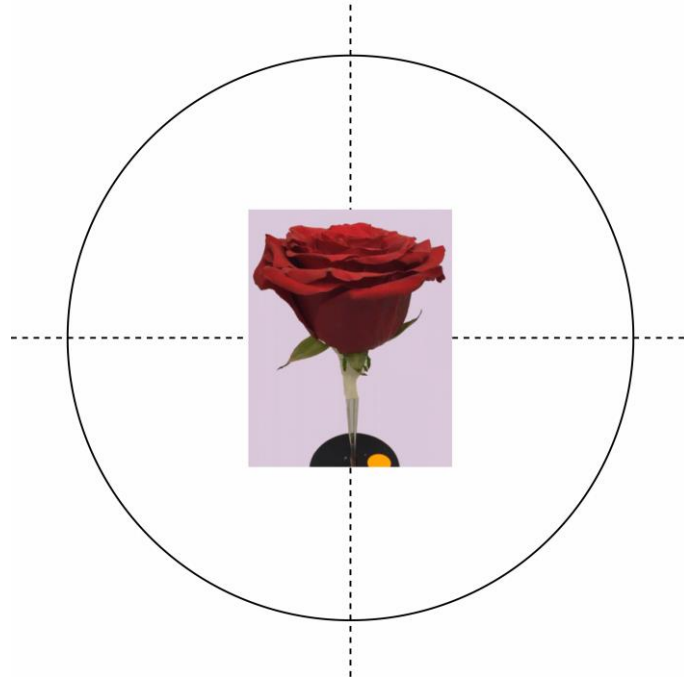
Classical acquisition strategy

Data acquisition



Classical acquisition strategy

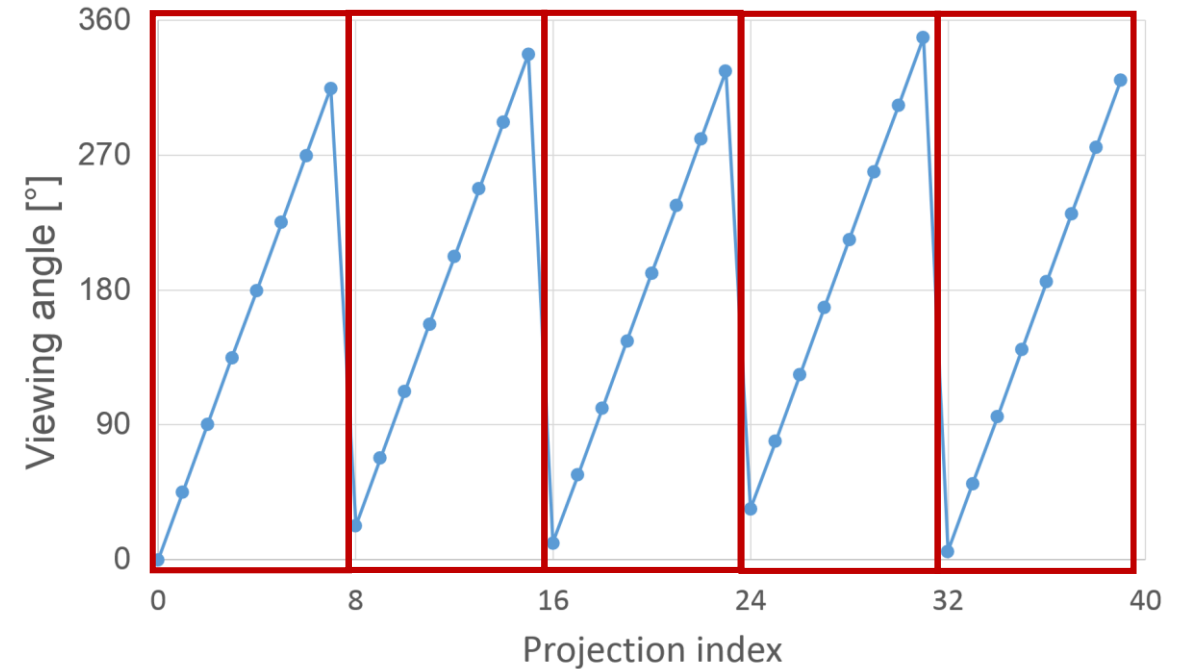
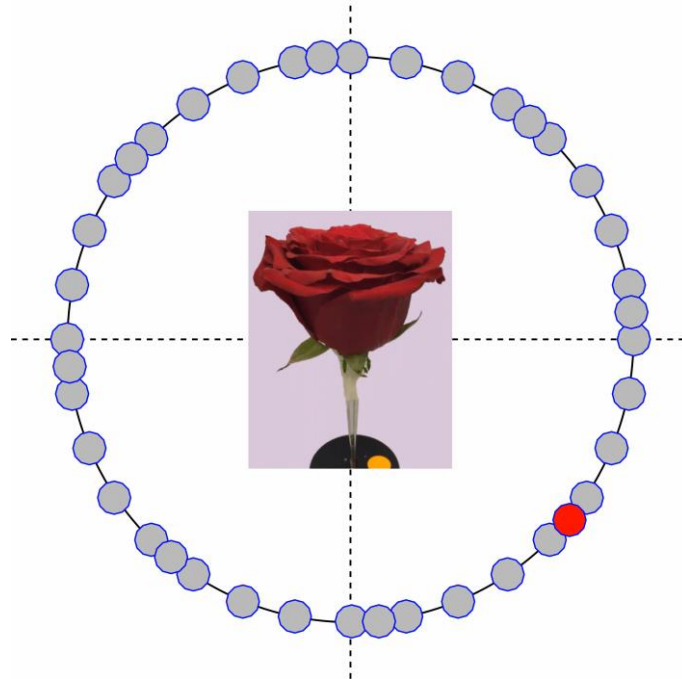
Data acquisition



Low-discrepancy acquisition strategy

(Van der Corrupt sequence)

Data acquisition



Low-discrepancy acquisition strategy

(Van der Corrupt sequence)



Overview

- Data acquisition
- 4D tomographic reconstruction
- Results



Image formation model

X-ray path: Ω_i

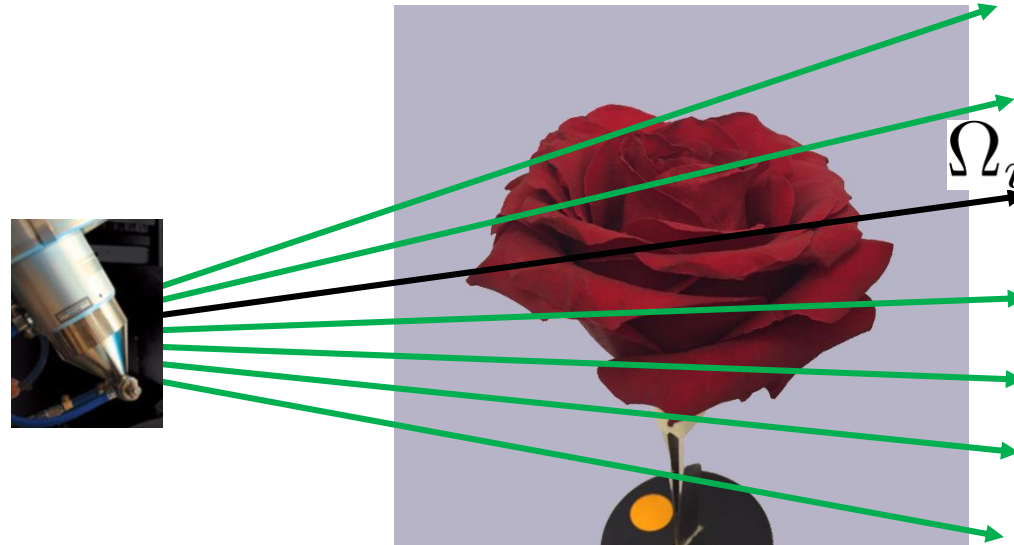




Image formation model

X-ray path: Ω_i

Unknown field: $f(x)$

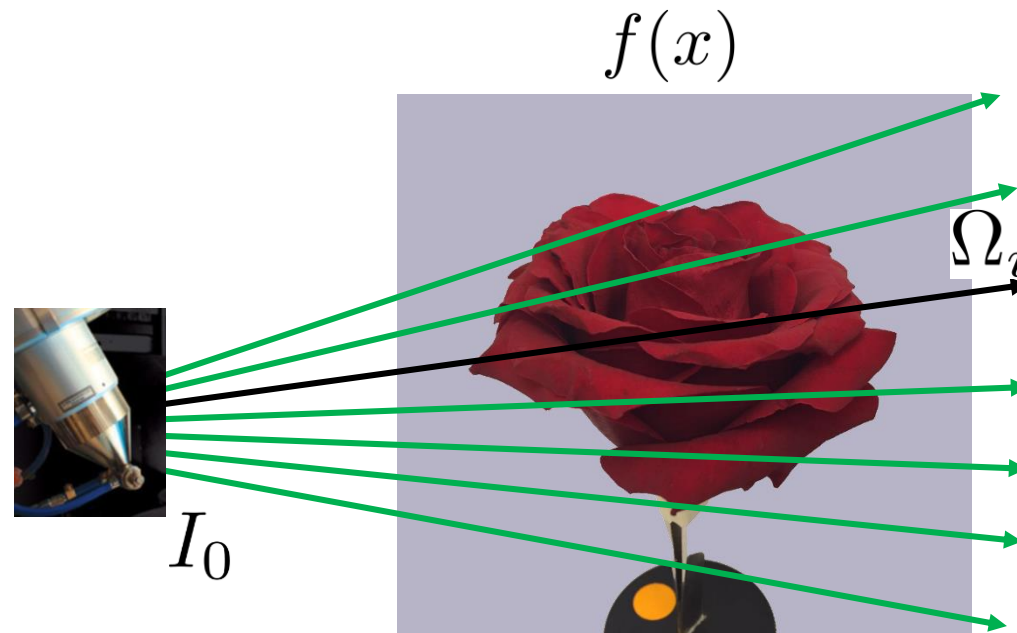


Image formation model

X-ray path: Ω_i

Unknown field: $f(x)$

Measurement: p_i

$$\int_{\Omega_i} f(x) d\Omega_i = \underbrace{-\log(I_i/I_0)}_{p_i}$$

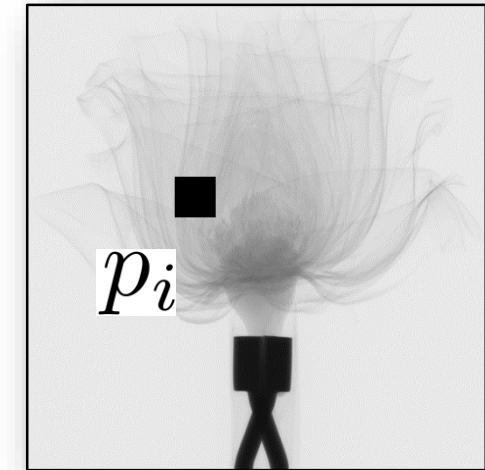
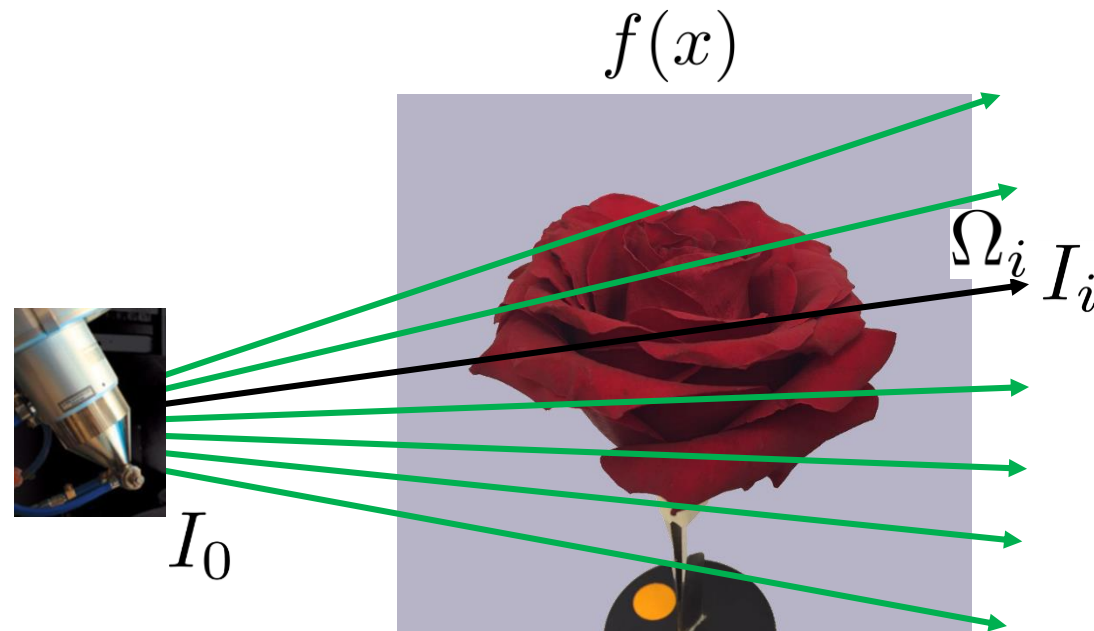


Image formation model

X-ray path: Ω_i

Unknown field: $f(x)$

Measurement: p_i

$$\sum_j f_j A_{ij} = p_i$$

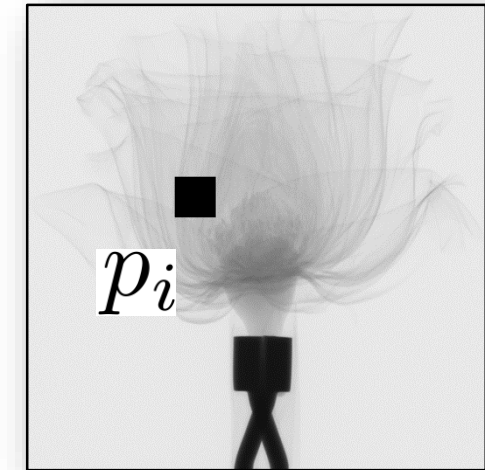
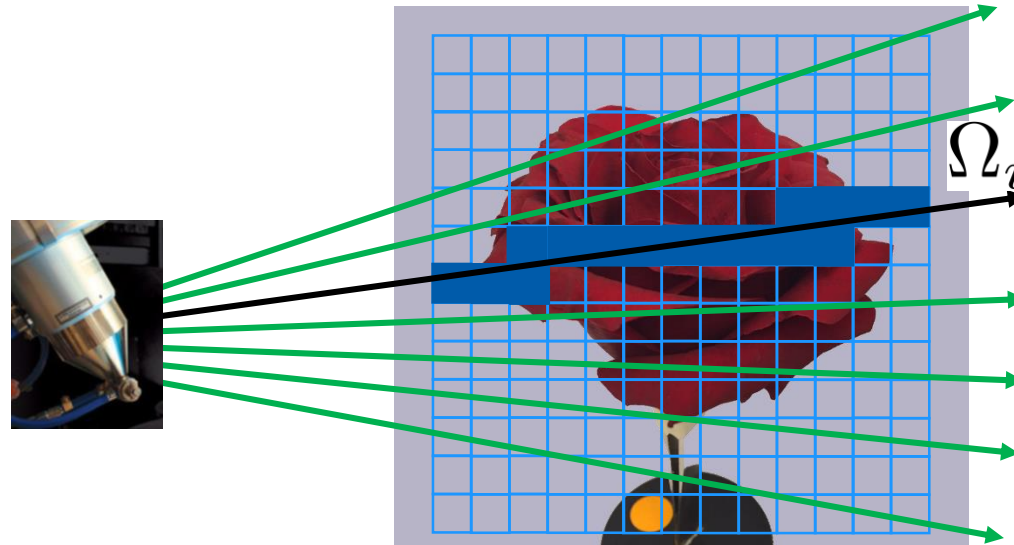




Image formation model

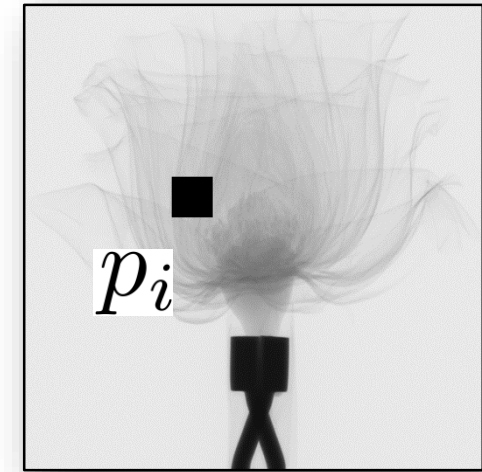
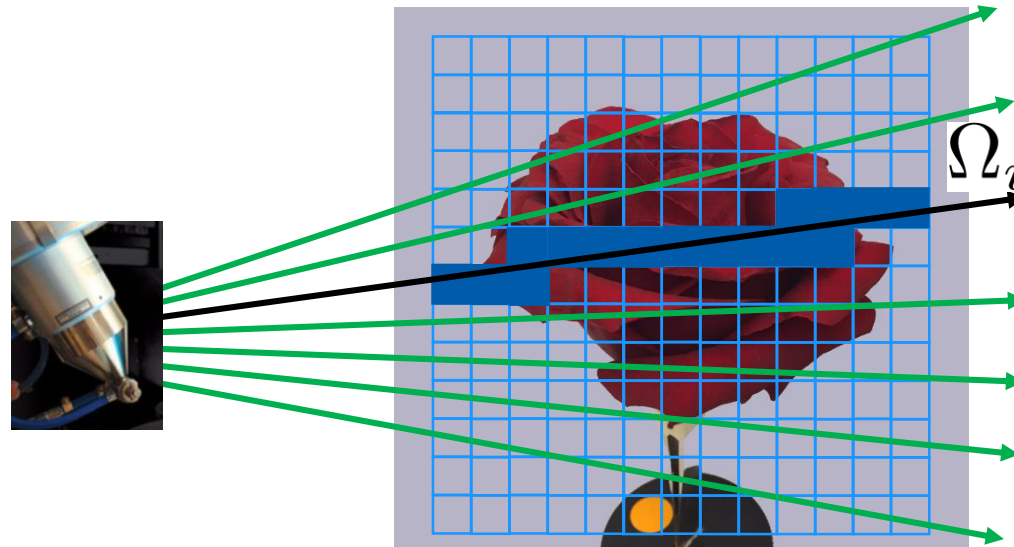
X-ray path: Ω_i

Unknown field: $f(x)$

Measurement: p_i

$$\sum_j f_j A_{ij} = p_i$$

$$A_t f_t = p_t$$



Negligible deformation in sequential projections!

Space-time tomography



Frame 01



Frame 02



Frame 46



Frame 91



Frame 92

$$\underbrace{\begin{pmatrix} \mathbf{A}_1 & & \\ & \ddots & \\ & & \mathbf{A}_t \\ & & & \ddots & \\ & & & & \mathbf{A}_{N_t} \end{pmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_t \\ \vdots \\ \mathbf{f}_{N_t} \end{pmatrix}}_{\mathbf{f}} = \underbrace{\begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_t \\ \vdots \\ \mathbf{p}_{N_t} \end{pmatrix}}_{\mathbf{p}}$$

Linear system

- Sparse system
- Memory consuming
- Ill-posed problem

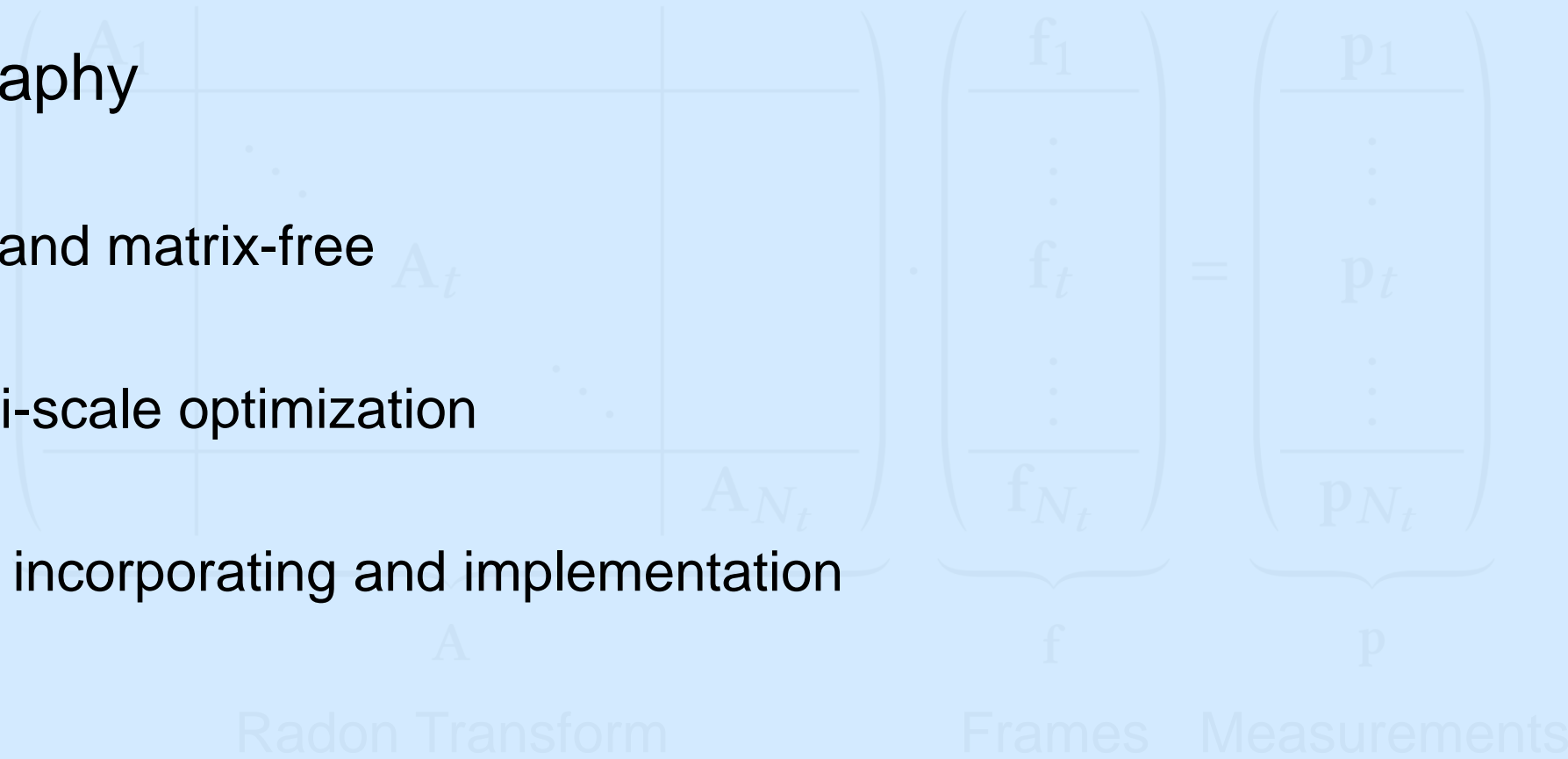
$$\underbrace{\begin{pmatrix} \mathbf{A}_1 & & \\ & \ddots & \\ & & \mathbf{A}_t & & \\ & & & \ddots & \\ & & & & & \mathbf{A}_{N_t} \end{pmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_t \\ \vdots \\ \mathbf{f}_{N_t} \end{pmatrix}}_{\mathbf{f}} = \underbrace{\begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_t \\ \vdots \\ \mathbf{p}_{N_t} \end{pmatrix}}_{\mathbf{p}}$$

Radon Transform
Frames
Measurements

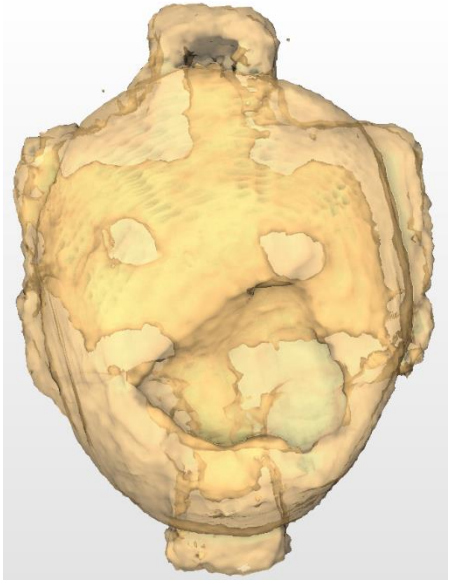
Linear system

Space time tomography

- Sparse system
 - Non-parametric and matrix-free
- Memory consuming
 - Alternating, multi-scale optimization
- Ill-posed problem
 - Flexibility: priors incorporating and implementation



Priors

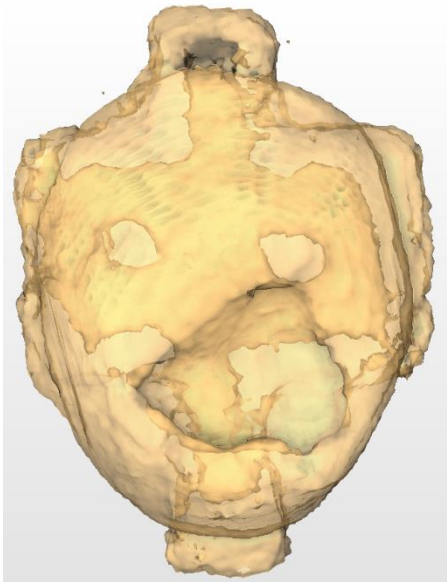


Tomography

$$\|A_t \mathbf{f}_t - \mathbf{p}_t\|_2^2$$

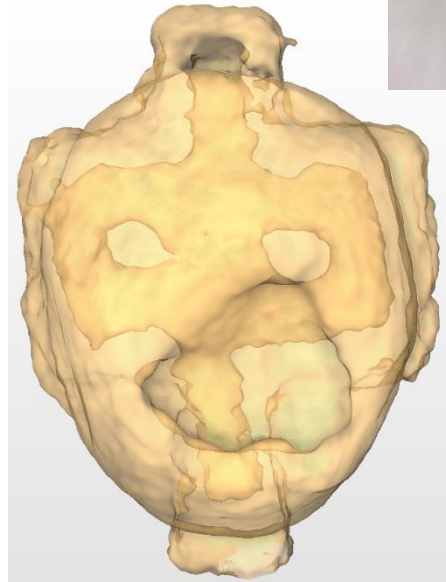


Priors



Tomography

$$\|\mathbf{A}_t \mathbf{f}_t - \mathbf{p}_t\|_2^2$$

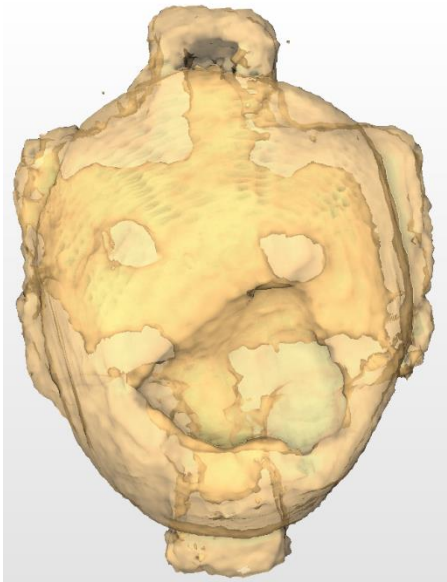


Spatial smoothness

$$\|\nabla_S \mathbf{f}_t\|_{H_\epsilon}$$

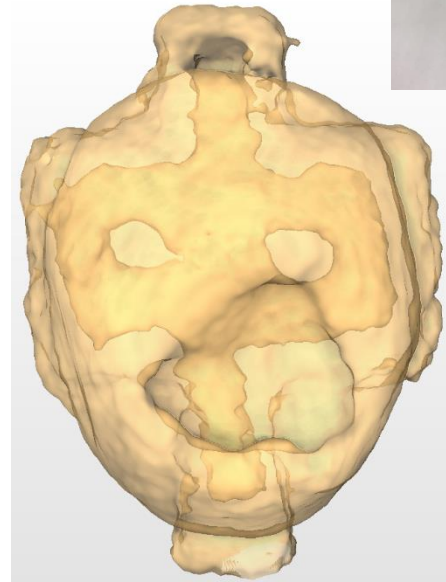


Priors



Tomography

$$\|\mathbf{A}_t \mathbf{f}_t - \mathbf{p}_t\|_2^2$$



Spatial smoothness

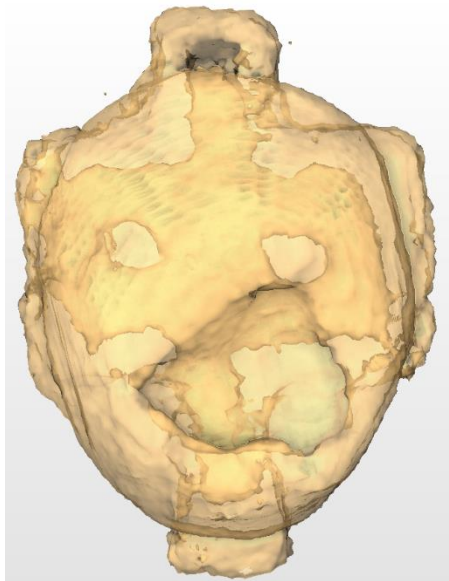
$$\|\nabla_S \mathbf{f}_t\|_{H_\epsilon}$$



Temporal smoothness

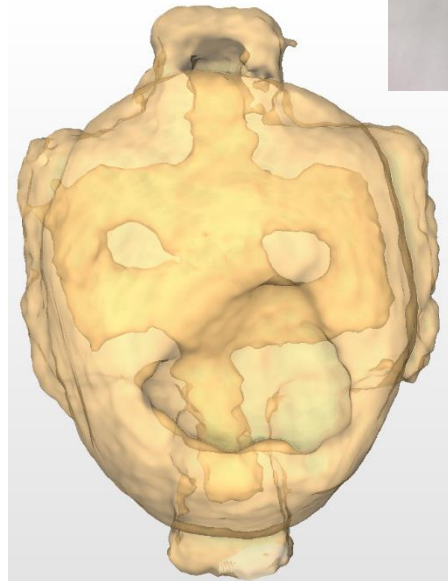
$$\|\nabla_T \mathbf{f}_t\|_2^2$$

Priors



Tomography

$$\|\mathbf{A}_t \mathbf{f}_t - \mathbf{p}_t\|_2^2$$



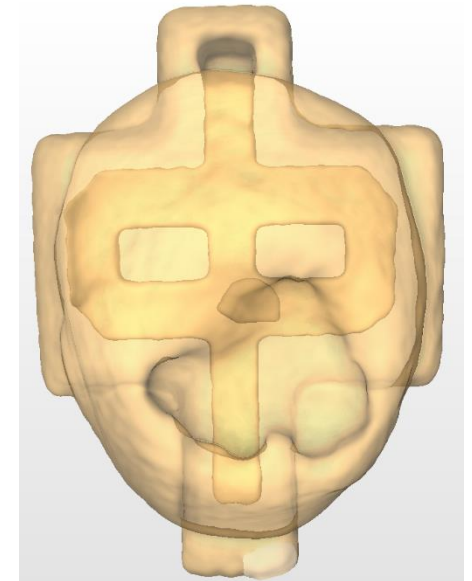
Spatial smoothness

$$\|\nabla_S \mathbf{f}_t\|_{H_\epsilon}$$



Temporal smoothness

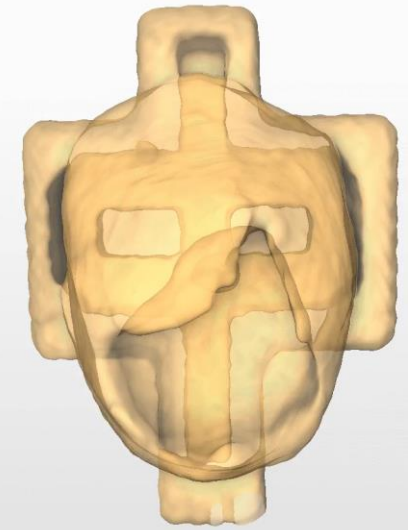
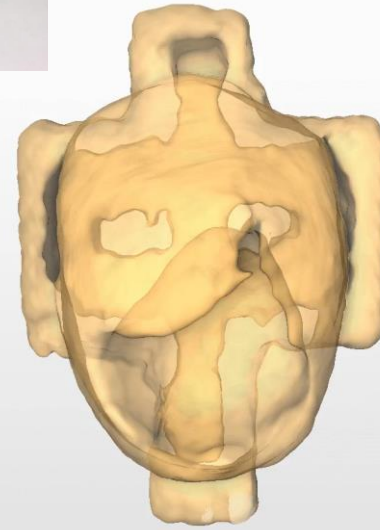
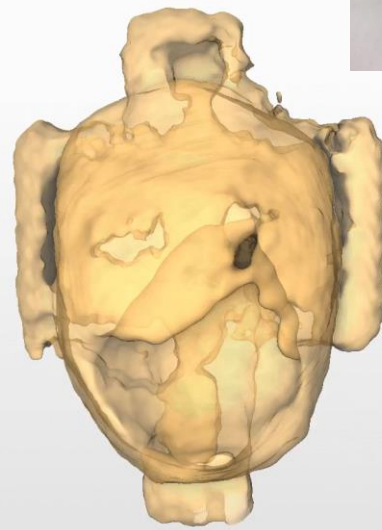
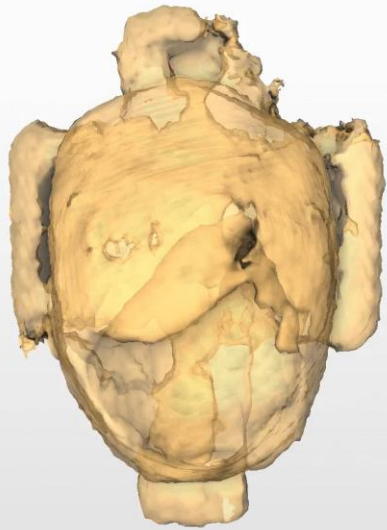
$$\|\nabla_T \mathbf{f}_t\|_2^2$$



Flow correction

$$\|\nabla_T \mathbf{f}_t + \nabla_S \mathbf{f}_t \cdot \mathbf{u}_t\|_1$$
$$\sum_{i=x,y,z} \|\nabla_S \mathbf{u}_{t,i}\|_{H_\tau}$$

Priors



Tomography

$$\|\mathbf{A}_t \mathbf{f}_t - \mathbf{p}_t\|_2^2$$

Spatial smoothness

$$\|\nabla_S \mathbf{f}_t\|_{H_\epsilon}$$

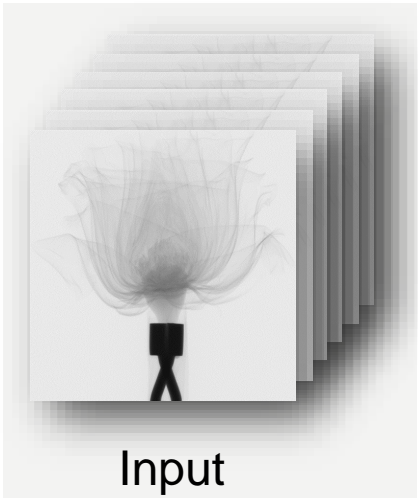
Temporal smoothness

$$\|\nabla_T \mathbf{f}_t\|_2^2$$

Flow correction

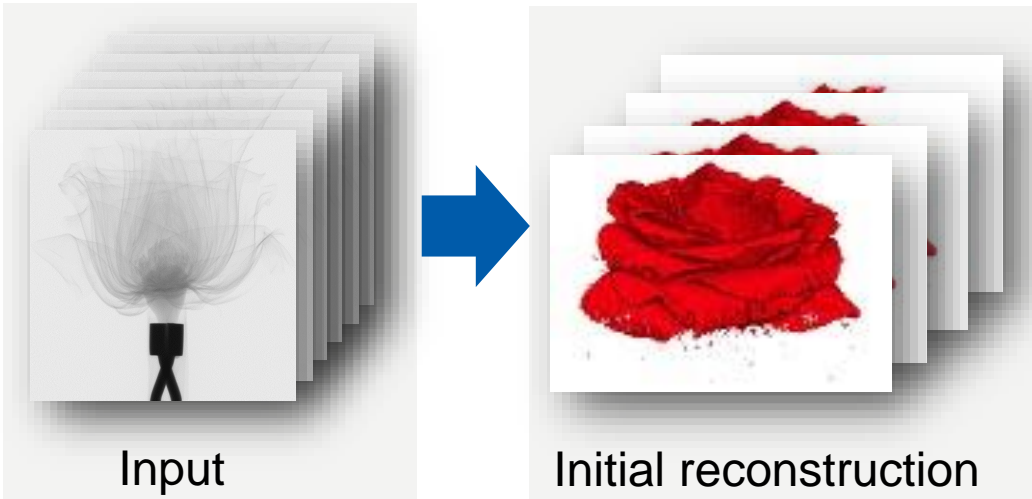
$$\|\nabla_T \mathbf{f}_t + \nabla_S \mathbf{f}_t \cdot \mathbf{u}_t\|_1$$
$$\sum_{i=x,y,z} \|\nabla_S \mathbf{u}_{t,i}\|_{H_\tau}$$

4D joint optimization



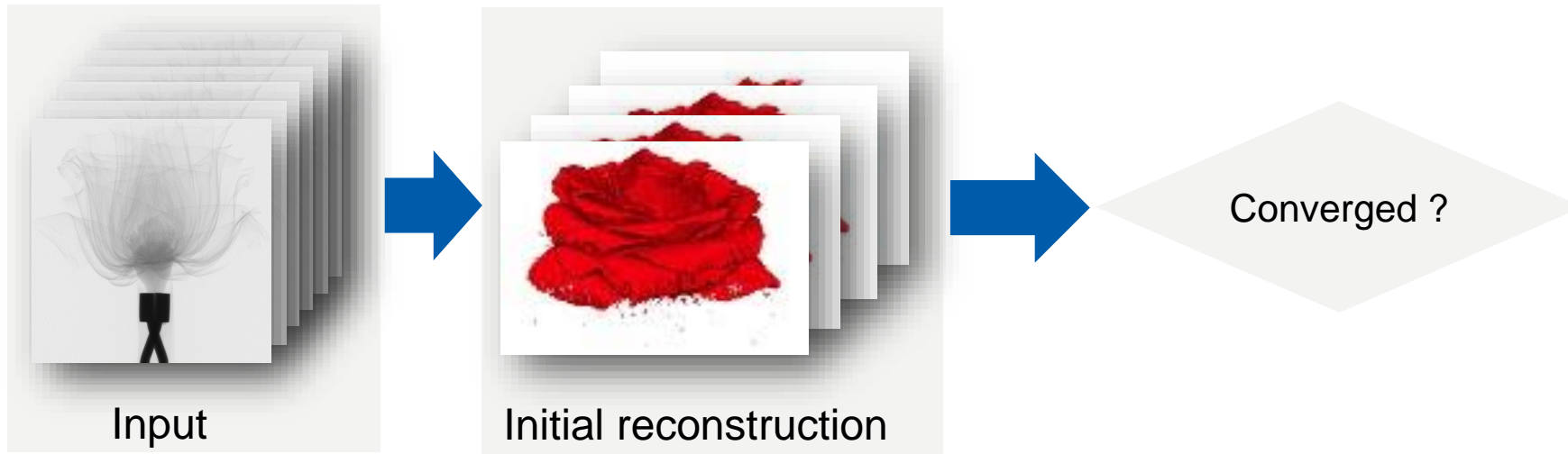


4D joint optimization



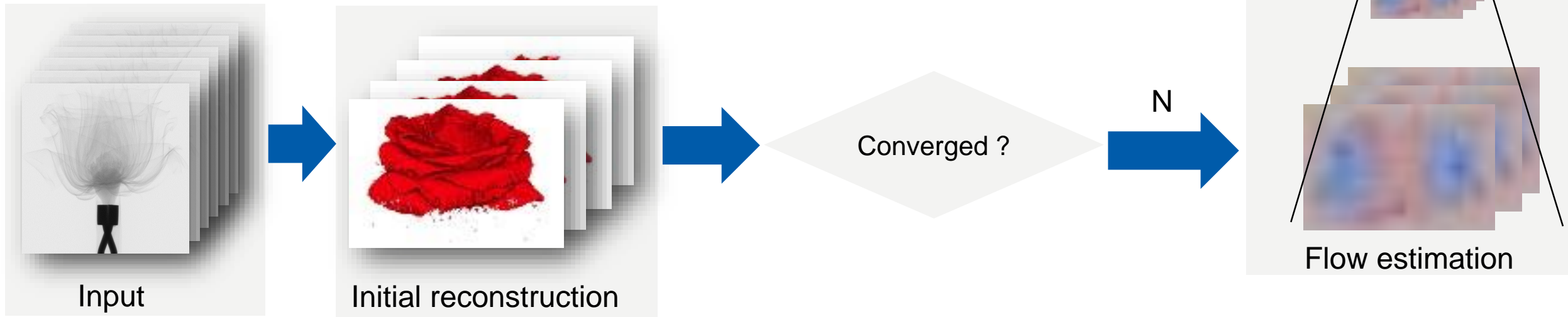


4D joint optimization



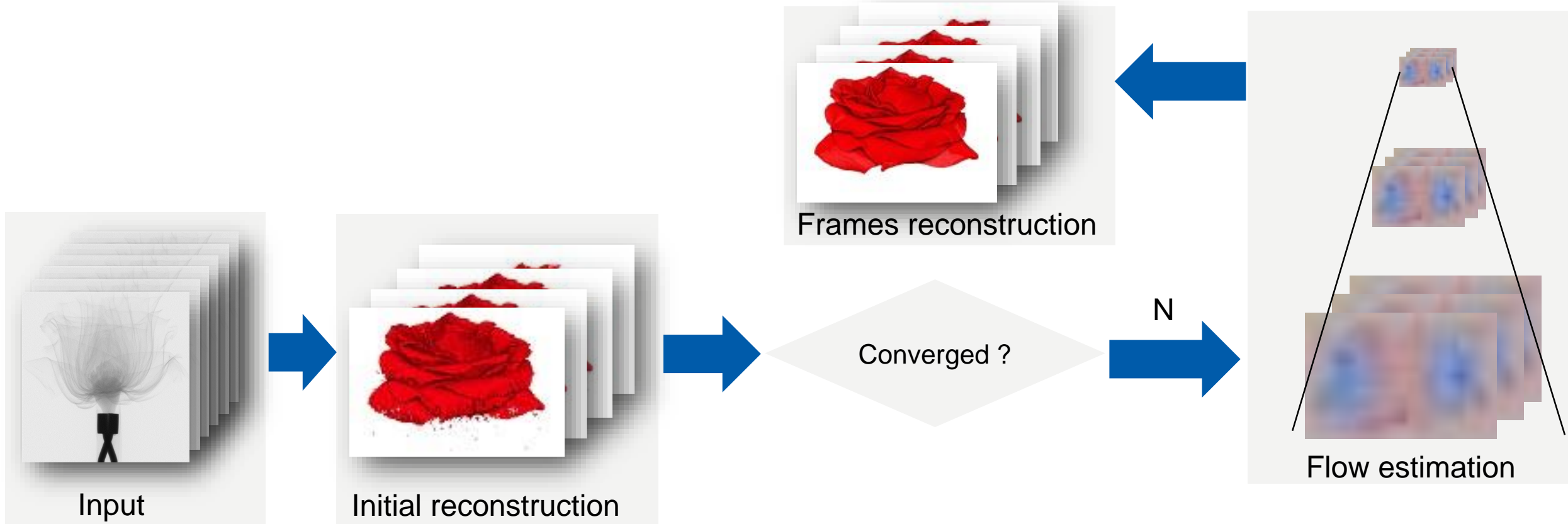


4D joint optimization



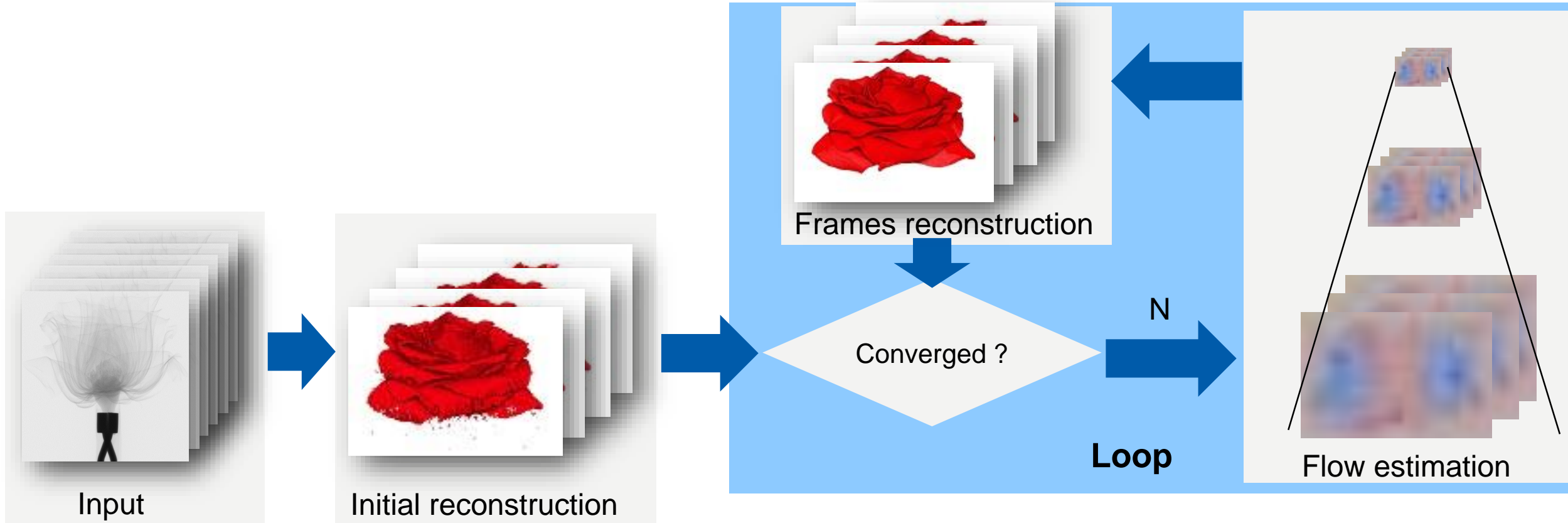


4D joint optimization



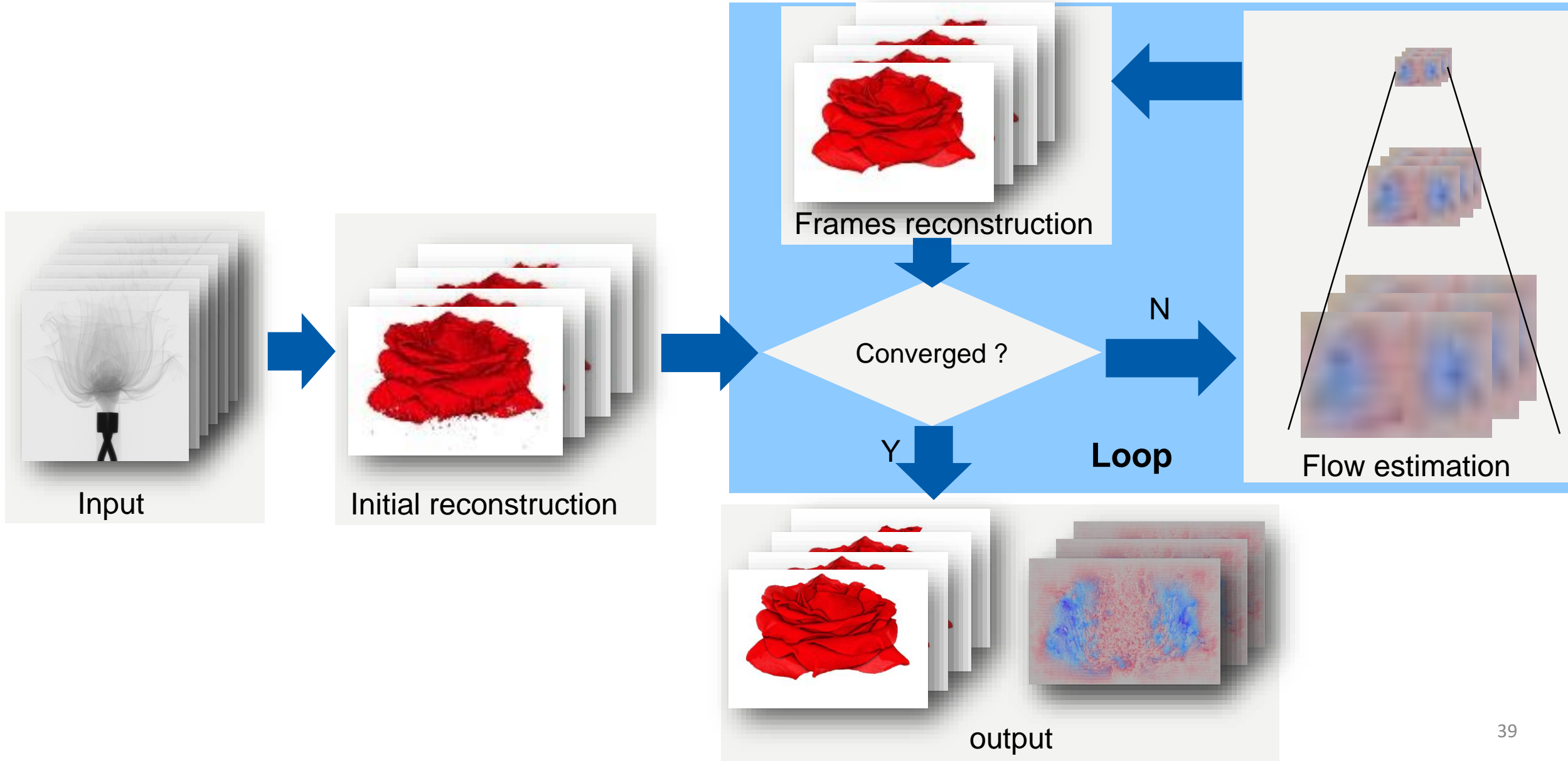


4D joint optimization





4D joint optimization





Overview

- Data acquisition
- 4D tomographic reconstruction
- Results



Wilting rose

Photo

Projection

Before



After



Wilting rose

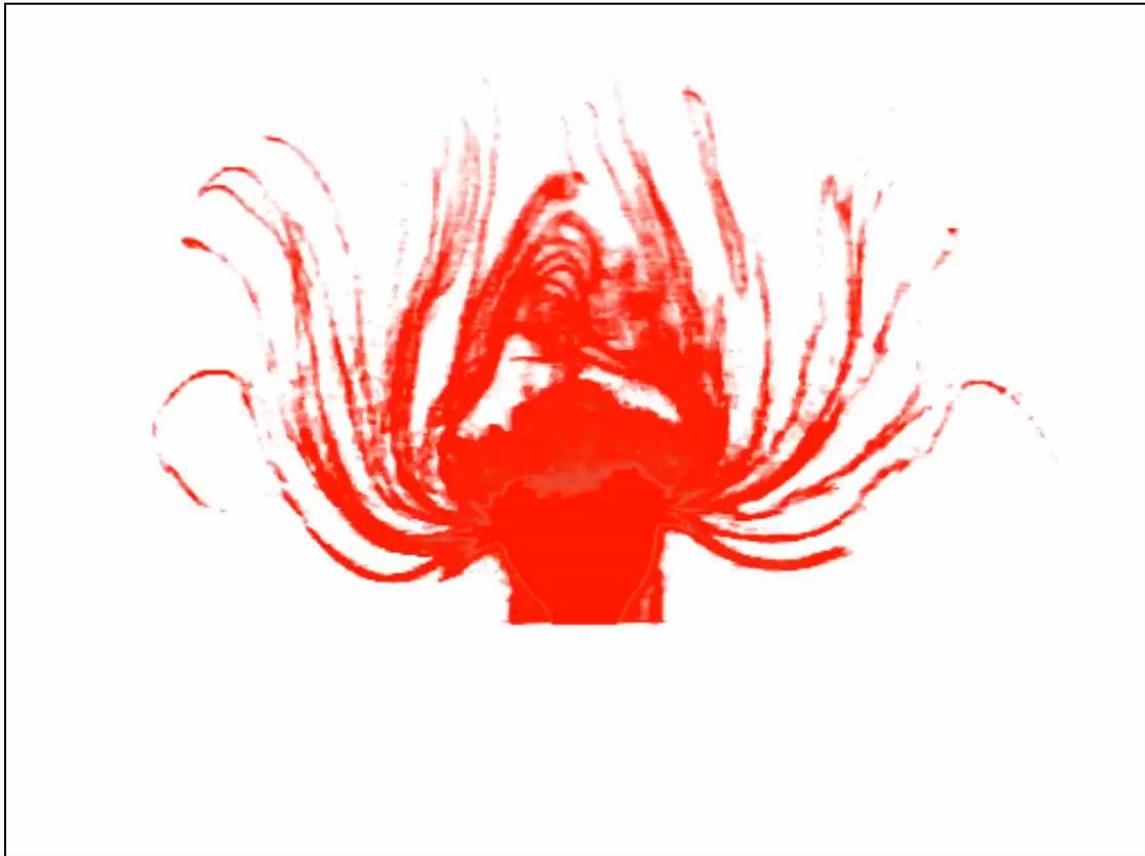


Baseline method

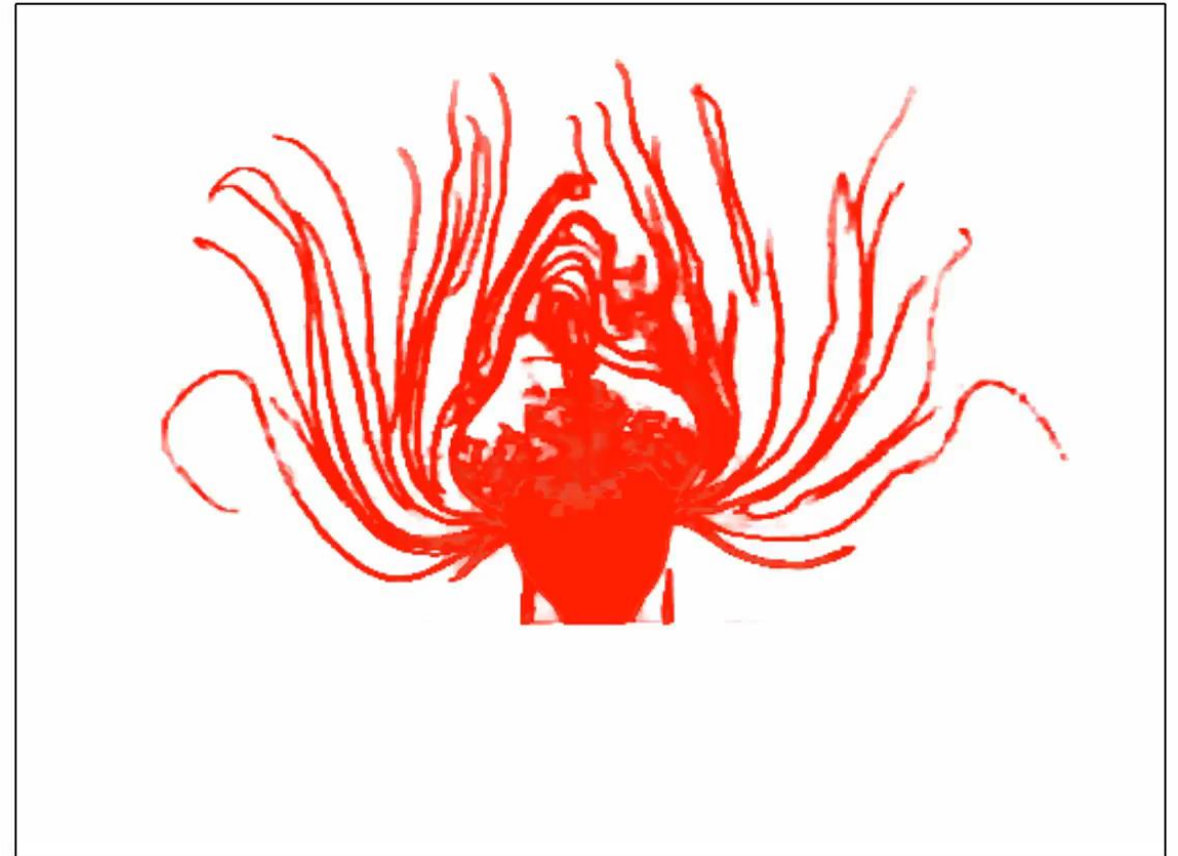


Ours

Wilting rose

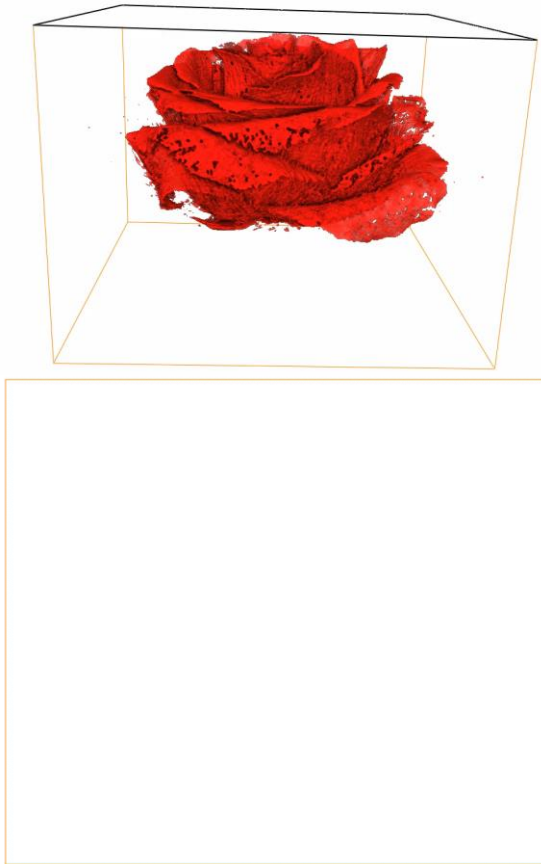


Baseline method

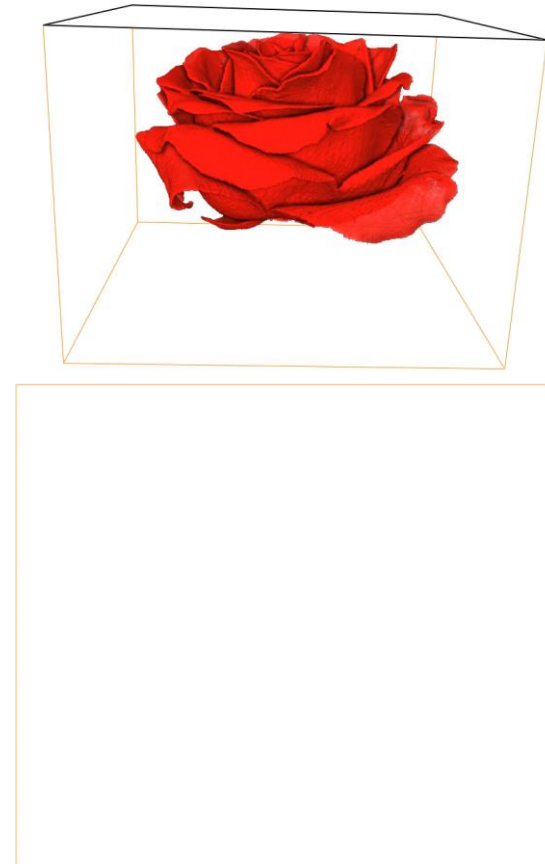


Ours

Wilting rose



Baseline method

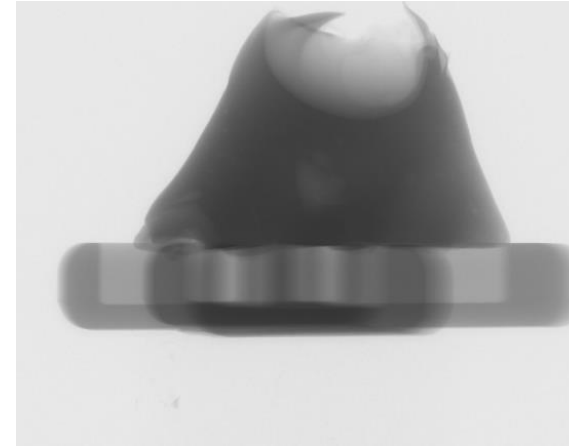


Ours



High viscosity fluid flow

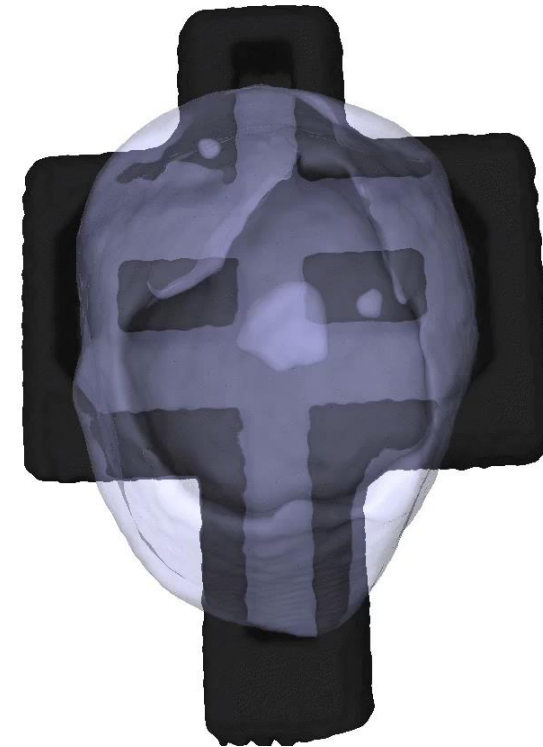
Before



After

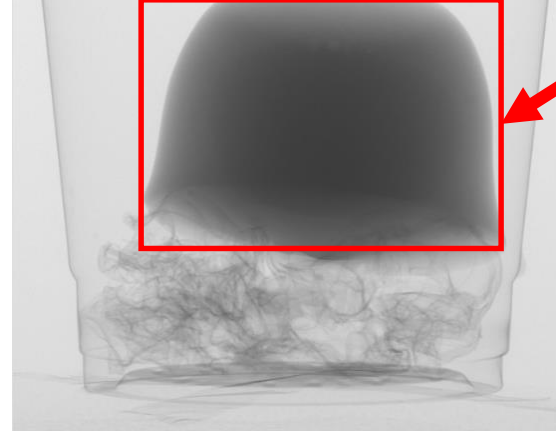
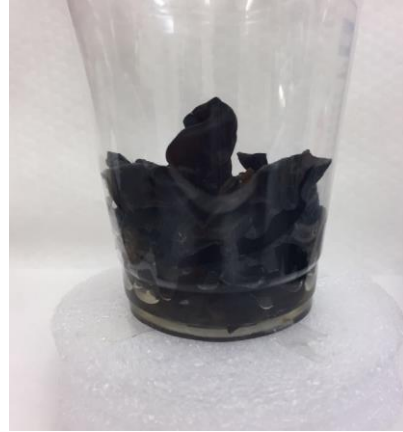


High viscosity fluid flow

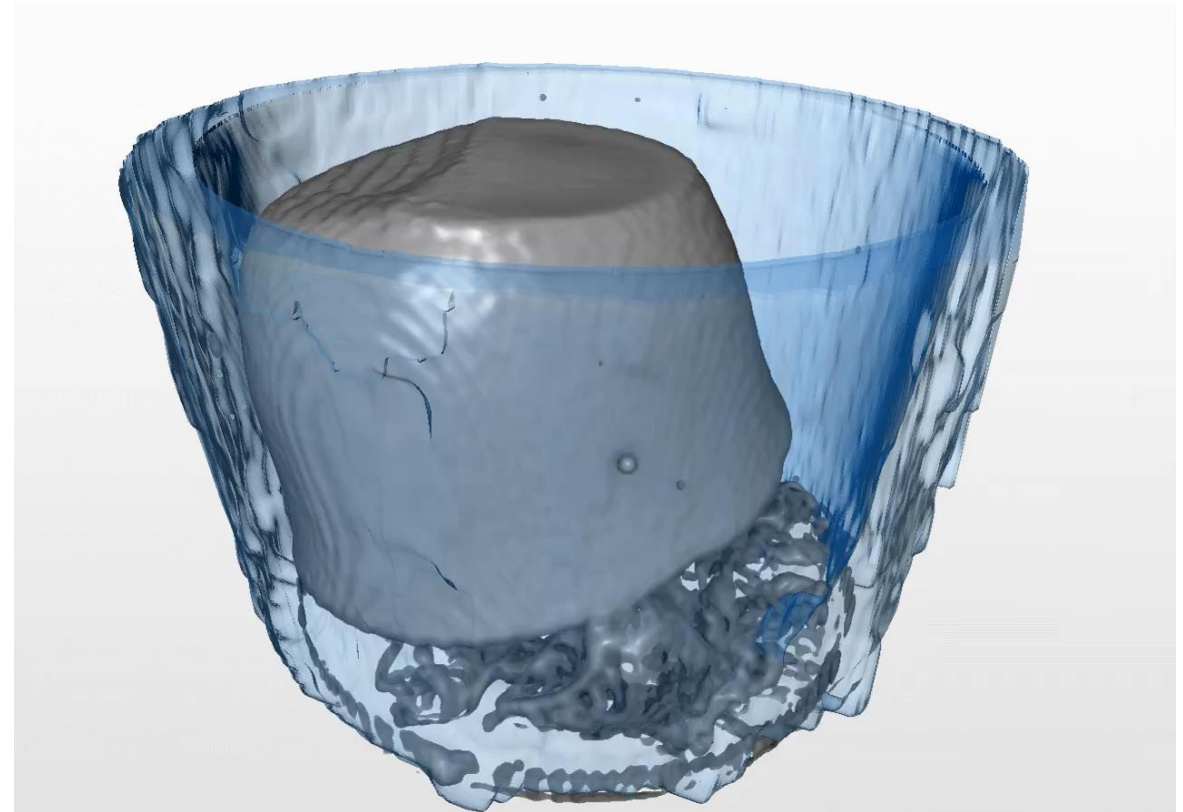
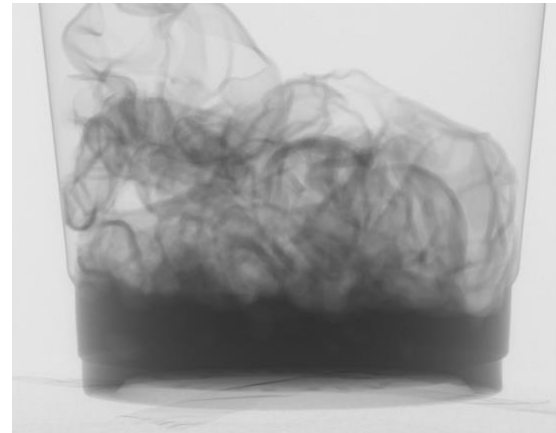




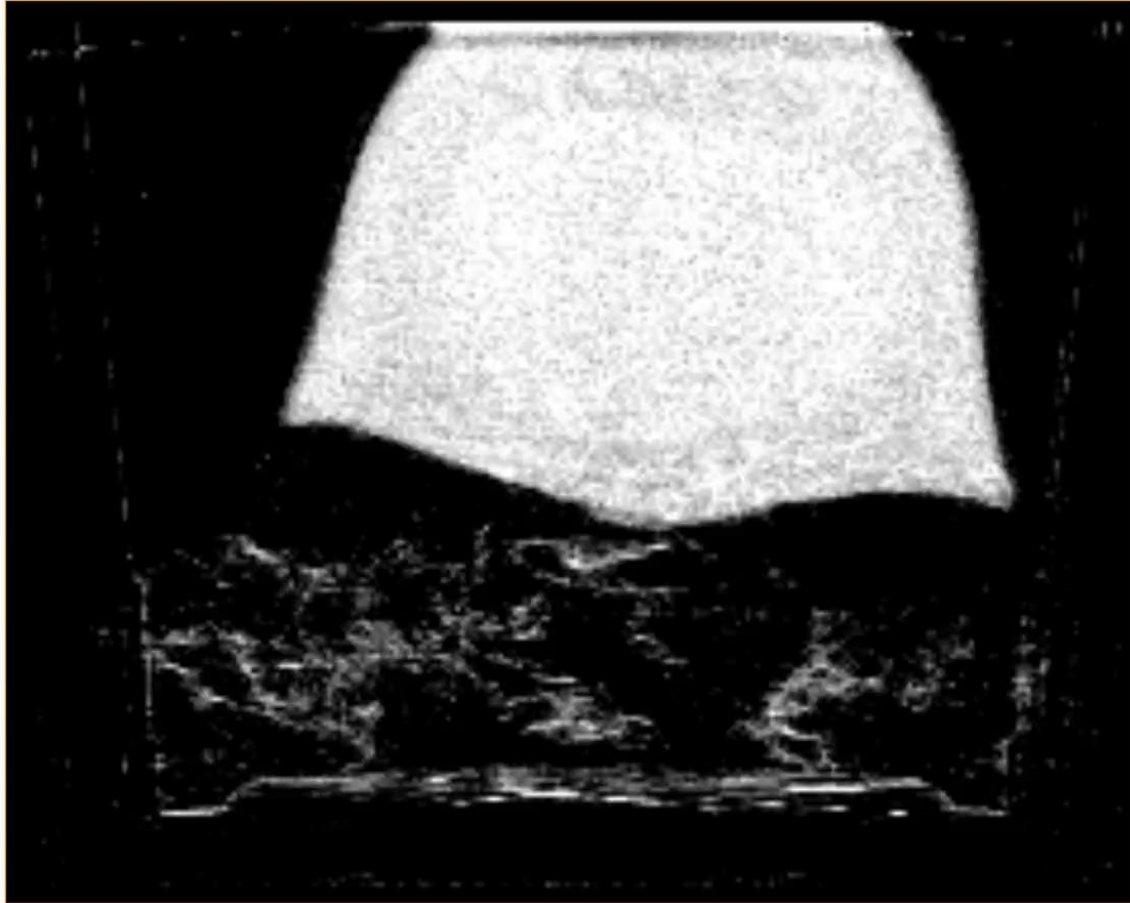
Mushroom re-hydration



Melting ice



Mushroom re-hydration



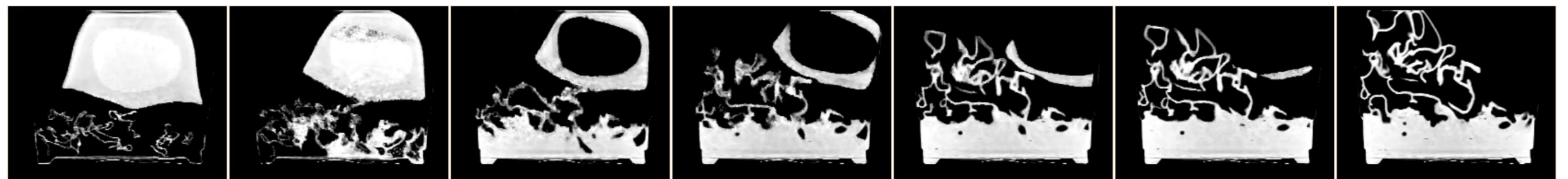
Baseline method



Ours



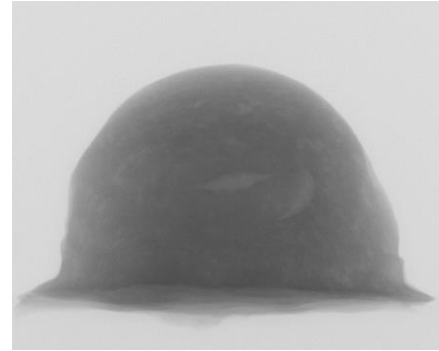
Mushroom re-hydration



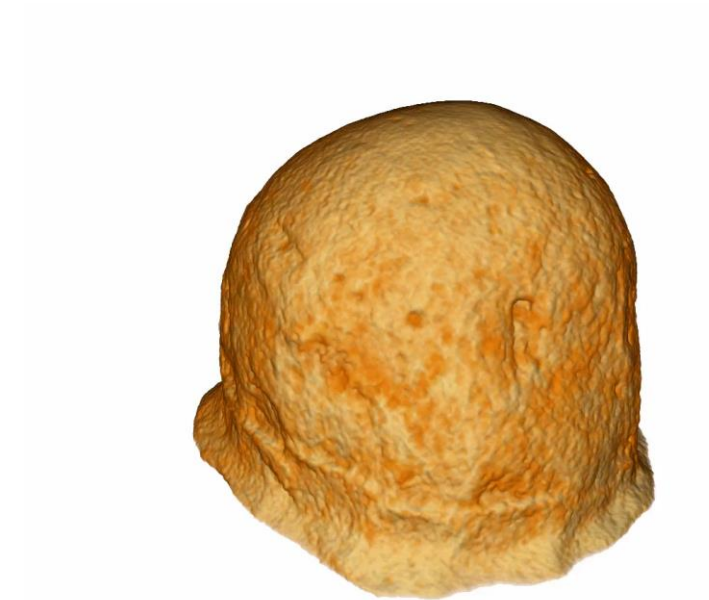
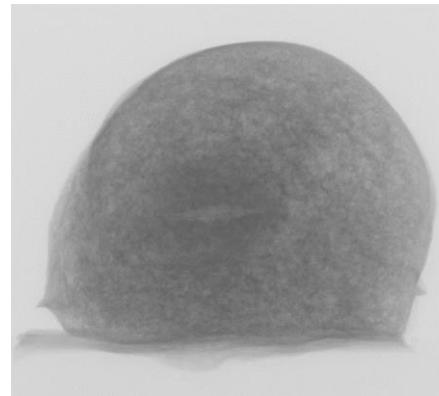
Rising dough



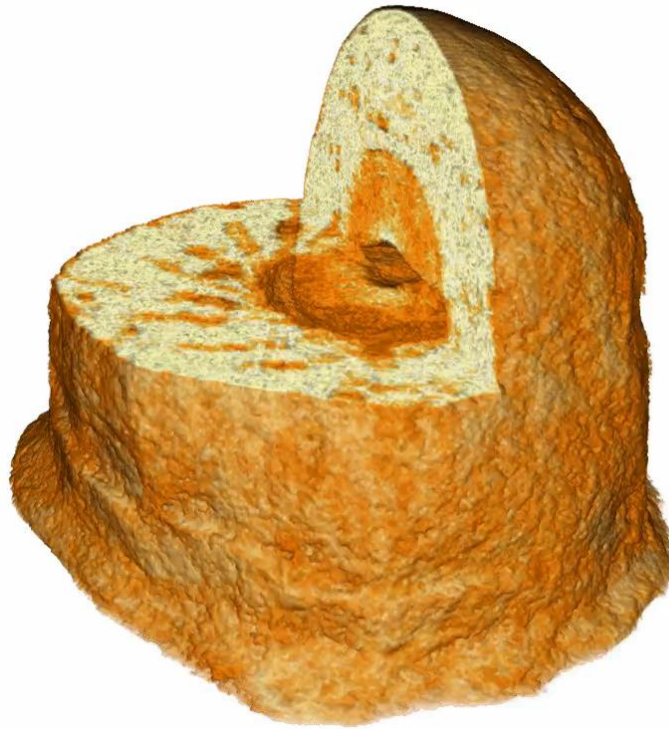
Before



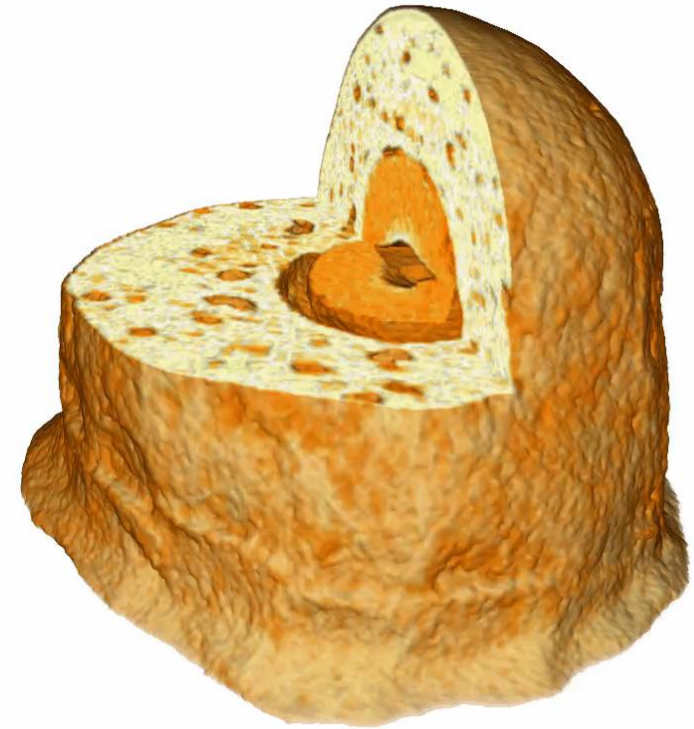
After



Rising dough

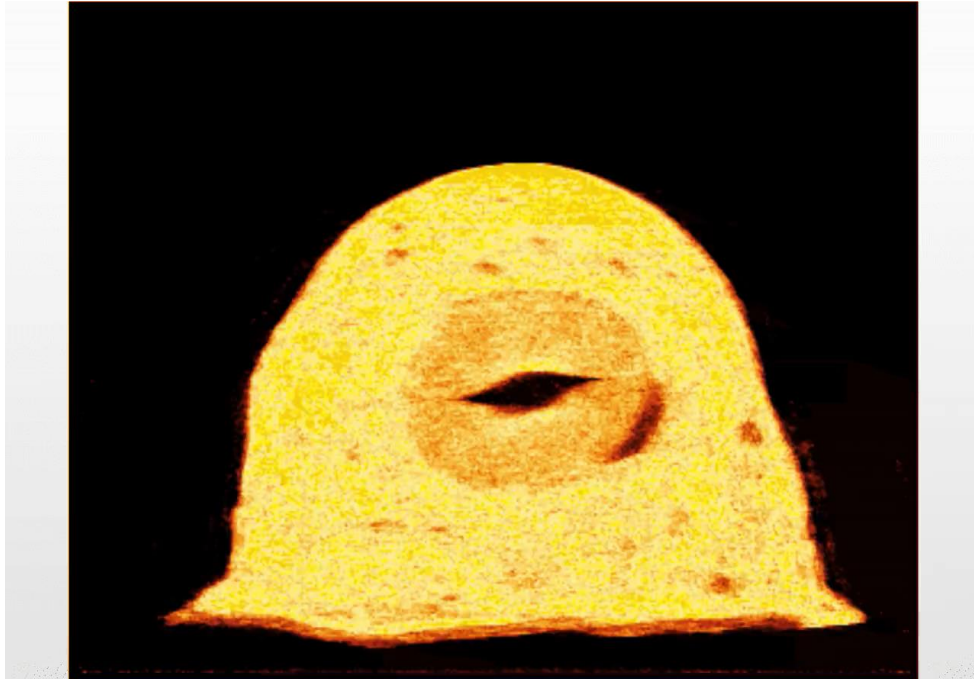


Baseline method

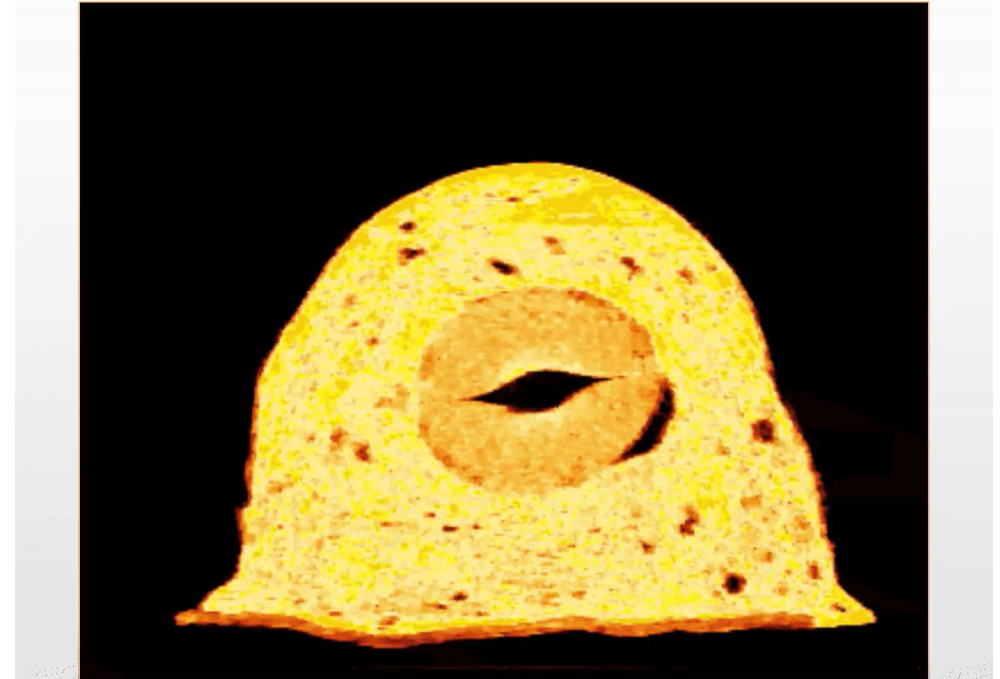


Ours

Rising dough



Baseline method

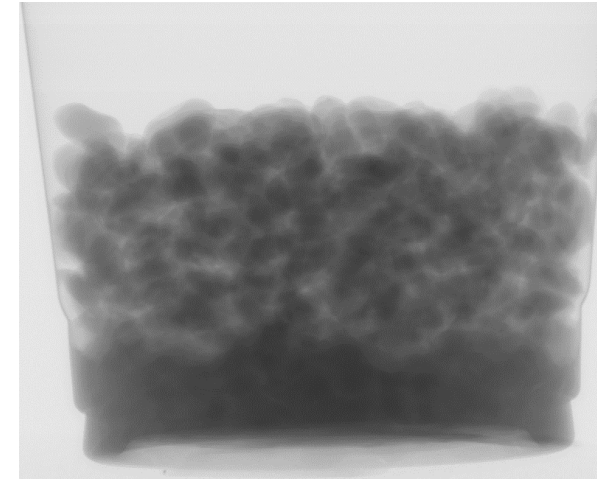


Ours

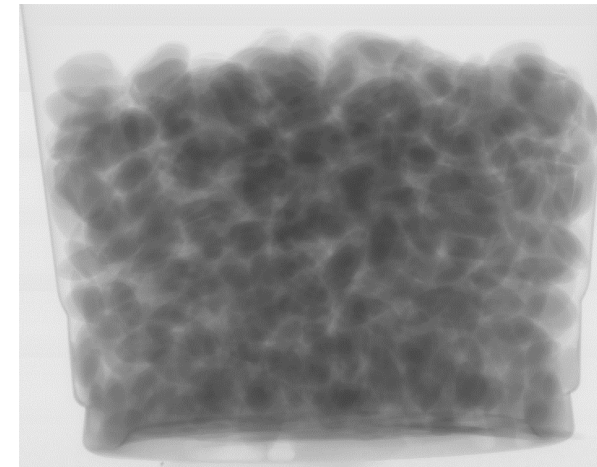


Lentil/lupin seeds hydration

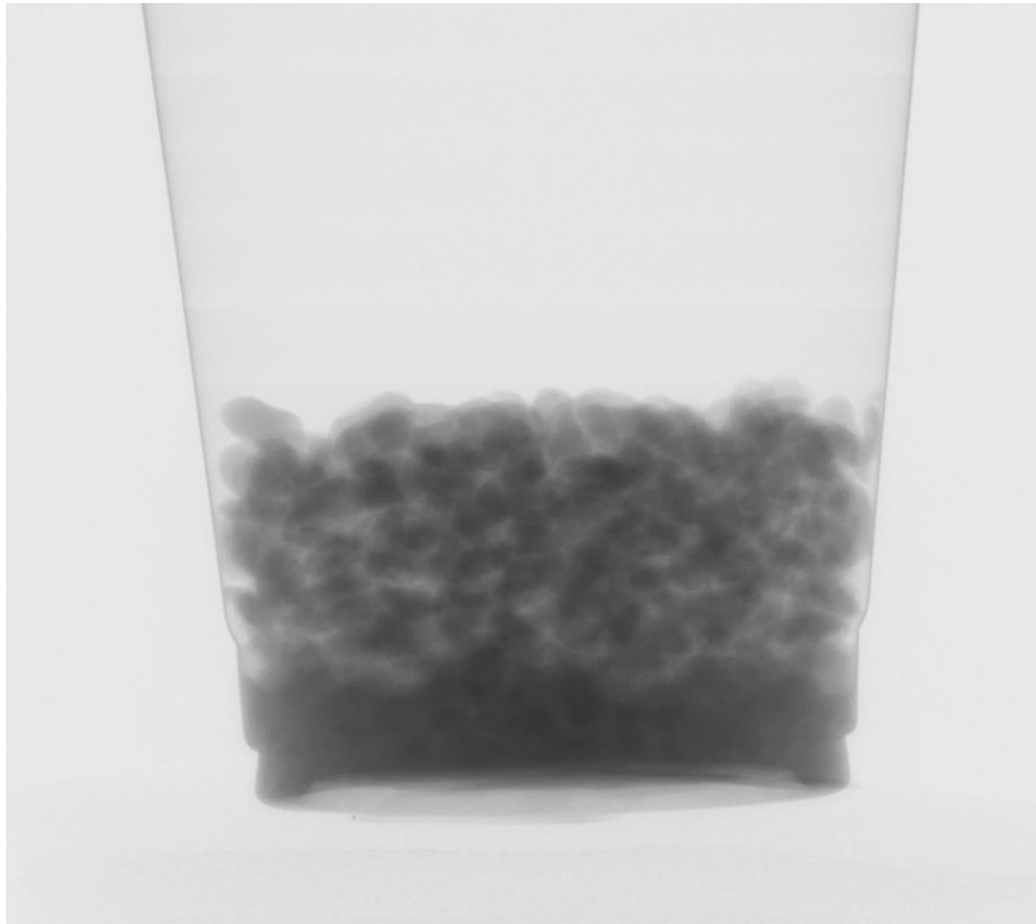
Before



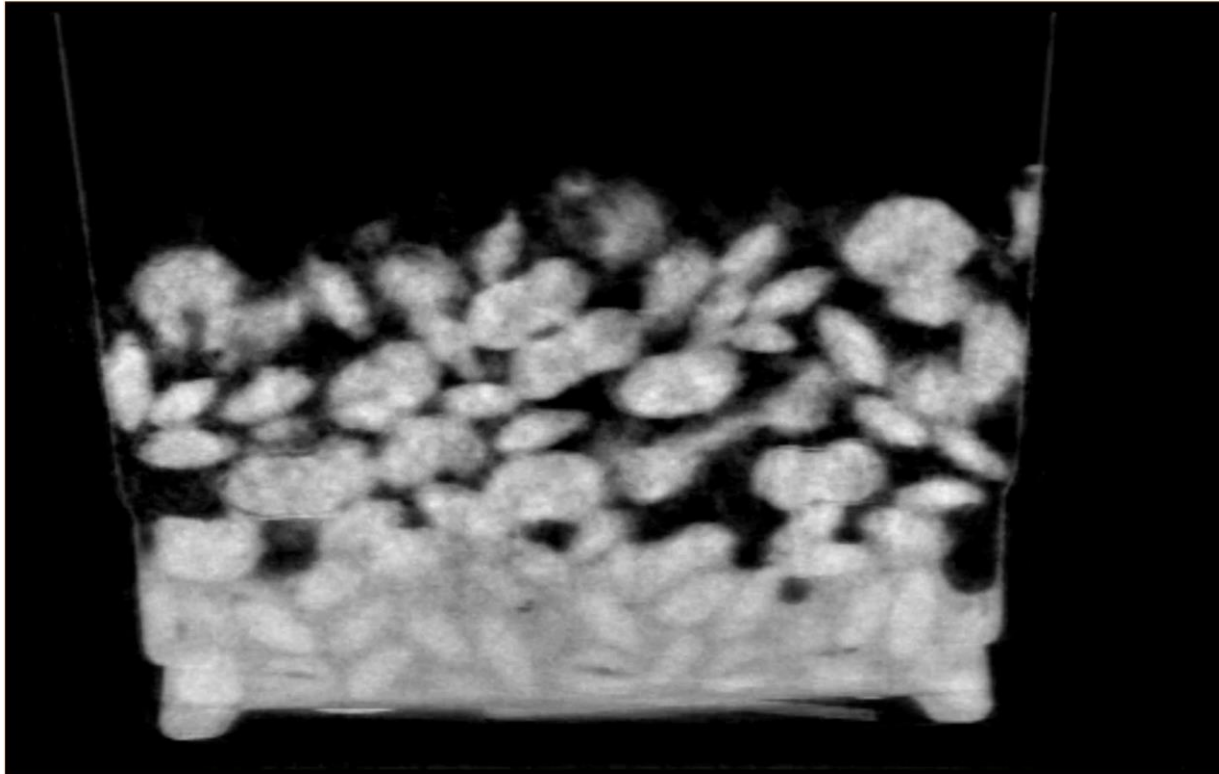
After



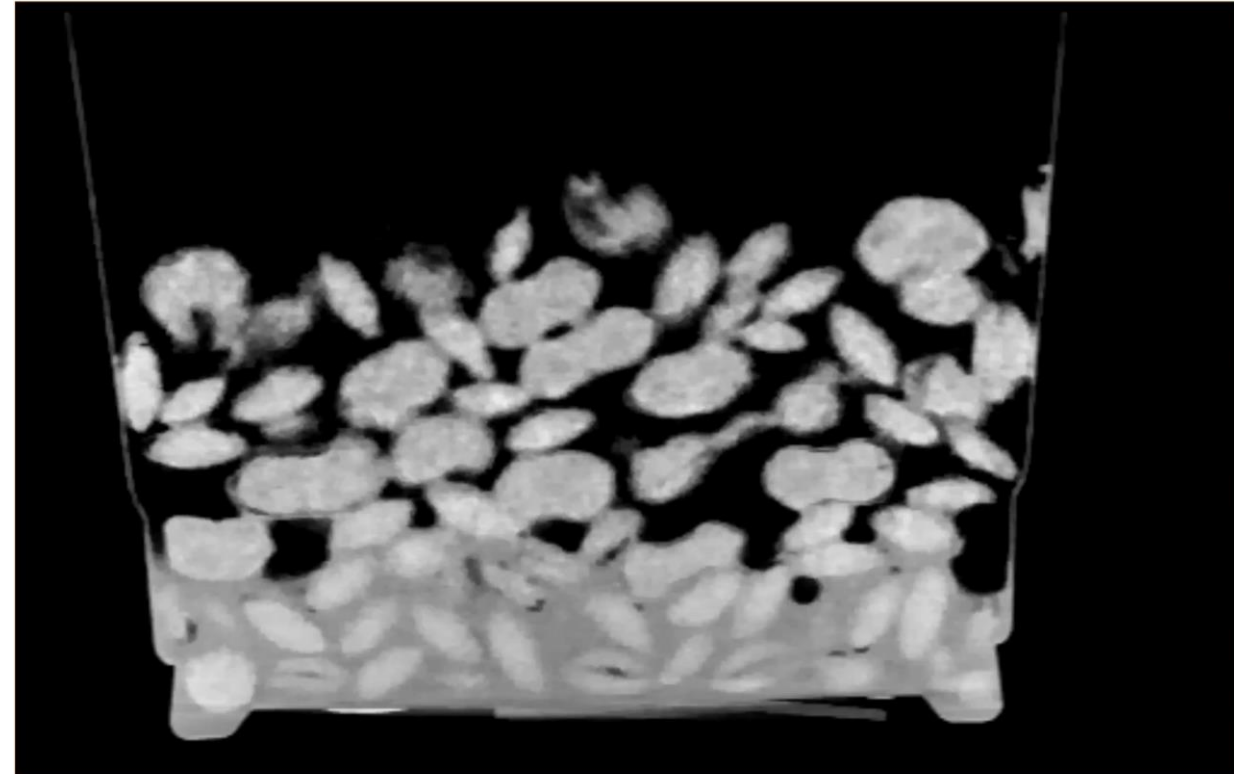
Lentil/lupin seeds hydration



Lentil/lupin seeds hydration



Baseline method

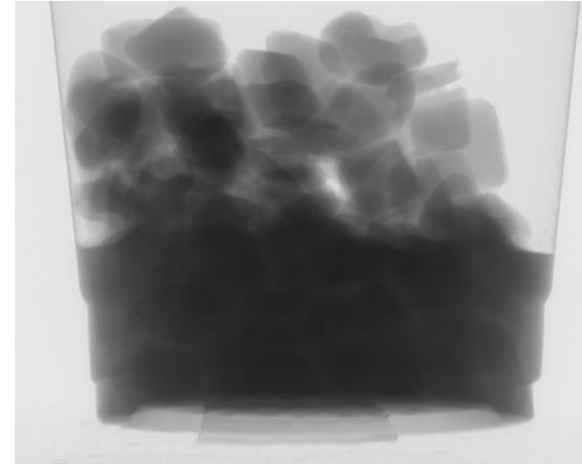
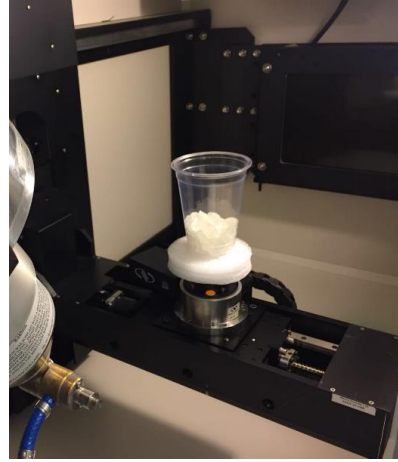


Ours

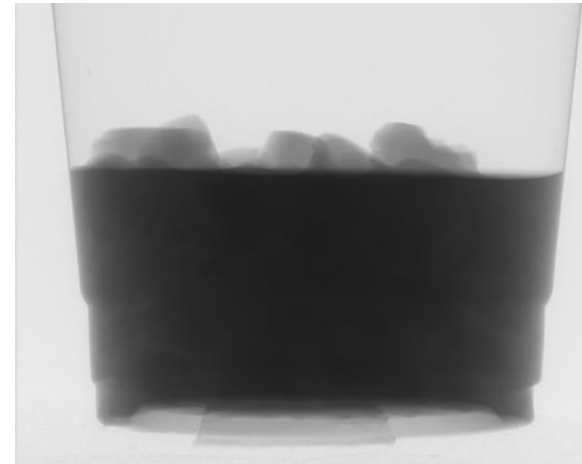


Crystal sugar dissolution

Before

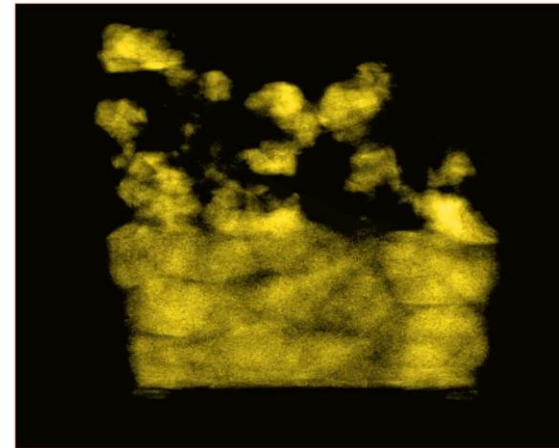
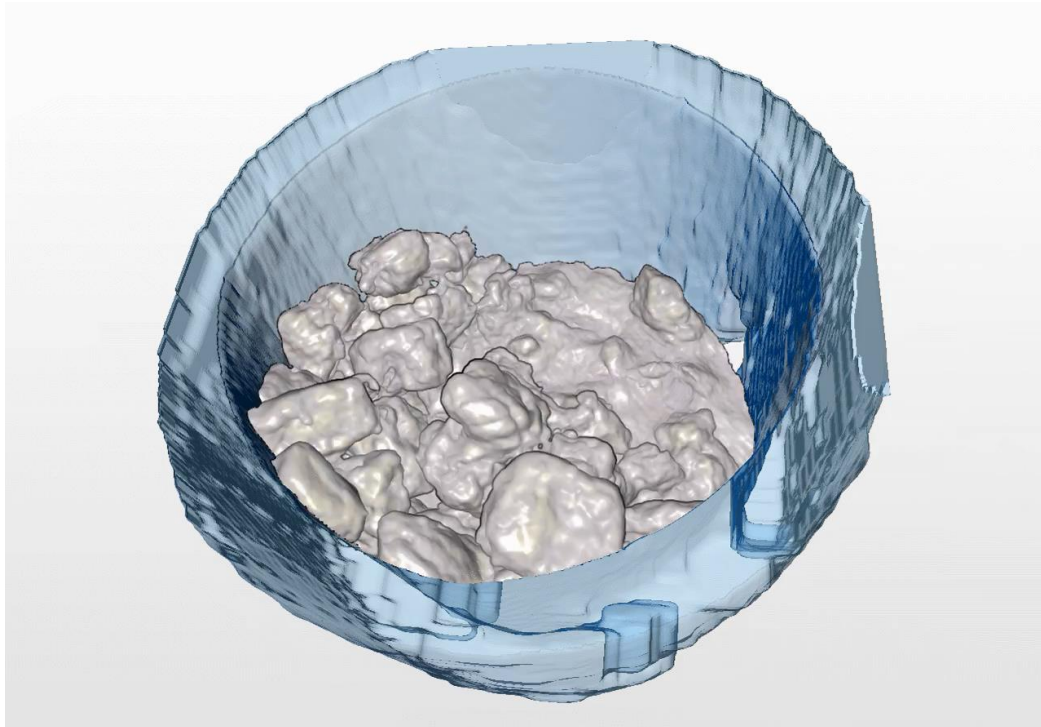


After

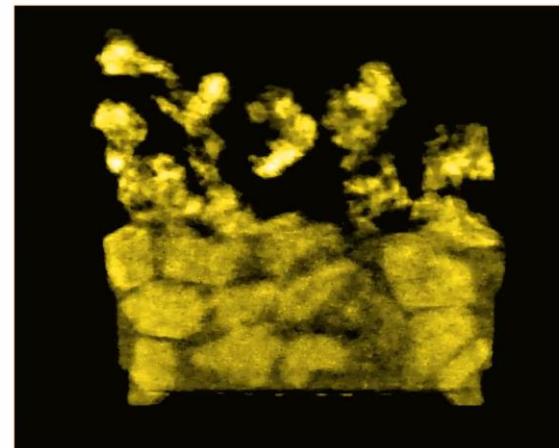




Crystal sugar dissolution



Baseline method

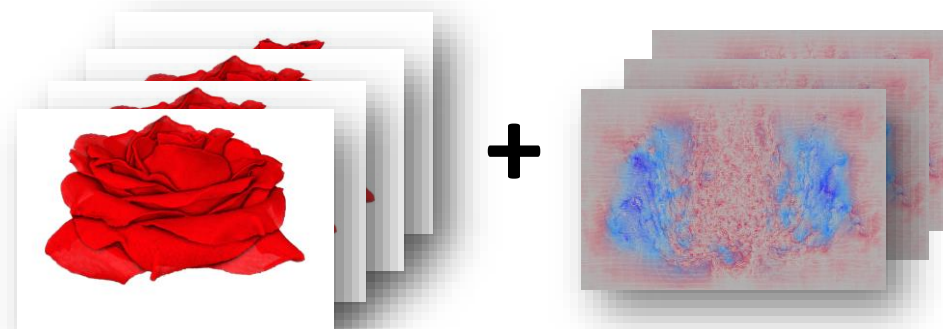
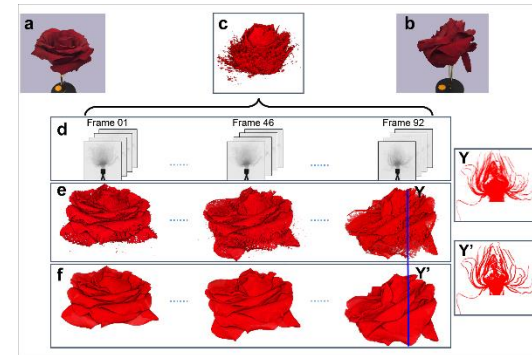
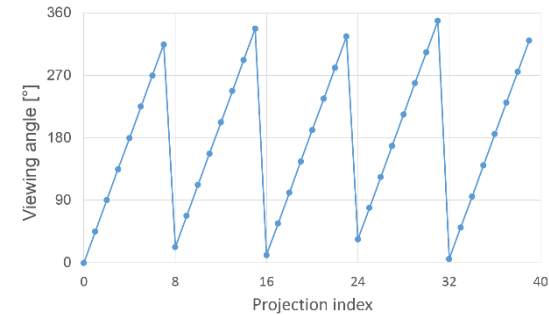


Ours



Summary

- A new sampling strategy
- A new 4D framework for tomography
- Well suited to graphics





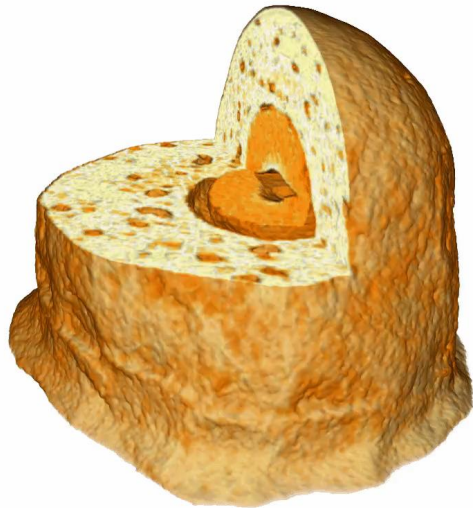
Next steps

- Extension to faster deformation
- Extension to other imaging modalities (e.g. electron microscope)
- Re-Simulation (e.g. domain modification)



Thank you !

Code & Data



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