GENERATIONS / VANCOUVER 12-16 AUGUST
SIGGRAPH2018

VOXEL CORES: EFFICIENT, ROBUST, AND PROVABLY GOOD APPROXIMATION OF 3D MEDIAL AXES

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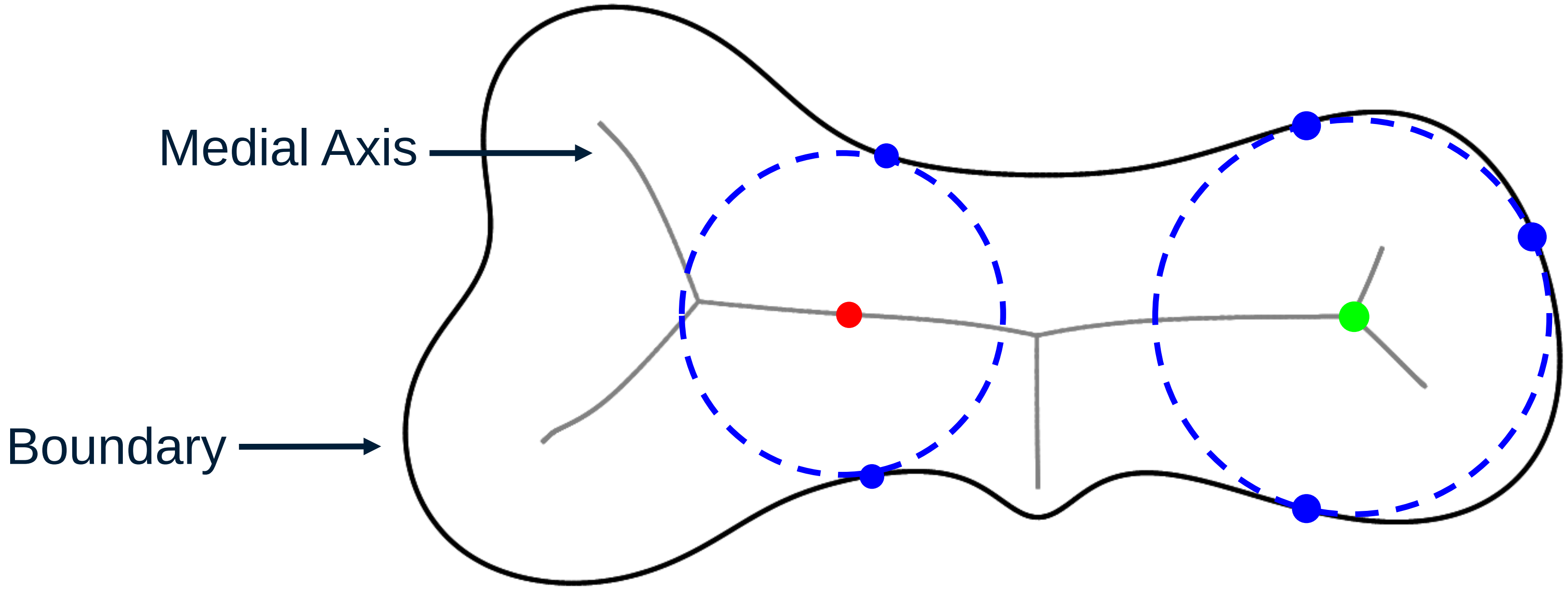
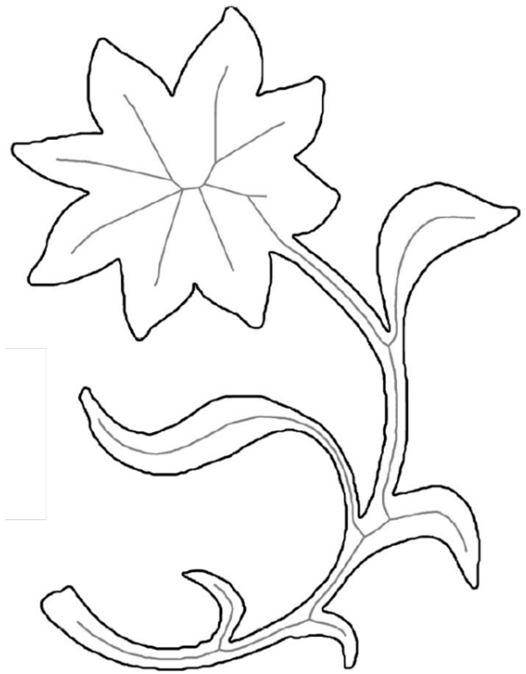
MEDIAL AXIS

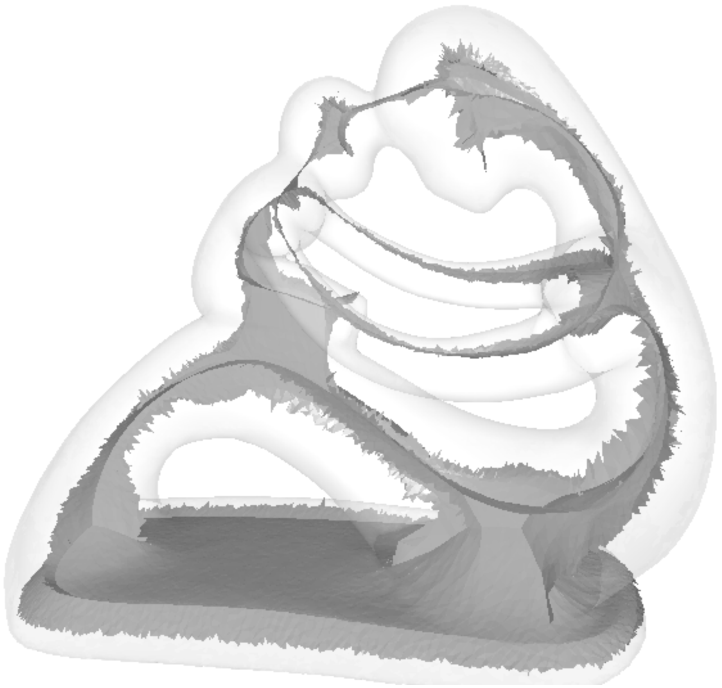
- All points inside the shape that have two or more nearest neighbors on the shape's boundary [Blum 1967]



MEDIAL AXIS: PROPERTIES

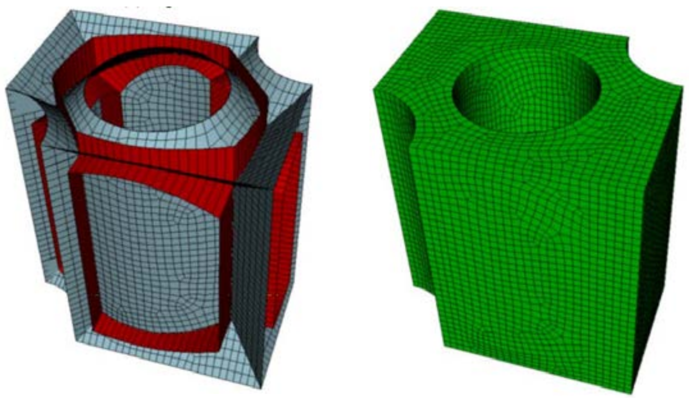
- Centered
- Compact
- Preserving topology
- Reconstructing shape



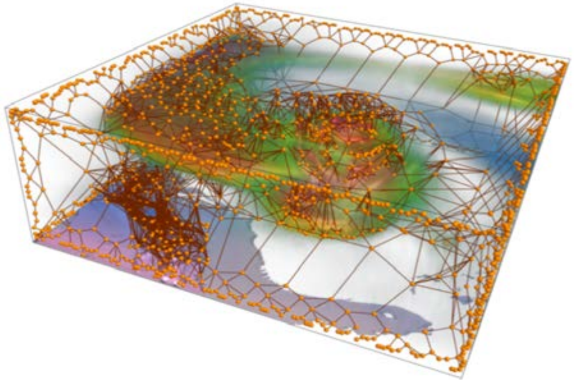


3D MEDIAL AXIS: APPLICATIONS

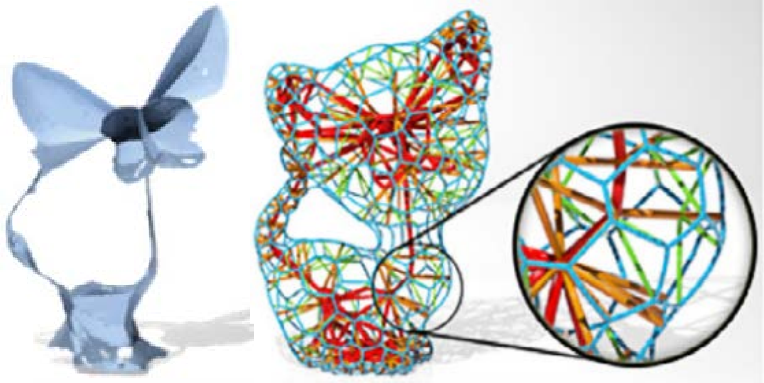
Washington University in St. Louis
School of Engineering & Applied Science



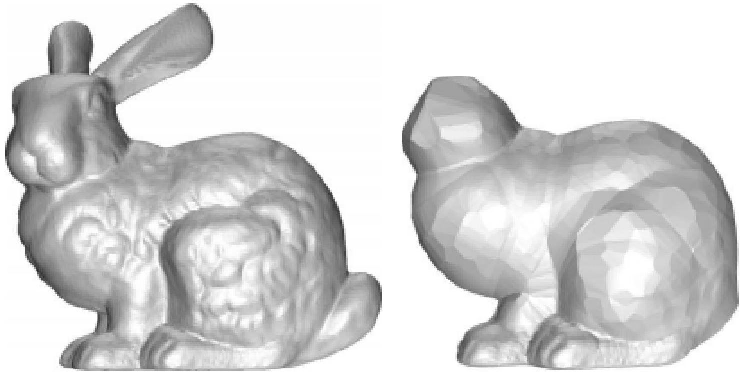
Meshing
[Quadros 14]



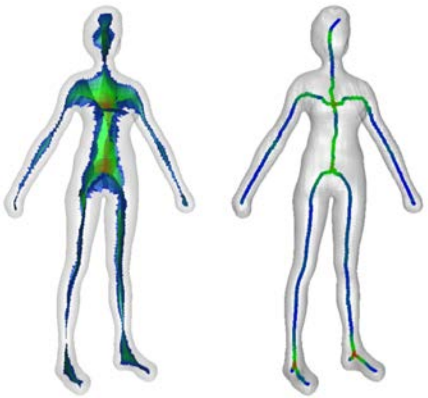
Camera navigation
[Hsu et al. 13]



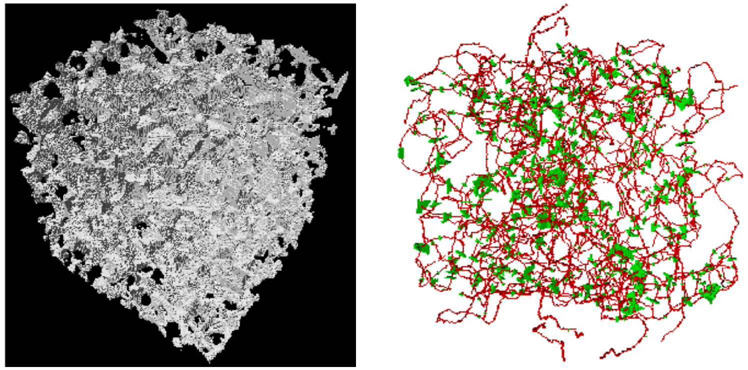
3D Printing
[Zhang et al. 15]



Shape simplification
[Tam & Hendrich 03]



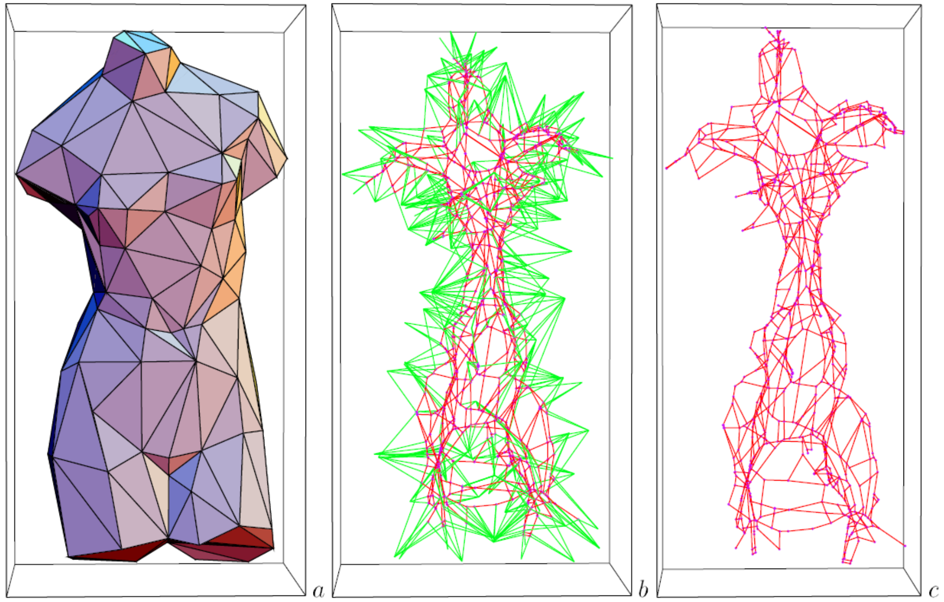
Computing skeletons
[Dey & Sun 06]



Structural analysis
[Lindquist et al. 99]

COMPUTING 3D MEDIAL AXES

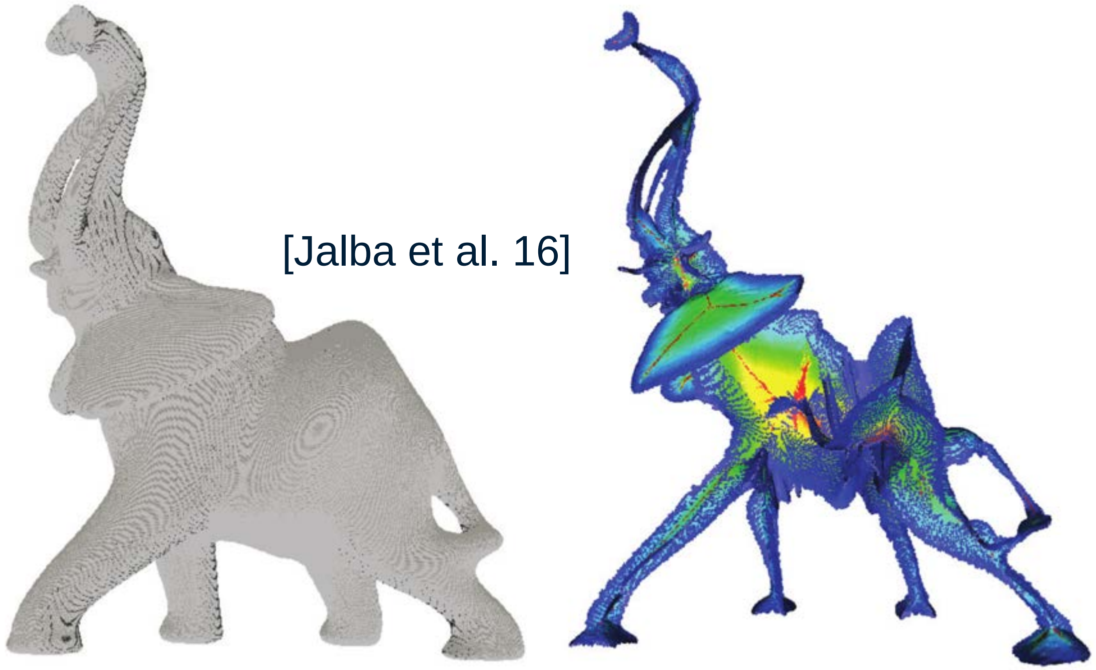
- Exact algorithms [Milenkovic 93; Sherbrooke et al. 96; Culver et al. 04]
  – Limited to simple polyhedra



[Culver et al. 04]

COMPUTING 3D MEDIAL AXES

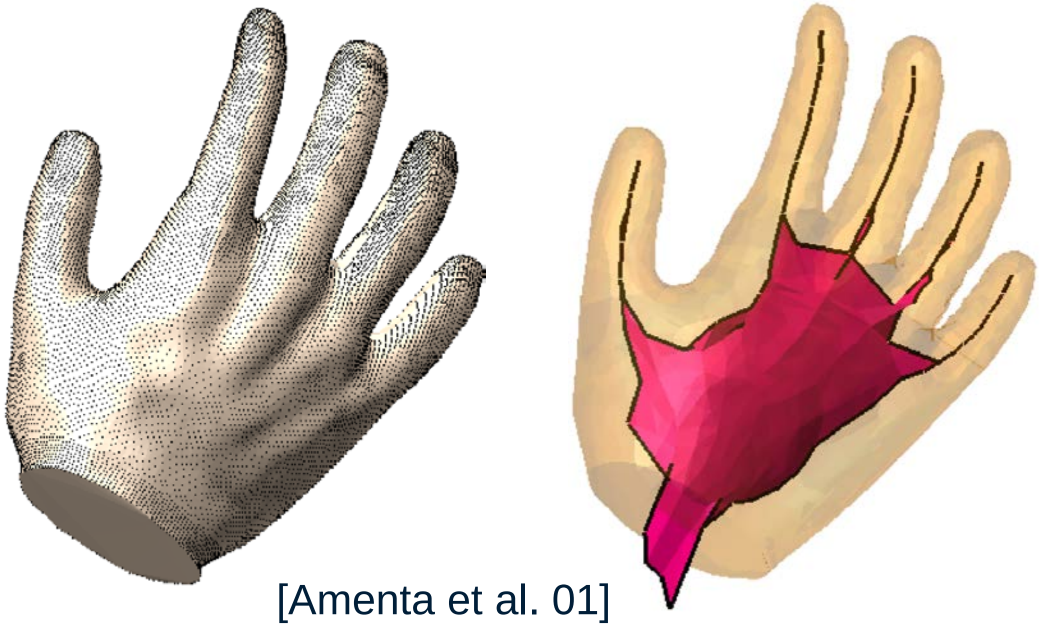
- Voxel-based approximation [Palágyi & Kuba 99, Siddiqi et al. 02, Jalba et al. 16, etc.]
  - Poor scalability with grid resolution
  - No bound on approximation error



[Jalba et al. 16]

COMPUTING 3D MEDIAL AXES

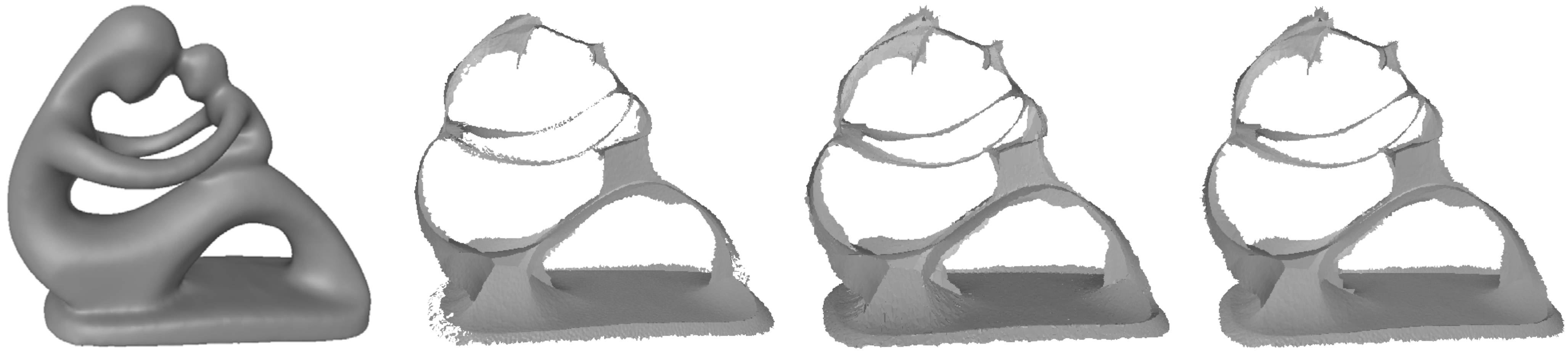
- Sampling-based approximation [Amenta et al. 01, Dey & Zhao 04, Giesen et al. 06, etc.]
  - More scalable
  - Bounded approximation error
  - But, often produces topological errors
    - Lacking topological guarantee
    - Numerical fragile



[Amenta et al. 01]

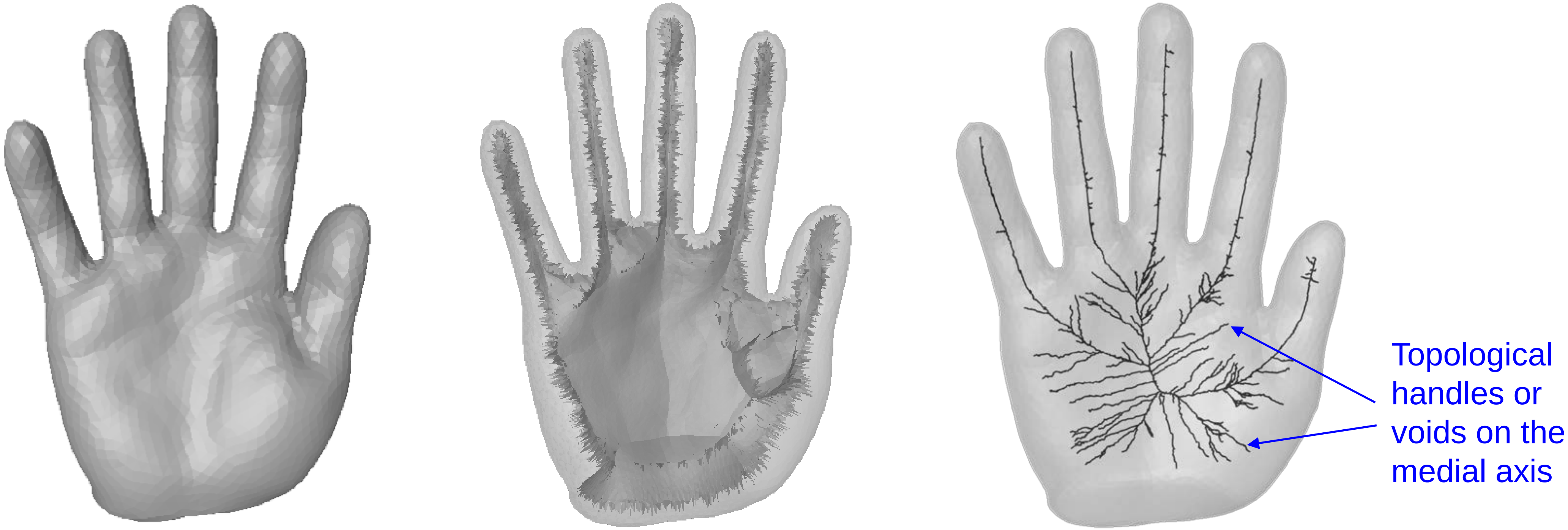
TOPOLOGICAL ERRORS

Input shape | [Dey & Zhao 04] | [Amenta et al. 01] | [Amenta & Kolluri 01] *(with topo. guarantee)*



	Input shape	[Dey & Zhao 04]	[Amenta et al. 01]	[Amenta & Kolluri 01]
Connected Components:	**1**	**235**	**23950**	**1**
Euler number:	**-3**	**78**	**213577**	**61**

TOPOLOGICAL ERRORS

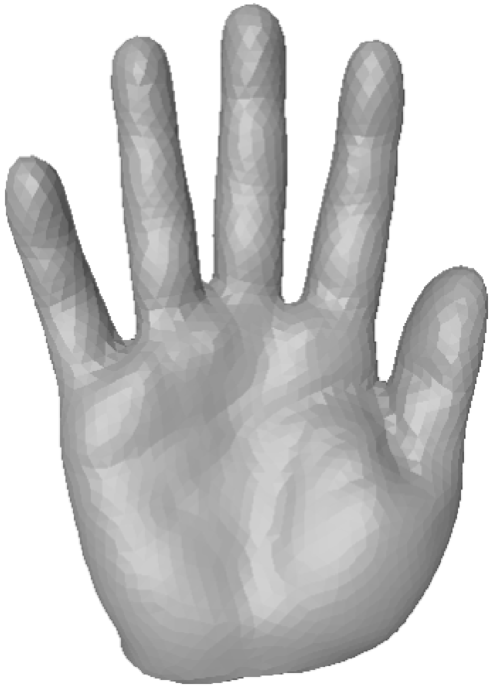


Input shape

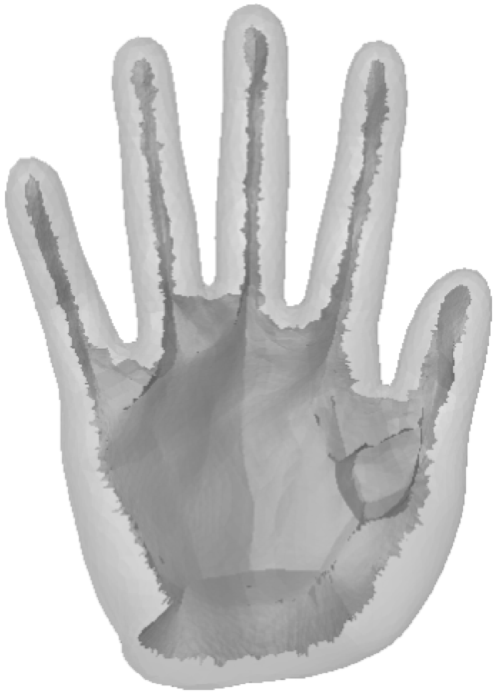
Medial axis
(w topo errors)

Curve skeleton
[Yan et al. 16]

TOPOLOGICAL ERRORS



Input shape

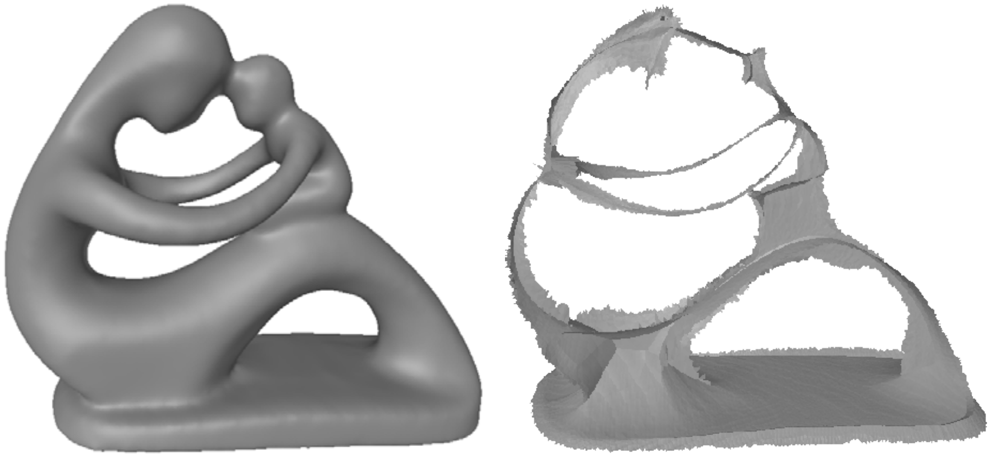


Medial axis
(w/o topo errors)

Curve skeleton
[Yan et al. 16]

OUR CONTRIBUTION

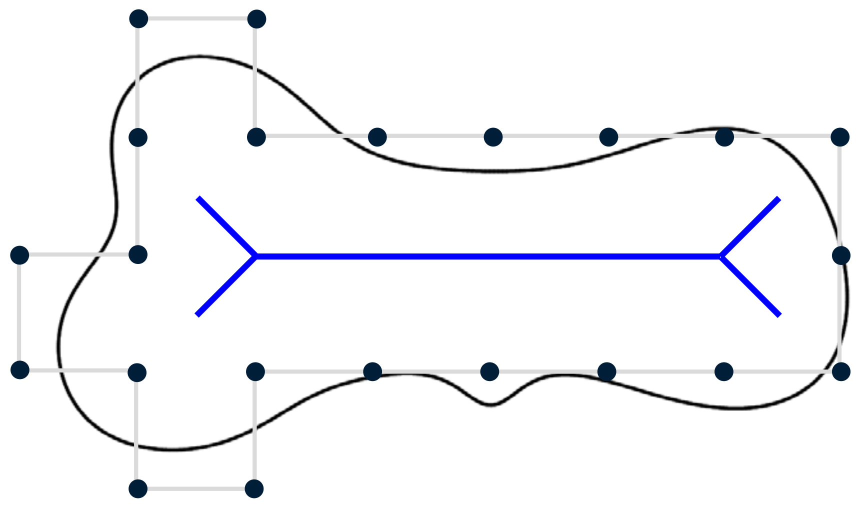
- A new sampling-based algorithm for approximating 3D medial axes
  - Scalable
  - Geometric and topological guarantees
  - Simple and numerically stable



Connected Components:	**1**	**1**
Euler number:	**-3**	**-3**

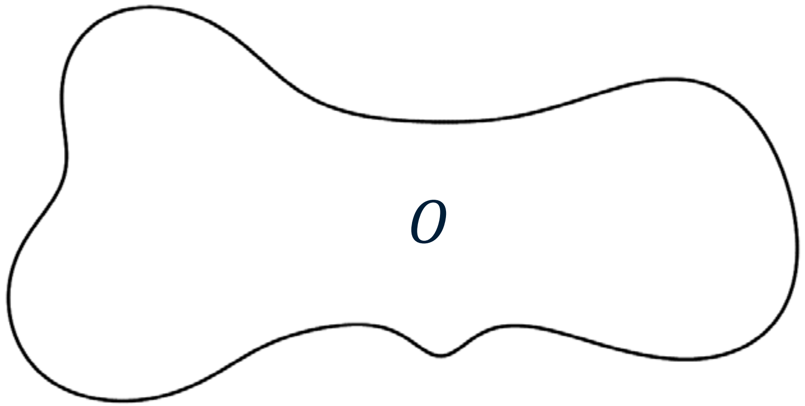
THE IDEA

- Sample not on, but *near*, the boundary, in a regular pattern
  - Voxelize the shape (similar to voxel based methods)
  - Take a subset of the Voronoi diagram of the boundary vertices of voxelization (similar to sampling based methods)



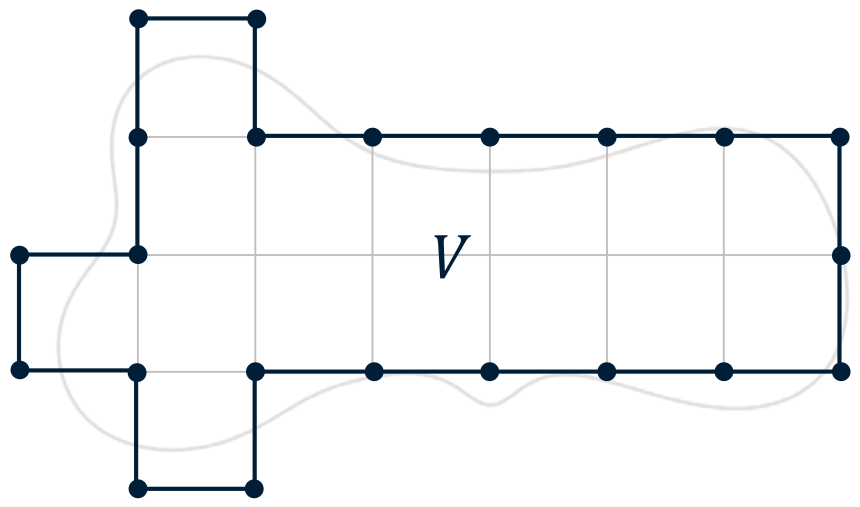
VOXELIZATION

- Given an input shape O

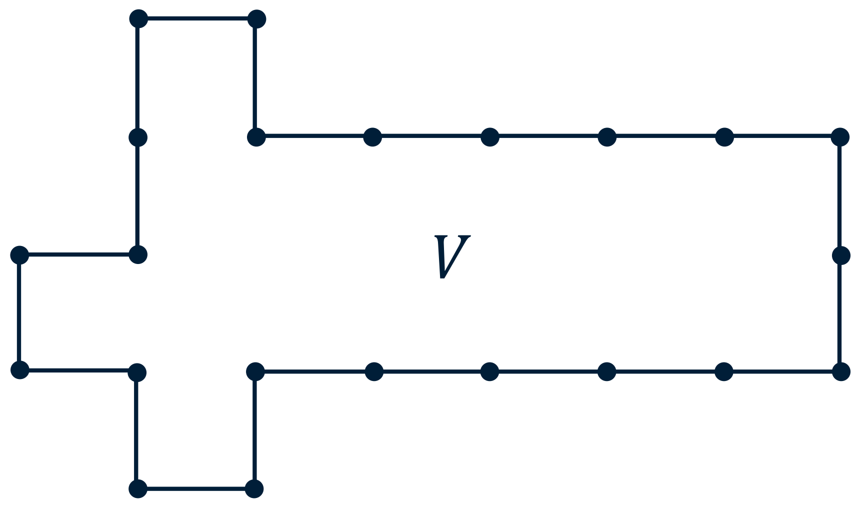


VOXELIZATION

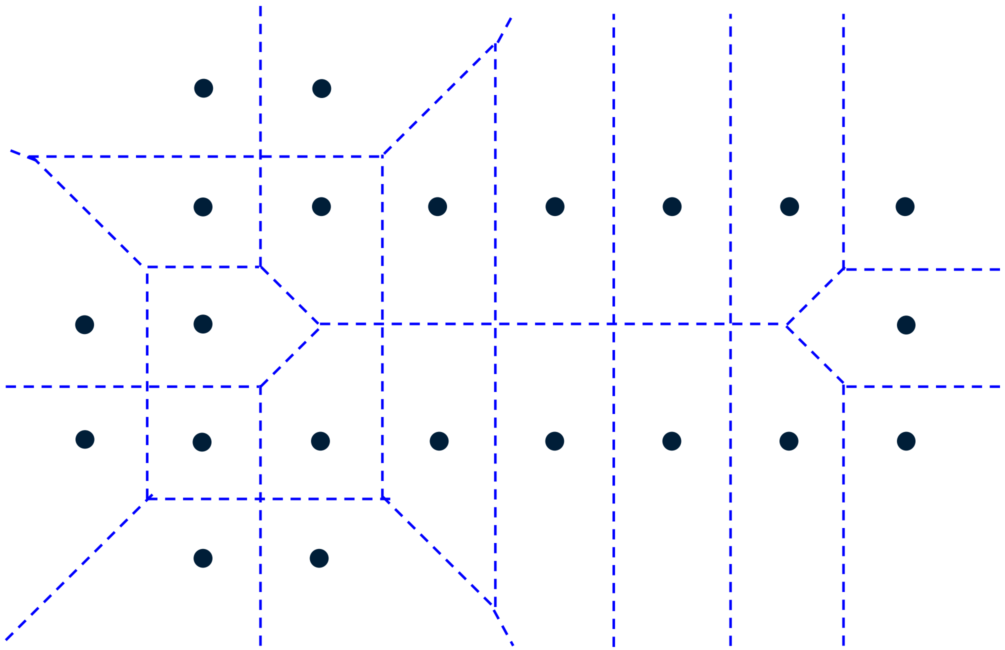
- Given an input shape O
- Voxelization V consists of voxels whose centers lie in O
  - "Gauss digitization"



MEDIAL AXIS OF VOXELIZATION

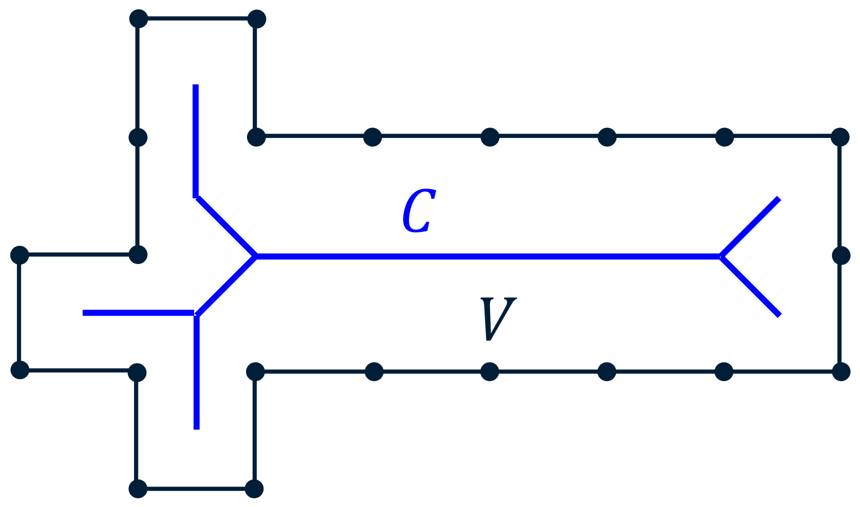


MEDIAL AXIS OF VOXELIZATION



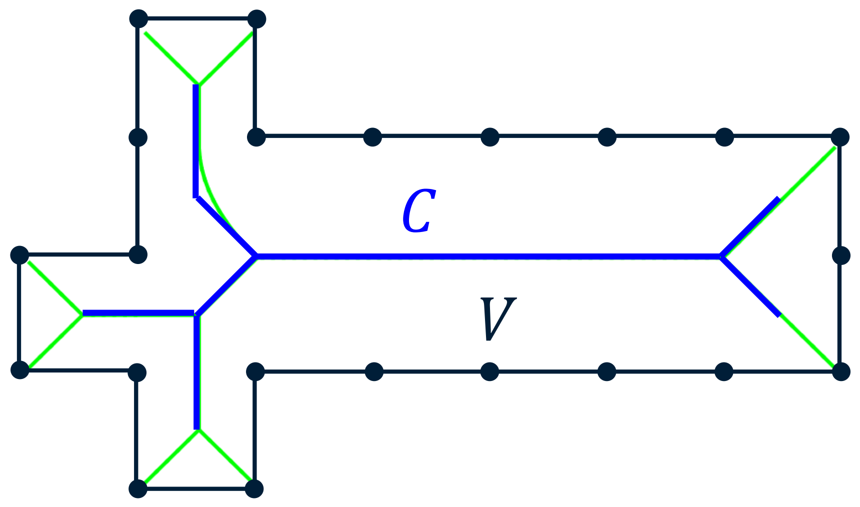
MEDIAL AXIS OF VOXELIZATION

- Voxel core C: faces of the Voronoi diagram of boundary vertices of V that lie inside V



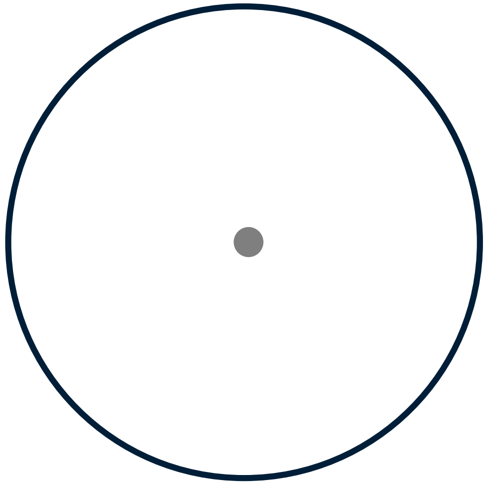
MEDIAL AXIS OF VOXELIZATION

- Voxel core C: faces of the Voronoi diagram of boundary vertices of V that lie inside V
  - Homotopy equivalent with V
  - Bounded Hausdorff distance to medial axis of V

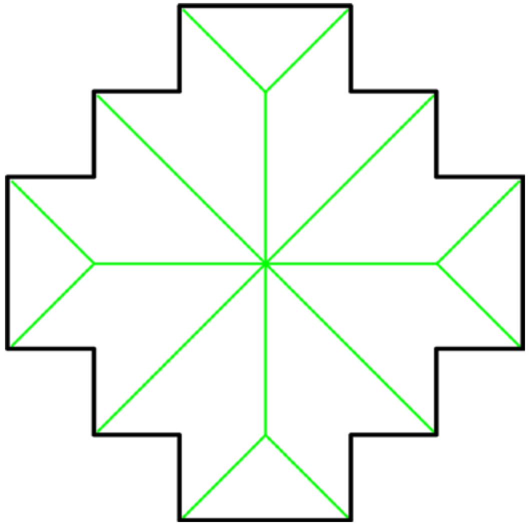


MEDIAL AXIS OF VOXELIZATION

- A shape and its voxelization can have very different medial axes
  - But a subset of the two medial axes are close [Chazal and Lieutier 05]



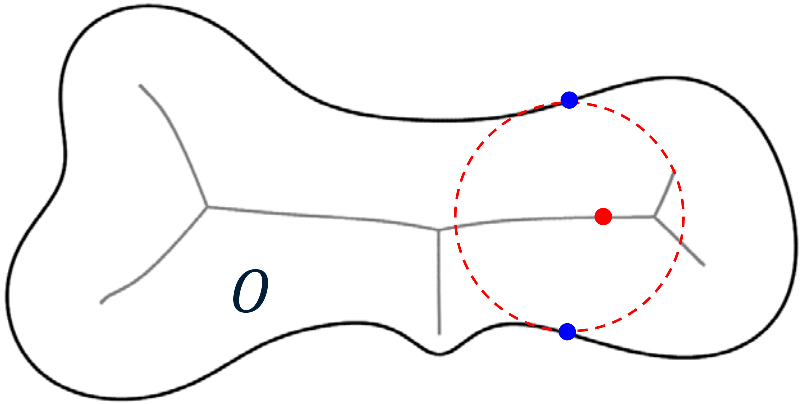
O and its medial axis



V and its medial axis

MEDIAL AXIS OF SHAPE

- λ-medial axis of O: medial axis points whose feature size $\geq \lambda$ [Chazal and Lieutier 05]



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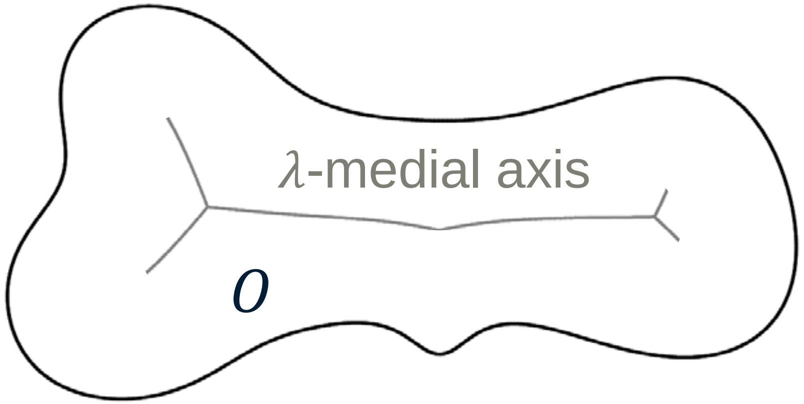
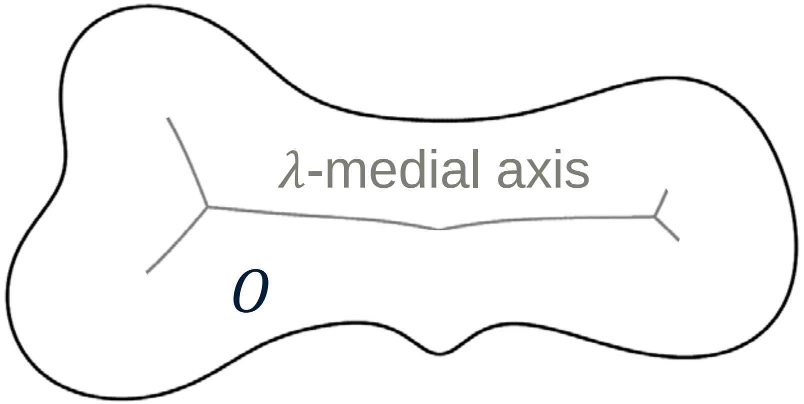
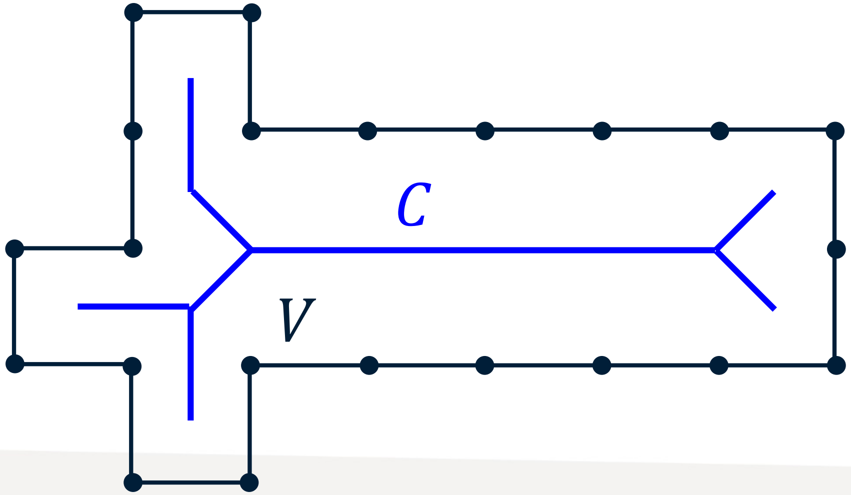


MEDIAL AXIS OF SHAPE

- λ-medial axis of O: medial axis points whose feature size $\geq \lambda$ [Chazal and Lieutier 05]
- λ-voxel core of V: set of all faces of C whose feature size $\geq \lambda$





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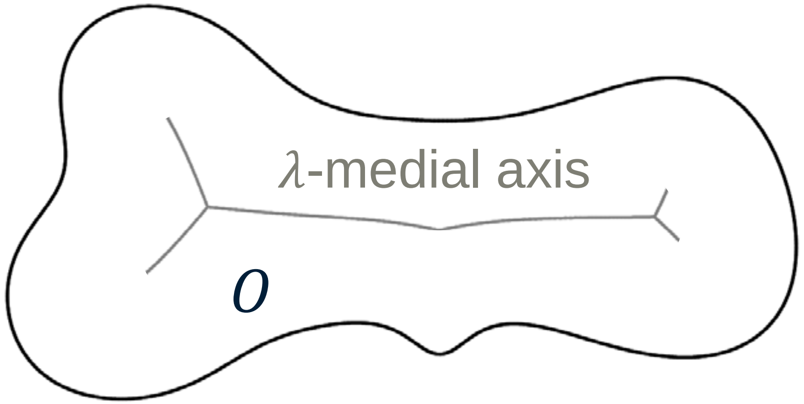

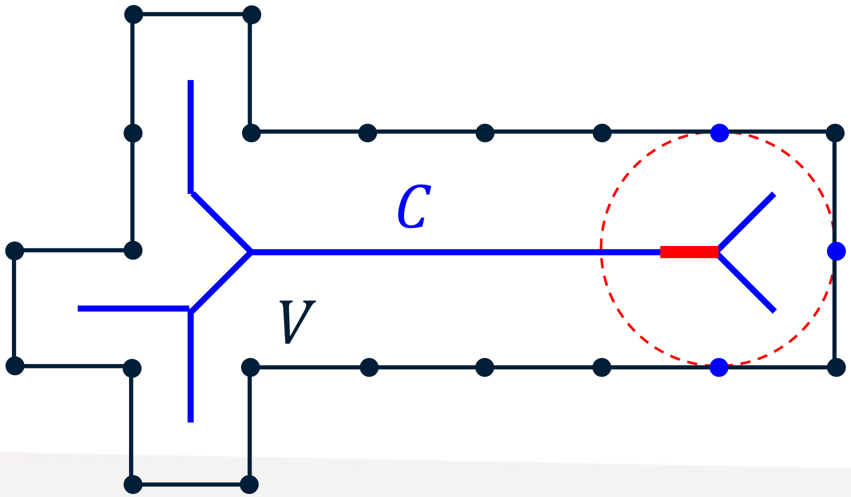

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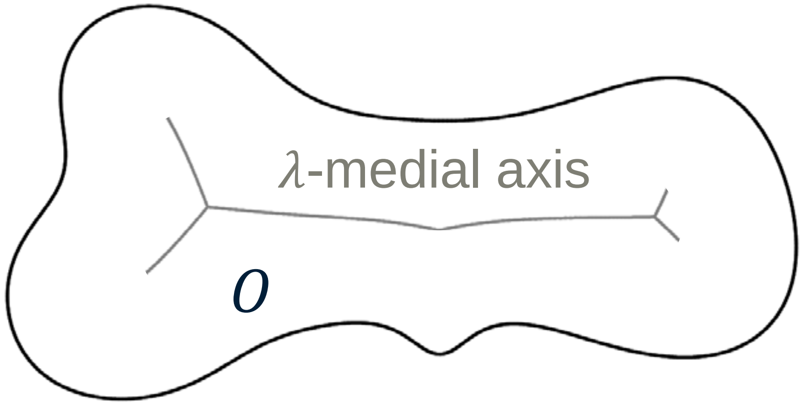
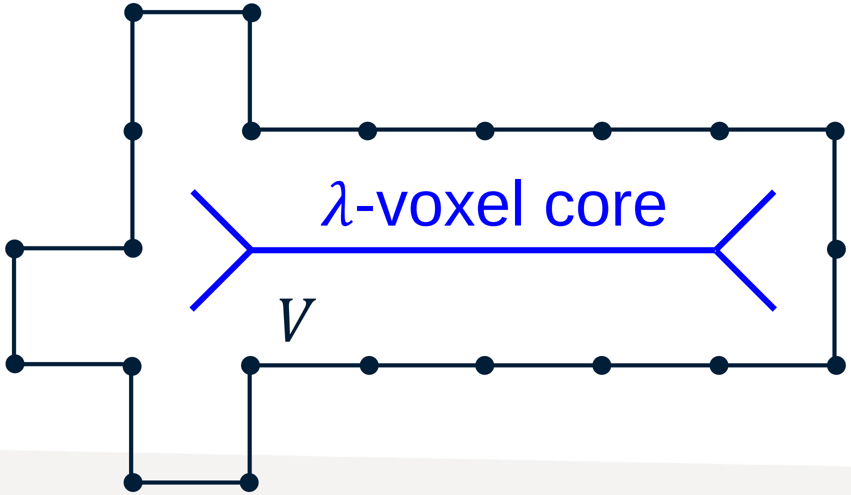


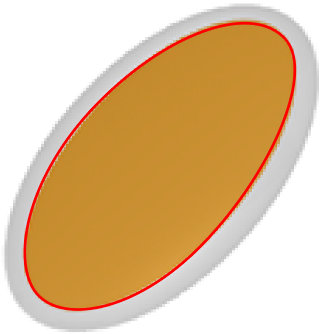


MAIN RESULTS

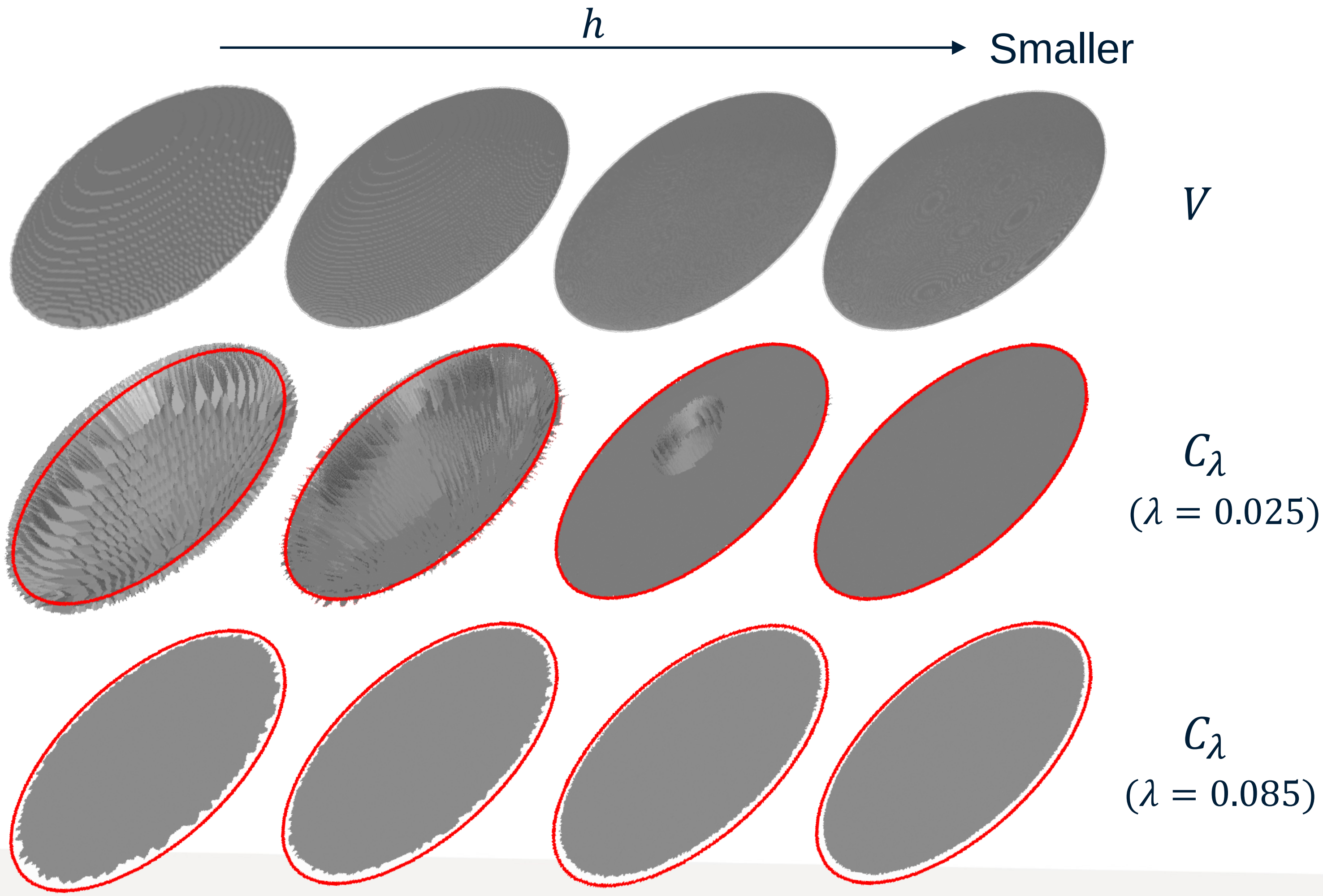
- Assuming O is smooth (C^2) and voxel size is sufficiently small
  - Voxel edge length $\leq \frac{2\sqrt{3}}{3}$ $\times$ minimum local feature size of O
- Voxel core of V is homotopy equivalent to medial axis of O
- λ-voxel core of V converges onto λ-medial axis of O as voxel size goes to 0
  - for any choice of $\lambda > 0$

CONVERGENCE OF λ-VOXEL CORE

- λ: balances convergence rate and coverage of the medial axis

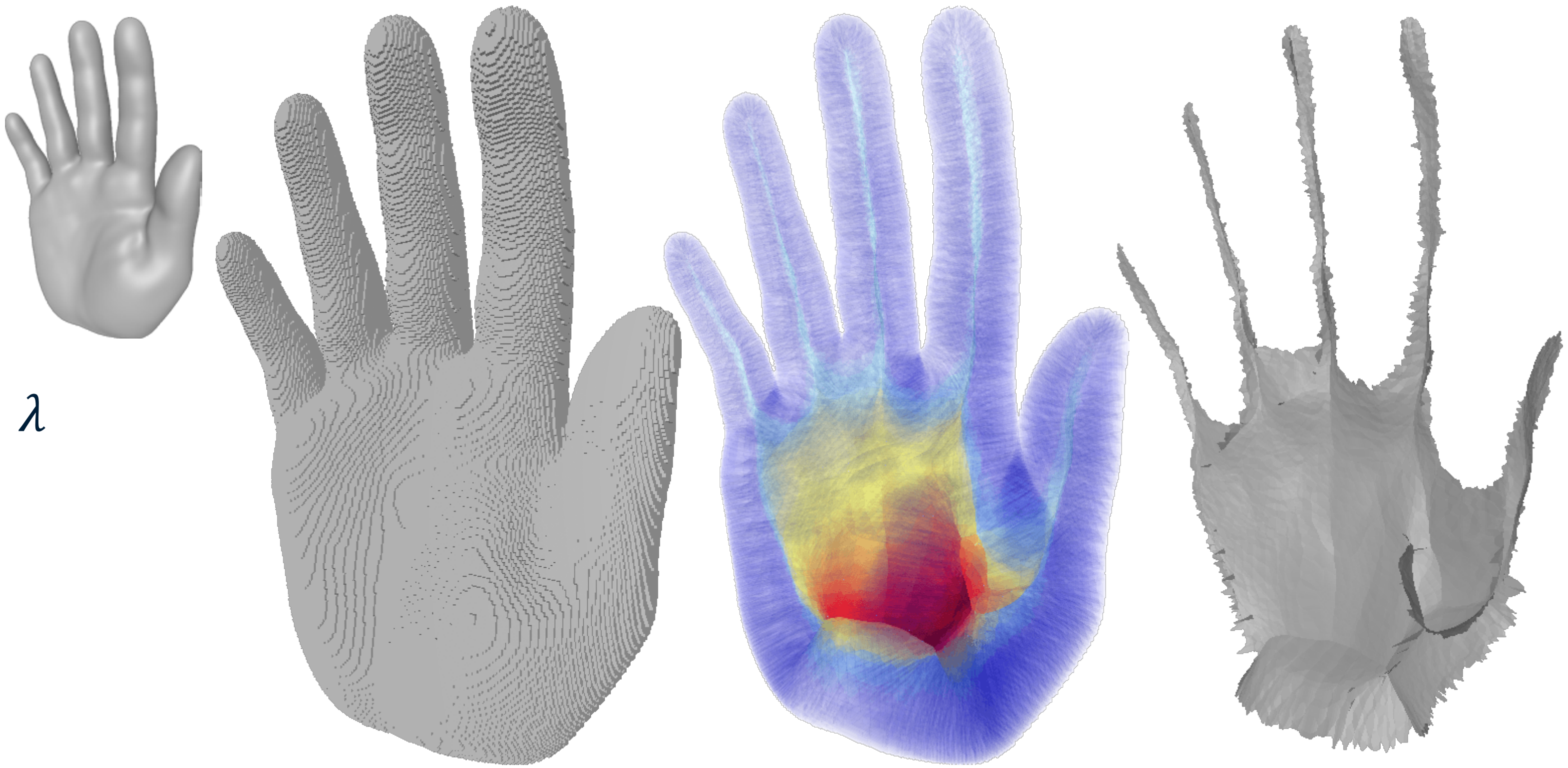


Ellipsoid and medial axis



ALGORITHM

1. Voxelization
2. Extracting voxel core
   - Computing Voronoi diagram
   - Keeping interior part
3. Pruning by feature size given λ
   - Topology-preserving contraction from voxel core to λ-voxel core

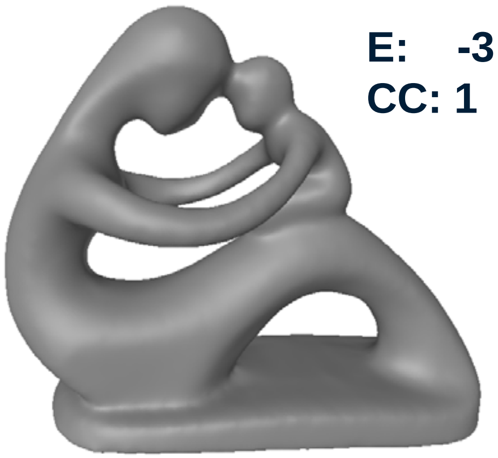


Voxelization

Voxel core (color: feature size)

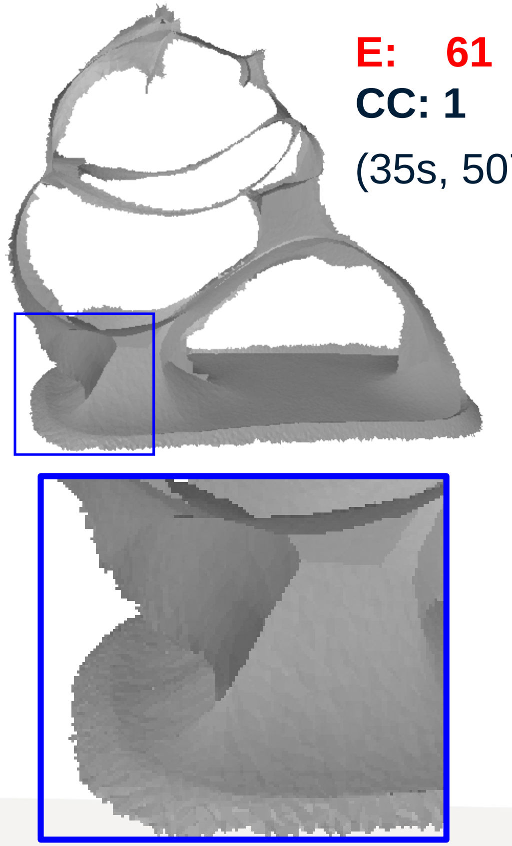
Approximate medial axis

COMPARISONS



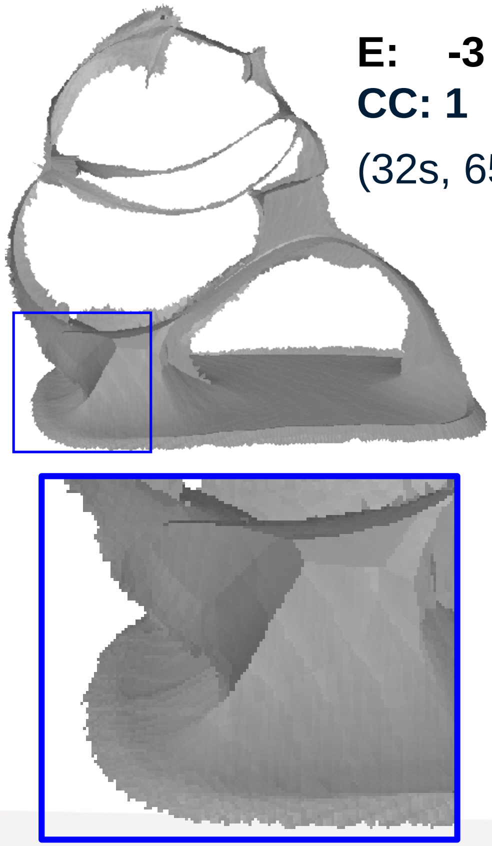
E: -3
CC: 1

[Amenta & Kolluri 01]



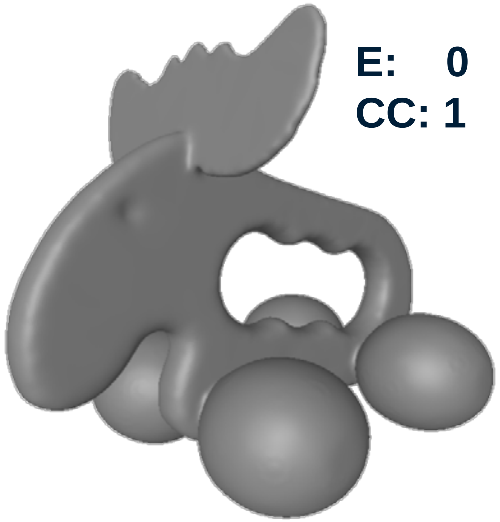
E: 61
CC: 1
(35s, 507MB)

Ours (512^3)



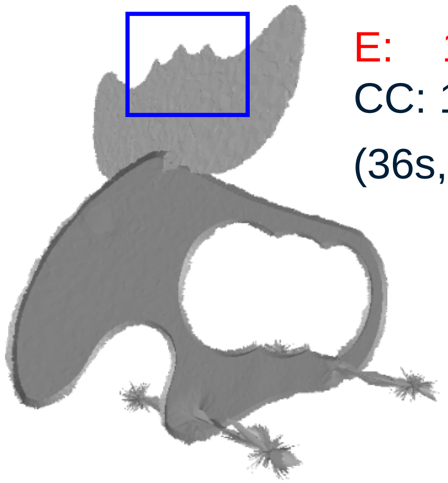
E: -3
CC: 1
(32s, 653MB)

COMPARISONS

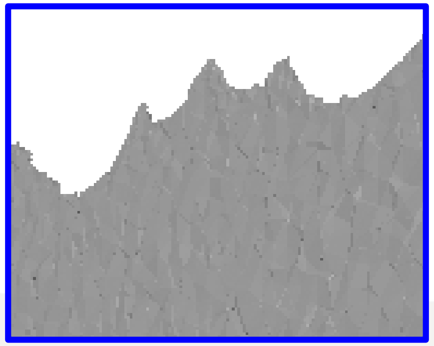


E: 0
CC: 1

[Amenta & Kolluri 01]



E: 133
CC: 1
(36s, 507MB)



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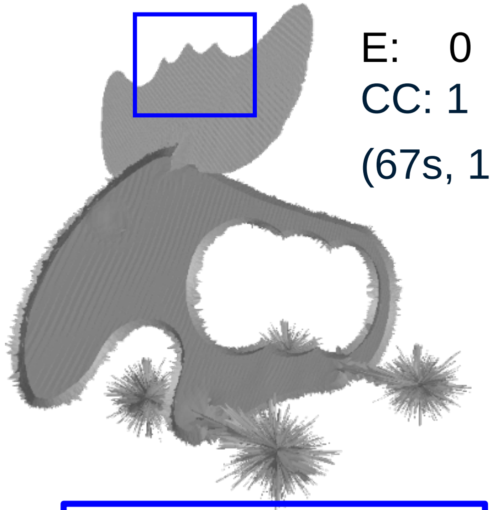
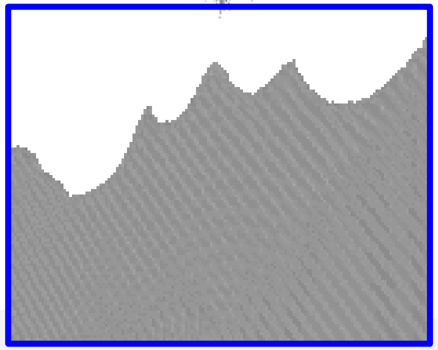


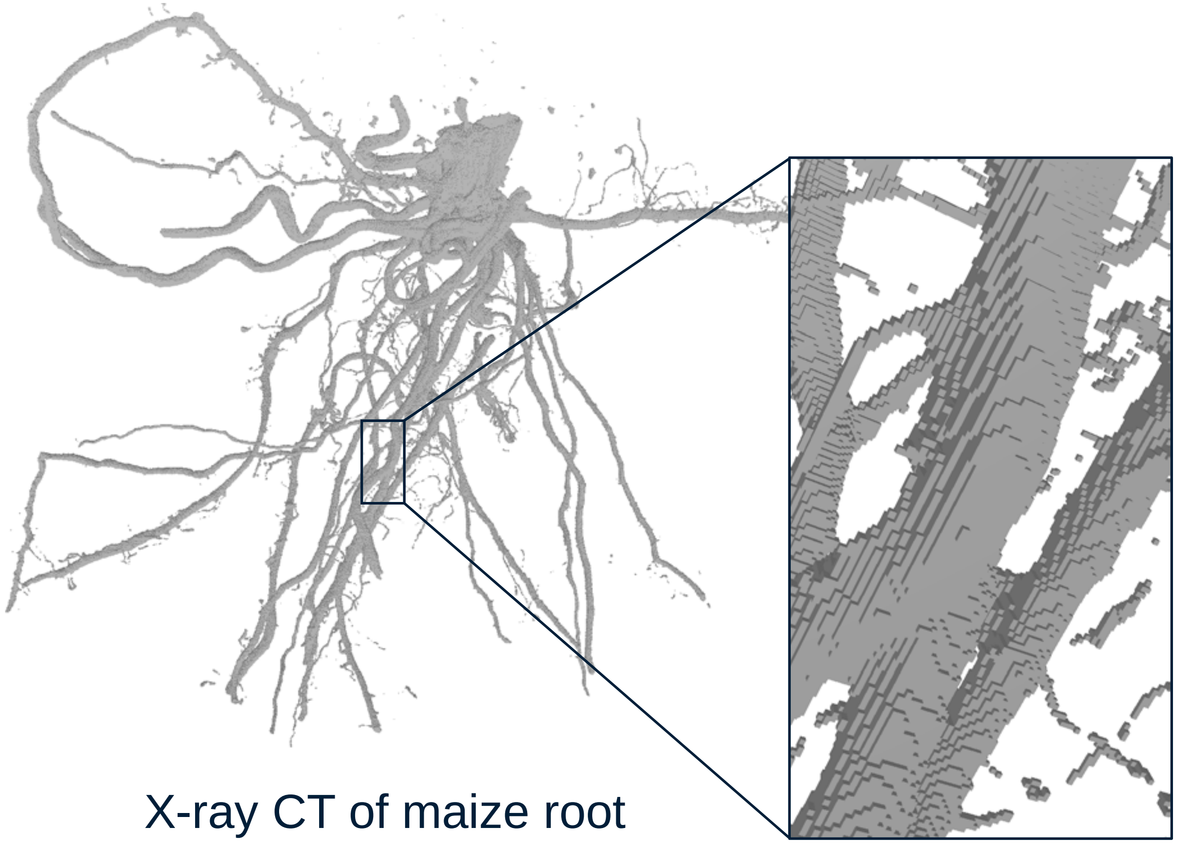
E: 0
CC: 1
(67s, 1.1GB)



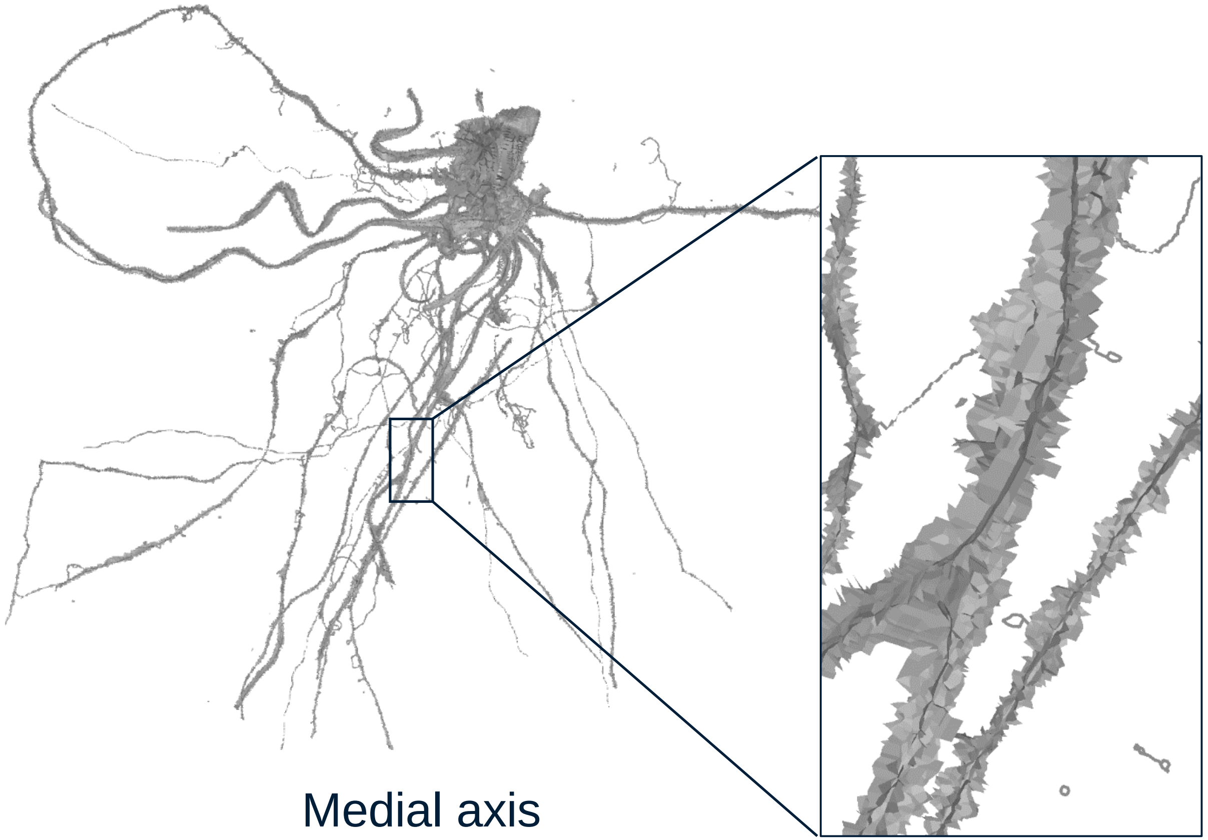
MORE EXAMPLES

- Triangle meshes processed at resolution 1024^3
  - Time < 3min, memory < 5GB

SEGMENTED 3D VOLUMES

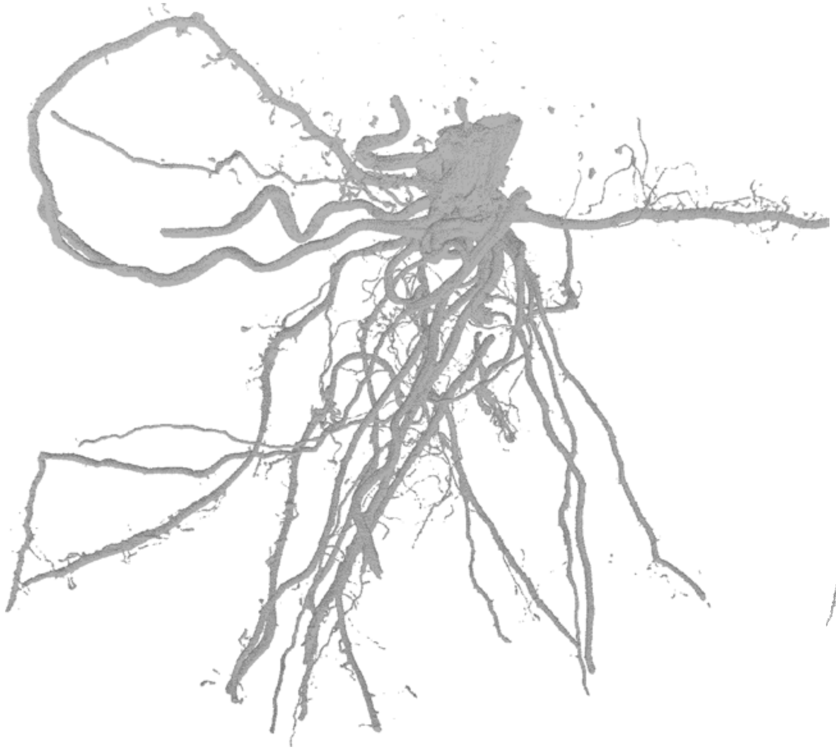


X-ray CT of maize root
(1560 × 789 × 1041)



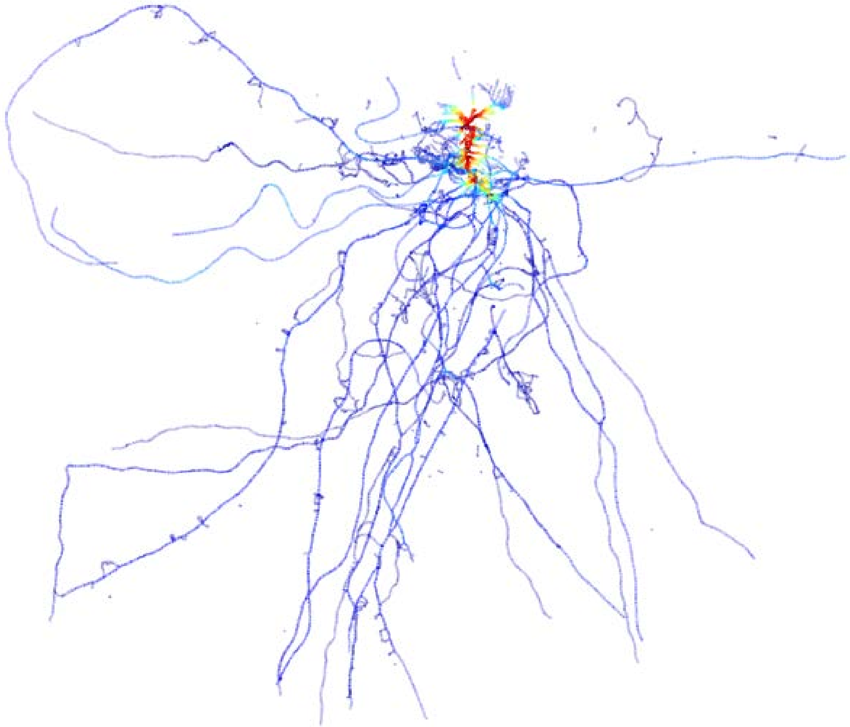
Medial axis
(<1min, 1.5GB)

SEGMENTED 3D VOLUMES



X-ray CT of maize root

Medial axis

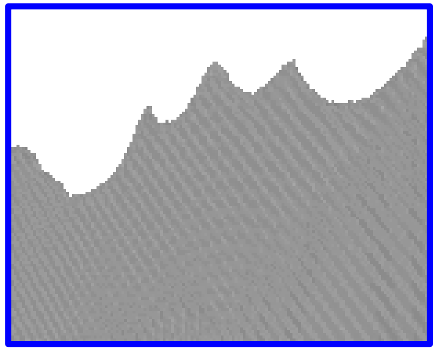


Curve skeleton

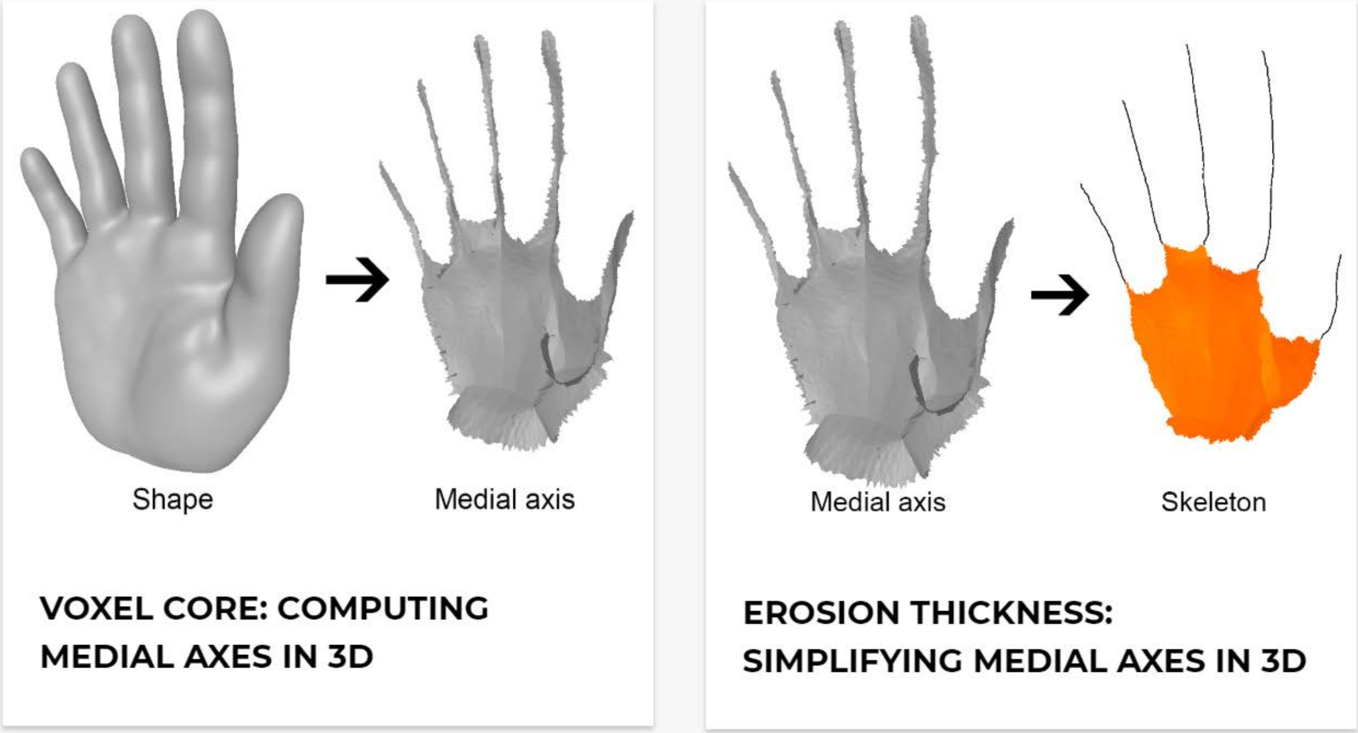
CONCLUSION

- A simple, theoretically sound and computation-friendly algorithm for computing 3D medial axes

- Limitations and future work:
  - Aliasing artifacts
  - Efficiency on very large volumes (e.g, adaptive grids)
  - Extension to higher dimensions



Exe + code: yajieyan.github.io/project/ma



Siggraph'18 | Siggraph'16