SIGGRAPH ASIA 2018 TOKYO
CONFERENCE 4 – 7 December 2018
EXHIBITION 5 – 7 December 2018
Tokyo International Forum, Japan
SA2018.SIGGRAPH.ORG

Construction and Fabrication of Reversible Shape Transforms

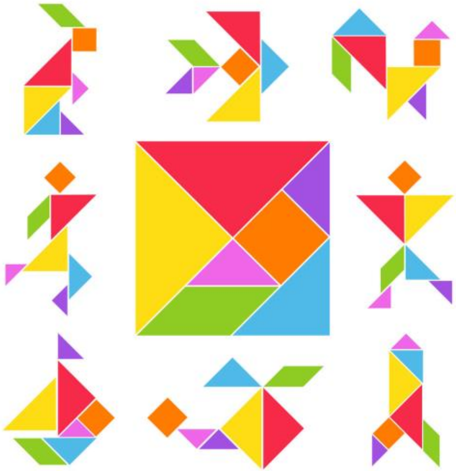
Shuhua Li[1, 2], Ali Mahdavi-Amiri[1], Ruizhen Hu[3], Han Liu[1, 4], Changqing Zou[5], Oliver Van Kaick[4], Xiuping Liu[2], Hui Huang[3], Hao Zhang[1]

[1]Simon Fraser University
[2]Dalian University of Technology
[3]Shenzhen University
[4]Carleton University
[5]University of Maryland, College Park

Geometric Dissection

The classic dissection between a square and a triangle by **Henry Dudeney** 1907

Ancient Dissection Puzzles



Tangram タングラム

Archimedes' Stomachion

Famous Questions And Theories

- 1807 Wallace-Bolyai-Gerwien Theorem
- 1900 Hilbert’s Third Problem
- 2007 Hinged Dissections Exist

Sponsored by

Wallace-Bolyai-Gerwien Theorem 1807

"One polygon can be cut into a finite number of pieces and rearranged into another polygon if and only if two polygons have the same area."

Sponsored by

Hilbert's Third Problem 1900

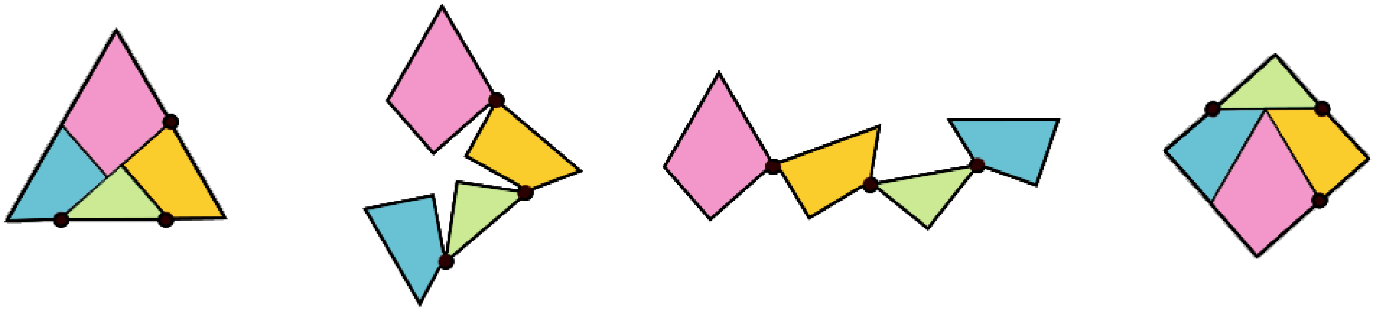
"Is it always possible to cut one polyhedral into finitely many polyhedral pieces and recompose pieces to form another of equal volume?"

--David Hilbert

23 Jan. 1862 - 14 Feb. 1943

Sponsored by

Hinged Dissections Exist 2007



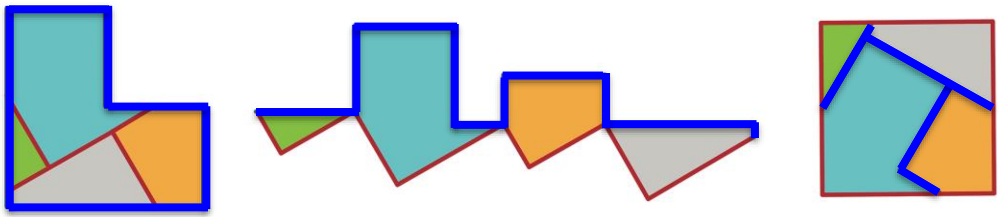
Two equal-area polygons must possess a hinged dissection*

*Hinged dissections exist. Abbott et al.. Discrete & Computational Geometry, 2012.

Sponsored by

Reversible Hinged Dissections

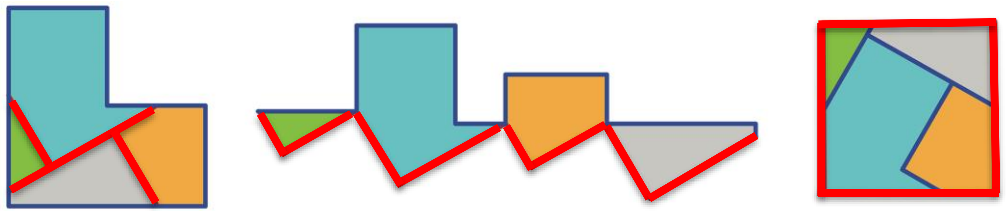
Reversible inside-out transform (RIOT)*



*Hinged Dissections: Swinging and Twisting. Greg N. Frederickson. Cambridge University Press. 2002.

Reversible Hinged Dissections

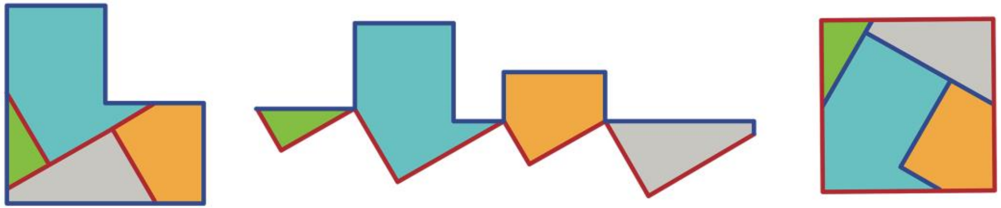
Reversible inside-out transform (RIOT)*



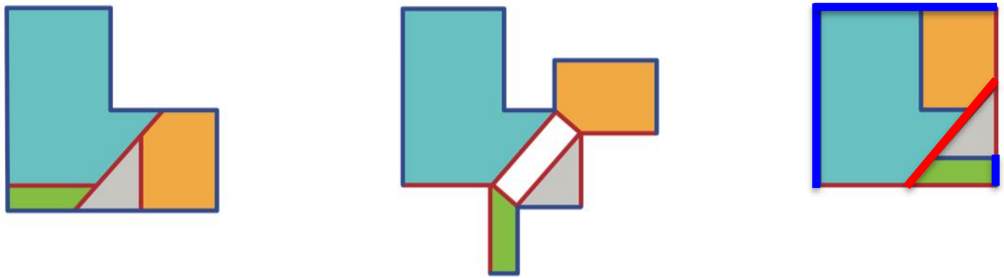
*Hinged Dissections: Swinging and Twisting. Greg N. Frederickson. Cambridge University Press. 2002.

Reversible Hinged Dissections

Reversible inside-out transform (RIOT)*



Non-reversible*



*Hinged Dissections: Swinging and Twisting. Greg N. Frederickson. Cambridge University Press. 2002.

SIGGRAPH ASIA 2018 TOKYO

Our Motivations: RIOT Calls For More Attentions

- No theories ensure that a RIOT always exists between shapes of equal area.
- No RIOT construction schemes between general shapes.
- Only a few manual RIOT designs between non-trivial shapes.

Our Work: From A Design And Modeling Perspective

- A **quick** RIOT exploration tool
- Fully automatic **approximate** RIOT construction
- Fabrication to make **collision-free** assembly puzzles

Related Works in Computer Graphics

3D Decompose-and-assemble

- Dapper: Decompose-and-Pack for 3D Printing. Chen et al., ToG, 2015.
- Reconfigurable Interlocking Furniture. Song et al., ToG, 2017.

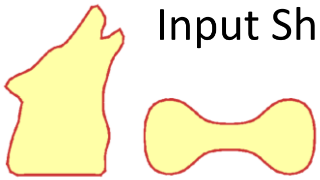
Approximate geometric dissections

- An Algorithm for Creating Geometric Dissection Puzzles. Zhou et al., In Proc. of Bridges Conf., 2012.
- Approximate Dissections. Duncan et al., ToG, 2017.

3D geometric puzzles

- Boxelization: folding 3D objects into boxes. Zhou et al., ToG, 2014.
- Computational Design of Twisty Joints and Puzzles. Sun et al., ToG, 2015.

The Most Related Work (2017)



Input Shapes

Pieces

Arrangements

Similarities & Differences

Similarities

- Natural shape pairs
- Approximate dissections
- User interaction
- Fabrication

Differences

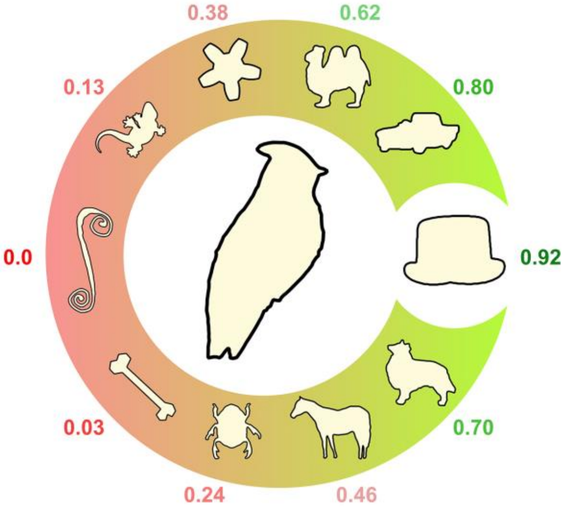
- Different Problems
  Hinged & inside-out reversibility
- Different Approaches
  Conjugate trunks
- Additional Tool
  Quick RIOT exploration tool

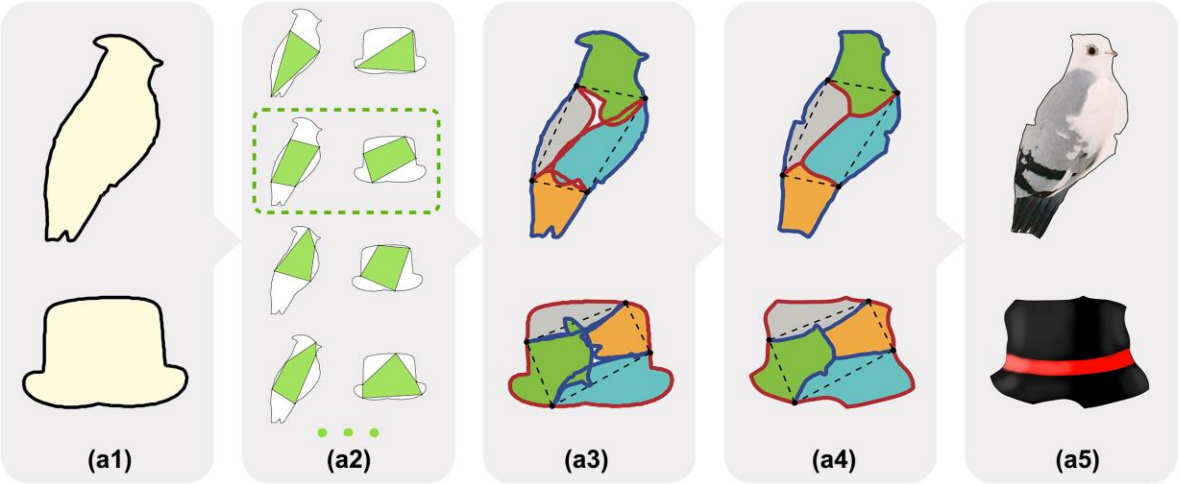
Approximate Dissections. Duncan et al., ACM Trans. on Graph., 2017.

Sponsored by

Algorithm Overview

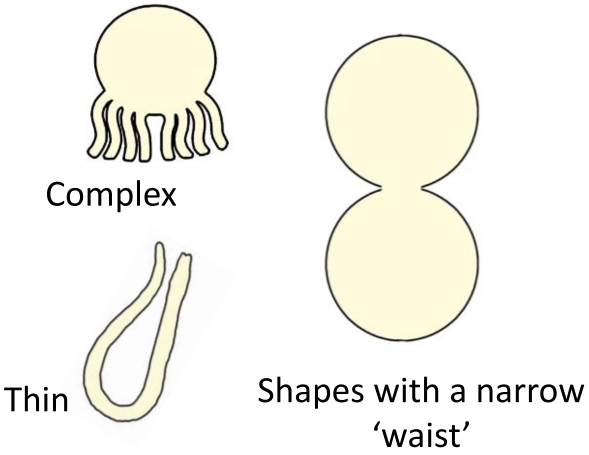
1 Quick RIOT Exploration — 2 Candidate Trunk Pair — 3 Deformation — 4 Fabrication



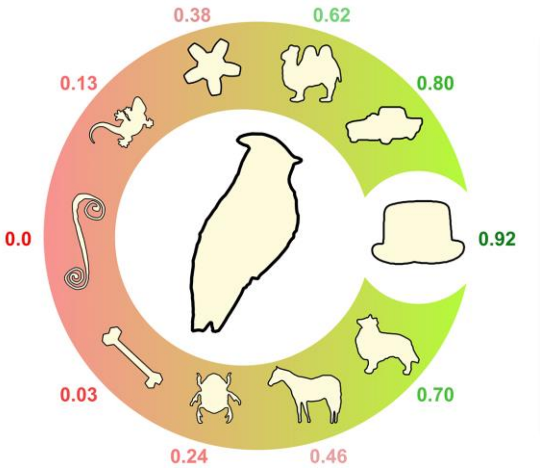


Quick RIOT Exploration

Quick Reversibility Scores (**QRS**) Of Shapes

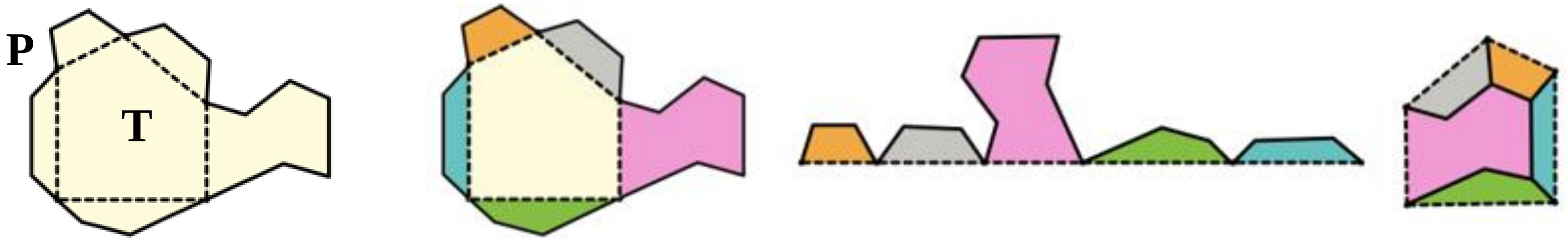


Quick Cross-Reversibility Score (**QCRS**) Of Shape Pairs



Candidate Trunk Pair —— What is Trunk ?

A Trunk T of Shape P



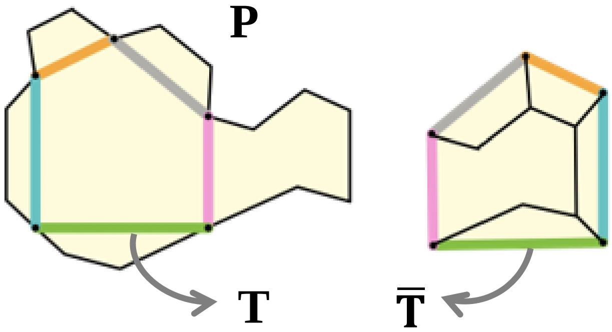
T: convex, inscribed polygon

$\overline{T}$: convex

Treks into Intuitive Geometry. Jin Akiyama and Kiyoko Matsunaga. 2015. Springer.

Candidate Trunk Pair —— What is Trunk ?

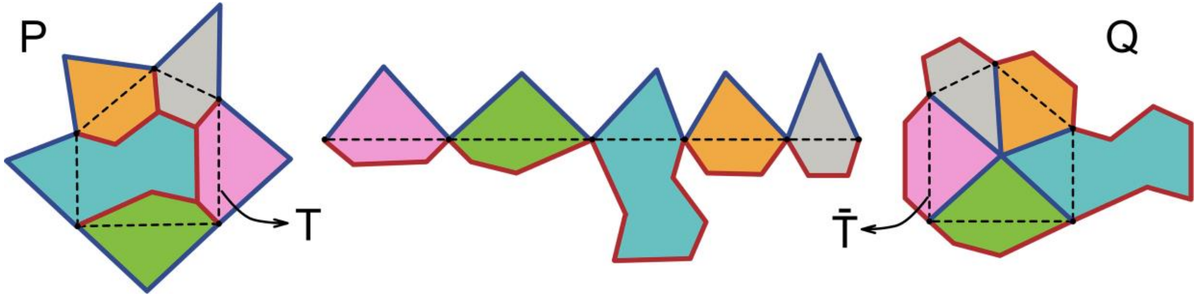
The Conjugate Trunk $\bar{T}$



- Two polygons sharing the same set of edges in reverse order are said to be **conjugate**.
- T and $\bar{\text{T}}$ are conjugate trunks of P

Treks into Intuitive Geometry. Jin Akiyama and Kiyoko Matsunaga. 2015. Springer.

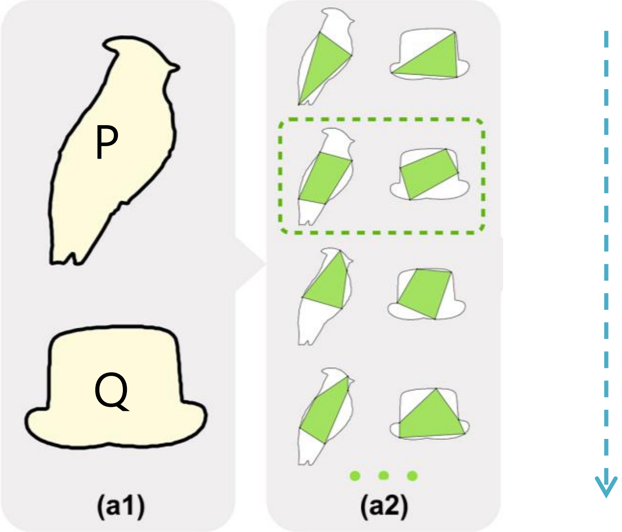
Candidate Trunk Pair —— A Sufficient Condition



Two shapes have a RIOT, if they possess a pair of conjugate trunks*

*Treks into Intuitive Geometry. Jin Akiyama and Kiyoko Matsunaga. 2015. Springer.

Candidate Trunk Pair

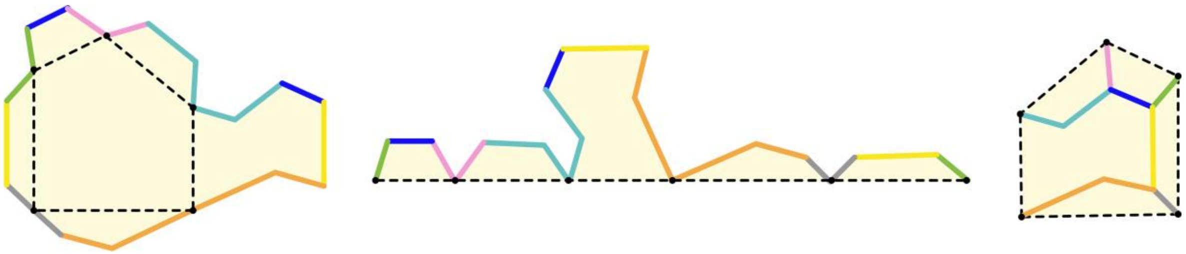


- Selecting Candidate Vertices
- Generating Candidate Trunks
- Trunk Pair Selection

Candidate Trunk Pair —— Selecting Candidate Vertices

Two criteria

- **Boundary Congruency**
- Area Compatibility



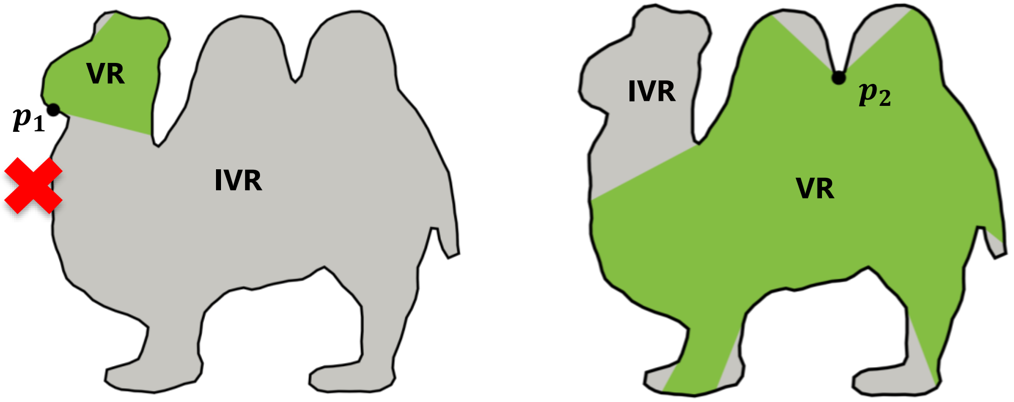
The boundary of a reversible shape can be divided into congruent segment pairs.

Candidate Trunk Pair —— Selecting Candidate Vertices

Two criteria

- Boundary Congruency
- Area Compatibility

Visible Region (VR) / Invisible Region (IVR)



$\sum_i Area(IVR_i(p)) < Area(VR(p)\text{-}Circle)$,

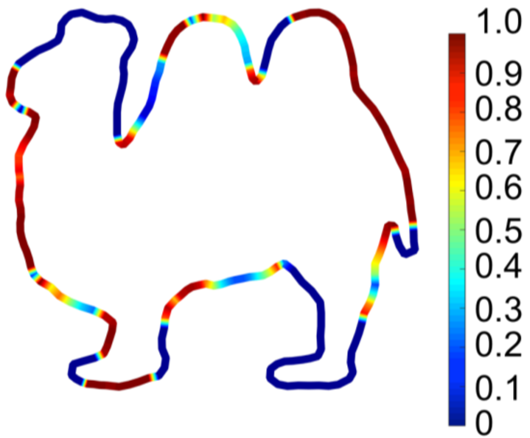
$VR(p)\text{-}Circle$: A circle with the same perimeter as VR(p)

Candidate Trunk Pair —— Selecting Candidate Vertices

- ❑ Binary score $S_{\mathrm{b}}(p)$ to exclude points
  - ▪Criterion: Area Compatibility

- ❑ Congruency score $S_c(p)$ for the remaining points
  - ▪Criterion: Boundary Congruency

- ❑ Candidate vertexes: $S_c(p) > 0.3$



Candidate Trunk Pair —— Generating Candidate Trunks

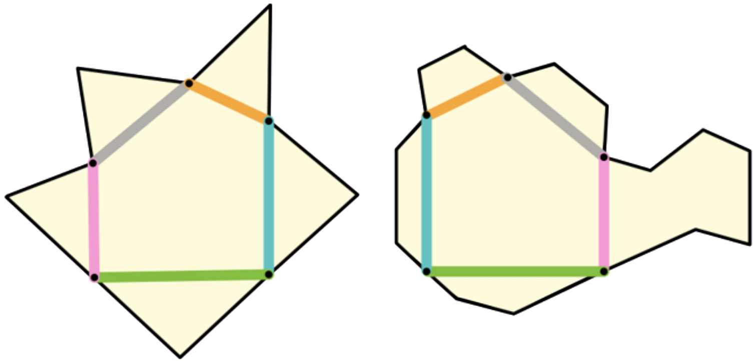
Candidate trunks

- A upper bound *K* of trunk edges (auto)
- From triangles to *K*-gons
- Heuristic conditions: inscribed, convex, boundary congruency, and area for fabrication

Candidate Trunk Pair —— Trunk Pair Selection

Three criteria

- Edge conjugacy
- Angle reversibility
- Area reversibility



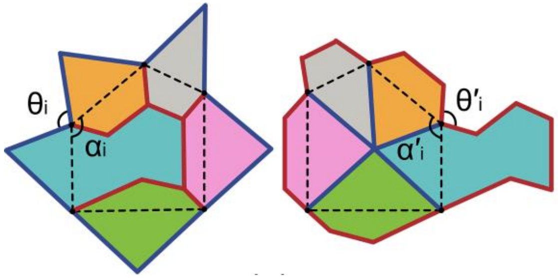
- An edge correspondence
- Corresponded edges surround in reverse order
- Corresponded edges have similar length

Candidate Trunk Pair —— Trunk Pair Selection

Three criteria

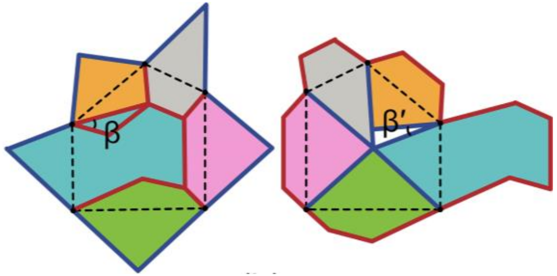
- Edge conjugacy
- **Angle reversibility**
- Area reversibility

- The angel relationships for reversible shapes:



$$2\pi - \theta_i - \alpha_i = \alpha_i' \ ,$$
$$2\pi - \theta_i' - \alpha_i' = \alpha_i$$

- The angle errors for approximate reversible shapes:

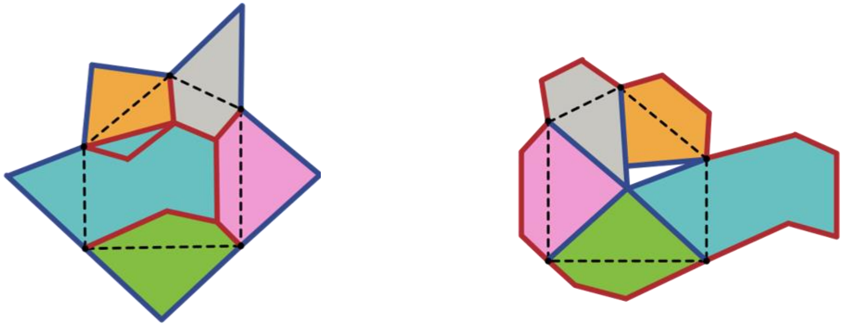


$$2\pi - \theta_i - \alpha_i = \alpha_i' - \boldsymbol{\beta'} \ ,$$
$$2\pi - \theta_i' - \alpha_i' = \alpha_i + \boldsymbol{\beta}$$

Candidate Trunk Pair —— Trunk Pair Selection

Three criteria

- Edge conjugacy
- Angle reversibility
- **Area reversibility**



$$Area(gaps)\ + Area(overlaps)$$

Candidate Trunk Pair —— Trunk Pair Selection

Three criteria

- Edge conjugacy
- Angle reversibility
- Area reversibility

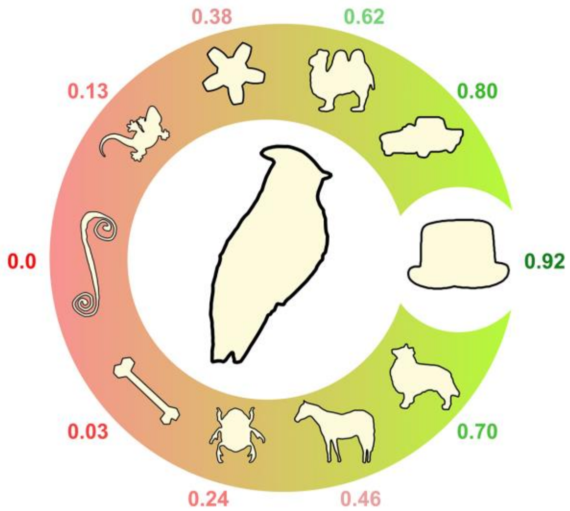
Cross-Reversibility Score (CRS)

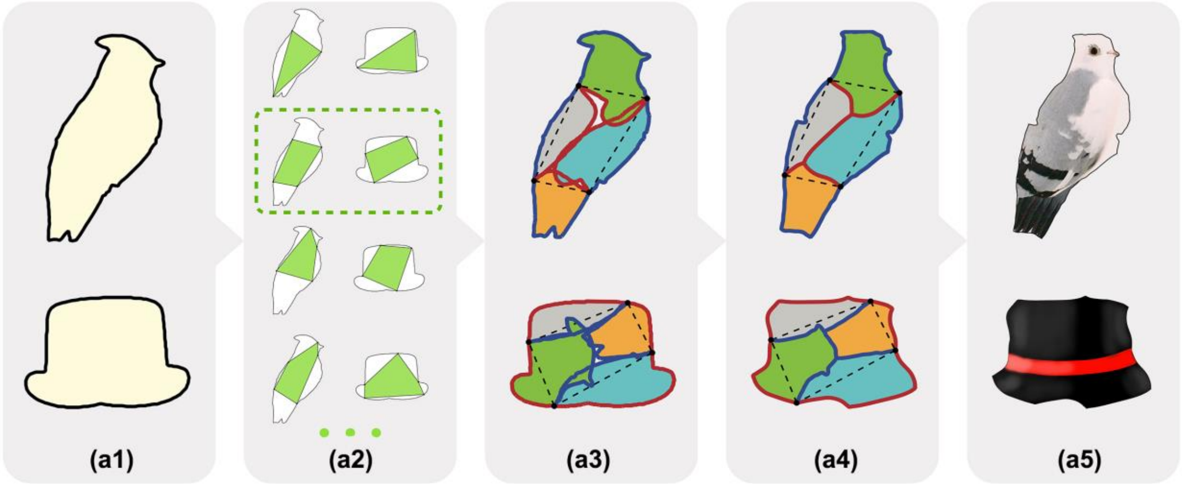
$$CRS(T,T') = \max_{i=0,\ldots,n-1} \min\{S_E^i, S_\angle^i, S_A^i\}$$

$$CRS(P,Q) = \max_{\{(T,T')\}} \mathrm{CRS}(T,T')$$

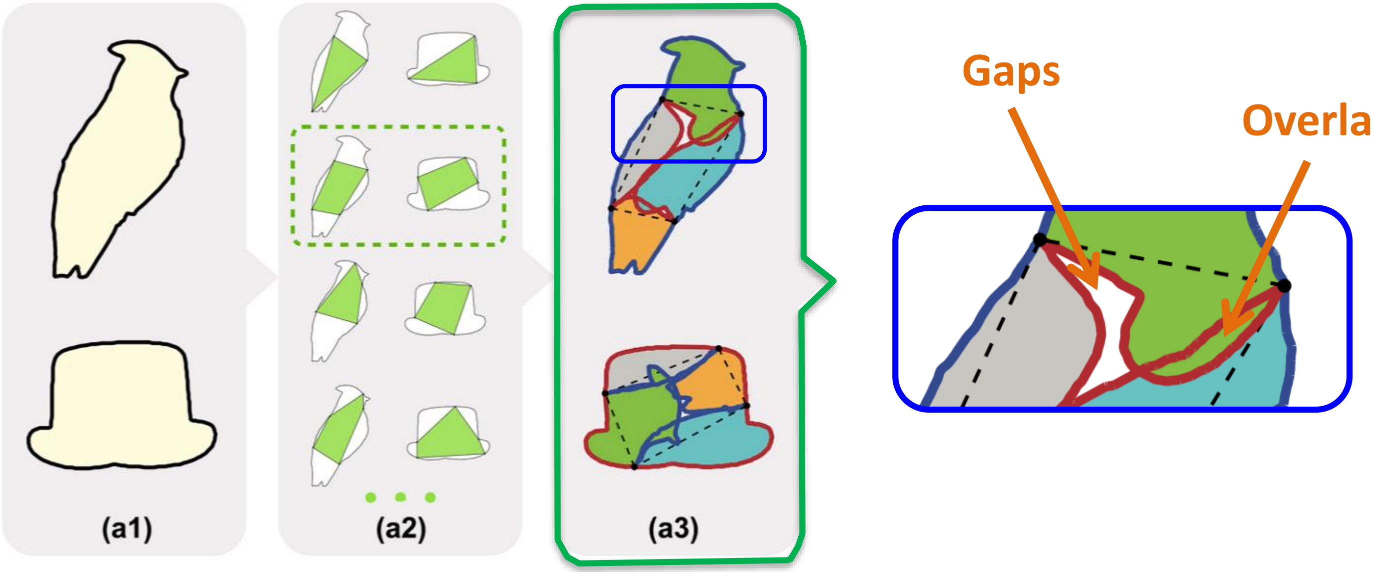
Approximate Construction Overview

1 — Quick RIOT Exploration
2 — Candidate Trunk Pair
3 — **Deformation**
4 — Fabrication



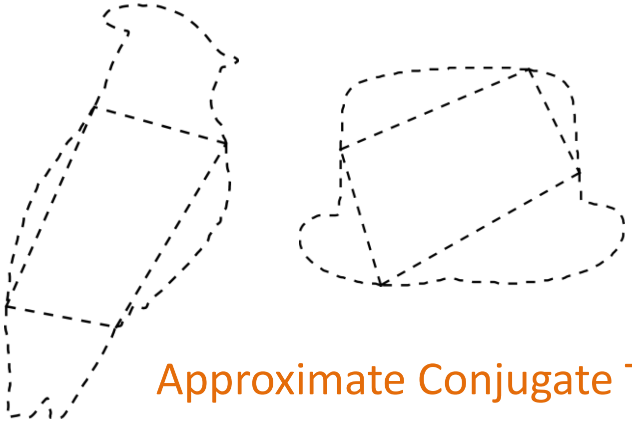


Boundary Deformation —— Approximate RIOT

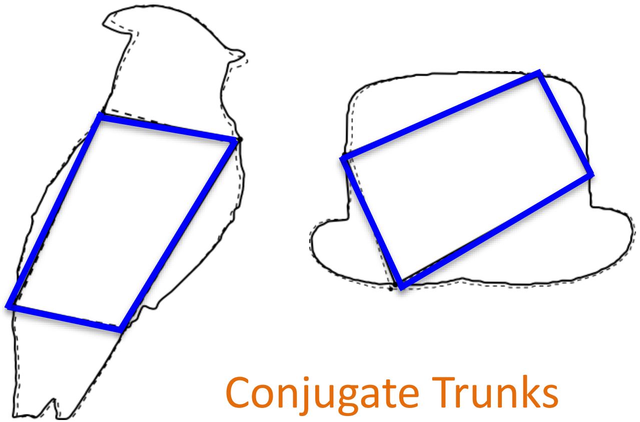


Sponsored by

Boundary Deformation — Conjugate Trunks Adjustment



Approximate Conjugate Trunks

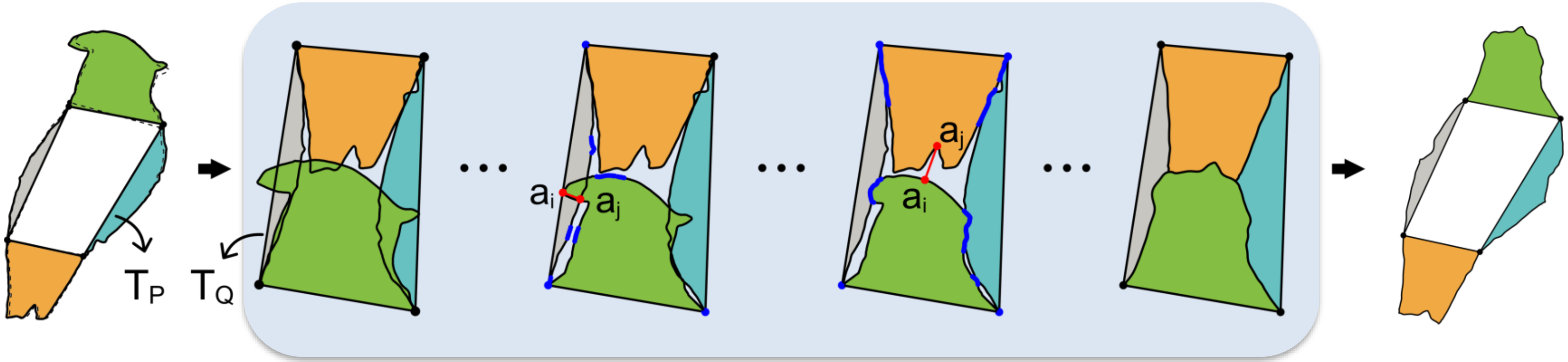


Conjugate Trunks

T_P and T_Q are fixed

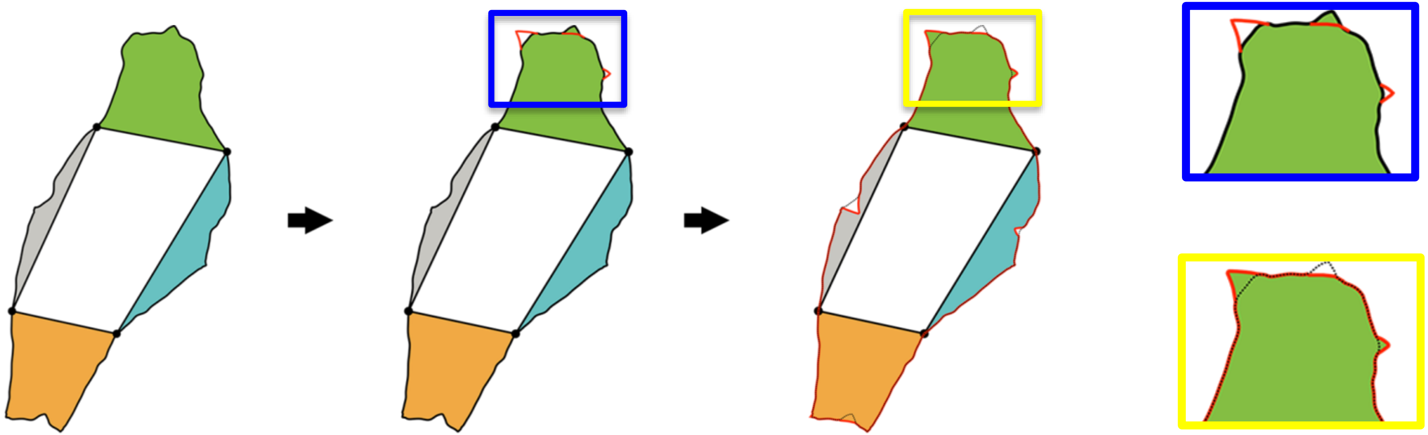
Boundary Deformation

Transformed curves are deformed by 2D Laplacian editing*

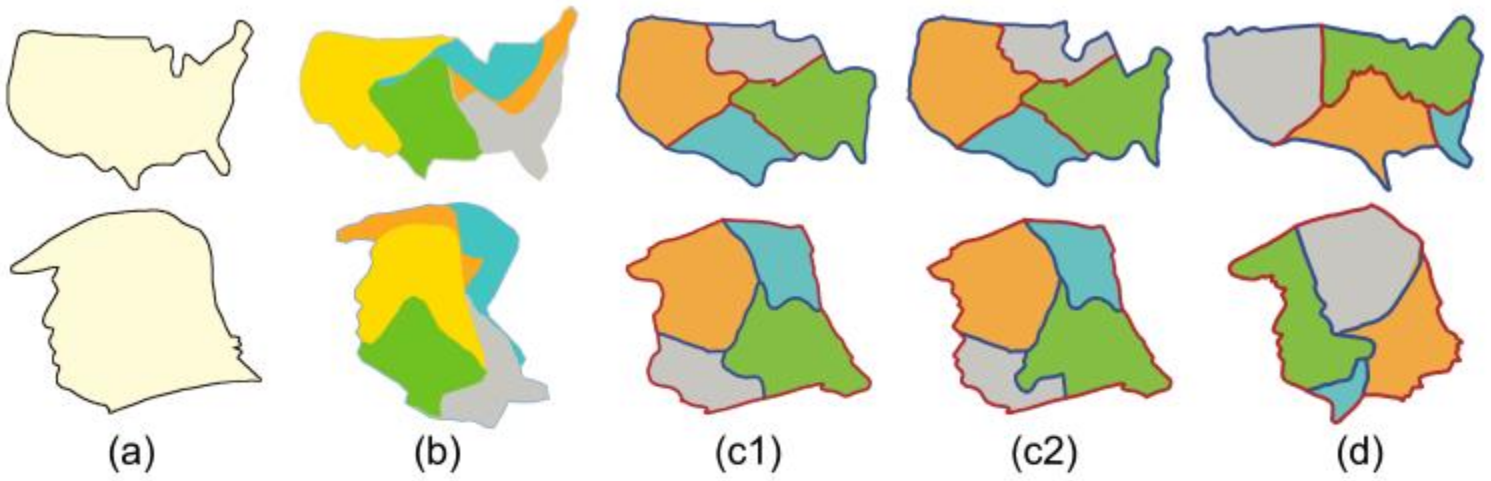


*Laplacian surface editing. Sorkine et al.. In Proceedings of the 2004 Eurographics/ACMSIGGRAPH symposium on Geometry processing.

Boundary Deformation — User Assistance



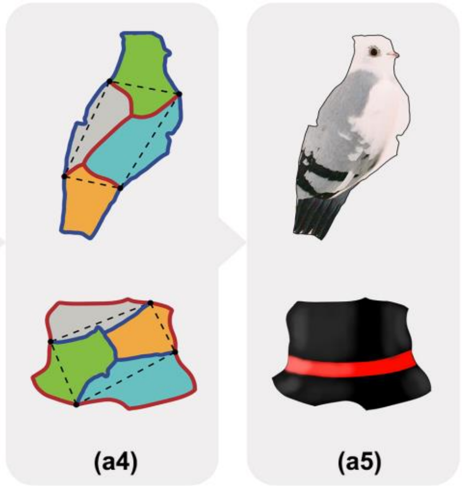
Boundary Deformation — User Assistance

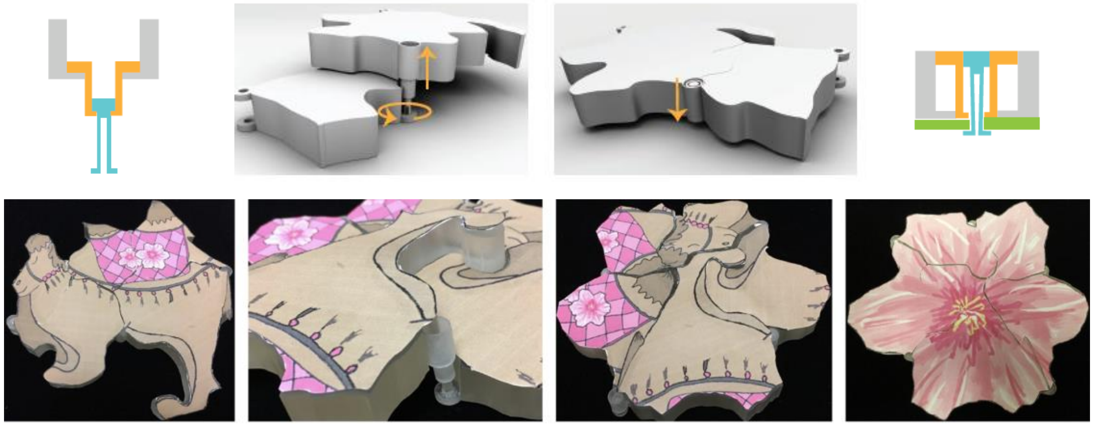


Approximate Dissections. Duncan et al., ACM Trans. on Graph., 2017.

Sponsored by

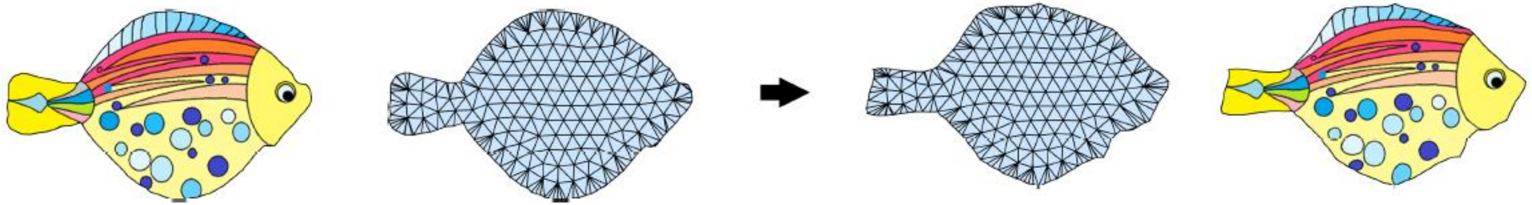
Texture and Fabrication





A telescopic structure can be used to lift a piece up, rotate it, and place it back to its base plane

Texture and Fabrication —— Automatic Texture Transfer



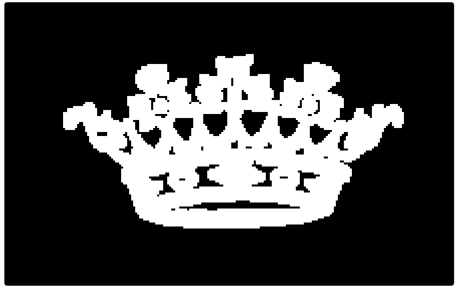
Available input texture | triangle meshing | Deformed mesh | Deformed texture

Results And Evaluation

Silhouette Image Collection

- Two public silhouette image datasets
  - MPEG-7 database
    http://www.dabi.temple.edu/~shape/MPEG7/dataset.html
  - Animal database
    https://sites.google.com/site/xiangbai/animaldataset
  - 81 shape classes and 3,400 shapes in total
- Other images found online

Silhouette Image Pre-processing

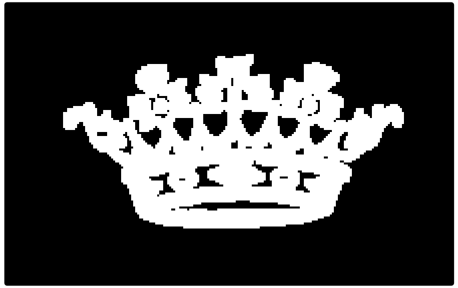


input silhouette image

fill interior holes

Sponsored by

Silhouette Image Pre-processing



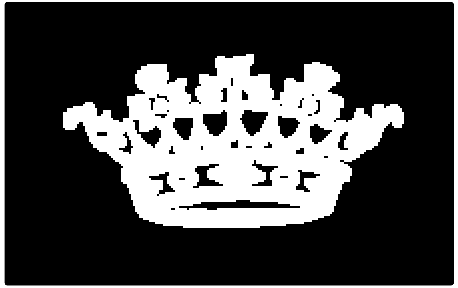
input silhouette image

extract a single closed contour

area normalization

Sponsored by

Silhouette Image Pre-processing



input silhouette image

extract a single closed contour

adaptive sampling

Sponsored by

Parameters

- Default parameter setting
  - Sampling distance for candidate vertices $d_{space} = \frac{L^c}{15}$
  - Distance tolerance for boundary simplification $\tau_s = 0.1$
  - Threshold for congruency score $\tau_c = 0.3$
  - Variances for reversibility score $\sigma_{PA} = 1, \sigma_W = 4$

Exact RIOT pairs manually designed by Jin Akiyama

Sponsored by

Parameters

- Default parameter setting
  - Sampling distance for candidate vertices $d_{space} = \frac{\mathrm{L}^c}{15}$
  - Distance tolerance for boundary simplification $\tau_s = 0.1$
  - Threshold for congruency score $\tau_\mathrm{c} = 0.3$
  - Variances for reversibility score $\sigma_{\mathrm{PA}} = 1, \sigma_{\mathrm{W}} = 4$

The large shape collection combing two public datasets

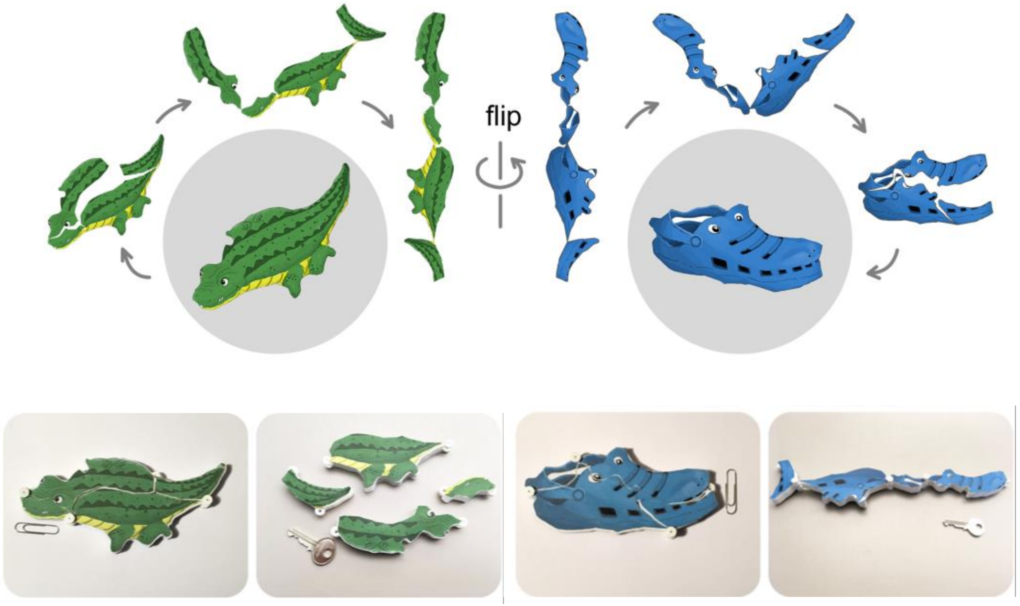
Sponsored by

Timing

- MATLAB implement on a 4 GHz desktop
- Average time
  - 0.12s/shape for QRS
  - 1.99s/pair for QCRS
  - 10.36s/shape for candidate trunks
  - 11.90s/pair for candidate conjugate trunks
  - 2.19 minutes per pair for boundary deformation

Results Gallery

crocodile and the Crocs shoe



Results Gallery
pine cone and squirrel
flip
Charlie Chaplin and his hat
flip
tortoise and hare
flip
cat and mouse
flip
Labrador and bowl
flip
beaver and maple leaf
flip
carrot and bunny
flip
ambulance car and alarm bell
flip
butterfly and fish
flip
Monkey king and banana
flip

QCRS Evaluation

How consistent it is with respect to CRS

- 1000 random pairs of shape pairs
- Ranking consistency between QCRS and CRS

$$QCRS(P_1, Q_1) \quad \geq / \leq \quad QCRS(P_2, Q_2)$$

$$CRS(P_1, Q_1) \quad \geq / \leq \quad CRS(P_2, Q_2)$$

Consistent Ranking?

Sponsored by

QCRS Evaluation

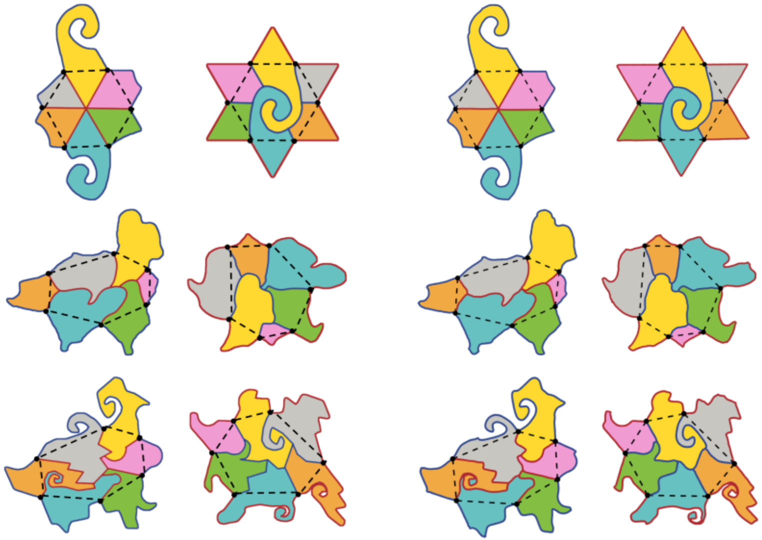
How consistent it is with respect to CRS

- 1000 random pairs of shape pairs
- Ranking consistency between QCRS and CRS

Ranking consistency: 77.4%

Comparisons With Manual Designs

- Nine manually designed pairs
- Success for seven pairs
- Fail for two pairs
  (too complex boundaries)



Our automatic RIOT solutions (right) are almost the same with manual designs (left).

Sponsored by

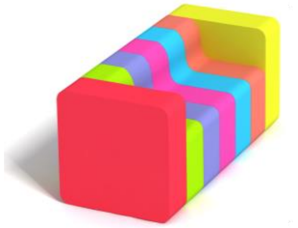
Application: Real Sofa Design

The Borghese sofa

The three back pieces of the Borghese sofa (left) can be transformed into different animals: bunny, bear, and fish (right)

Fabricated prototypes using a 3D printer

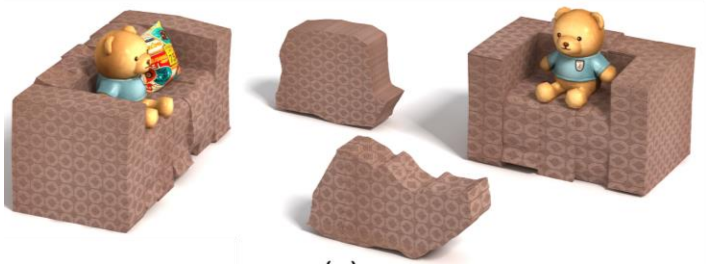
Application: 3D Sofa Design



A 3D input sofa is partitioned into parallel thick slices

The RIOT pairs for all the slices

Output deformed sofa



Two possible sofa configurations: double sofa & a loveseat

Sponsored by

Conclusions

- First approximate RIOT problem
- Fully automatic RIOT construction algorithm
- A quick RIOT exploration tool
- Numerous fascinating RIOT result pairs

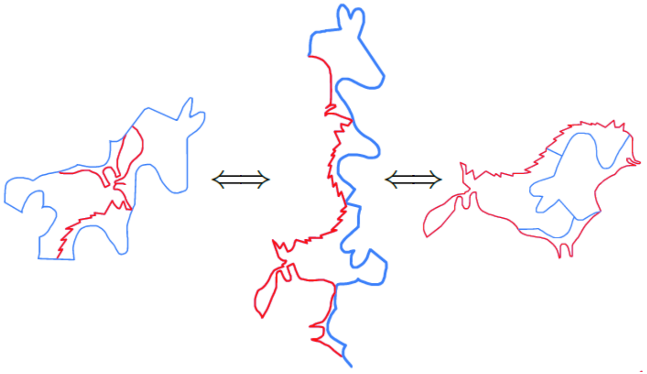
Sponsored by

Limitations

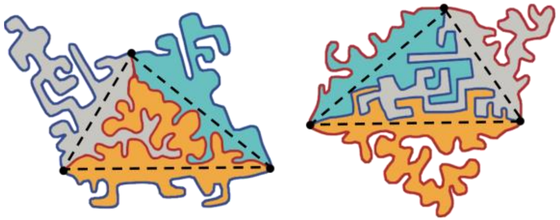
Conjugate trunks —— only a sufficient condition

Limited types of trunks*

Failures on shapes with excessive boundary complexity†



*Reversible nets of polyhedral. Akiyama et al.. In Japanese Conference on Discrete and Computational Geometry and Graphs. 2015.

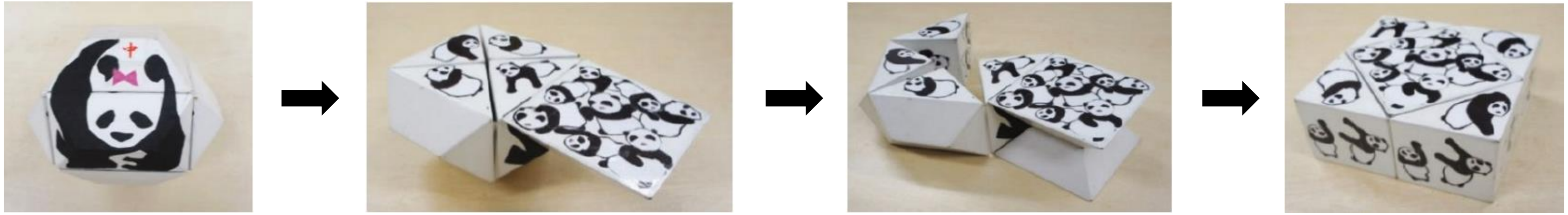


†Treks into Intuitive Geometry. Jin Akiyama and Kiyoko Matsunaga. 2015. Springer.

Future Work

More difficult dissection puzzles

Extension to 3D shapes*



*Treks into Intuitive Geometry. Jin Akiyama and Kiyoko Matsunaga. 2015. Springer.

Acknowledgements

- All Authors
- Prof. Richard Bartels
- Dr. Kai Yang
- Dr. Akshay Gadi Patil
- Anonymous reviewers
- Funds

NSERC CRSNG *NSFC*

Gift funds from Adobe

China Scholarship Council

Shenzhen Innovation Program

GD Science and Technology Program

SIGGRAPH ASIA 2018 TOKYO

CONFERENCE 4 – 7 December 2018
EXHIBITION 5 – 7 December 2018
Tokyo International Forum, Japan
SA2018.SIGGRAPH.ORG

Sponsored by

THANK YOU !

Q&A

Project homepage

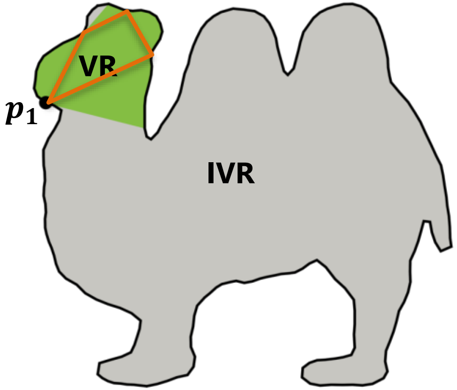
http://vcc.szu.edu.cn/research/2018/RIOT

Appendix

Candidate Trunk Pair —— Selecting Candidate Vertices

Two criteria

- Boundary Congruency
- Area Compatibility



Visible Region (VR)

Invisible Region (IVR)

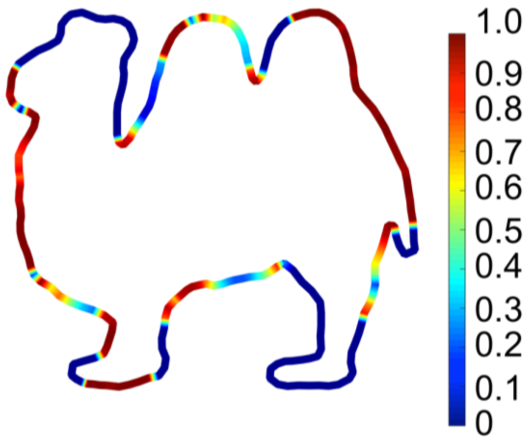
Convex trunk T

$Area(T)$ and $Area(T\text{'s exterior pieces})$ are incompatible.

Candidate Trunk Pair —— Selecting Candidate Vertices

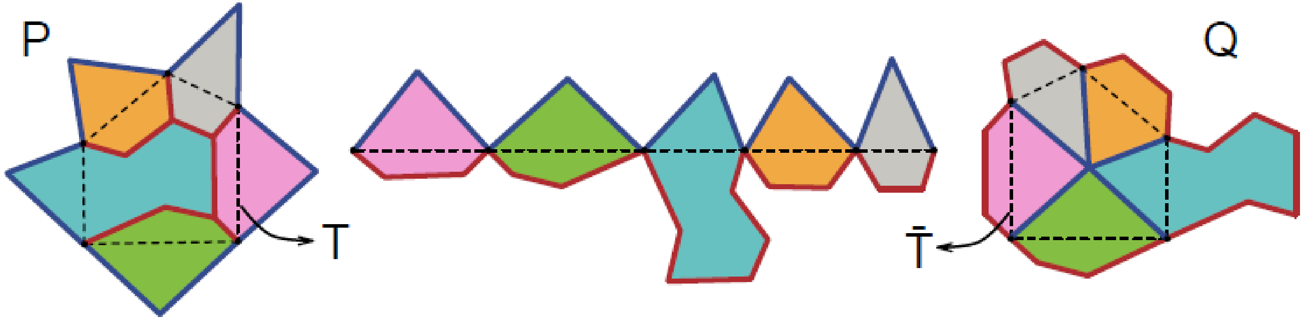
$$S_b(p) = \begin{cases} 0, & \text{if } \sum_i \text{Area}(IVR_i(p)) \geq \text{Area}(VR(p)\text{-Circle}), \\ 0, & \text{if } L(IVR_i(p)) \geq L/2, \\ 1, & \text{otherwise.} \end{cases}$$

$$S_c(p) = \begin{cases} 0, & \text{if } L(C_l^p) + L(C_r^p) \leq 0.03L, \\ exp\left(-\frac{d_c^2(C_l^p, C_r^p)}{2\sigma_c^2}\right), & \text{otherwise,} \end{cases}$$



Candidate Trunk Pair —— Generating Candidate Trunks

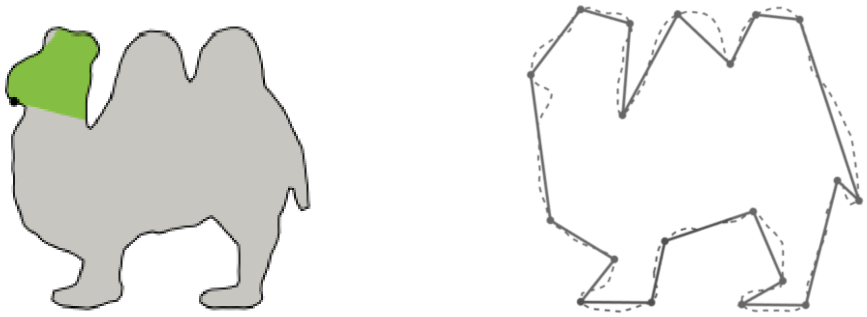
The upper bound $\boldsymbol{K}$ of trunk edges



$$K = \#|convex\ points|$$

Candidate Trunk Pair —— Generating Candidate Trunks

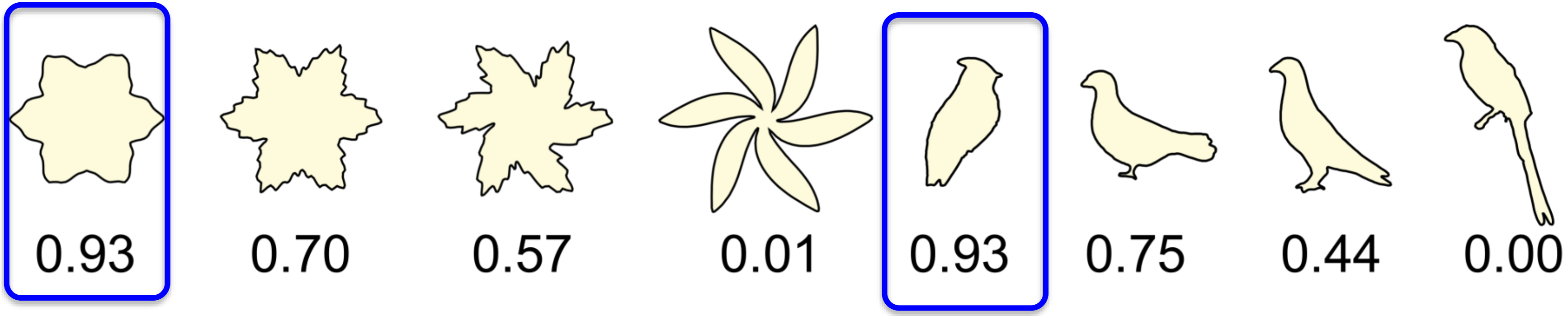
The upper bound $\boldsymbol{K}$ of trunk edges



$$\boldsymbol{K = \#|convex\ points\ of\ simplified\ shape|}$$

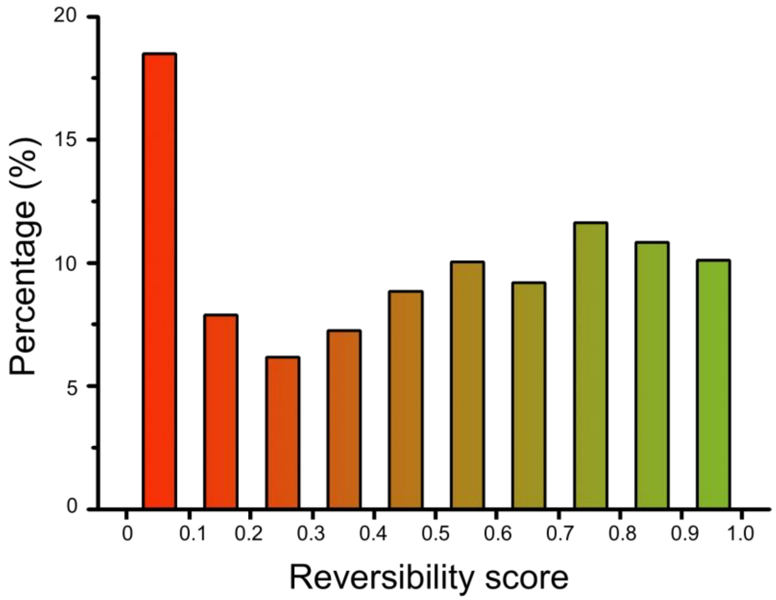
Sponsored by

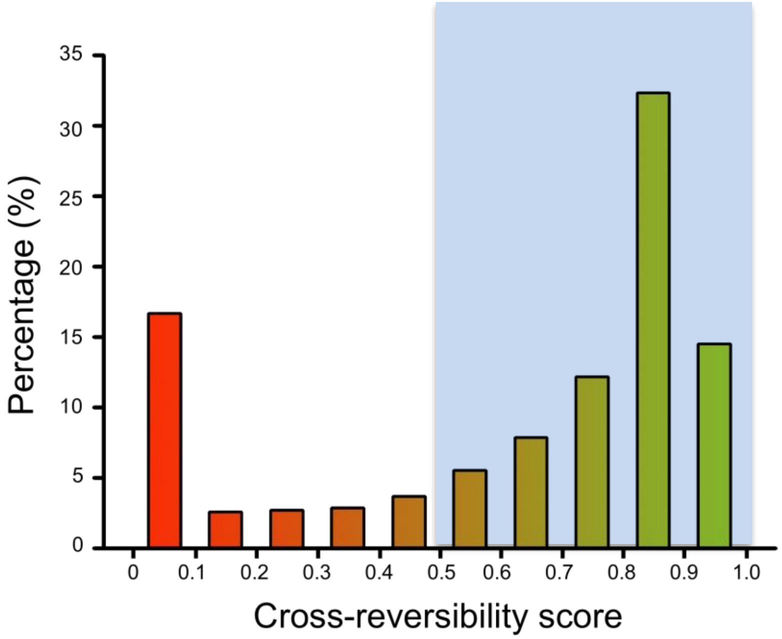
Quick Reversibility Scores (QRS)



Shapes from two classes and their QRS

Distribution of QRS and QCRS





Sponsored by