# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

Zhongshi Jiang, New York University
Scott Schaefer, Texas A&M University
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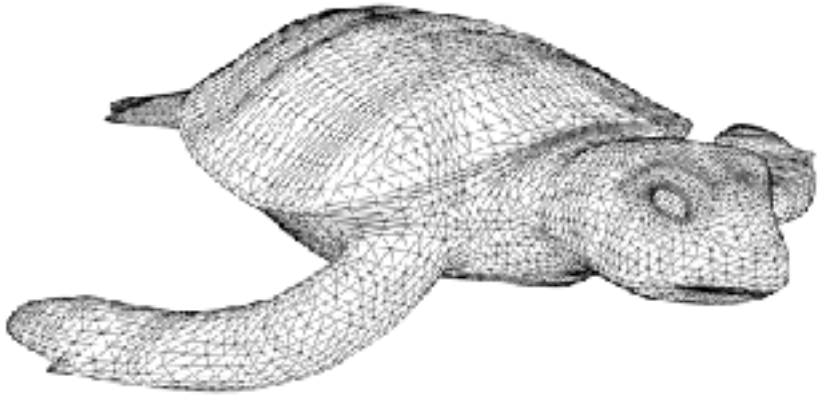
# Maps



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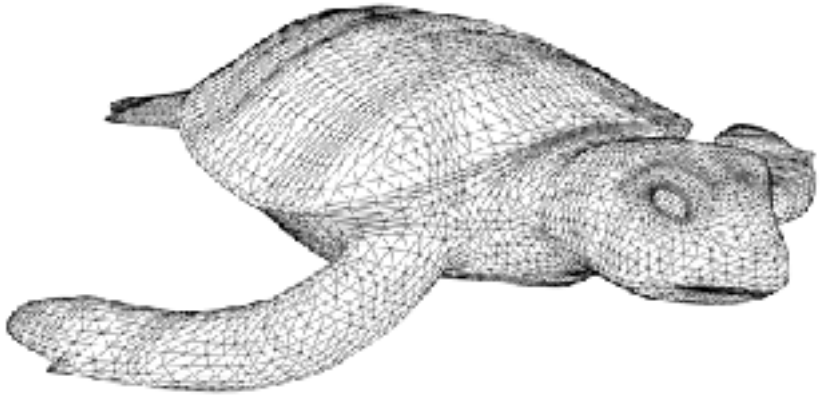

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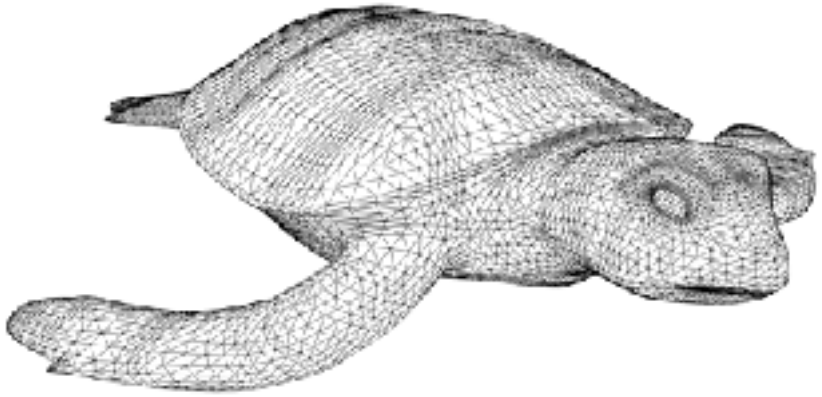

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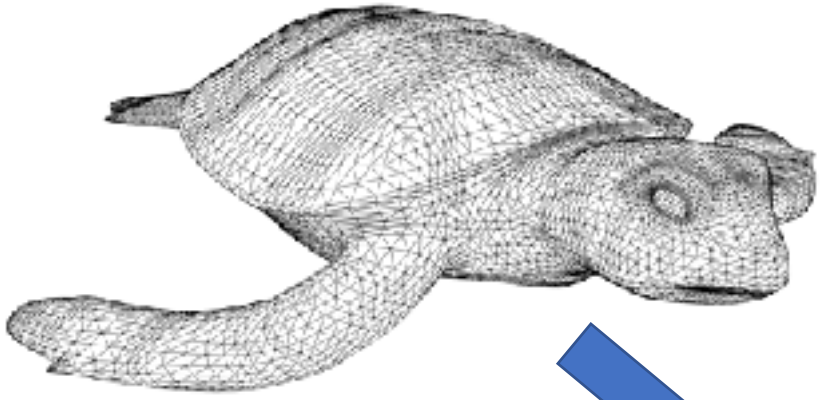

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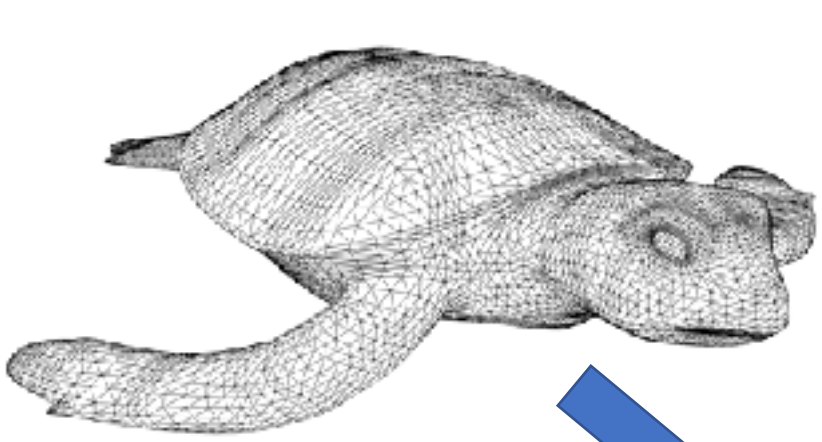

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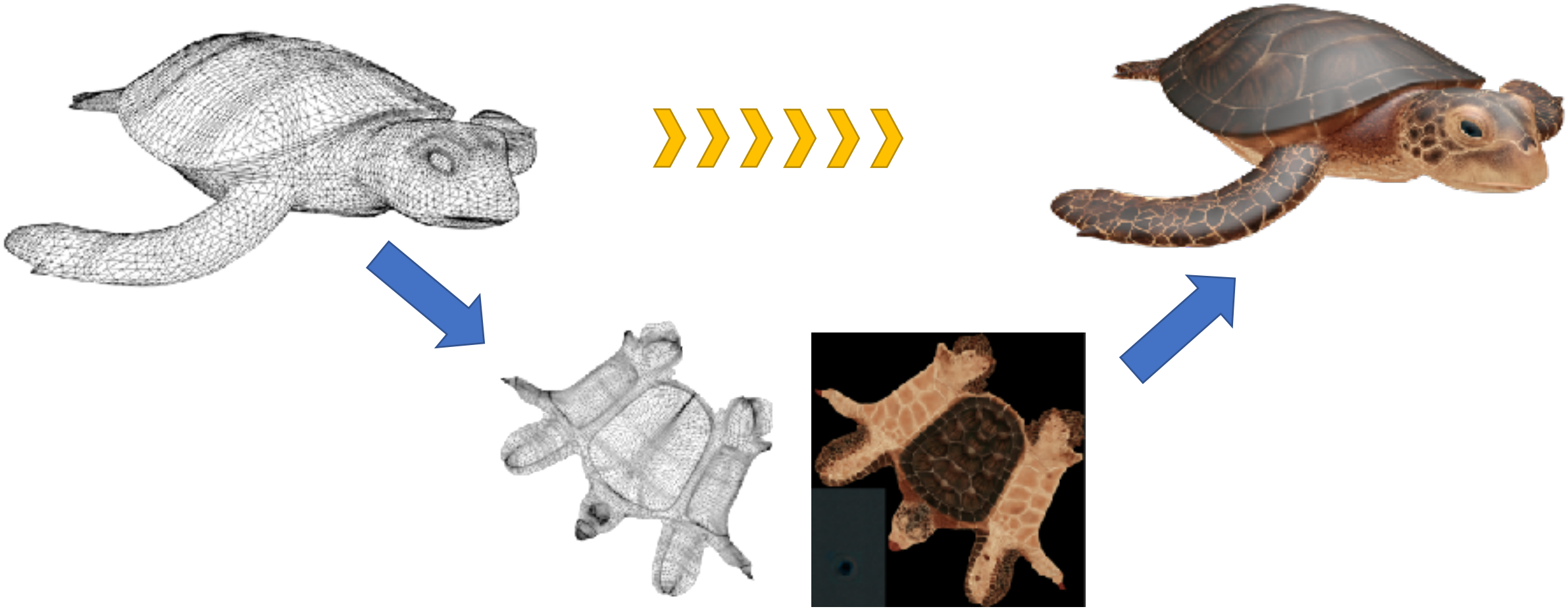

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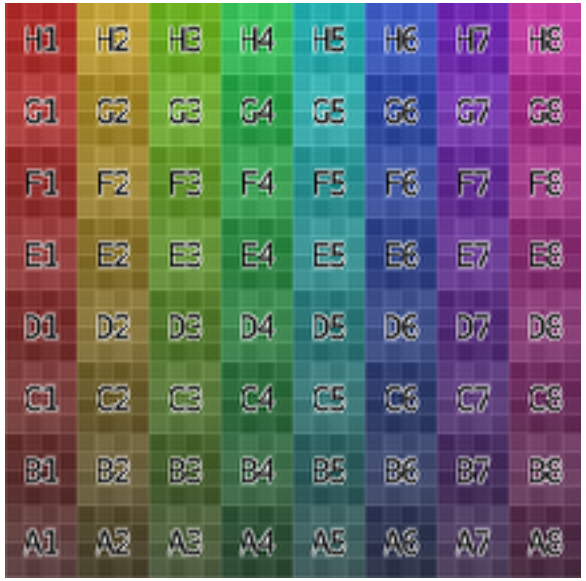
H1 H2 H3 H4 H5 H6 H7 H8
G1 G2 G3 G4 G5 G6 G7 G8
F1 F2 F3 F4 F5 F6 F7 F8
E1 E2 E3 E4 E5 E6 E7 E8
D1 D2 D3 D4 D5 D6 D7 D8
C1 C2 C3 C4 C5 C6 C7 C8
B1 B2 B3 B4 B5 B6 B7 B8
A1 A2 A3 A4 A5 A6 A7 A8

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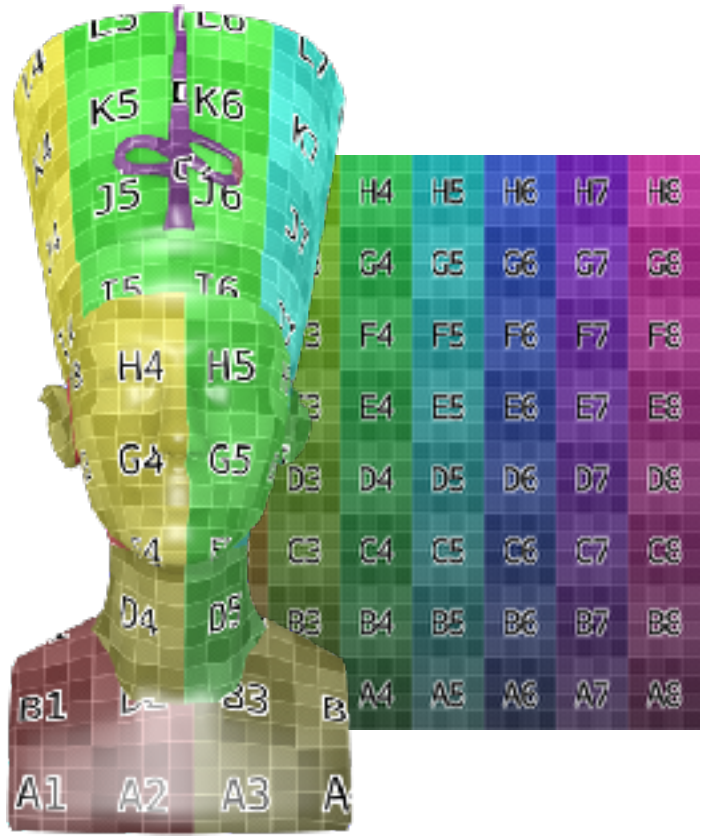

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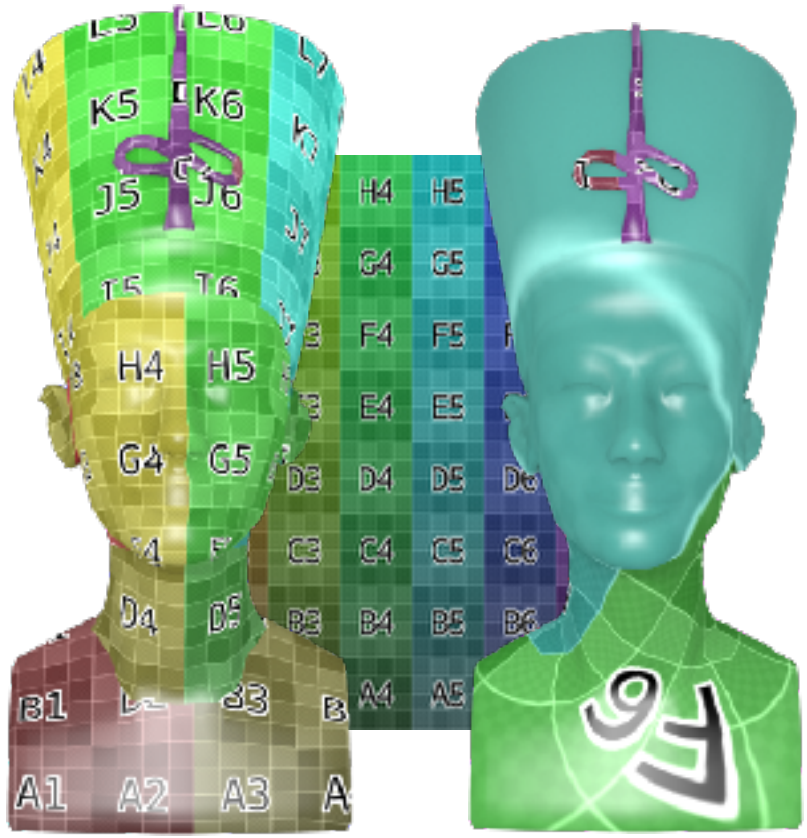

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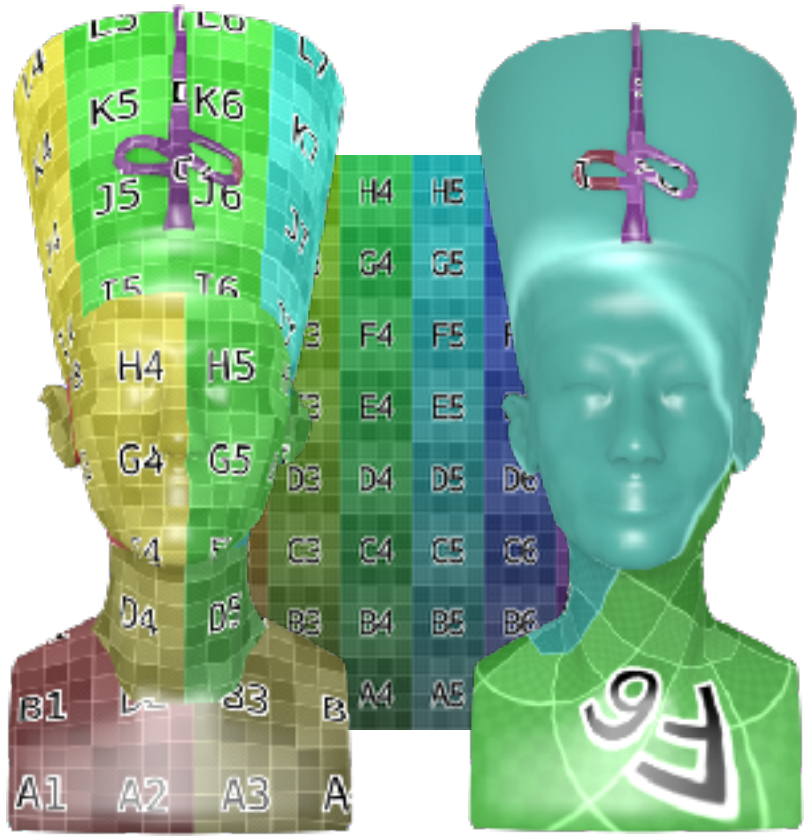
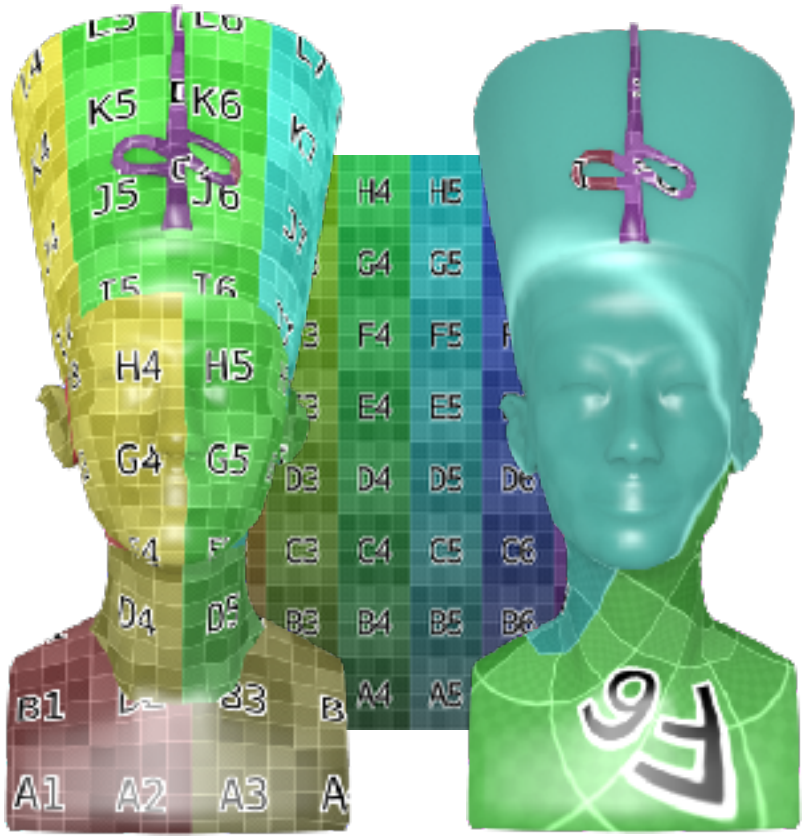



Distortion

# Better Maps



Distortion

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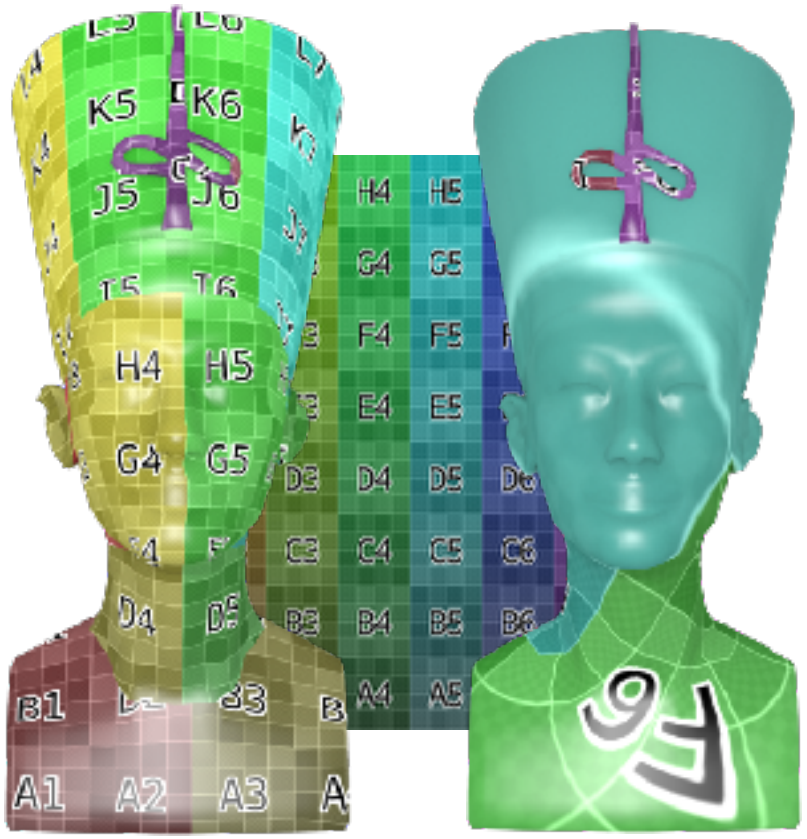


Distortion

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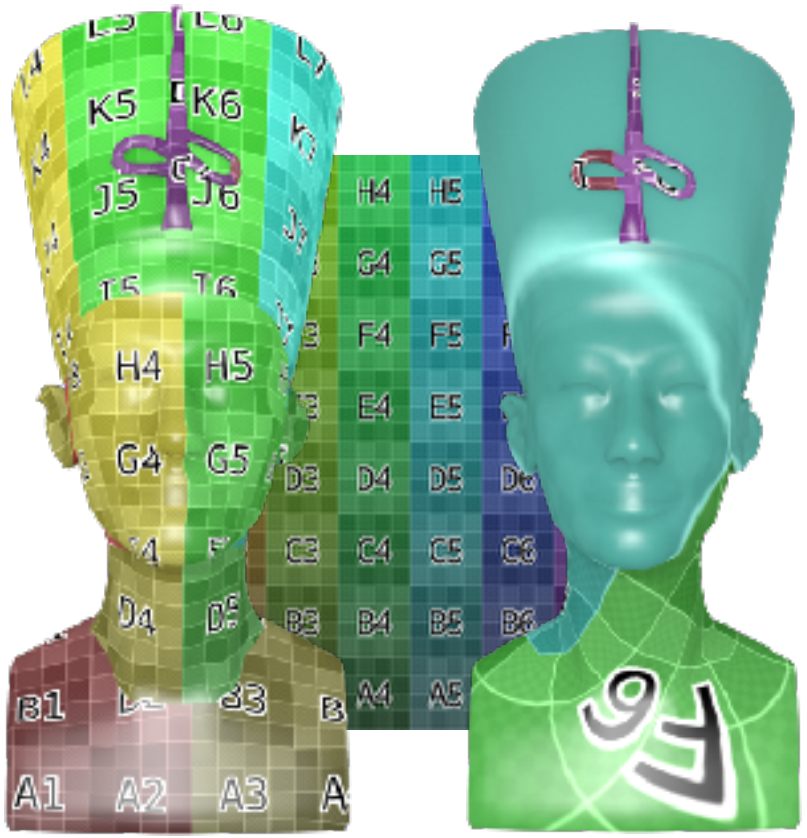
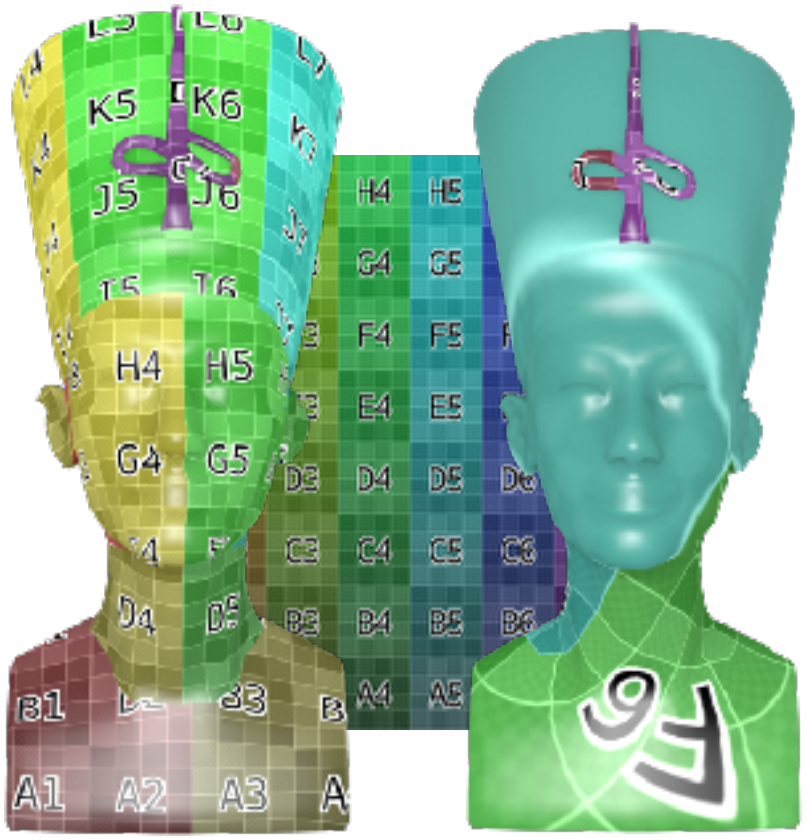



Distortion

Overlap

Overlapping

# Better Maps

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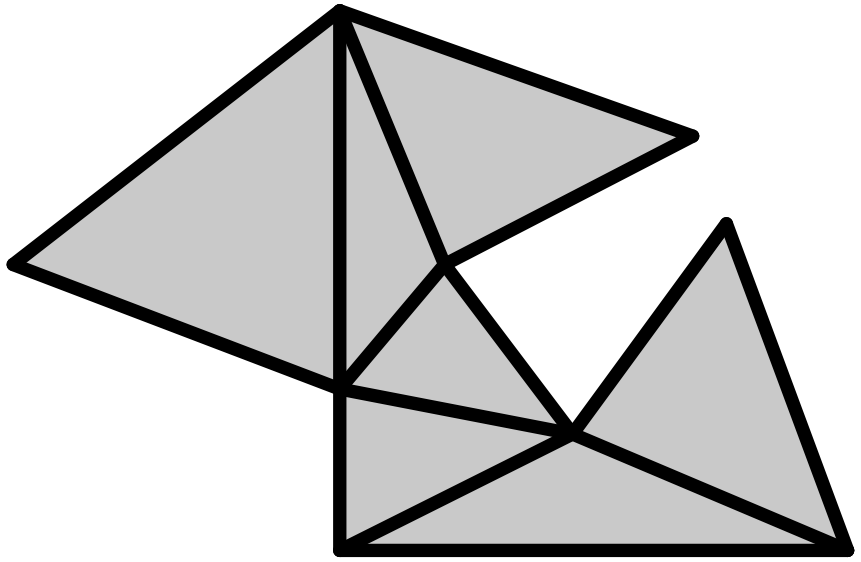

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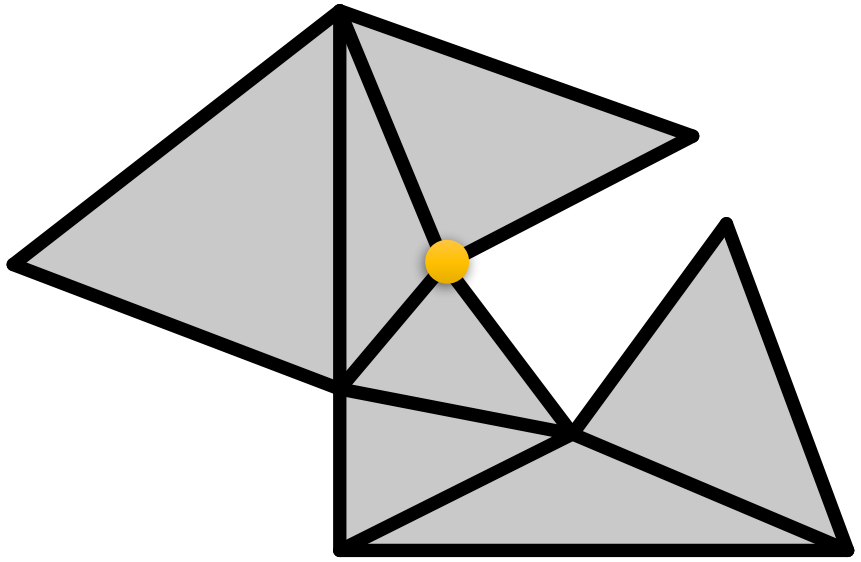

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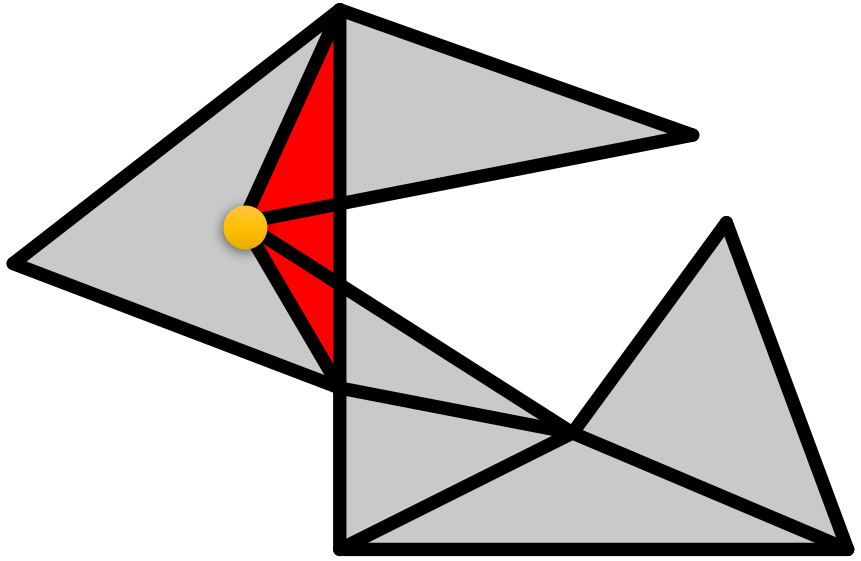

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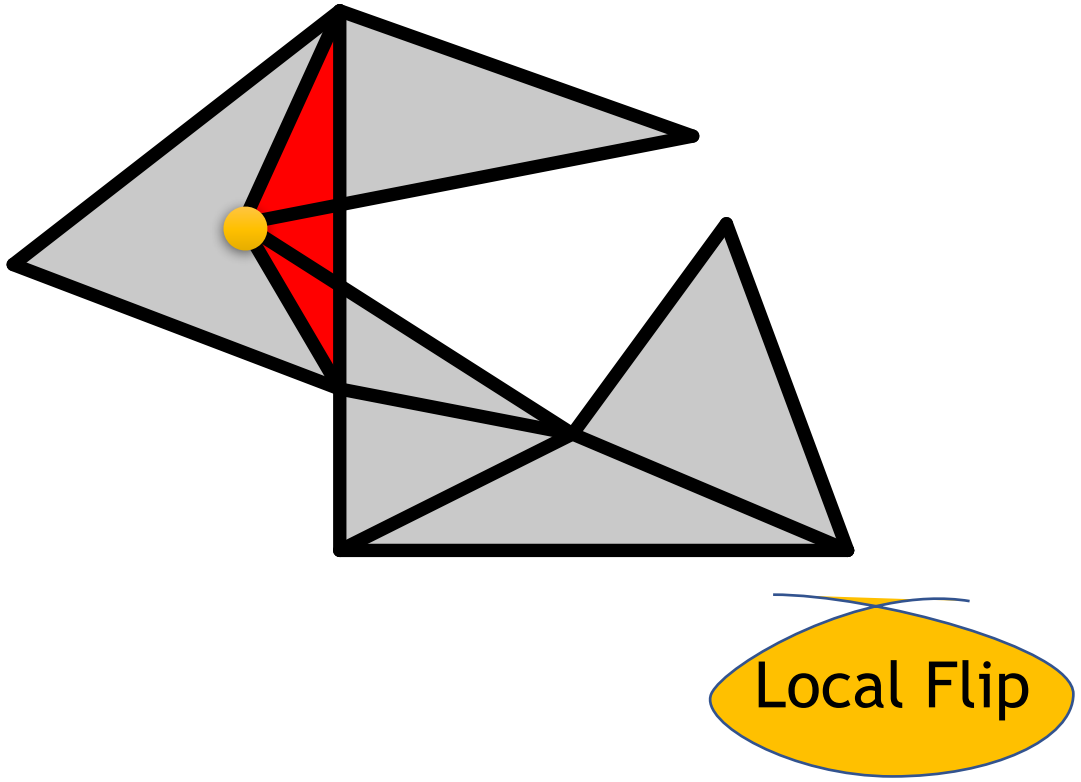

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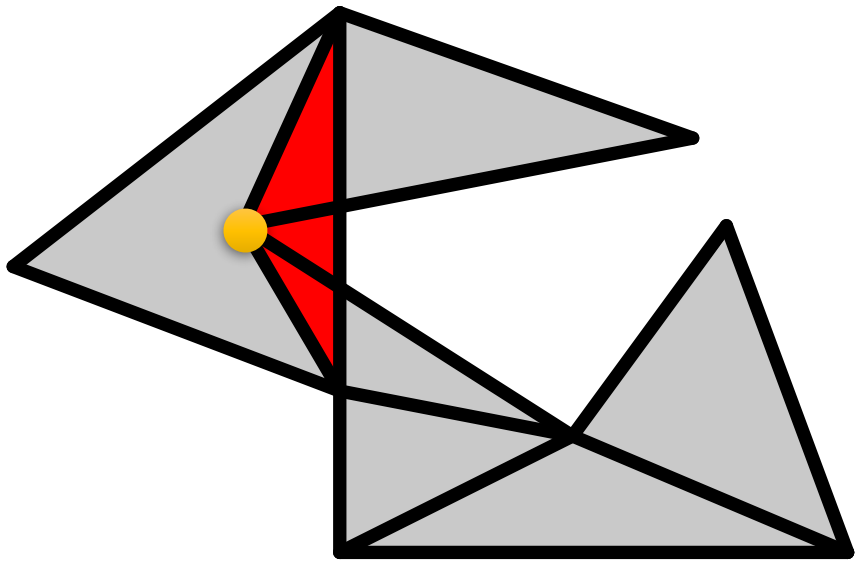
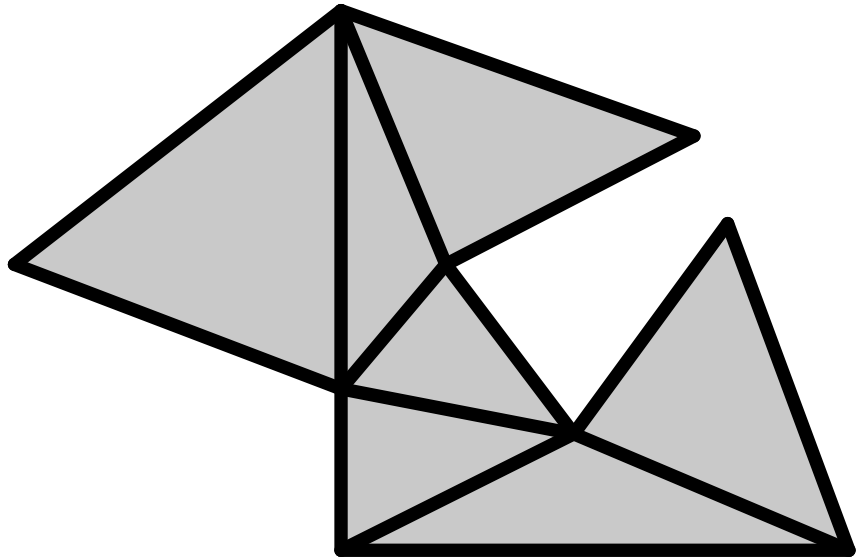
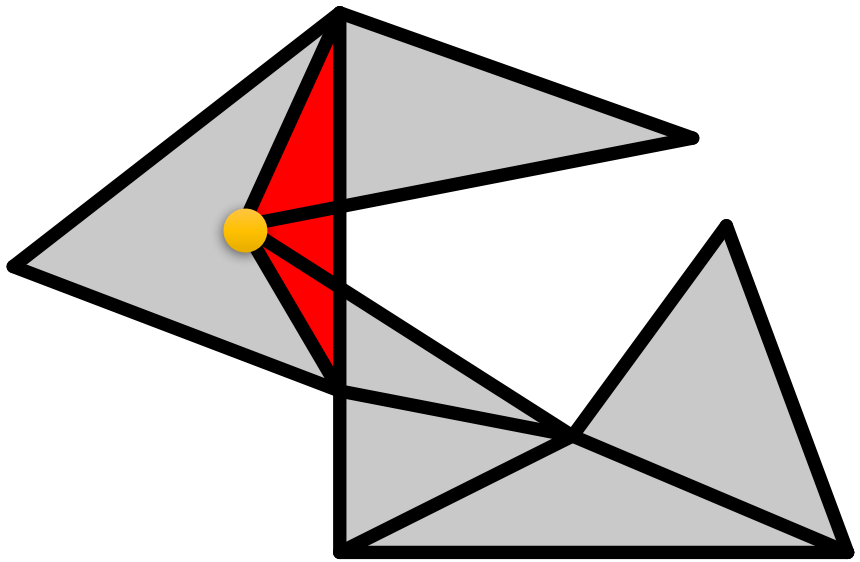


Local Flip



# Overlapping



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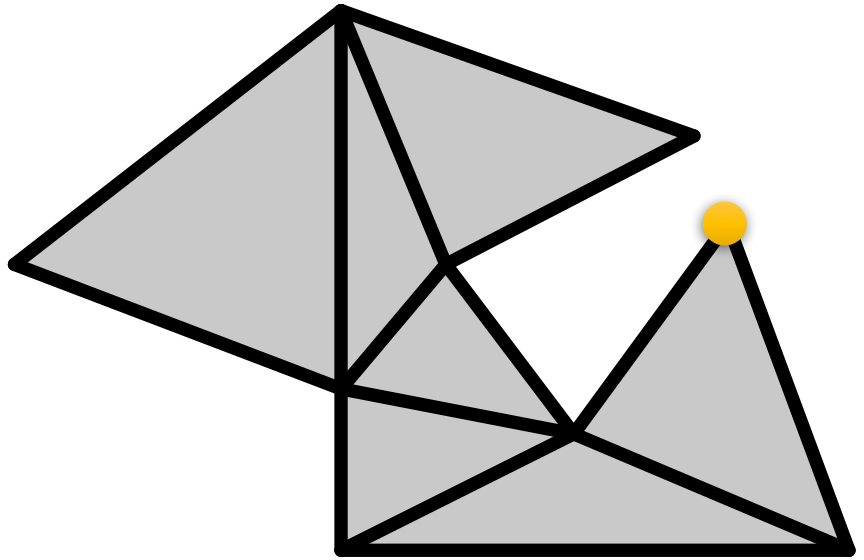

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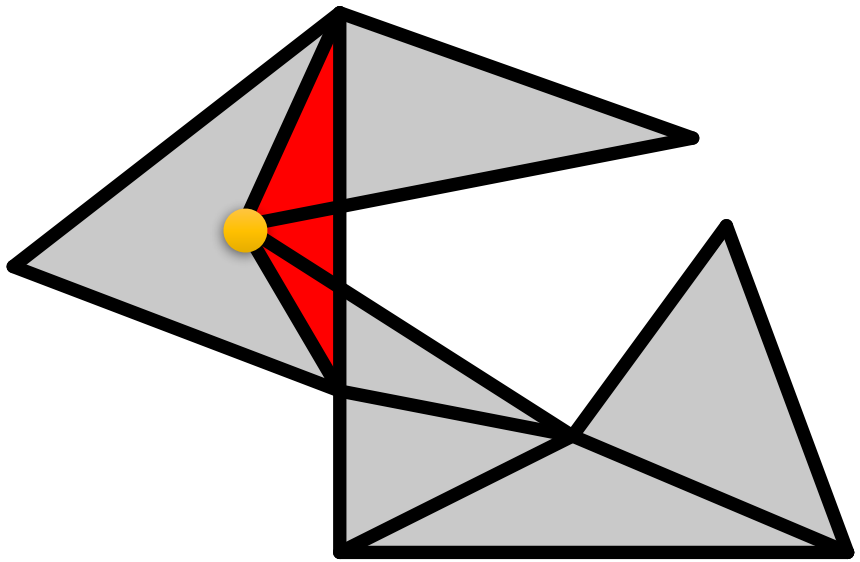

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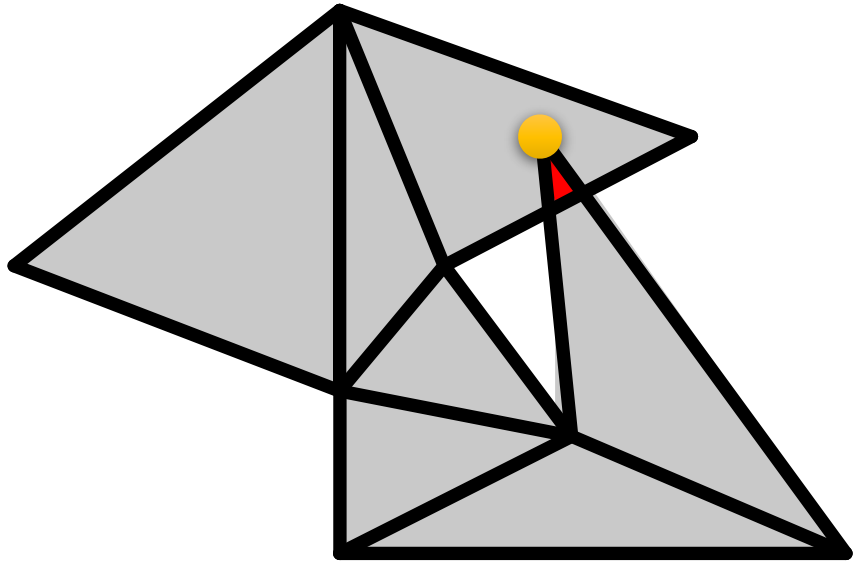

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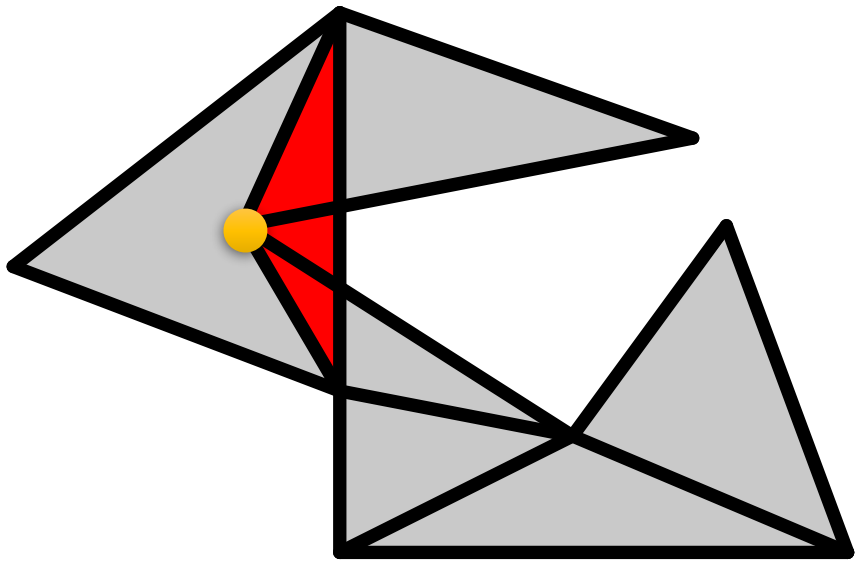
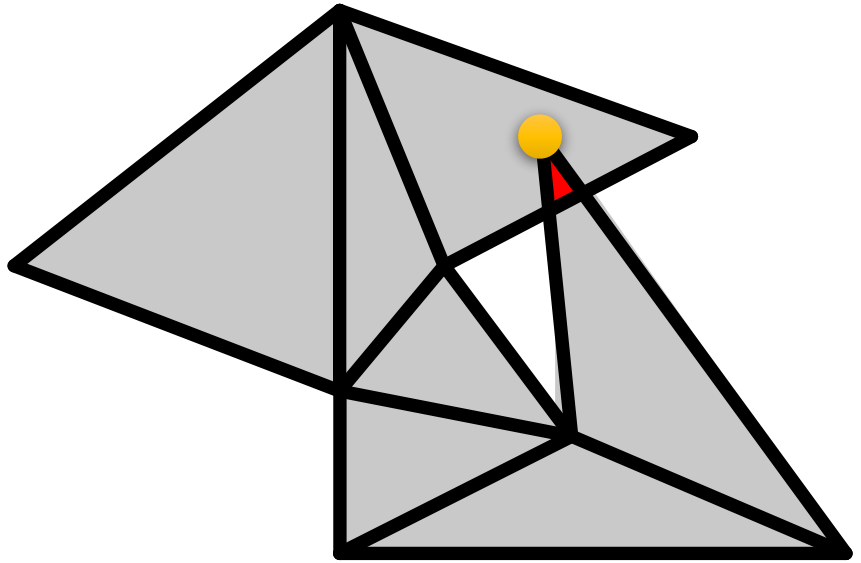
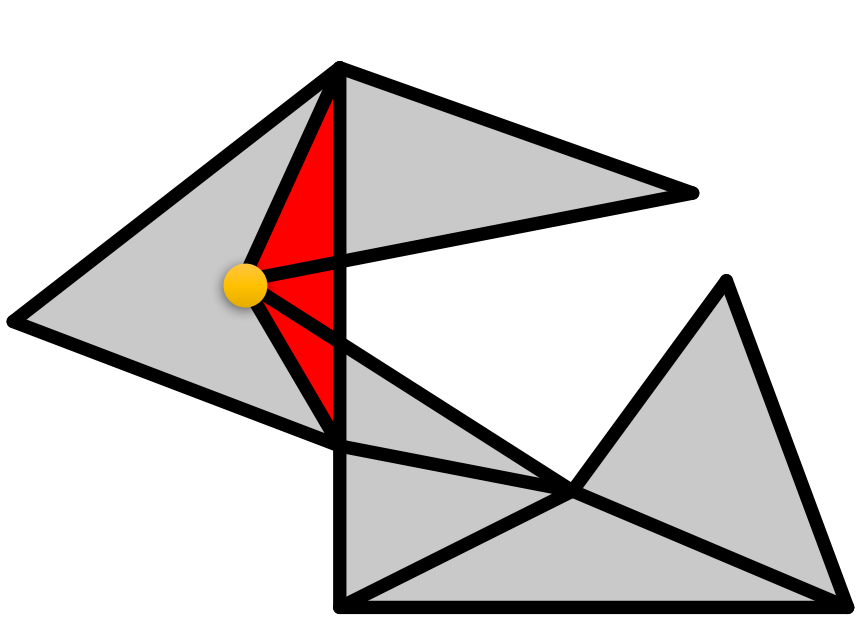


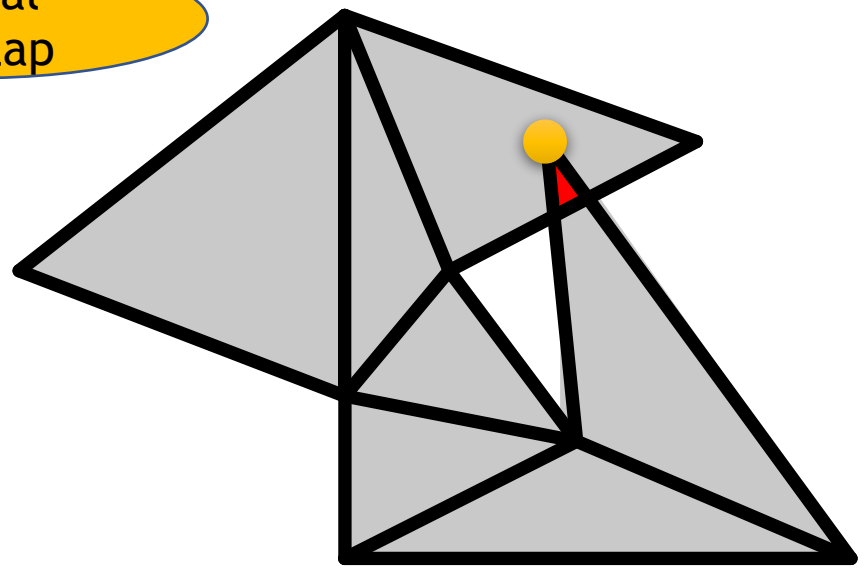
Local Flip



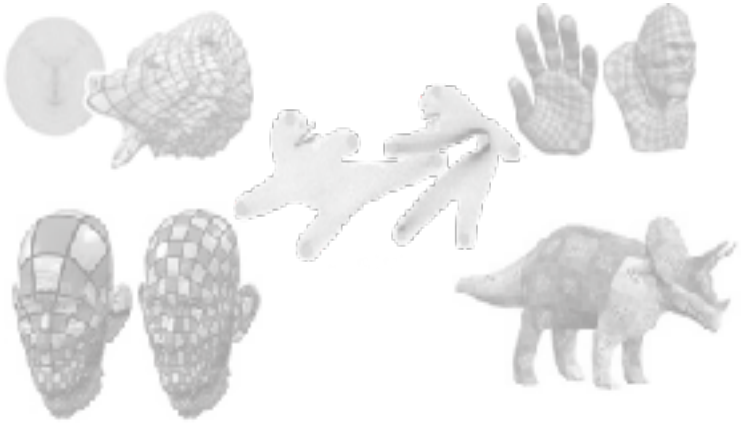
# Overlapping



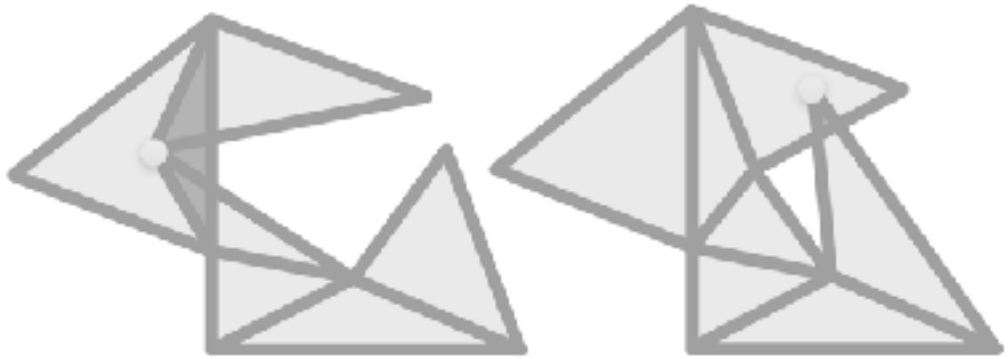
Global Overlap



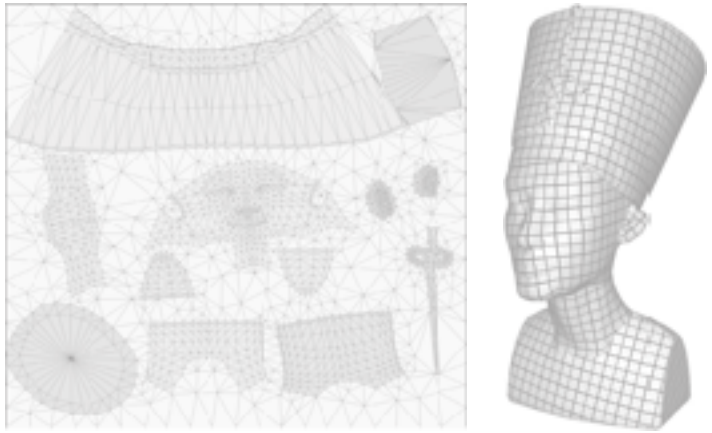
Local Flip



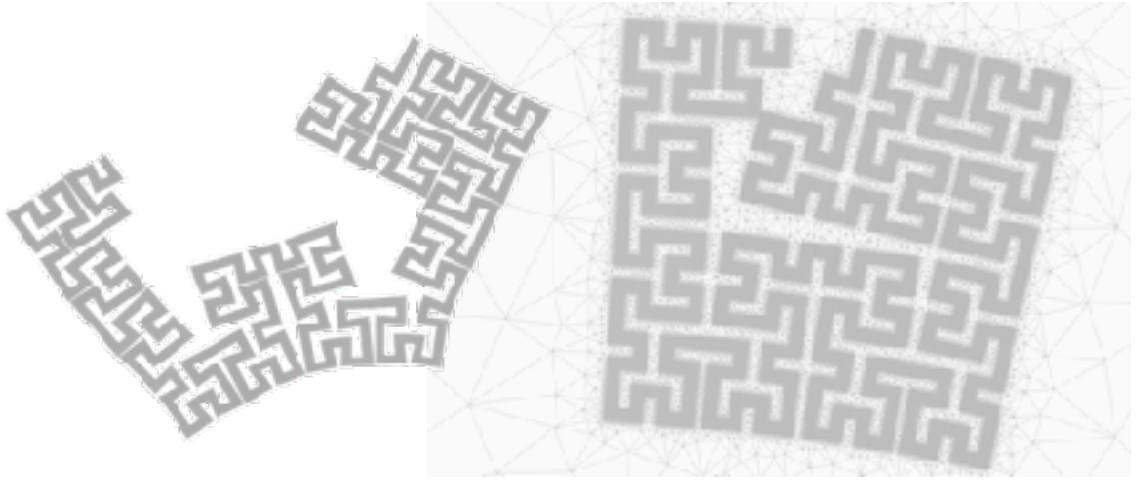
Locally Injective Maps



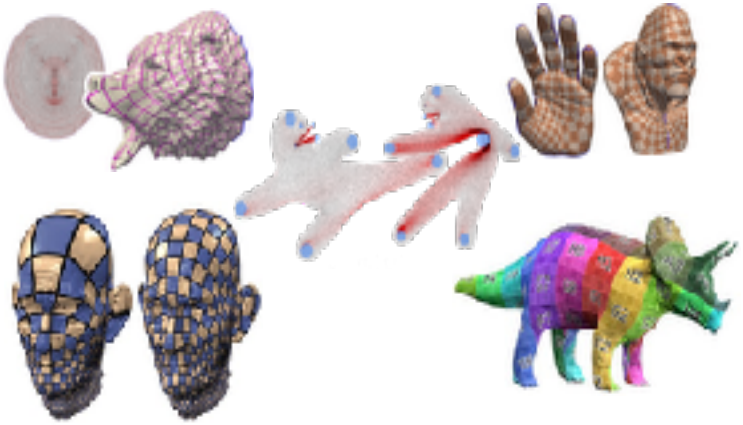
Bijective Maps



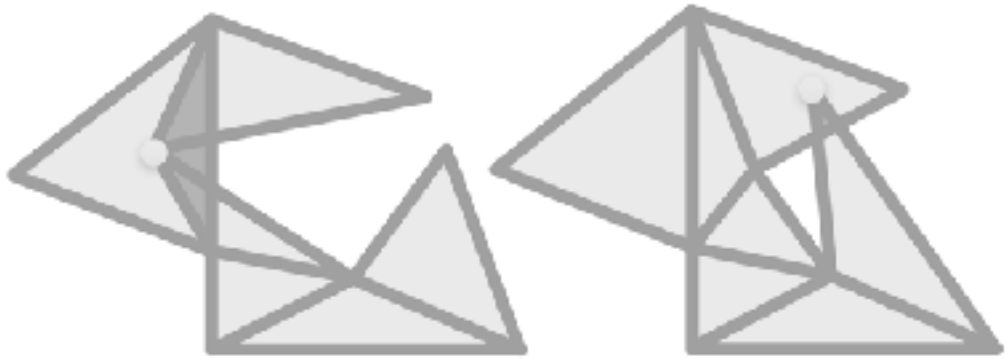
Results



Comparisons



Locally Injective Maps



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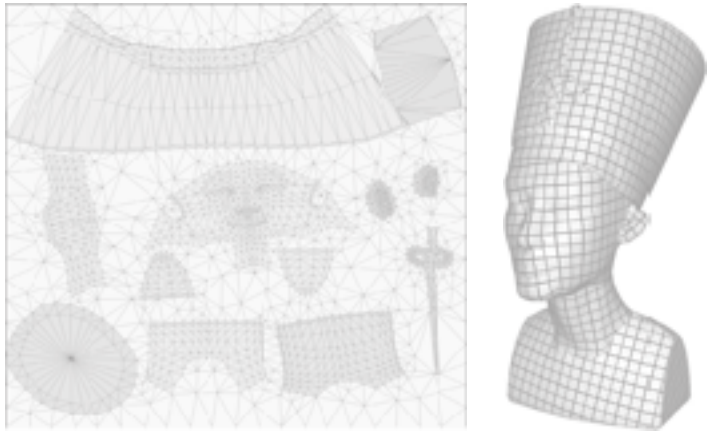

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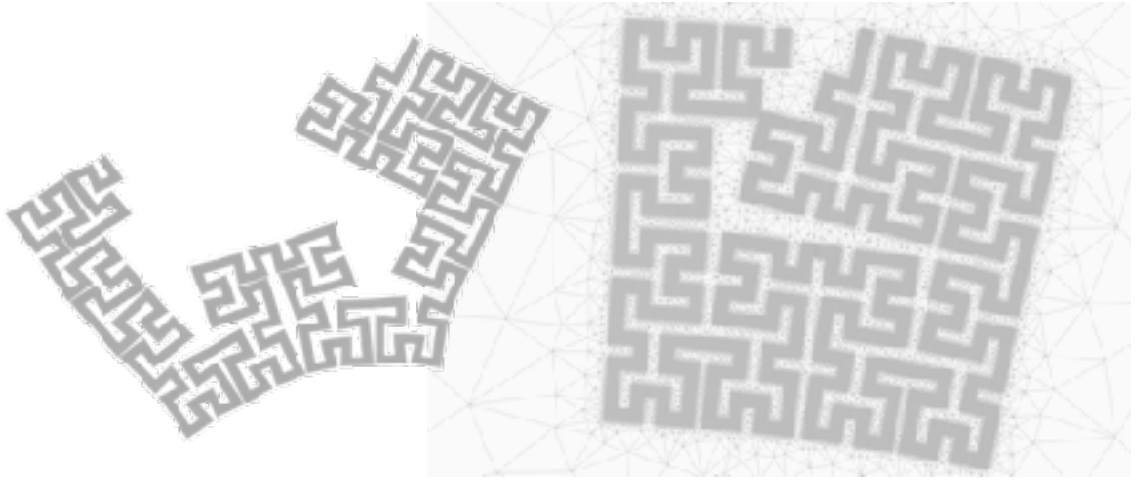
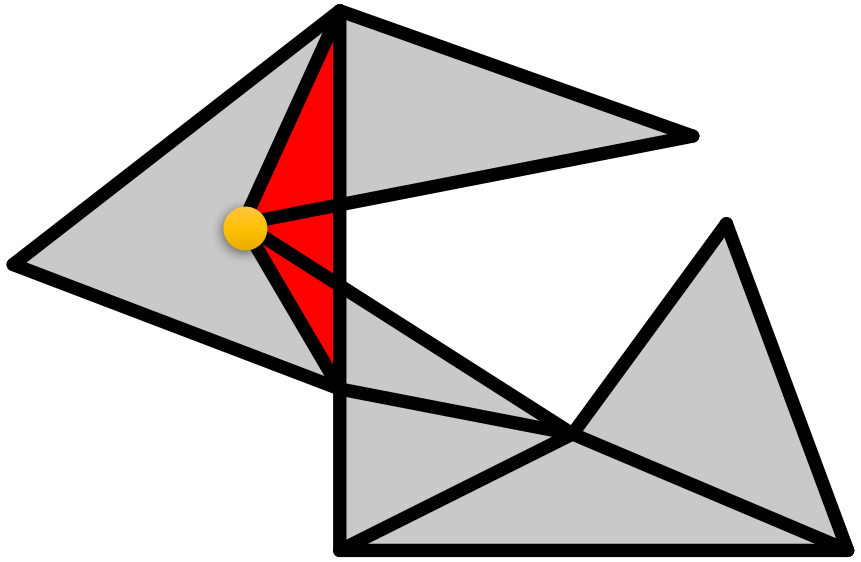


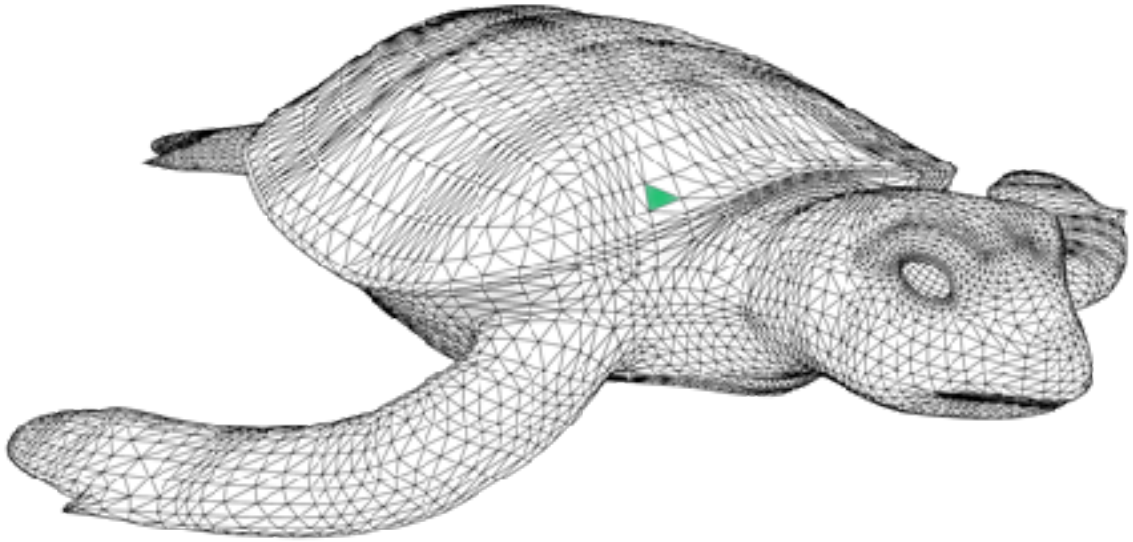
Comparisons

# Bijective Condition: Local



$$det(\Delta) > 0 \quad \forall \Delta \in \mathcal{M}$$

# Symmetric Dirichlet Energy



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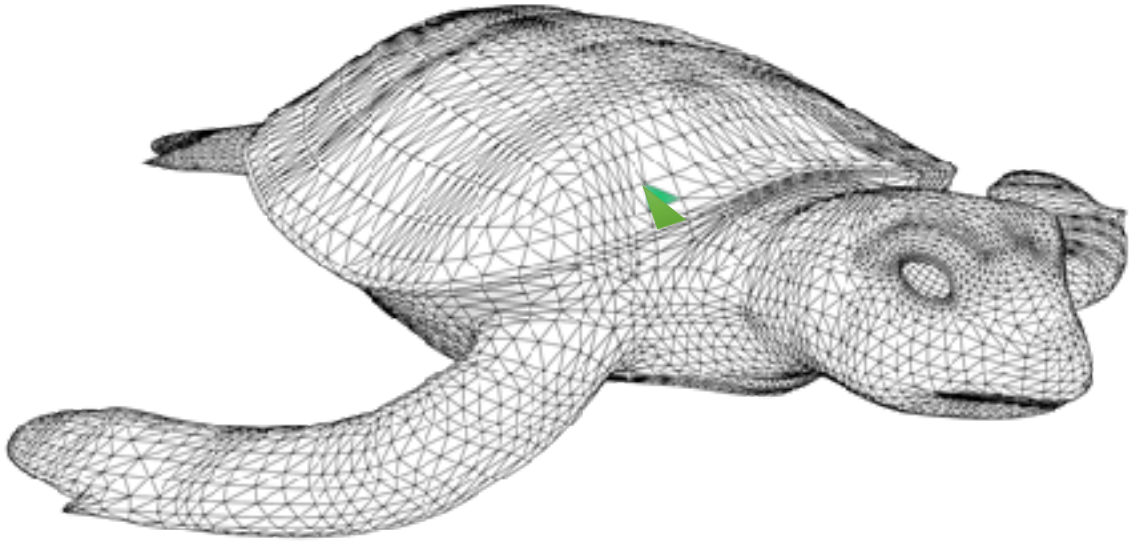

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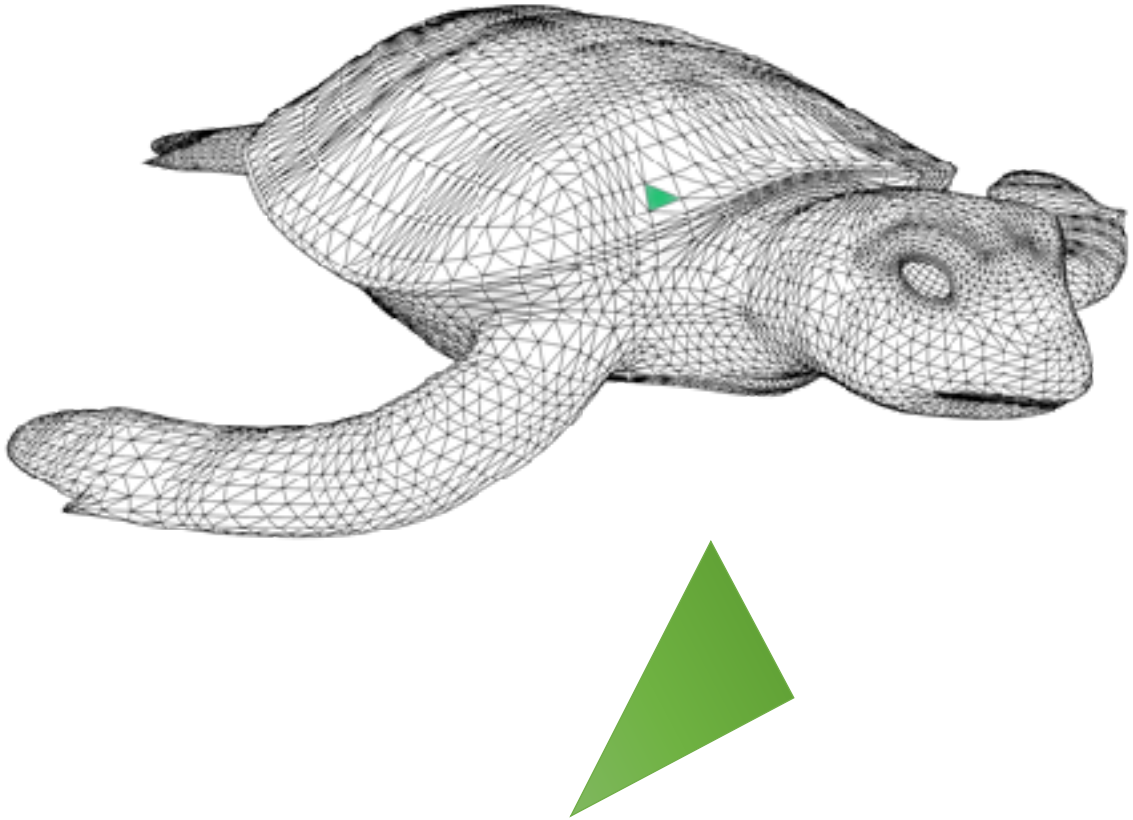

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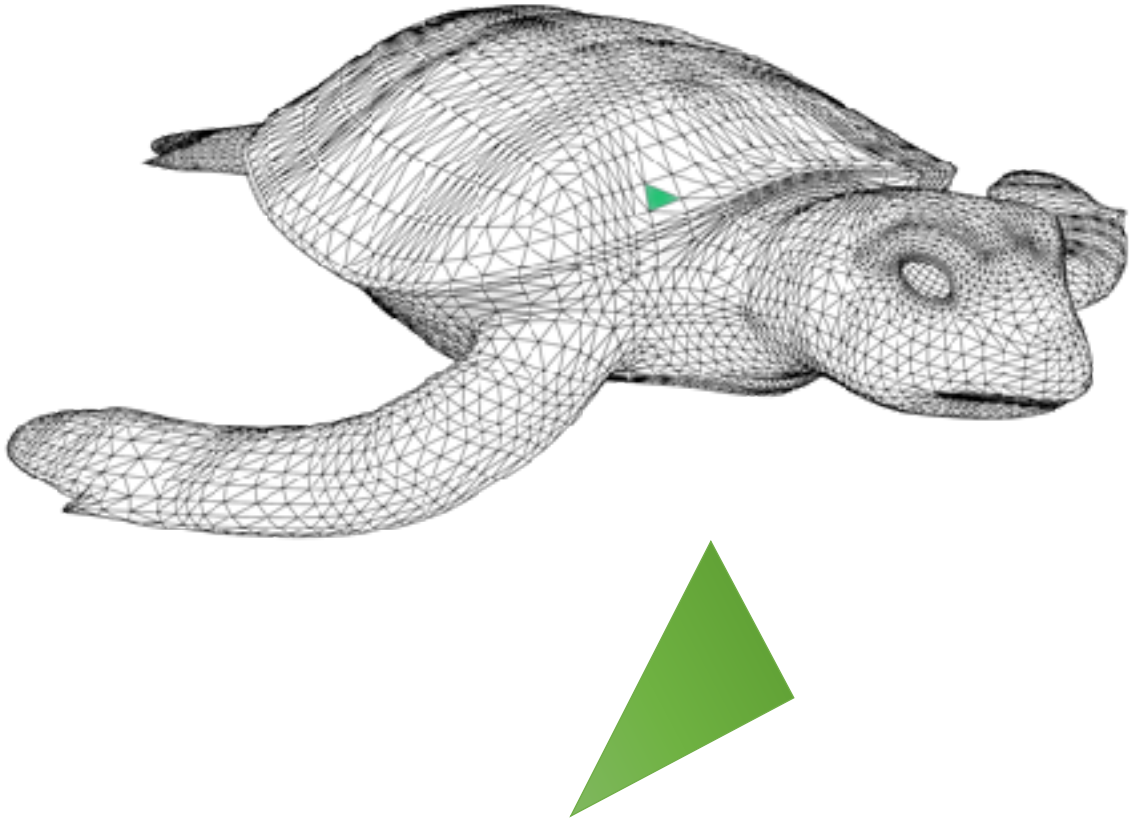

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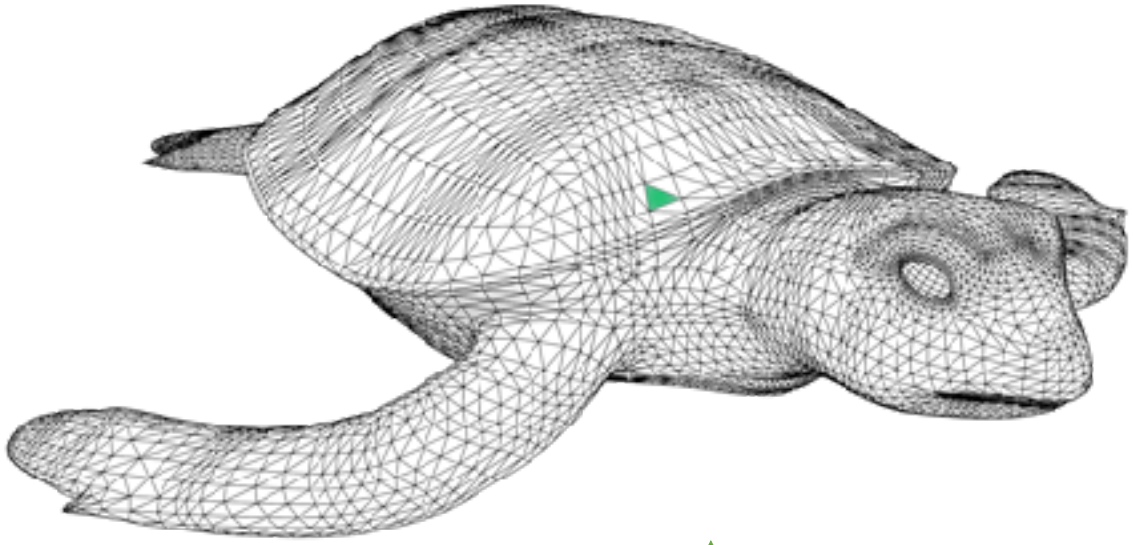

Reference
Triangle

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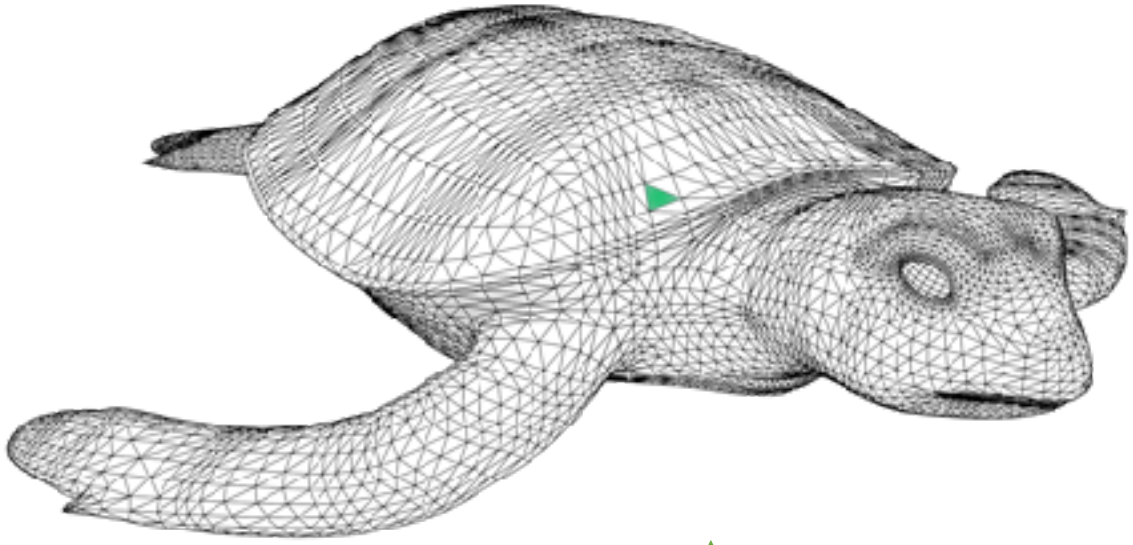

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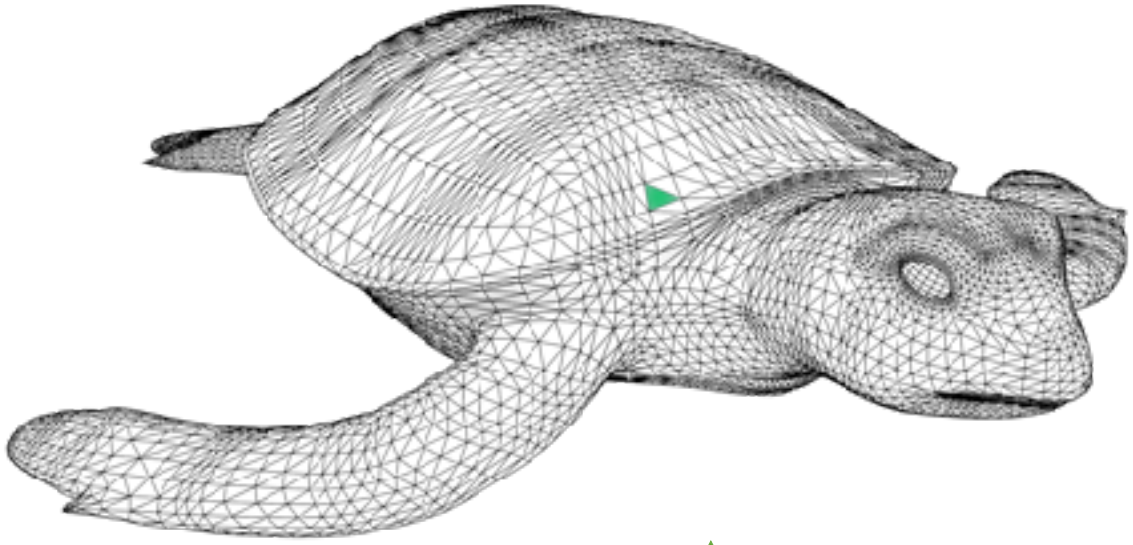

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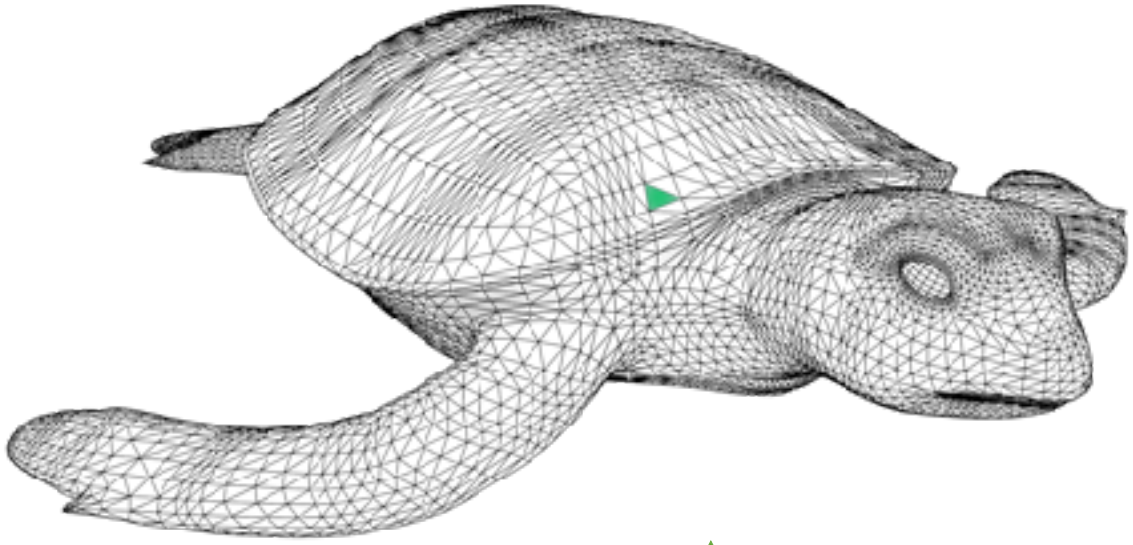

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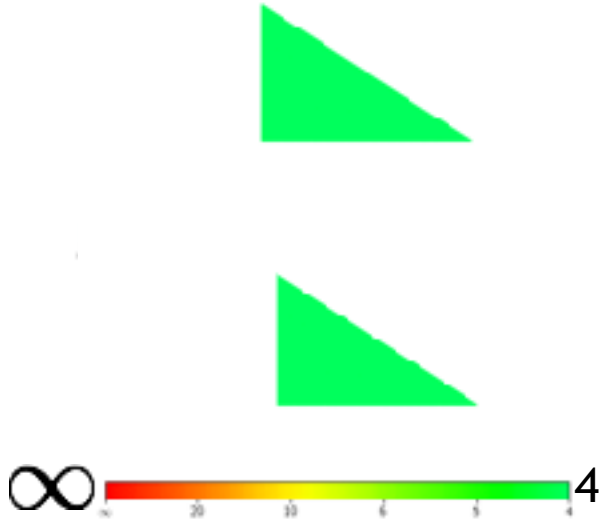

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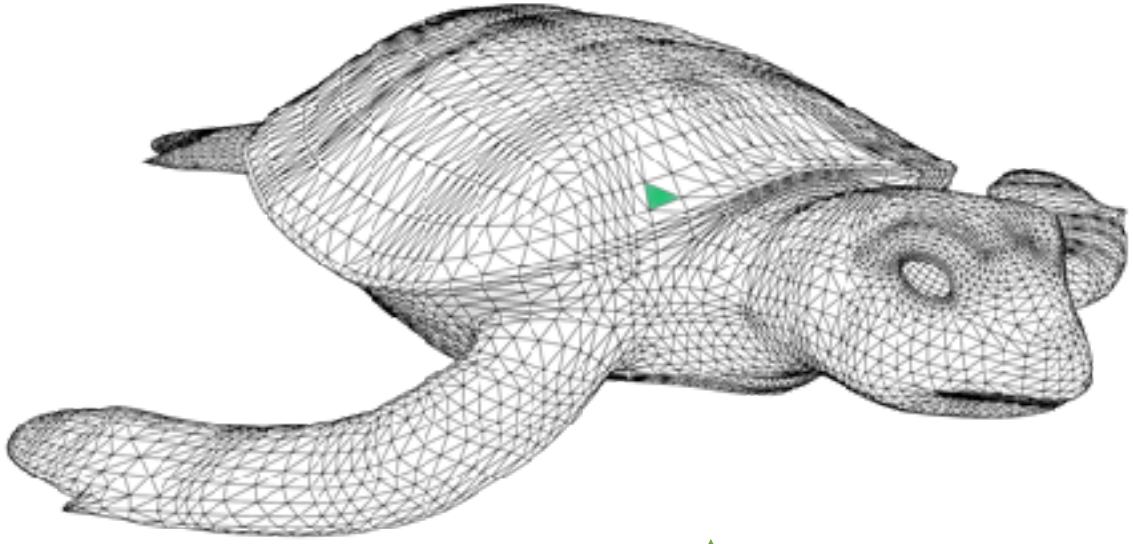
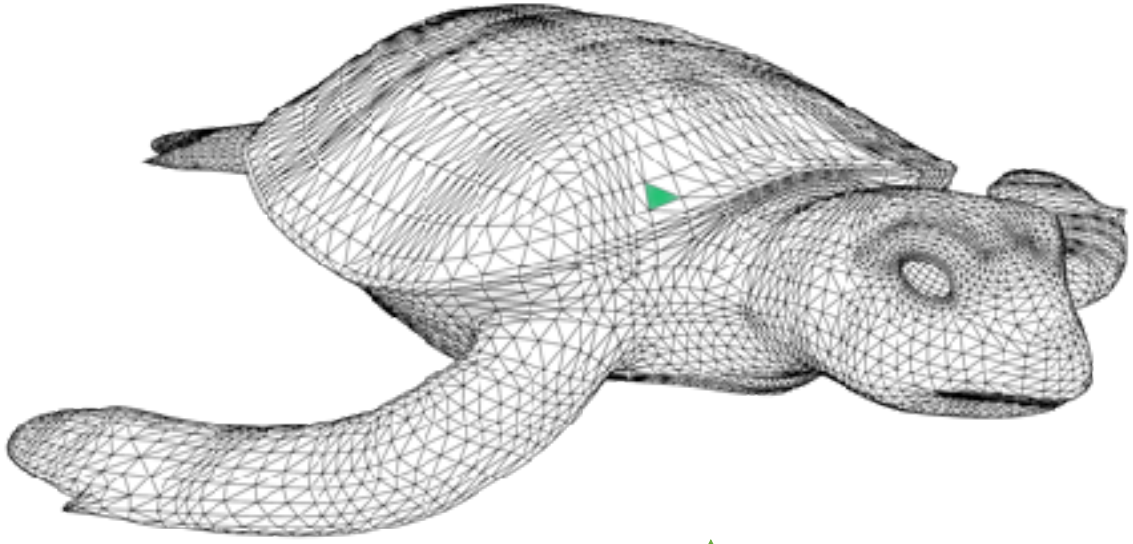


$$E = ||J||_F^2 + ||J^{-1}||_F^2$$

Reference Triangle

# Symmetric Dirichlet Energy

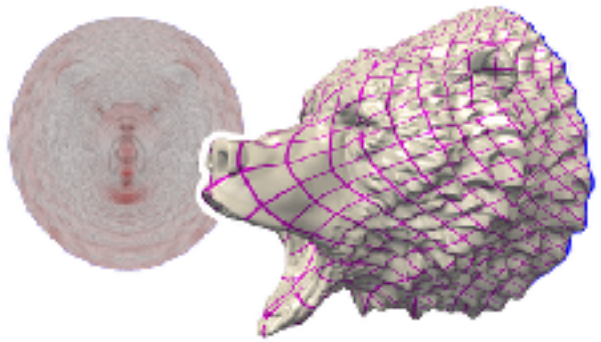


$$E = ||J||_F^2 + ||J^{-1}||_F^2$$
$$= {\sigma_1}^2 + {\sigma_2}^2 + {\sigma_1}^{-2} + {\sigma_2}^{-2}$$

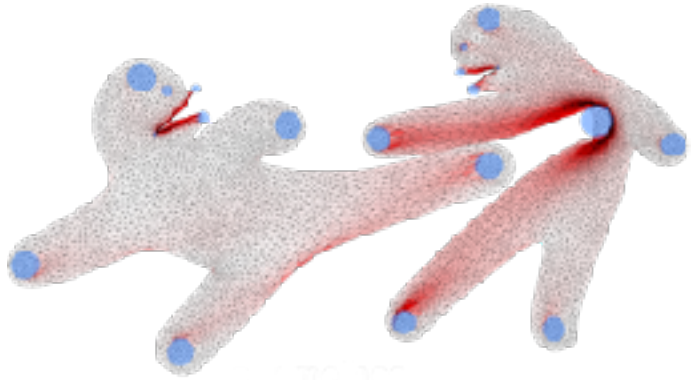
Reference Triangle

$J$

# Locally Injective Mappings



[ CM: Shtengel et al 2017]



[LIM: Schueller et al 2013]

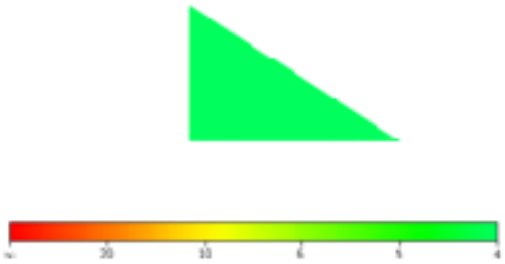
[AKVP: Claci et al 2016]

[SLIM: Rabinovich et al 2017]

[AQP: Kovalsky et al 2016]

# Scalable Locally Injective Mappings

$$\mathfrak{P}_{[X_{k-1}]}(\mathbf{x})$$



Rotation Invariance Energies

[Rabinovich et al 2017]

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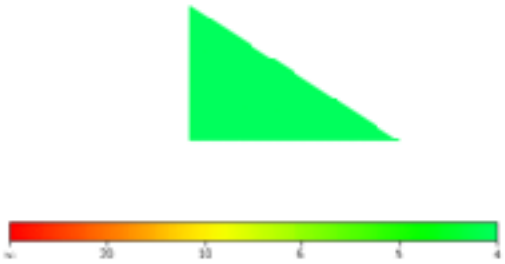
$$\mathfrak{P}_{[X_{k-1}]}(\mathbf{x})$$

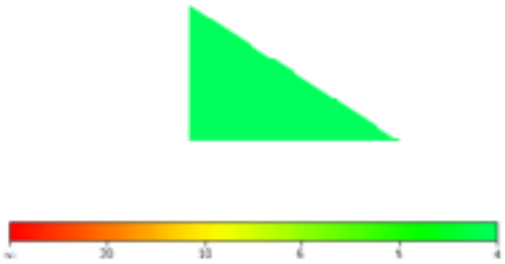


Rotation Invariance Energies

[Rabinovich et al 2017]

# Scalable Locally Injective Mappings

$$\mathfrak{P}_{[X_{k-1}]}(\mathbf{x})$$

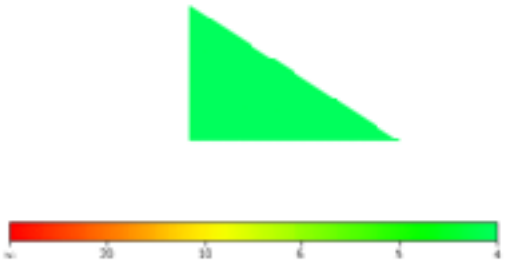


Rotation Invariance Energies

[Rabinovich et al 2017]

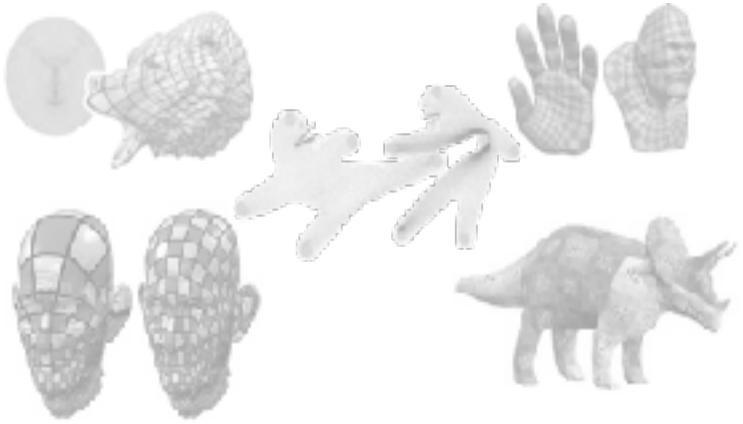
# Scalable Locally Injective Mappings

$$X_k = \text{argmin}_{\mathbf{x}} \mathfrak{P}_{[X_{k-1}]}(\mathbf{x})$$

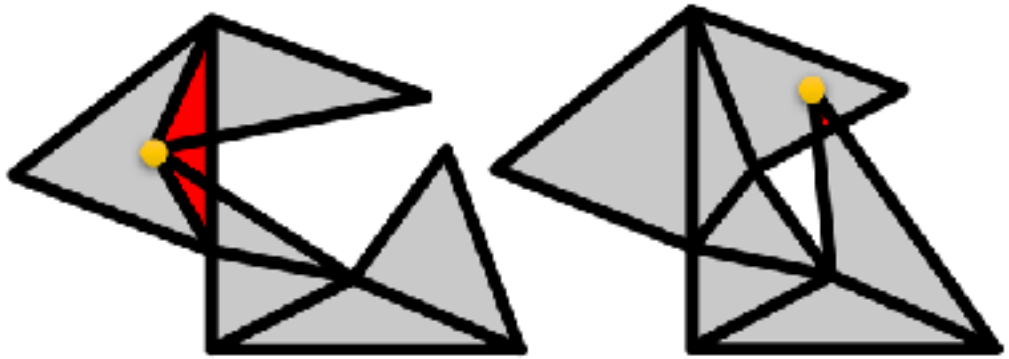


Rotation Invariance Energies

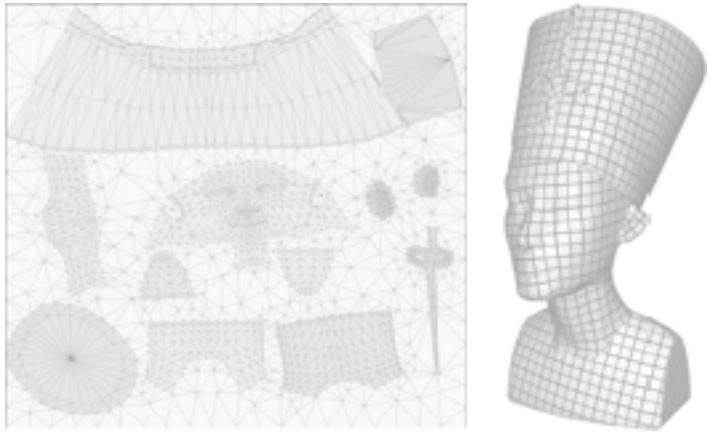
[Rabinovich et al 2017]



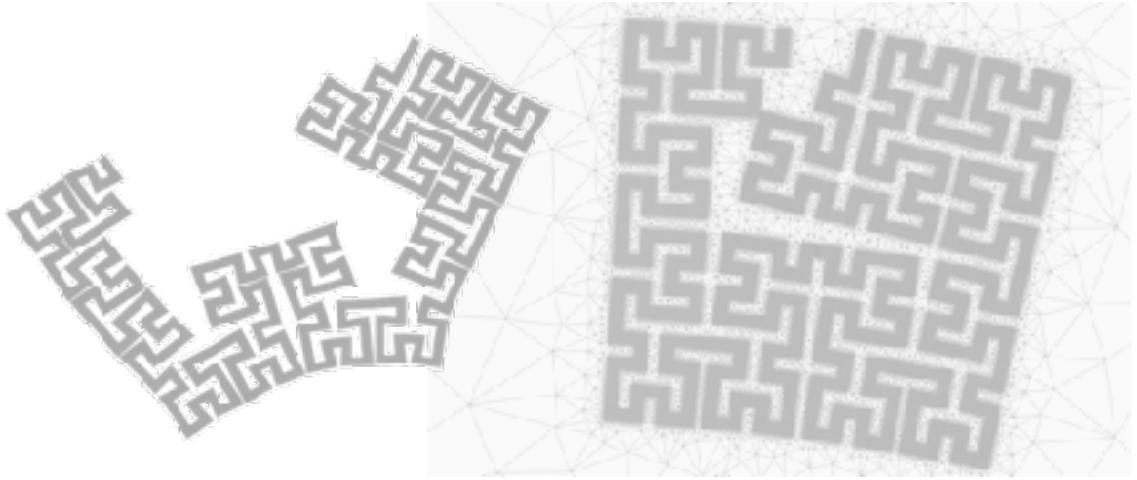
Locally Injective Maps



Bijective Maps



Results

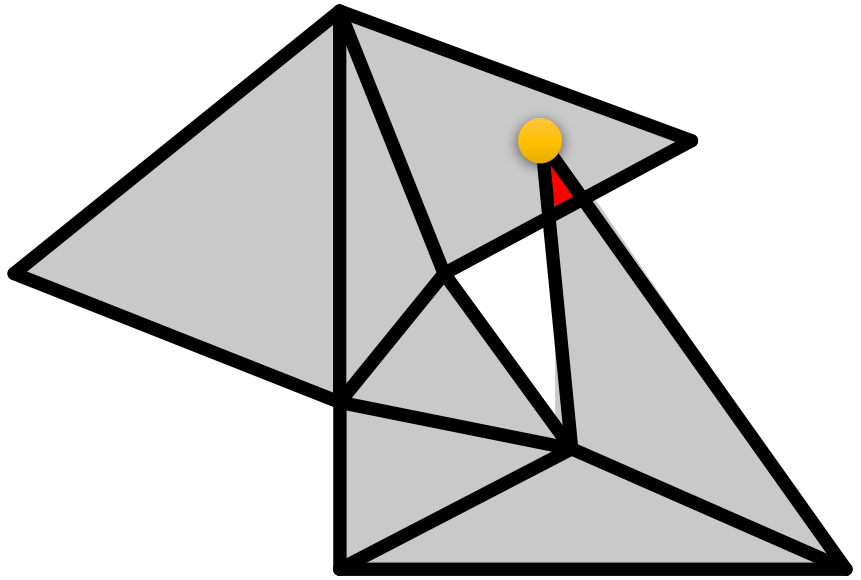


Comparisons

# Bijective Condition: Global

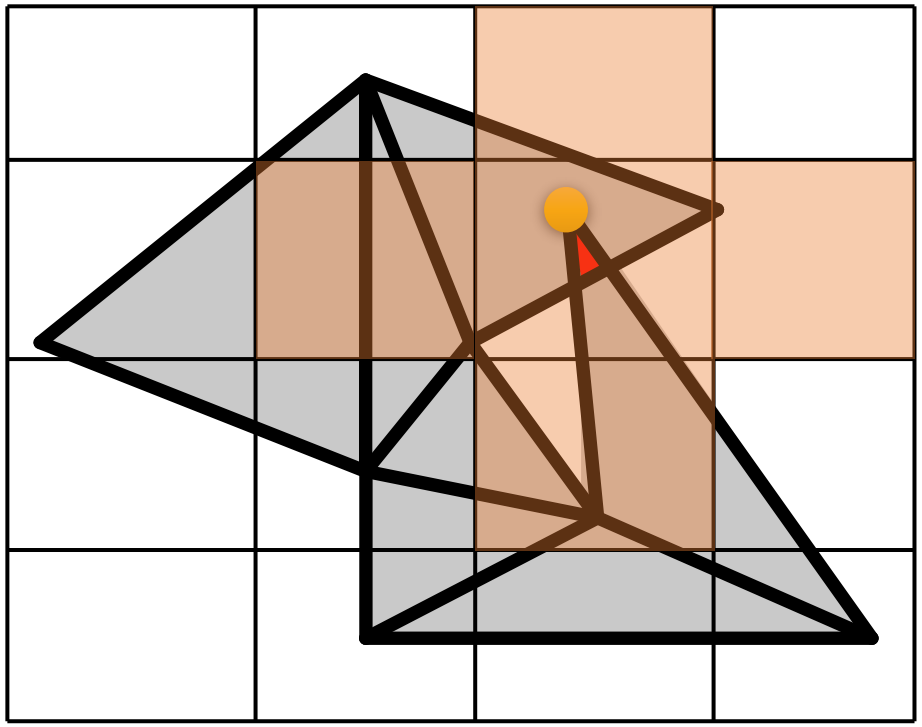
$$\Delta_1 \cap \Delta_2 = \emptyset$$

$$\forall (\Delta_1, \Delta_2) \in \mathcal{M} \times \mathcal{M}$$



# Bijective Maps: Smith and Schaefer 2015

- Collision Detection on Boundary
- Local Support Energy



# Scaffold: Natural Collision Detector

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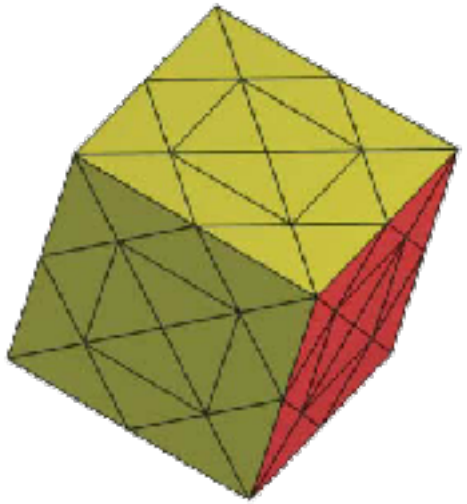
# Scaffold Pipeline

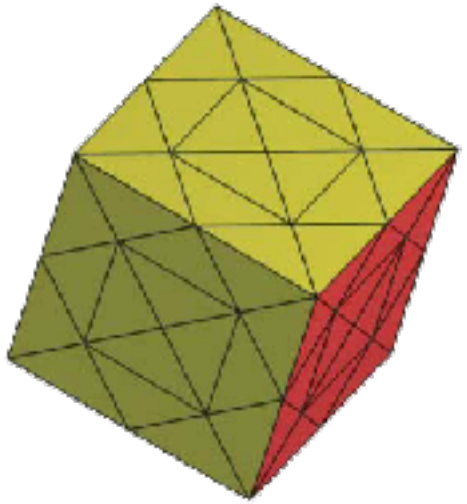
[Zhang et al 2005]

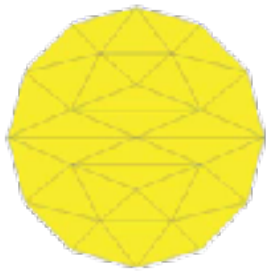


# Scaffold Pipeline

[Zhang et al 2005]

[Tutte 1963]





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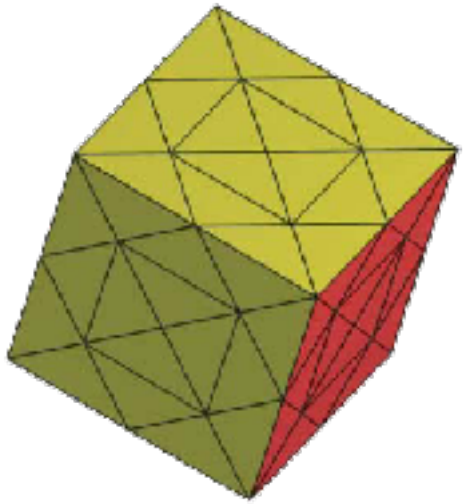
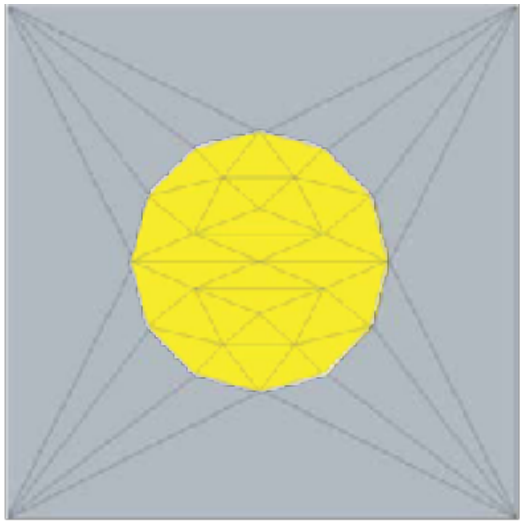


- Build Scaffold



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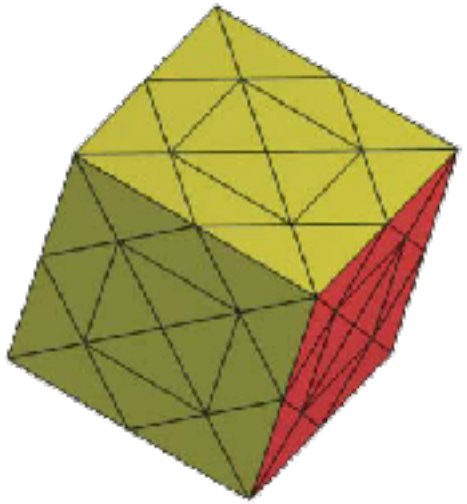
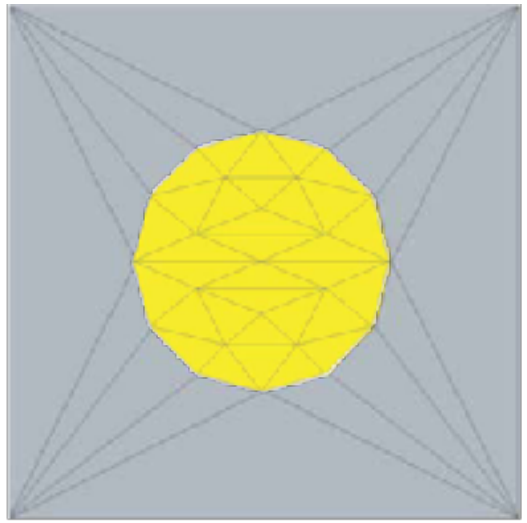
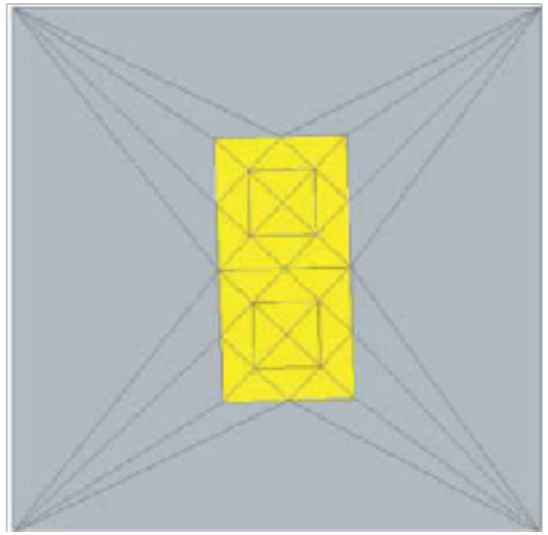
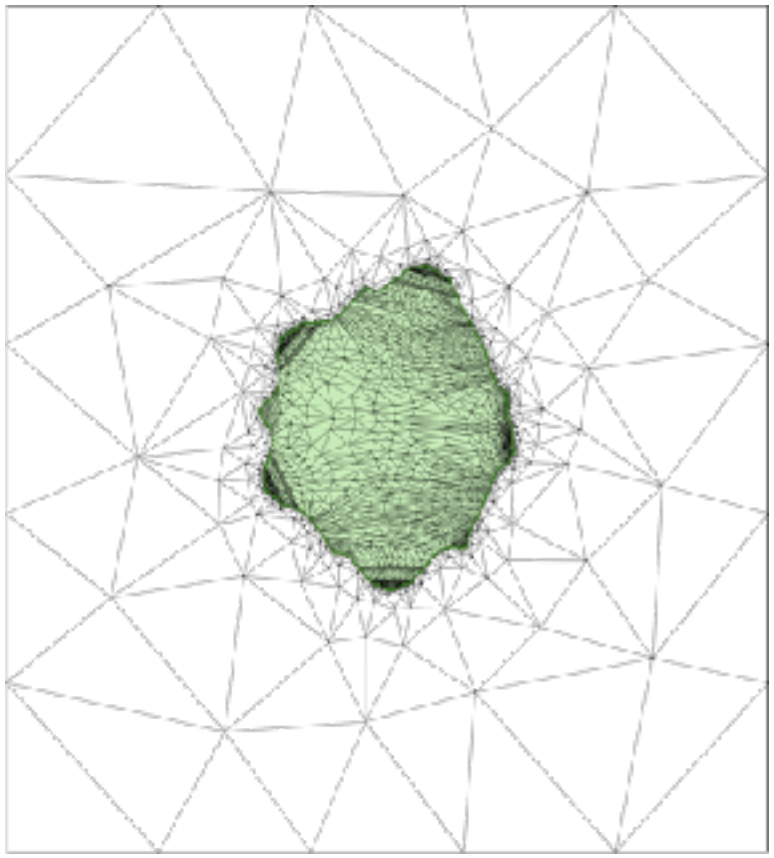


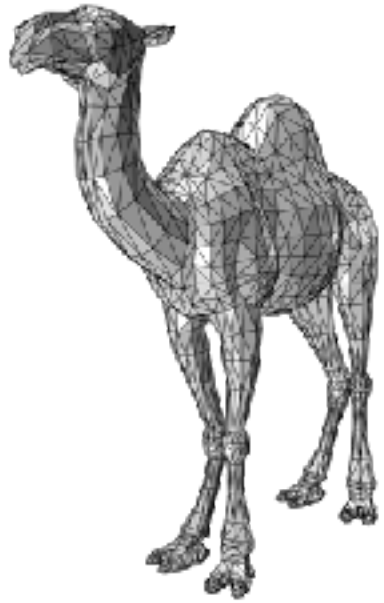
- Tutte Embedding [Tutte 1963]
  - Build Scaffold
  - Valid Interior Deformation [Zhang et al 2005]





# Scaffold Restricts





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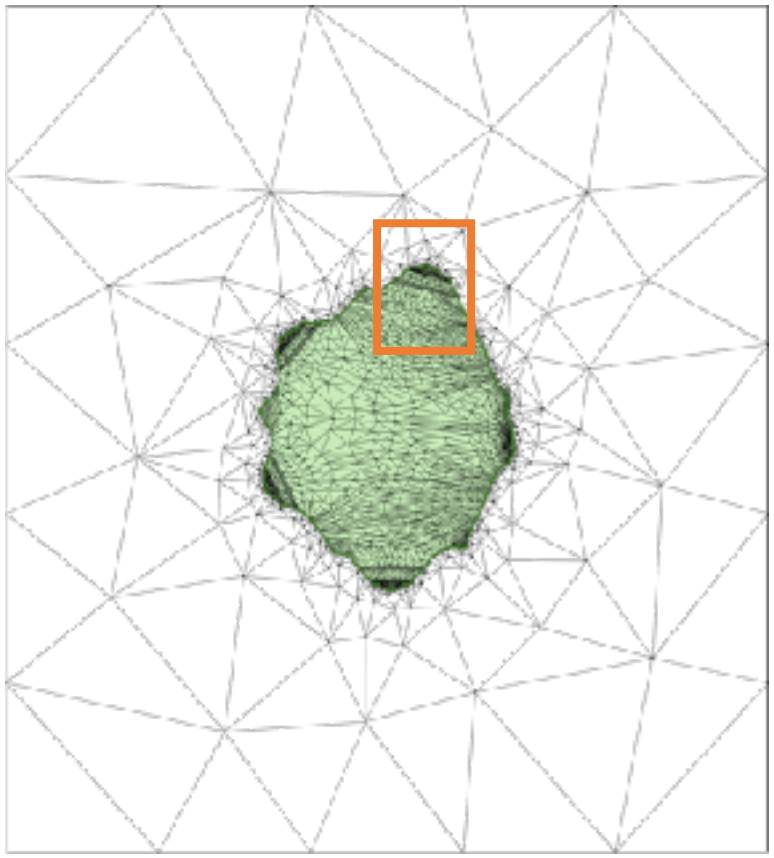

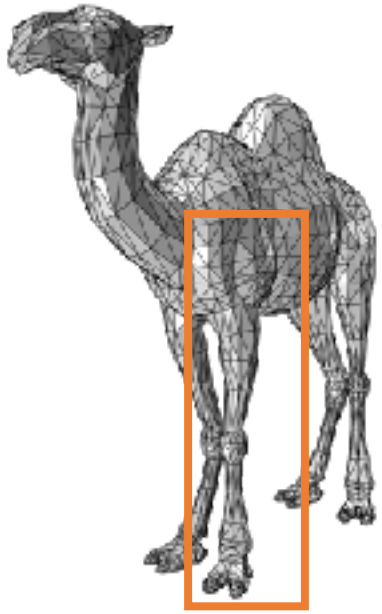

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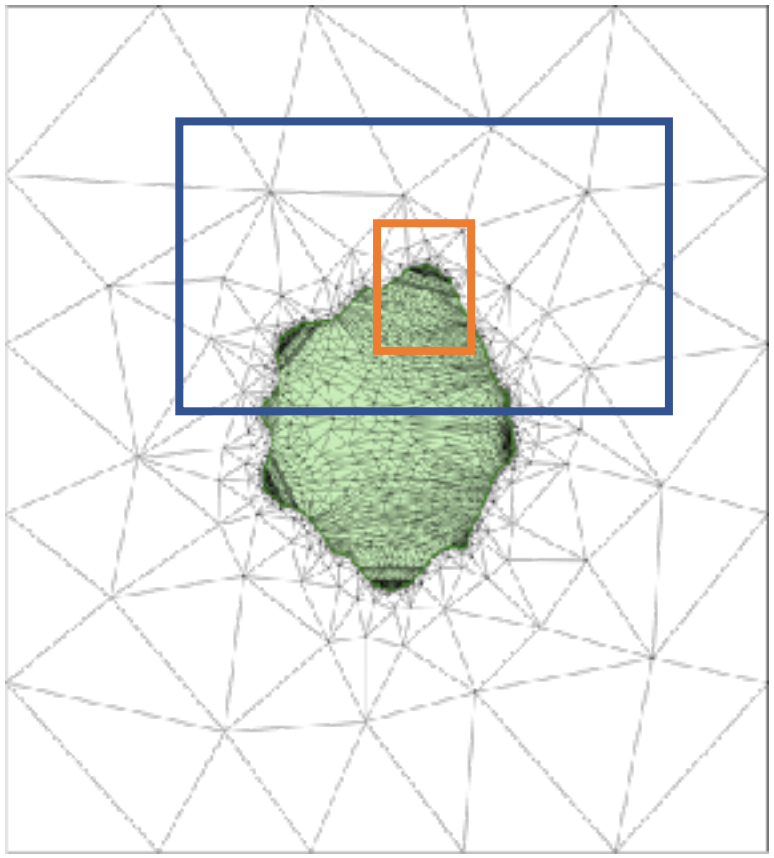

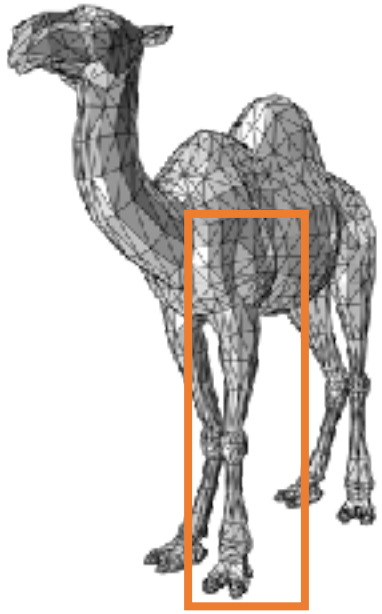

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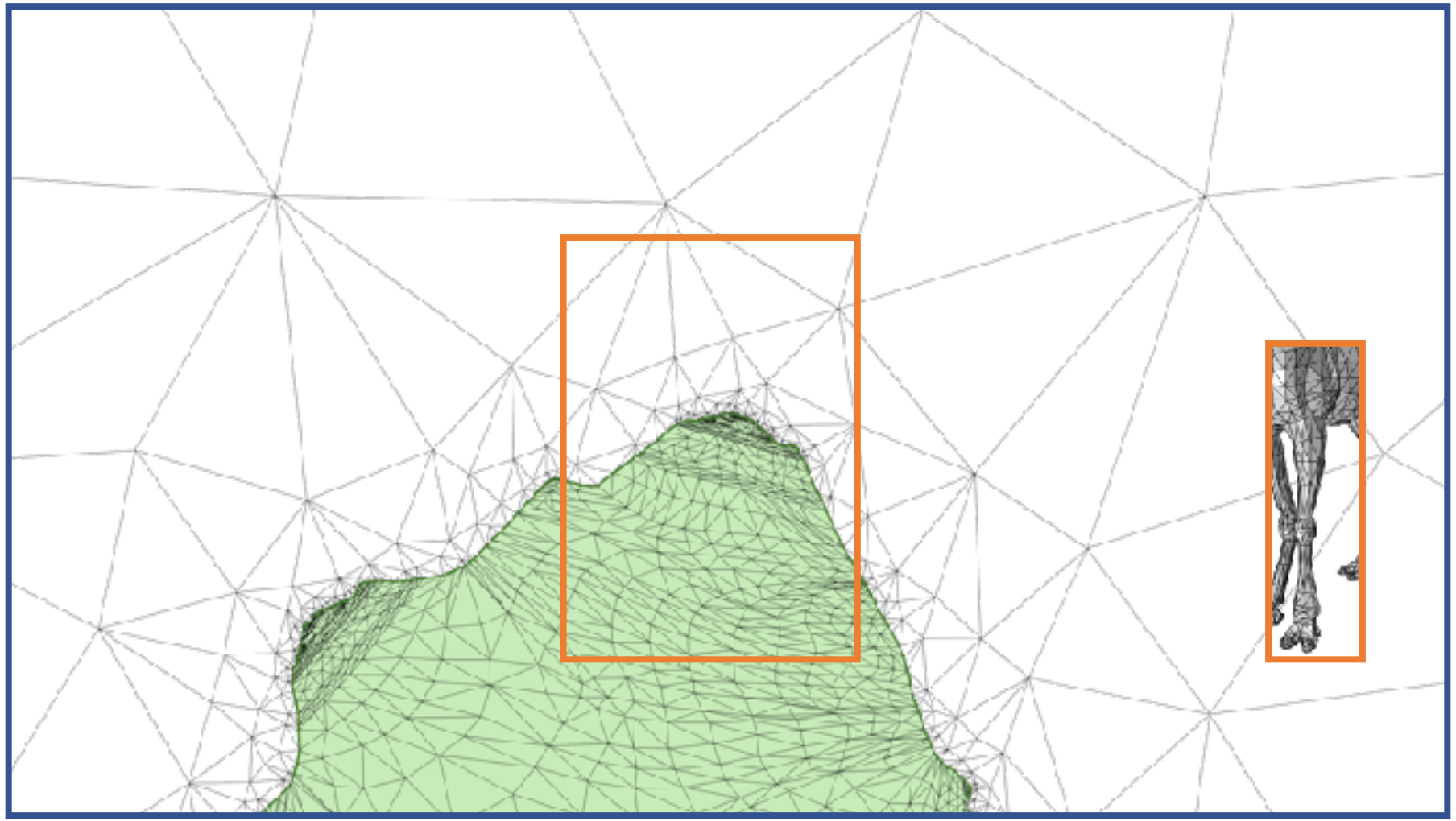

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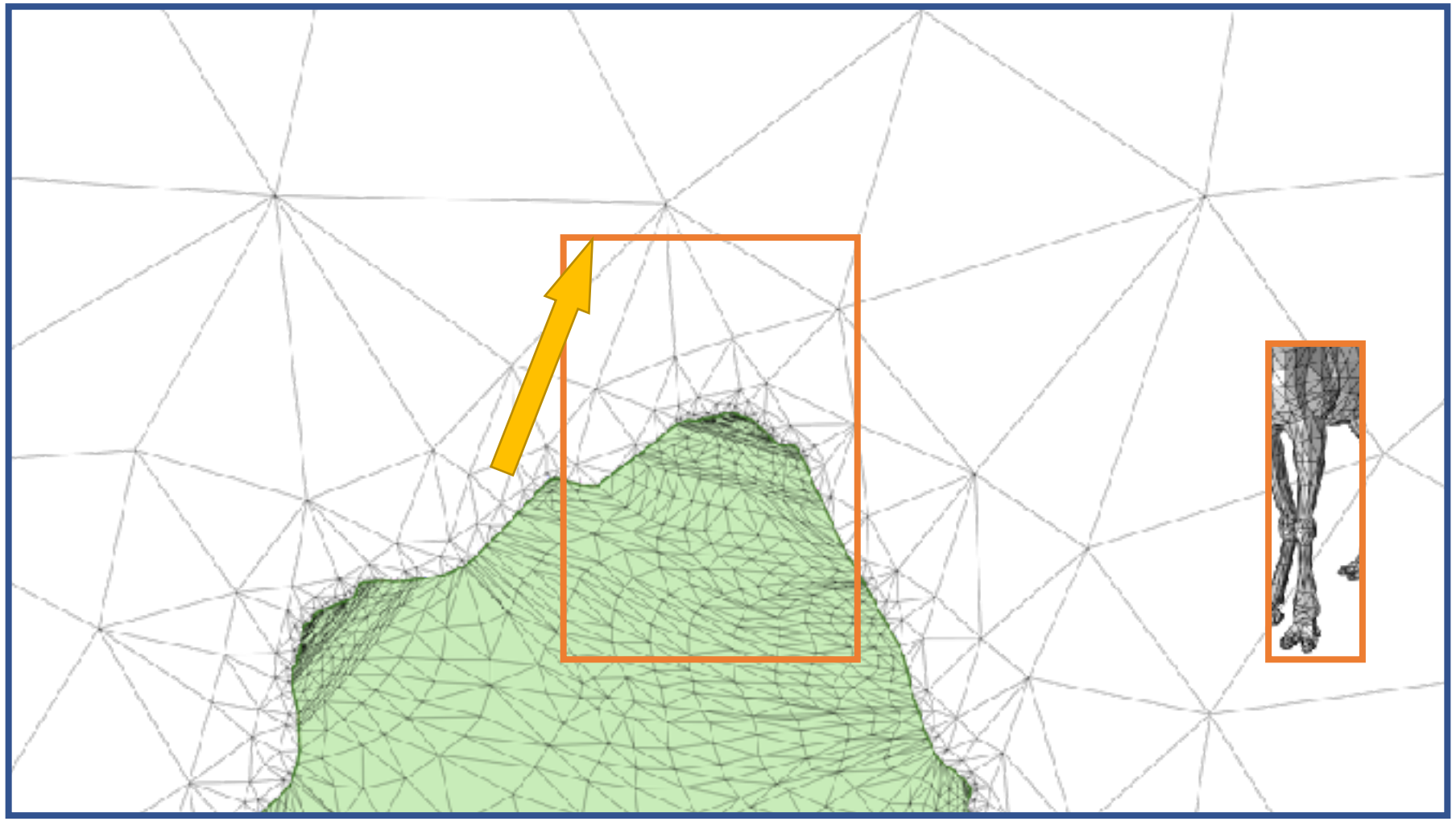

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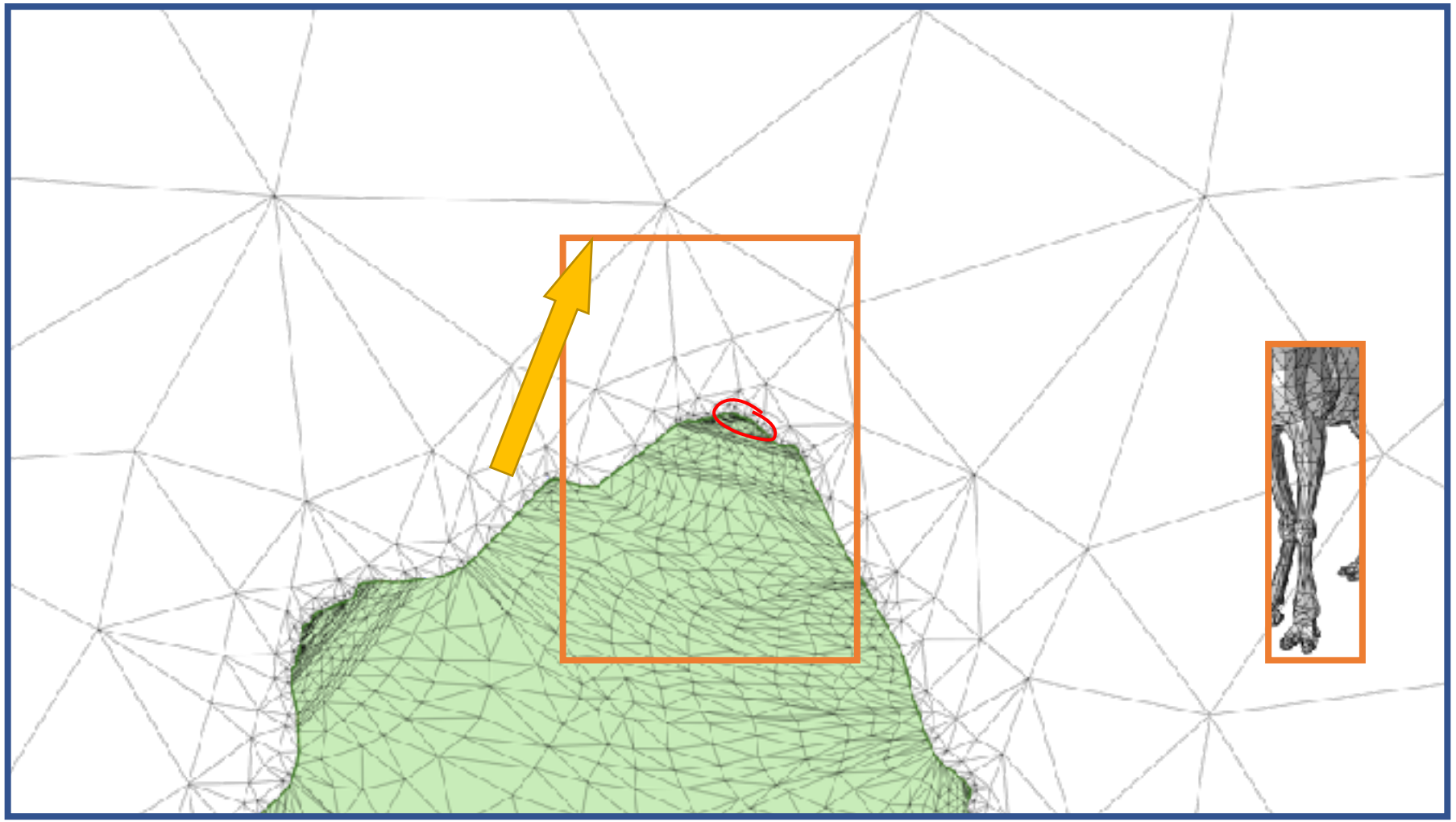

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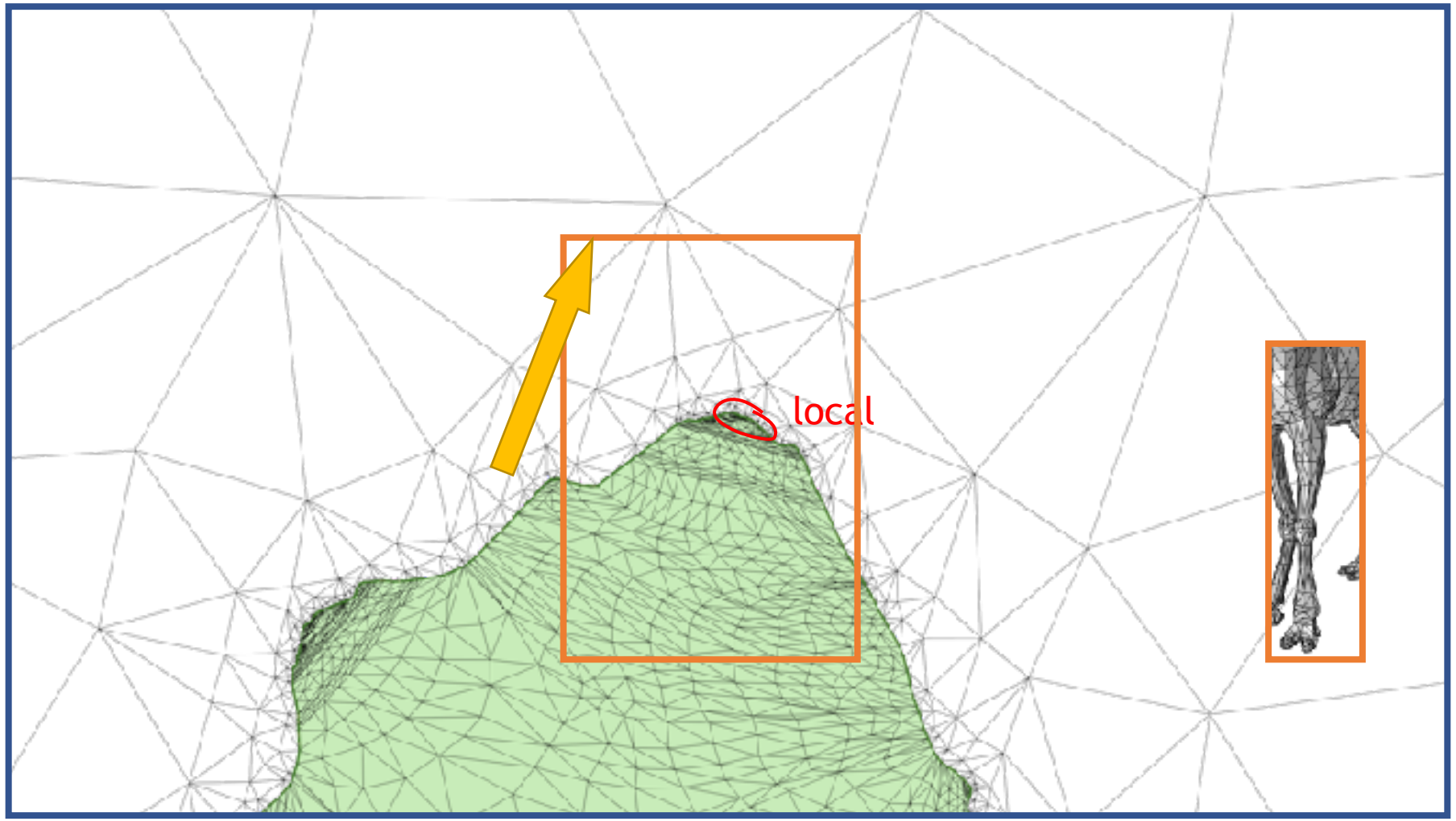
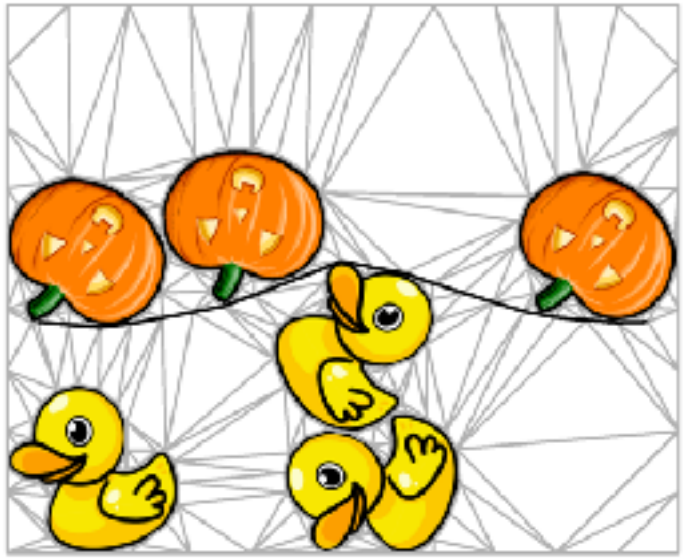


# More Progressive

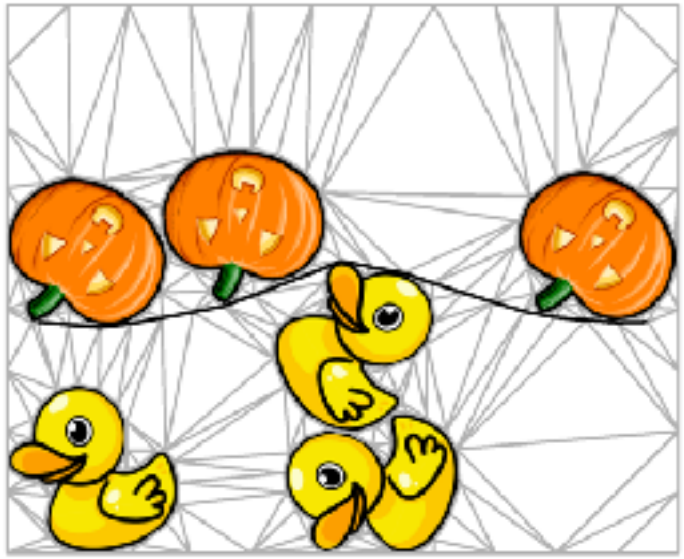
[Müller et al 2015]



# More Progressive

[Müller et al 2015]

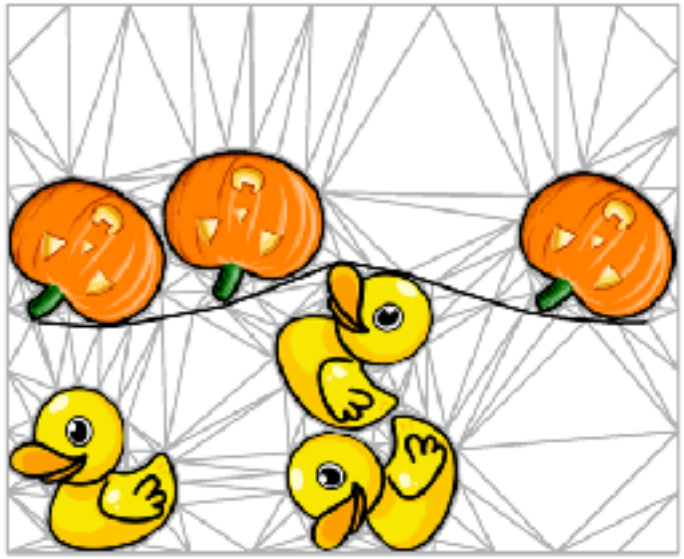
- Start from Identity

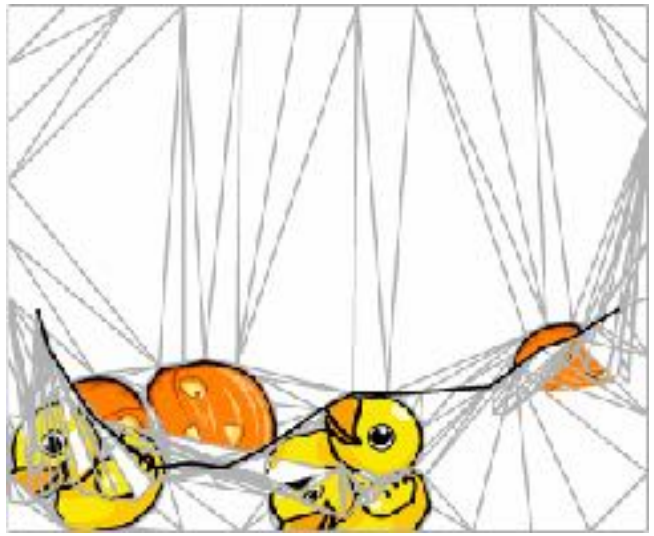


# More Progressive

[Müller et al 2015]

- Start from Identity
  - Deform Arbitrarily





# More Progressive

[Müller et al 2015]

- Start from Identity
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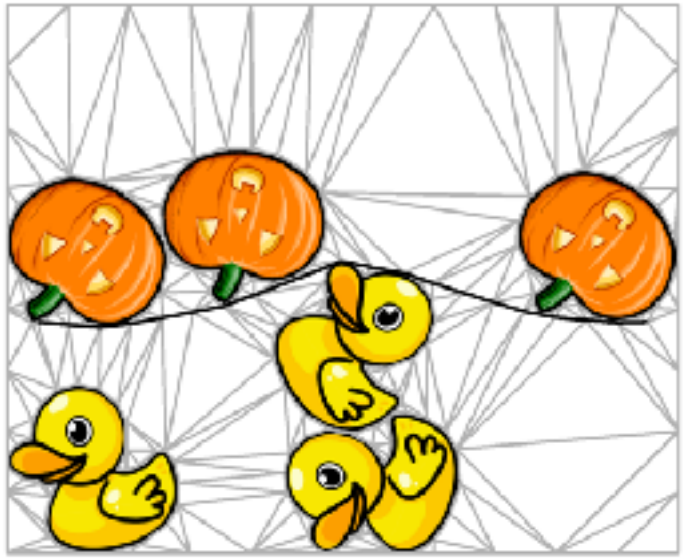
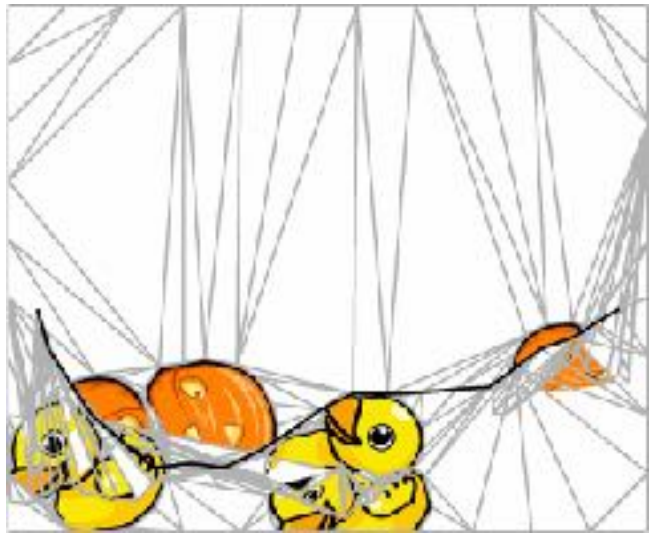
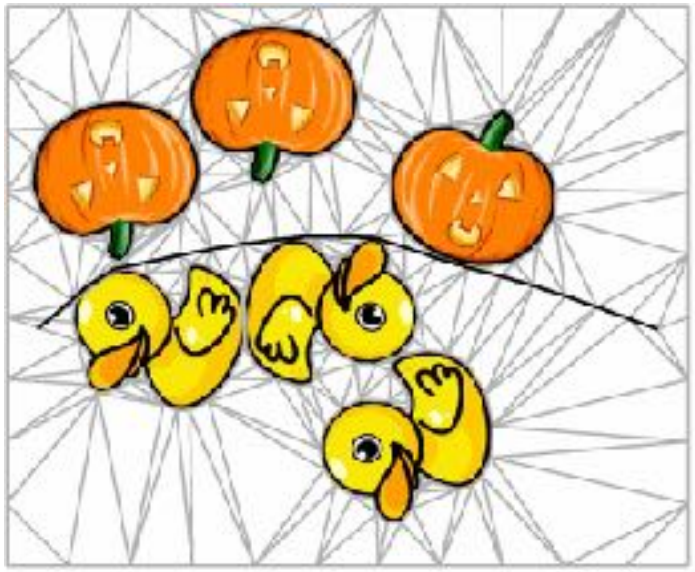
  - Bounce Back







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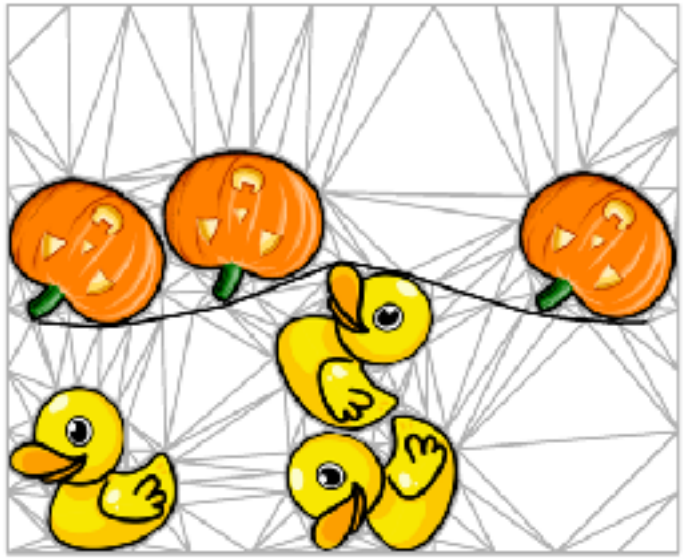

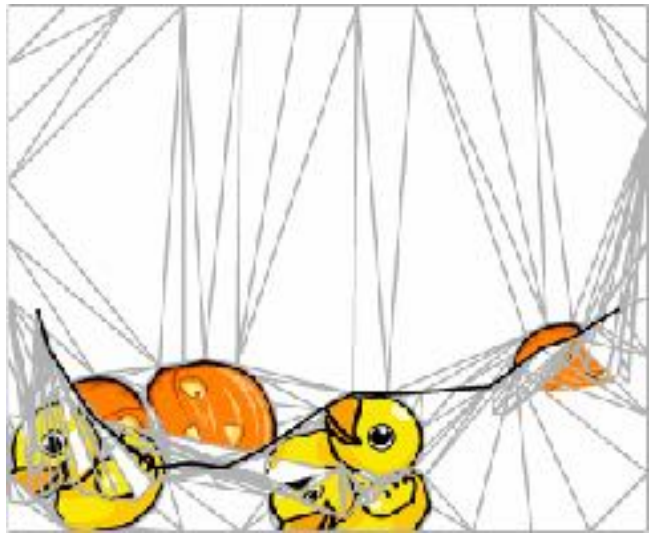

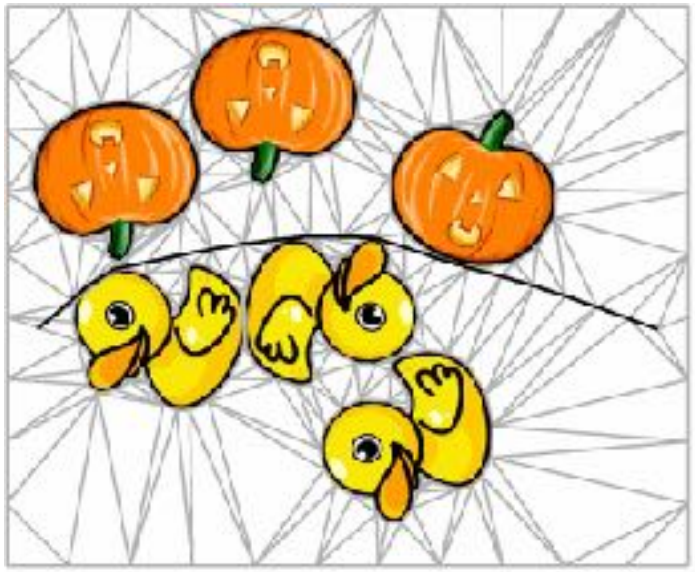

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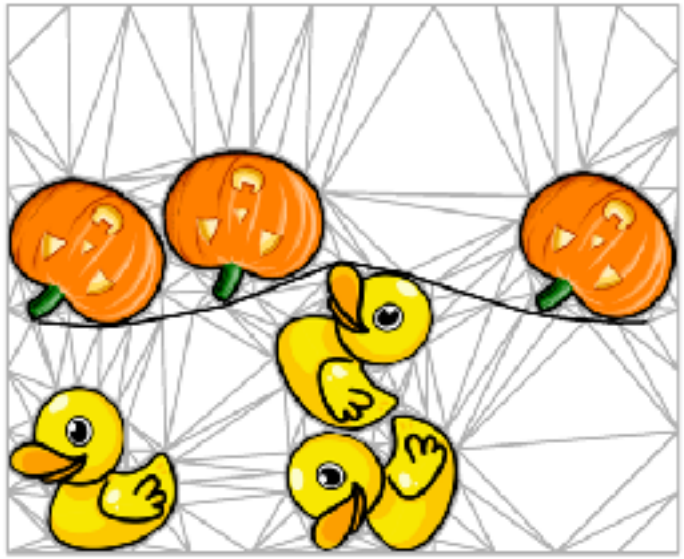
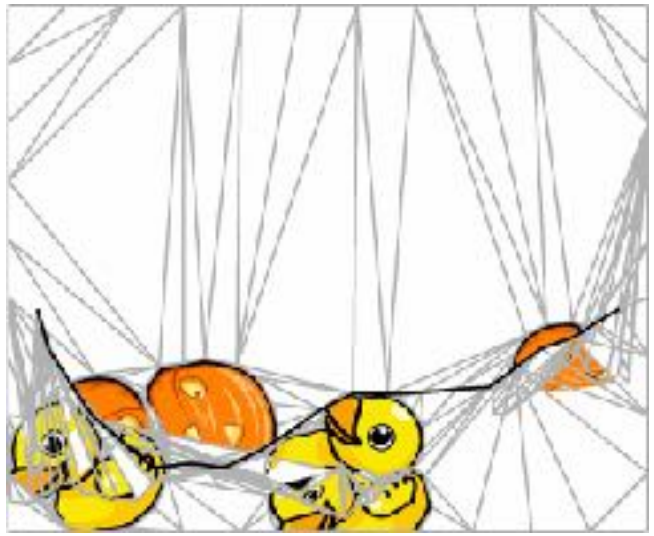
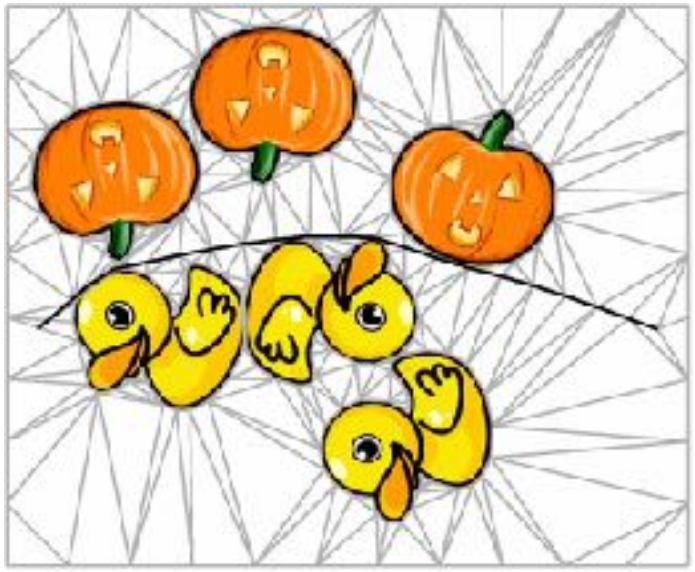
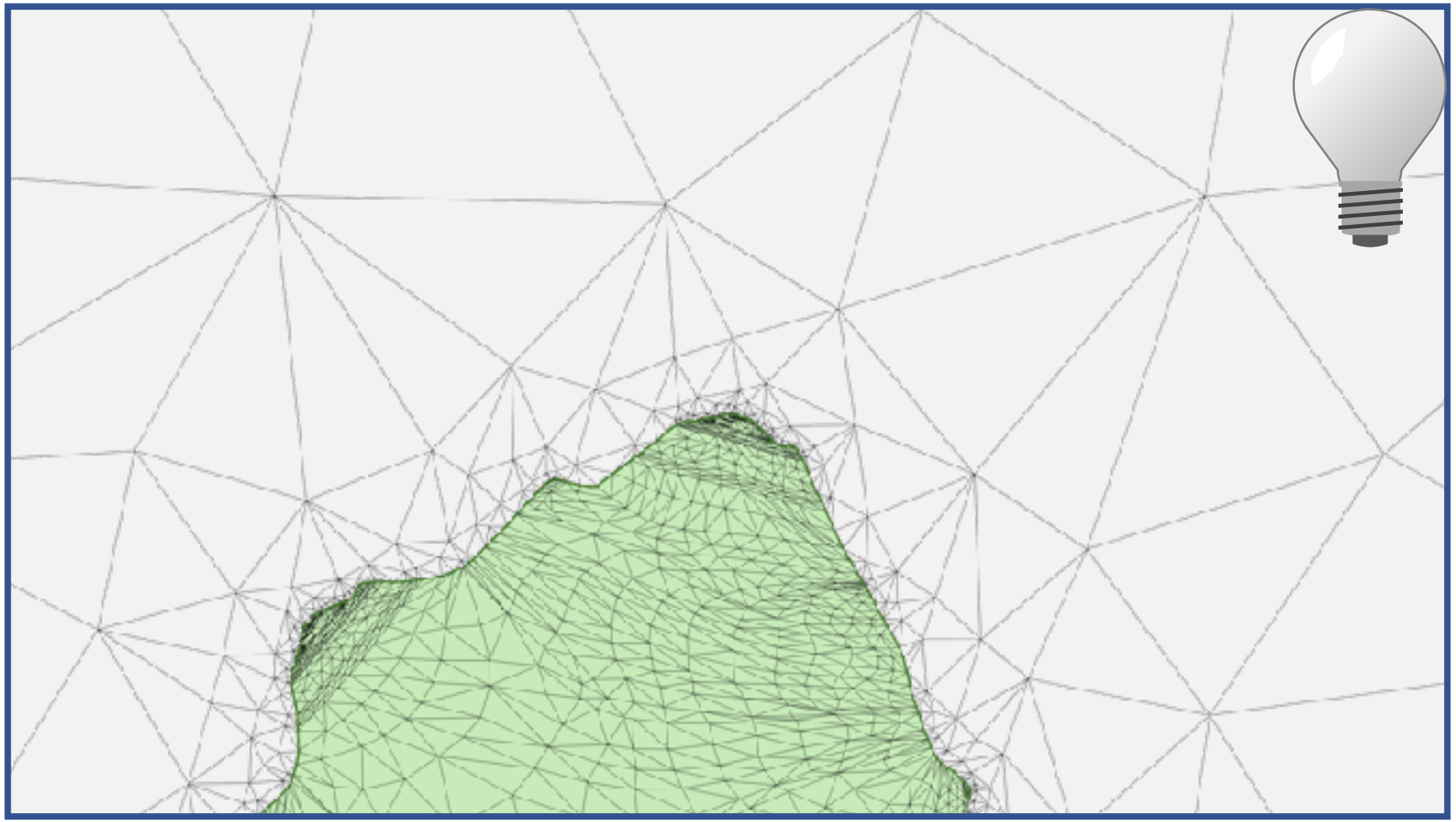


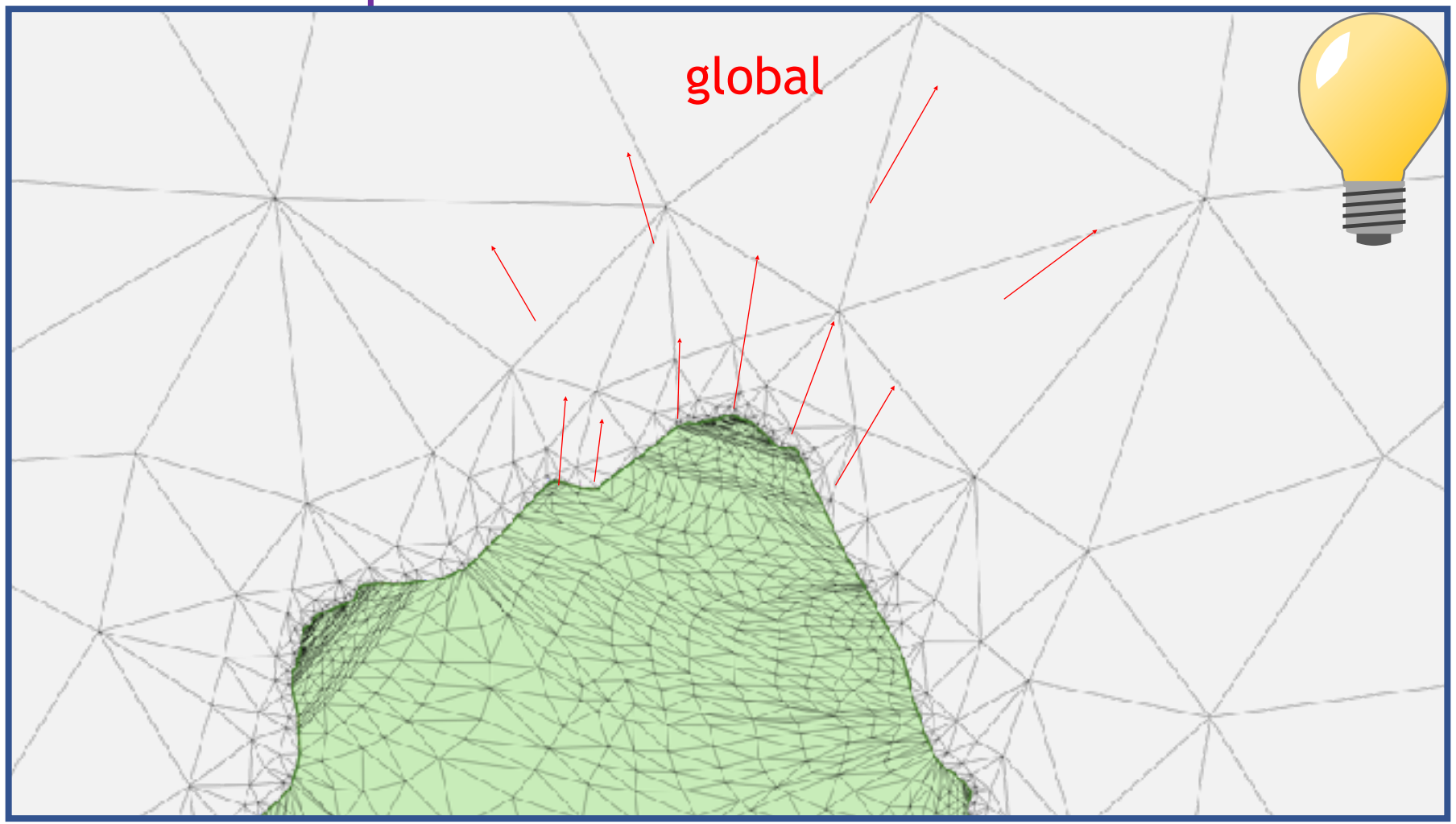




# Scaffold



# Scaffold Helps



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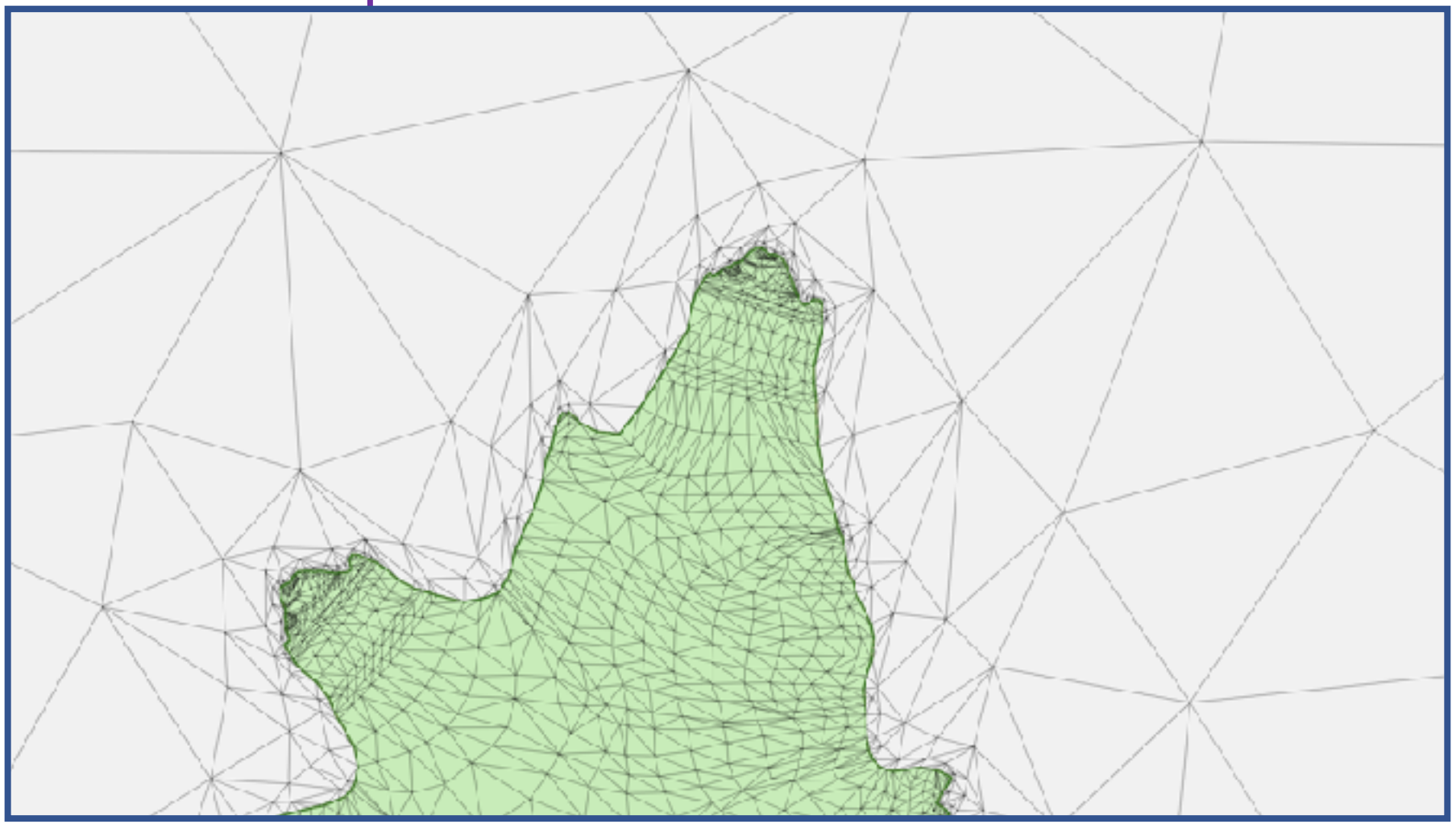
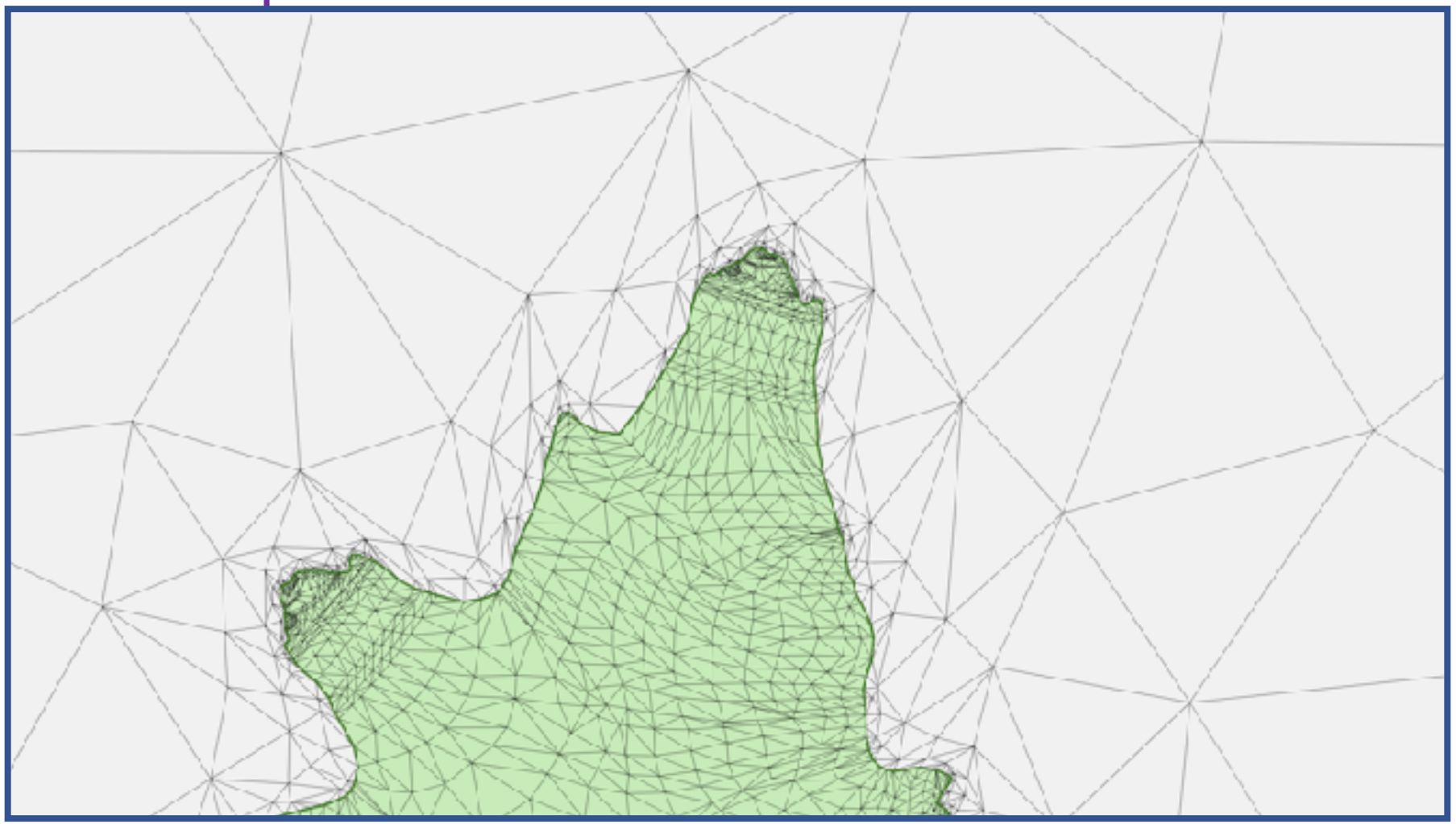
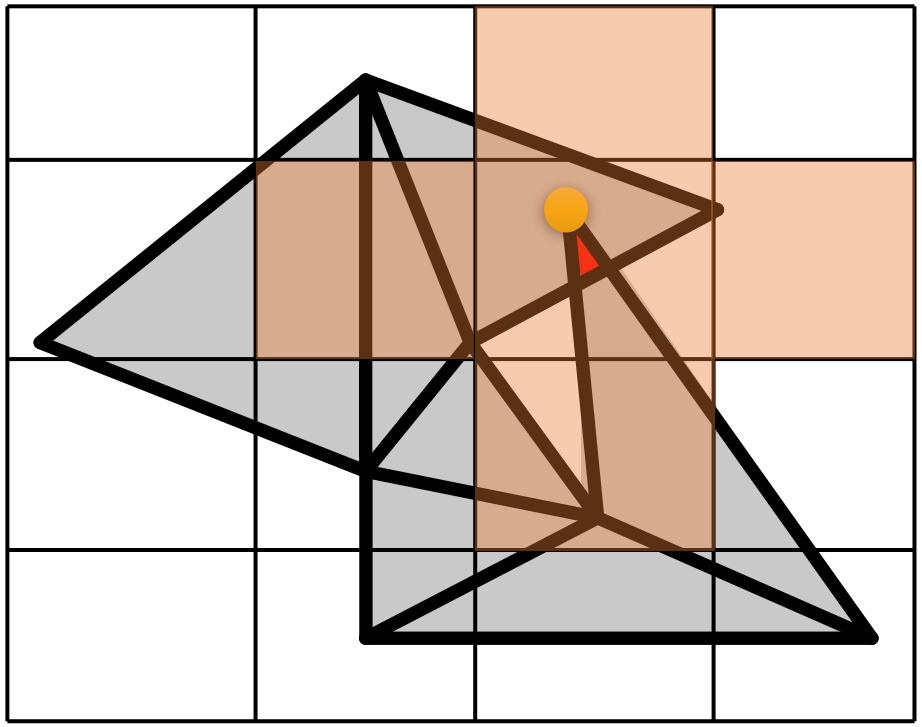


# SCAF Helps



# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Previous
  - Local Knowledge
  - Collision Detection



# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution

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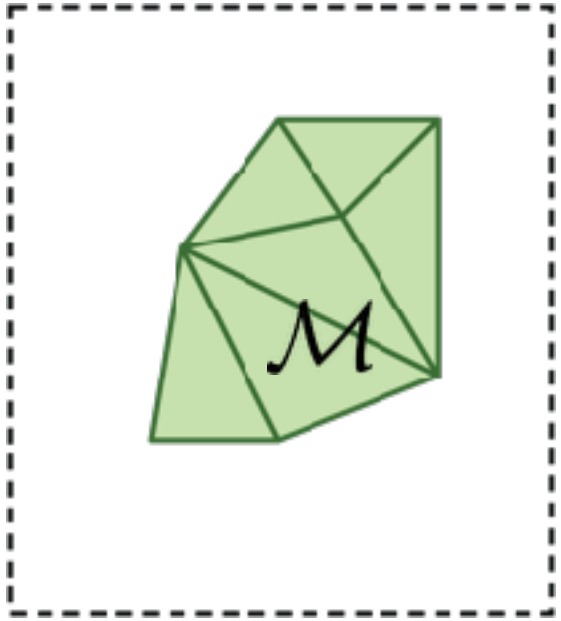

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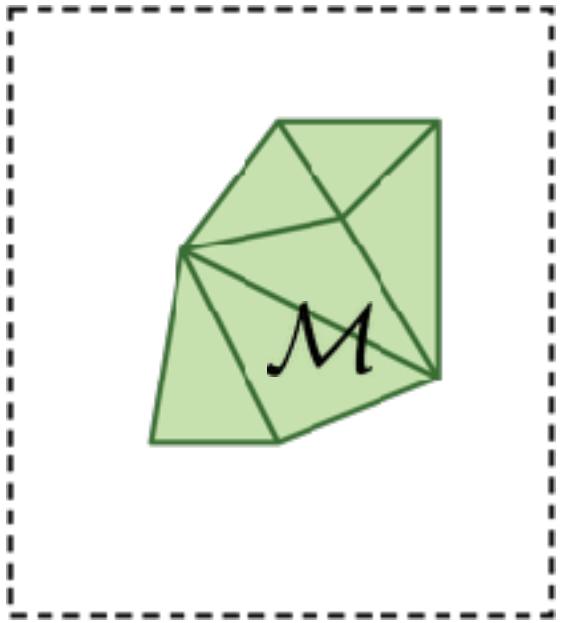


$E_{\mathcal{M}}$

# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
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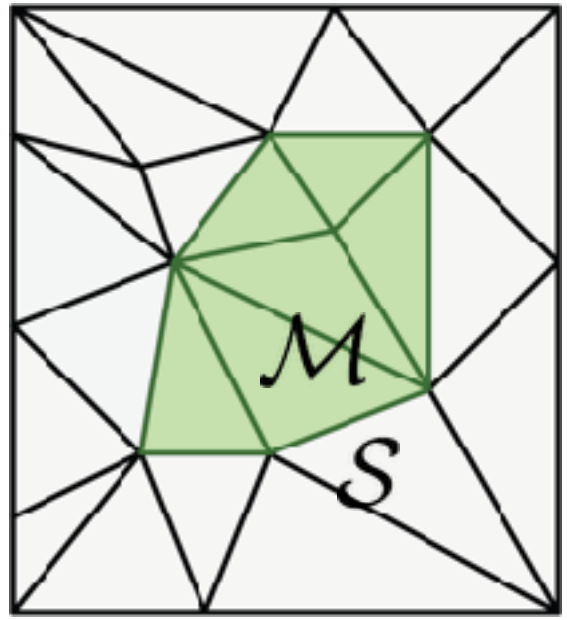


$$E = E_{\mathcal{M}} + E_{\mathcal{S}}$$

# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

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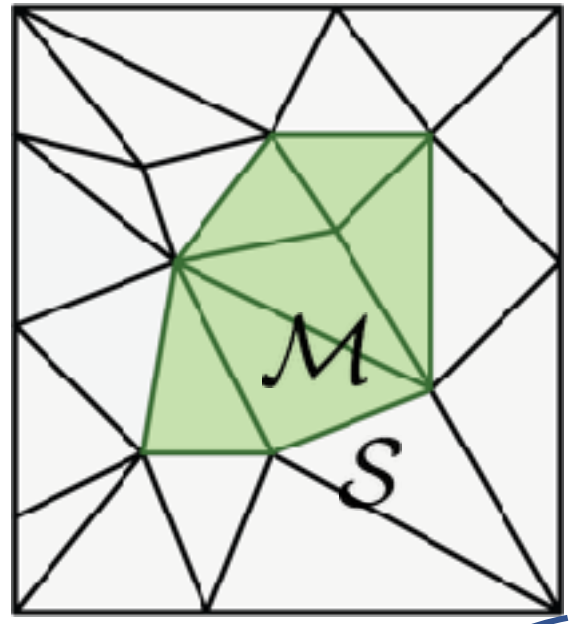
  - Global Knowledge
  - Collision Resolution

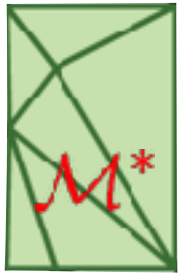



$$E = E_{\mathcal{M}} + E_{\mathcal{S}}$$

# Formulation

$$E_{\mathcal{M}}(X)$$

coordinates



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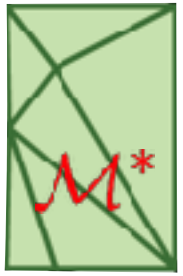
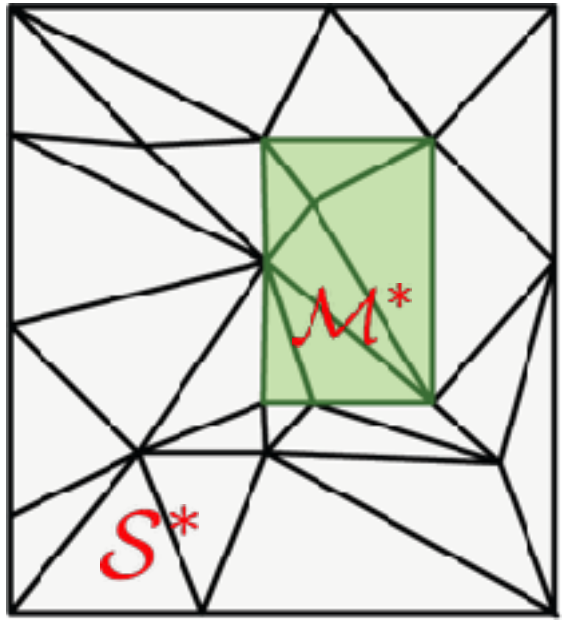


# Formulation

$$E_{\mathcal{M}}(X) + E_{\mathcal{S}^*}(X)$$

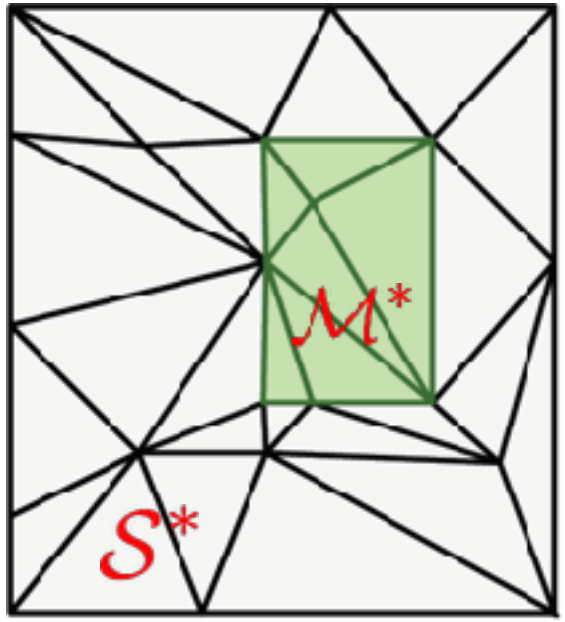
coordinates



# Formulation

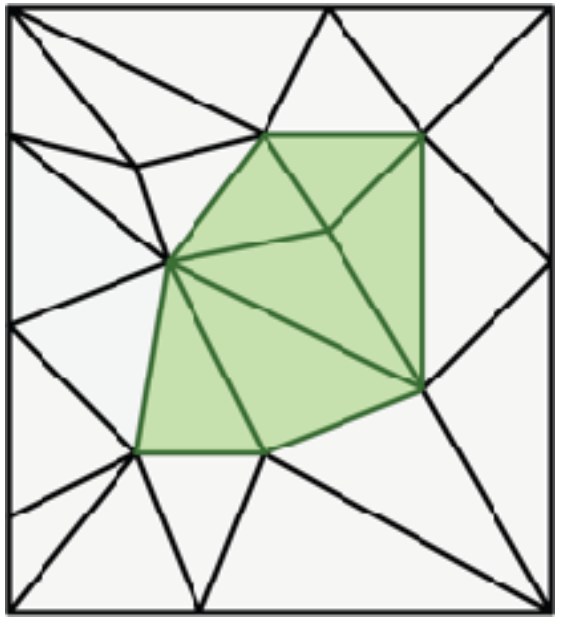
$$X^* = \operatorname{argmin}_X \; E_{\mathcal{M}}(X) + E_{\mathcal{S}^*}(X)$$

coordinates



# Formulation

$$\min_{X_k} E(X_k) = E_{\mathcal{M}}(X_k) + E_{\mathcal{S}_{k-1}}(X_k) \text{ from } X_{k-1}$$



$$E = E_{\mathcal{M}} + E_{\mathcal{S}}$$

# Formulation

[Rabinovich et al 2017]

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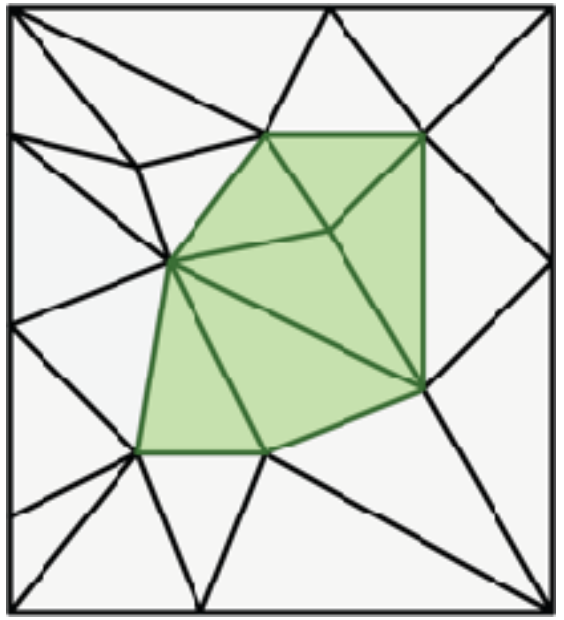

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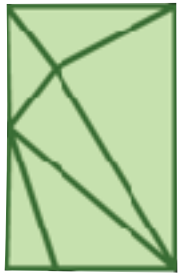

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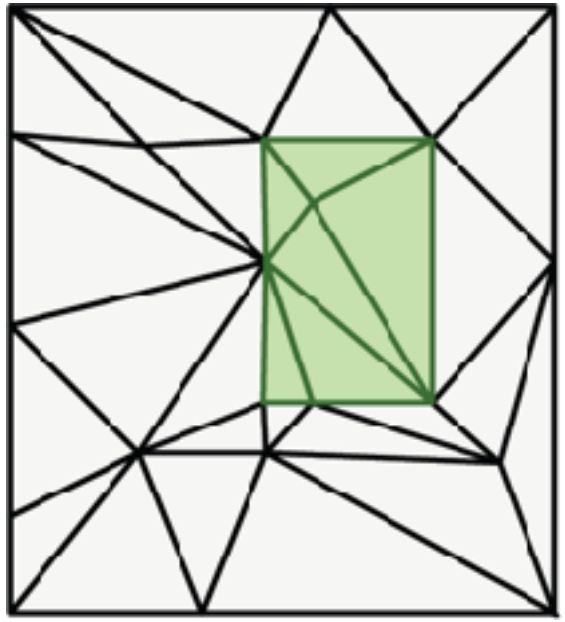

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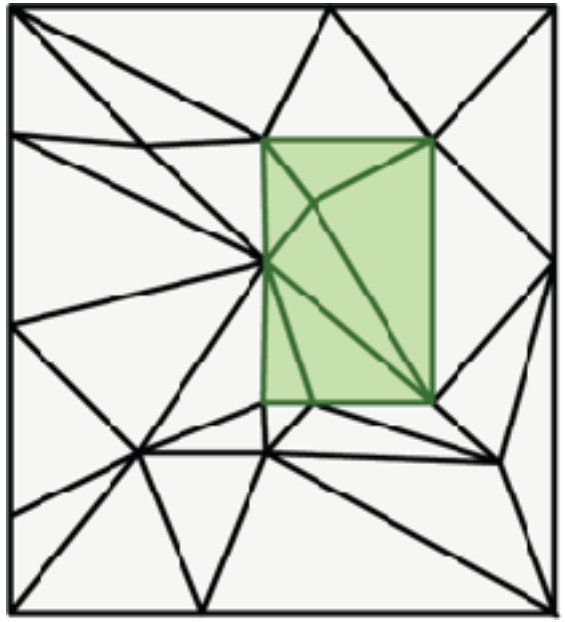

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# Triangulation

- Local Update
  - Edge Flip [Zhang et al 2015] [Müller et al 2015]

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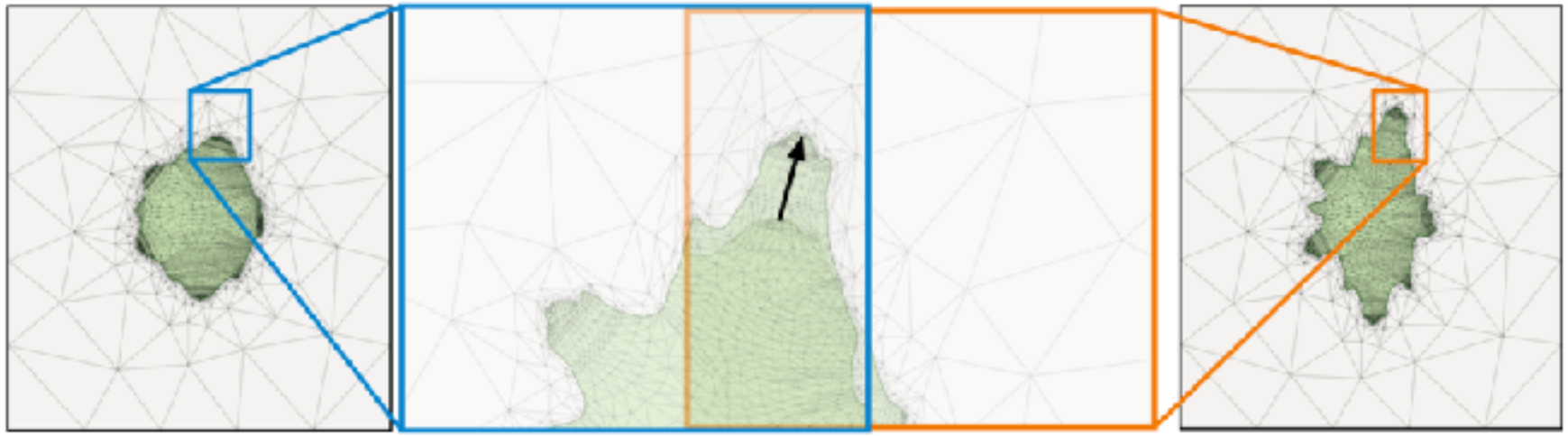

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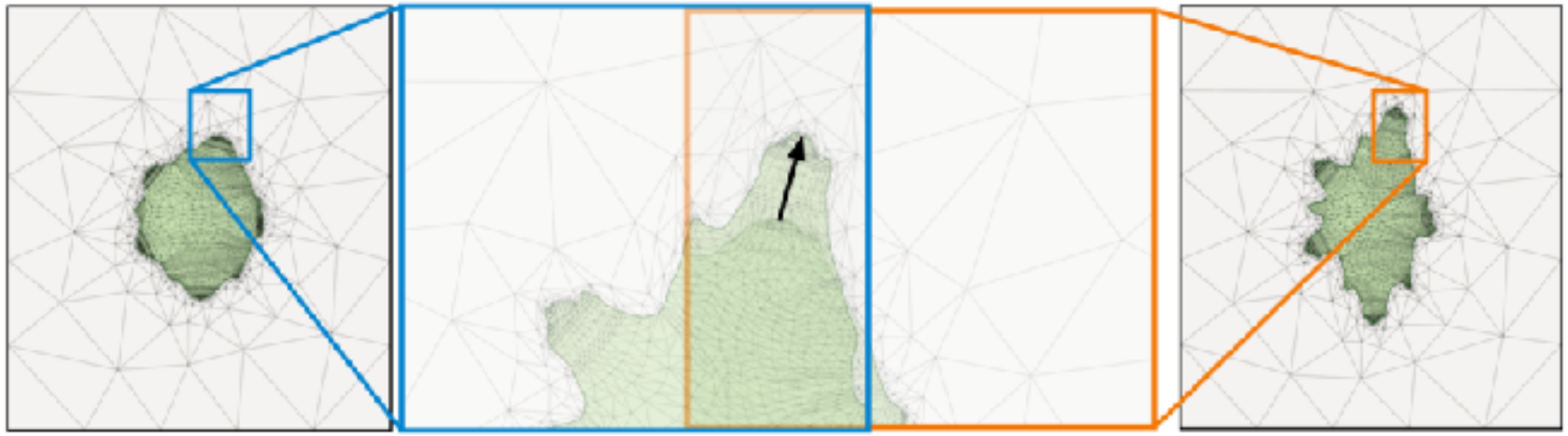
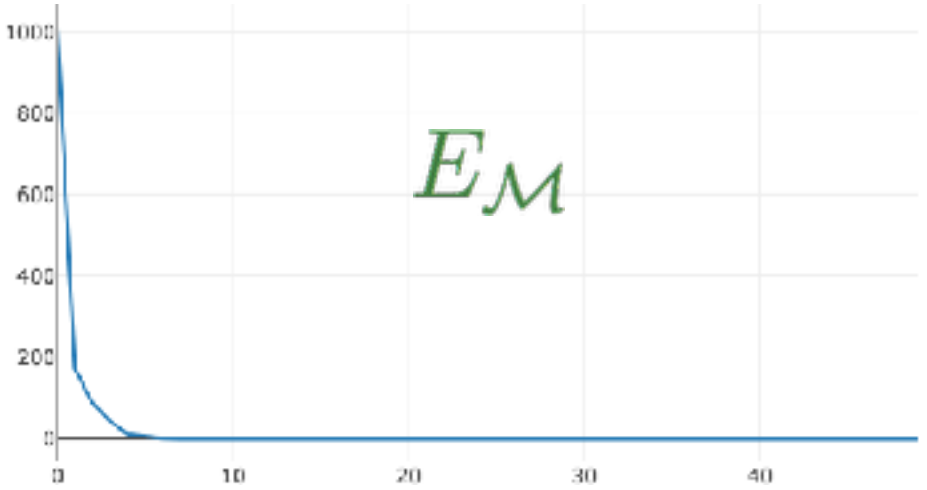
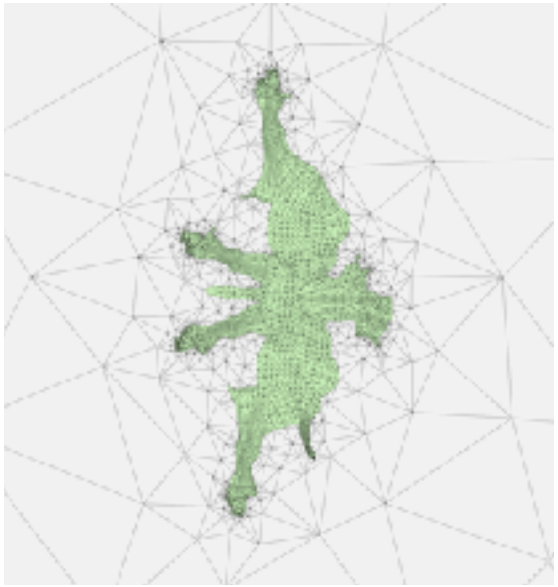
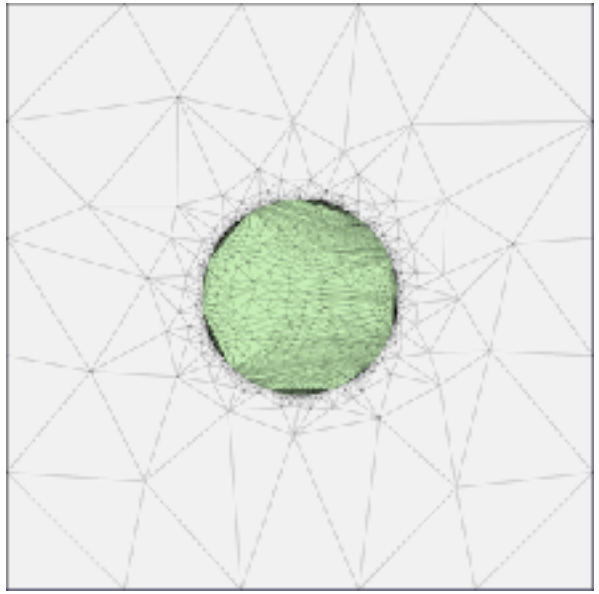


# Triangulation

- Direct reconstruct
- Coarse





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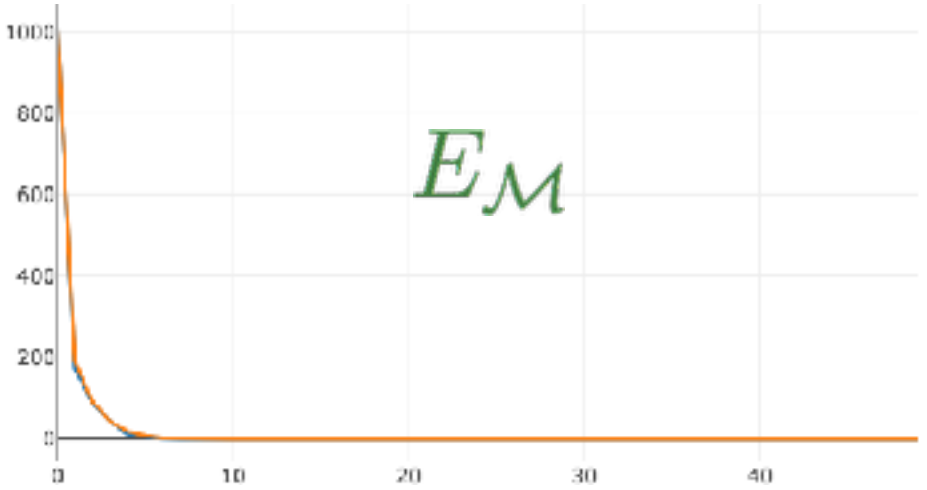


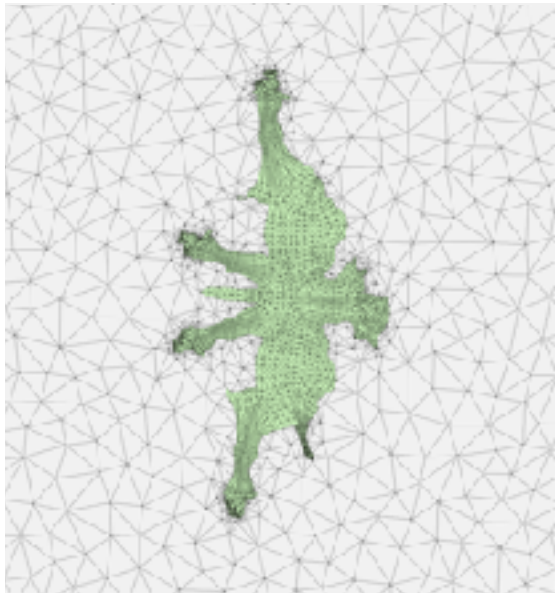

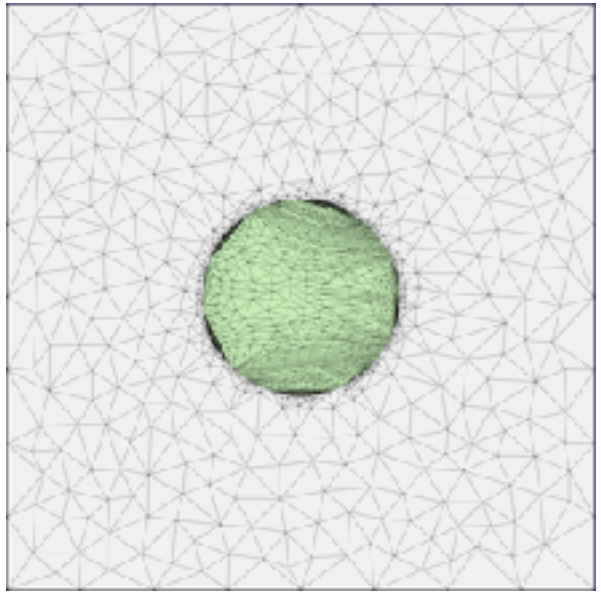

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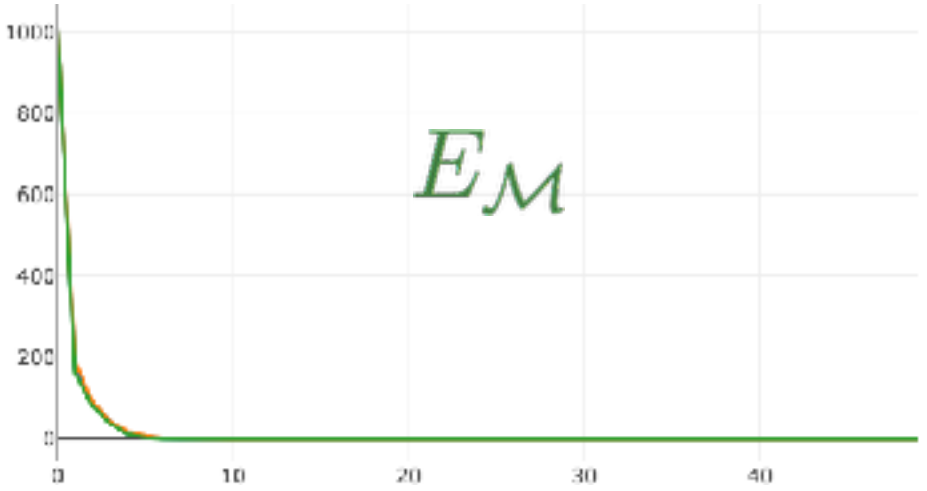


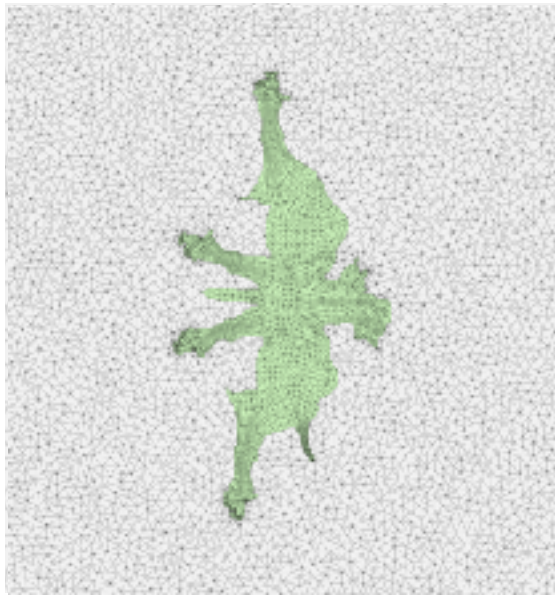

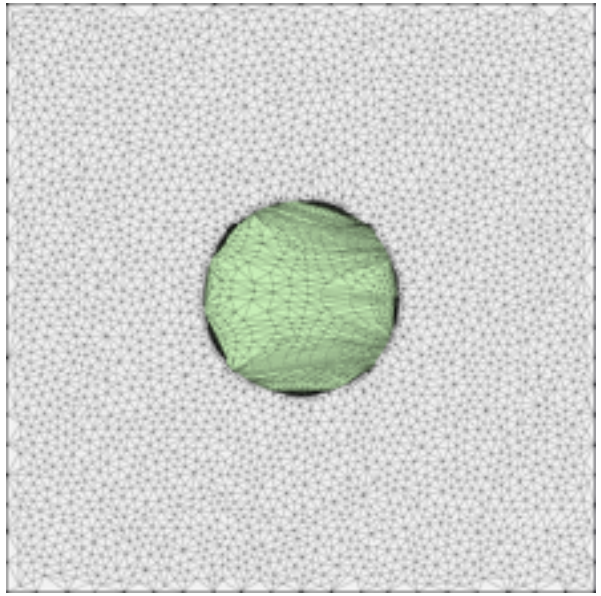

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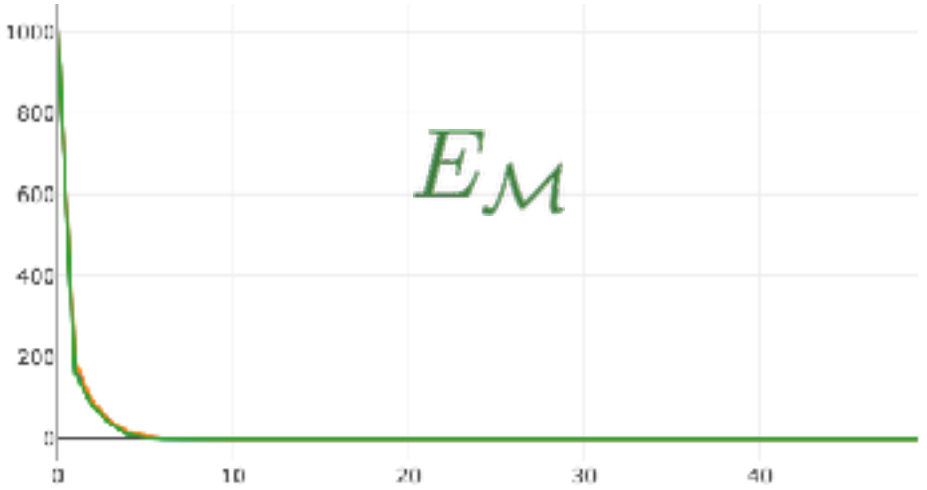
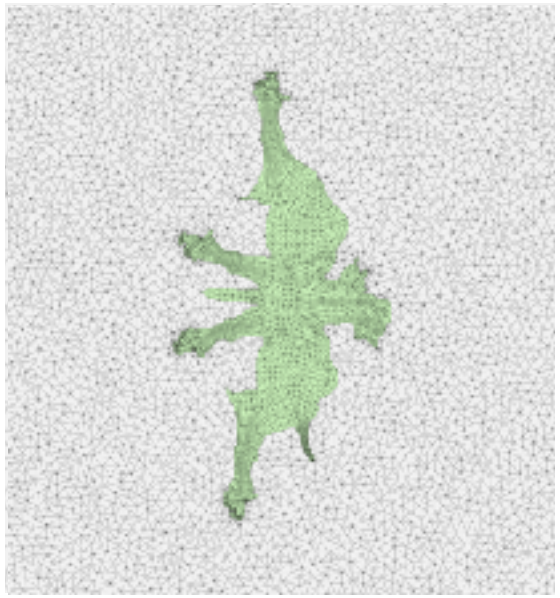
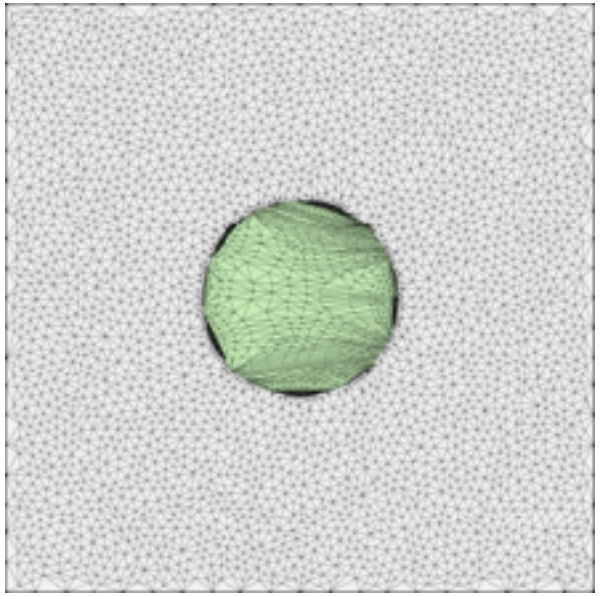



Triangle





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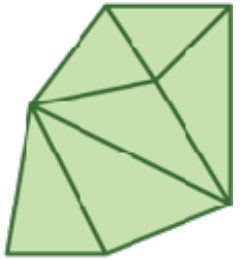

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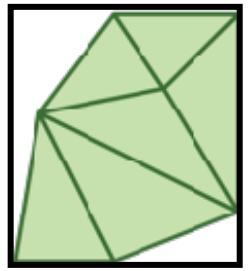

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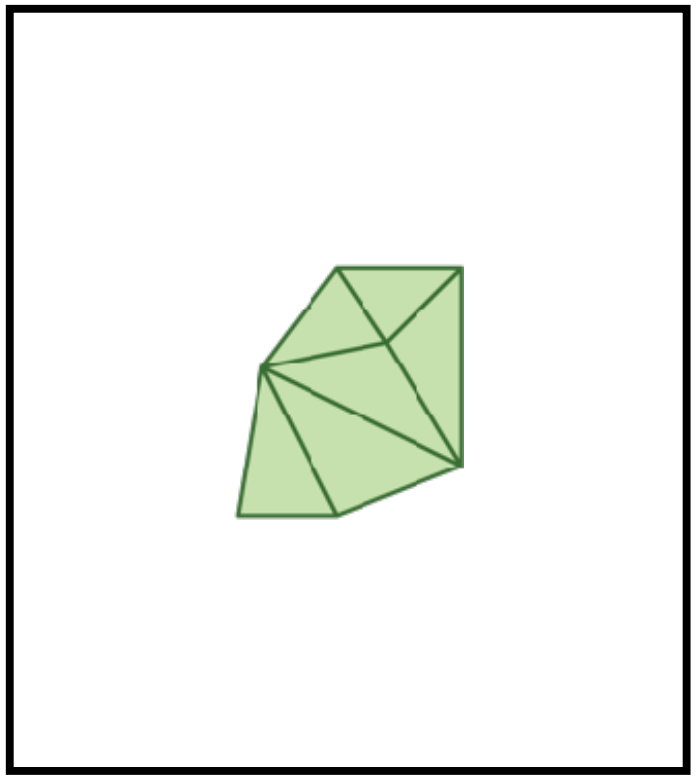

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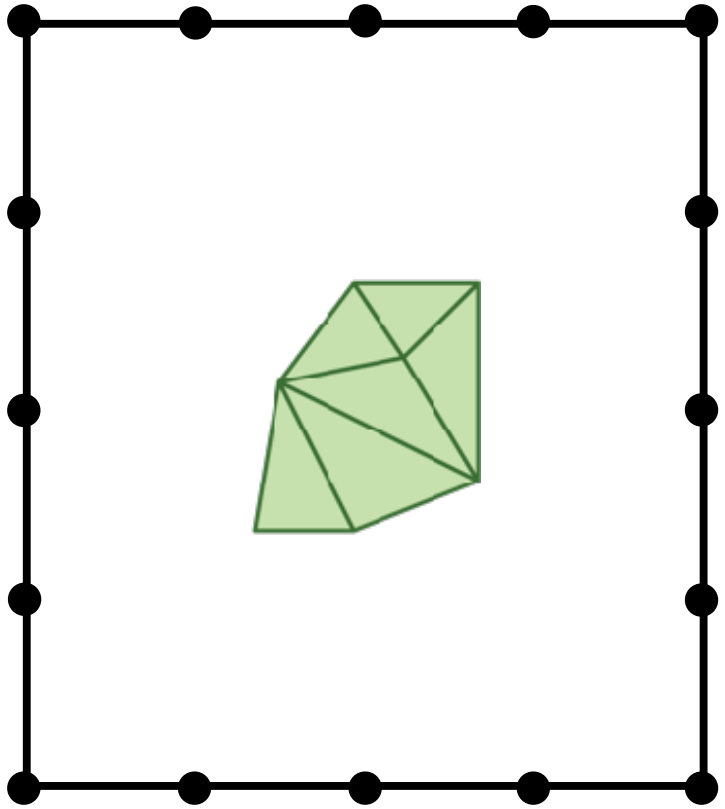

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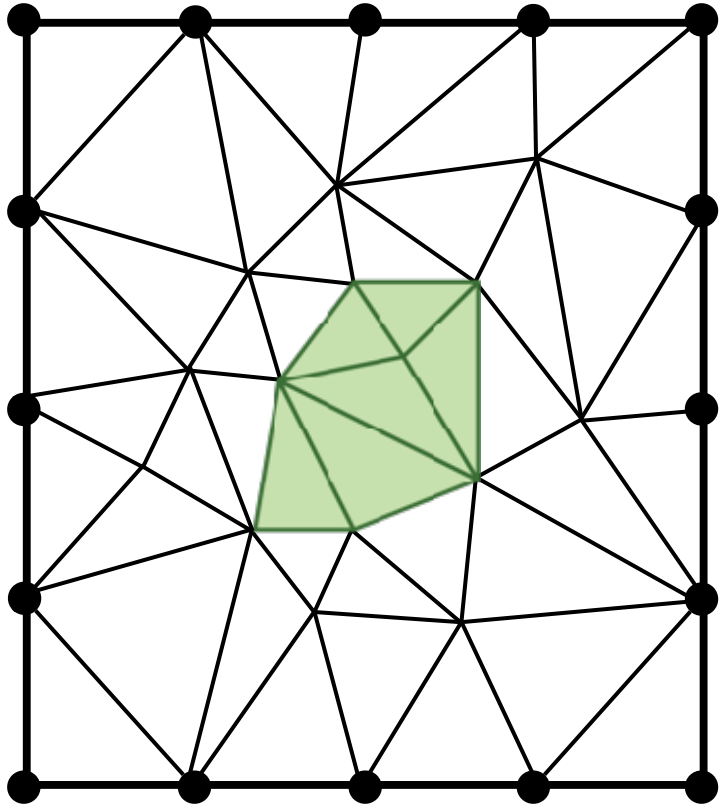
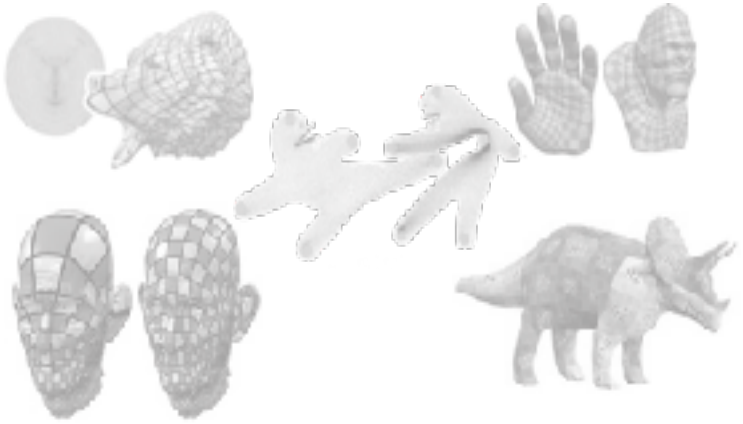
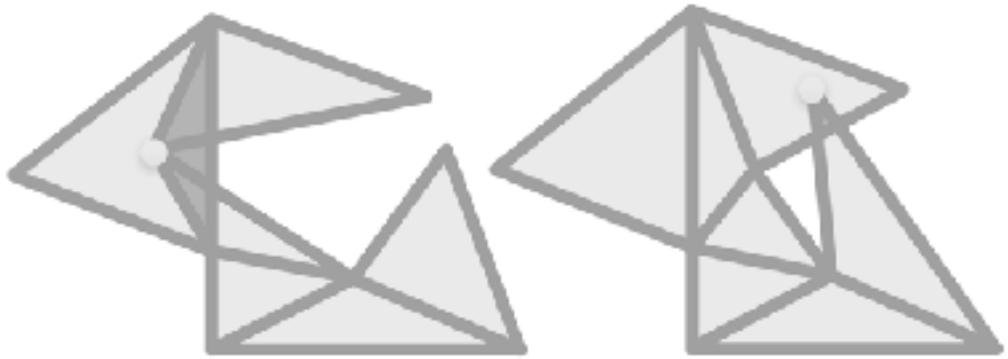




Locally Injective Maps



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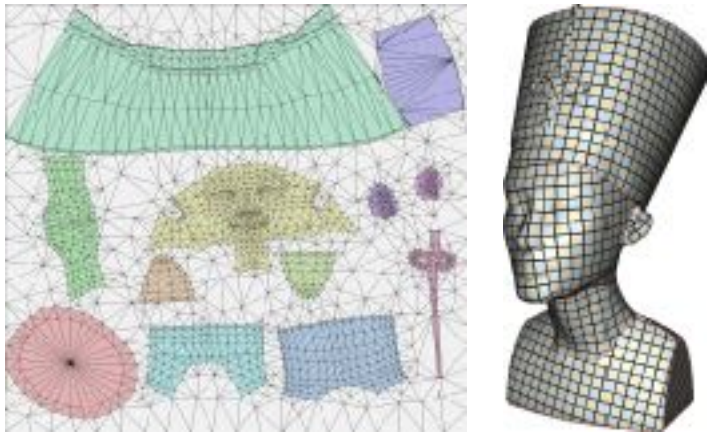
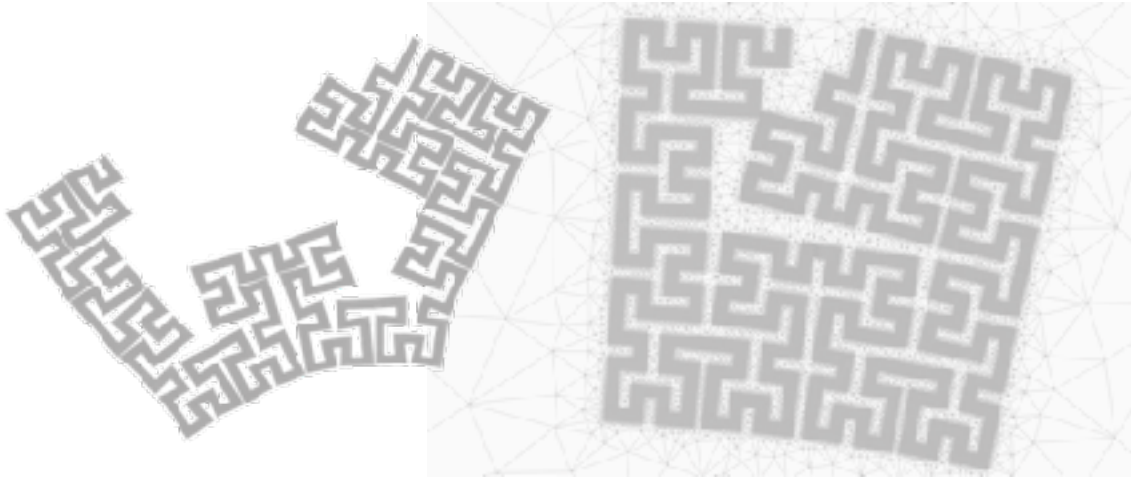




Results



Comparisons

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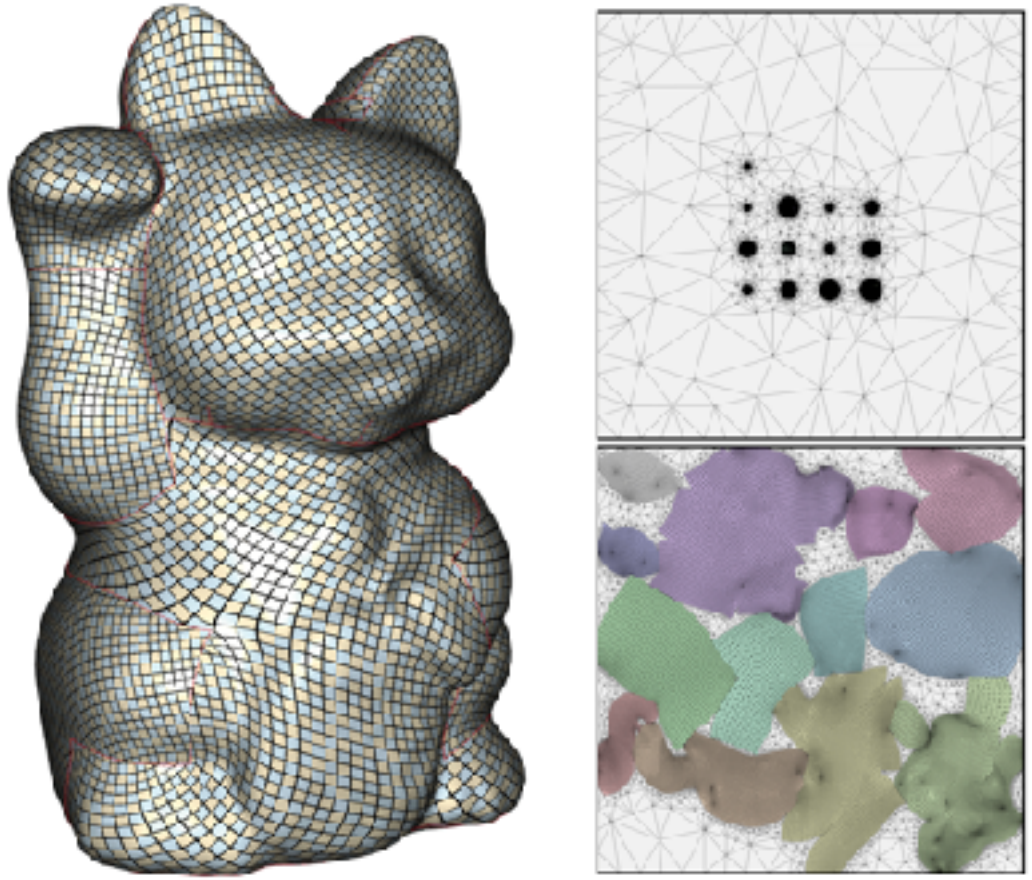

# Robustness

- Global Parameterization Dataset [Myles et al 2014]
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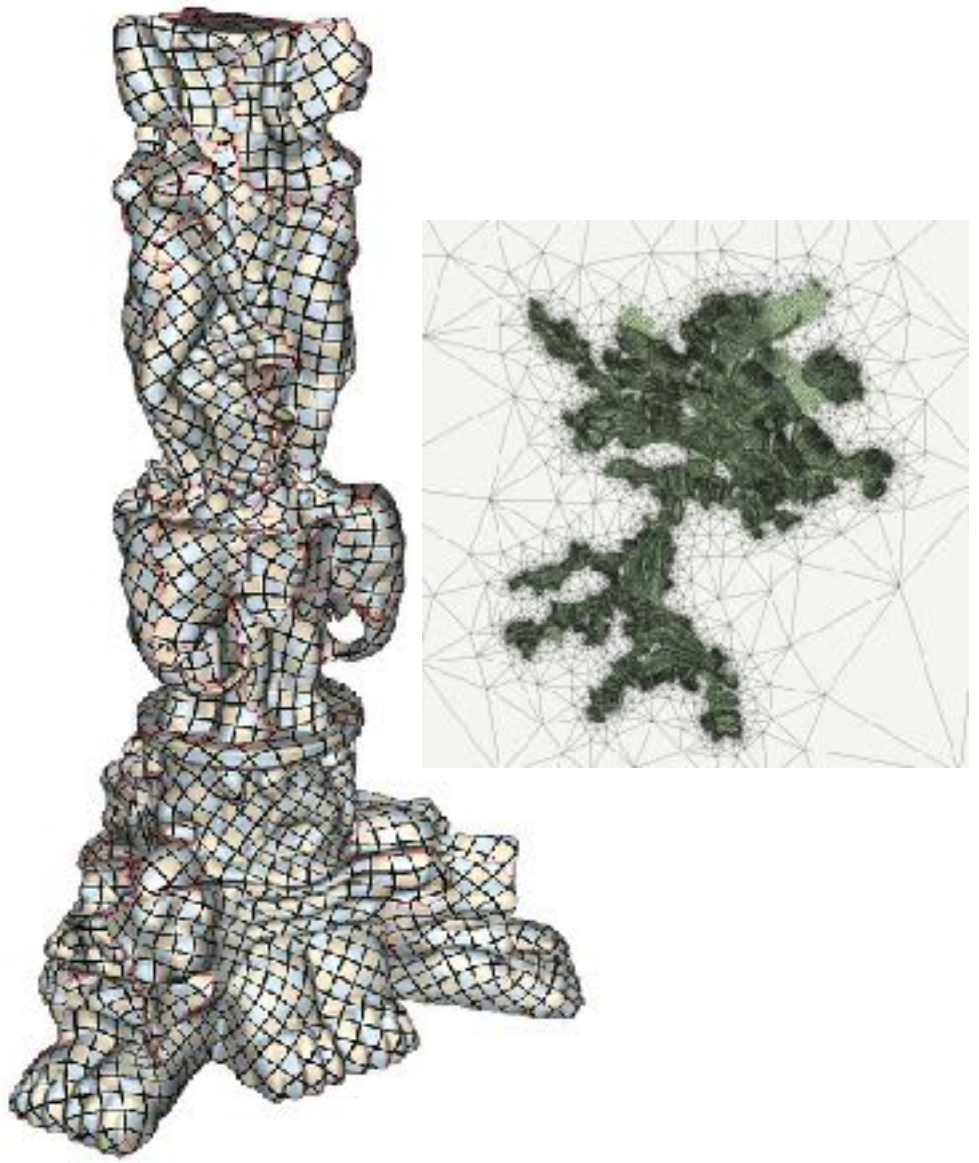

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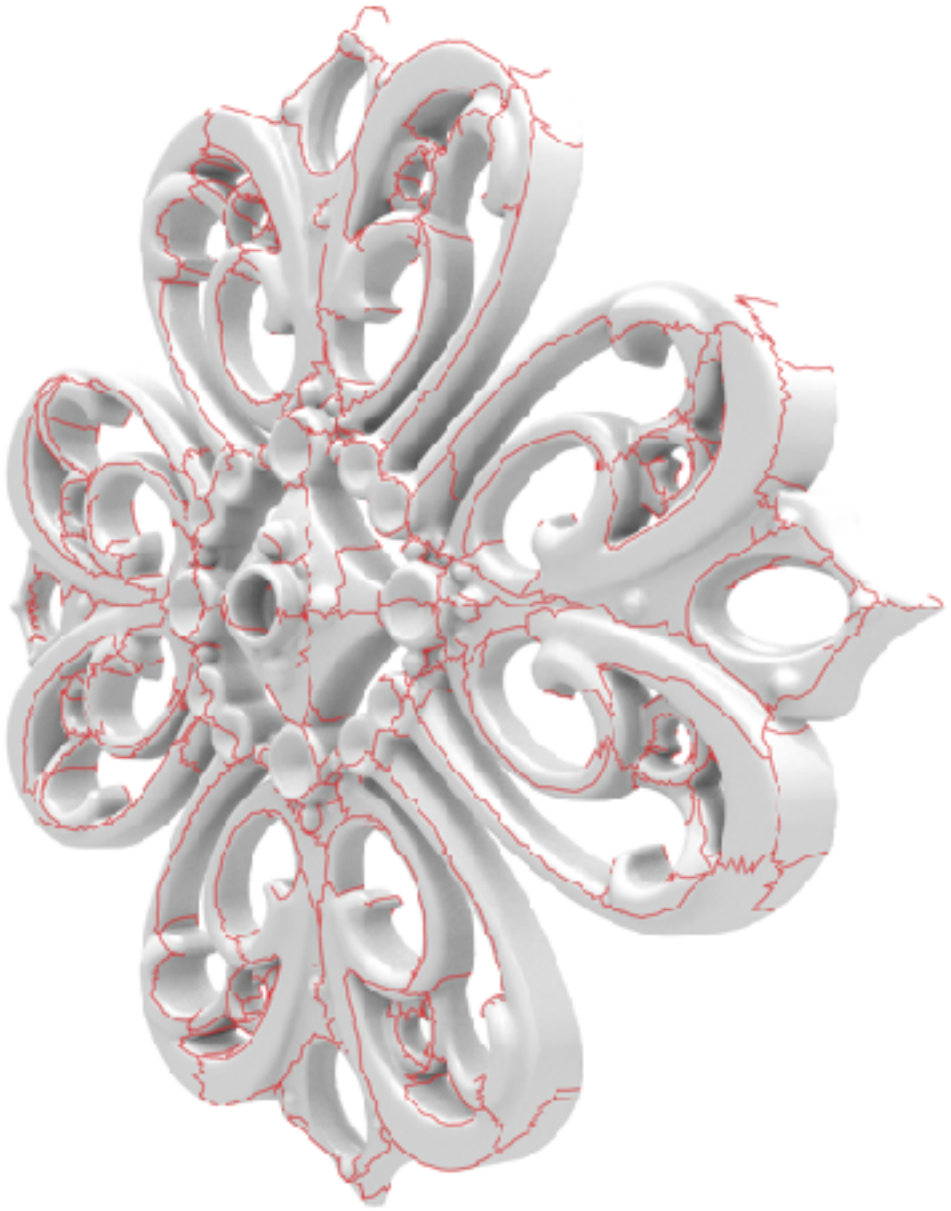

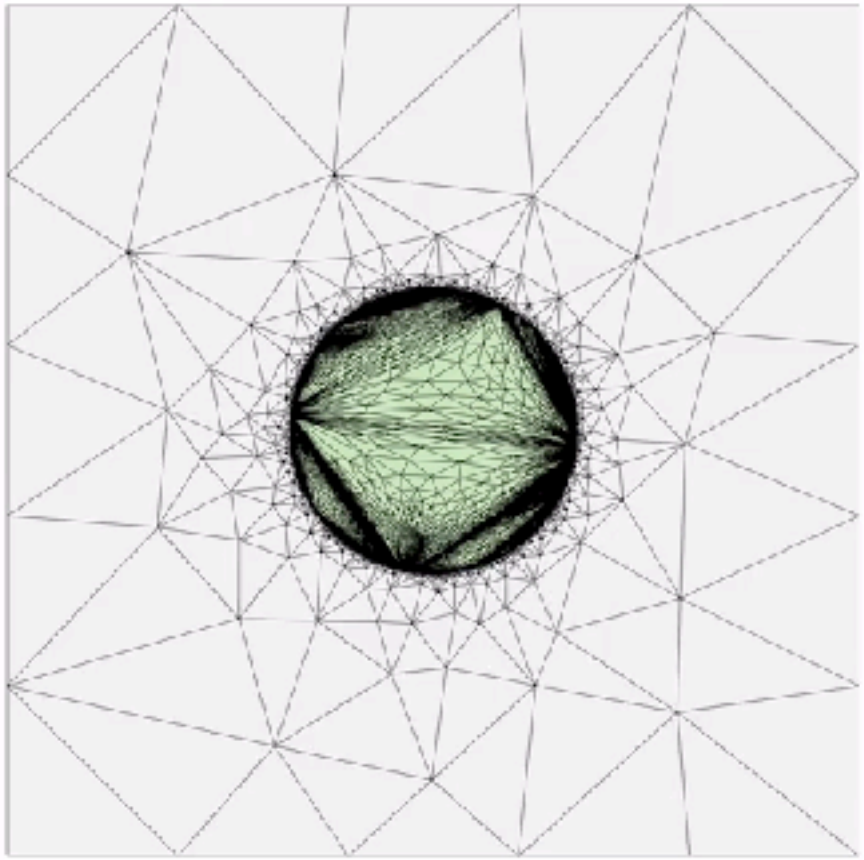

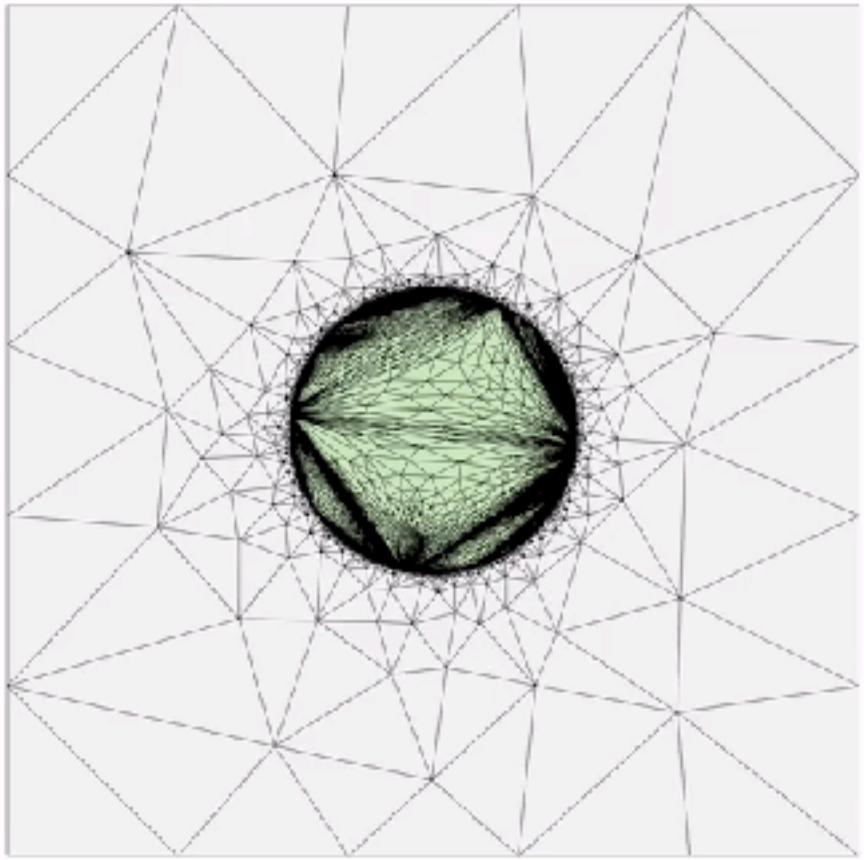

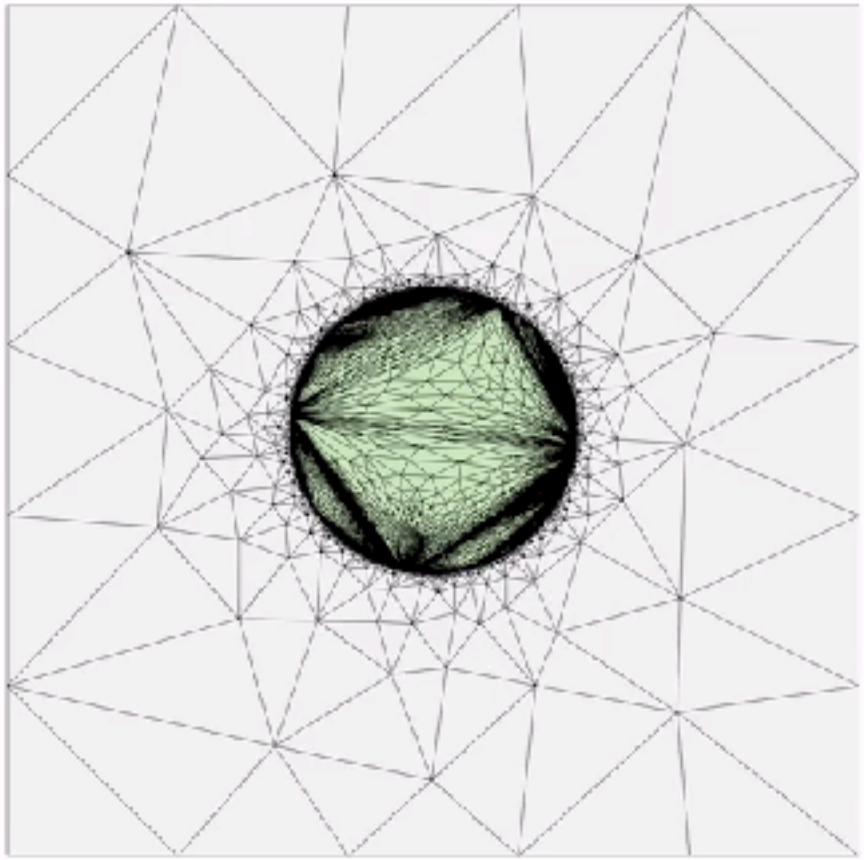

# Volume

- Same Formulation
- Different Tessellation
  - TetGen [Si 2015]
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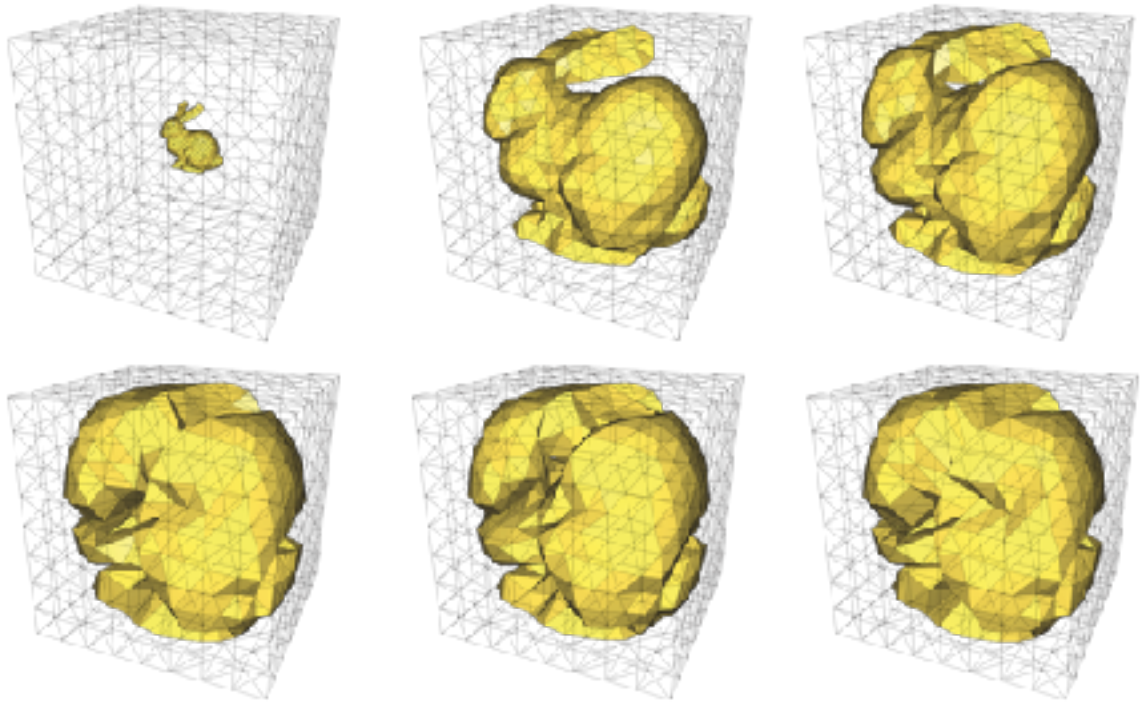

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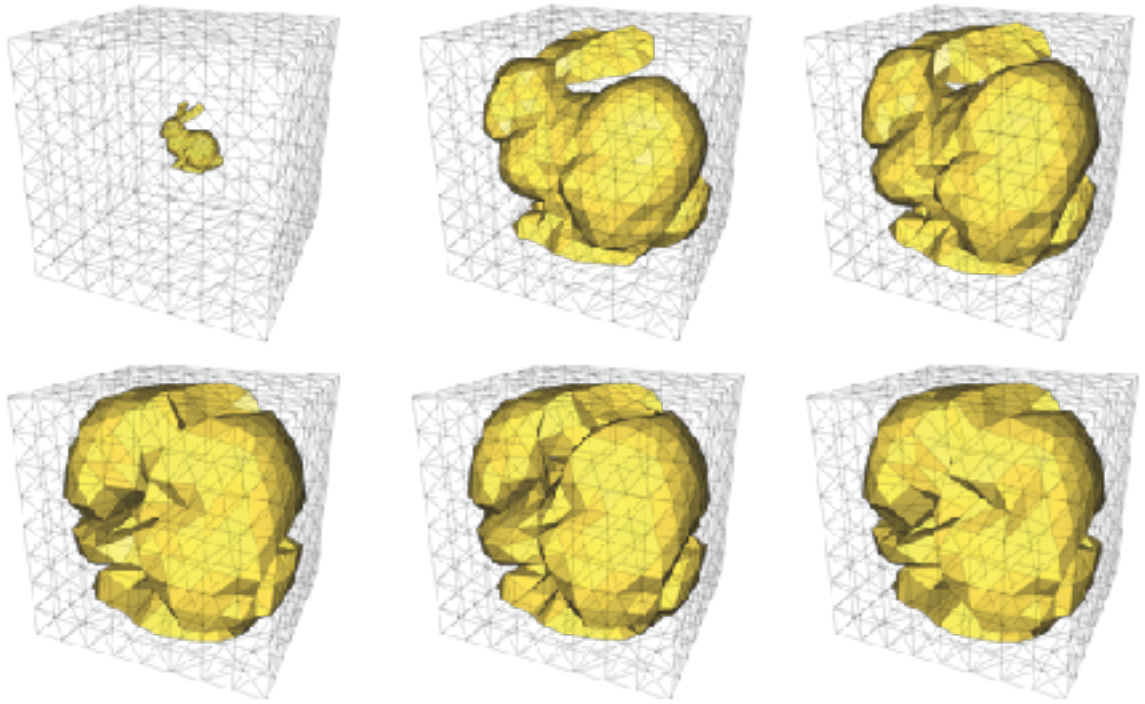
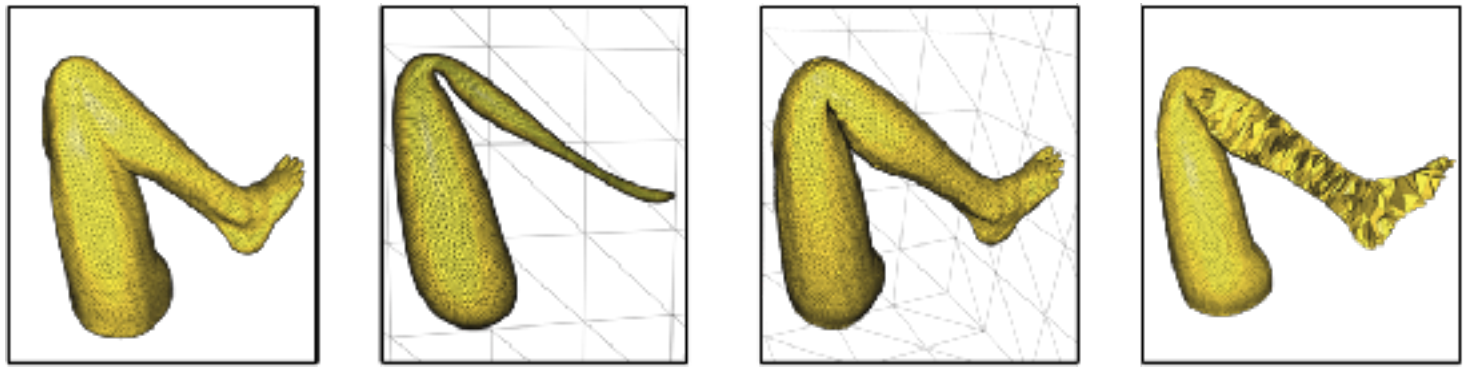
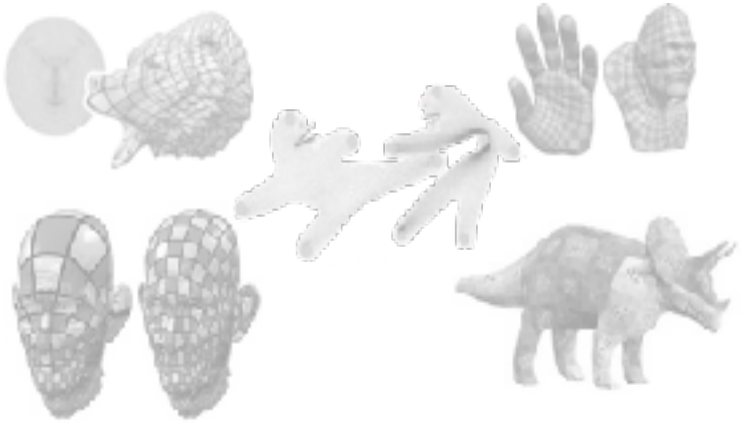
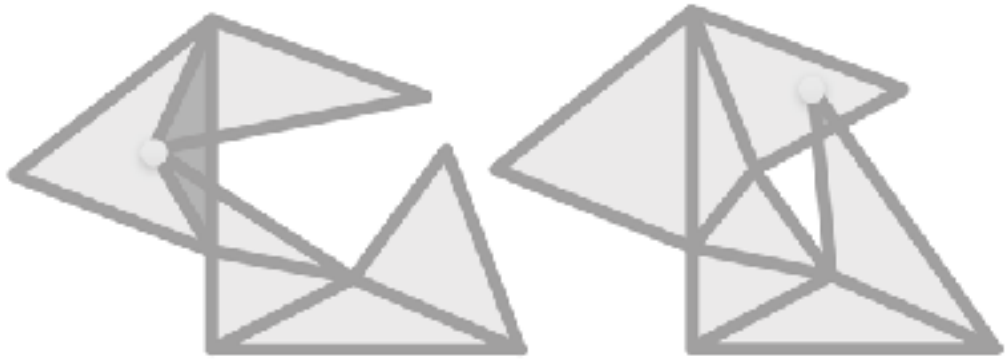






Locally Injective Maps



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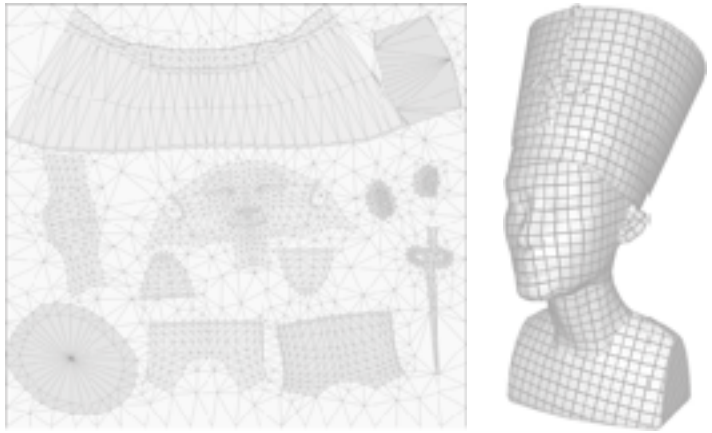
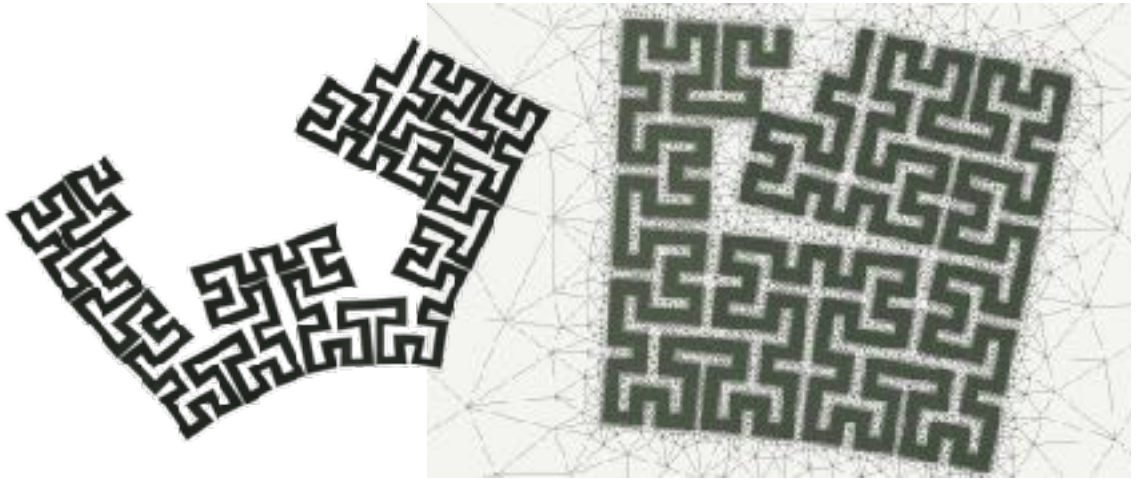
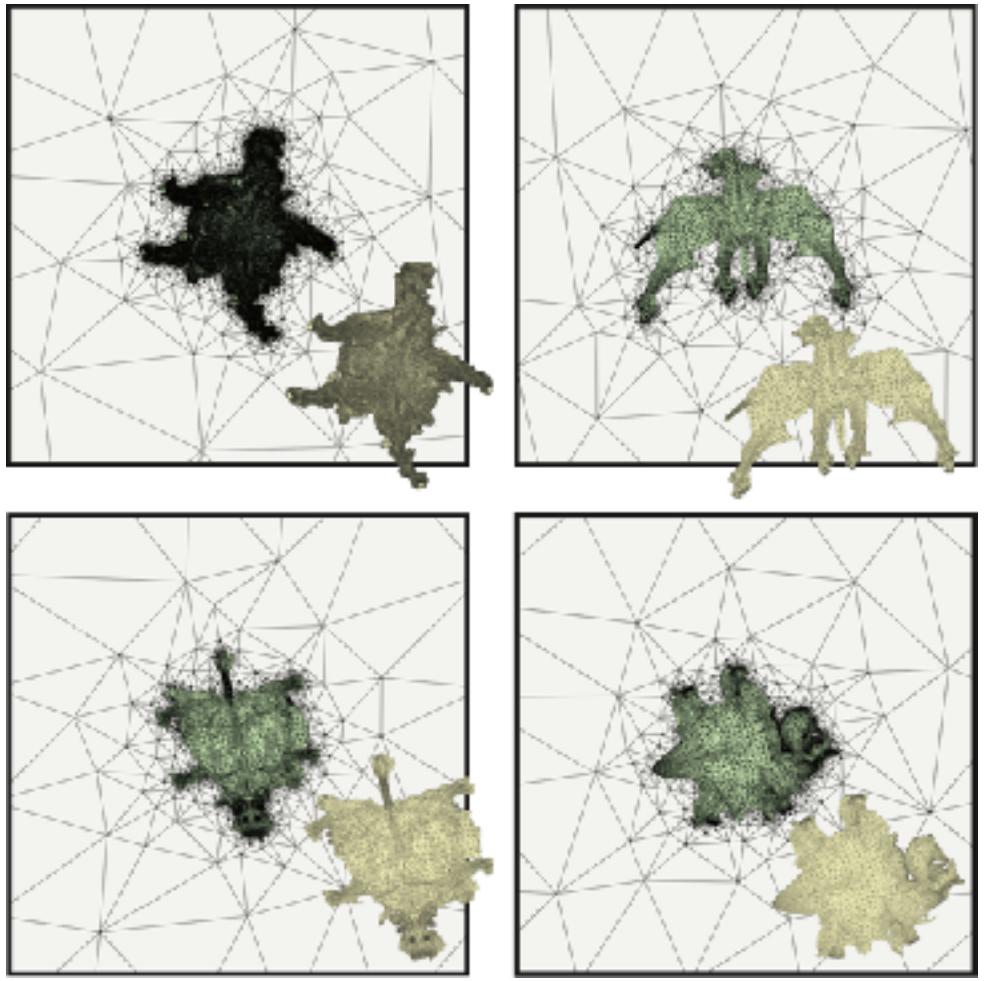


Results



Comparisons

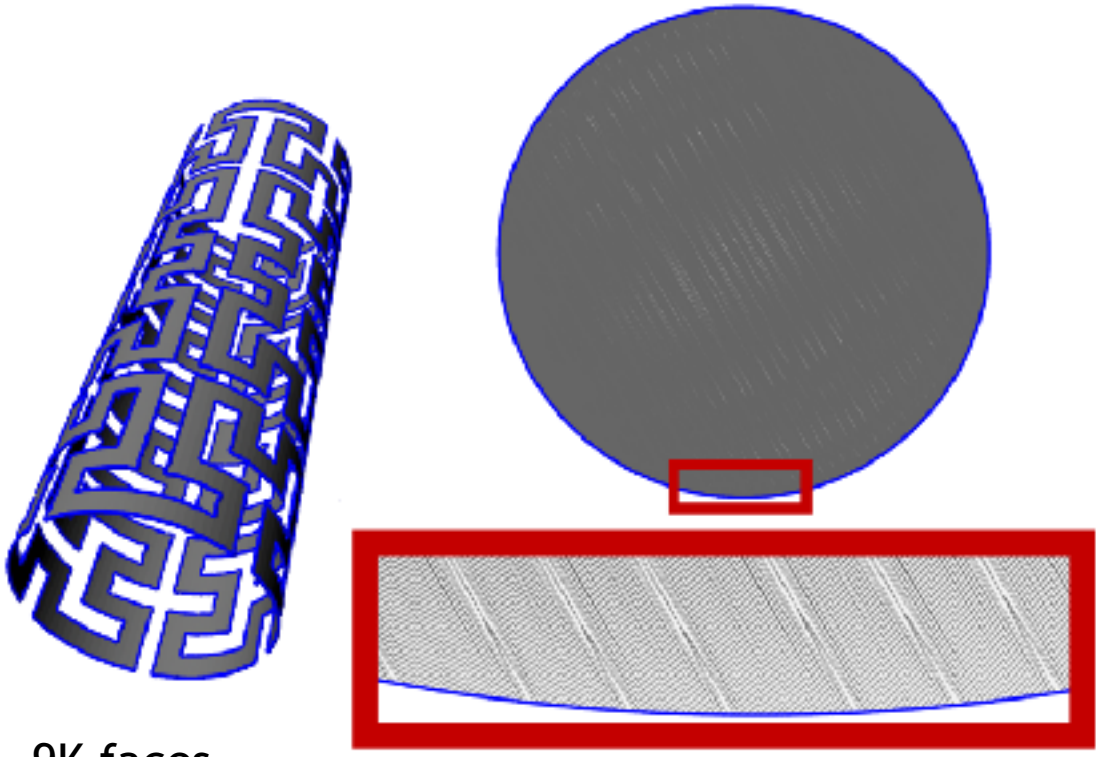
# Quality



Ours

[Smith and Schaefer 2015]

# Quality



9K faces

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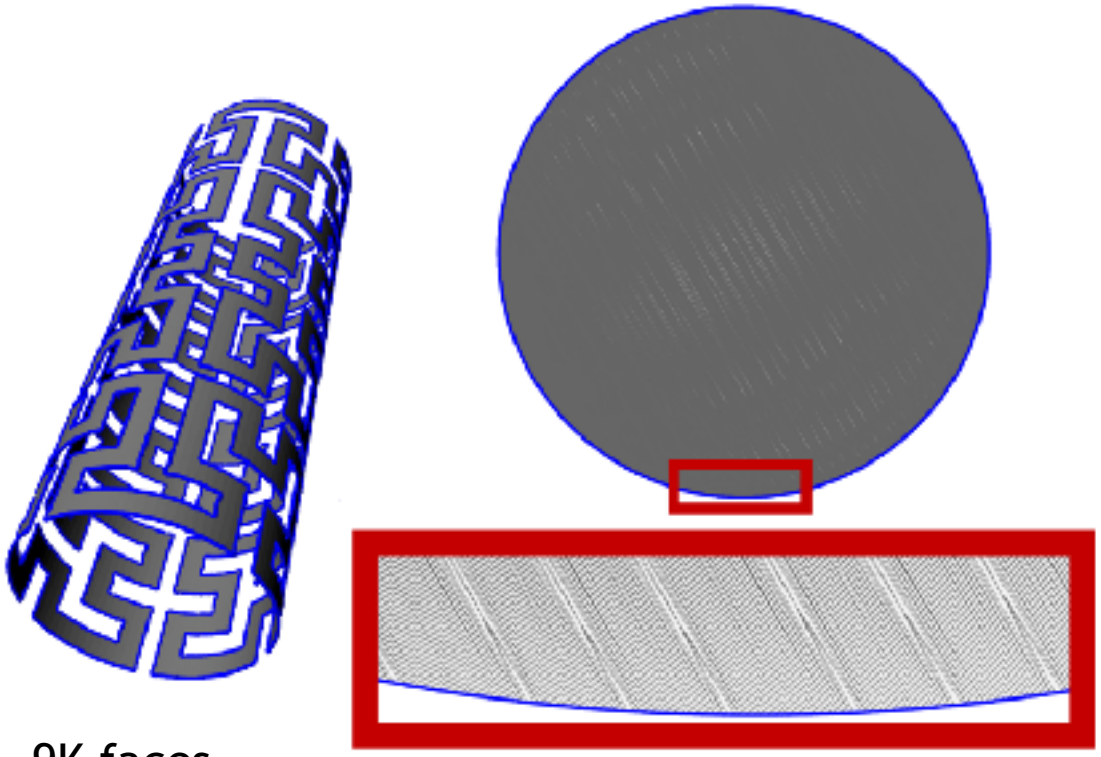

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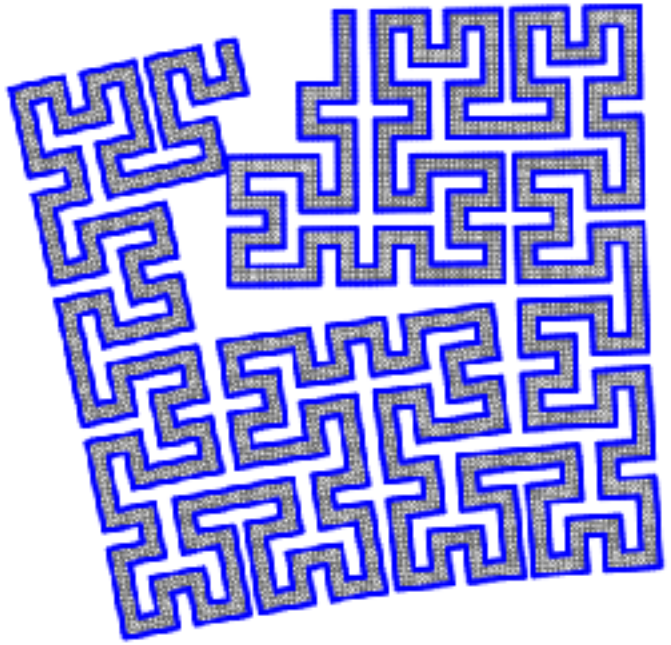

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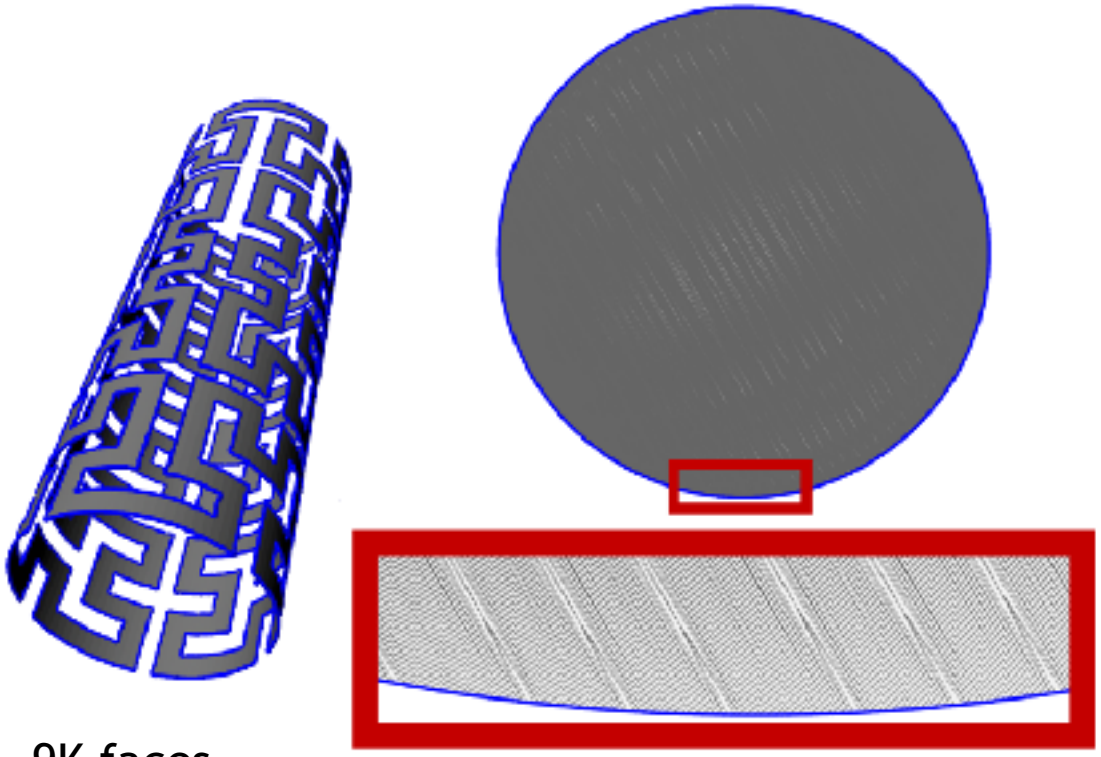

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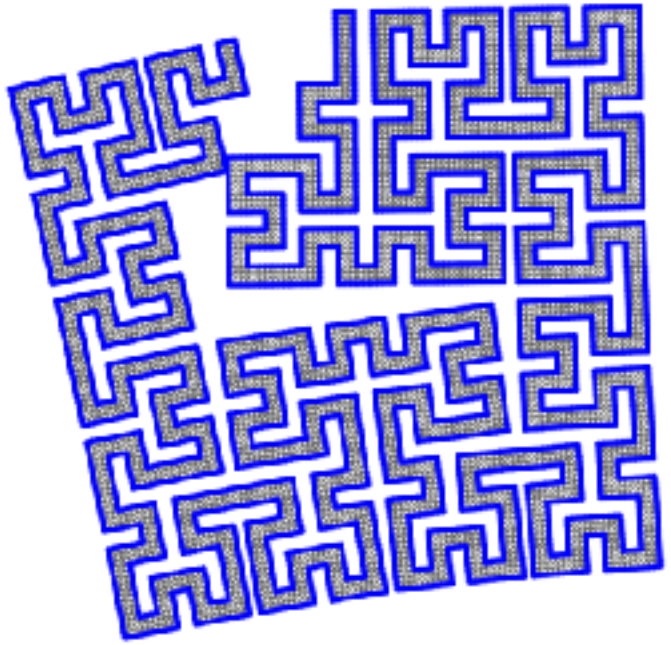
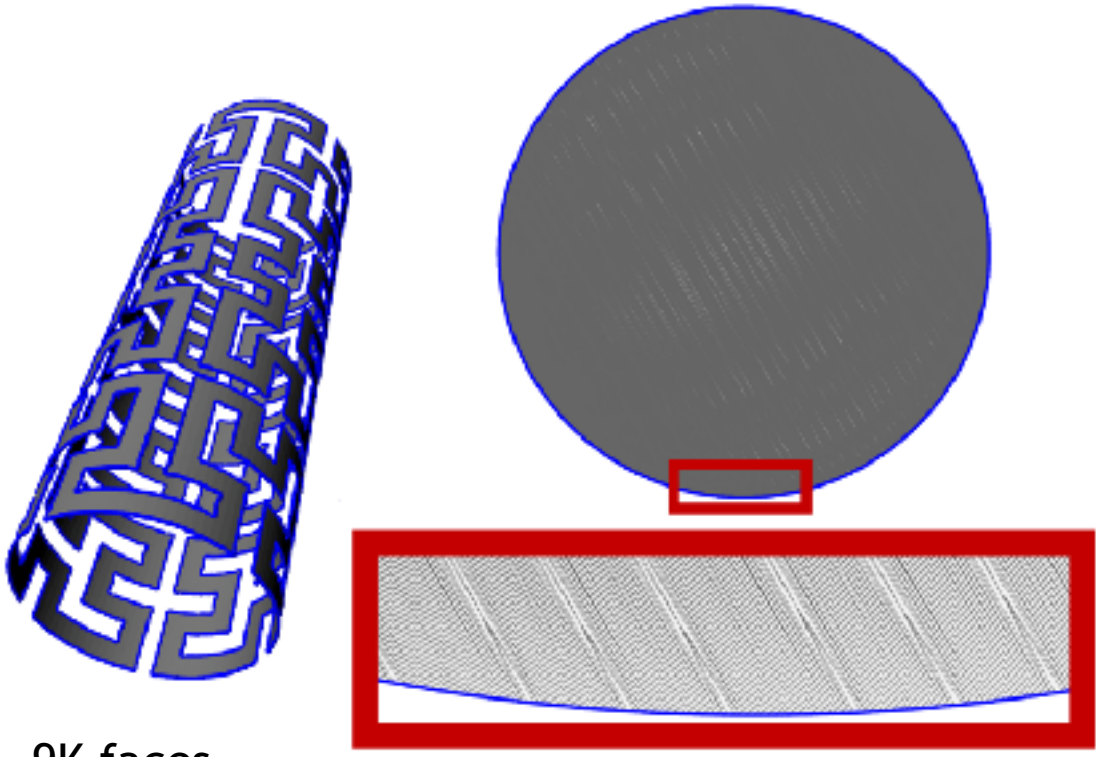
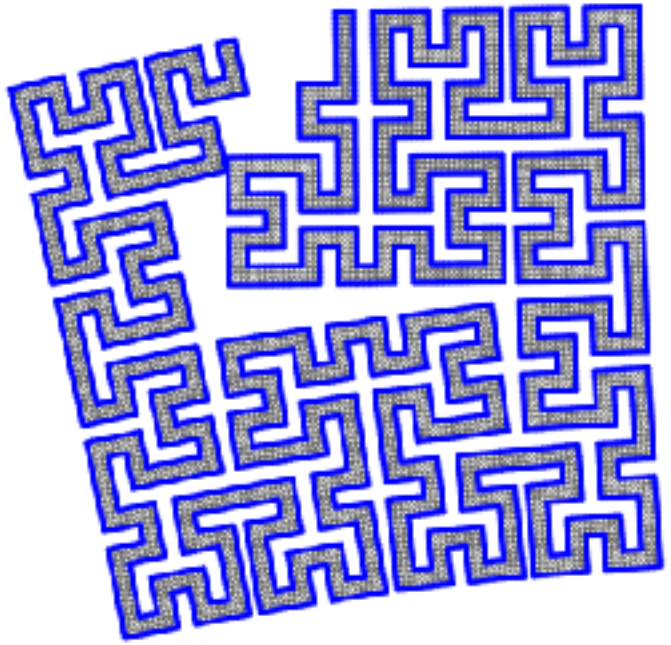


[Smith and Schaefer 2015]: 2 hour 20 min

# Quality



9K faces

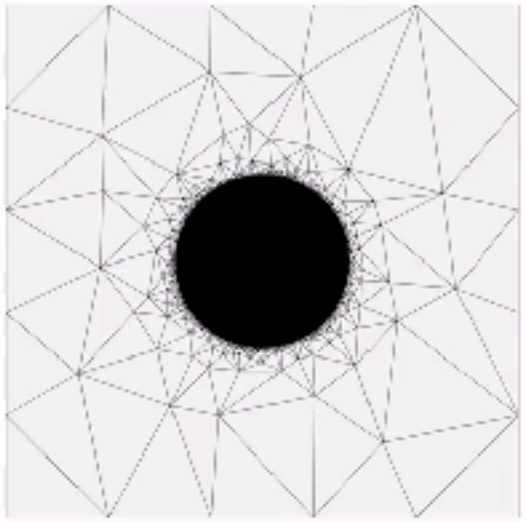


[Smith and Schaefer 2015]: 2 hour 20 min

< 2 min

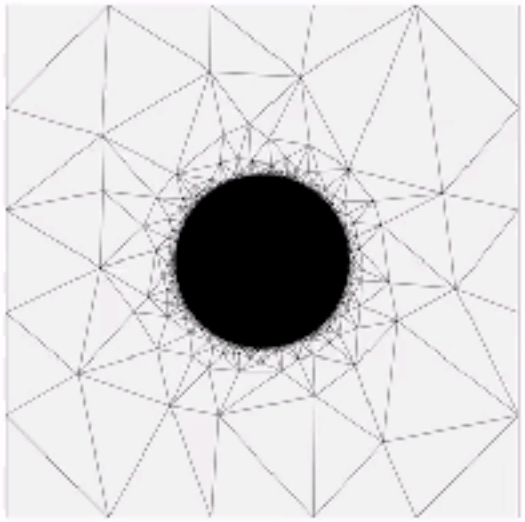
# Efficiency

80K faces



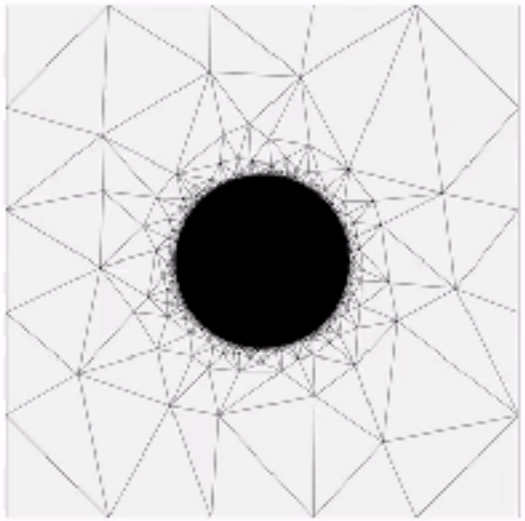
# Efficiency

80K faces



# Efficiency

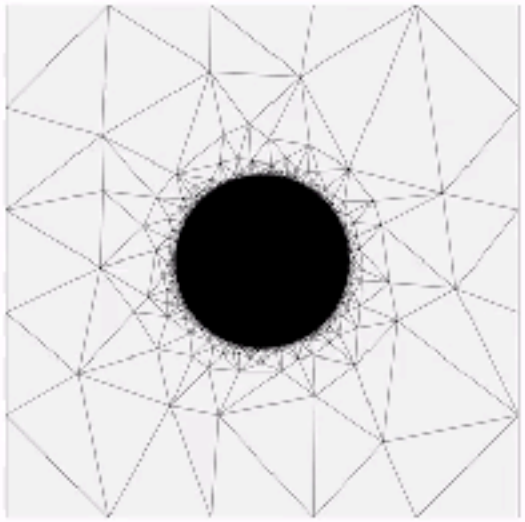
< 40 min



# Efficiency

< 40 min

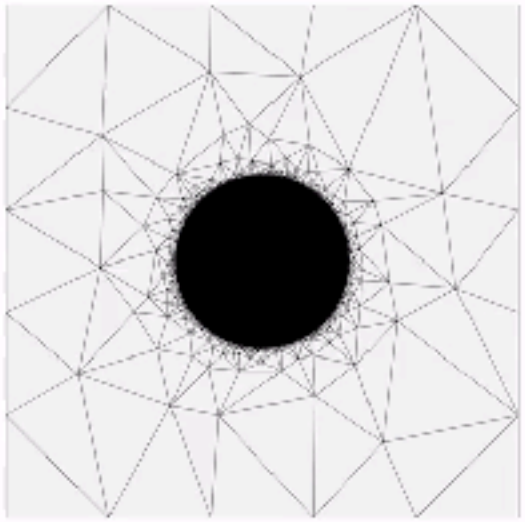
Over 5 days!



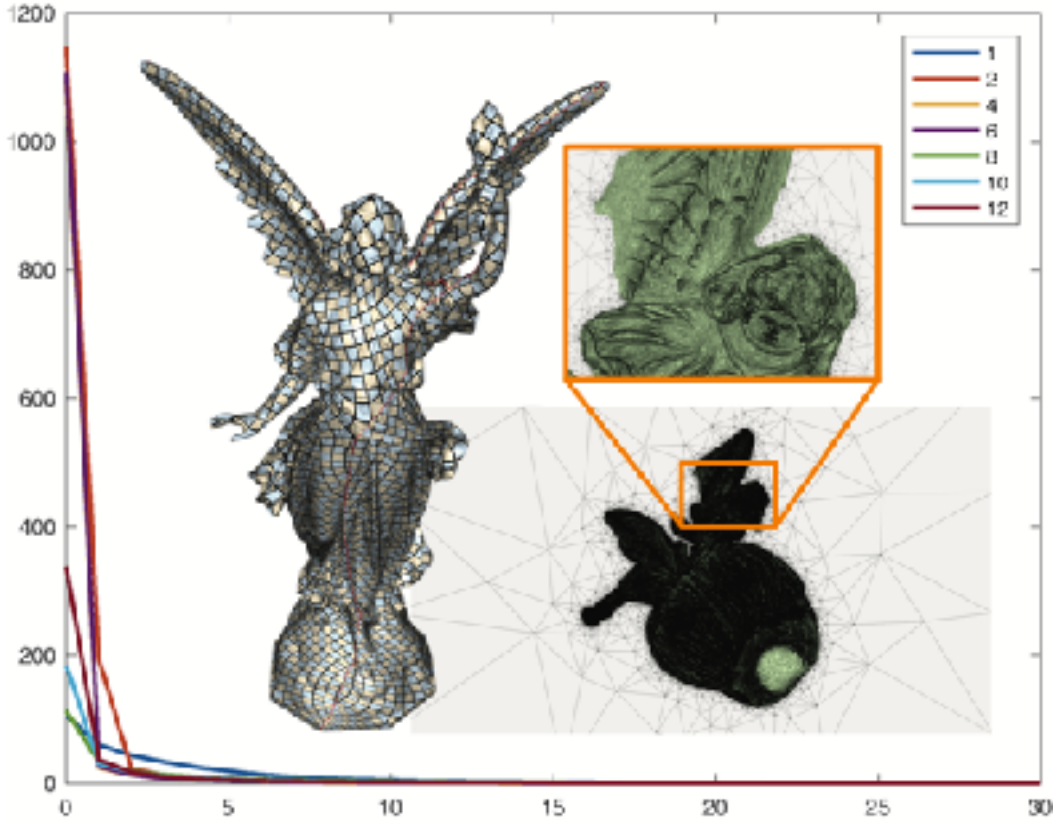
# Efficiency

< 40 min

Over 5 days!



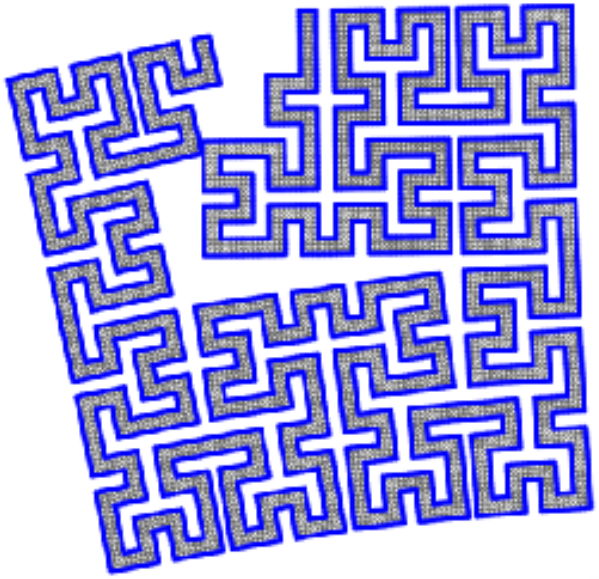
# Scalability




- Inherited from SLIM [Rabinovich et al. 2017]

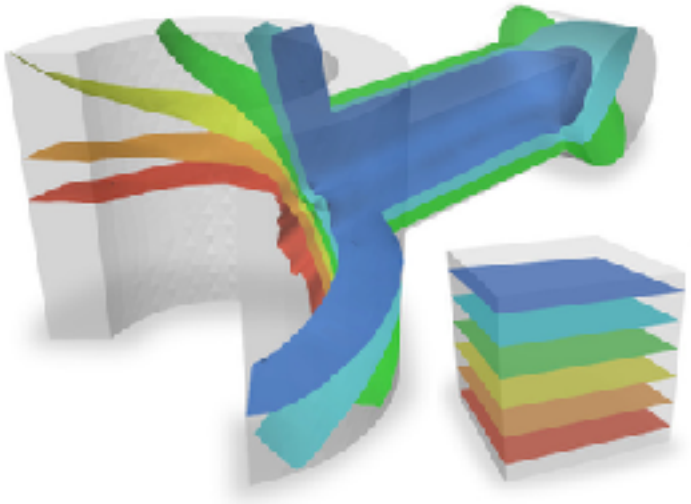
# Limitations

- Local Minimum

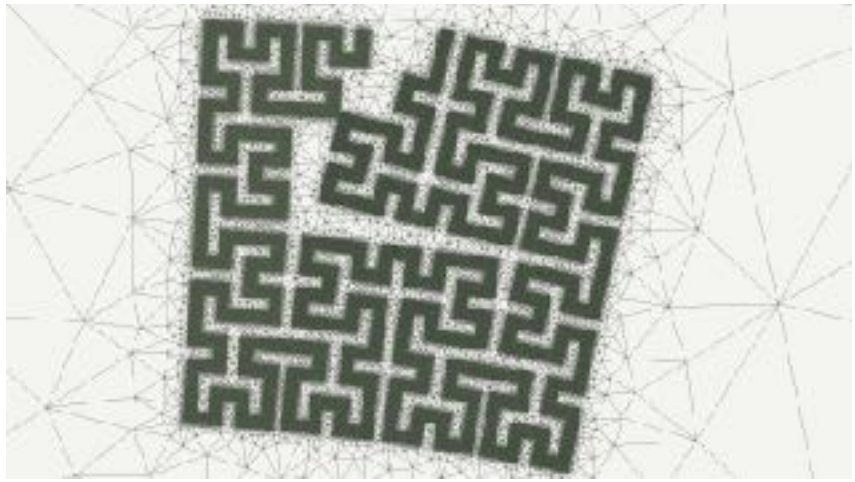


- Valid Initialization
  - Easy in 2D
  - Hard in 3D

- Implementation in 3D
  - Less Efficient
  - More Involved



# SCAF: Simplicial Complex Augmentation Framework for Bijective Maps



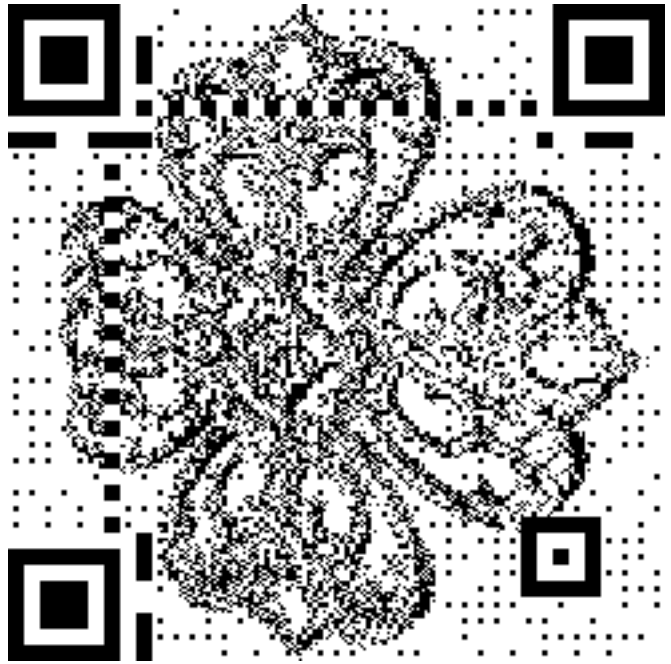
Efficient

Scalable

Robust

# Thank You!

The authors acknowledge funding from the NSF CAREER awards IIS-1652515 and IIS- 1148976, and a gift from Adobe. We would like to thank Michael Rabinovich and Roi Poranne for providing the source code and Lucy models for [Rabinovich et al. 2017], Leonardo Sacht for providing the source code and Leg model for [Sacht et al. 2013], and the anonymous reviewers for their insightful comments and suggestions.



https://github.com/jiangzhongshi/scaffold-map