SCUT

基于等几何分析的结构拓扑优化

王英俊 副教授

wangyj84@scut.edu.cn

华南理工大学 机械与汽车工程学院

提纲

拓扑优化简介

----结构优化设计分类

结构优化

尺寸优化

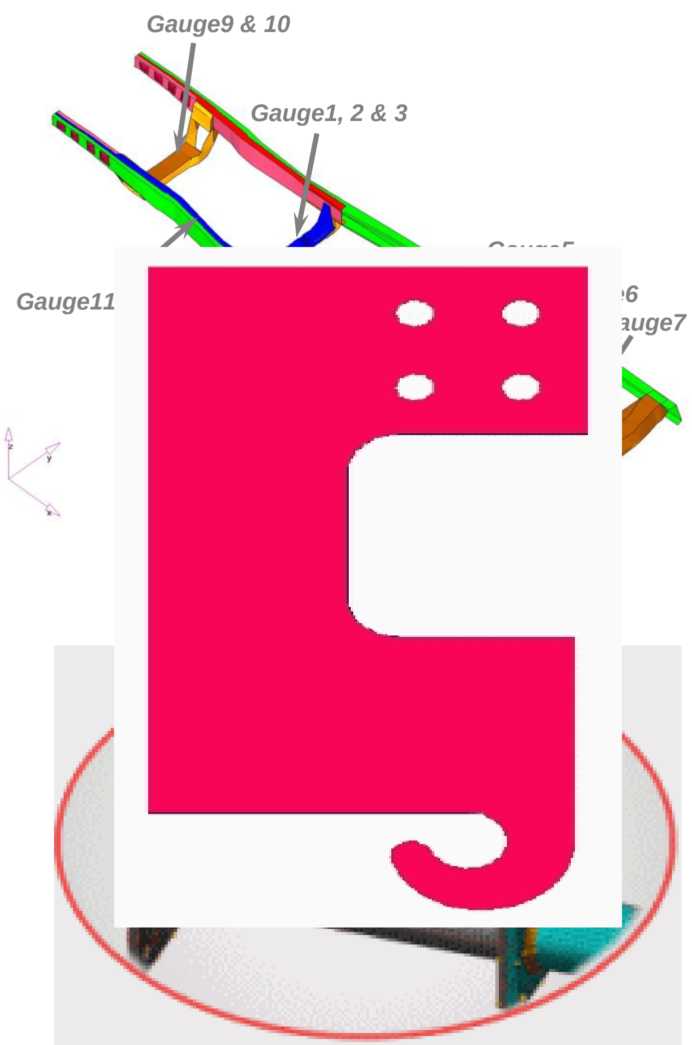
结构构型、形状不变，对各处结构尺寸（大小）进行优化。一种参数优化技术，寻找最优的设计参数组合，如横截面尺寸、厚度等，*技术简单*、*普遍应用*。

拓扑优化

给定设计区域，对材料分布情况进行优化设计。一种具有**创新性的概念设计**技术。

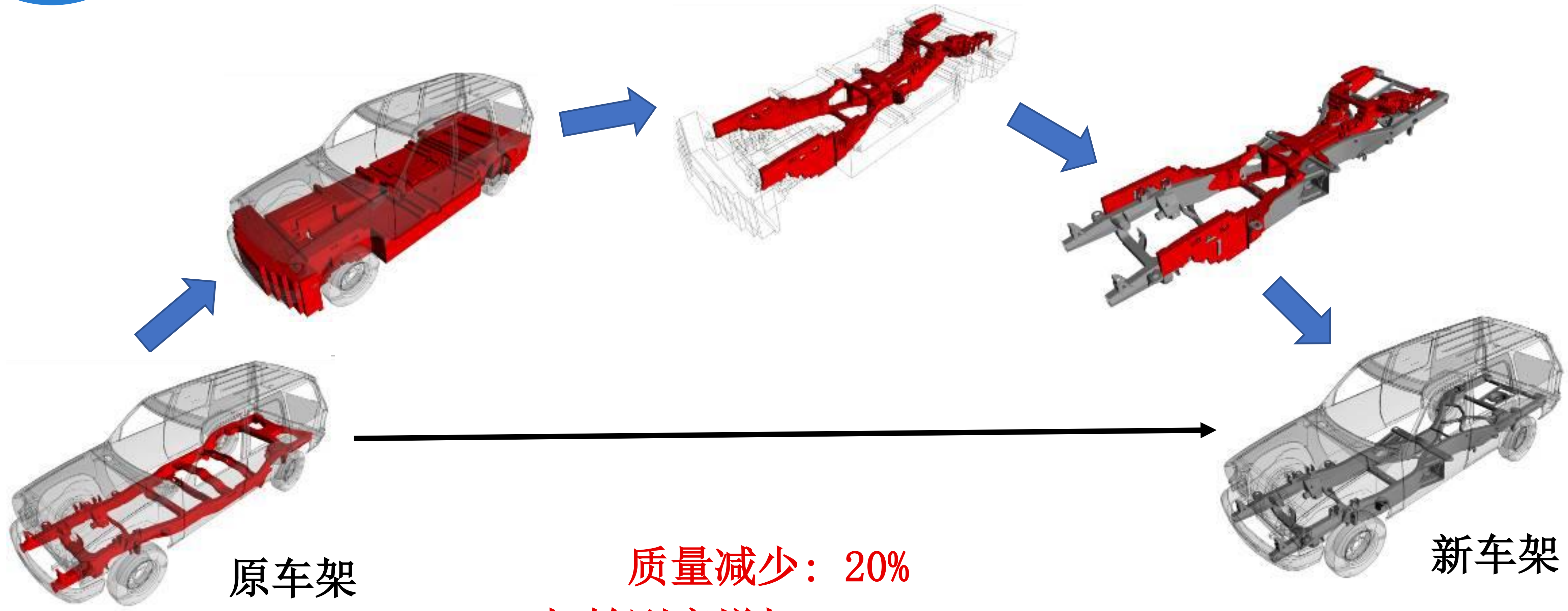
形状优化

结构构型不变，对各结构形状进行优化、*适合于详细设计阶段*。



1 拓扑优化简介

----拓扑优化实例



质量减少：20%

扭转刚度增加：31%

焊缝长度降低：50%

The Auto/Steel Partnership
Member Companies

Bethlehem Steel Corporation	Daimler Chrysler Corporation
Dofasco Inc.	Ford Motor Company
General Motors Corporation	Ispat Inland Inc.
National Steel Corporation	Rouge Steel Corporation
Stelco Inc.	United States Steel Corporation

1 拓扑优化简介

----拓扑优化的特点

- 是将寻求结构的最优拓扑问题转化为在给定的设计区域内寻求最优材料分布的问题。

- 通过拓扑优化分析，设计人员可以全面了解产品的结构和功能特征，可以有针对性地对总体结构和具体结构进行设计。

- 拓扑优化的最大优点是能在不知道结构拓扑形状的前提下，根据已知边界条件和载荷条件确定出较合理的结构形式，它不涉及具体结构尺寸设计，但可以提出最佳设计方案。

目标函数（想要的是什么？）

$$\min f(X)$$

设计变量 (我可以改变什么来改进结构性能?)

$$X_i^L \le X_i \le X_i^U \quad i = 1, 2, 3, \ldots N$$

设计约束 (必须保证哪些性能指标?)

$$g_j(X) \le 0 \qquad j = 1, 2, 3, \ldots, M$$

注：函数$f(x)$, $g_i(x)$是连续的, 可以是线性或非线性, 显式或隐式的,

例子:

显式 $y(x) = x^2 - 2x$

隐式 $y^3 - y^2x + yx = 0$

1 拓扑优化简介

----拓扑优化的建模与求解

优化建模方法

- 变密度法：虚材料、密度惩罚函数、最普遍
- Level Set法 (水平集法)：水平集函数隐式表达
- ESO(进化法)：启发式、进化策略、高σ增低σ删
- MMC(移动可变形组件法)：显示构件参数、形状拓扑统一
- ICM(独立映射法)：独立拓扑变量、映射为连续变量
- GA(遗传算法)：非梯度、复杂问题
- ……

优化求解方法

- OC法(优化准则法)
- MMA法(移动渐进线法)
- SLP(序列线性规划法)
- SQP(序列二次规划法)
- …………

提纲

1 拓扑优化简介

2 等几何分析

3 等几何结构拓扑优化

4 总结与展望

等几何分析

----传统产品设计分析流程

产品设计—形状设计 & 结构性能

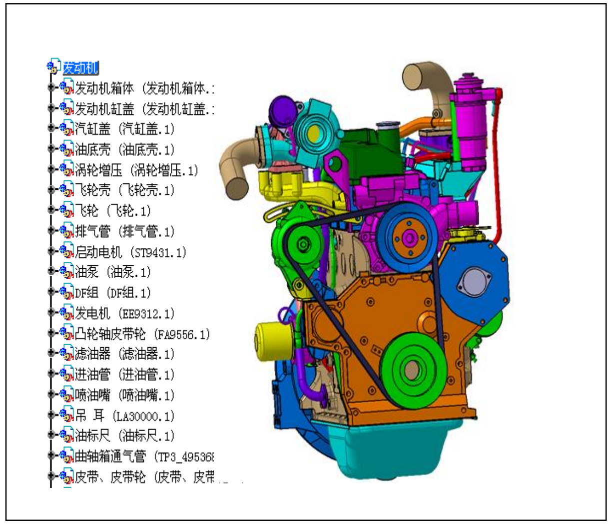
设计工具—功能完备 & 效率高

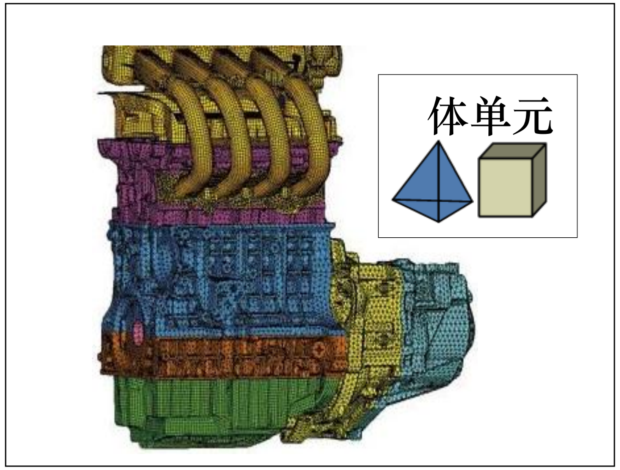
设计
- CAD (较完备)
  - CSG 特征历史
  - BREP 边界形状
- 有限元法 CAE (较完备) — 有限单元

表达差异大

两软件：一设计、一分析 ——→ 效率低

如何高效率的设计工具?





CAD 系统

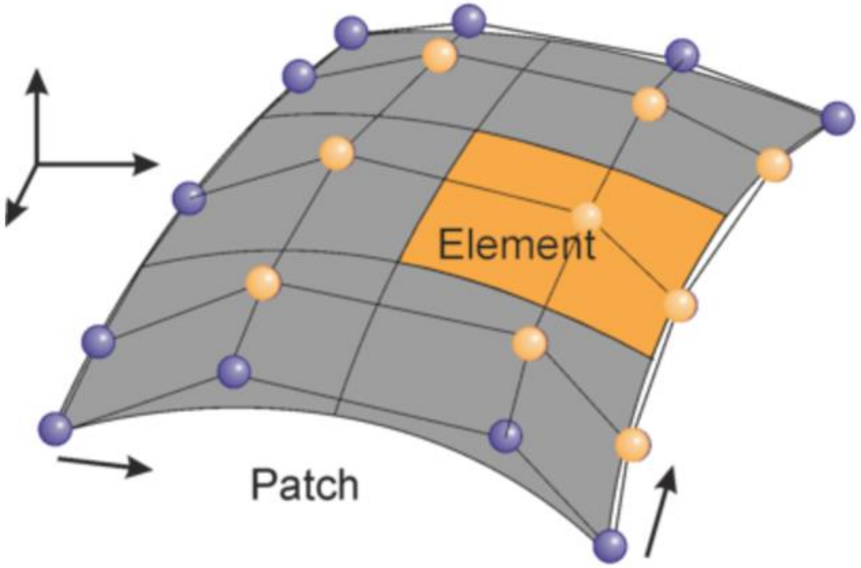
开始 ↓ 几何建模 4% ↓ 模型转换、简化、修复 53% ↓ 网格生成 20% → 边界条件、参数设置 6% ↑ 计算求解 4% ↑ 后处理 5% ↑ 其他 8% ↑ 结束

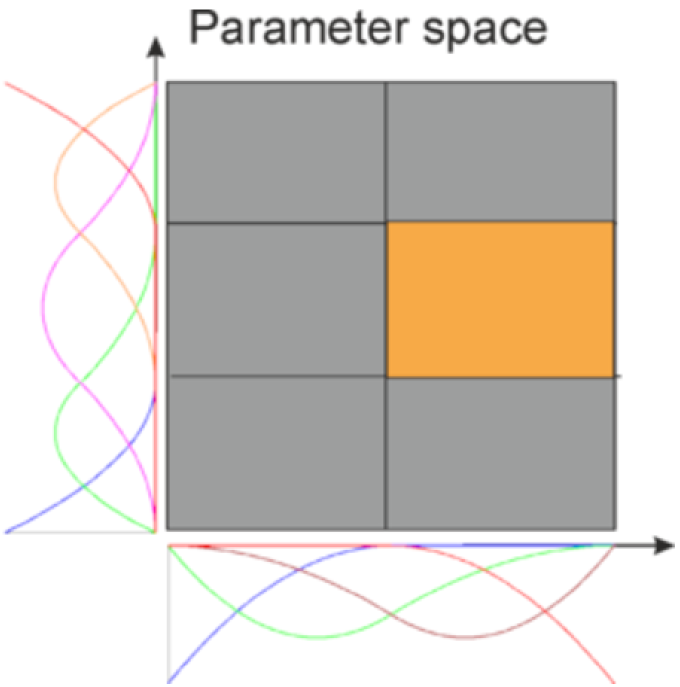
CAE 系统

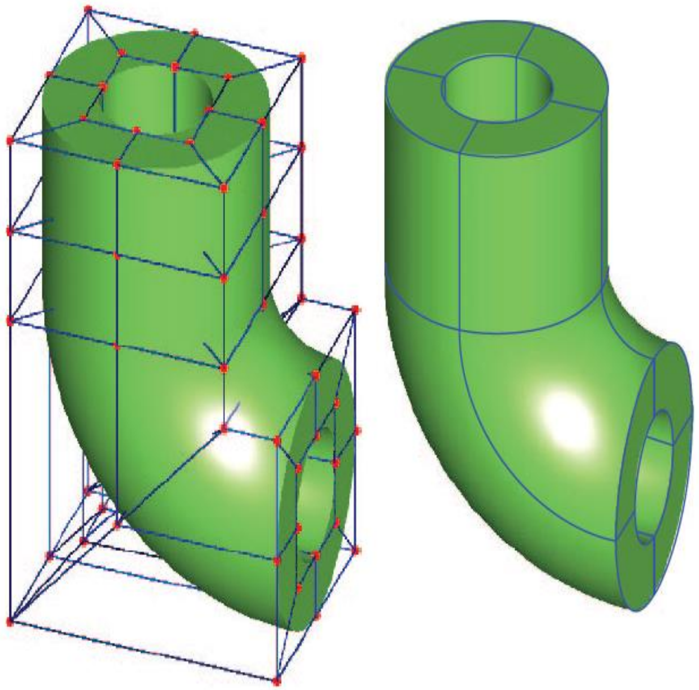
2 等几何分析 ----简介

美国三院院士Thomas J.R. Hughes 等2005年提出等几何分析（Isogeometric Analysis，IGA）

是一种直接采用计算机CAD样条(B样条、NURBS等)基函数作为形函数，代替有限元法的插值函数（拉格朗日插值函数），对精确几何模型进行性能分析的数值方法，实现了CAD与CAE的无缝融合。







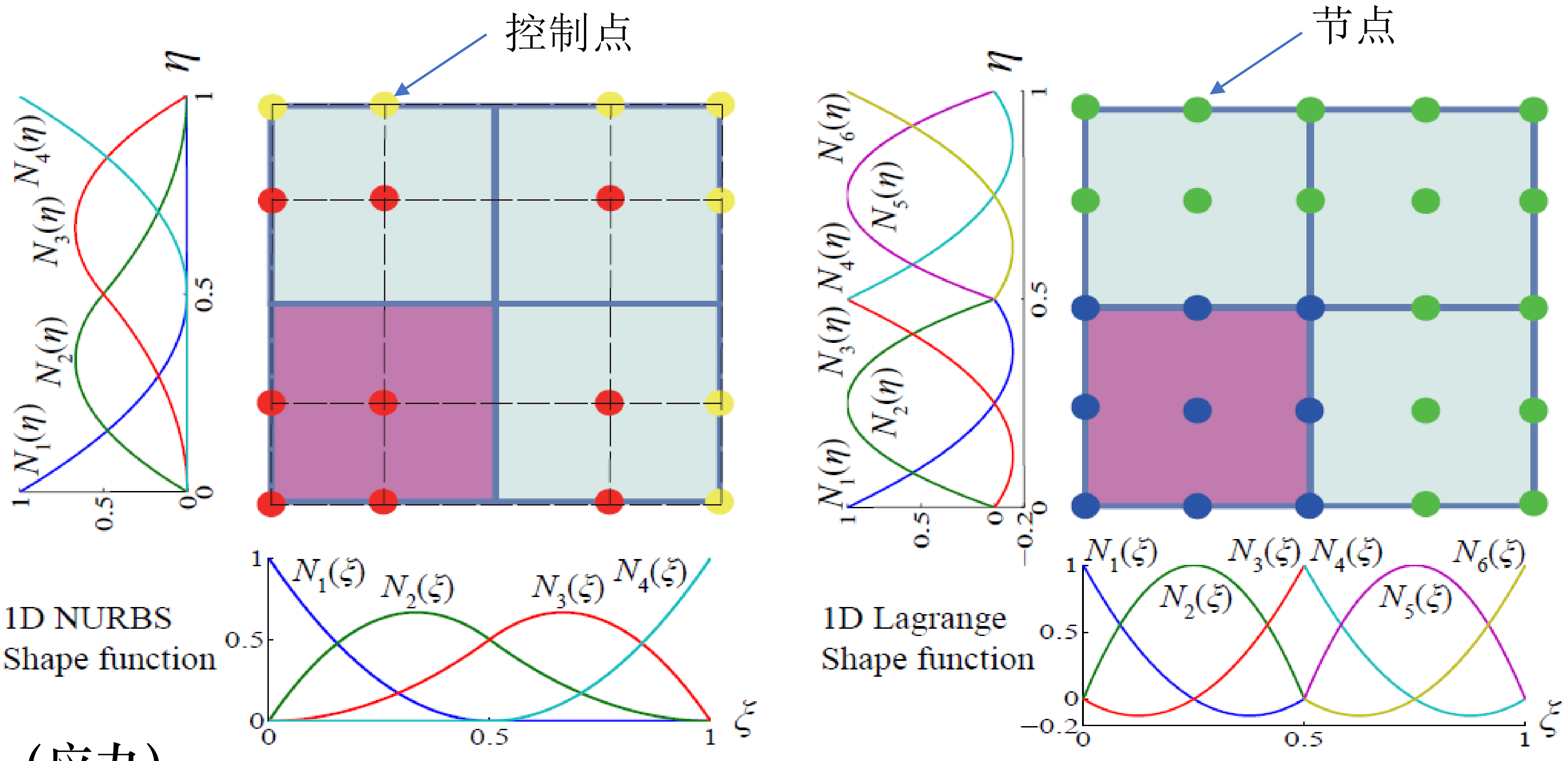
等几何分析 ----优点与特点

NURBS

- 单元间的高连续性：C^{p-k}
- 单位分解：$\sum_A N_A = 1$
- 局部支撑：（p+1）span
- 非端点插值：N_A may $\neq 1$ at A

Lagrange

- 单元间 C^0 连续(位移)，不连续（应力）
- 单元内部支撑
- 端点插值：$N_A = 1$ at A



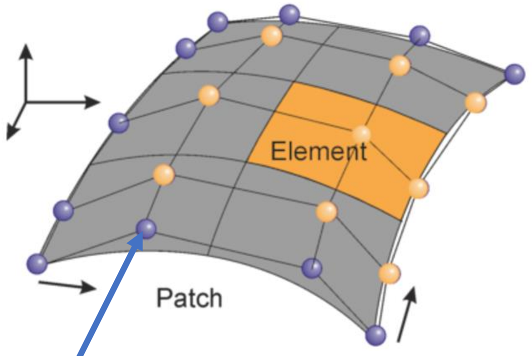
(a) Isogeometric element (b) Lagrange element

精度高、自由度少（高效率）

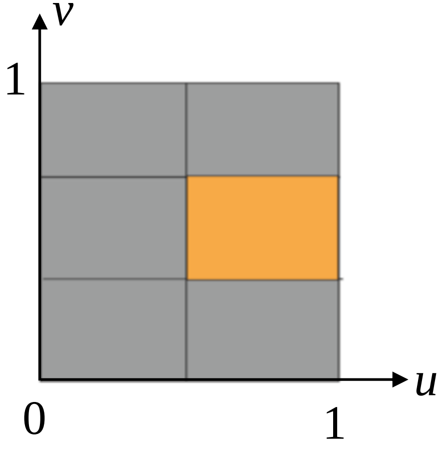
2 等几何分析

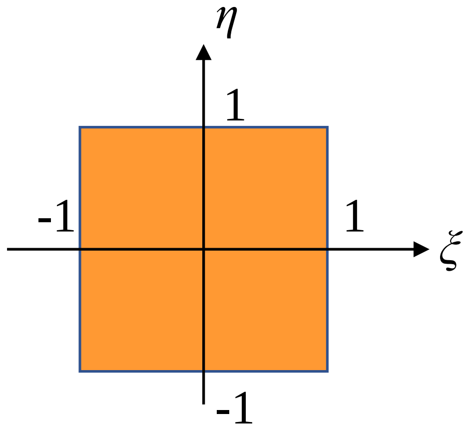
----IGA计算单元刚度矩阵

$$K_e = \int_{\Omega_e} B^T D B d\Omega = \int_{\widehat{\Omega}_e} B^T D B |J_1| d\widehat{\Omega} = \int_{\overline{\Omega}_e} B^T D B |J_1||J_2| d\overline{\Omega}$$



载荷、约束
控制点





$$J_1 = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} \end{bmatrix}$$

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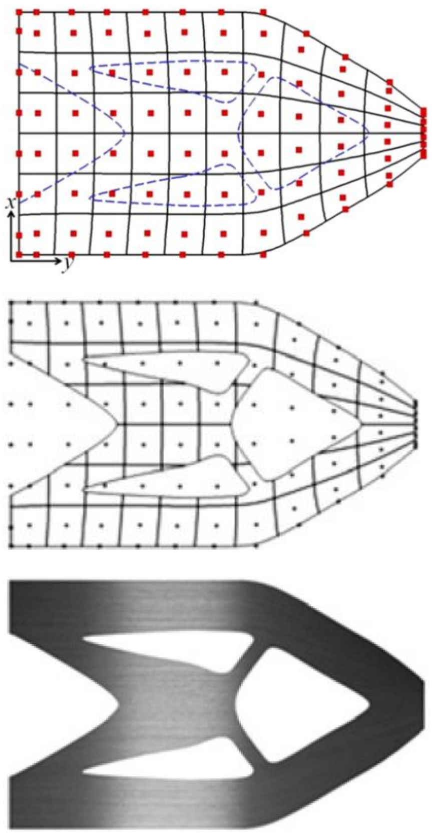
等几何结构拓扑优化

----简介

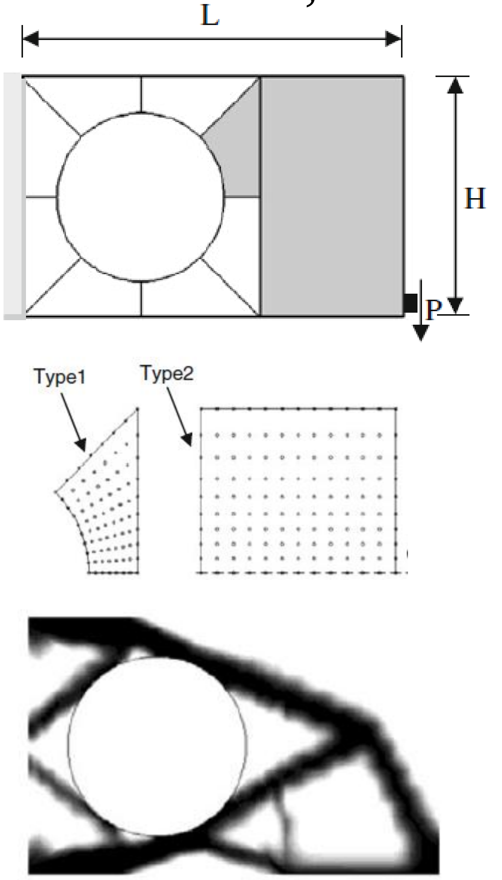
等几何结构拓扑优化：以等几何分析代替有限元法进行结构拓扑优化的方法

Seo et al., 2010



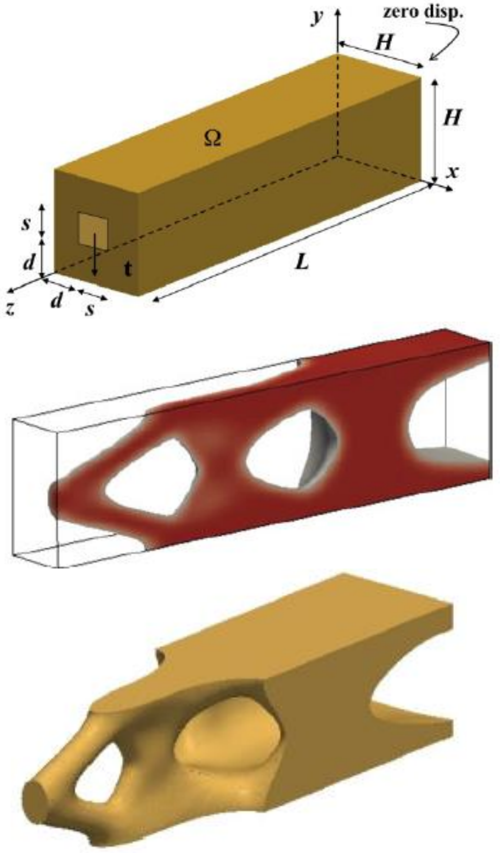
Trimmed spline

Hassani et al., 2012



IGA+SIMP

Dedè et al., 2012

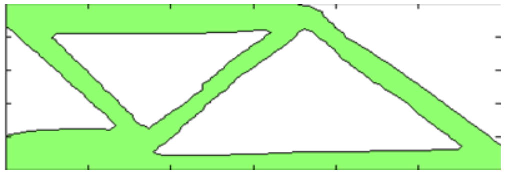


IGA+Phase Field Model

Wang et al., 2016-2018

IGA+Level Set

IGA+SIMP+Multiscale

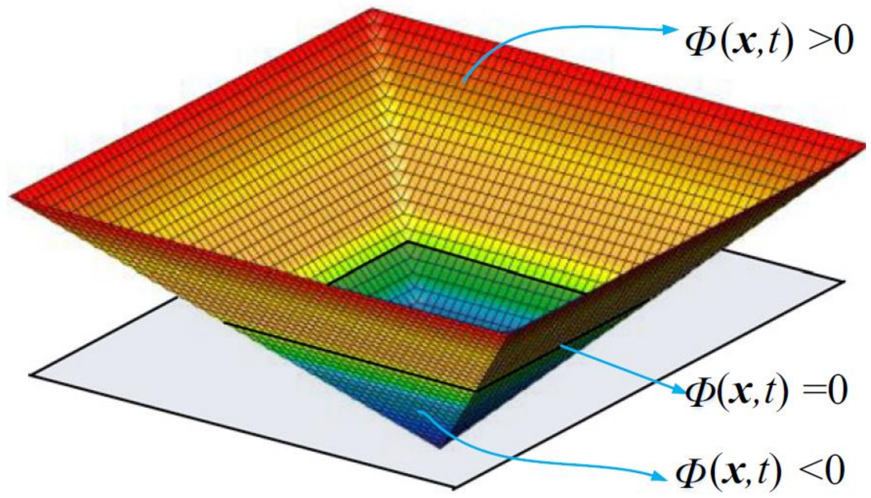


IGA+MMC

3 等几何结构拓扑优化

----IGA+Level set—水平集法

$$
\begin{cases}
\Phi(\boldsymbol{x},t) > 0 & \forall \boldsymbol{x} \in \Omega \backslash \partial\Omega & \text{(inside)},\\
\Phi(\boldsymbol{x},t) = 0 & \forall \boldsymbol{x} \in \partial\Omega \bigcap D & \text{(boundary)},\\
\Phi(\boldsymbol{x},t) < 0 & \forall \boldsymbol{x} \in D \backslash \Omega & \text{(outside)}.
\end{cases}
$$



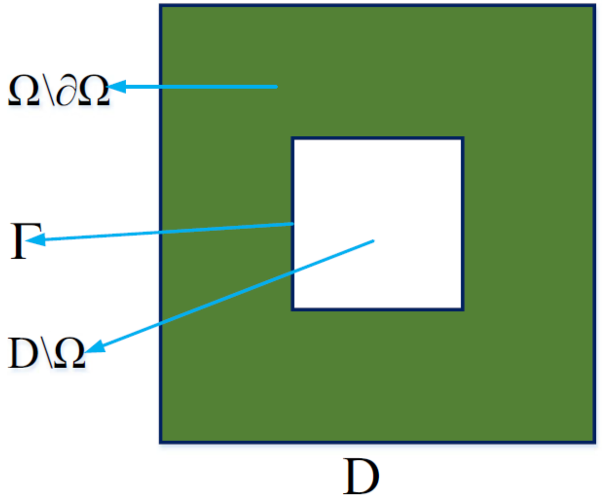
用高一维的水平集函数描述几何形状
零水平集线表示实体部分与空白部分的分界线

Hamilton-Jacobi 方程

$$\frac{\partial \Phi(\boldsymbol{x},t)}{\partial t} - v_n |\nabla \Phi(\boldsymbol{x},t)| = 0, \quad \Phi(\boldsymbol{x},0) = \Phi_0(\boldsymbol{x})$$

$$v_n = -\nabla \Phi(\boldsymbol{x},t) / |\nabla \Phi(\boldsymbol{x},t)|$$

求解该方程，使边界沿法线方向移动，改变形状



3 等几何结构拓扑优化

--- IGA+Level set —NURBS参数化水平集

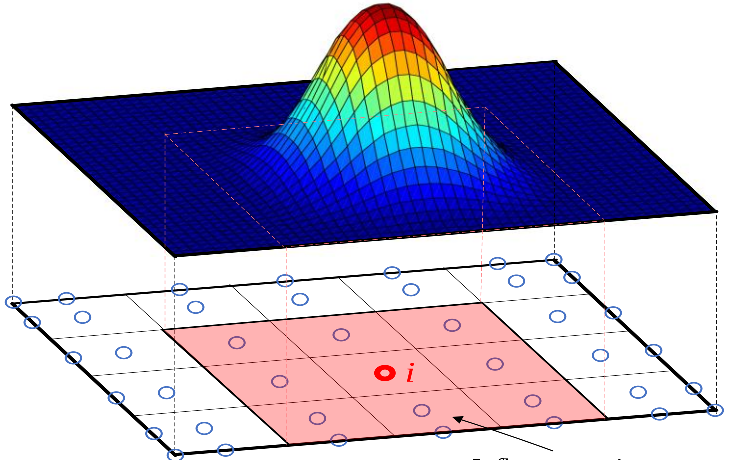
$$\frac{\partial \Phi(\boldsymbol{x},t)}{\partial t} - v_n |\nabla \Phi(\boldsymbol{x},t)| = 0$$ 是一个与时间t和空间x都相关的偏微分方程

方程难求解，求解效率低

RBF参数化水平集：通过紧支径向基函数（CS-RBF）参数化插值水平集函数分离时间和空间变量　（开源代码见Wei .et al 2018, SMO）

$$\boldsymbol{\Phi}(\boldsymbol{x},t) = \boldsymbol{R}(\boldsymbol{x})\boldsymbol{\phi}(t) = \sum_i R_i(\boldsymbol{x})\phi_i(t)$$

非负、局部支撑（仅计算一次）



NURBS参数化水平集：$\boldsymbol{\Phi}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\boldsymbol{\phi}(t)$

设计变量由水平集值Φ变为水平集扩展系数ϕ

等几何结构拓扑优化

--- IGA+Level set —优化模型

最小柔度问题

$$\text{Minimize}: J(\boldsymbol{u}, \Phi) = \int_{\Omega} \boldsymbol{\varepsilon}^T(\boldsymbol{u}) \boldsymbol{E} \boldsymbol{\varepsilon}(\boldsymbol{u}) H(\Phi) d\Omega,$$

$$\text{Subject to:} \quad V(\Omega) = \int_{\Omega} H(\Phi) d\Omega \leq V_{\max},$$

where $H(\Phi)$ is the Heaviside function

$$H(\Phi) = \begin{cases} 1 & if\ \Phi \geq 0 \\ 0 & if\ \Phi < 0 \end{cases}$$

水平集扩展系数，
设计变量

NURBS插值函数，
保持不变

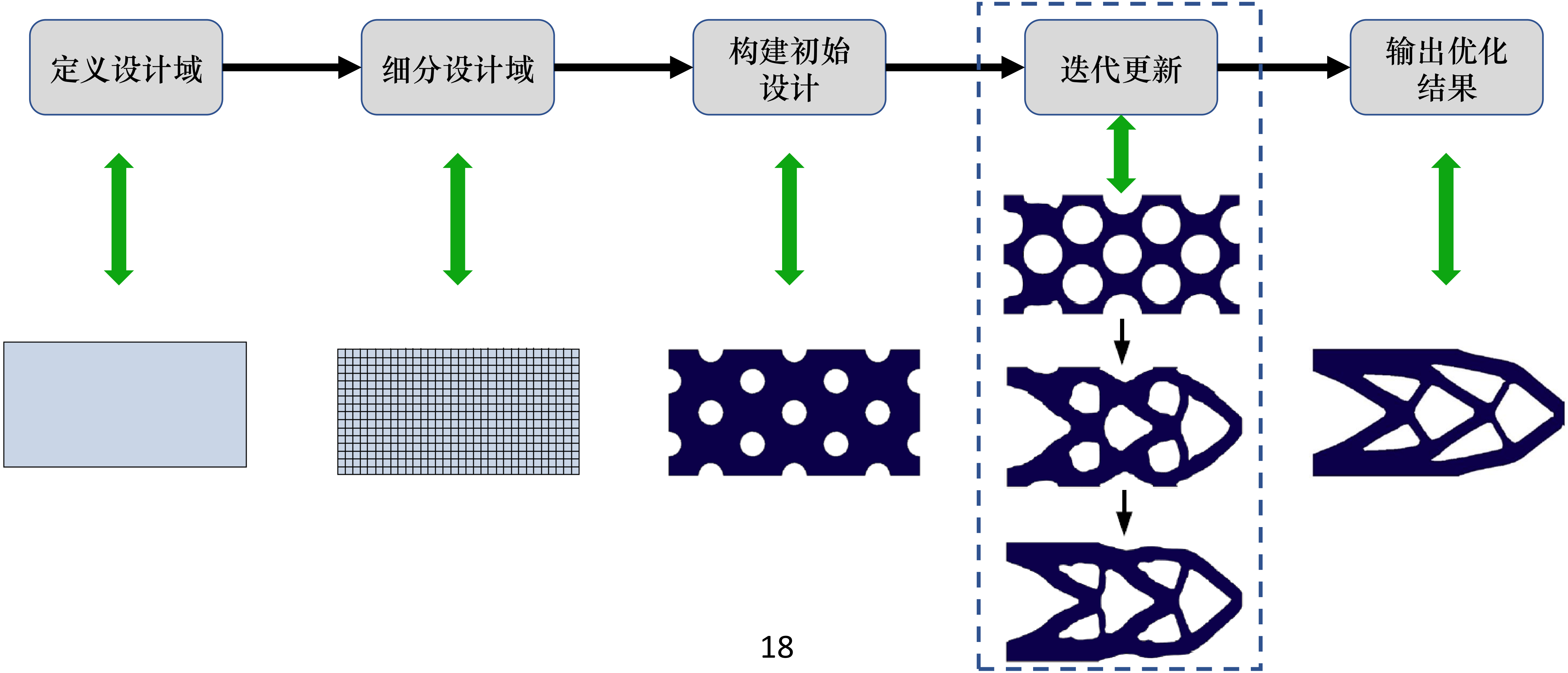
$$\boldsymbol{\Phi} = \boldsymbol{N} \boldsymbol{\phi}$$

$$\boldsymbol{K}_e' = \int_{\overline{\Omega}_e} \boldsymbol{B}^T \boldsymbol{D} \boldsymbol{B} |\boldsymbol{J}_1| |\boldsymbol{J}_2| \rho(\Phi) d\overline{\Omega} = \boldsymbol{K}_e \cdot \rho_e$$

$$\rho(\Phi) = \frac{(E_s - E_w) H(\Phi) + E_w}{E_s}$$

等几何结构拓扑优化

--- IGA+Level set —流程



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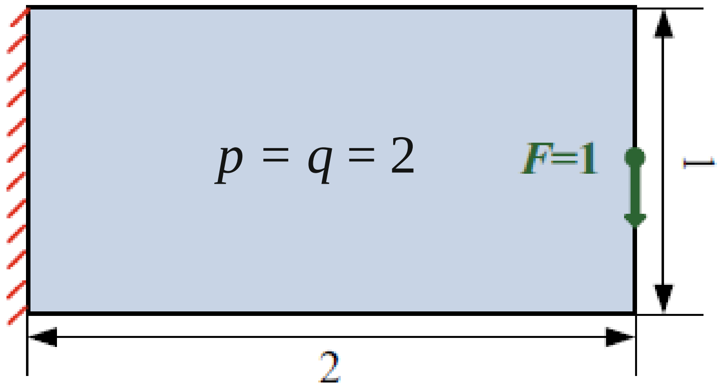
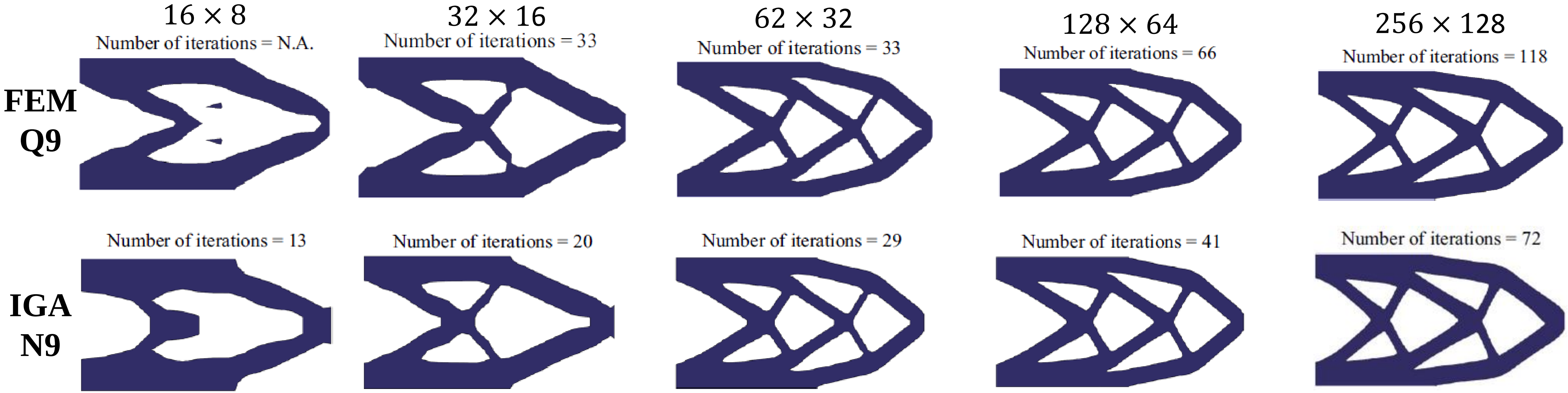
等几何结构拓扑优化

--- IGA+Level set —算例--CM

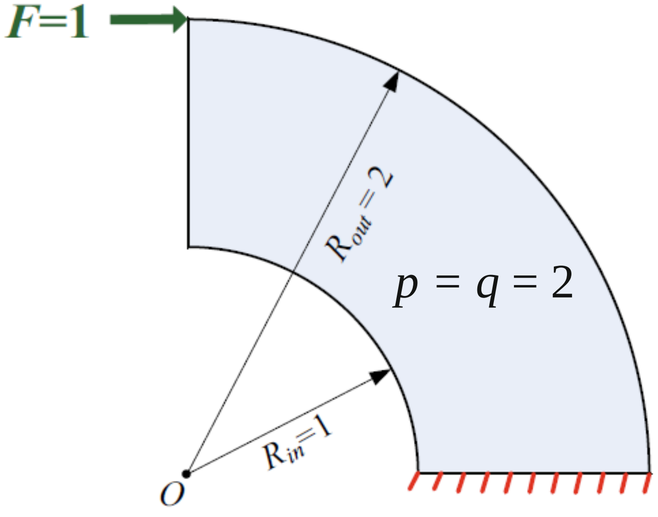


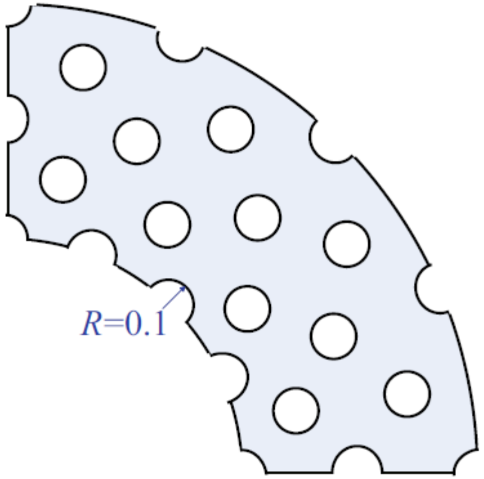
Cases	N_{FEM}	N_{IGA}	FEM time (s)	IGA time (s)	Speedups (IGA/FEM)
16 × 8	1122	360	0.62	0.20	3.10
32 × 16	4290	1224	2.62	0.83	3.16
64 × 32	18,354	4488	13.30	4.11	3.23
128 × 64	69,426	17,160	101.88	33.50	3.04
256 × 128	269,874	67,080	1489.36	360.22	4.13

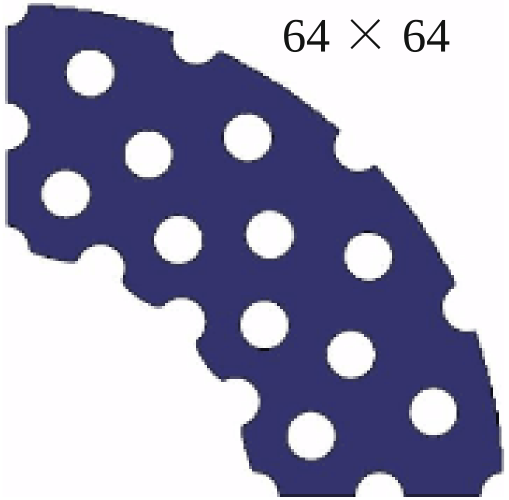


等几何结构拓扑优化

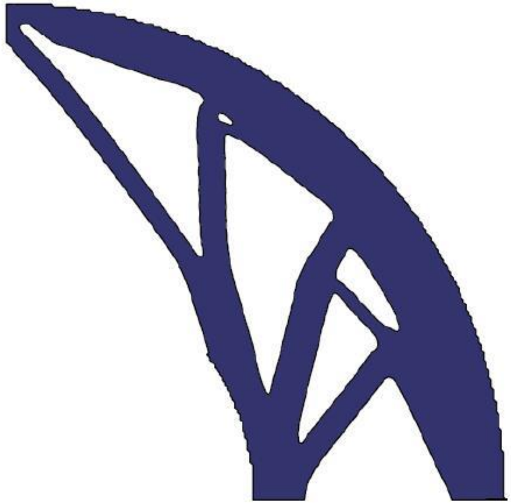
--- IGA+Level set —算例--CM

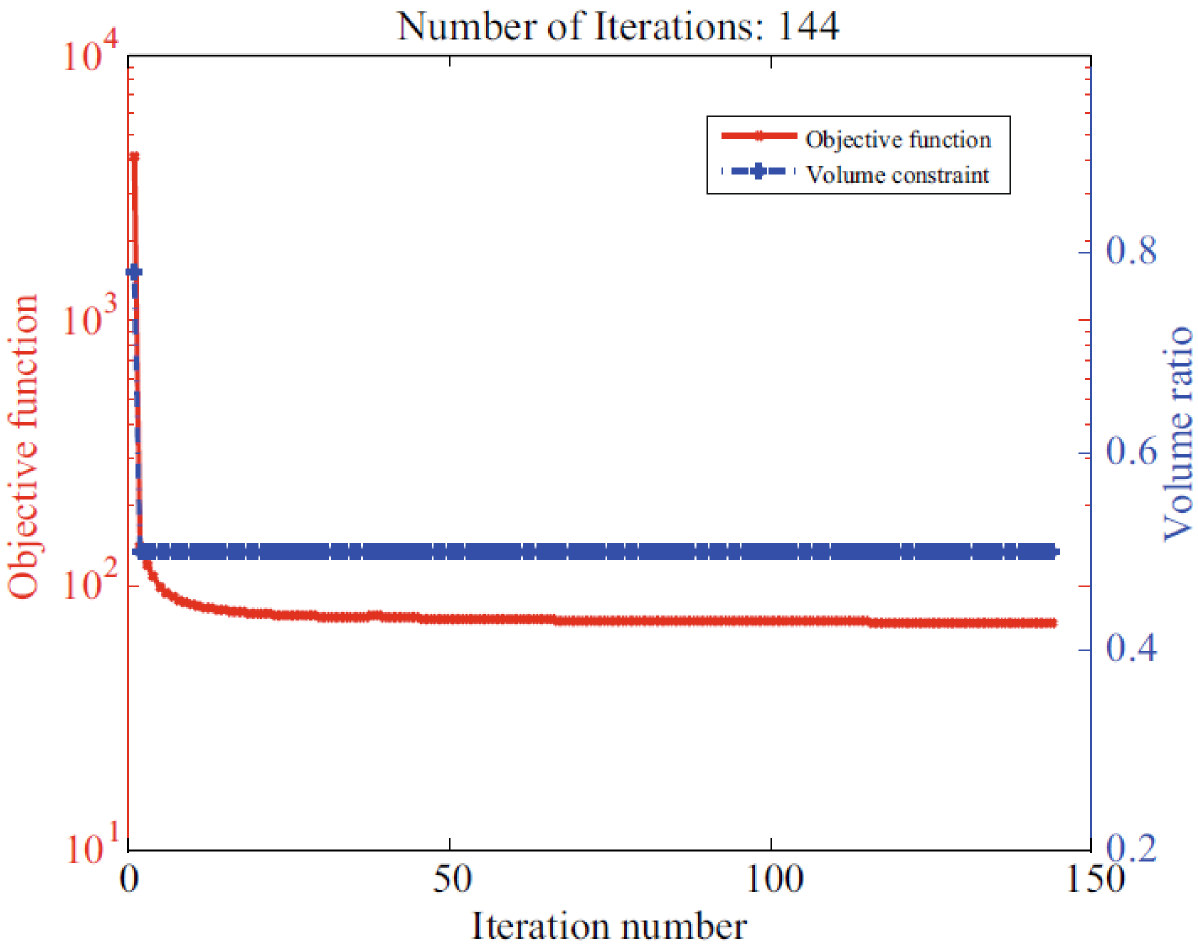






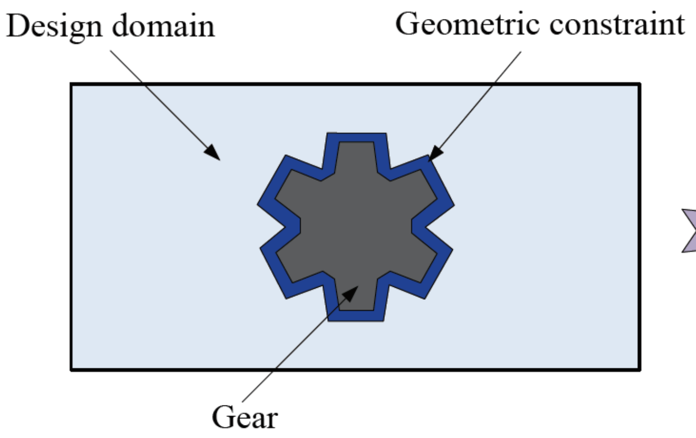
64 × 64

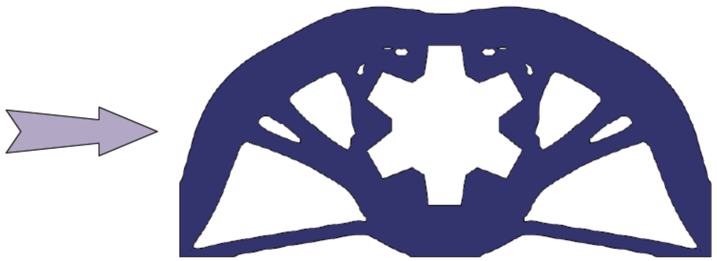


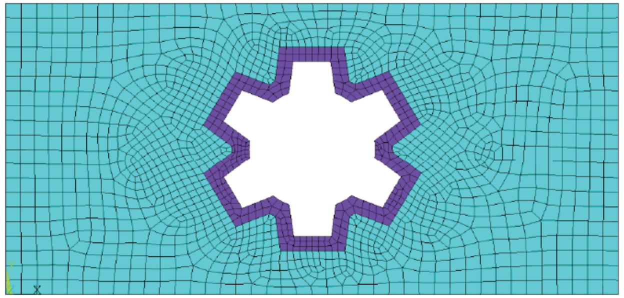


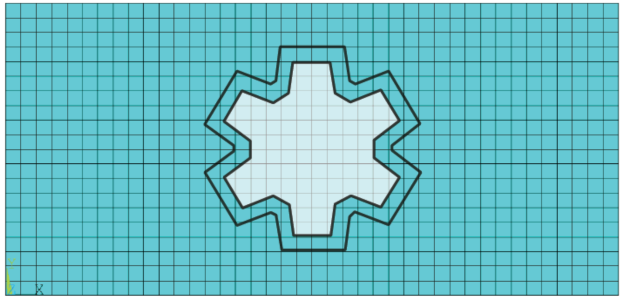
等几何结构拓扑优化

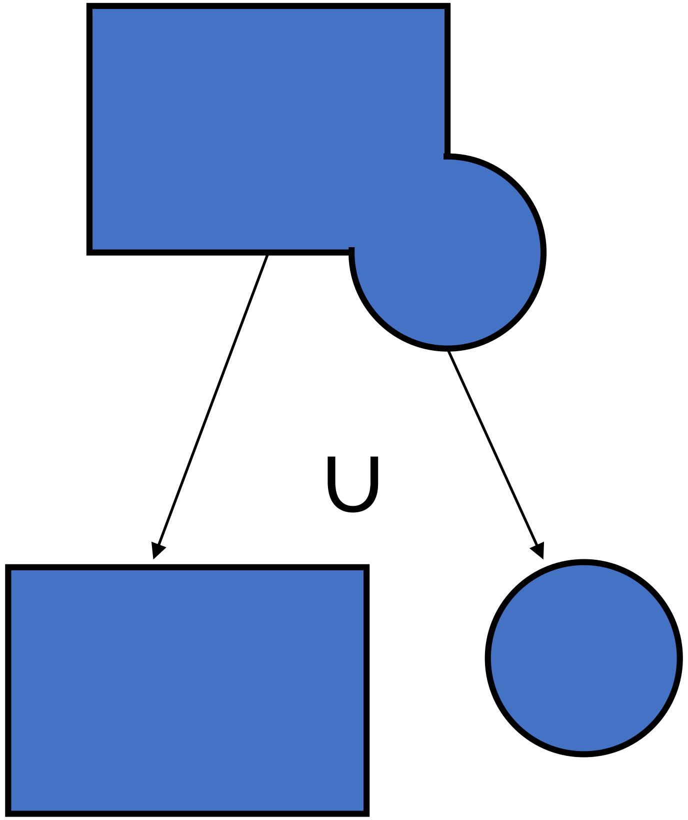
--- IGA+Level set —几何约束











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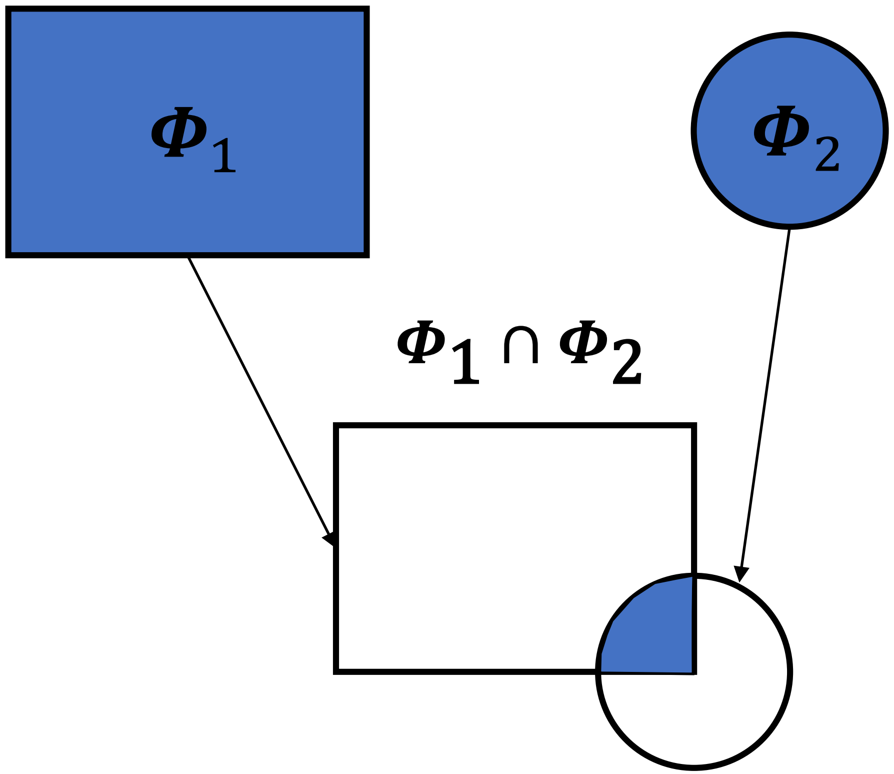
等几何结构拓扑优化

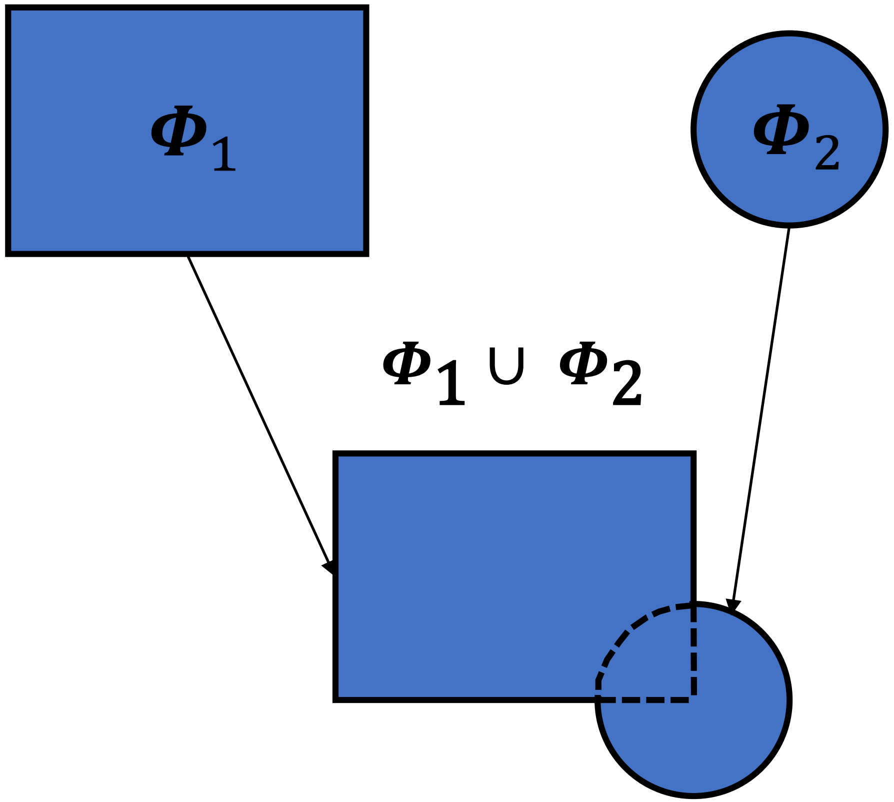
--- IGA+Level set —R-函数

R-函数

$$\Phi_1 \cap \Phi_2 = \frac{1}{1+a}\left(\Phi_1 + \Phi_2 - \sqrt{\Phi_1^2 + \Phi_2^2 - 2a\Phi_1\Phi_2}\right)$$

$$\Phi_1 \cup \Phi_2 = \frac{1}{1+a}\left(\Phi_1 + \Phi_2 + \sqrt{\Phi_1^2 + \Phi_2^2 - 2a\Phi_1\Phi_2}\right)$$

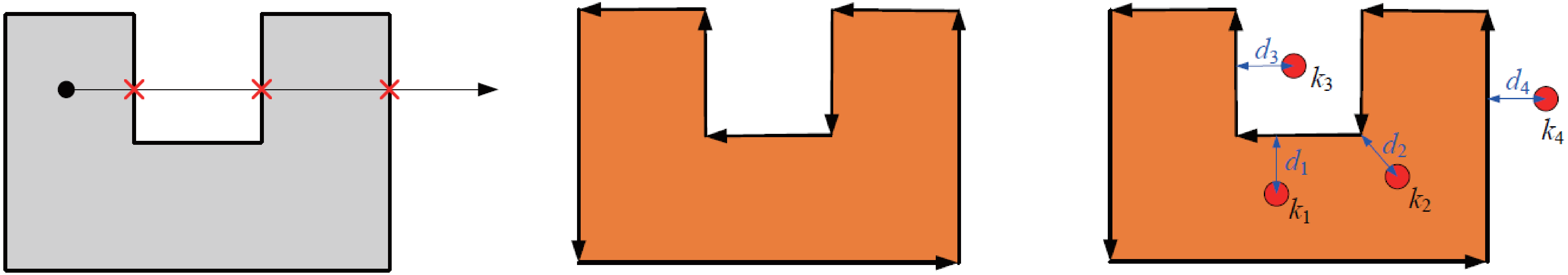


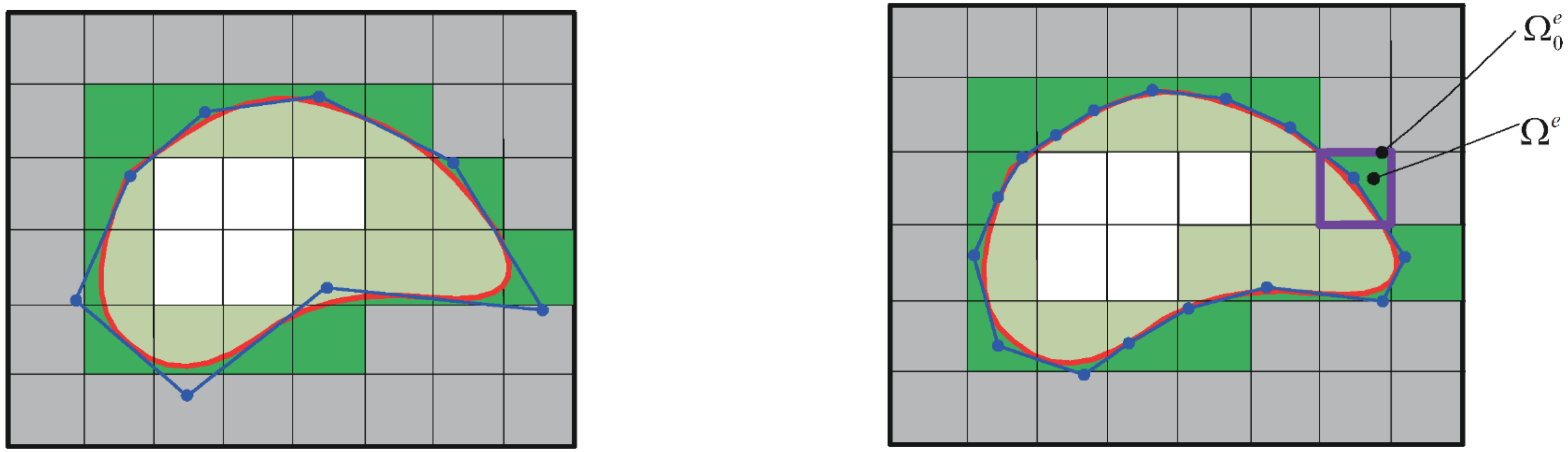


等几何结构拓扑优化

--- IGA+Level set —任意几何约束

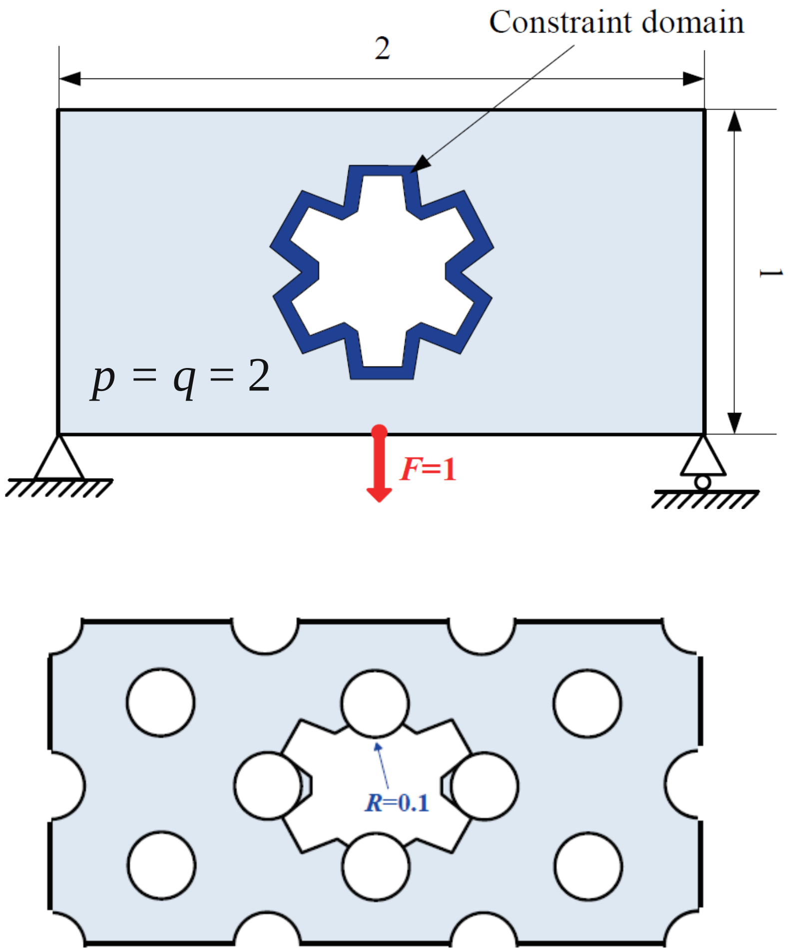
Point-in-polygon (PIP) algorithm

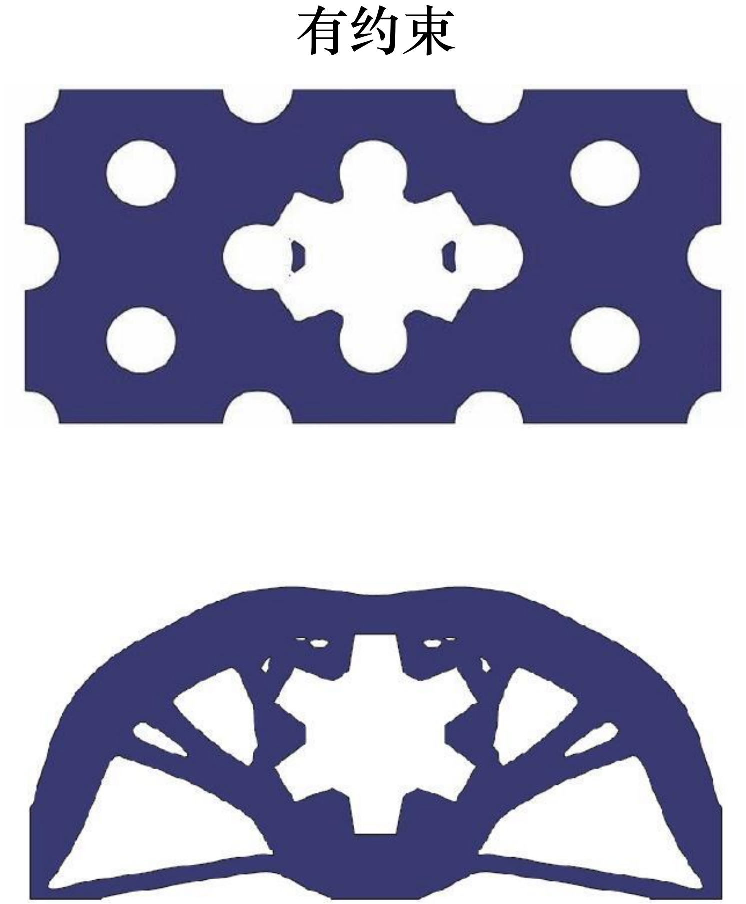


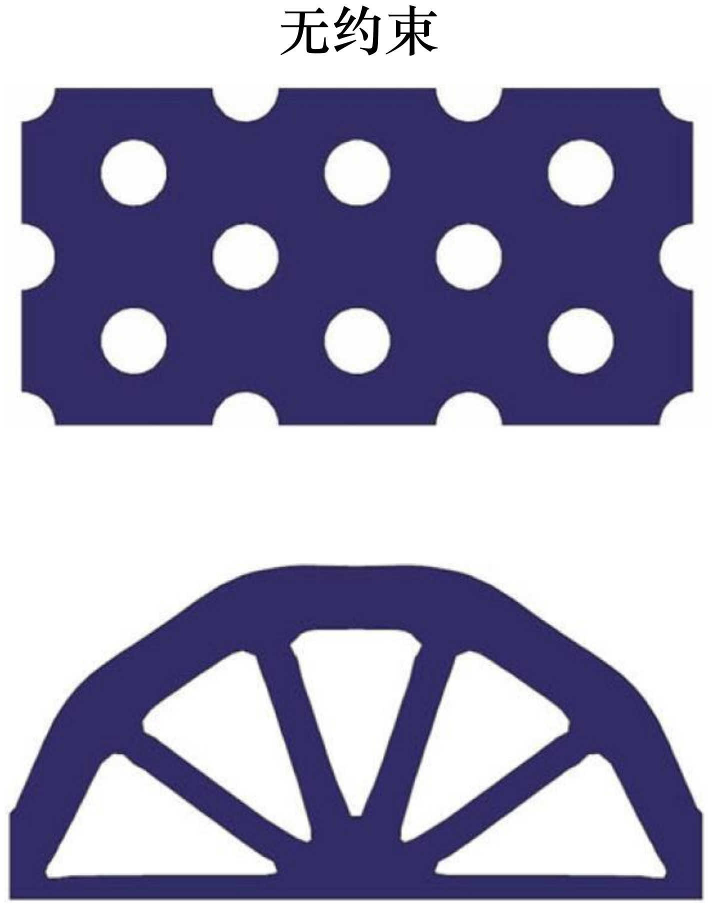


等几何结构拓扑优化

--- IGA+Level set —任意几何约束—算例--FME

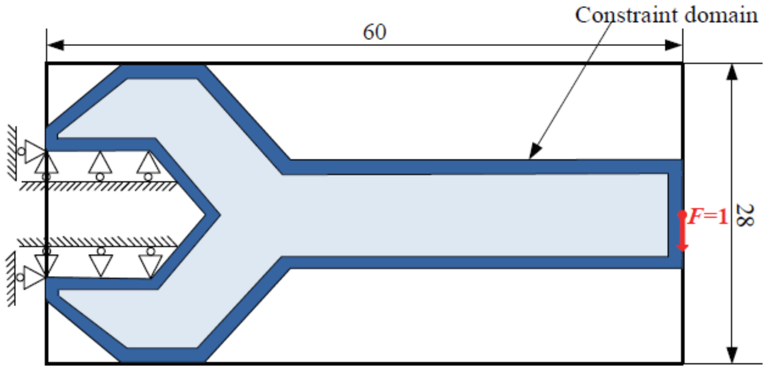


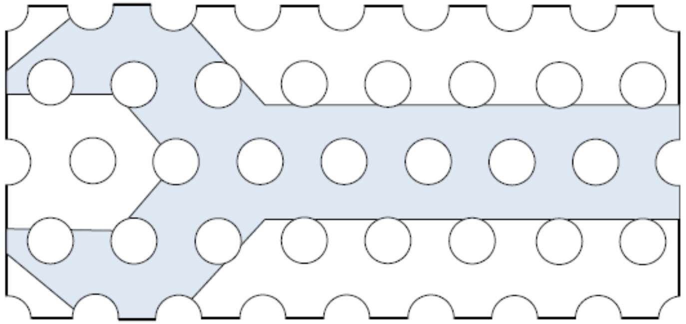


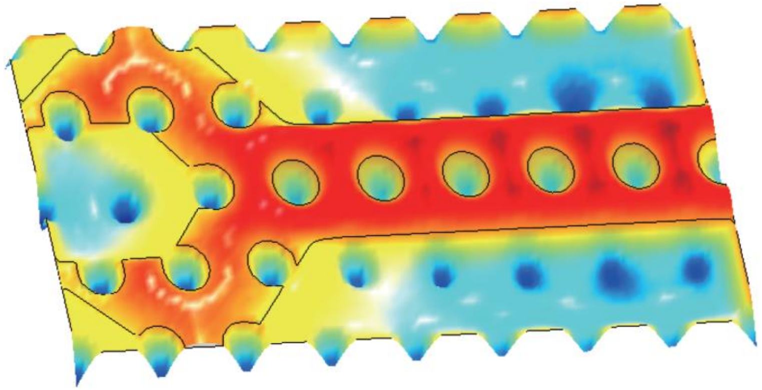


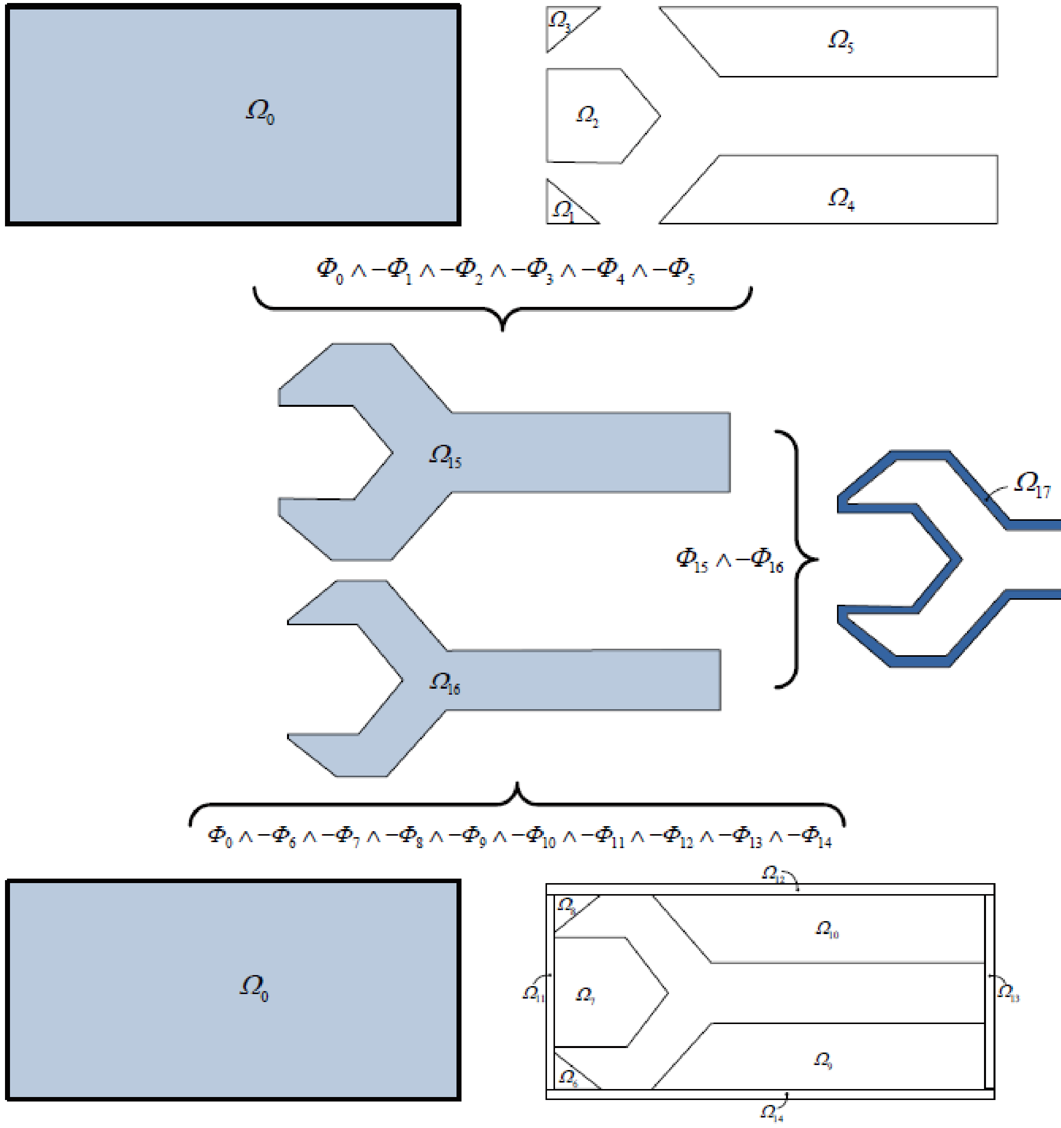
3 等几何结构拓扑优化

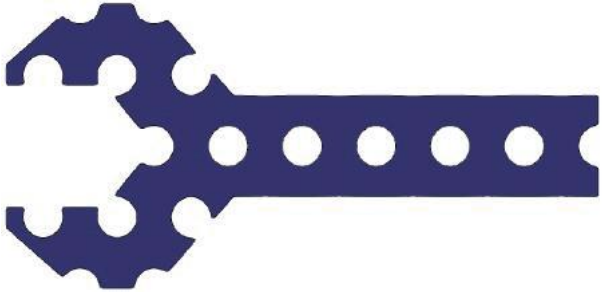
--- IGA+Level set —任意几何约束—算例--FME

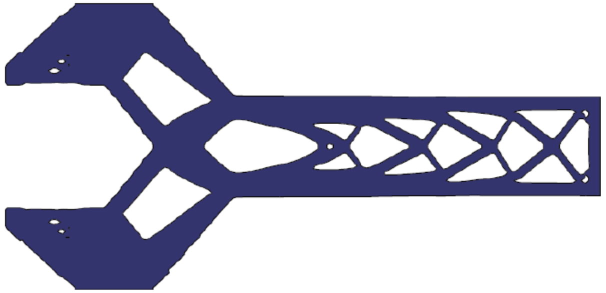








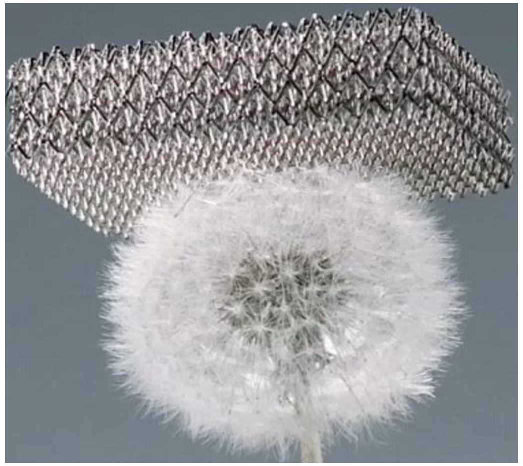




3 等几何结构拓扑优化

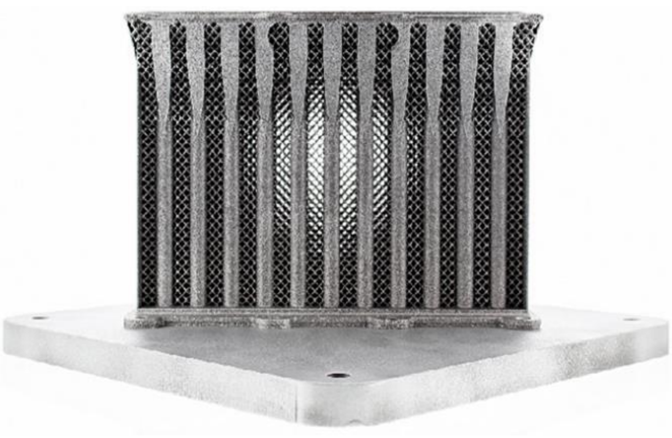
--- IGA+Multiscale—多孔点阵材料

超轻

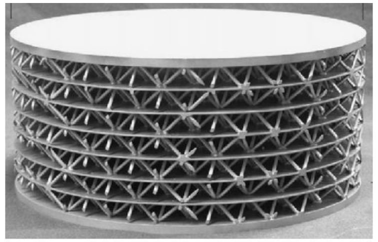


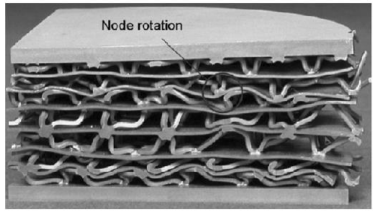
高比刚度

高散热



高能量吸收



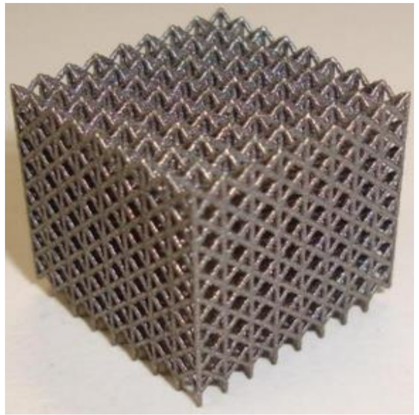


等几何结构拓扑优化

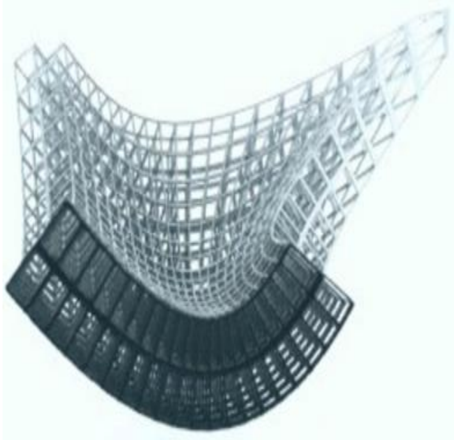
--- IGA+Multiscale—变密度多孔点阵

点阵结构体

等密度结构体

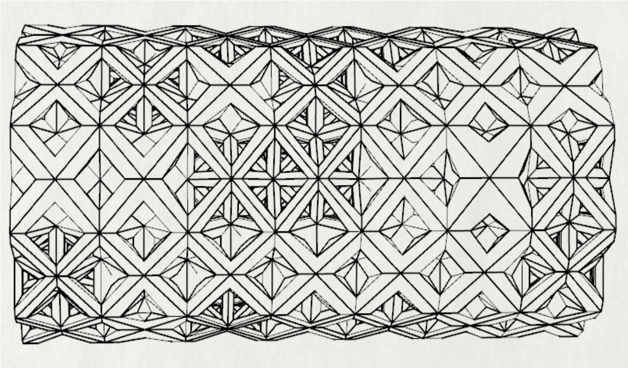


相同几何外形



不同几何外形

变密度结构体



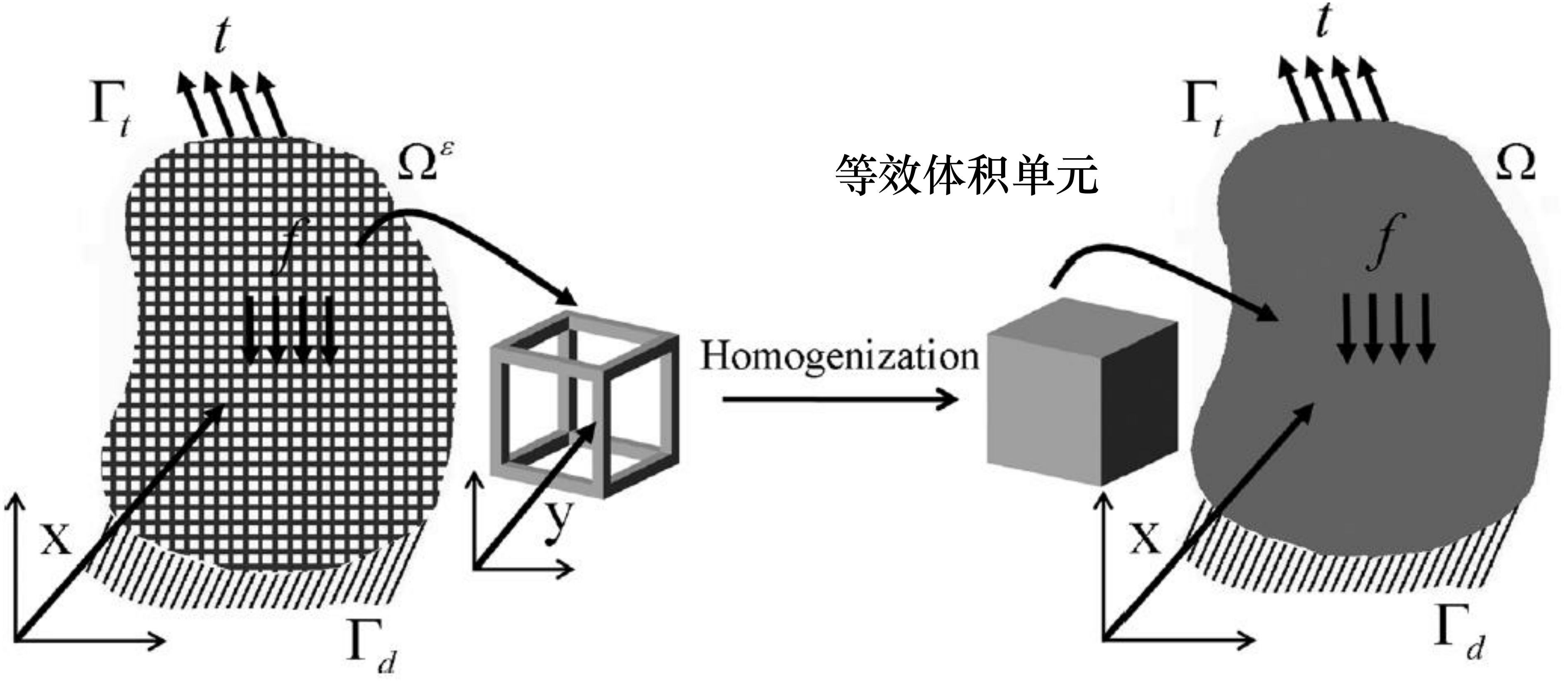
拓扑优化

设计最优密度分布

高性能点阵结构件

等几何结构拓扑优化

--- IGA+Multiscale—等效体积单元



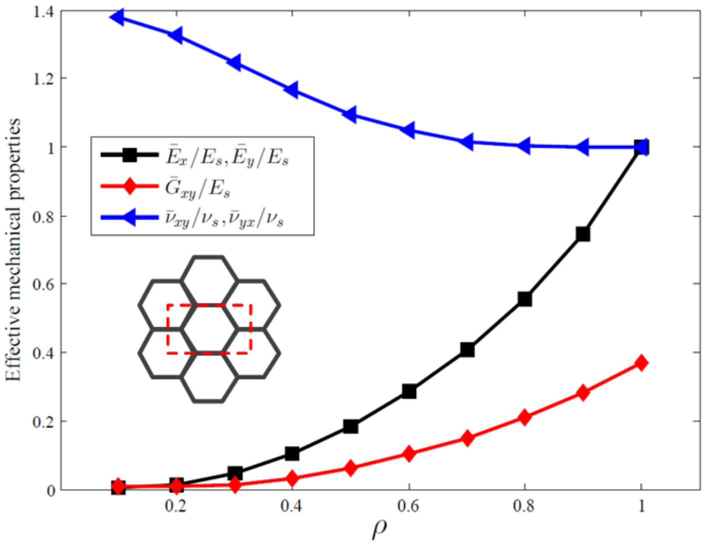
优化：微结构胞体拓扑不变，密度变

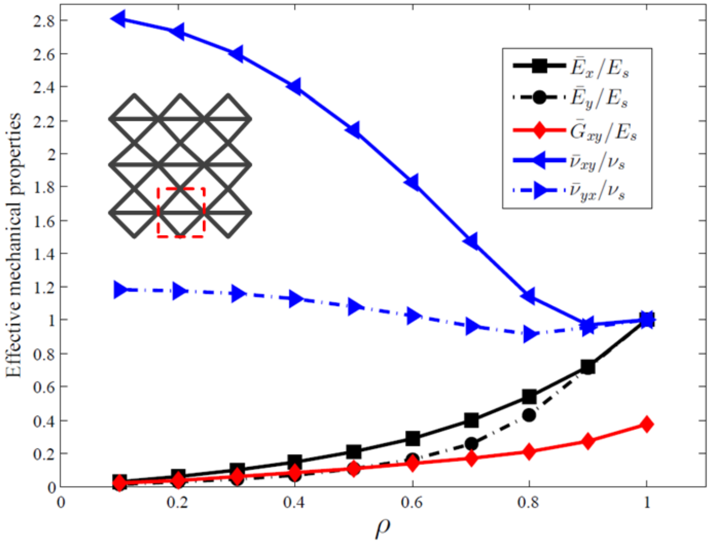
单元密度 = 胞体密度

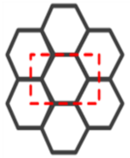
等几何结构拓扑优化

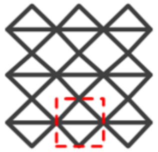
--- IGA+Multiscale—拟合材料属性

根据均匀化方法，拟合点阵材料胞体跟相对密度的关系函数





	Fitting function ($\rho \le 0.1$)	Fitting function ($0.1 < \rho \le 1$)	R-squared (R^2)
$\frac{\bar{E}_x}{E_s} = \frac{\bar{E}_y}{E_s}$	0.066ρ	$0.5824\rho^3 + 0.3737\rho^2 + 0.0432\rho - 0.0087$	0.9995
$\frac{\bar{G}_{xy}}{E_s}$	0.06ρ	$0.0845\rho^3 + 0.389\rho^2 - 0.1174\rho + 0.0137$	0.9998
$\frac{\bar{\nu}_{xy}}{\nu_s} = \frac{\bar{\nu}_{yx}}{\nu_s}$	$-0.5139\rho + 1.4287$	$0.4921\rho^3 - 0.2276\rho^2 - 0.7177\rho + 1.4611$	0.9957

	Fitting function ($\rho \le 0.1$)	Fitting function ($0.1 < \rho \le 1$)	R-squared (R^2)
$\frac{\bar{E}_x}{E_s}$	0.277ρ	$1.461\rho^3 - 1.0976\rho^2 + 0.6599\rho - 0.035$	0.9992
$\frac{\bar{E}_y}{E_s}$	0.117ρ	$2.3064\rho^3 - 1.9059\rho^2 + 0.6493\rho - 0.0399$	0.9987
$\frac{\bar{G}_{xy}}{E_s}$	0.19ρ	$0.6203\rho^3 - 0.6607\rho^2 + 0.4267\rho - 0.0214$	0.9984
$\frac{\bar{\nu}_{xy}}{\nu_s}$	$-0.8333\rho + 2.8961$	$6.2067\rho^3 - 10.618\rho^2 + 2.7732\rho + 2.5951$	0.9965
$\frac{\bar{\nu}_{yx}}{\nu_s}$	$-0.082\rho + 1.1913$	$1.8941\rho^3 - 2.9063\rho^2 + 0.9008\rho + 1.1094$	0.9825

3

等几何结构拓扑优化

--- IGA+Multiscale—灵敏度分析

目标函数灵敏度： $$\frac{\partial c}{\partial \rho_e} = -\boldsymbol{u}_e^T \frac{\partial \bar{\boldsymbol{k}}_e}{\partial \rho_e} \boldsymbol{u}_e = -\boldsymbol{u}_e^T \frac{\partial \int_{\Omega_e} \boldsymbol{B}_e^T \overline{\mathbf{D}}_e \boldsymbol{B}_e d\Omega}{\partial \rho_e} \boldsymbol{u}_e$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \cdots & \frac{\partial N_{nc}}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & \cdots & 0 & \frac{\partial N_{nc}}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \cdots & \frac{\partial N_{nc}}{\partial y} & \frac{\partial N_{nc}}{\partial x} \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} \frac{E_x}{1-v_{xy}v_{yx}} & \frac{v_{yx}E_x}{1-v_{xy}v_{yx}} & 0 \\ \frac{v_{xy}E_y}{1-v_{xy}v_{yx}} & \frac{E_y}{1-v_{xy}v_{yx}} & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}$$

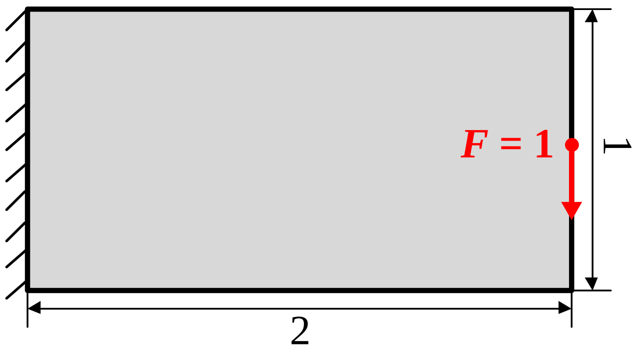
$$\frac{\partial c}{\partial \rho_e} = -\boldsymbol{u}_e^T \left(\int_{\Omega_e} \boldsymbol{B}_e^T \frac{\partial \overline{\mathbf{D}}_e}{\partial \rho_e} \boldsymbol{B}_e d\Omega \right) \boldsymbol{u}_e$$

$\frac{\partial \overline{\mathbf{D}}_e}{\partial \rho_e}$ 等效材料属性通过均匀化方法拟合为**材料属性—相对密度**函数，通过该函数计算。

3 等几何结构拓扑优化

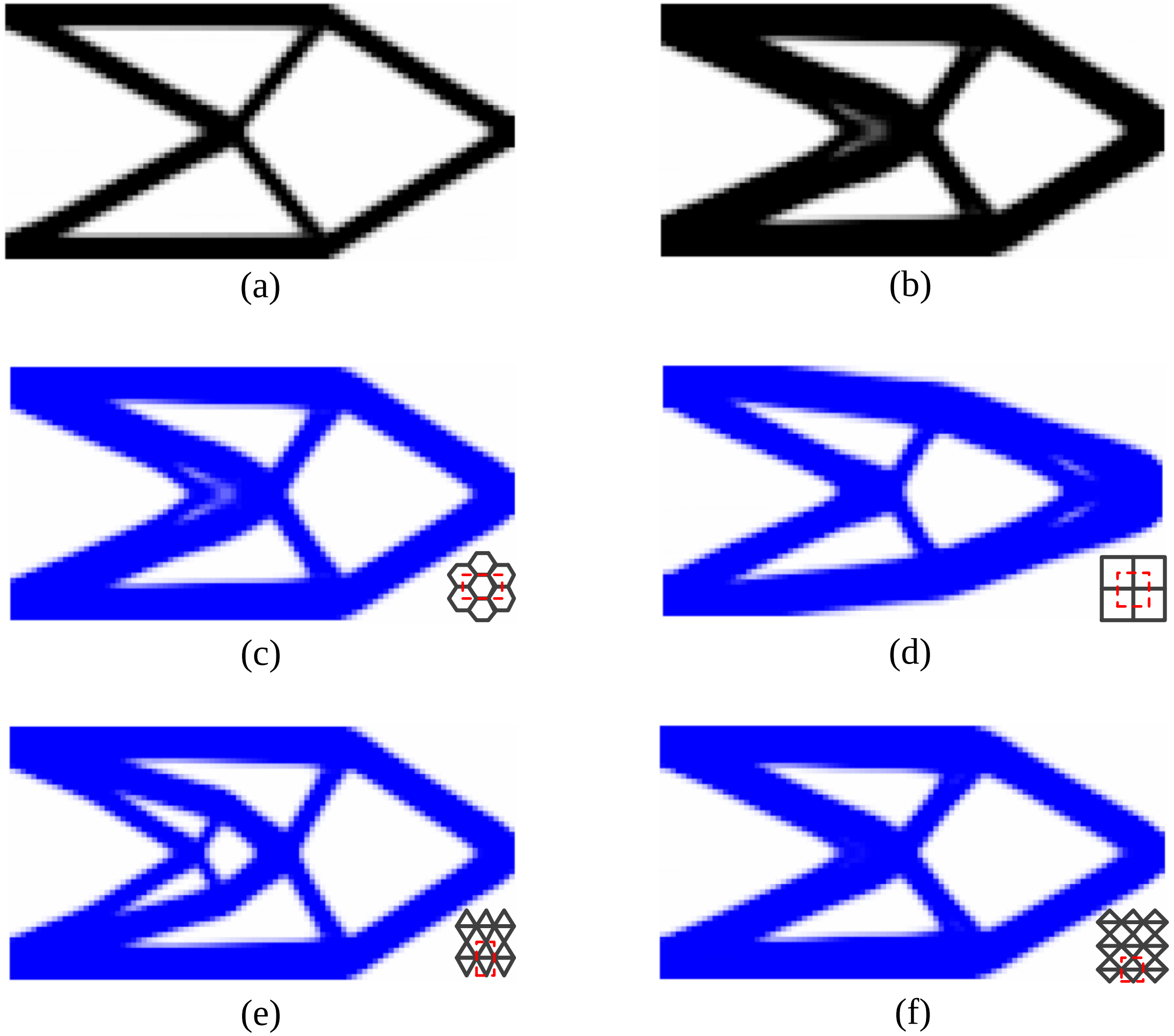
--- IGA+Multiscale—算例---CMAME

悬臂梁：96 ×48 二次单元



SIMP，不采用材料拟合函数 *VR*= 0.5
MR= *VR**0.6 =0.3

$$\rho_{cell} = 0.6$$

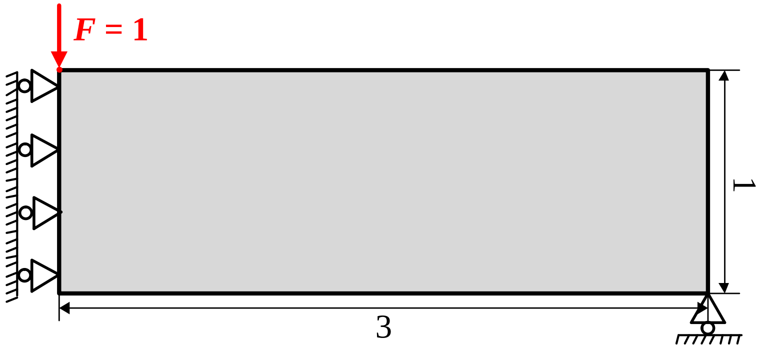


(a) 实体VR=MR = 0.3, (b) 实体VR =MR= 0.5, (c) 六边形, (d) 方形, (e) 锐角三角形组合, (f) 钝角三角形组合

等几何结构拓扑优化

--- IGA+Multiscale—算例---CMAME

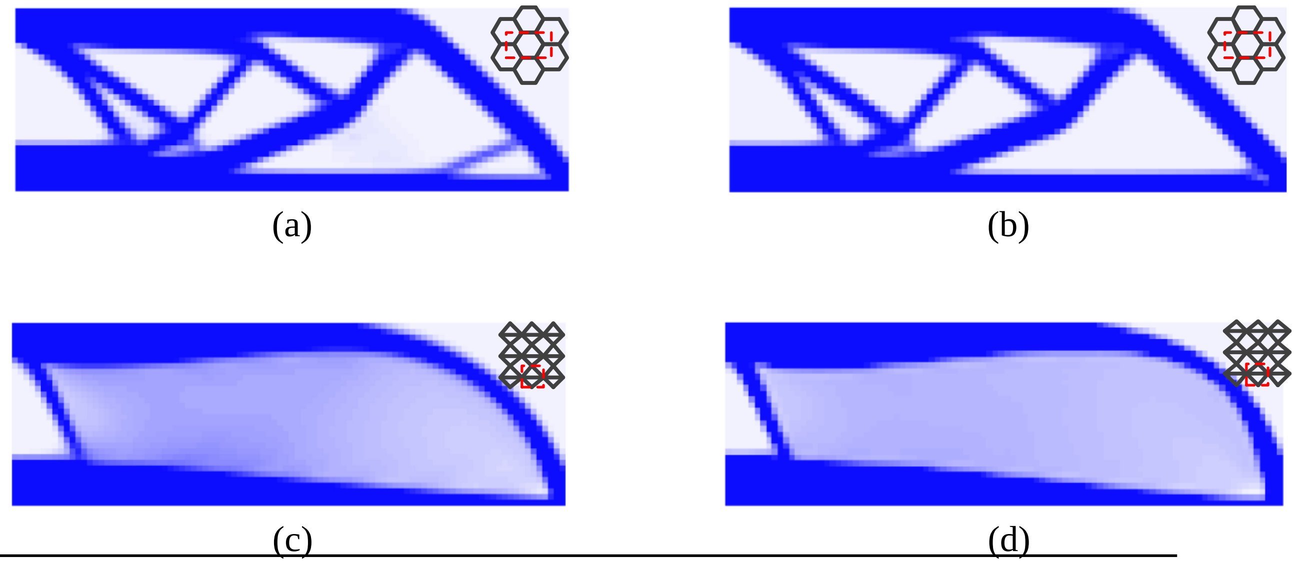
半MBB梁:96 ×32 二次单元



$VR = VR_0 = 0.5$

SIMP

采用拟合函数

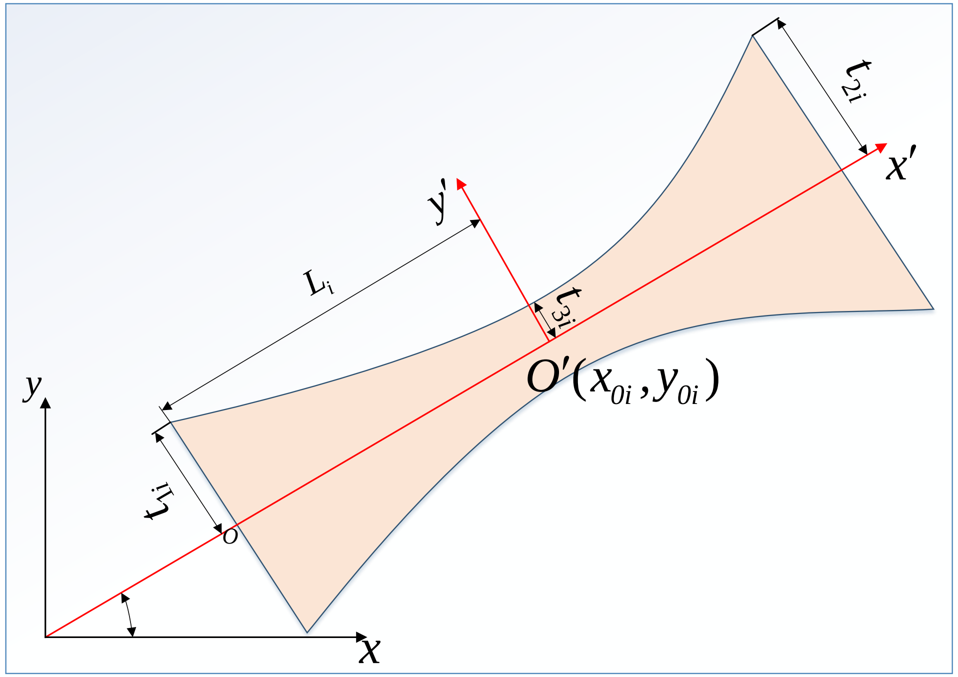


	六边形 (a)	六边形 (b)	钝角三角形 (c)	钝角三角形 (d)
Compliance	223.64	221.84	221.46	219.68
Number of iterations	68	68	70	68

等几何结构拓扑优化

--- IGA+MMC

MMC: Moving Morphable Components 移动可变形组件法 （开源代码见Zhang .et al 2016,53, SMO）



组件显示公式描述

每个组件有一个水平集函数

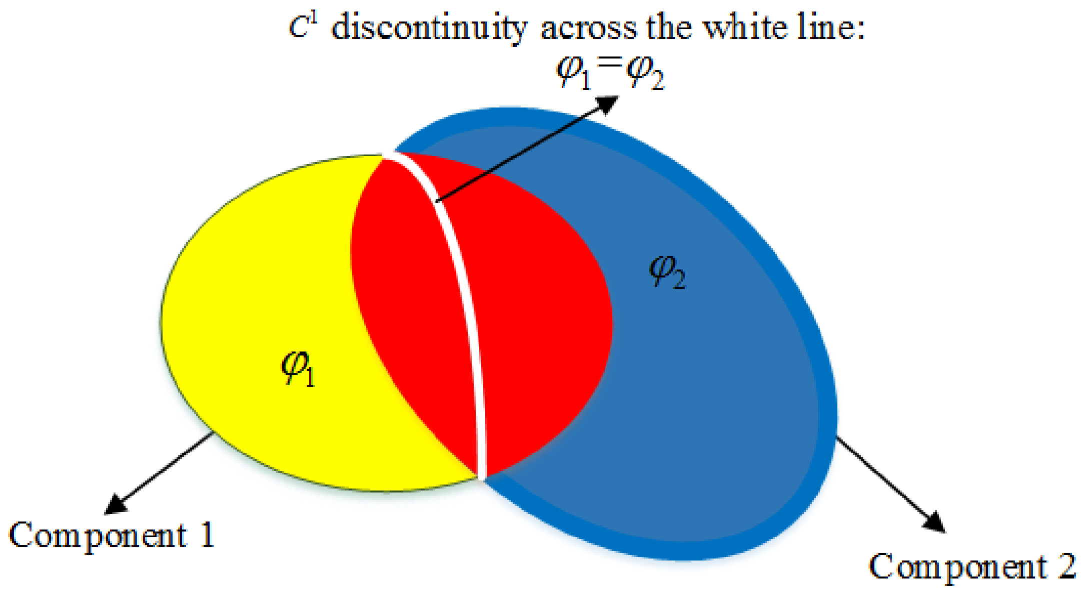
组件移动、变形来优化结构

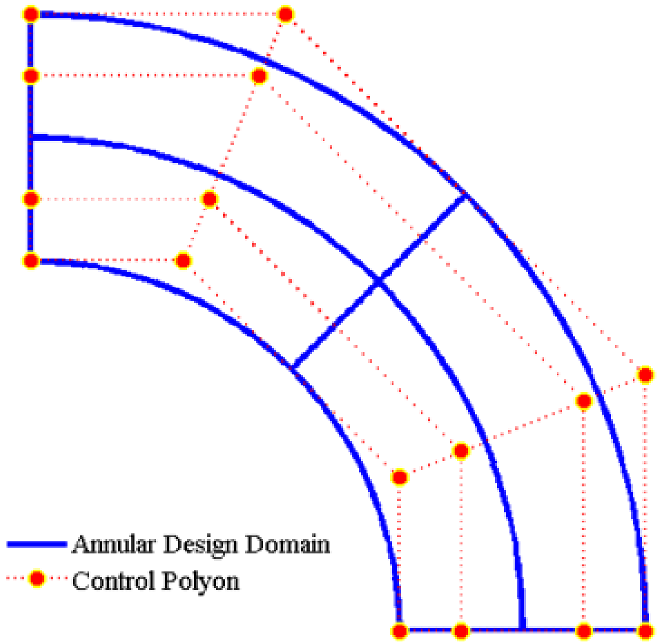
设计变量为组件参数，数量少

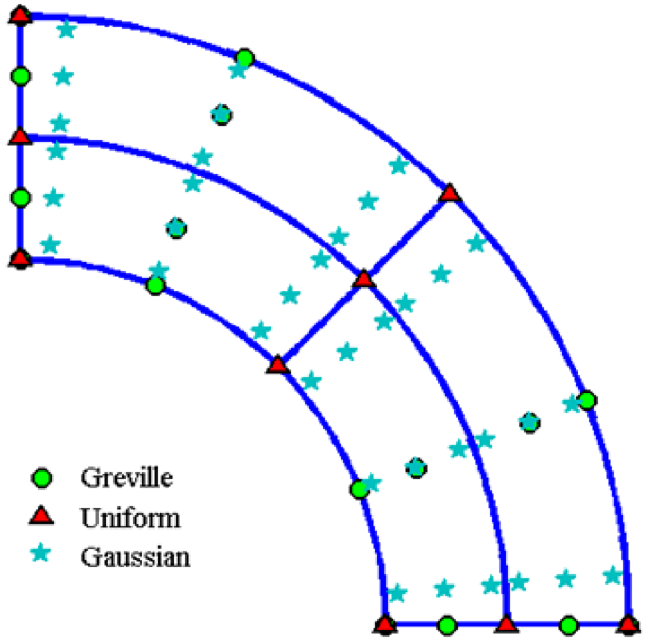
$$\boldsymbol{D}_i = (x_{0i}, y_{0i}, L_i, t_{1i}, t_{2i}, t_{3i}, \theta_i)$$

等几何结构拓扑优化

--- IGA+MMC---改进





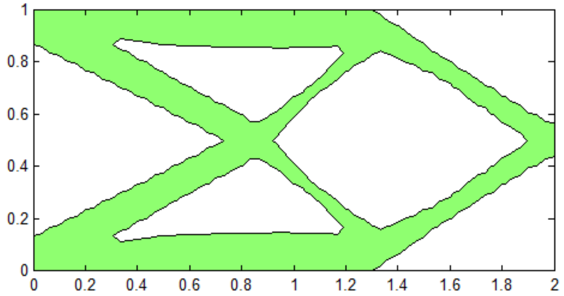


Max函数，构件重合不连续

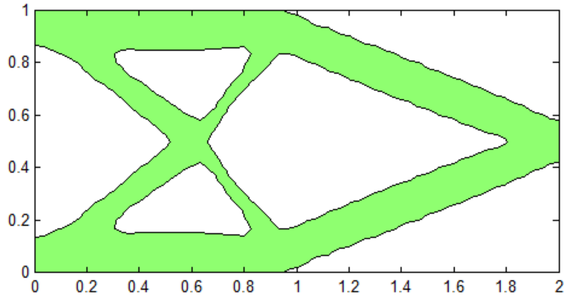
R函数，连续、可导

等几何结构拓扑优化

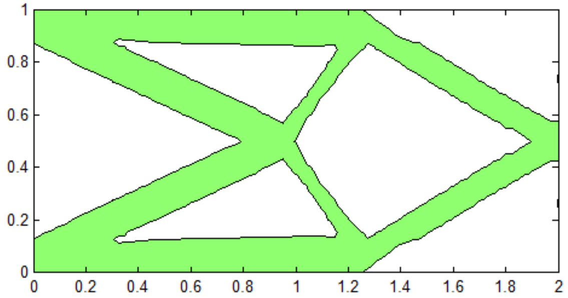
--- IGA+MMC—算例--投稿CMAME



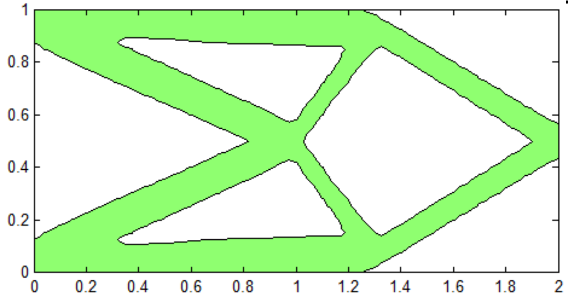
(a) Optimal structure of 60×30 FE mesh using max function
the value of objective functional C=76.52



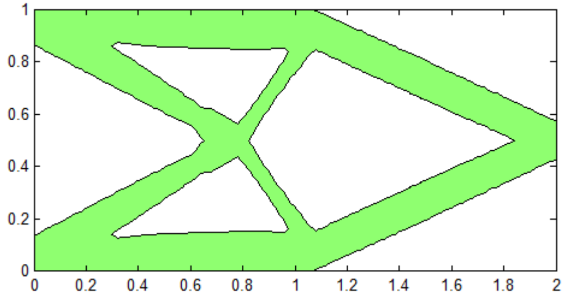
(b) Optimal structure of 60×30 FE mesh using R- functions
the value of objective functional C=76.65



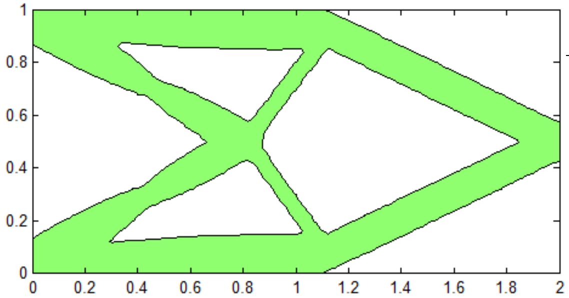
(c) Optimal structure of 80×40 FE mesh using max function
the value of objective functional C=74.68



(d) Optimal structure of 80×40 FE mesh using R-functions
the value of objective functional C=74.43



(e) Optimal structure of 100×50 FE mesh using max function
the value of objective functional C=74.66



(f) Optimal structure of 100×50 FE mesh using R-functions
the value of objective functional C=74.49

FEM: R函数与Max函数的比较

mesh size	60×30	80×40	100×50
Total iteration steps using max function	464	244	841
Total iteration steps using R-functions	291	184	542
The improvement of the convergence rate ($\frac{step_{max}-step_{R-functions}}{step_{max}}\times 100\%$)	37.3%	24.6%	35.6%

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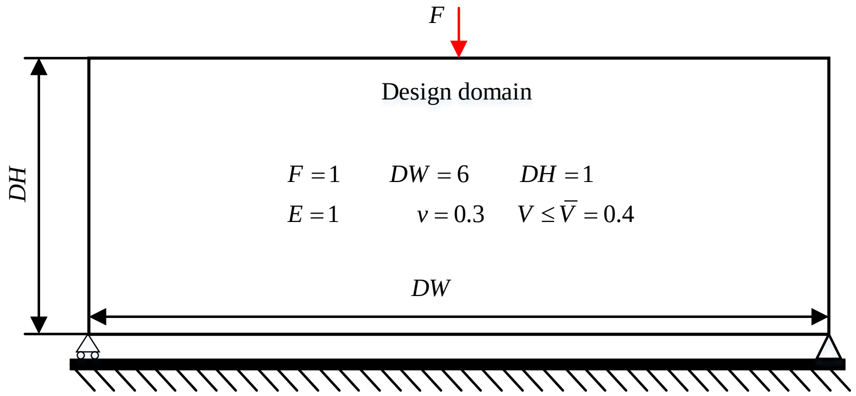
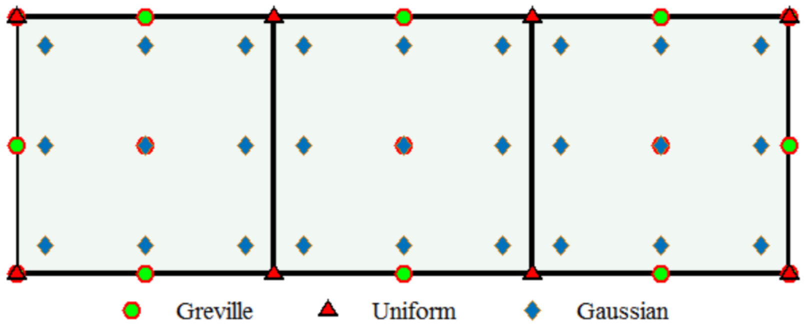
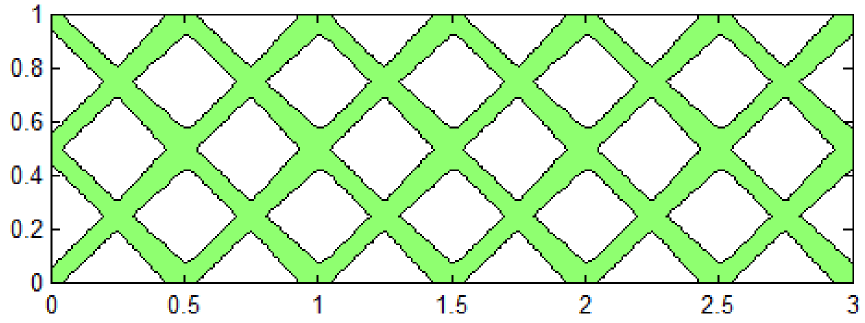
等几何结构拓扑优化

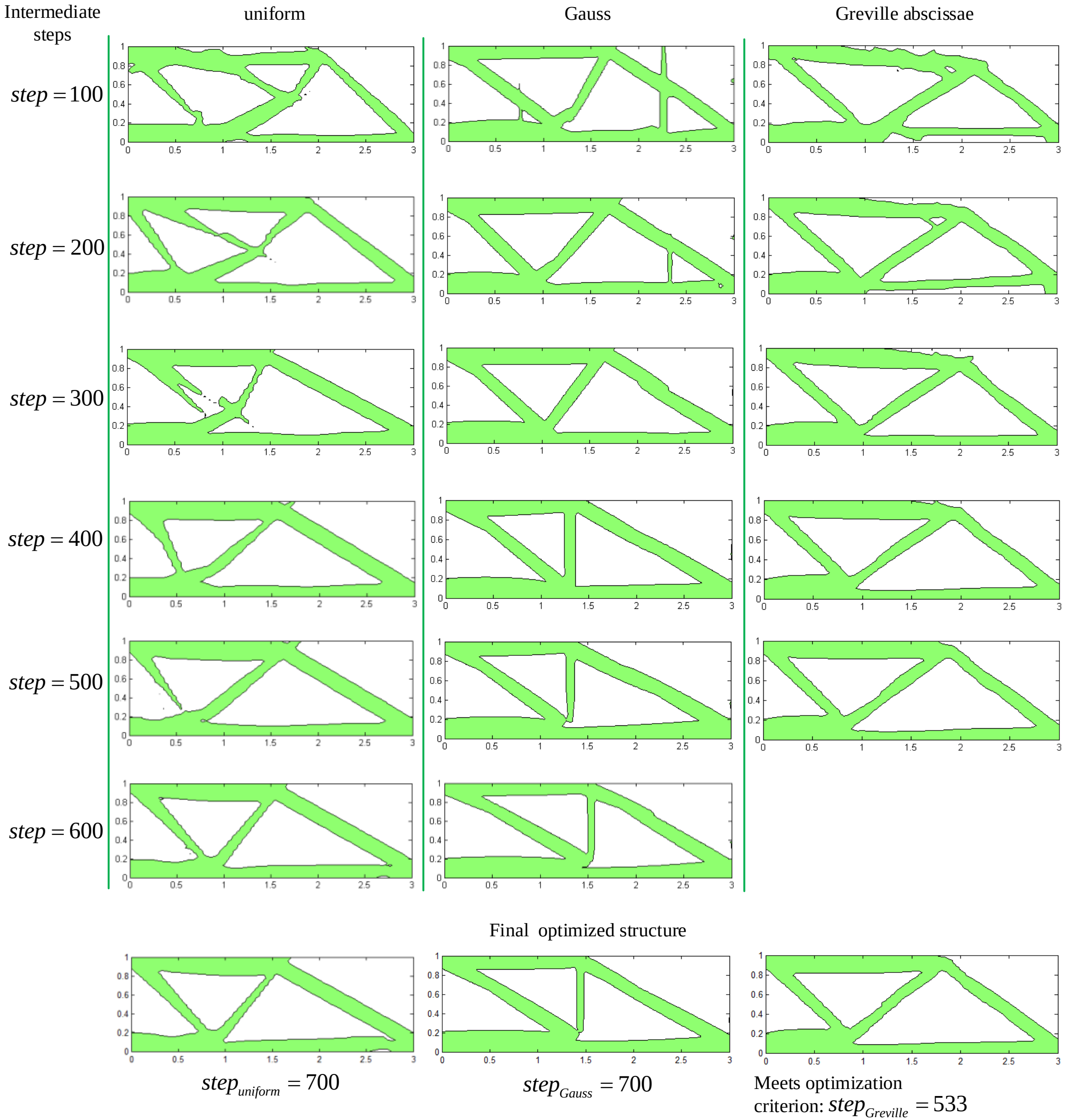
--- IGA+MMC—算例--投稿CMAME







IGA: 不同配点比较



提纲

总结与展望

等几何分析：热点
高精度、高阶效率高
难处理复杂模型
接触问题难处理
尚无商业软件

拓扑优化：方法多
各有优劣
能解决问题就是好方法

3D打印+拓扑优化：

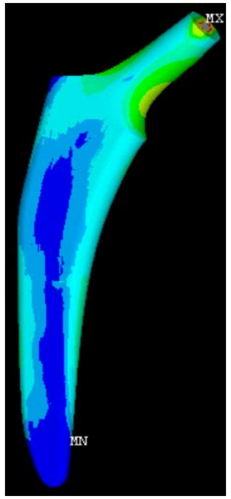
仰望天空

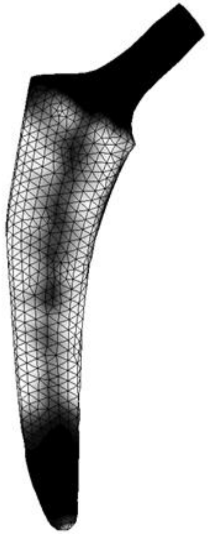
脚踏实地

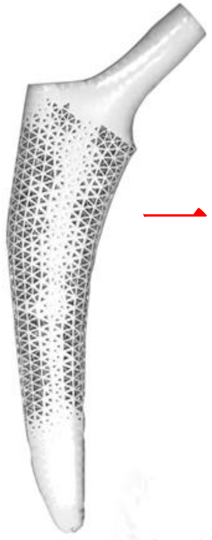
||

理论+应用

设计 ⟶ 分析 ⟶ 优化 ⟶ 3D打印制造







一体化

SCUT

Thank you