Blended Cured Quasi-Newton

for Distortion Optimization

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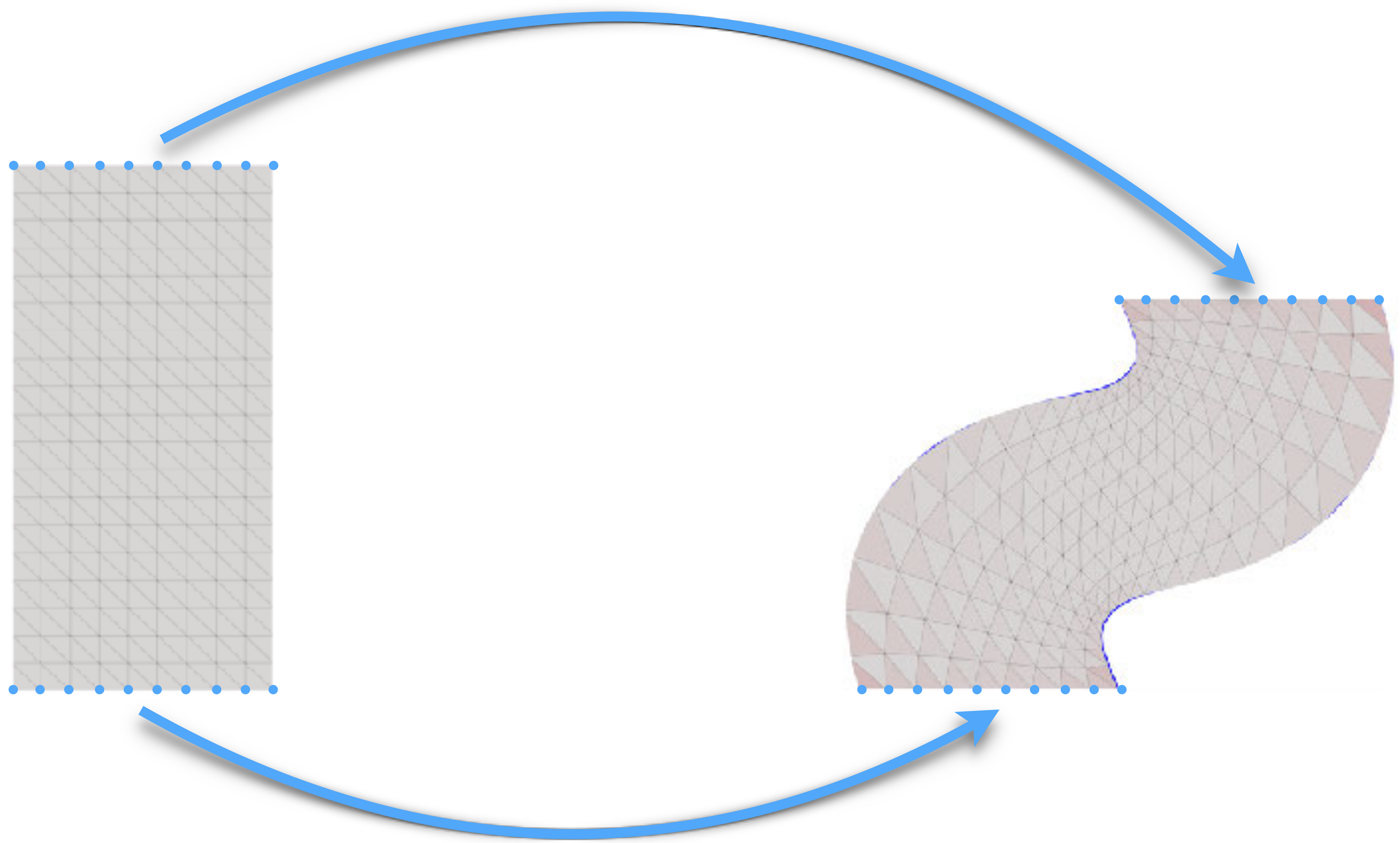
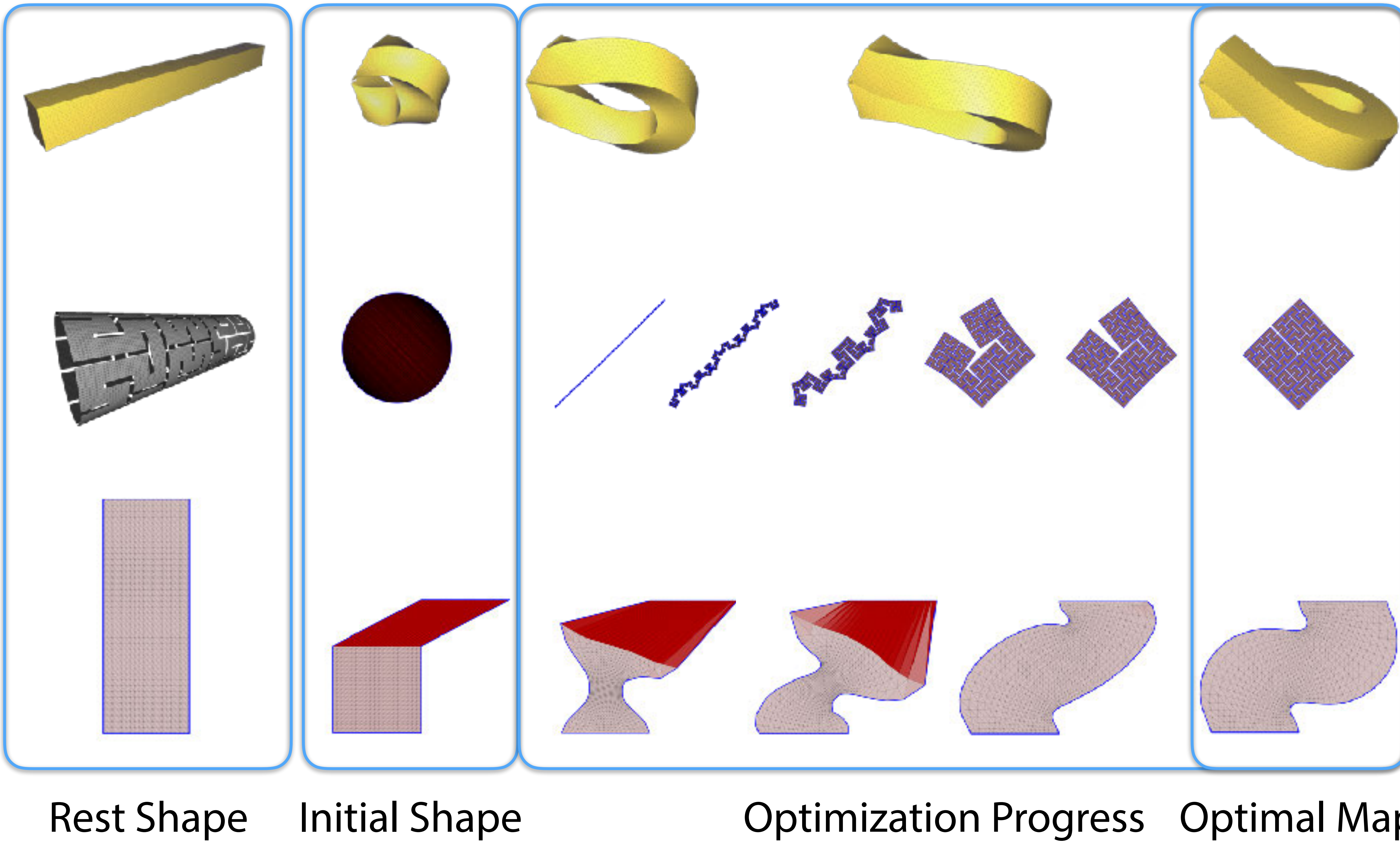
Background

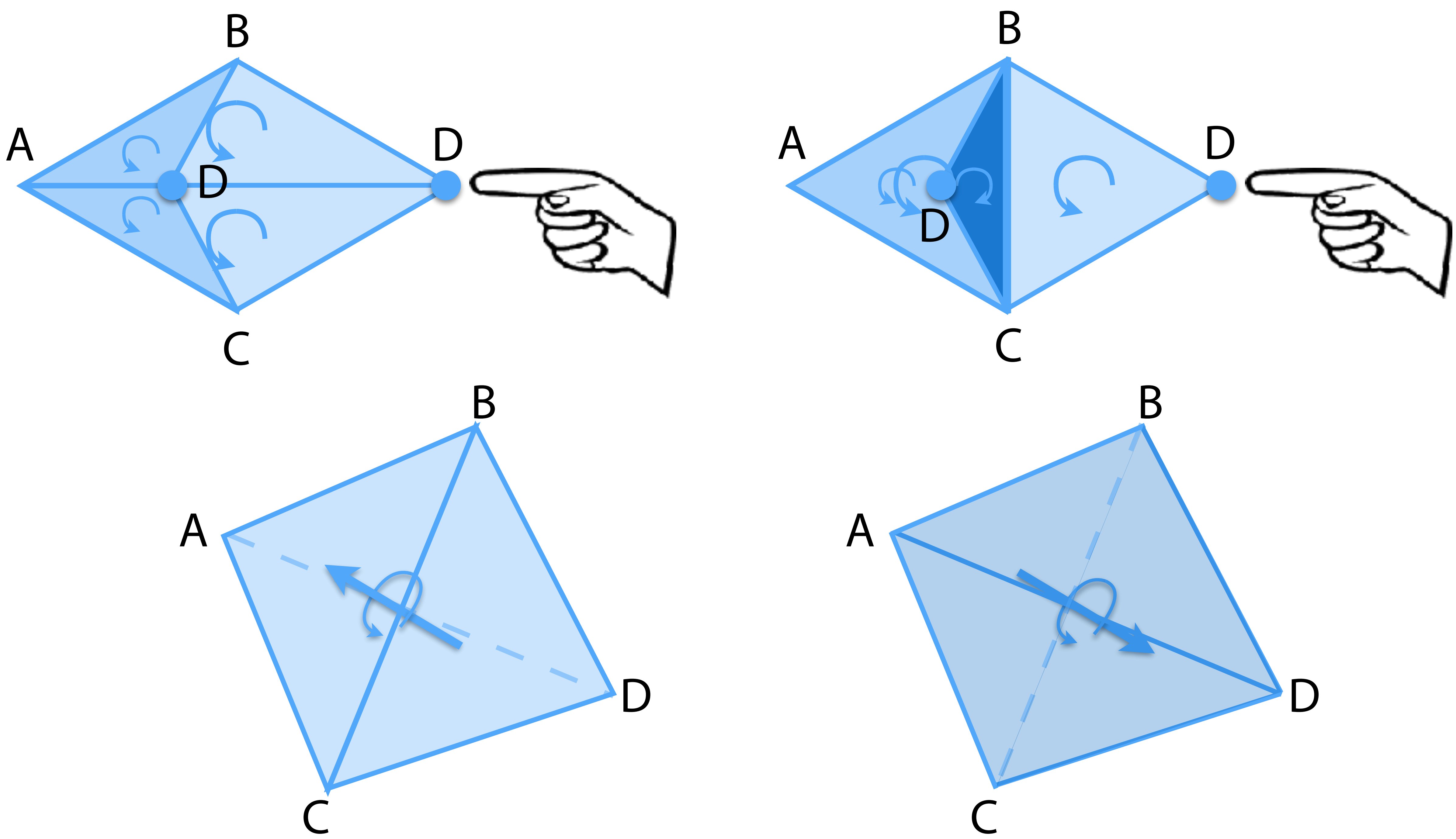
Distortion Optimization



Local Optimization



Non-Flip Constraint

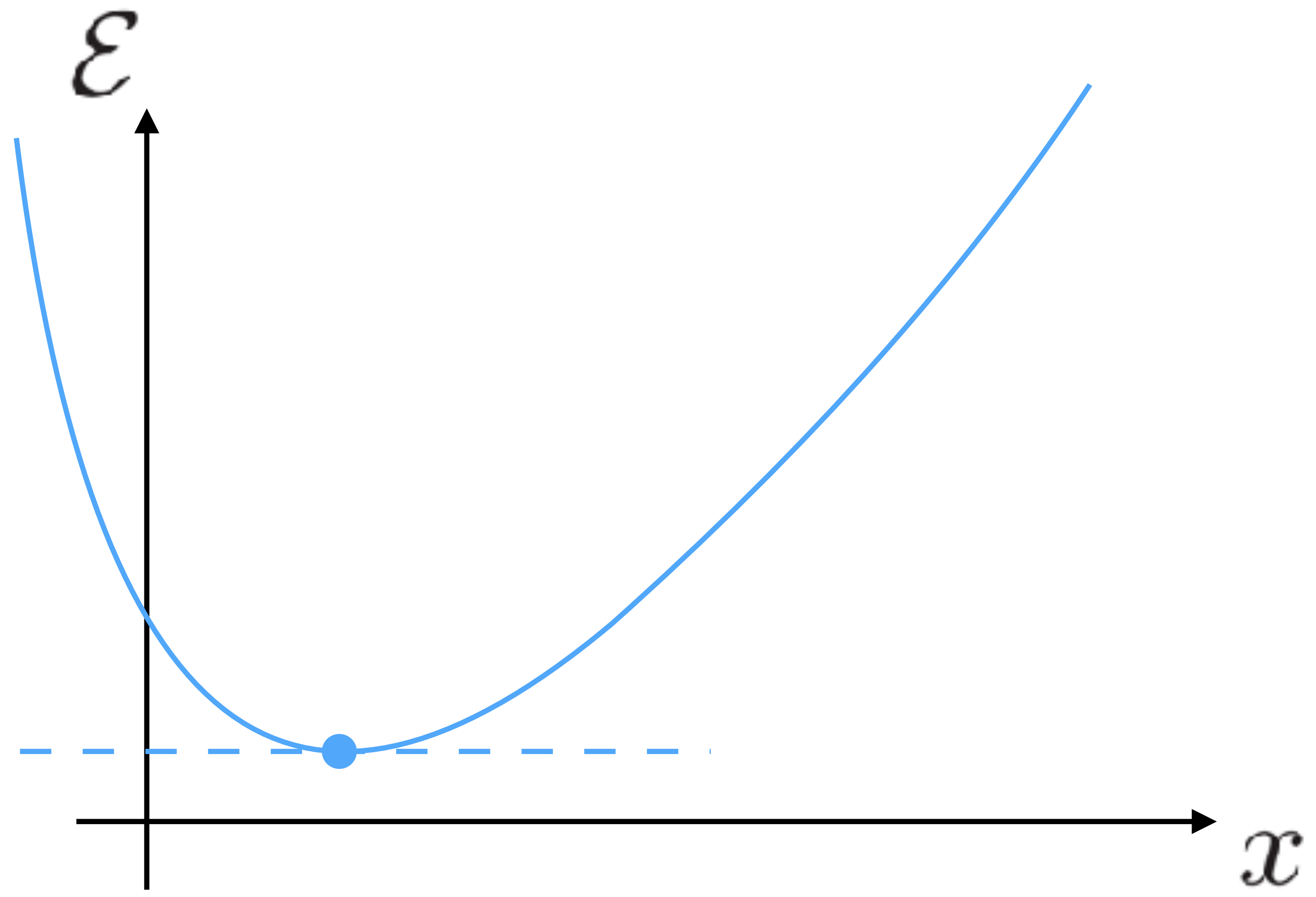


How To Optimize

Optimality Condition

$$\min_x \mathcal{E}(x)$$

$$\nabla \mathcal{E} = 0$$

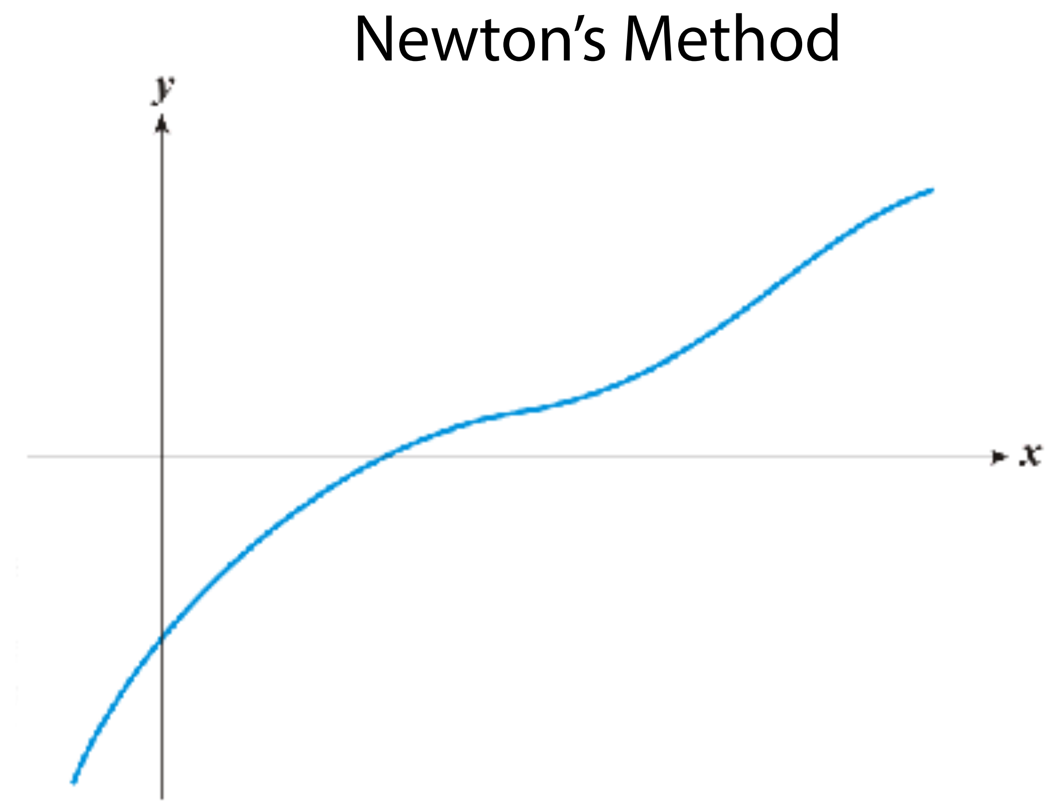


Newton's Method

$$\min_x \mathcal{E}(x)$$

$$Hp + \nabla\mathcal{E} = 0$$

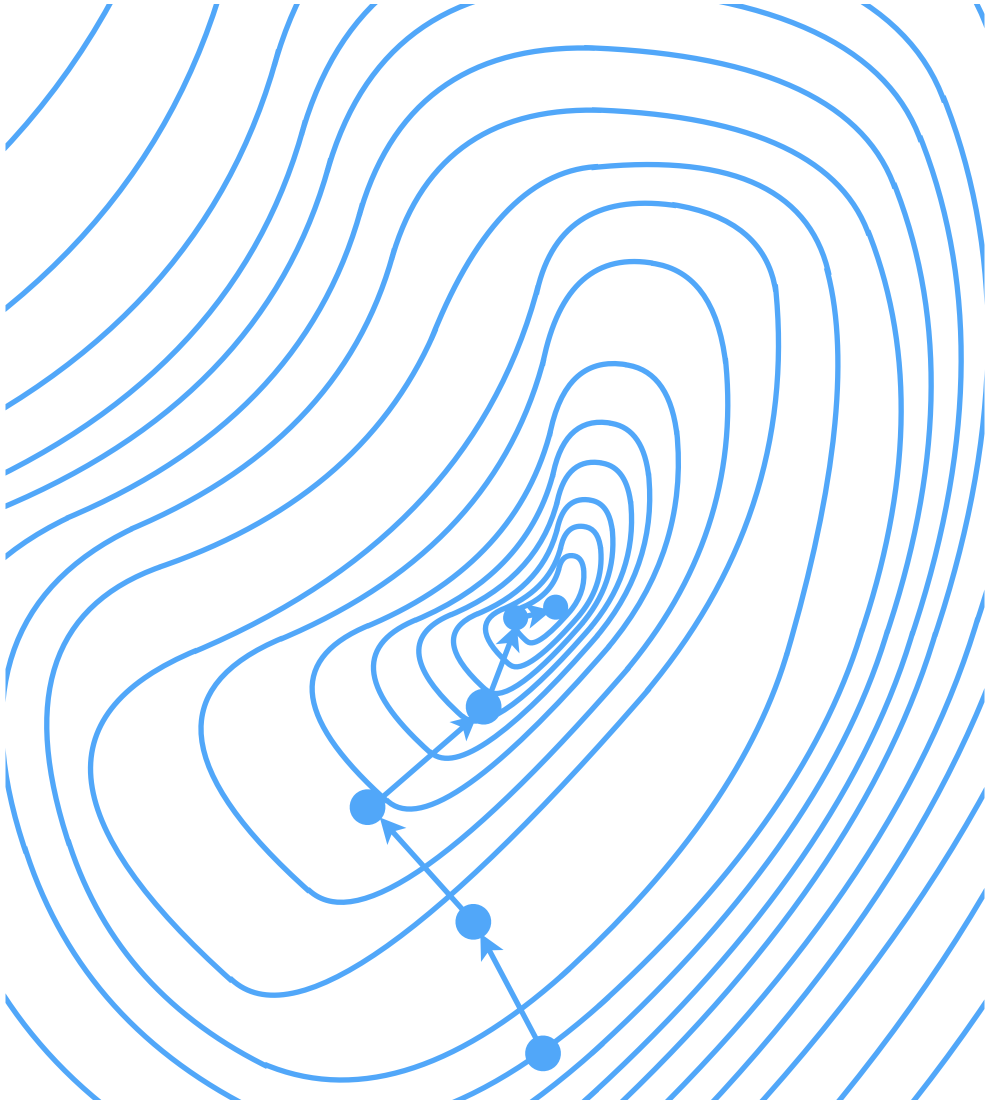
$$\frac{\partial^2 \mathcal{E}}{\partial x_i \partial x_j} \qquad\qquad \frac{\partial \mathcal{E}}{\partial x}$$



Gradient Descent

$$\min_{x} \mathcal{E}(x)$$

$$x_{n+1} = x_n - \alpha \nabla \mathcal{E}$$



Pros & Cons

Newton's Method

✓ Good Convergence Rate

✗ Stability

✗ Expensive Per Iteration Cost

Gradient Descent

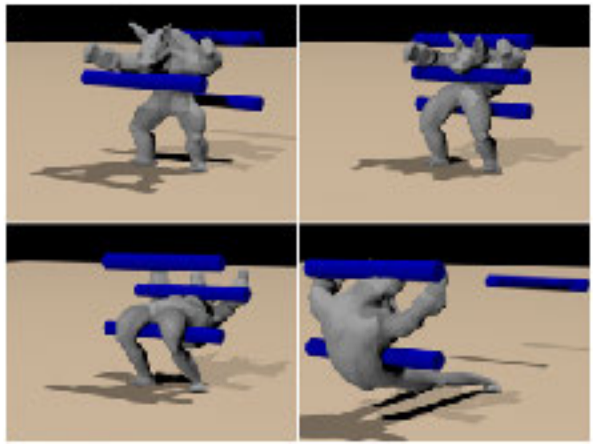
✓ Cheap Per Iteration Cost

✗ Poor Convergence Rate

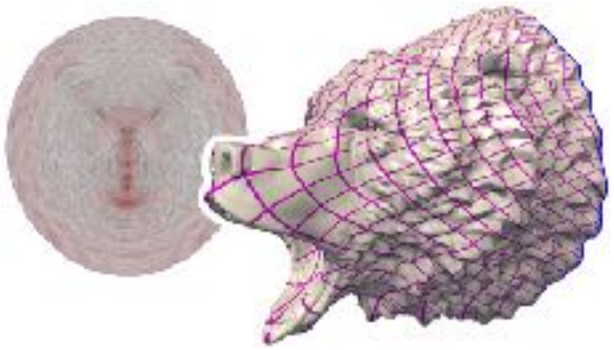
Review

Newton's Method

- ✓ Good Convergence Rate
- ✗ Stability
- ✗ Expensive Per Iteration Cost



[Teran et al. 2005]



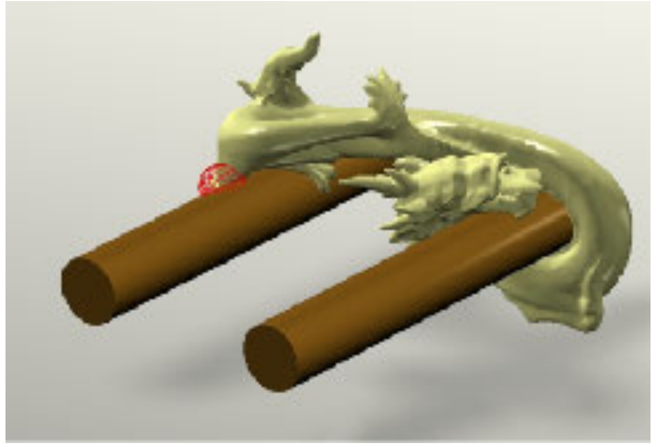
[Shtengel et al. 2017]

Review

Newton's Method

[Shtengel et al. 2015]

- ✓ Good Convergence Rate
- ✗ Stability
- ✗ Expensive Per Iteration Cost



[Wang & Yang 2016]

[Chen & Weber 2016]

Review

[Bouaziz et al. 2014]

Proxy

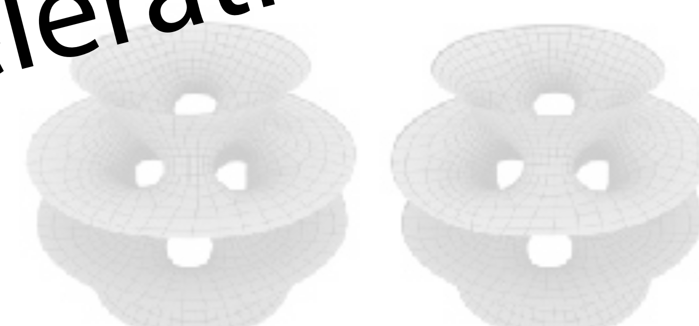
[Rabonovich et al. 2016]

[Claici et al. 2017]

[Kovalsky et al. 2016]

Acceleration

[Liu et al. 2017]



[Peng et al. 2018]

Gradient Descent

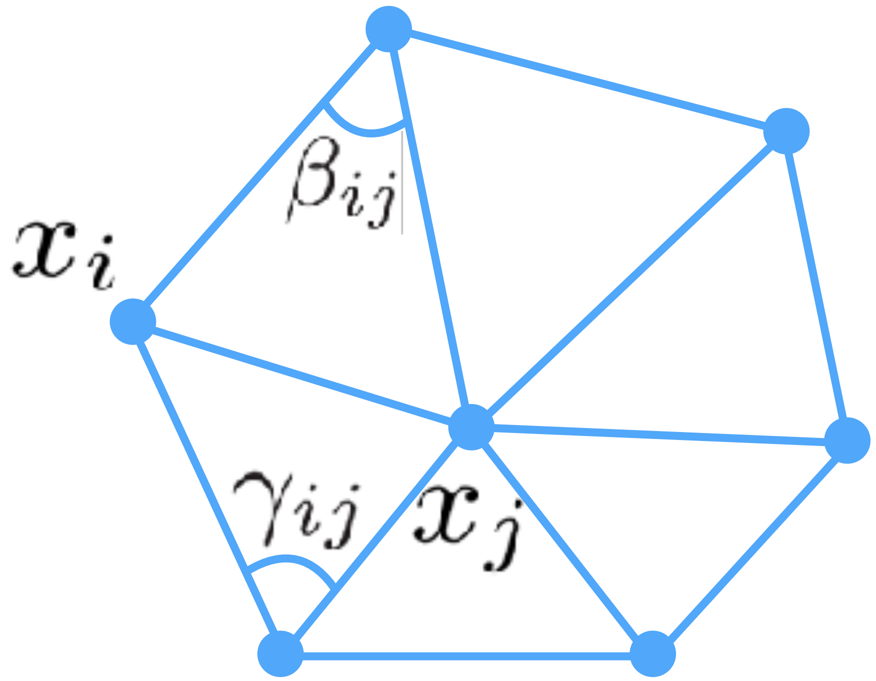
- ✓ Cheap Per Iteration Cost
- ✗ Poor Convergence Rate

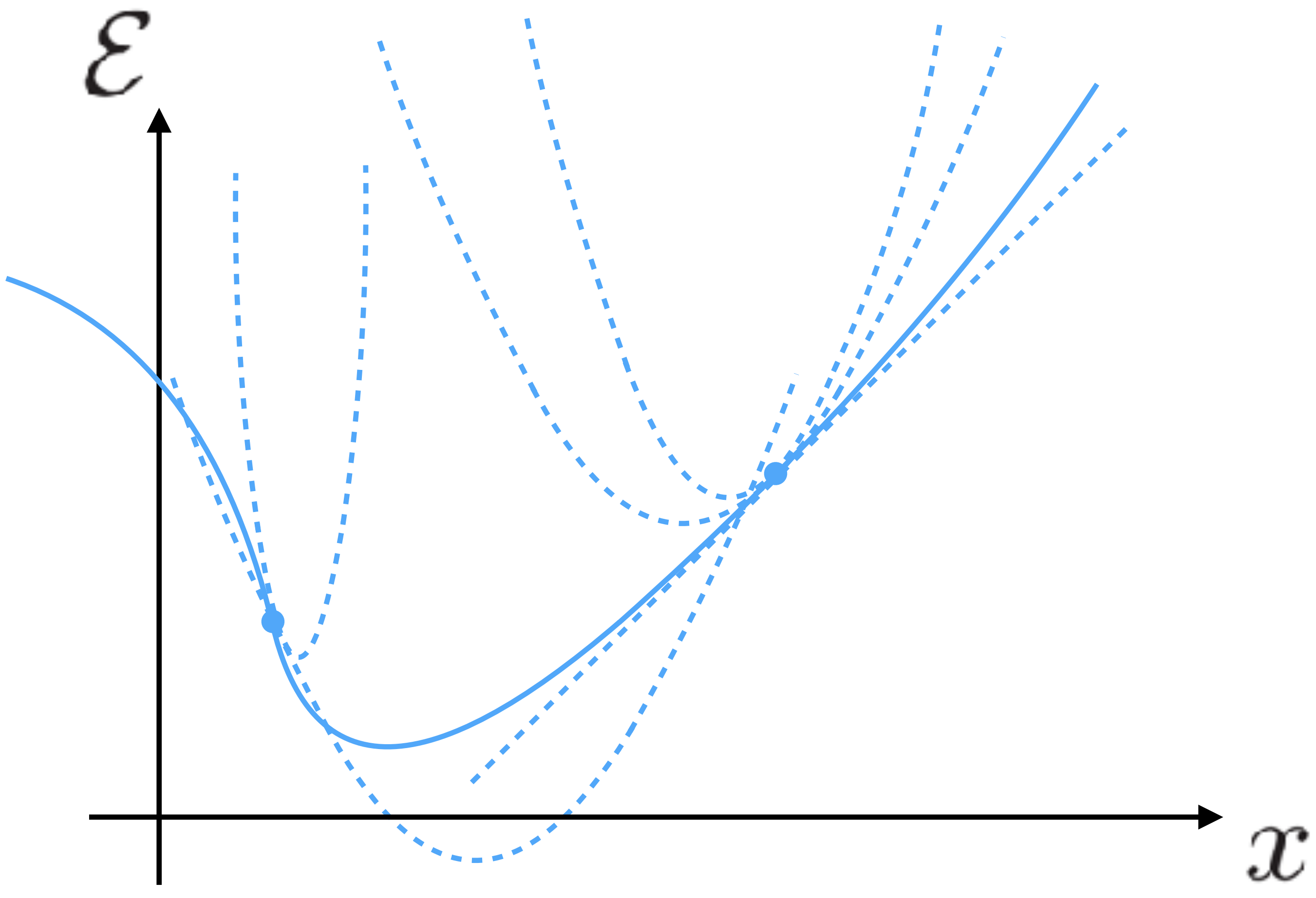
Proxy

Proxy

$$\min_x \mathcal{E}(x)$$

$$Lp + \nabla \mathcal{E} = 0$$

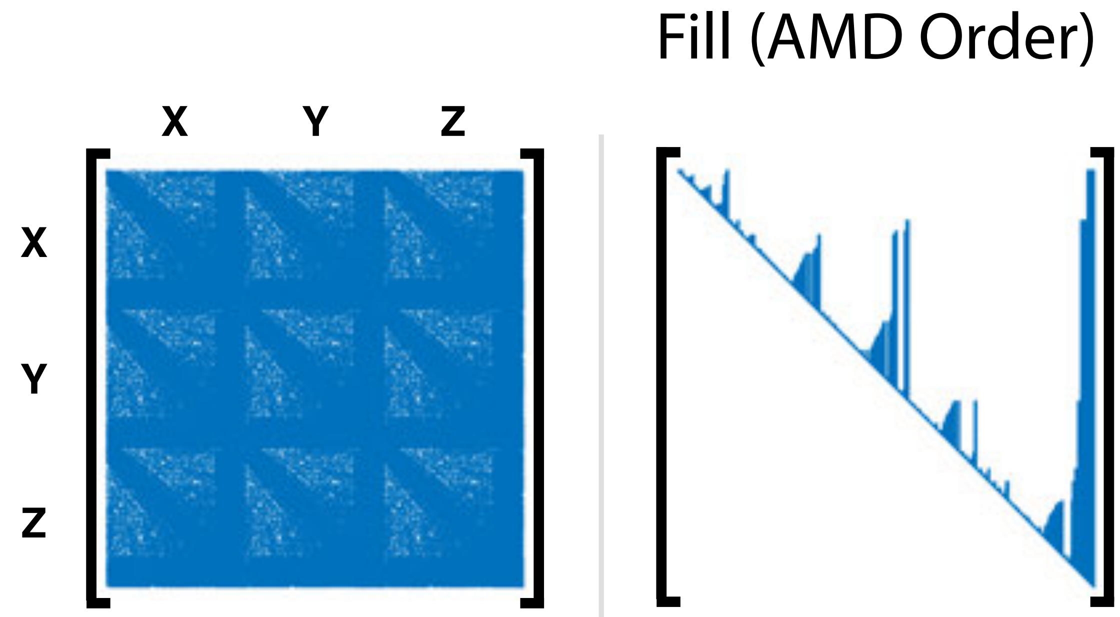


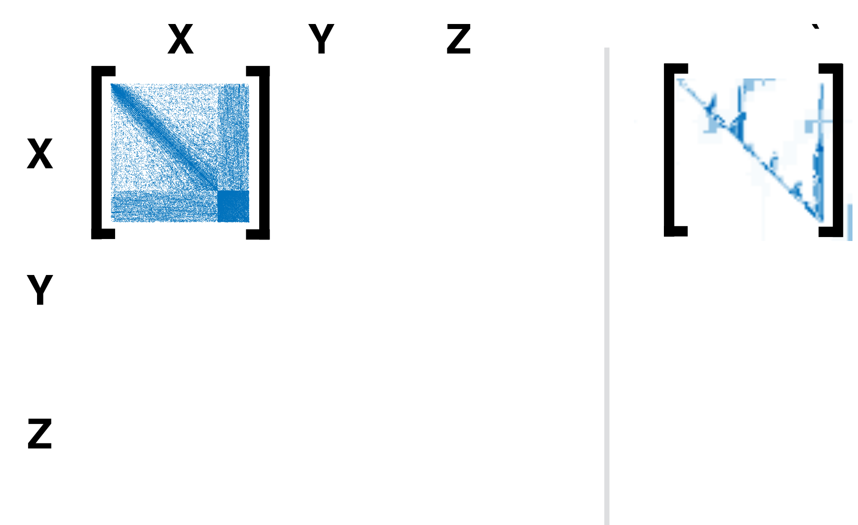


Decoupling

$$\min_{x} \mathcal{E}(x)$$

$$Lp + \nabla \mathcal{E} = 0$$



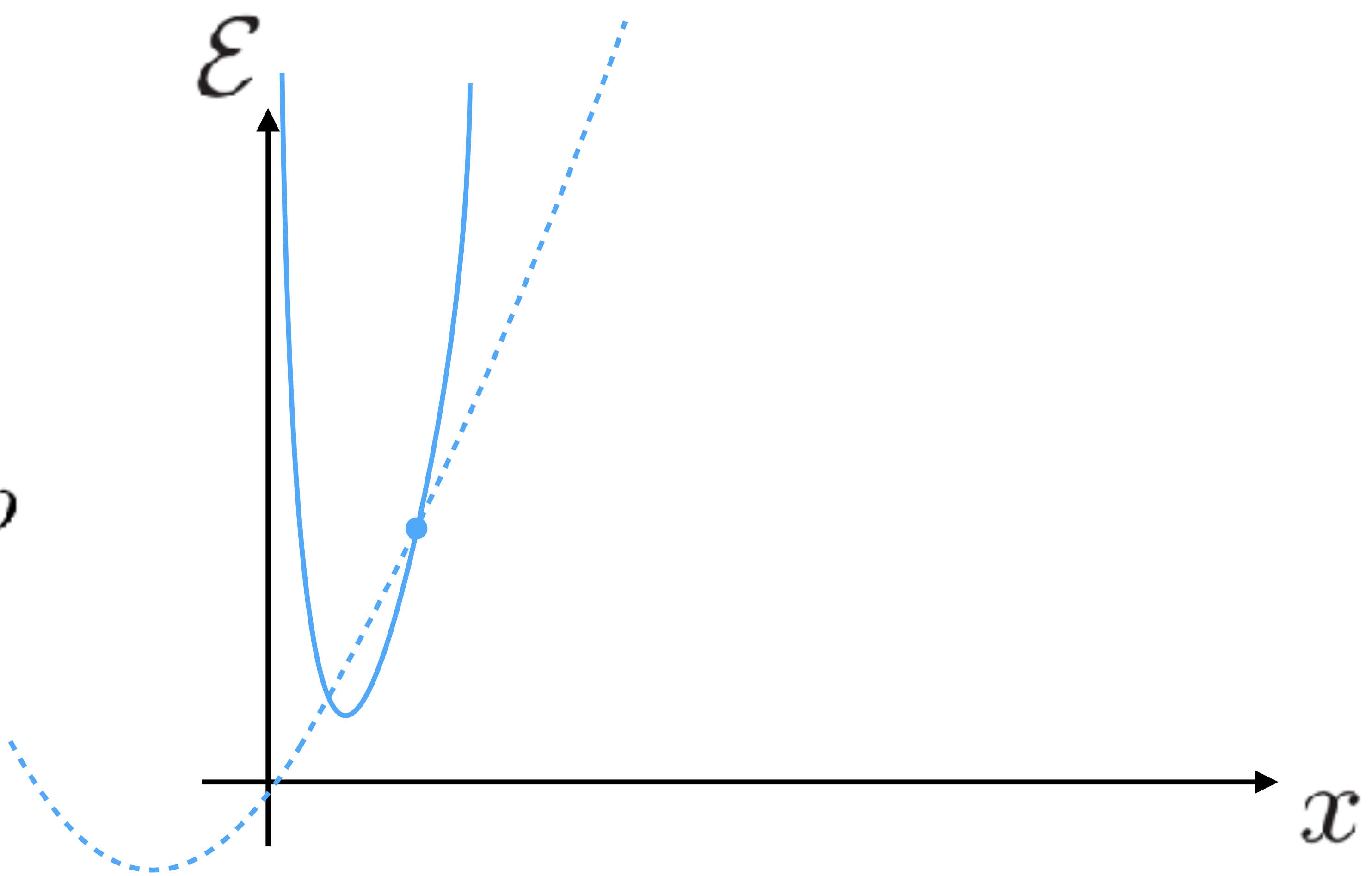


Barrier Type Energy

$$\min_x \mathcal{E}(x)$$

$$Lp + \nabla\mathcal{E} = 0$$

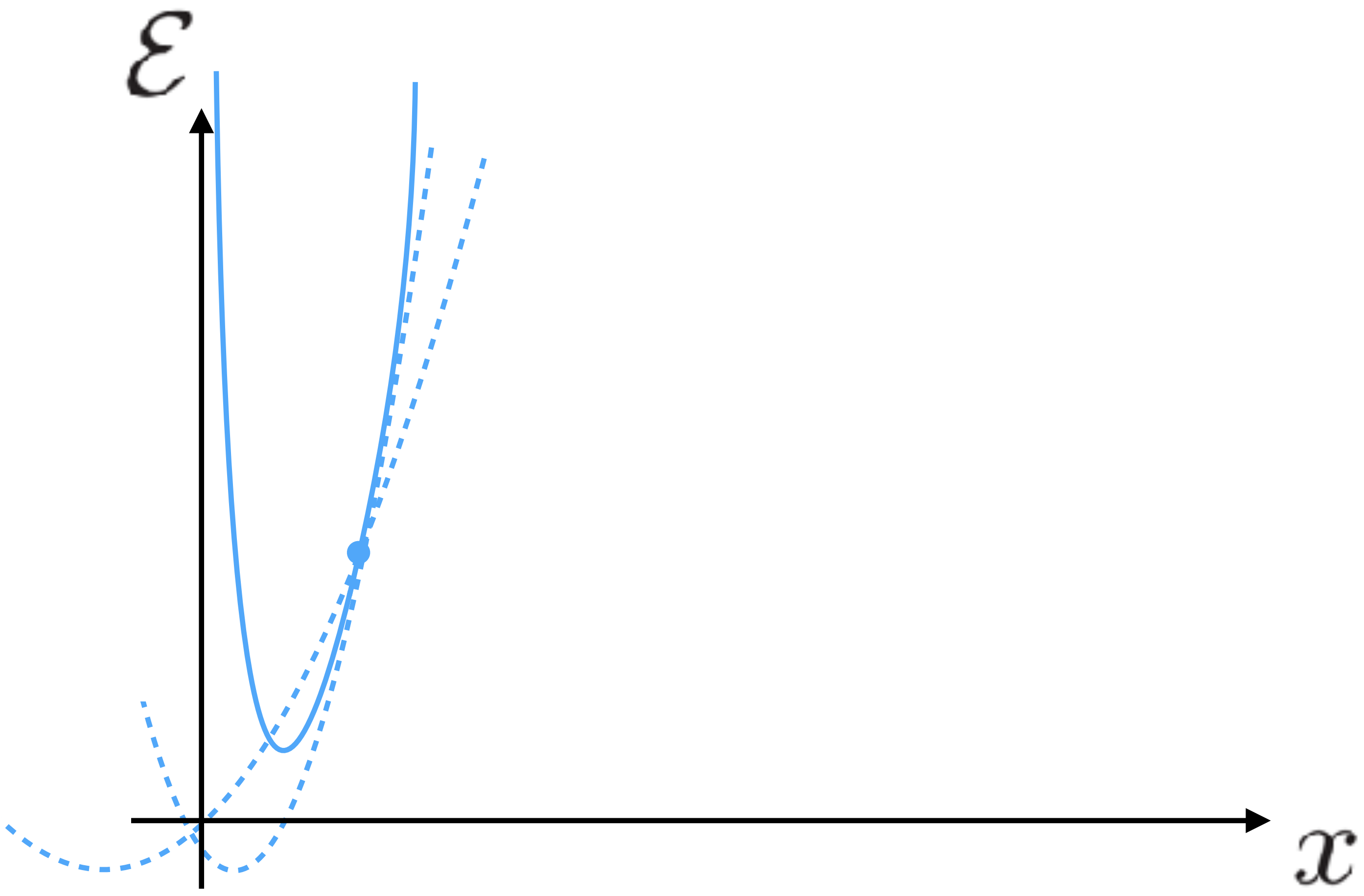
$$\Delta x = x_{n+1} - x_n = \alpha p$$



Reweighting

$$\min_x \mathcal{E}(x)$$

$$\tilde{L}p + \nabla\mathcal{E} = 0$$



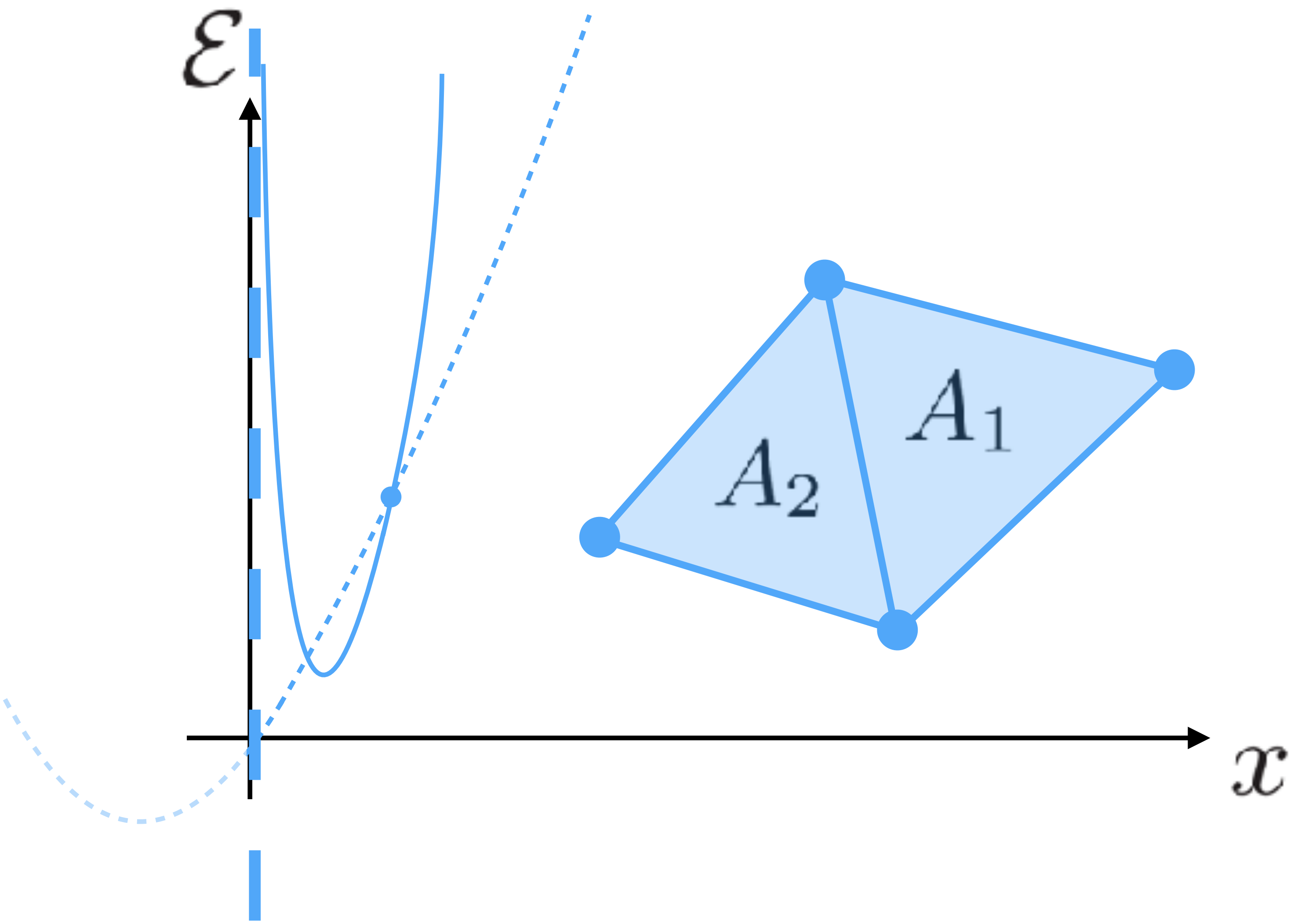
Adding Constraints

$$\min_x \mathcal{E}(x)$$

$$Lp + \nabla\mathcal{E} = 0$$

$$A(x_n + p) > 0$$

$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \end{bmatrix} > 0$$



Working? Good Enough?

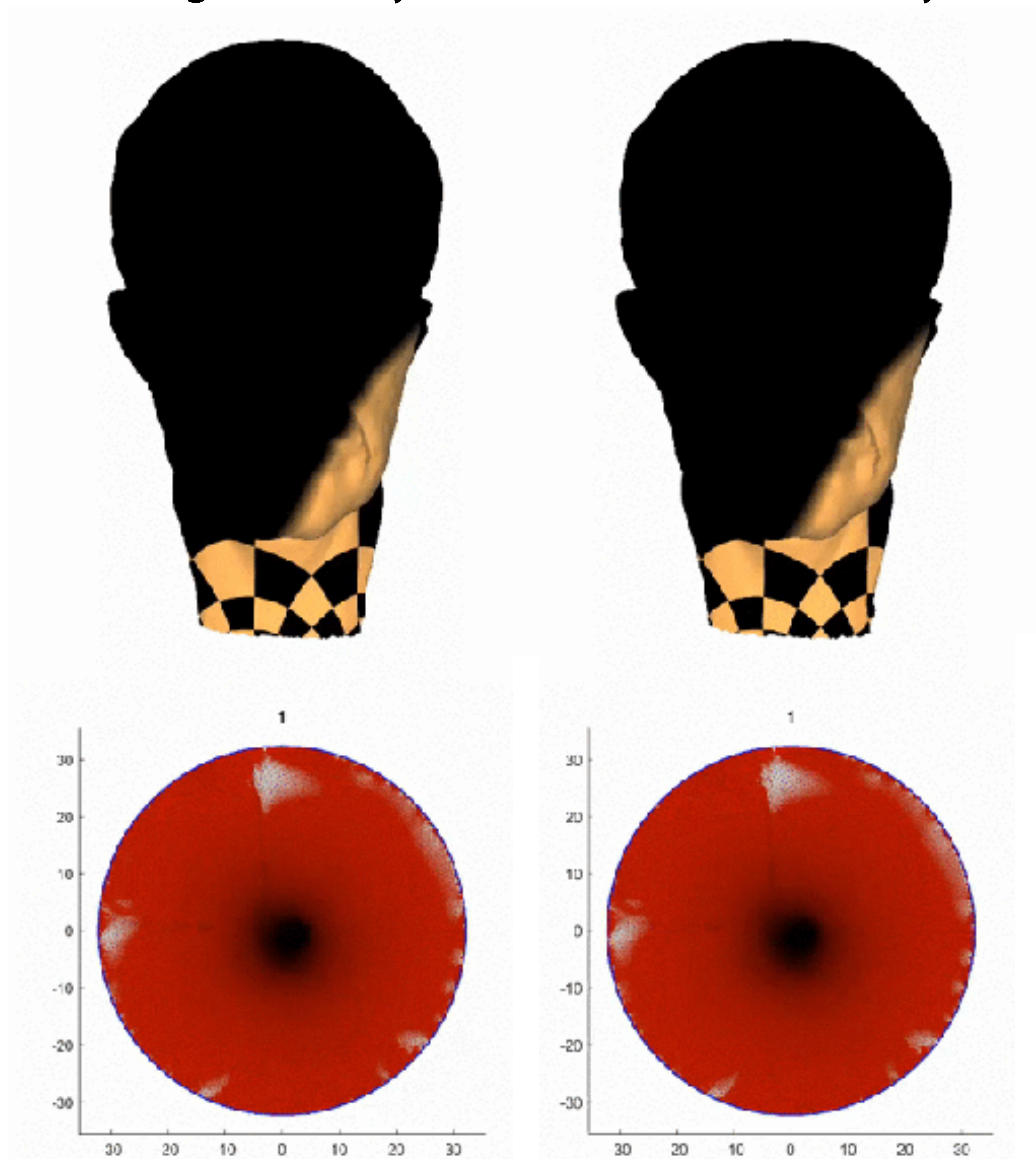
$$\min_x \mathcal{E}(x)$$

$$Lp + \nabla\mathcal{E} = 0$$

$$A(x_n + p) > 0$$

Original Proxy

Enhanced Proxy



Per Iteration Cost

$$\min_x \mathcal{E}(x)$$

$$\min_p \frac{1}{2} p^T L p + p^T \nabla \mathcal{E}$$

$$A(x_n) + \nabla A(x_n)^T p > 0$$

$$Lp + \nabla \mathcal{E} - \nabla A(x_n) \lambda = 0$$

$$0 \leq \lambda \perp A(x_n) + \nabla A(x_n)^T p > 0$$

$$0 \leq \lambda \perp \boxed{\nabla A(x_n)^T \boxed{L^{-1}} \nabla A(x_n)} \lambda + \boxed{A(x_n) - \nabla A(x_n)^T L^{-1} \nabla \mathcal{E}} > 0$$

Don't Give Up Easily

$$\min_x \mathcal{E}(x)$$

$$\min_q \frac{1}{2}\|p - q\|_2$$

$$\min_p \frac{1}{2} p^T L p + p^T \nabla \mathcal{E}$$

$$A(x_n + q) > 0$$

$$q = p + \nabla A \lambda$$

$$0 \leq \lambda \perp \boxed{\nabla A(x_n)^T \nabla A(x_n)} \lambda + \boxed{A(x_n) + \nabla A(x_n)^T p} > 0$$

Acceleration

Acceleration

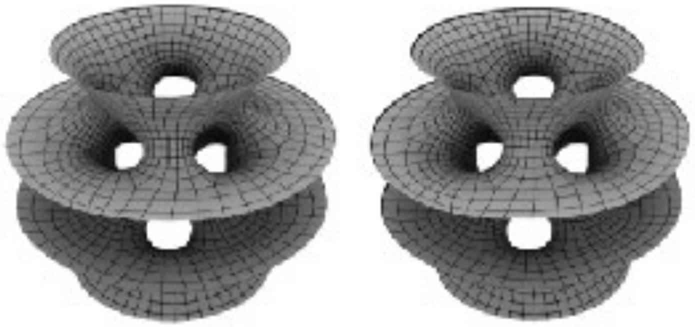
Nesterov Acceleration

[Kovalsky et al. 2016]

BFGS

[Liu et al. 2017]

Anderson Acceleration



[Peng et al. 2018]

BFGS

$$\min_x \mathcal{E}(x)$$

$$B_0 = L$$

$$Bp + \nabla\mathcal{E} = 0$$

$$s_k = x_{k+1} - x_k$$

$$y_k = \nabla\mathcal{E}_{k+1} - \nabla\mathcal{E}_k$$

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k}$$

$$y_k = B_{k+1} s_k$$

Blending

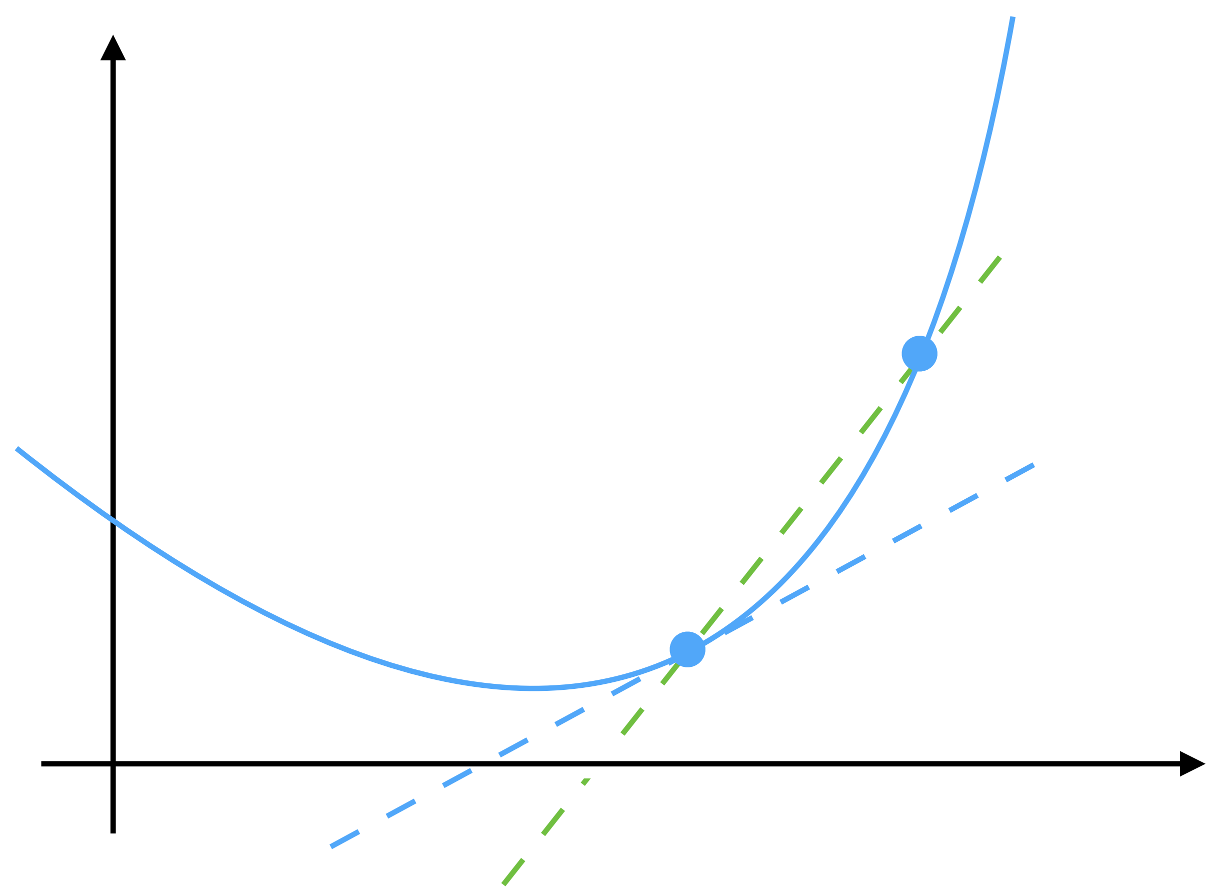
$$\min_x \mathcal{E}(x)$$

$$Bp + \nabla \mathcal{E} = 0$$

$$tH + (1-t)P$$

I $\quad L$

$\nabla \mathcal{E} \nabla \mathcal{E}^T$



Blending

$$\min_x \mathcal{E}(x)$$

$$\begin{tikzcd} B \arrow[r, bend left, leftrightarrow] & B \end{tikzcd}$$

$$Bp + \nabla \mathcal{E} = 0$$

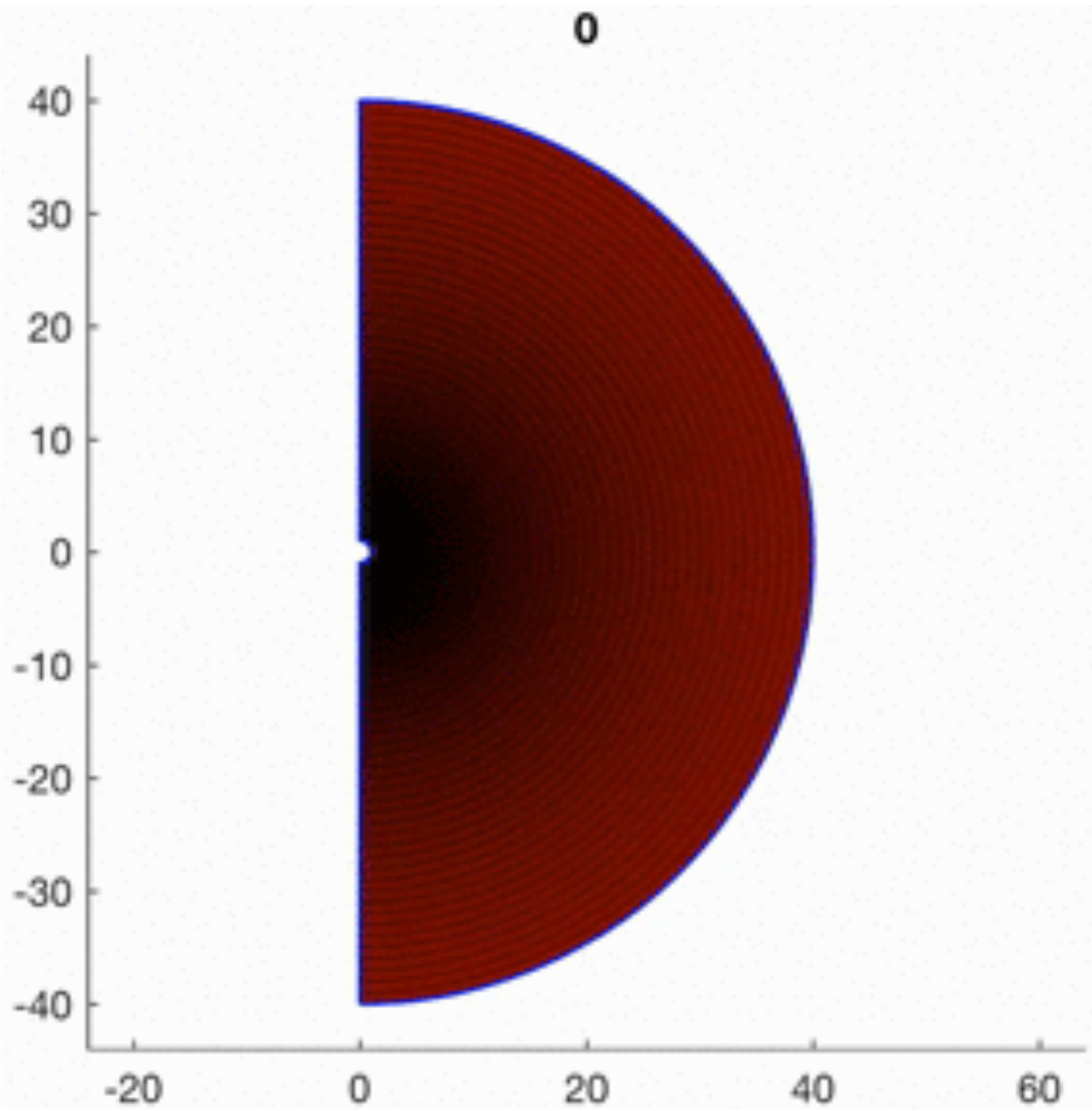
$$\tilde{y}_k = L s_k$$

$$z_k = t y_k + (1 - t) \tilde{y}_k$$

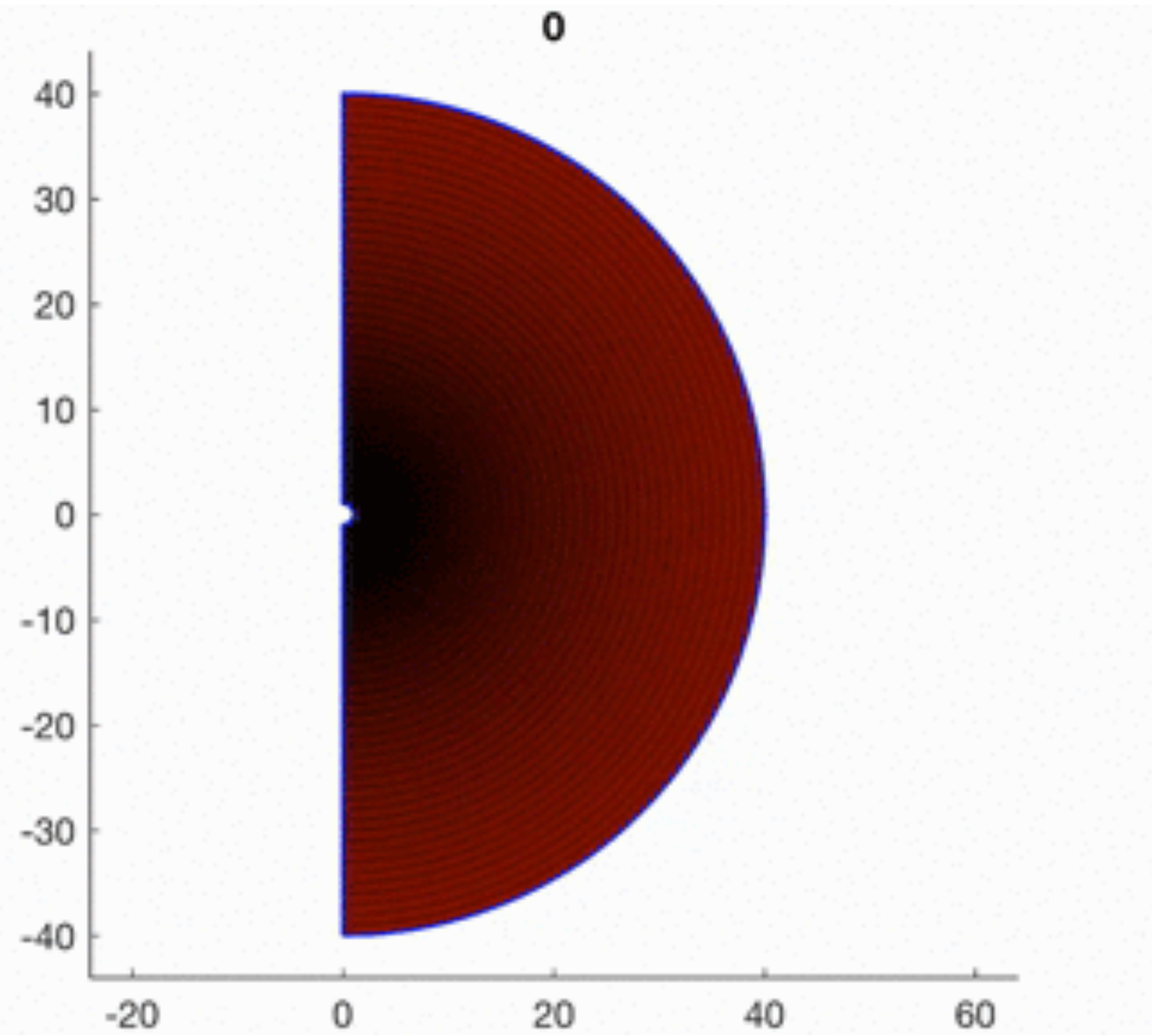
$$B_{k+1}^{-1} = (I - \frac{s_k z_k^T}{z_k^T s_k}) B_k^{-1} (I - \frac{z_k s_k^T}{z_k^T s_k}) + \frac{s_k s_k^T}{z_k^T s_k}$$

Comparison

SL-BFGS



Ours

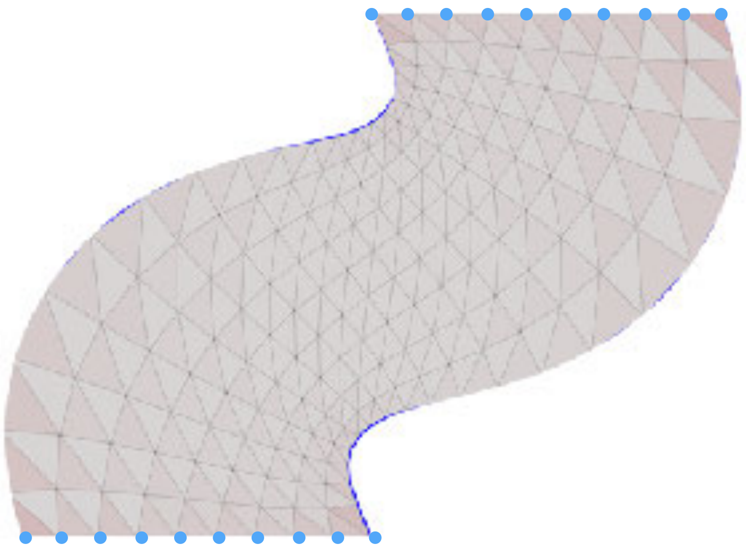


Constraint & Termination

Position Constraint

$$\min_x \mathcal{E}(x)$$

$$Kx + b = 0$$



$$x = \tilde{K}\tilde{x} + x_0$$

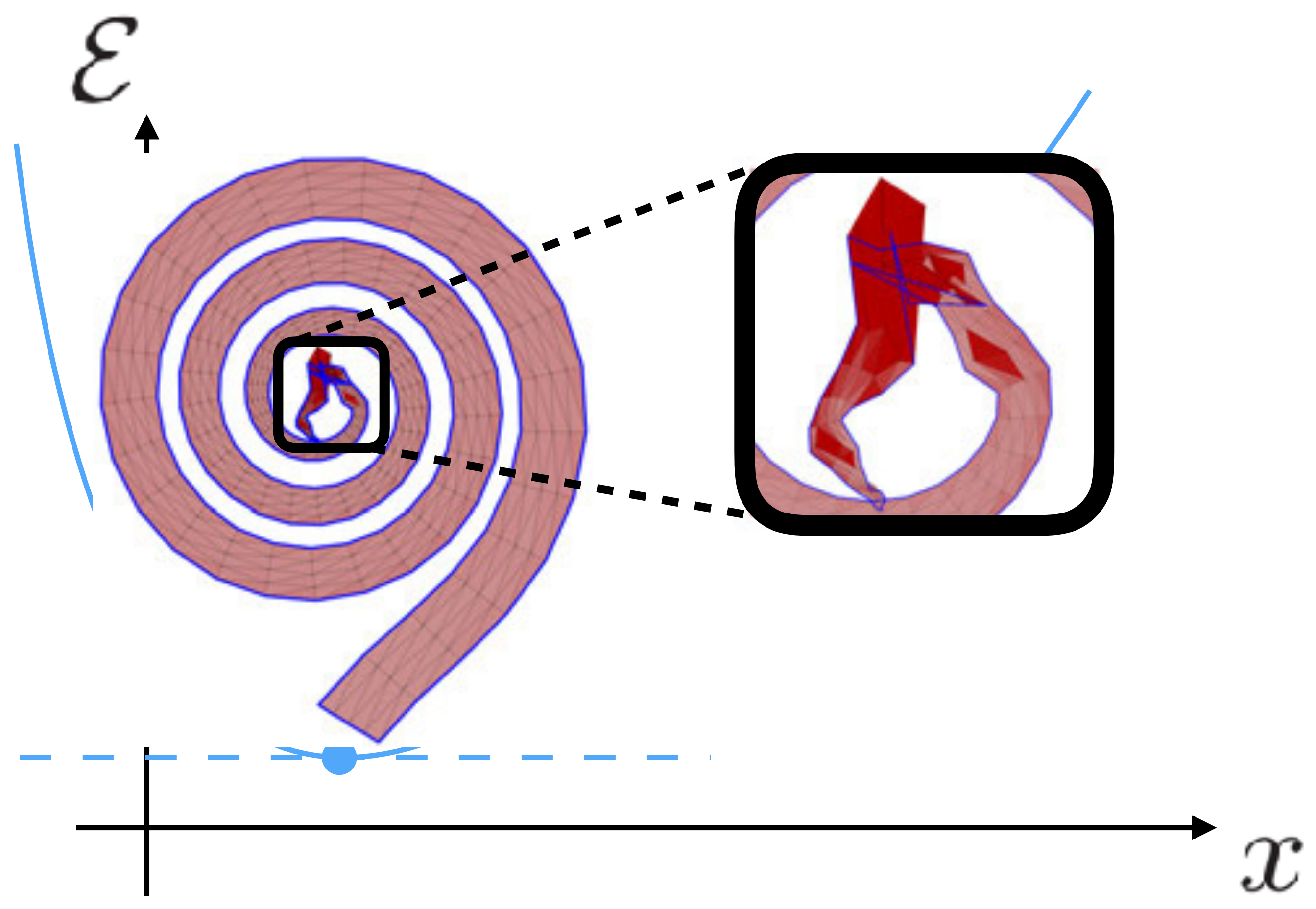
$$col(\tilde{K}) = null(K)$$

$$\underset{K}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}} \qquad \underset{\tilde{K}}{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Stopping Criteria

$$\min_x \mathcal{E}(x)$$

$$\nabla \mathcal{E} = 0$$



$$rel(\mathcal{E}) < \epsilon$$

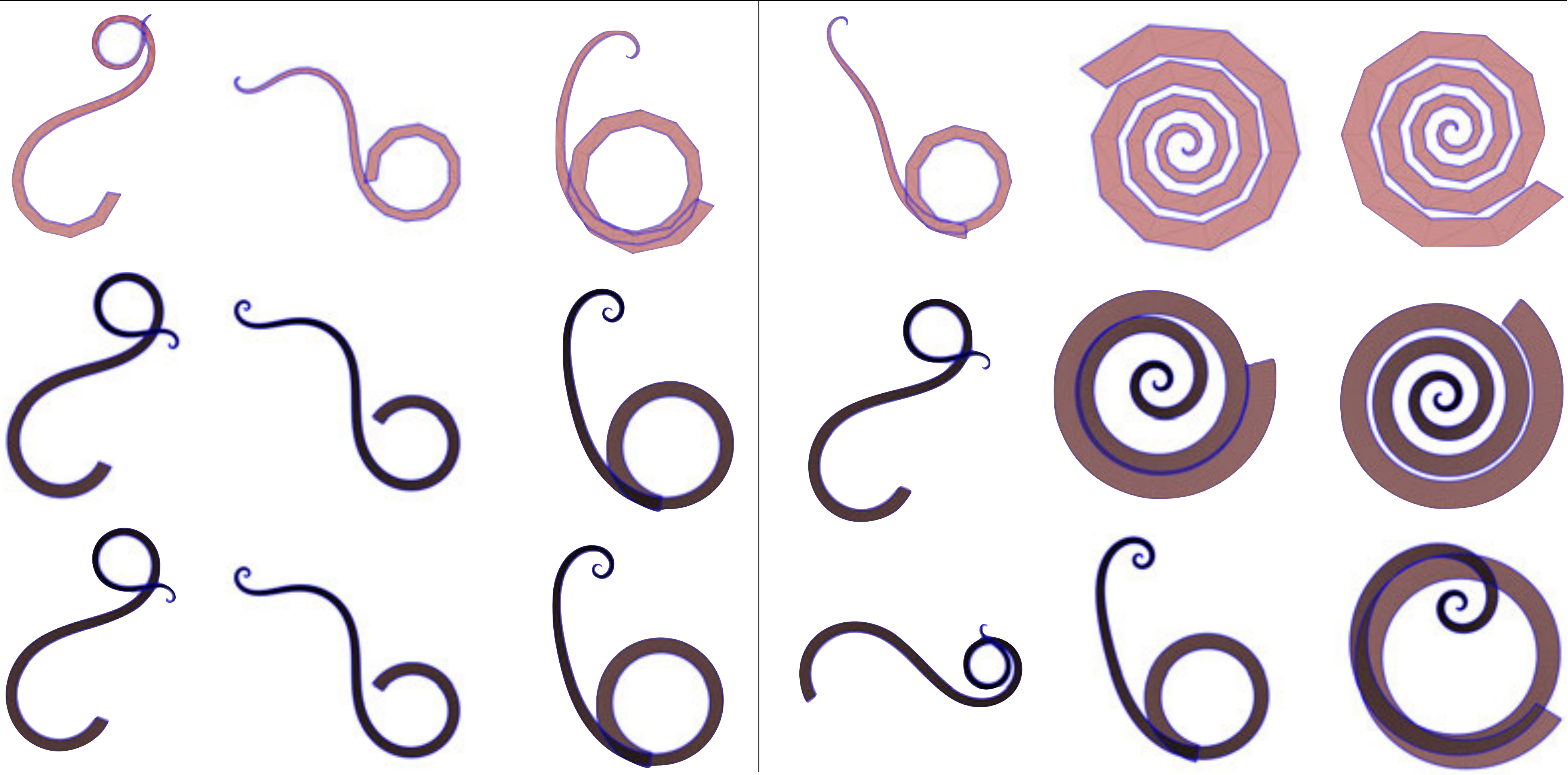
$$rel(x) < \epsilon$$

Stopping Criteria

$$\min_{x} \mathcal{E}(x)$$

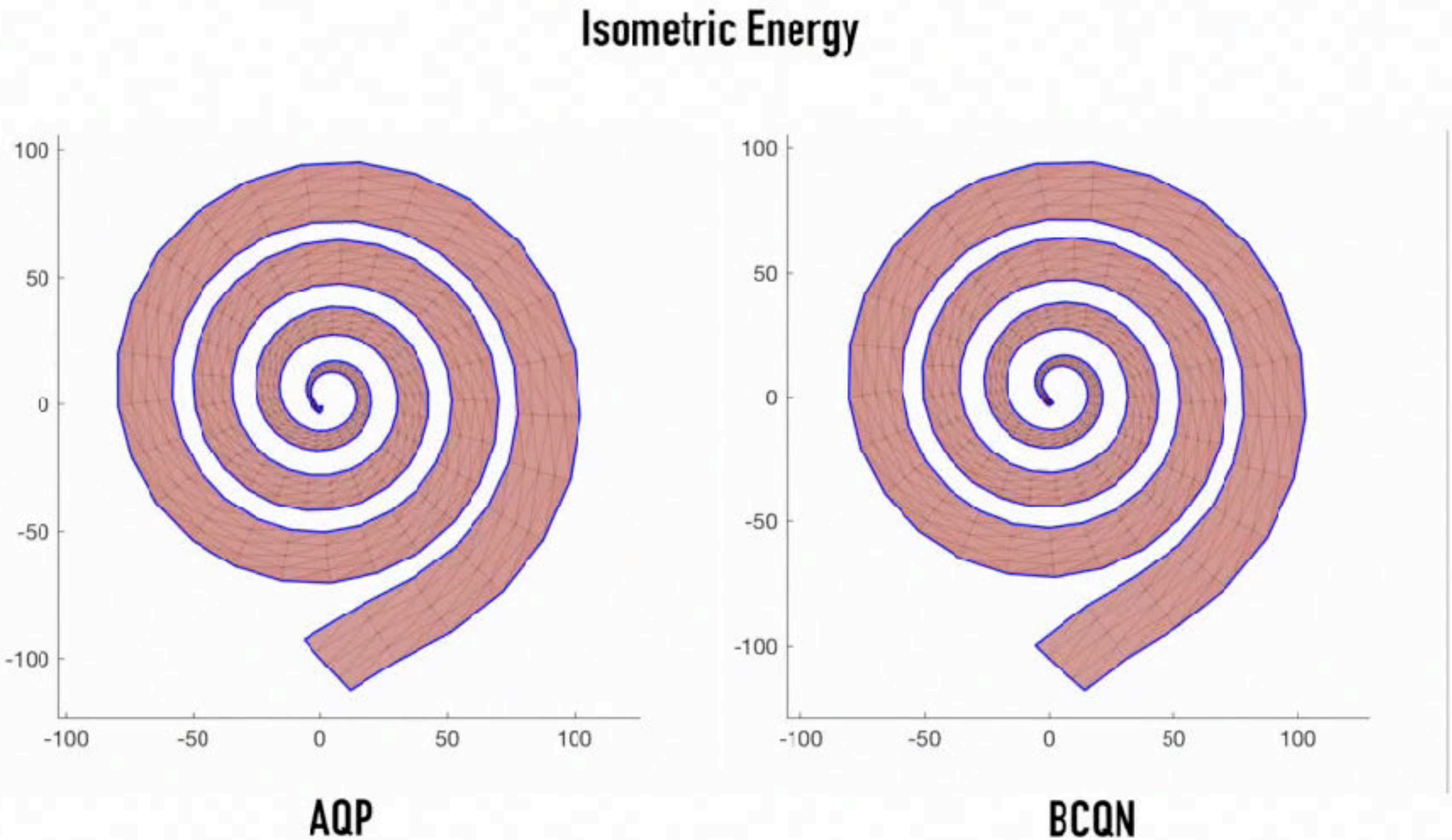
$$\beta \|\nabla \mathcal{E}\| < \epsilon$$

Tolerance	1e-3	1e-4	1e-5	1e-3	1e-4	1e-5
Coarse						
Fine						
Scaled						

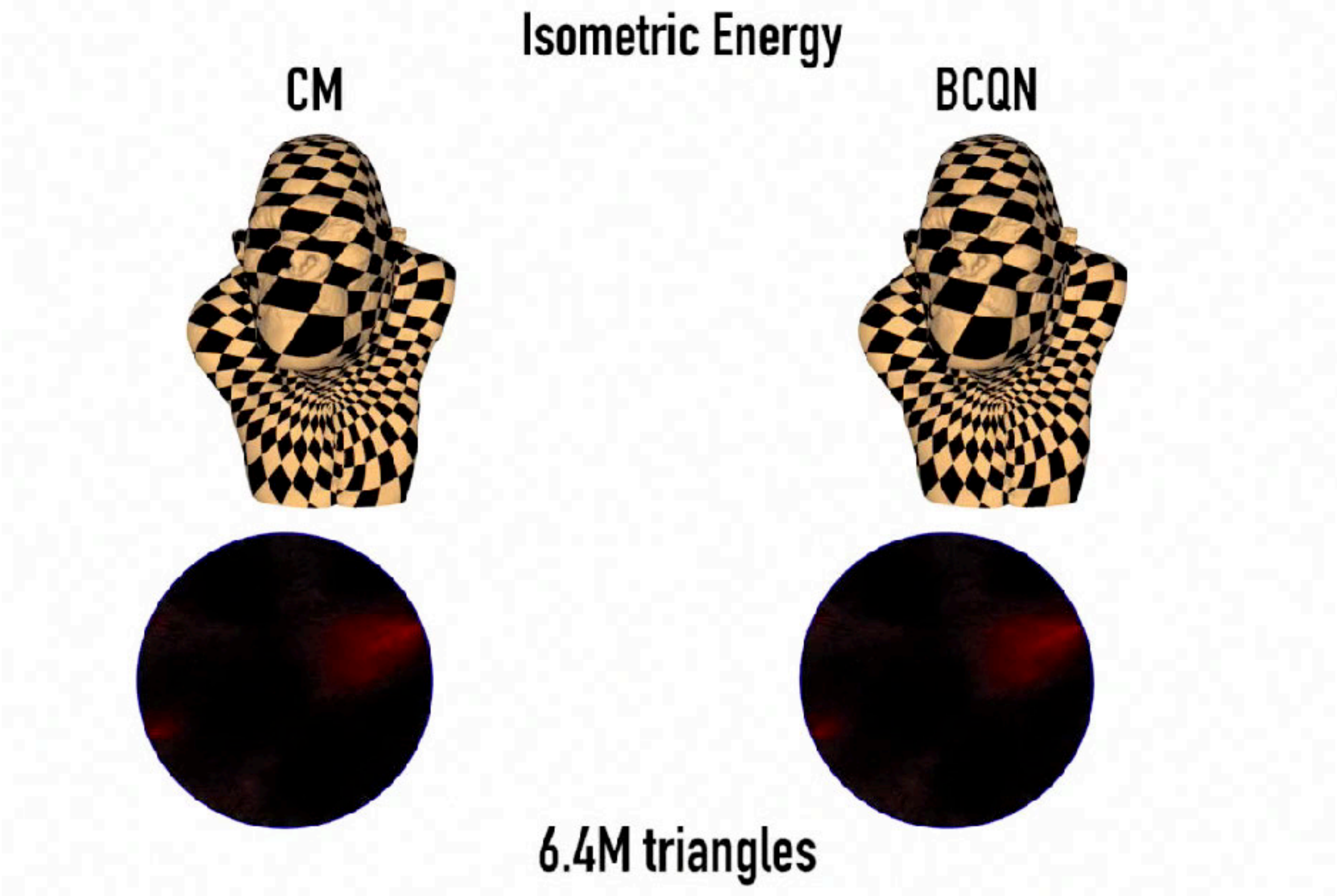


Results

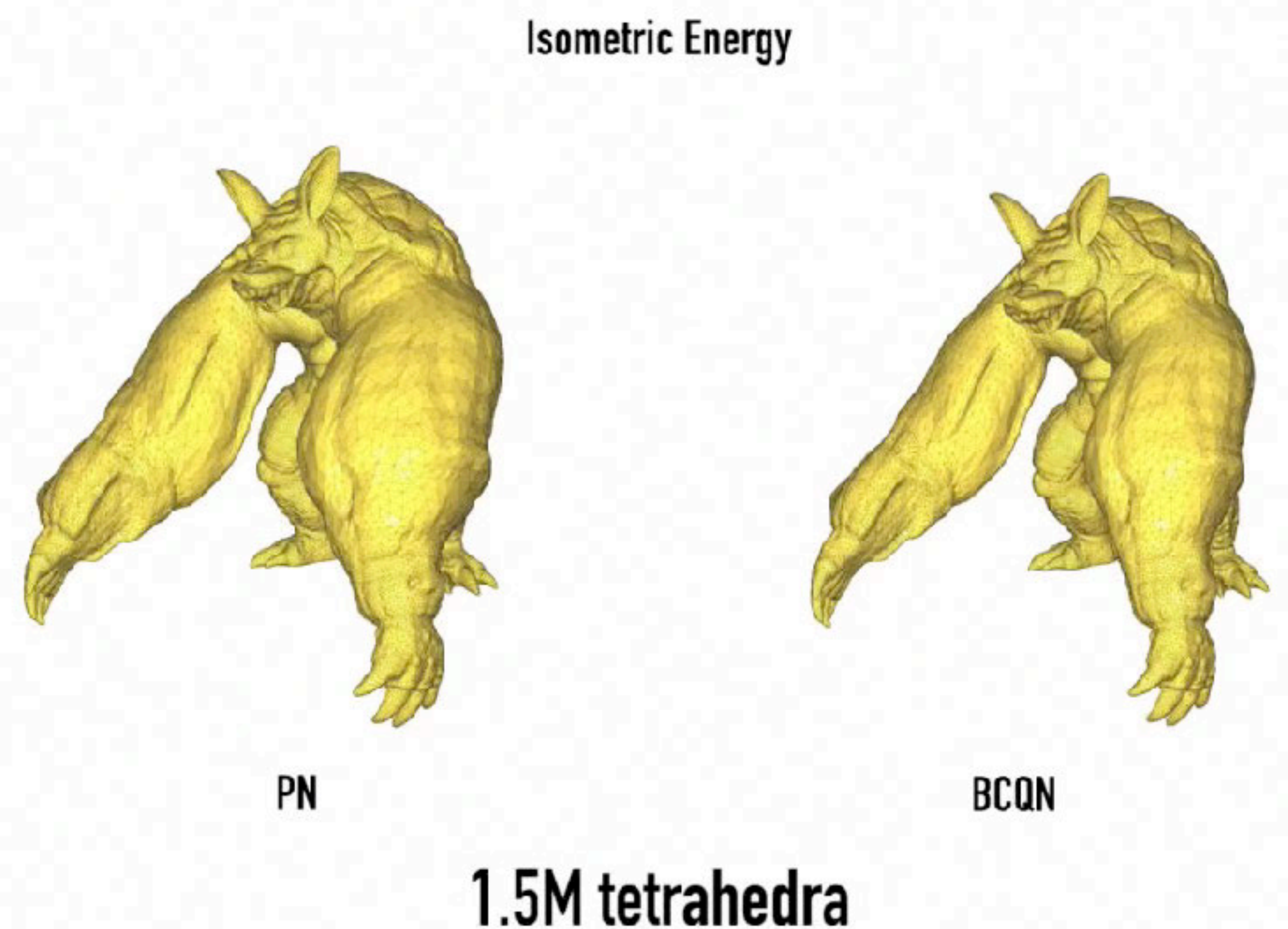
Results


Isometric Energy
100
50
0
-50
-100
-100
-50
0
50
100
AQP
100
50
0
-50
-100
-100
-50
0
50
100
BCQN

Results

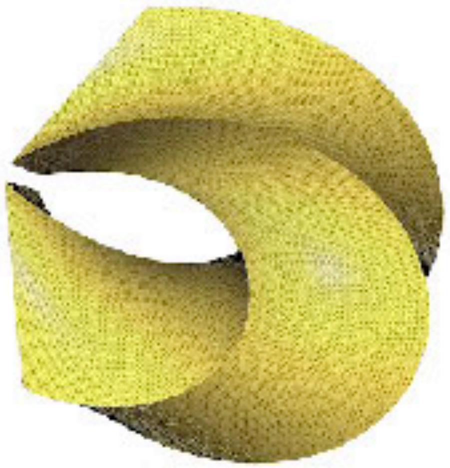


Results

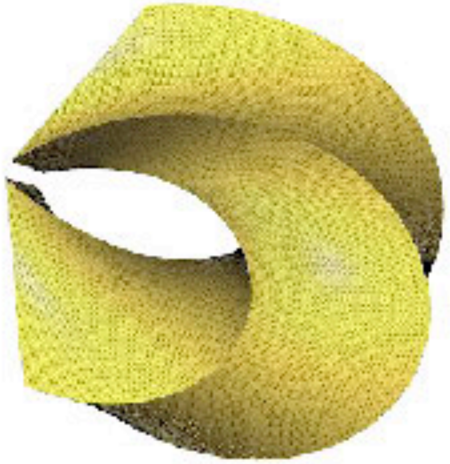


Results

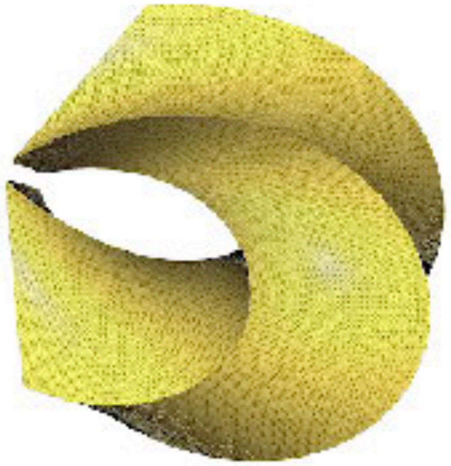
Isometric Energy



BCQN



AQP



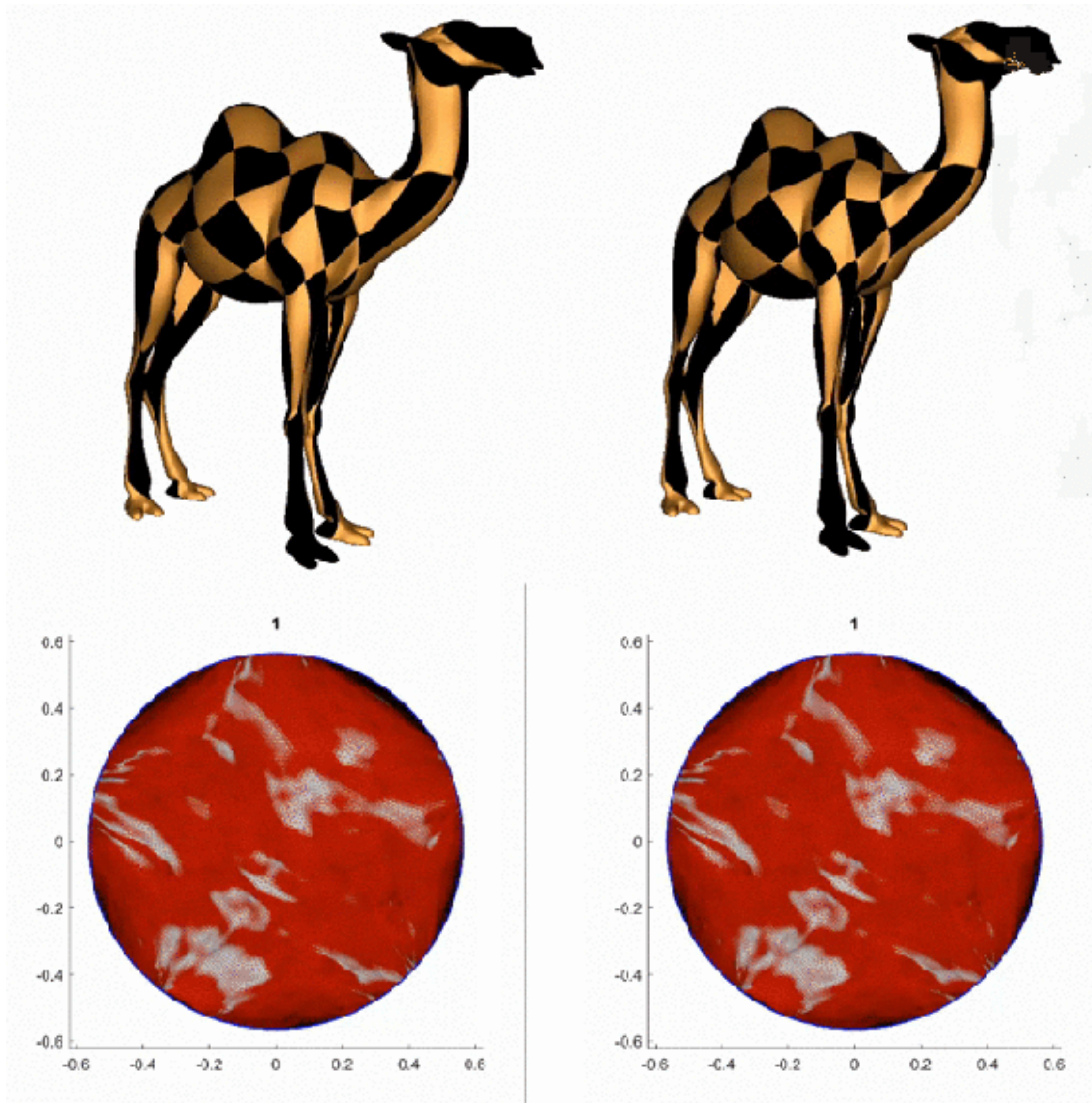
PN

1.5M tetrahedra

Results

AQP

Ours



Q & A