Space-time Tomography for Continuously Deforming Objects

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Tomography in graphics

Gas Flows capture  
[Atcheson et al. 2008]

Fluid scanning  
[Gregson et al. 2012]

Velocity reconstruction  
[Gregson et al. 2014]

Transparent reconstruction  
[Trifonov et al. 2006]

Multi-layer 3D display  
[Wetzstein et al. 2011]

Flower modeling  
[Ijiri et al. 2014]
Motivation

Unknowns: $\sim 500^3$ voxels

Equations: $2M$ pixels $\times$ Number of projections

Large number of projections + priors
Motivation

Time for one projection: ~ 5 s

Is it possible to scan continuously deforming objects?
Motivation

Classical reconstruction using 5520 projections (duration: 9h32m)

Before scan

Wilting rose

After scan
Related work

Discrete deformations / Key framing

Growing plant animation

[Kato et al. 2017]

Digital Volume Correlation to retrieve the deformation field

[Hild et al. 2014]
Related work

Periodic deformations (Medical field)

Electrocardiography (ECG)

[Chen et al. 2012]
Related work

General deformations (parametric approach)

Use of basis-functions approach (e.g. finite element framework)

The volume is advected from a reference, using an estimated displacement.

[Taillandier-Thomas et al. 2016]
Objective

Space-time tomography reconstruction

Before scan

Wilting rose

After scan
Space-time tomography

Assumption: relatively slow deformations (less than 6 voxels/minute)
Space-time tomography

Frame 01
Frame 02
Frame 46
Frame 91
Frame 92
Space-time tomography

Frame 01
Frame 02
Frame 46
Frame 91
Frame 92
Space-time tomography
Overview

• Data acquisition

• 4D tomographic reconstruction

• Results
Data acquisition

Classical acquisition strategy
Data acquisition

Classical acquisition strategy
Data acquisition

Low-discrepancy acquisition strategy

(Van der Corupt sequence)
Data acquisition

Low-discrepancy acquisition strategy

(Van der Corupt sequence)
Overview

• Data acquisition

• 4D tomographic reconstruction

• Results
Image formation model

X-ray path: $\Omega_i$
Image formation model

X-ray path: $\Omega_i$
Unknown field: $f(x)$
Image formation model

X-ray path: $\Omega_i$
Unknown field: $f(x)$
Measurement: $p_i$

$$\int_{\Omega_i} f(x) \, d\Omega_i = -\log\left(\frac{I_i}{I_0}\right)$$
Image formation model

X-ray path: $\Omega_i$
Unknown field: $f(x)$
Measurement: $p_i$

$$\sum_j f_j A_{ij} = p_i$$
Image formation model

X-ray path: $\Omega_i$
Unknown field: $f(x)$
Measurement: $p_i$

$$\sum_j f_j A_{ij} = p_i$$

$$A_t f_t = p_t$$

Negligible deformation in sequential projections!
Space-time tomography

\[
\begin{pmatrix}
A_1 & \cdots & A_t & \cdots & A_{N_t}
\end{pmatrix}
\begin{pmatrix}
f_1 \\
\vdots \\
f_t \\
\vdots \\
f_{N_t}
\end{pmatrix} =
\begin{pmatrix}
p_1 \\
\vdots \\
p_t \\
\vdots \\
p_{N_t}
\end{pmatrix}
\]
Linear system

- Sparse system
- Memory consuming
- Ill-posed problem

$$\begin{pmatrix}
A_1 & \cdots & A_t & \cdots & A_{N_t} \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\end{pmatrix} \cdot 
\begin{pmatrix}
f_1 \\
\vdots \\
f_t \\
\vdots \\
f_{N_t} \\
\end{pmatrix} = 
\begin{pmatrix}
p_1 \\
\vdots \\
p_t \\
\vdots \\
p_{N_t} \\
\end{pmatrix}$$

Radon Transform
Frames Measurements
Linear system

Space time tomography

- Non-parametric and matrix-free
- Alternating, multi-scale optimization
- Flexibility: priors incorporating and implementation
Priors

Tomography

$$\|A_t f_t - p_t \|_2^2$$
Priors

Tomography

$$\| A_t f_t - p_t \|_2^2$$

Spatial smoothness

$$\| \nabla_S f_t \|_{H_\epsilon}$$
Priors

Tomography
\[ \| A_t f_t - p_t \|_2^2 \]

Spatial smoothness
\[ \| \nabla_S f_t \|_{H^\epsilon} \]

Temporal smoothness
\[ \| \nabla_T f_t \|_2^2 \]
Priors

Tomography
\[ \| A_t f_t - p_t \|^2_2 \]

Spatial smoothness
\[ \| \nabla_S f_t \|_{H_\epsilon} \]

Temporal smoothness
\[ \| \nabla_T f_t \|^2_2 \]

Flow correction
\[ \| \nabla_T f_t + \nabla_S f_t \cdot u_t \|_1 \]
\[ \sum_{i=x,y,z} \| \nabla_S u_{t,i} \|_{H_\tau} \]
Priors

Tomography
\[ \| A_t f_t - p_t \|^2_2 \]

Spatial smoothness
\[ \| \nabla_S f_t \|_{H^\epsilon} \]

Temporal smoothness
\[ \| \nabla_T f_t \|^2_2 \]

Flow correction
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\[ \sum_{i=x, y, z} \| \nabla_S u_{t, i} \|_{H^\tau} \]
4D joint optimization
4D joint optimization

Input → Initial reconstruction
4D joint optimization

Input

Initial reconstruction

Converged?
4D joint optimization

Input

Initial reconstruction

Converged?

N

Flow estimation
4D joint optimization

Input → Initial reconstruction → Frames reconstruction

Converged?

N → Flow estimation
4D joint optimization

Input → Initial reconstruction → Frames reconstruction → Converged? → Loop → N → Flow estimation
4D joint optimization

Input

Initial reconstruction

Frames reconstruction

Converged?

Loop

N

Y

Flow estimation

output
Overview

• Data acquisition

• 4D tomographic reconstruction

• Results
Wilting rose

Before

Photo

After

Projection
Wilting rose

Baseline method

Ours
Wilting rose

Baseline method

Ours
Wilting rose

Baseline method

Ours
High viscosity fluid flow

Before

After
High viscosity fluid flow
Mushroom re-hydration

Melting ice
Mushroom re-hydration

Baseline method

Ours
Mushroom re-hydration
Rising dough

Before

After
Rising dough

Baseline method

Ours
Rising dough

Baseline method

Ours
Lentil/lupin seeds hydration

Before

After
Lentil/lupin seeds hydration
Lentil/lupin seeds hydration

Baseline method

Ours
Crystal sugar dissolution

Before

After
Crystal sugar dissolution

Baseline method

Ours
Summary

• A new sampling strategy

• A new 4D framework for tomography

• Well suited to graphics
Next steps

• Extension to faster deformation

• Extension to other imaging modalities (e.g. electron microscope)

• Re-Simulation (e.g. domain modification)
Thank you!

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Code & Data