An Implicit Frictional Contact Solver for Adaptive Cloth Simulation

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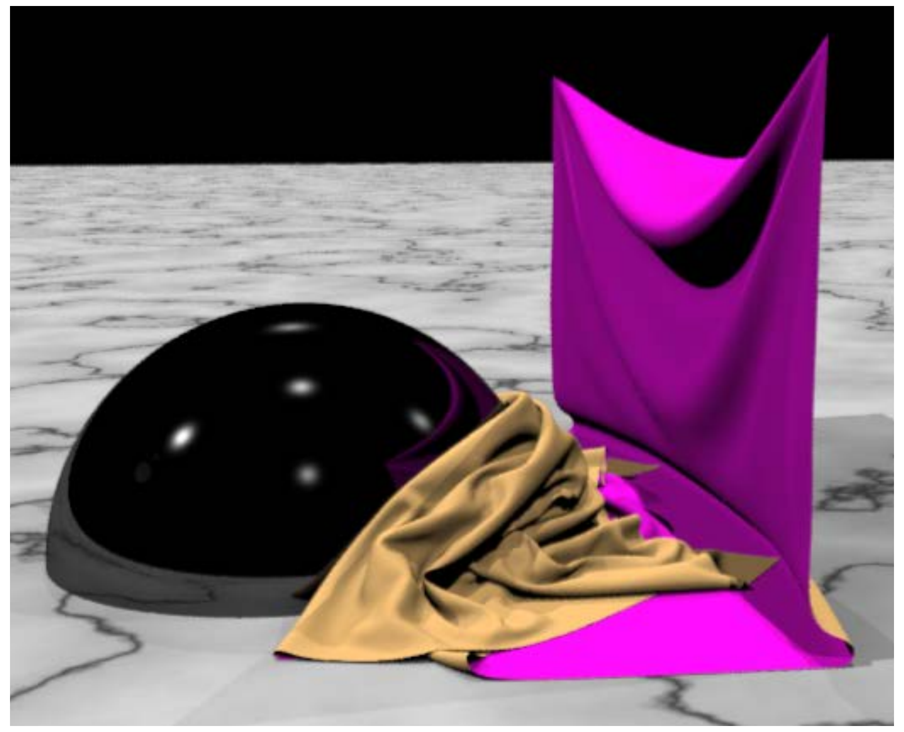
INVENTEURS DU MONDE NUMÉRIQUE


μ = 0
μ = 0.3

PREVIOUS WORK

- Traditional methods

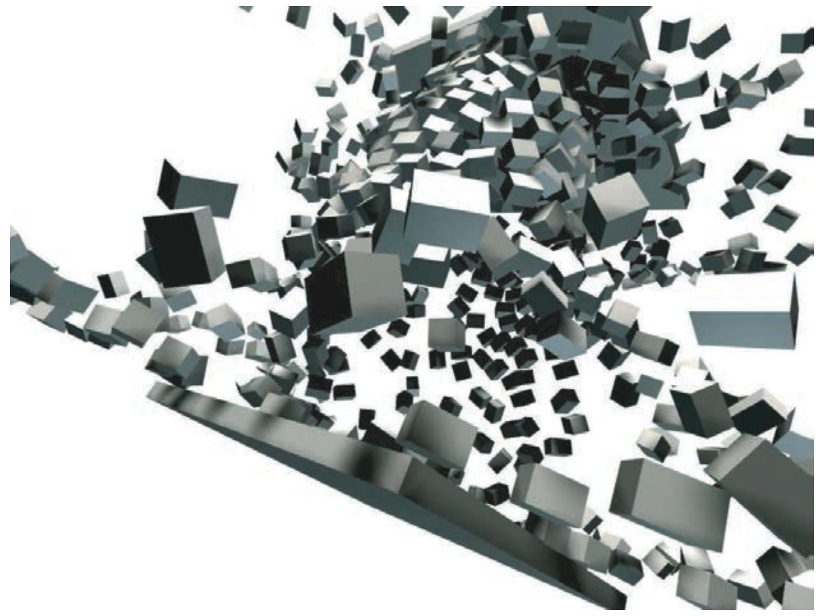
- [Provot 1997]



- [Bridson et al. 2002]

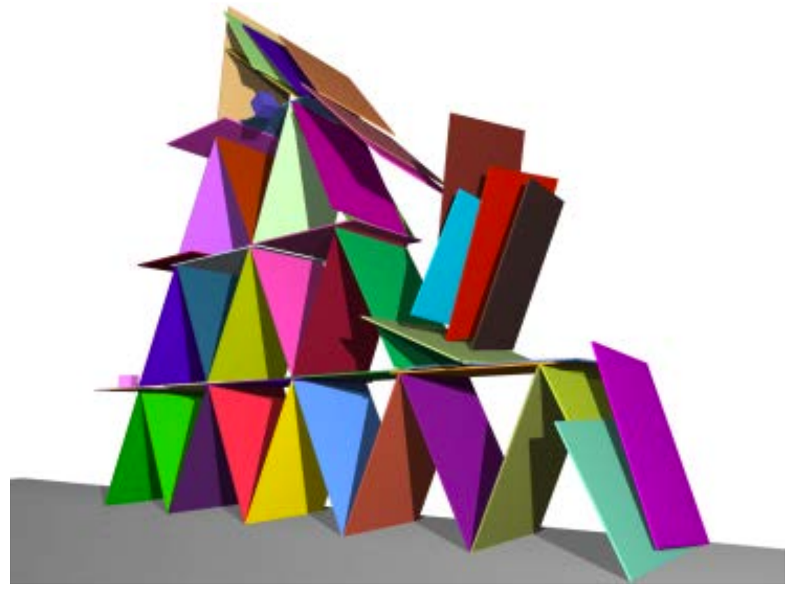
- [Harmon et al. 2008]

- Implicit solver



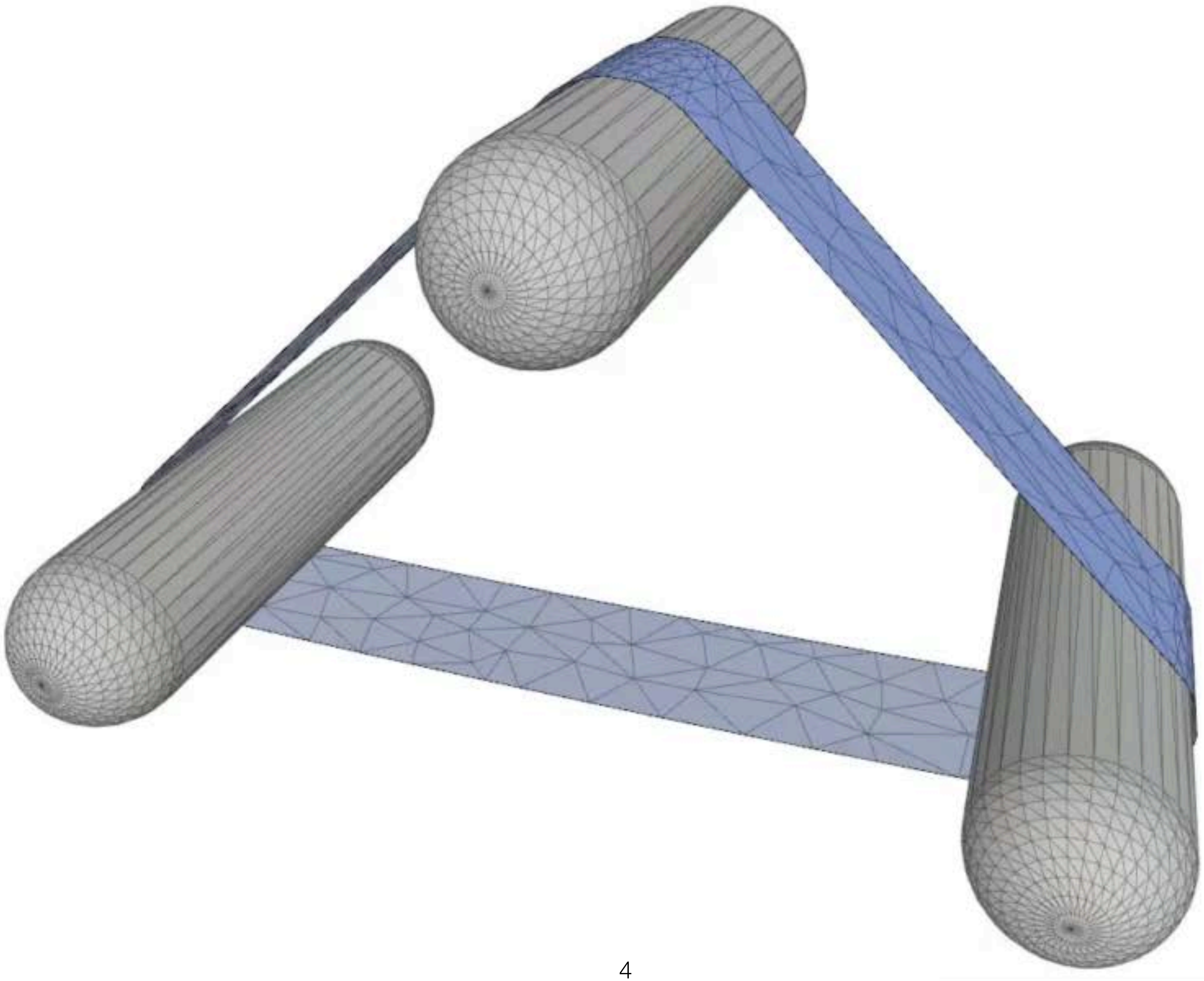
- [Erleben et al. 2007]

- [Otaduy et al. 2009]



- [Kaufman et al. 2008]

- [Daviet et al. 2011]


$\mu = 0.4$

CLOTH DYNAMICS (WITHOUT CONTACTS)

Newton second's law at vertices

$\mathbf{x}$ vertex positions, $\mathbf{v}$ vertex velocities

$$\dot{\mathbf{x}} = \mathbf{v}$$

$$M\dot{\mathbf{v}} = f(\mathbf{x}, \mathbf{v})$$

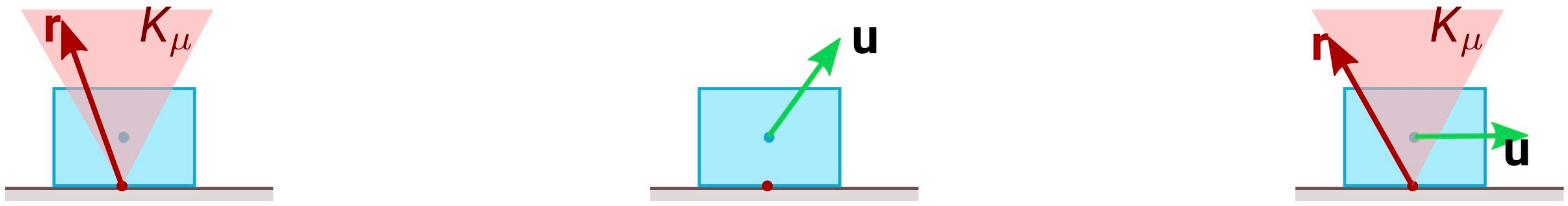
Linearize over a timestep Δt [Baraff et al. 98]:
sequence of linear systems on vertex velocities

$$\mathbf{A}\mathbf{v} = \mathbf{f}$$

SIGNORINI-COULOMB FRICTIONAL CONTACTS

Defines feasible set for $\mathbf{r}_i$ contact force and $\mathbf{u}_i$ relative velocity:

- $\mathbf{r}_i$ in friction cone K_μ (friction force bounded by normal applied load)
- Obeys maximum dissipation principle



(Sticking) (Separating) (Sliding)

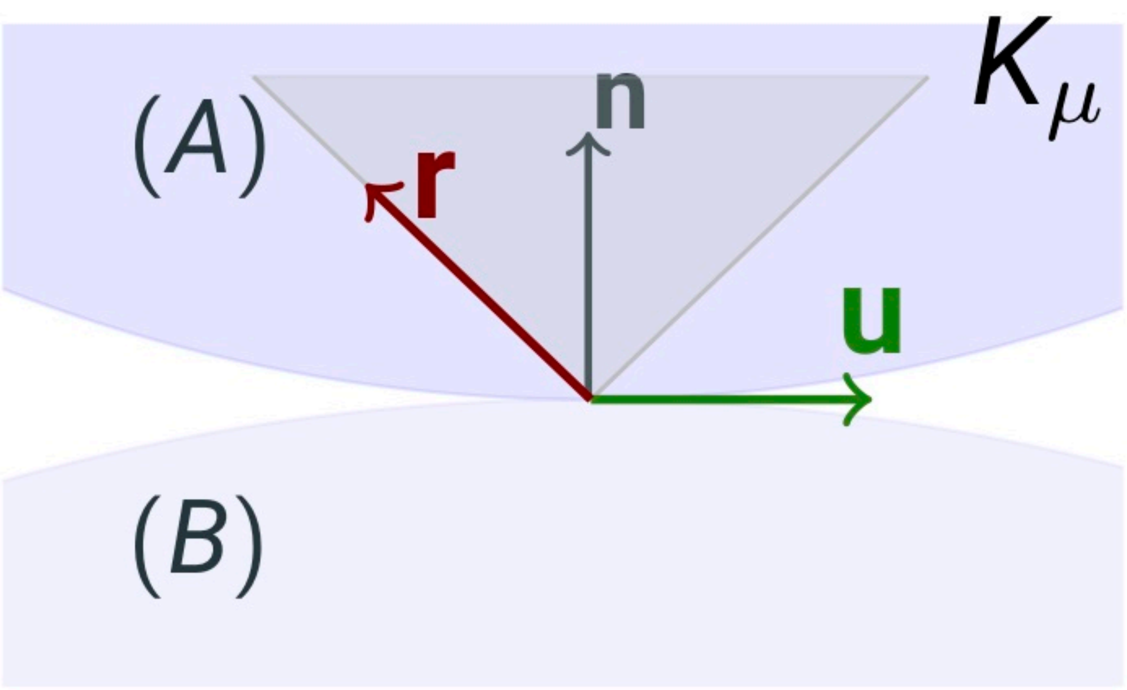
Abstract expression

$$(\mathbf{u}_i, \mathbf{r}_i) \in \mathcal{C}_{\mu_i}$$

LOCAL RELATIVE VELOCITY

Velocity difference of contacting points in basis aligned with contact normal:

$$\mathbf{u}_i = E_i^T (v_{(A)} - v_{(B)}), \quad E_i \text{ 3d rotation matrix.}$$



For c contacts and n vertices

Concatenated vector **u** linear combination of rotated vertex velocities:

$$\mathbf{u} = \mathbf{H}\mathbf{v} + \mathbf{w}, \quad \mathbf{H} \text{ sparse block matrix of 3d rotations.}$$

LINEARIZED DYNAMICS WITH COULOMB FRICTION

$$\mathbf{A}\mathbf{v} = \mathbf{f} + H^T\mathbf{r}$$

$$\mathbf{u} = H\mathbf{v} + \mathbf{w}$$

$$(\mathbf{u}_i, \mathbf{r}_i) \in \mathcal{C}_{\mu_i}$$

Two new variables

- **r** dual variable (Lagrange multiplier, used to enforce the constraint)
- **u** primal variable
  - Directly observable from the system
  - What we are really interested in

DUAL APPROACH

Express $\mathbf{v}$ and $\mathbf{u}$ as linear functions of $\mathbf{r}$

$$\left(H\mathbf{A}^{-1}H^T\mathbf{r} + H\mathbf{A}^{-1}\mathbf{f} + \mathbf{w}, \mathbf{r}\right) \in \mathcal{C}_\mu$$

For small independent objects (rigid-bodies, hair, ...)

- $\mathbf{A}$ block-diagonal, easy to inverse
- Widely successful [Erleben et al. 2007, Daviet et al. 2011, Mazhar et al. 2015, ...]

For fully connected object (cloth, elastic bodies...)

- $\mathbf{A}^{-1}$ dense $\implies$ inefficient
- [Otaduy et al. 2009]: successive block-diagonal approximations of $\mathbf{A}$

TOWARDS A PRIMAL APPROACH

Problem

H not invertible: $\mathbf{r}$, $\mathbf{v}$ cannot be recovered from $\mathbf{u}$

Solution

Make H invertible!

RESTRICTED SETTING

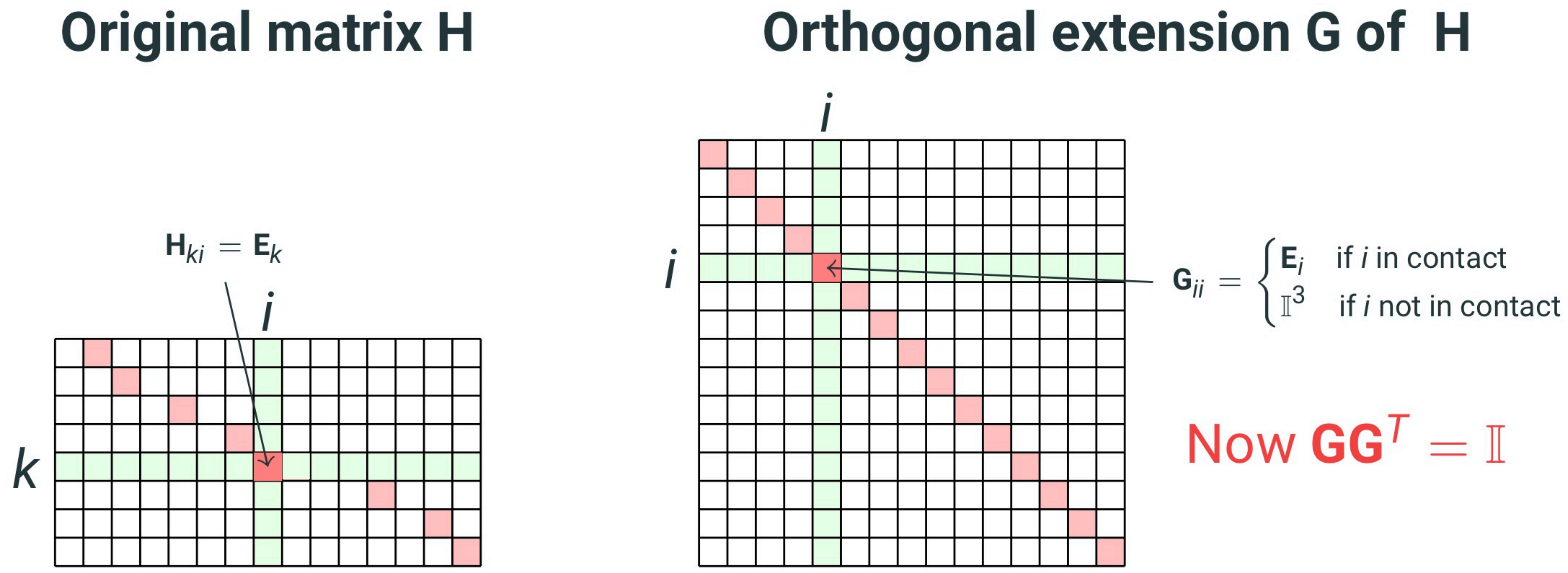
We assume for now that:

1. Contacts happen at vertices only
2. Contacts are against a kinematic object
3. There is at most one contact per vertex

(2. and 3. will be lifted later)

NODAL CONTACTS

- Row k of $\mathbf{H}$ contain exactly one non-zero block, rotation matrix $\mathbf{E}_k^T$
- $\mathbf{E}_k$ orthonormal: $\mathbf{E}_k \mathbf{E}_k^T = \mathbb{I}_3$
- Rows of $\mathbf{H}$ are independent



$\mathbf{H}$ is $3c \times 3n$ with rank $3c$

$\mathbf{G}$ is $3n \times 3n$ with rank $3n$

Now $\mathbf{G}\mathbf{G}^T = \mathbb{I}$

SYSTEM WITH ORTHOGONAL EXTENSION

$$\mathbf{A}\mathbf{v} = \mathbf{f} + \mathbf{G}^T\mathbf{r} \quad (1)$$

$$\mathbf{u} = \mathbf{G}\mathbf{v} + \mathbf{w} \quad (2)$$

$$(\mathbf{u}_i, \mathbf{r}_i) \in \widehat{\mathcal{C}_{\mu_i}} \quad (3)$$

$$\widehat{\mathcal{C}_{\mu_i}} := \begin{cases} \mathcal{C}_{\mu_i} & \text{if vertex } i \text{ is in contact} \\ \{\mathbf{r}_i = 0\} & \text{otherwise.} \end{cases}$$

Can now express **v** and **r** as linear functions of **u**:

Using (2) $\mathbf{v} = \mathbf{G}^T(\mathbf{u} - \mathbf{w})$

Using (1) $\mathbf{r} = \mathbf{G}\mathbf{A}\mathbf{G}^T\mathbf{u} - \mathbf{G}\left(\mathbf{f} + \mathbf{A}\mathbf{G}^T\mathbf{w}\right)$

SOLVING THE PRIMAL SYSTEM

$$\left(\mathbf{u}, \mathbf{GAG}^T\mathbf{u} - \mathbf{G}\left(\mathbf{f} + \mathbf{AG}^T\mathbf{w}\right)\right) \in \widehat{\mathcal{C}_\mu}$$

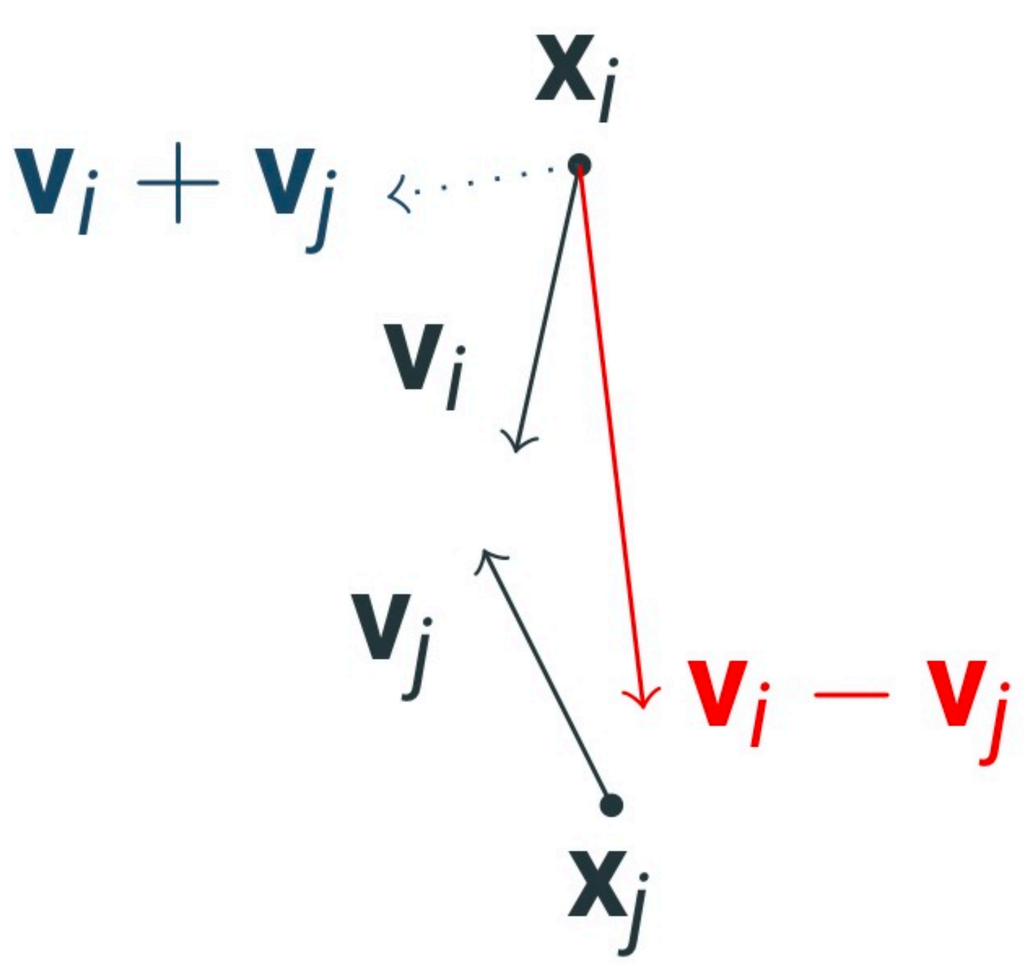
Structure similar to the dual problem, but:

- Roles of **u** and **r** reversed
- **W** sparse, positive definite

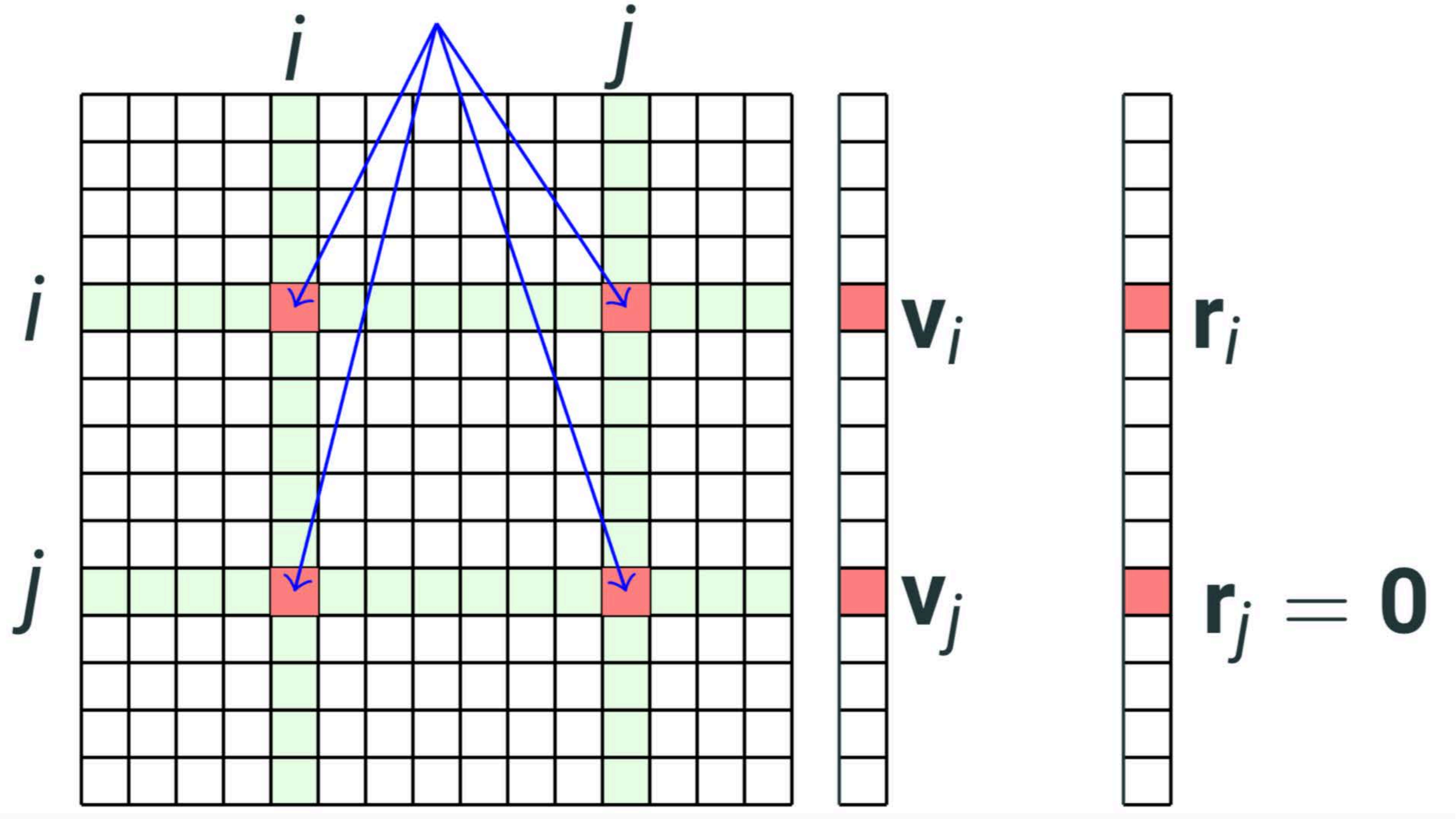
We can easily adapt the solver from [Daviet et al. 2011] (details in the paper)

ORTHOGONAL EXTENSION FOR SELF CONTACTS

Contact between vertices *i* and *j*



$$\begin{bmatrix} \mathbf{G}_{ii} & \mathbf{G}_{ij} \\ \mathbf{G}_{ji} & \mathbf{G}_{jj} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{E}_k & -\mathbf{E}_k \\ \mathbf{E}_k & \mathbf{E}_k \end{bmatrix}$$

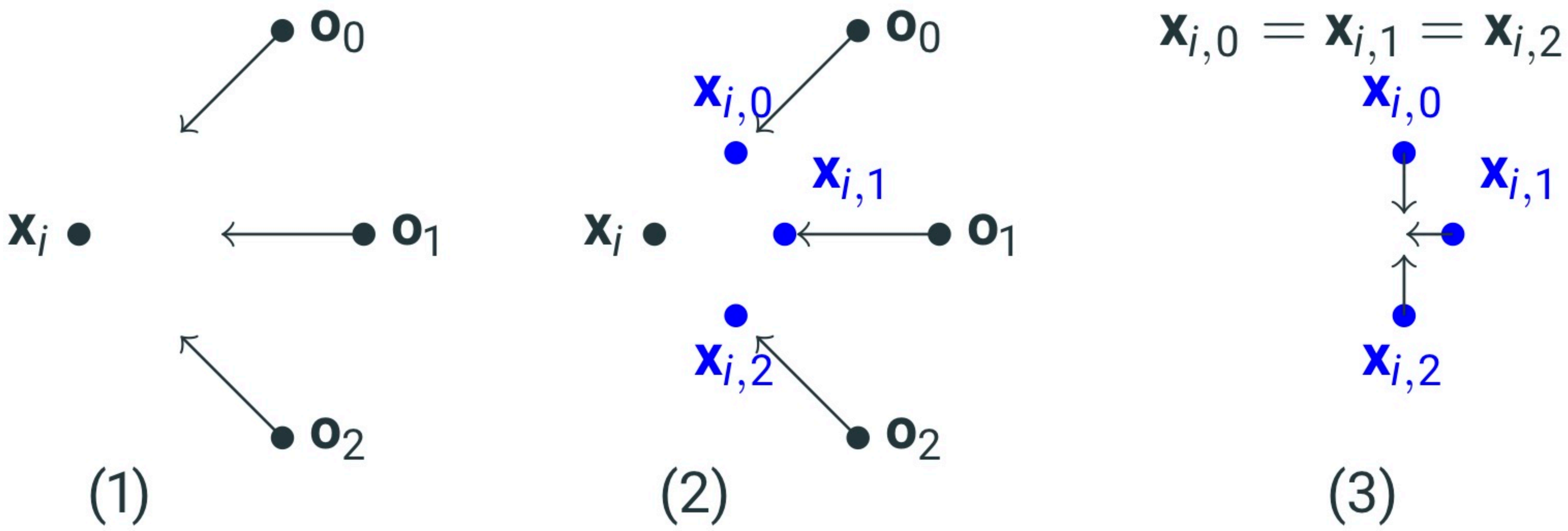


- Original row **H** computes velocity difference $E_i^T\left(\mathbf{v}_i - \mathbf{v}_j\right)$
- Add row in **G** and ensure zero-force on velocity sum $E_i^T\left(\mathbf{v}_i + \mathbf{v}_j\right)$

LAYERED CONTACTS

Handling multiple contacts per vertex

1. Duplicate vertex $\mathbf{x}_i$.
2. Each copy handles a contact.
3. Enforce equality of duplicated positions



PIN CONSTRAINT

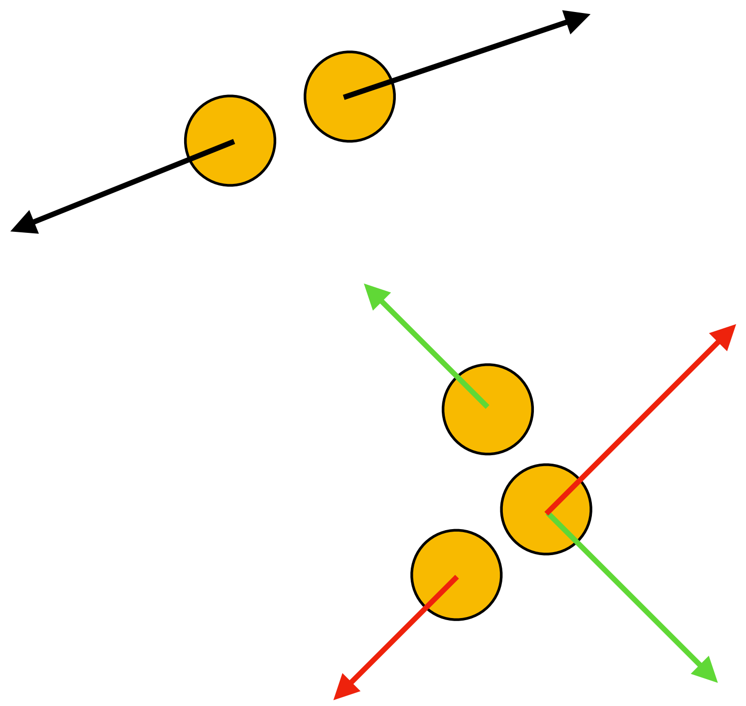
- Constraint $\mathbf{v}_{i,1} = \mathbf{v}_{i,2} = \cdots = \mathbf{v}_{i,p}$ enforced through Lagrange multiplier λ.
- Explicit expression for λ given $\mathbf{r}$:

$$\lambda = -\mathbf{G}^T \mathbf{r}$$

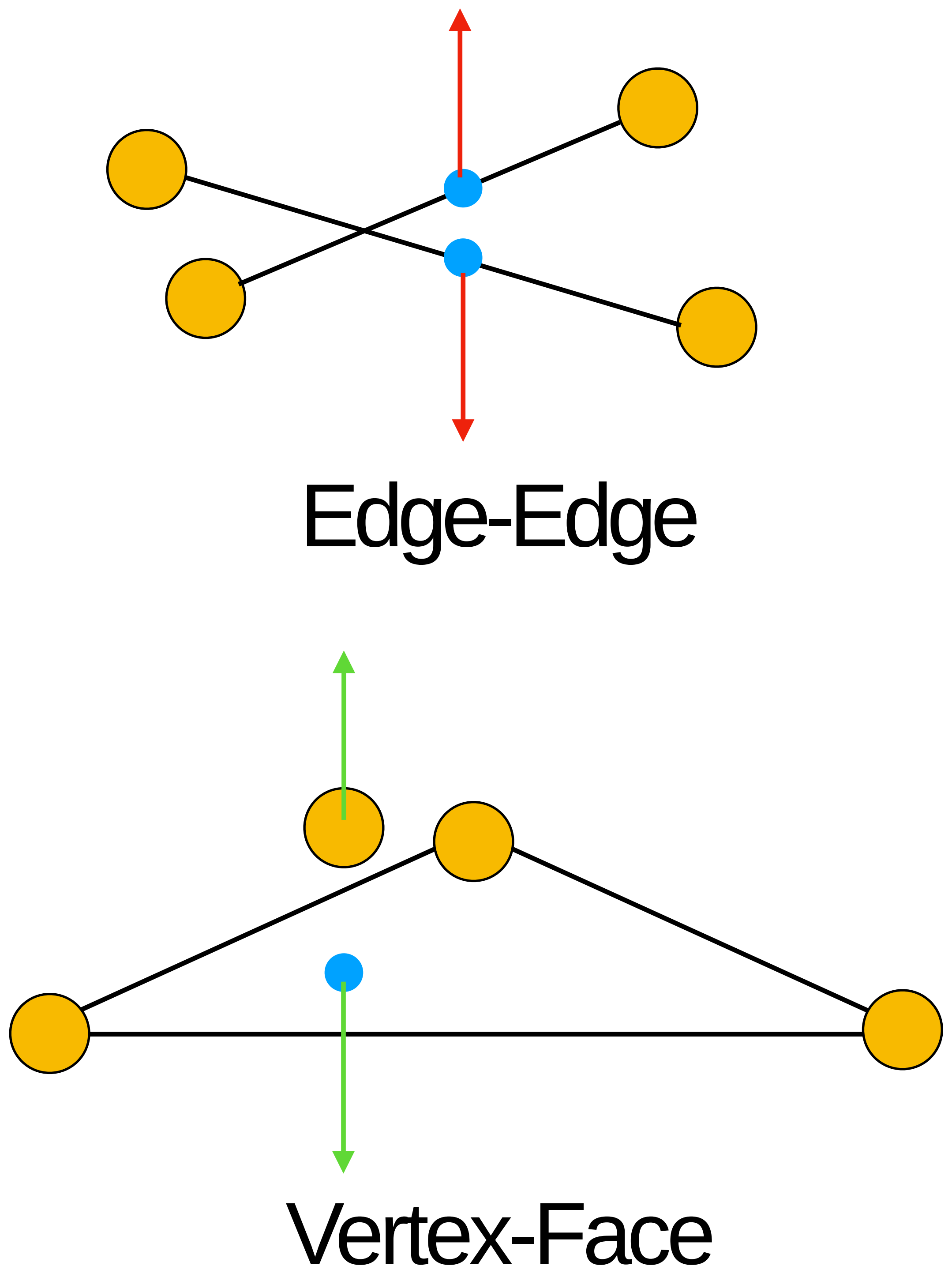
- Operator splitting to solve friction and pin constraints iteratively (see details in paper)

THE ISSUE WITH NODAL SOLVER

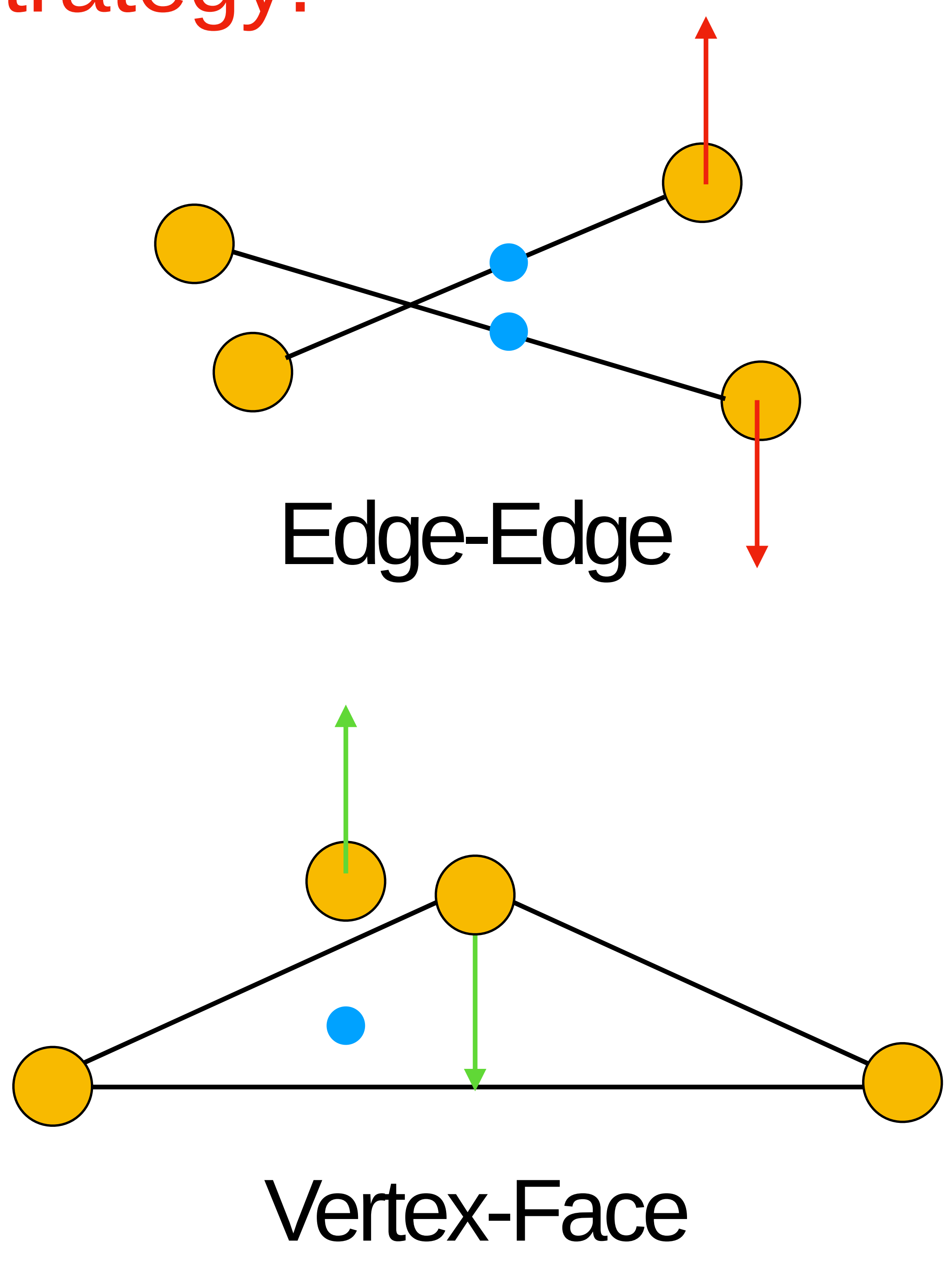
Nodal solver



In cloth mesh

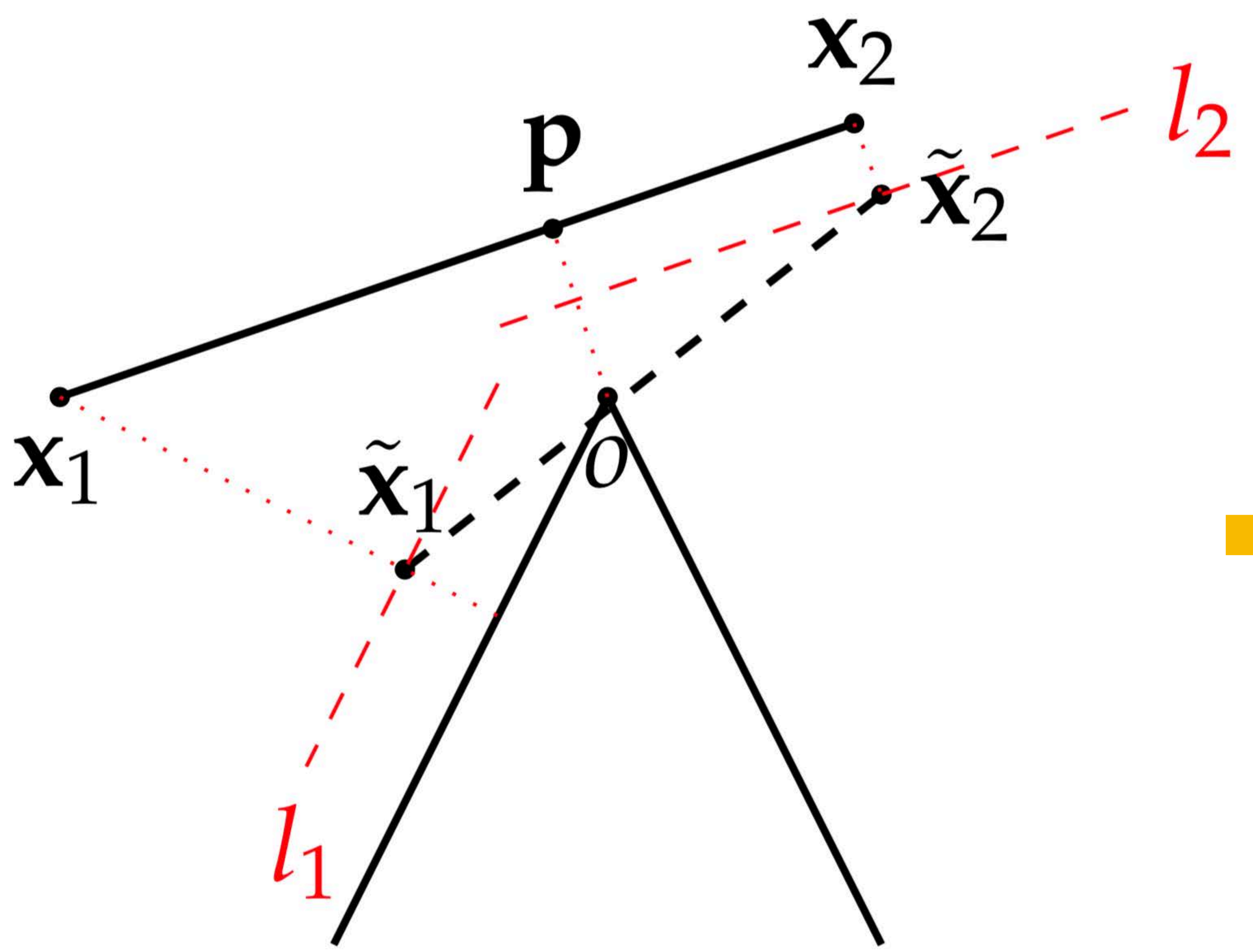


Strategy:



Move the contact to the closest vertex.

MOVING CONTACTS INTRODUCES PROBLEMS



Solution

Adaptive refinement

[Narain et al. 2012]

ADAPTIVE REMESHING

Remeshing metrics [Narain et al. 2012]

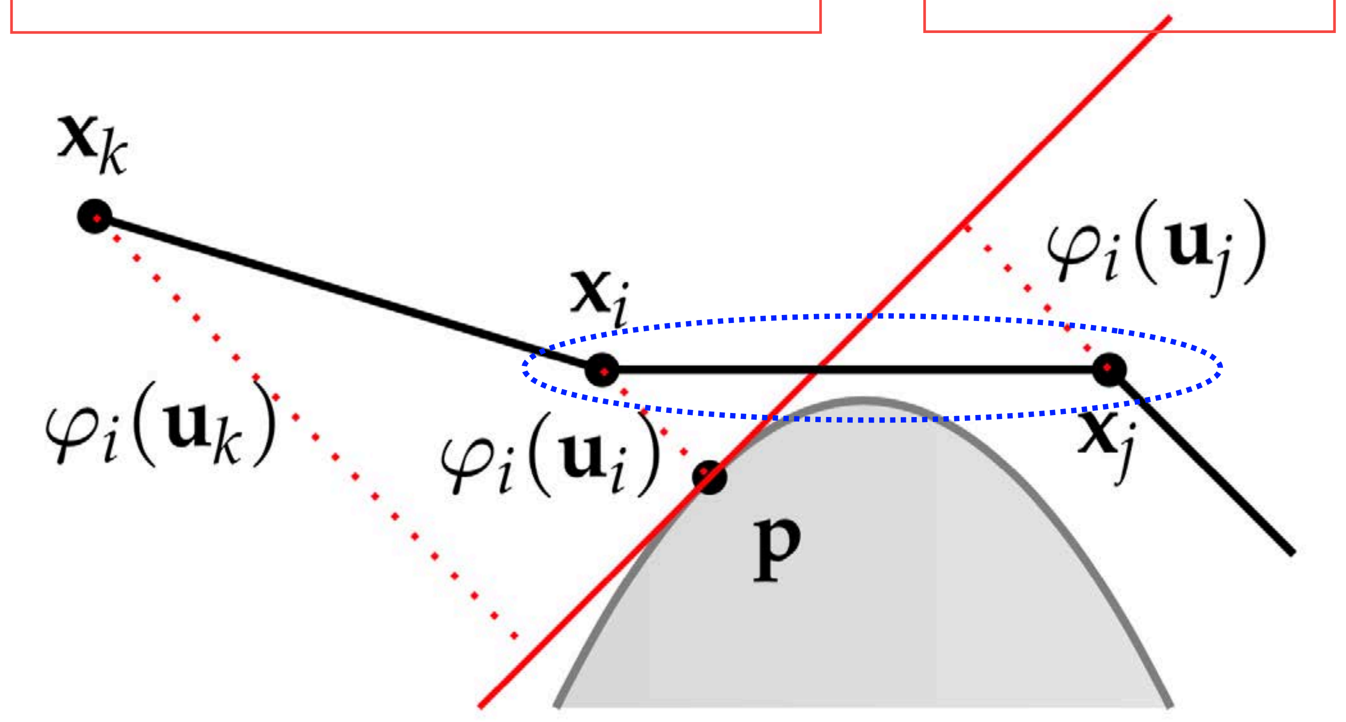
- velocity
- curvature
- distance to obstacle
- etc.

Obstacle metric (for each vertex close to obstacle)

The projection of the edge length in contact normal direction

Distance to the obstacle

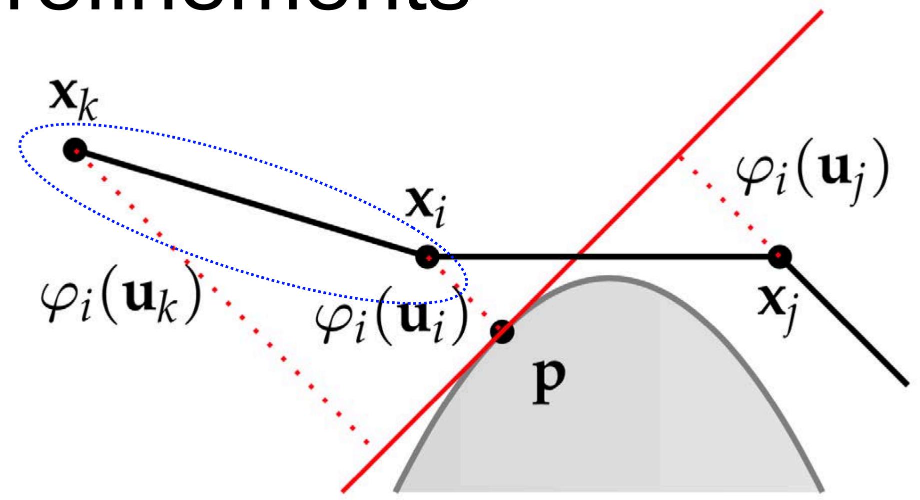
$$|\varphi_i(\mathbf{u}_i) - \varphi_i(\mathbf{u}_j)| < |\varphi_i(\mathbf{u}_i)|$$



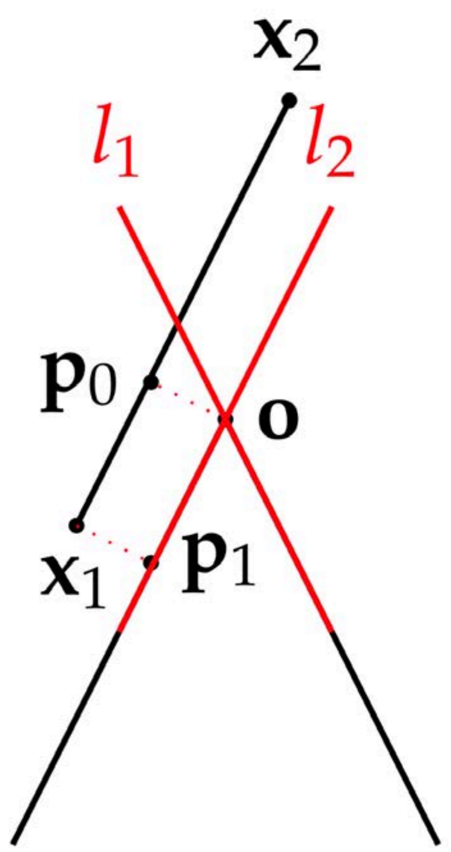
ISSUES OF THE OBSTACLE METRIC

$$|\varphi_i(\mathbf{u}_i) - \varphi_i(\mathbf{u}_j)| < \frac{\max_k |\varphi_i(\mathbf{u}_k)|}{|\varphi_i(\mathbf{u}_i)|}$$

Unnecessary refinements

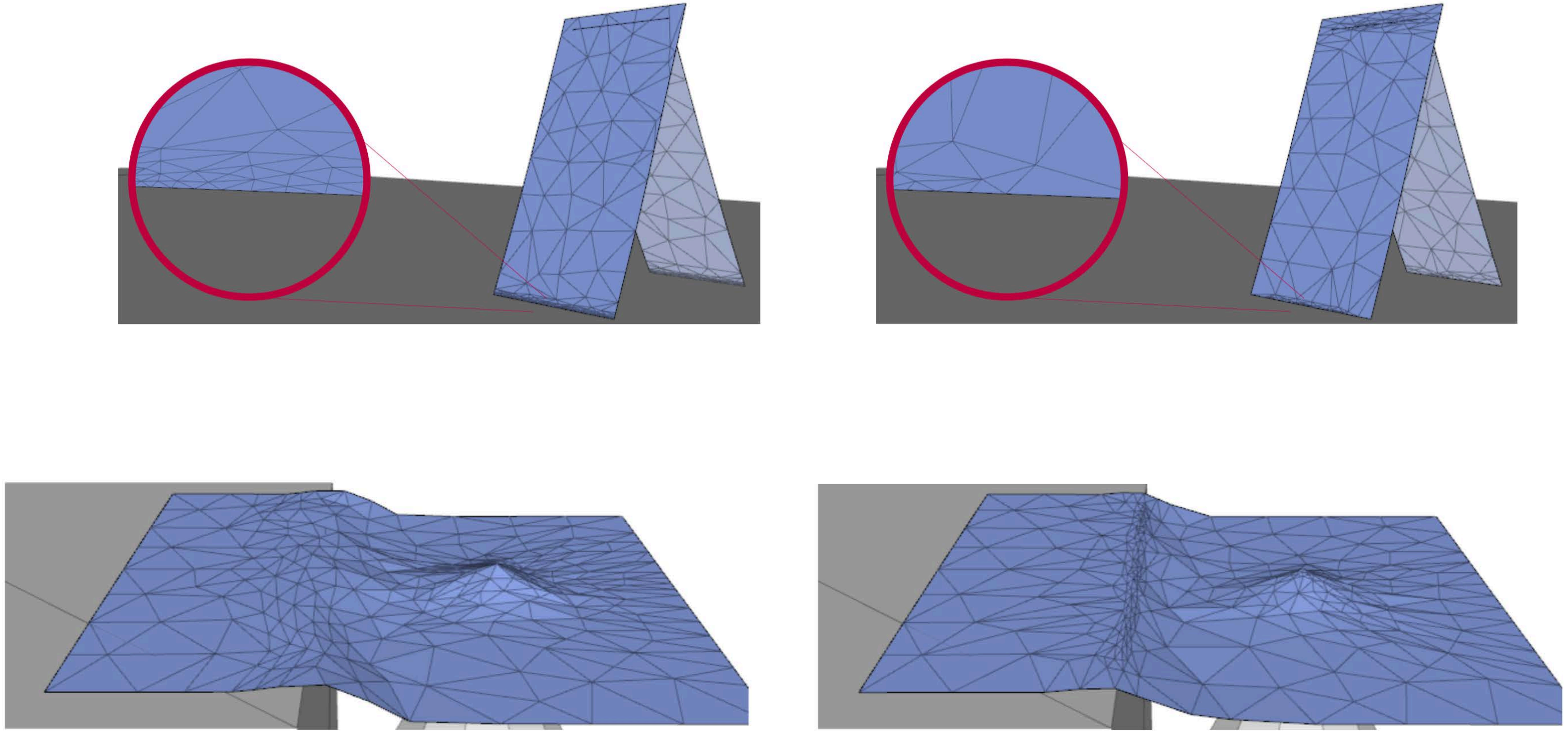


Inadequate refinements

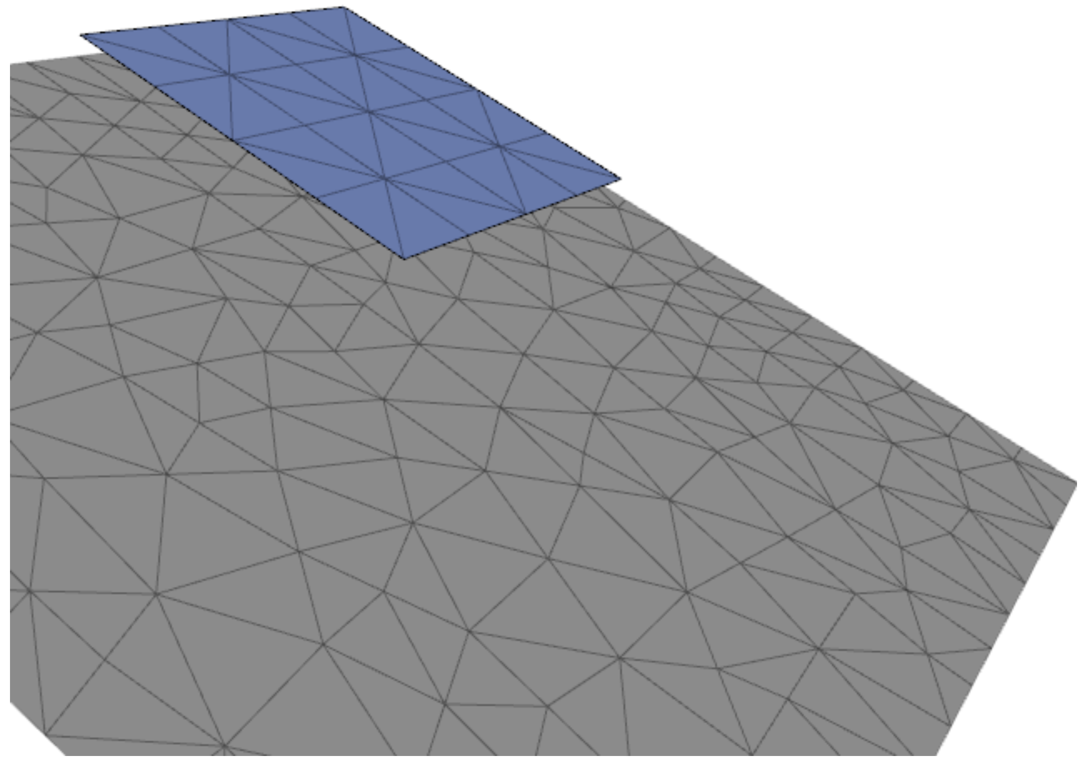


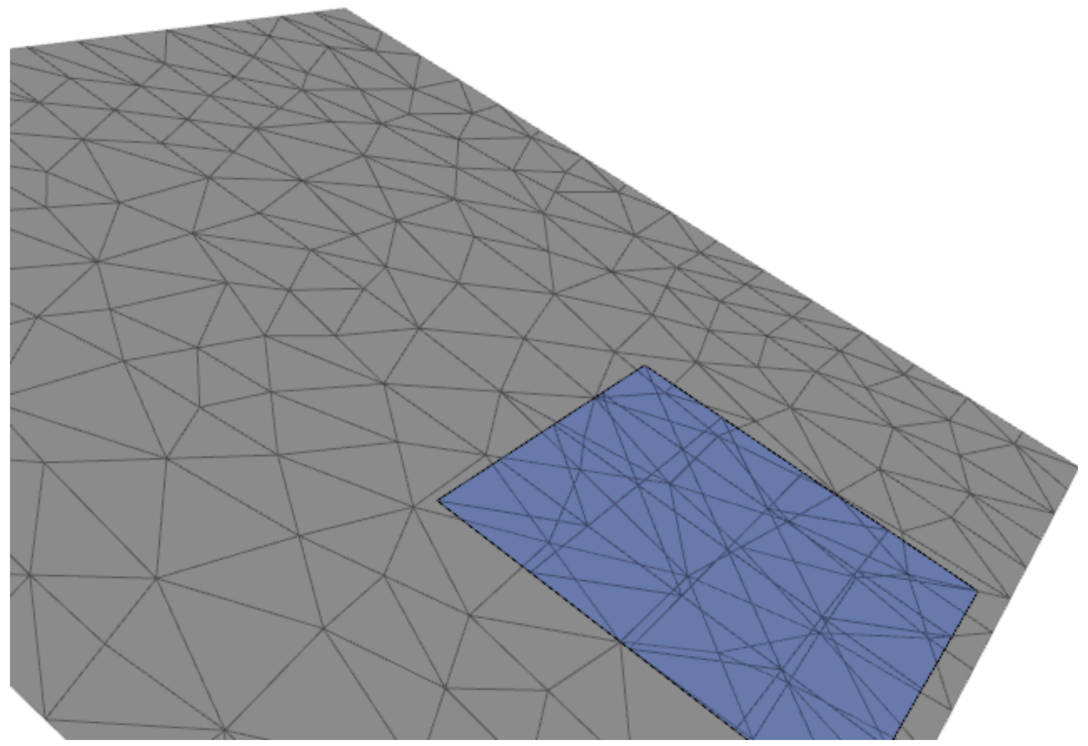
Old metric [Narain et al. 2012]

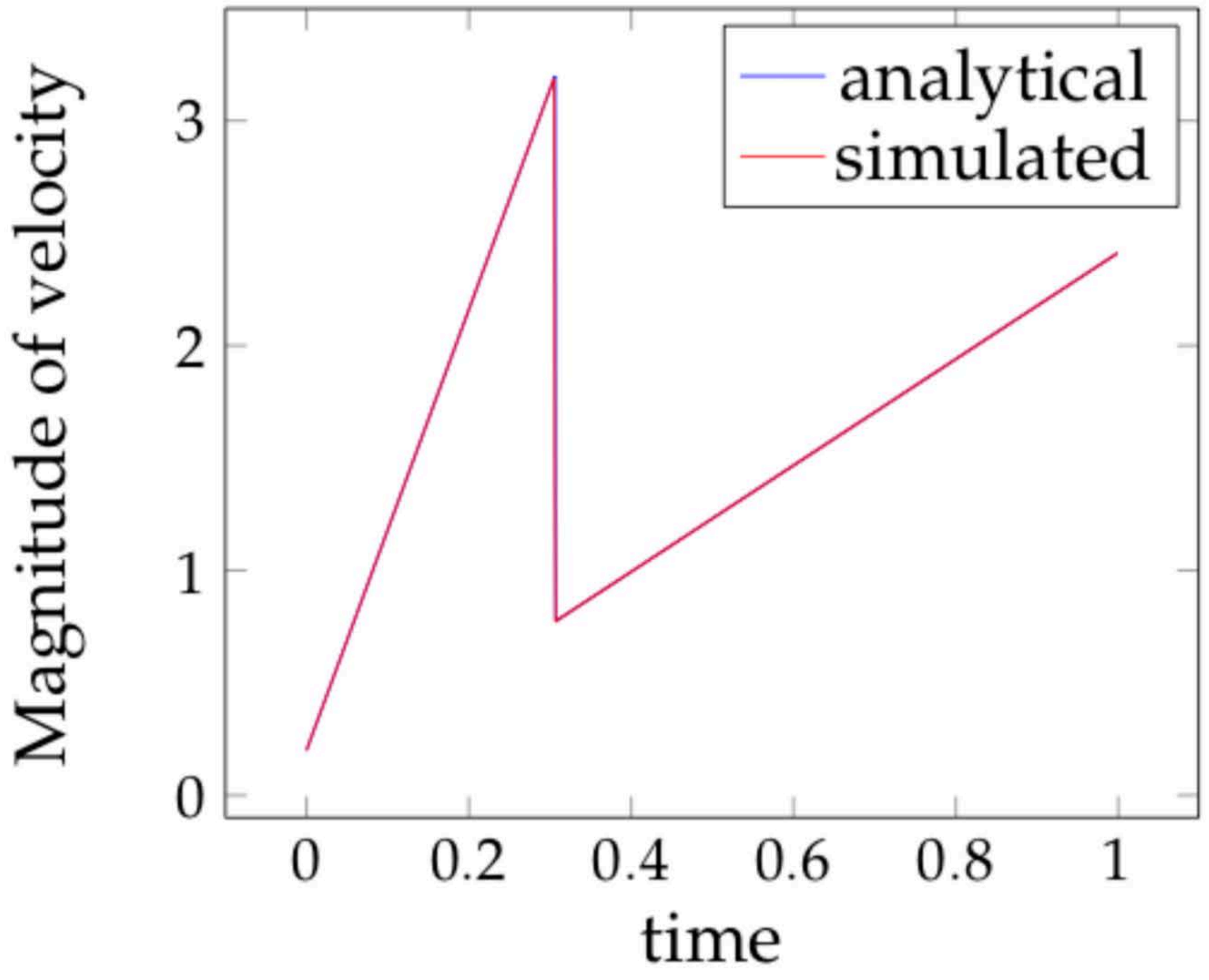
New metric

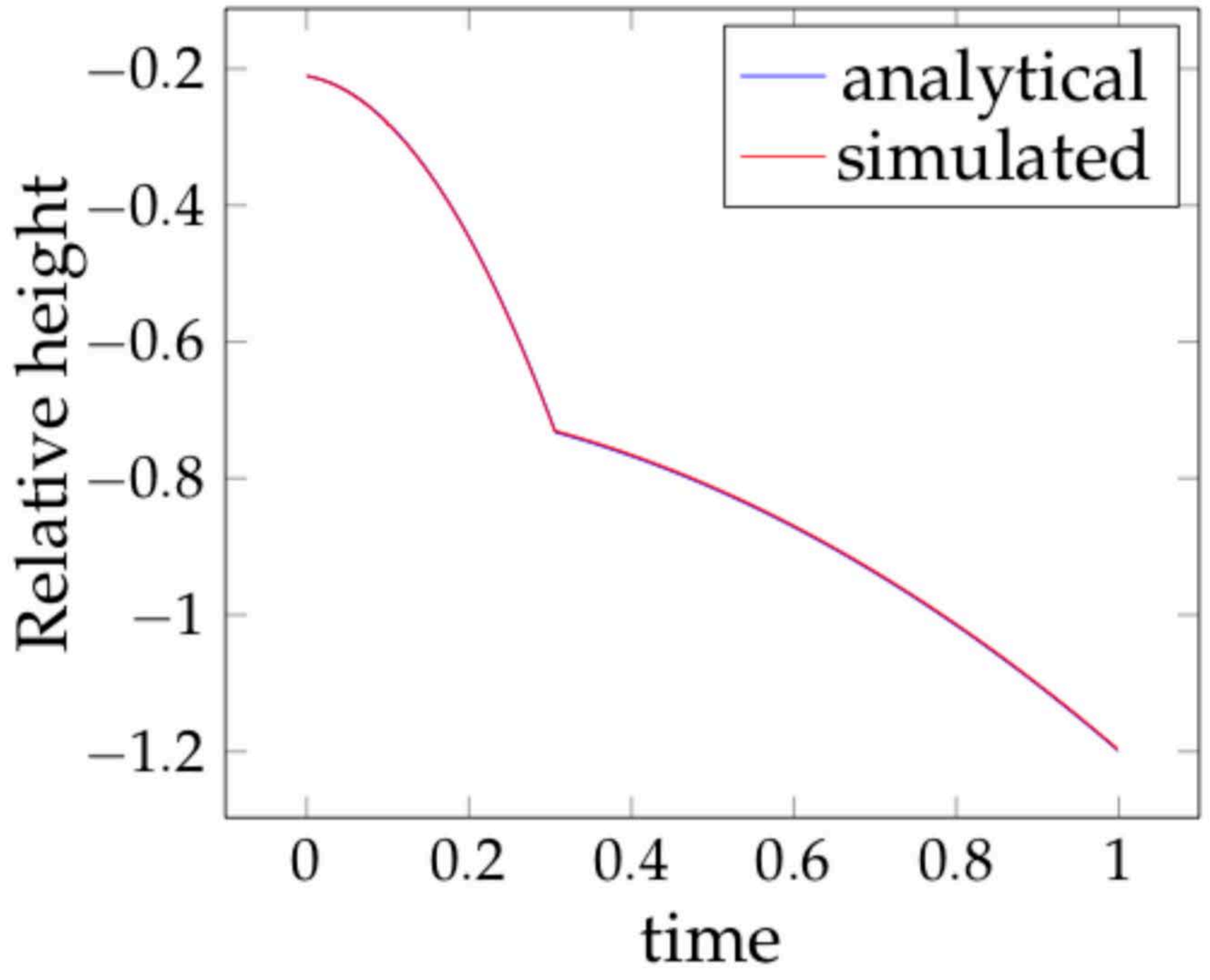


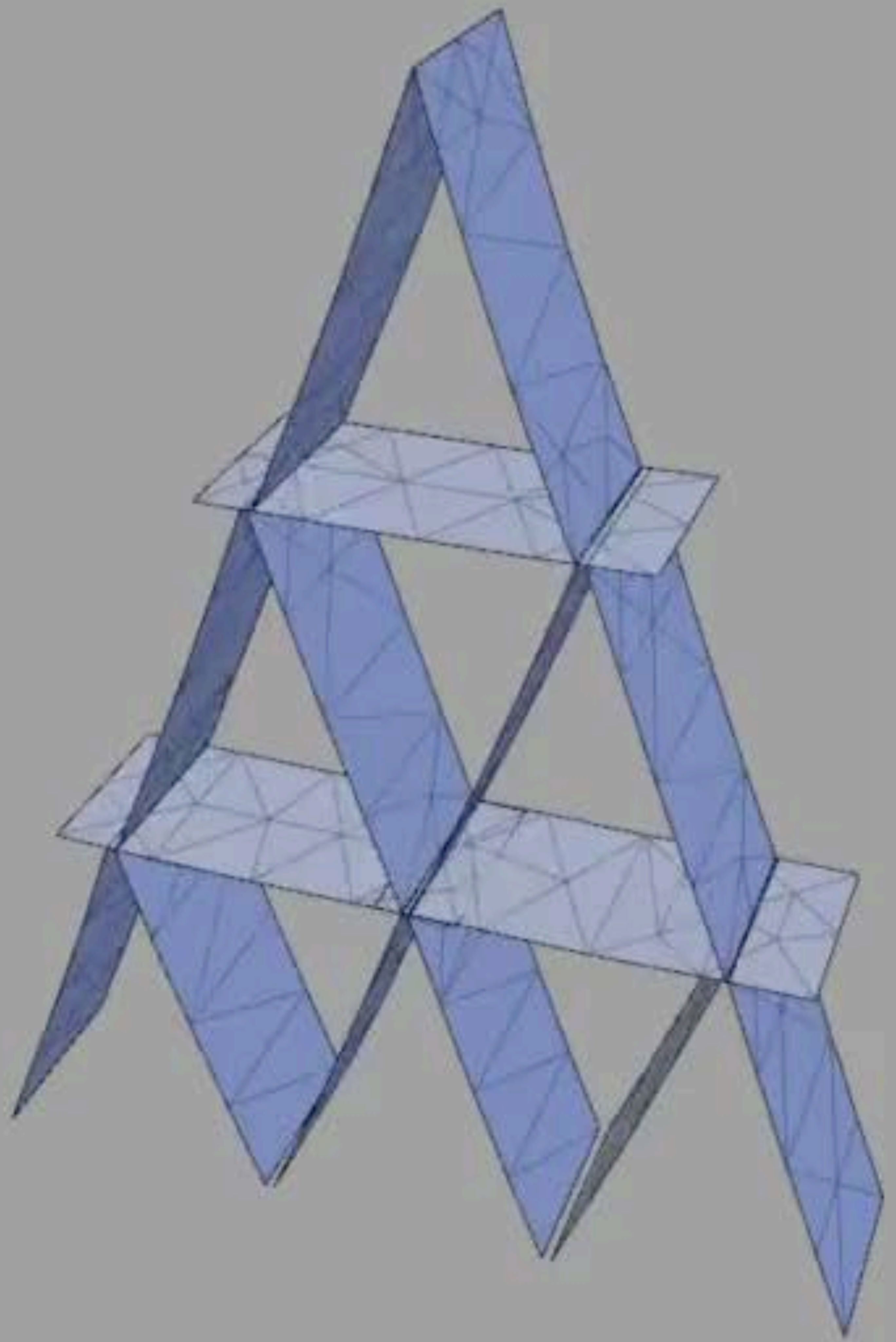
COMPARISON WITH ANALYTICAL RESULTS



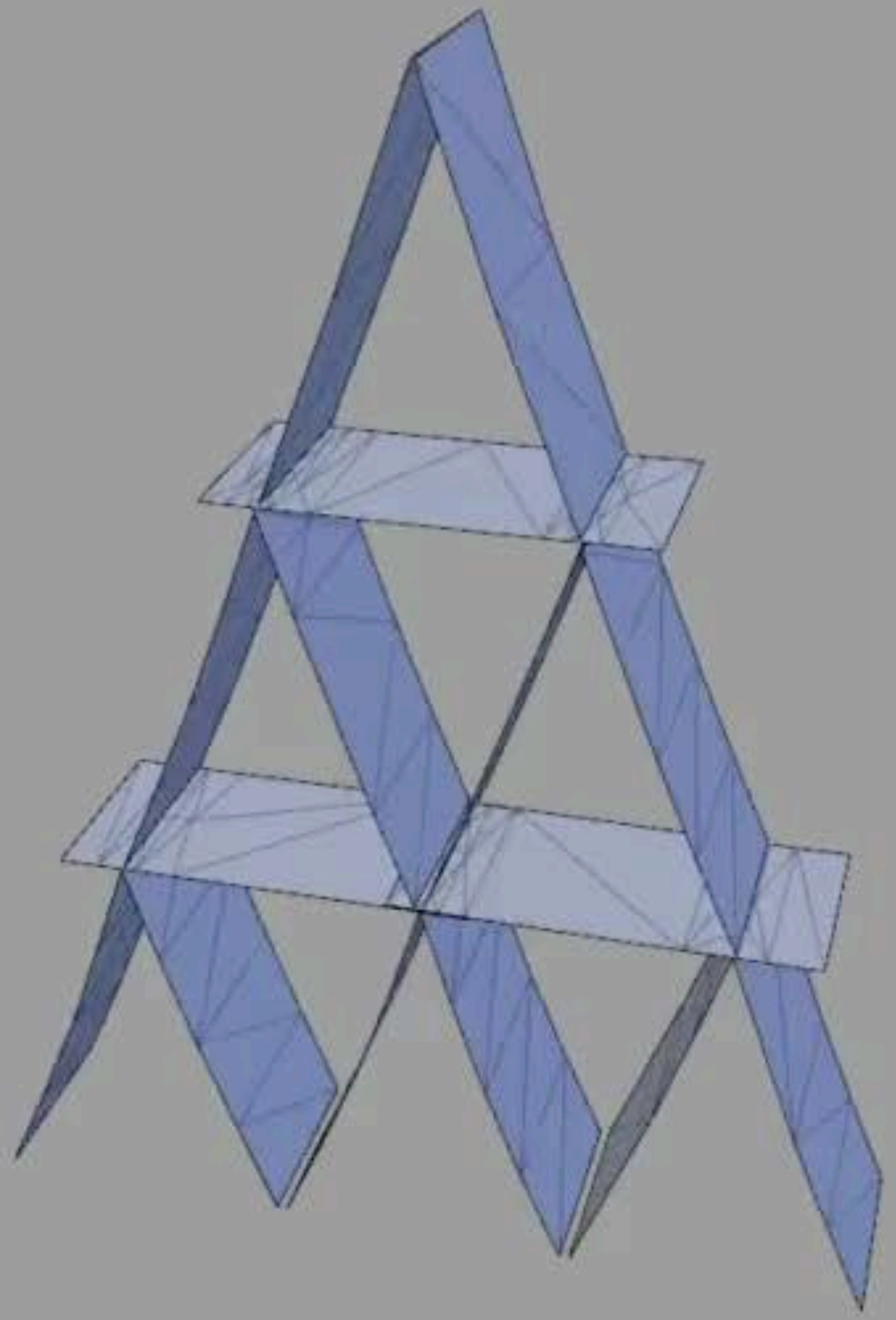








Bridson/Harmon



Our method


Otaduy
Our method


$\mu = 0.1$


x 0.5
μ = 0.3
μ = 0.6

CONCLUSIONS AND DISCUSSIONS

Conclusion

- Implicit frictional contact solver with exact Coulomb's law
- Nodal solver with adaptive refinement
- Extended to self contacts and layered contacts.

Discussion and limitations

- Nonexistence of solutions
- On Theoretical guarantees of robustness
- Accuracy of nodal constraints
- Future generalization

THANK YOU

Example	μ	$\bar{n}_v$ [1]	$\bar{n}_c$ [1]	$\bar{n}_i$ [2]	$\bar{n}_{ir}$ [2]	$\bar{t}_p$ [3]	$\bar{t}_r$ [3]	$\bar{t}_{cd}$ [3]	$\bar{t}_m$ [3]	$\bar{t}_s$ [3]	$\bar{T}$ [4]	$\bar{e}$ [5]	$\bar{n}_{iter}$ [5]
Table and Pin	0.2	4692	2550	0.10	0	3.29	0.35	0.14	0.06	2.38	52.6	5.7×10^{-9}	178
High-tension belt	0.4	502	476	0.00	0	2.48	0.03	0.01	2×10^{-3}	2.42	36.5	5.7×10^{-9}	261
House of cards	0.2	75	176	5×10^{-4}	0	0.09	2×10^{-3}	9×10^{-4}	4×10^{-4}	0.09	1.4	2.6×10^{-12}	387
	0.6	218	119	0.00	0	0.29	6×10^{-3}	2×10^{-3}	3×10^{-4}	0.28	4.6	3.9×10^{-11}	1778
Circular dragging	0.6	59	26	0.00	0	0.04	2×10^{-3}	6×10^{-5}	0.00	0.04	0.6	1.5×10^{-10}	25
Twist	0.0	5874	2557	0.02	0	2.10	0.64	0.21	0.09	0.59	33.6	7.6×10^{-9}	192
	0.1	5827	2521	0.01	0	2.08	0.61	0.20	0.09	0.66	33.3	7.4×10^{-9}	180
	0.3	5801	2647	0.51	0	6.22	0.67	0.20	0.09	4.58	99.5	1.0×10^{-4}	378
	0.6	6052	2663	1.63	0	8.12	0.90	0.20	0.09	5.91	129.9	1.3×10^{-3}	484
Arabesque	0.0	5224	2845	0.03	0	3.45	0.69	0.19	0.14	1.67	55.2	6.8×10^{-8}	262
	0.3	5532	2996	0.85	0	4.37	0.70	0.19	0.15	2.50	69.9	1.1×10^{-3}	294
Clubbing	0.0	7008	2803	0.02	0	2.36	0.71	0.21	0.13	0.68	37.8	7.7×10^{-9}	254
	0.1	6907	2833	0.04	0	2.41	0.69	0.21	0.13	0.77	38.6	4.7×10^{-9}	199
	0.3	7153	2827	0.03	0	4.75	0.75	0.18	0.14	2.95	76.0	2.3×10^{-8}	242
HipHop	0.0	5577	2597	0.05	0	11.01	0.57	0.18	0.09	9.59	176.2	8.8×10^{-9} *	353
	0.3	5500	2508	0.09	0	6.64	0.81	0.23	0.11	4.52	106.2	1×10^{-4}	558
Shawl	0.3	1381	328	0.03	0	1.10	0.31	0.09	0.003	0.34	17.6	3.3×10^{-9}	83
	0.6	3733	1571	1.79	0	2.18	0.46	0.16	0.03	0.89	34.9	4×10^{-4}	219