

1

与标量场相适应的网格剖分及其应用

山东大学计算机学院交叉研究中心

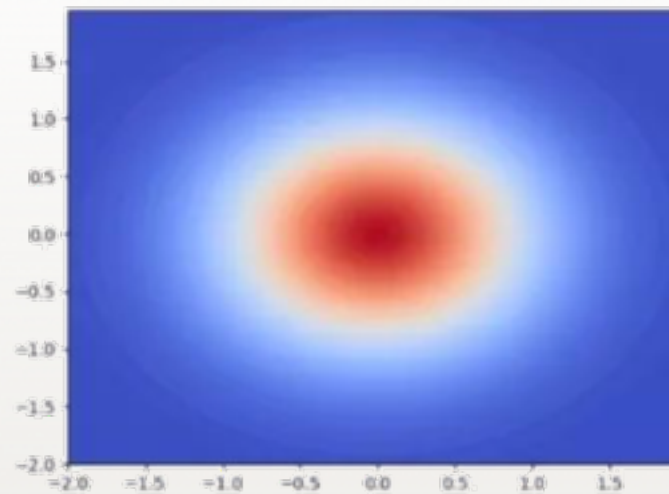
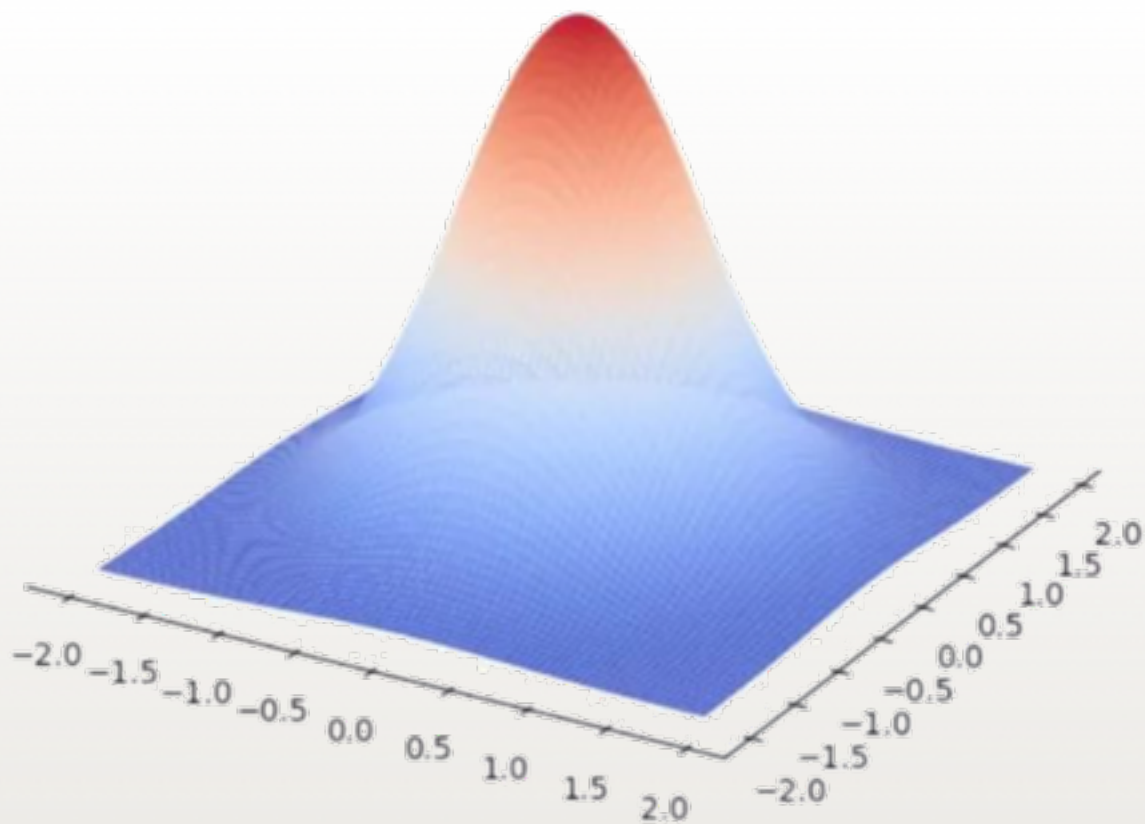
辛士庆

2

标量场

3

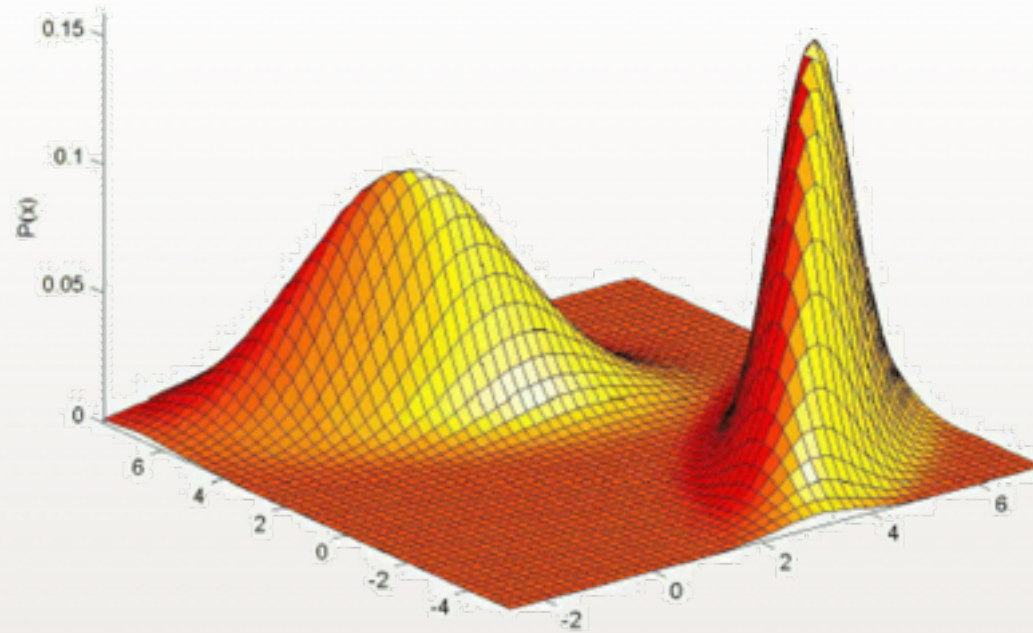
标量场无处不在



4

连续标量场

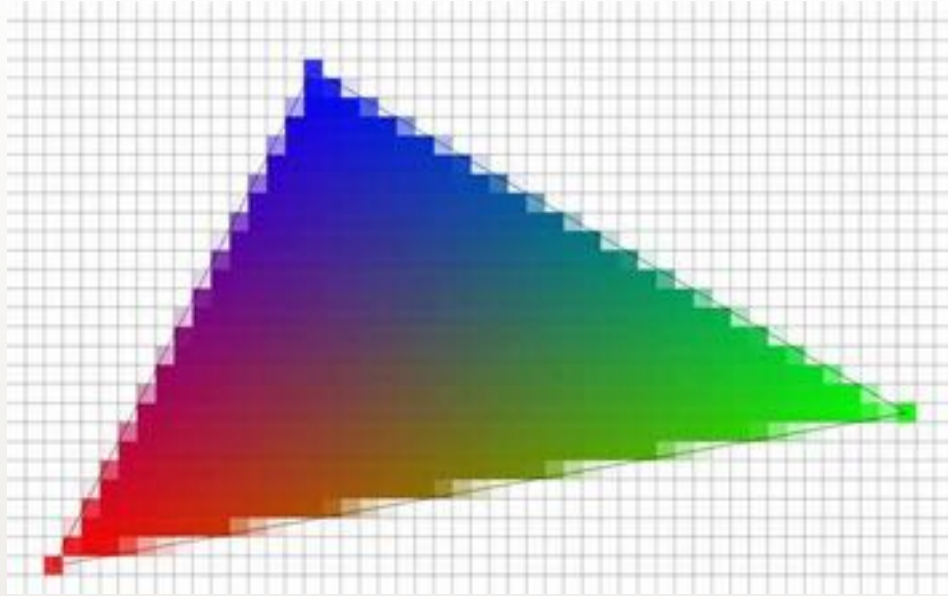
- $f = f(x,y)$, where (x,y) is in a closed domain, and f is continuous or even smooth.



5

离散标量场

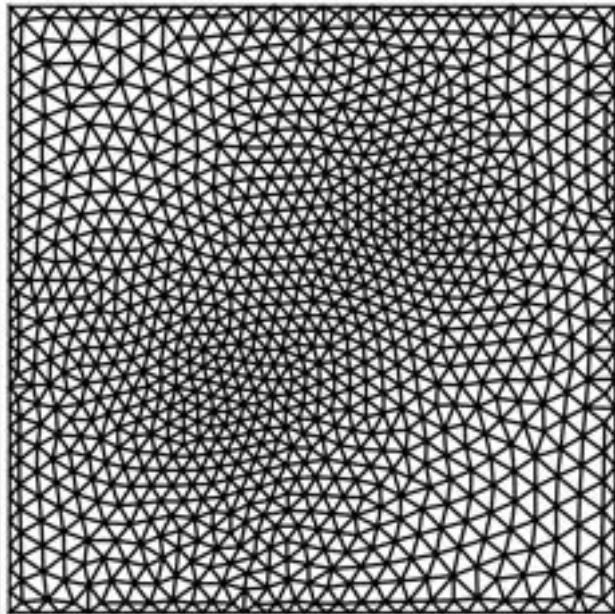
- Discrete domain; Potentially encode a continuous (or even smooth) scalar field



6

离散标量场

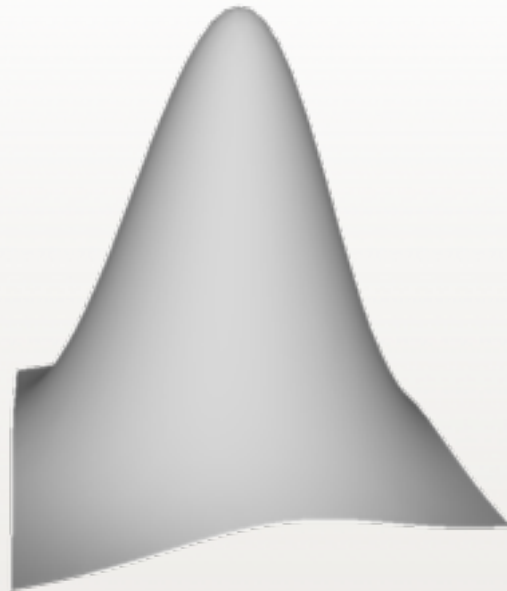
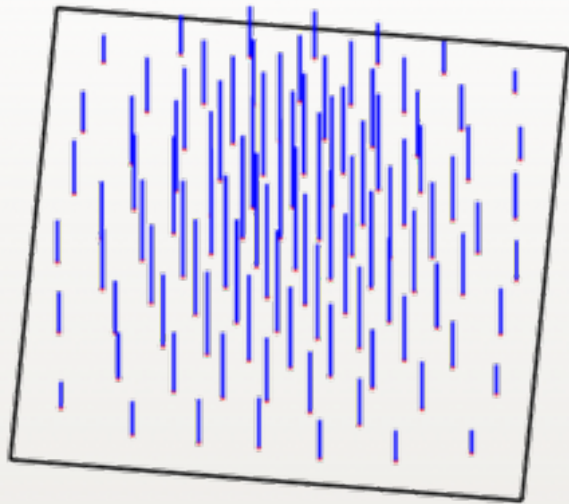
- 三角化的定义域



7

离散标量场

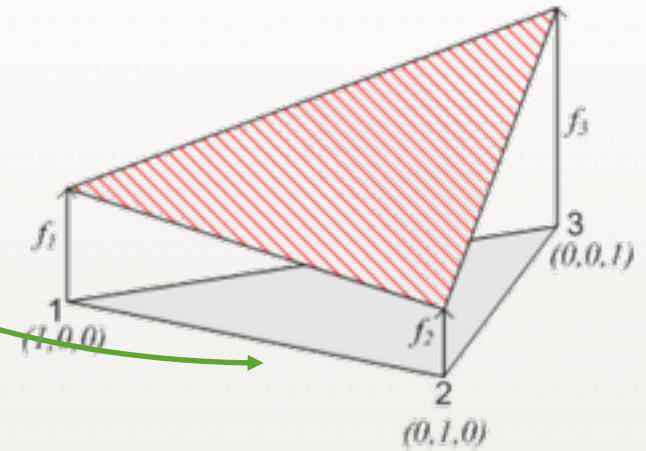
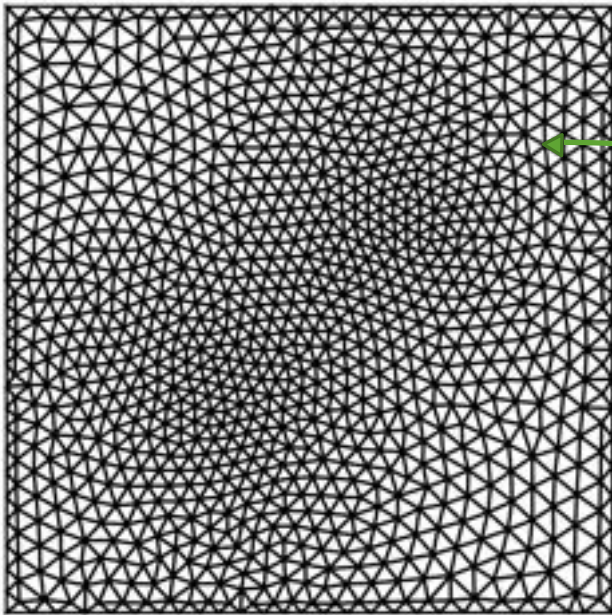
- 散乱点形成的离散化



8

离散标量场

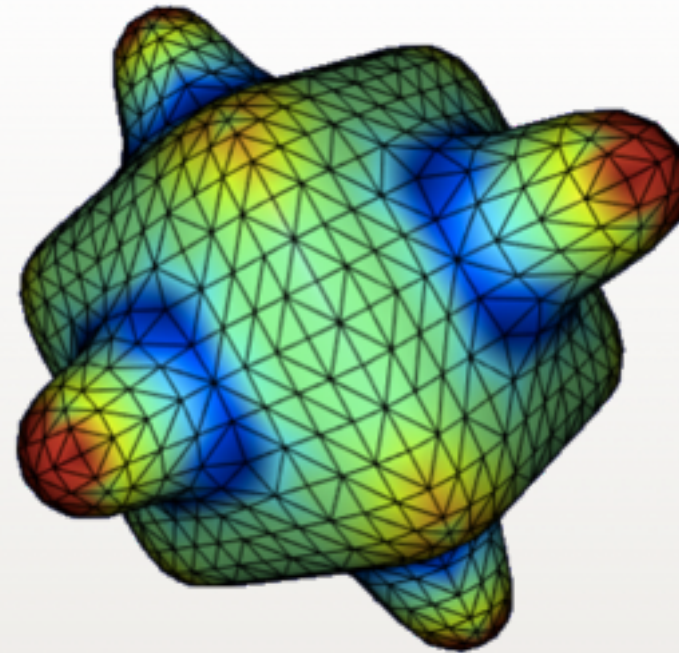
- 分片线形



9

网格曲面表面的密度场

- $f > 0$
- 每一个三角形内部是线形的



以密度场作为驱动的几何计算

不同密度场诱导的采样

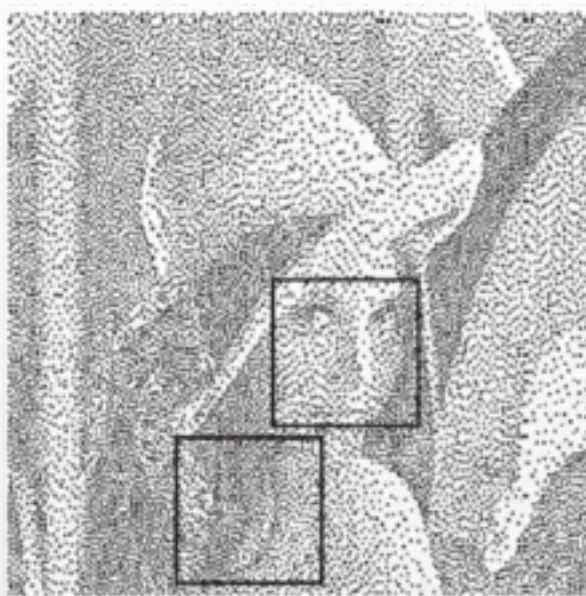
- 陈中贵等, IEEE TVCG, 2012



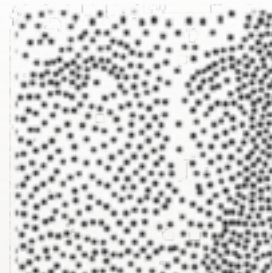
(a)



(b)



(c)



(d)

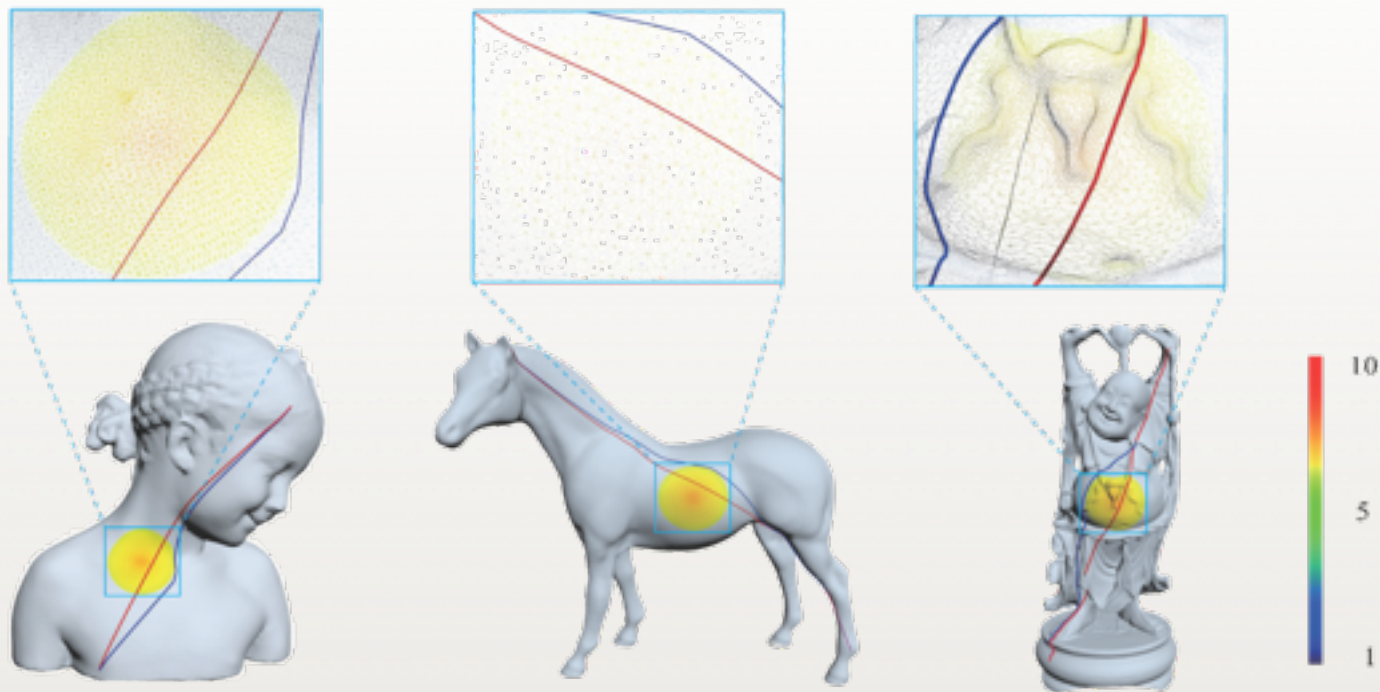


(e)

12

不同密度场诱导的测地线

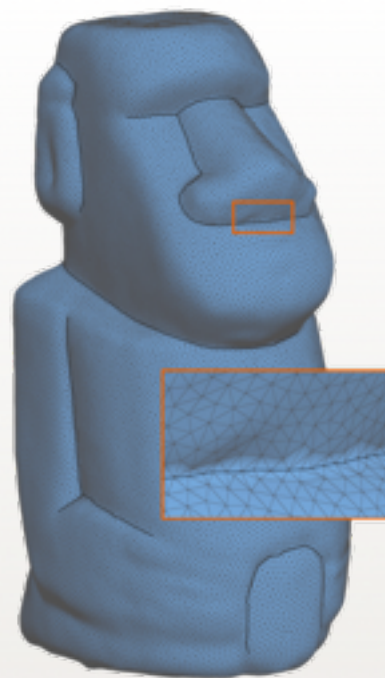
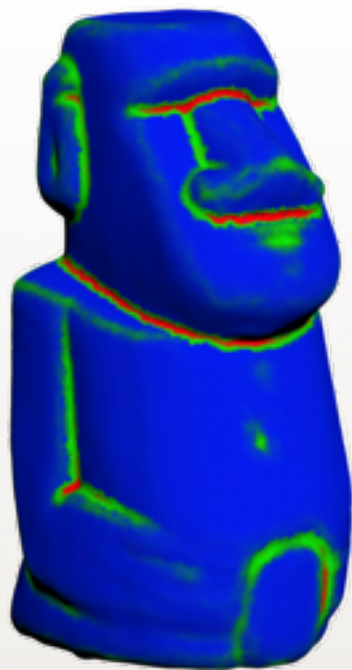
- 刘邦权等, SPM'17



13

不同密度场诱导的三角化

- 孟文龙等, CG'17

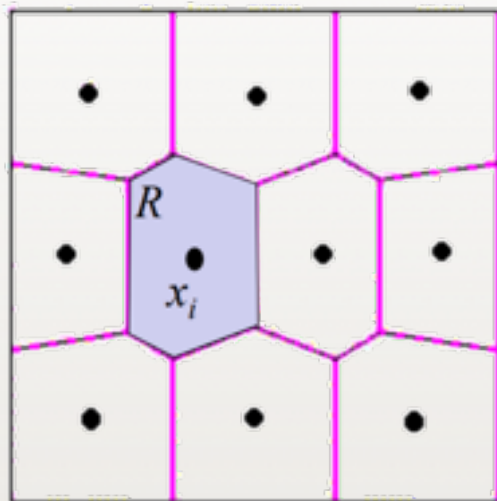


14

不同密度场诱导的CVT (Centroidal Voronoi Tessellation)

- Yang Liu等, SIGGRAPH 2009

$$E(x_1, \dots, x_k, R_1, \dots, R_k) = \sum_{i=1..k} \int_{x \in R_i} \|x - x_i\|^2 dx$$

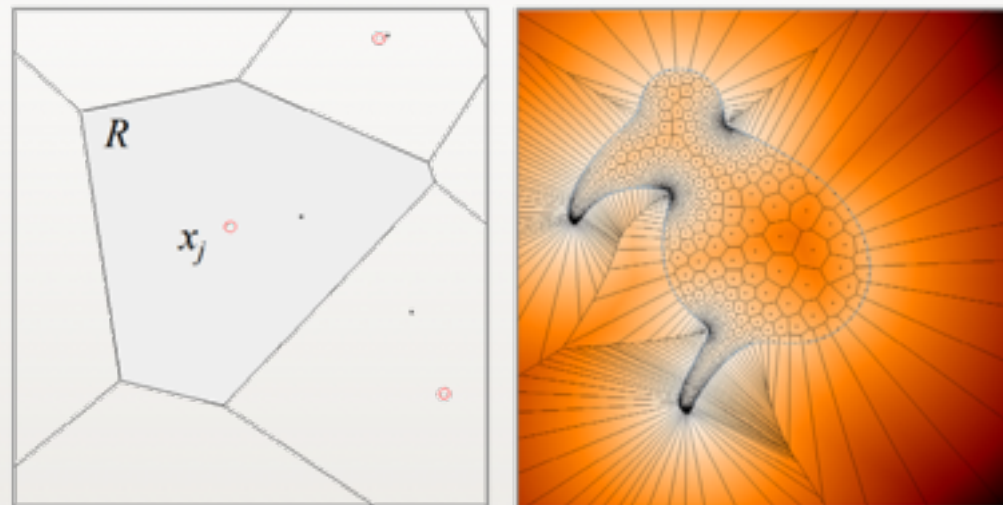


15

不同密度场诱导的CVT (Centroidal Voronoi Tessellation)

- Yang Liu等, SIGGRAPH 2009

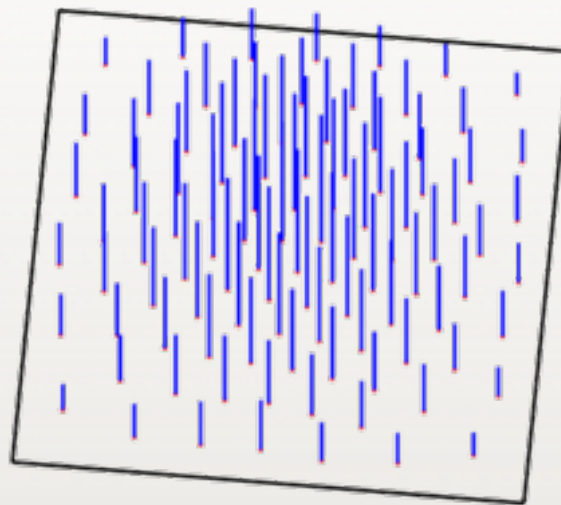
$$E(x_1, \dots, x_k, R_1, \dots, R_k) = \sum_{i=1..k} \int_{x \in R_i} \rho(x) \|x - x_i\|^2 dx$$



从Wasserstein Metric解释CVT的合理性

- Minimize the transport cost

$$E(x_1, \dots, x_k, R_1, \dots, R_k) = \sum_{i=1..k} \int_{x \in R_i} \rho(x) \|x - x_i\|^2 dx$$

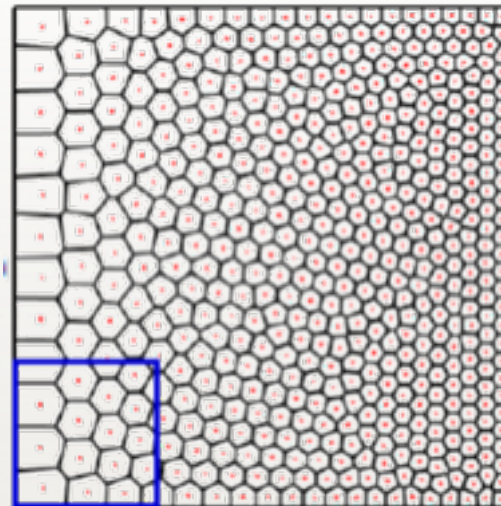
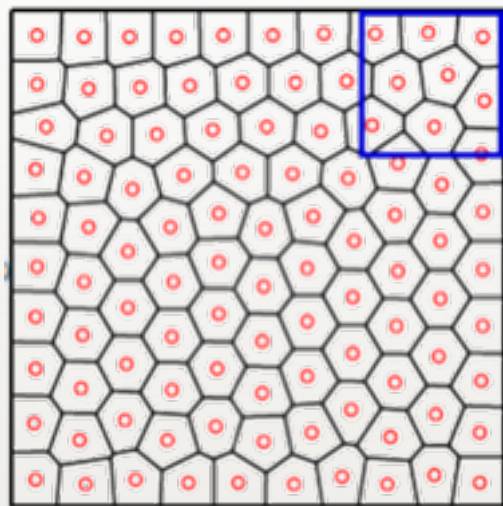


只关注质点的最终位置是不对的

17

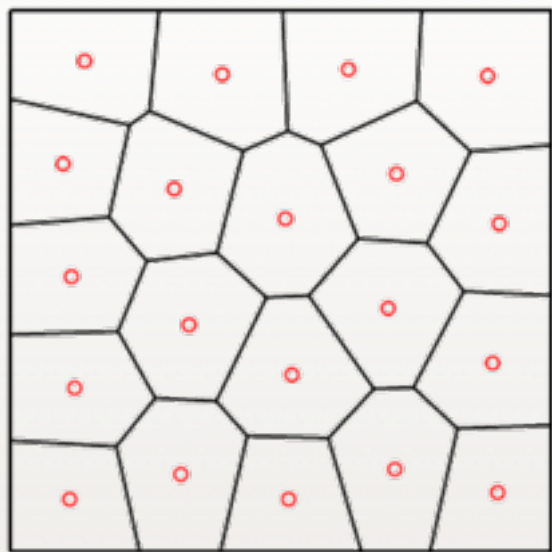
不同密度场诱导的CPD (Centroidal Power Diagram)

- 辛士庆等, SIGGRAPH 2016

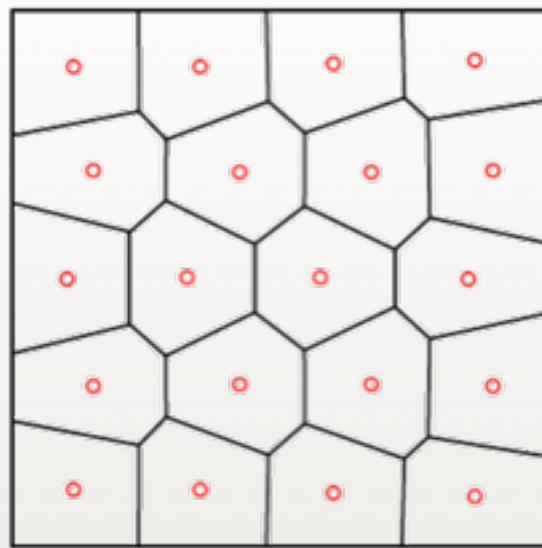


不同密度场诱导的CPD (Centroidal Power Diagram)

- 辛士庆等, SIGGRAPH 2016



CVT



CPD

小结

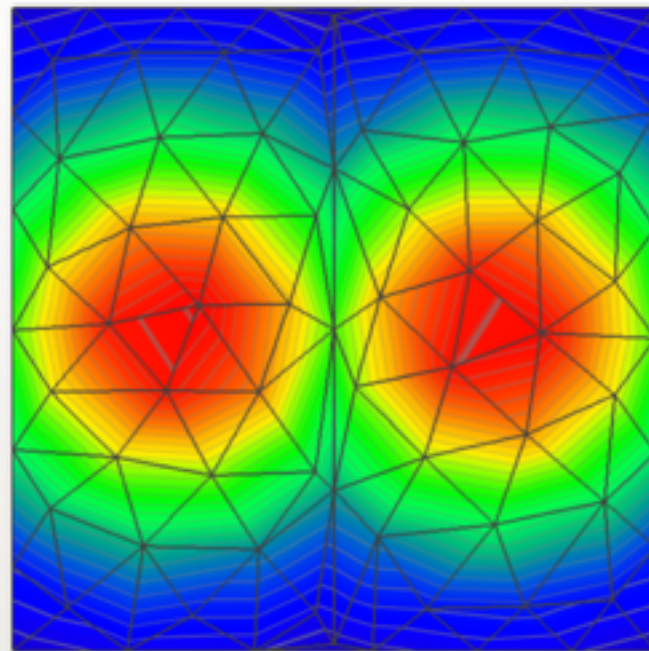
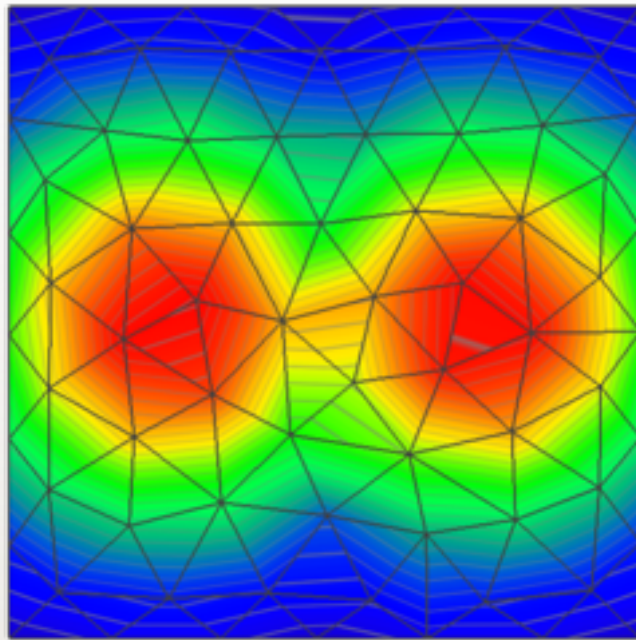
- 标量场/密度场的定义与问题相关，具有驱动能力
- 在均匀采样的问题上，如果把 $\{(x_i, m_i)\}$ 作为密度函数的离散表达，CVT更加合理；如果仅记录位置信息 $\{x_i\}$ 的话，那么CPD更加合理。

表达离散标量场的tessellation
往往与标量场的特点有关

21

例子

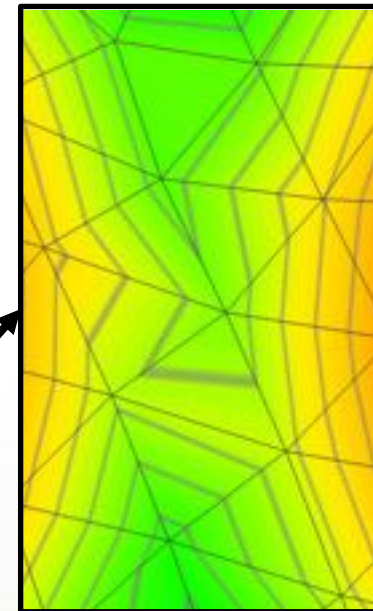
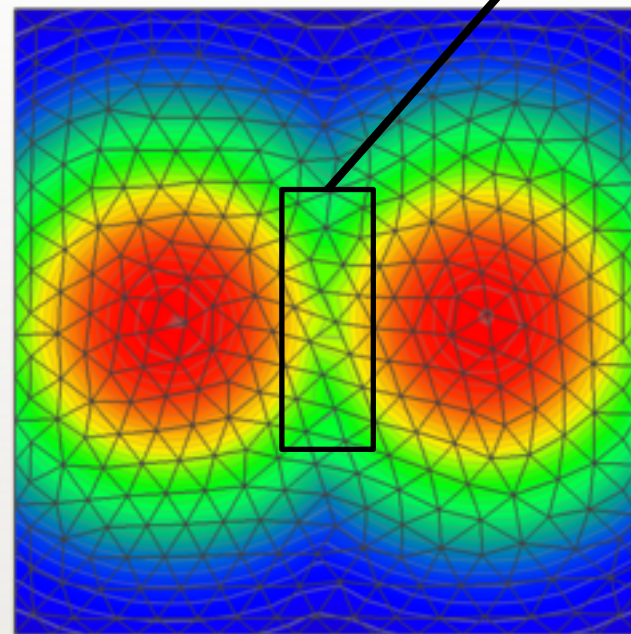
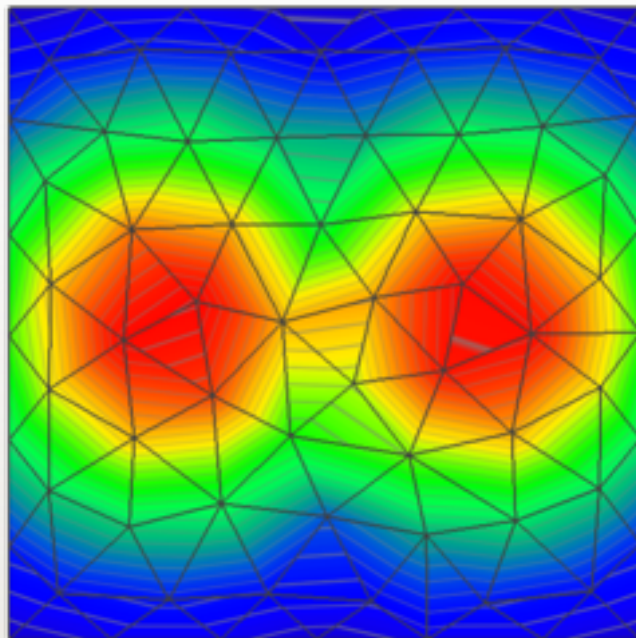
- 哪种区域的离散化方式更好呢？



22

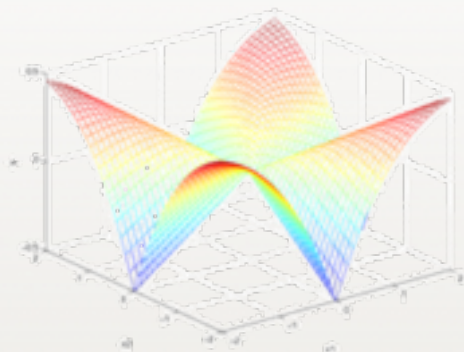
例子

- 加细也不见得管用

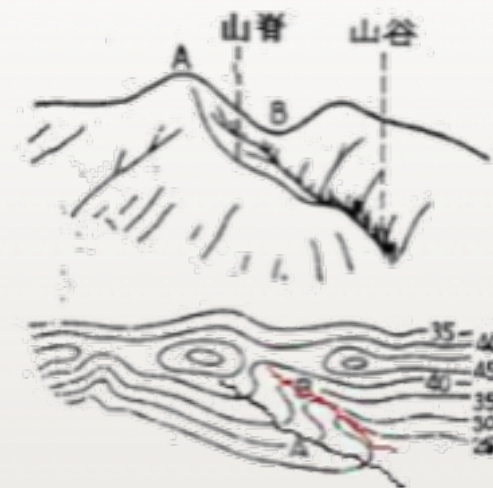
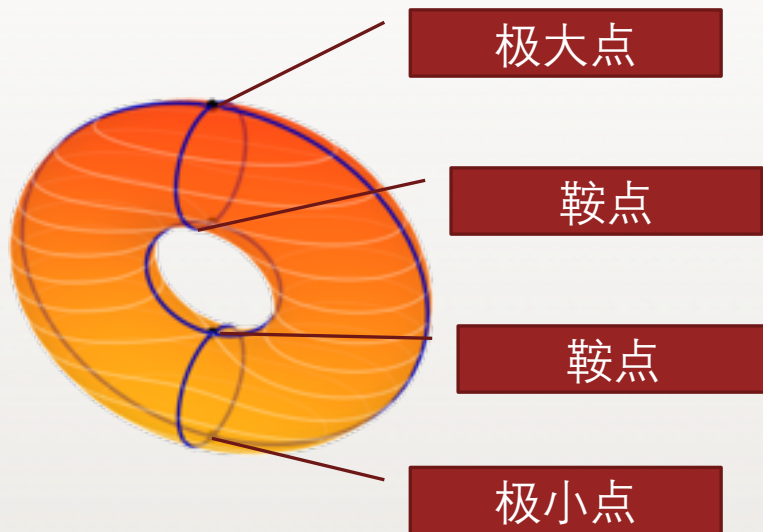


问题在哪里？

- 分片线性假设出了问题
 - 在某处有可能不光滑
 - 有某处有可能有极大点，极小点，鞍点，山脊，山谷



不光滑



应用之一：测地等值线

25

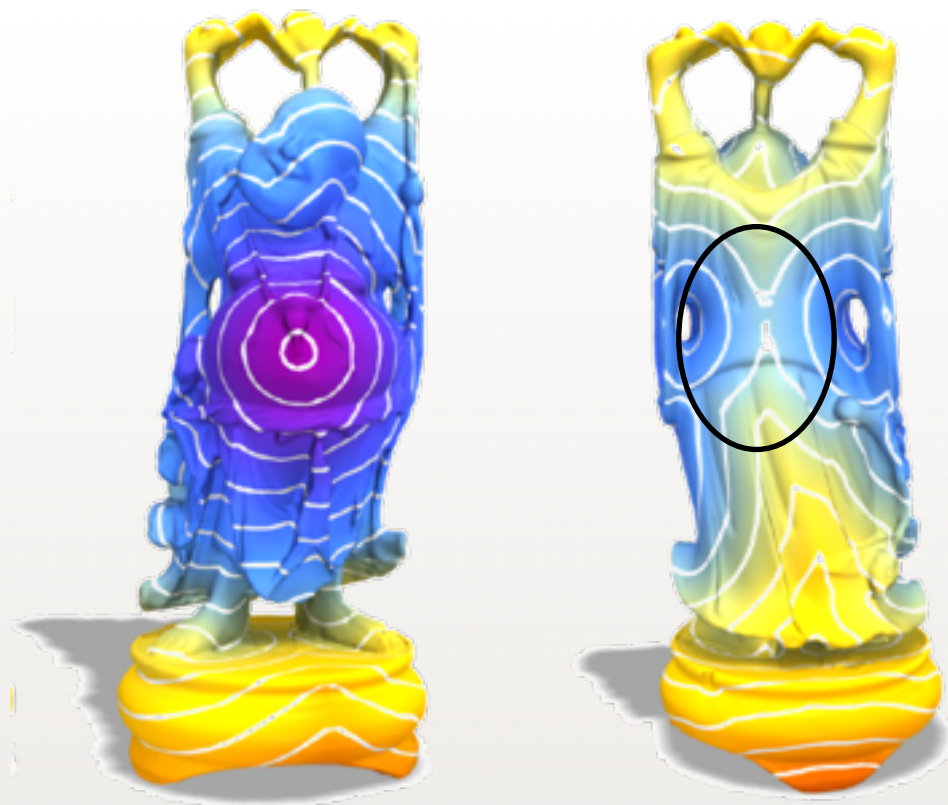
测地距离场

- 连续但并不处处光滑
- 辛士庆等, SPM '18



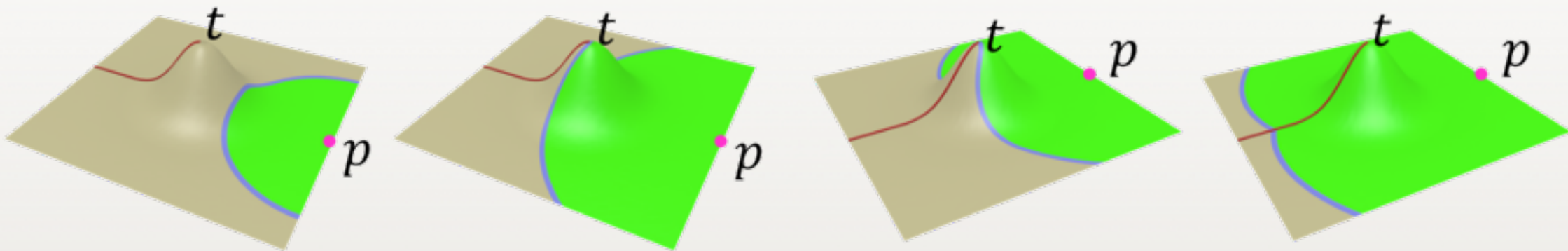
测地距离场

- 即使光滑模型+精确距离场, 等值线质量也不见得好



测地脊线

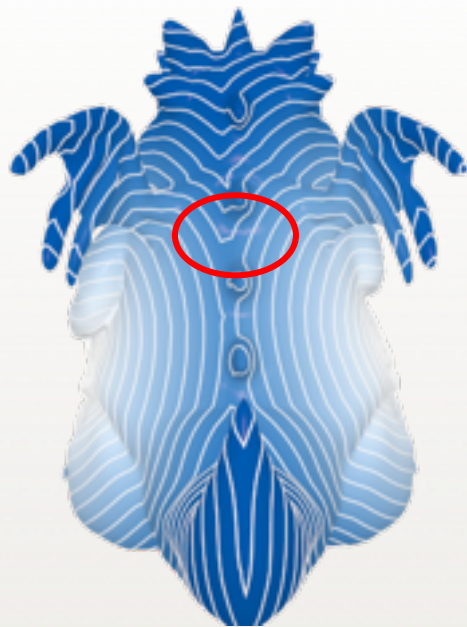
- 辛士庆等, ACM TOG 2016
- 测地脊点: 如果 q 到源点 p 有两条走法不同的最短路, 那么 q 为脊点
- 脊线处, 测地距离场连续但不光顺



28

从等值线观察线性插值带来的问题

- 该尖它不尖



29

从等值线观察线性插值带来的问题

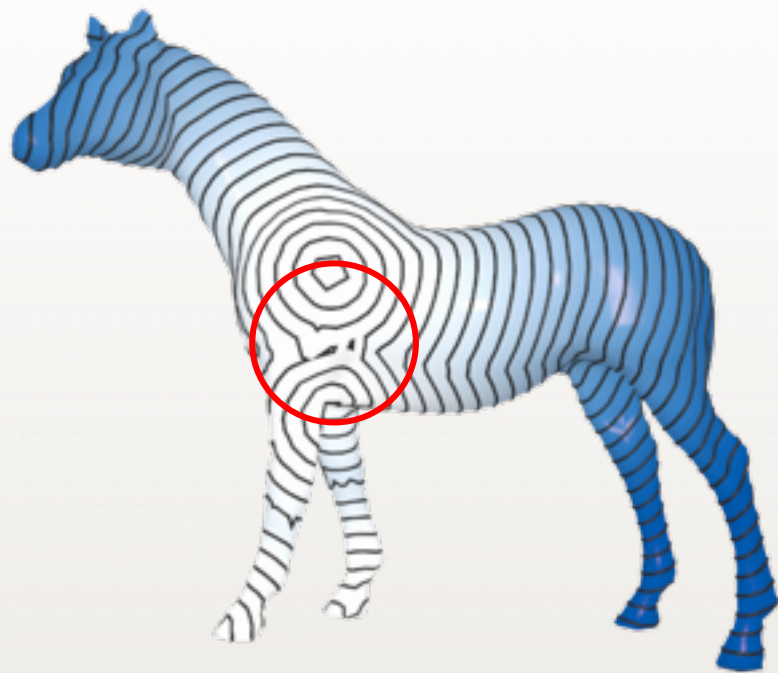
- 不该尖它尖



30

从等值线观察线性插值带来的问题

- 该连它不连



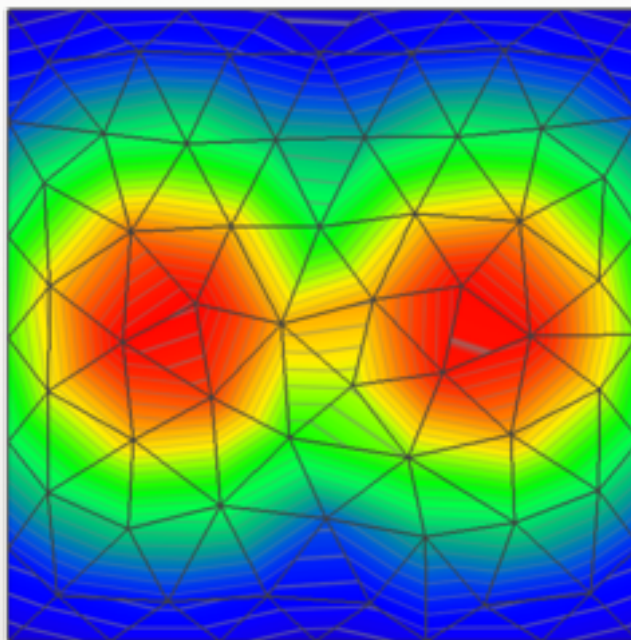
Linear interpolation



Real geodesic isolines

31 | 从等值线观察线性插值带来的问题

- 不该连它连

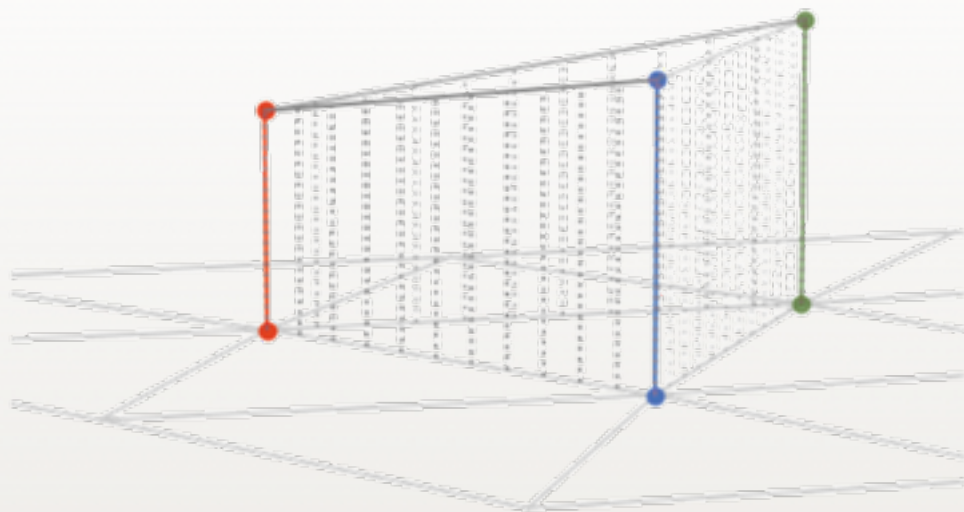


我们的目标

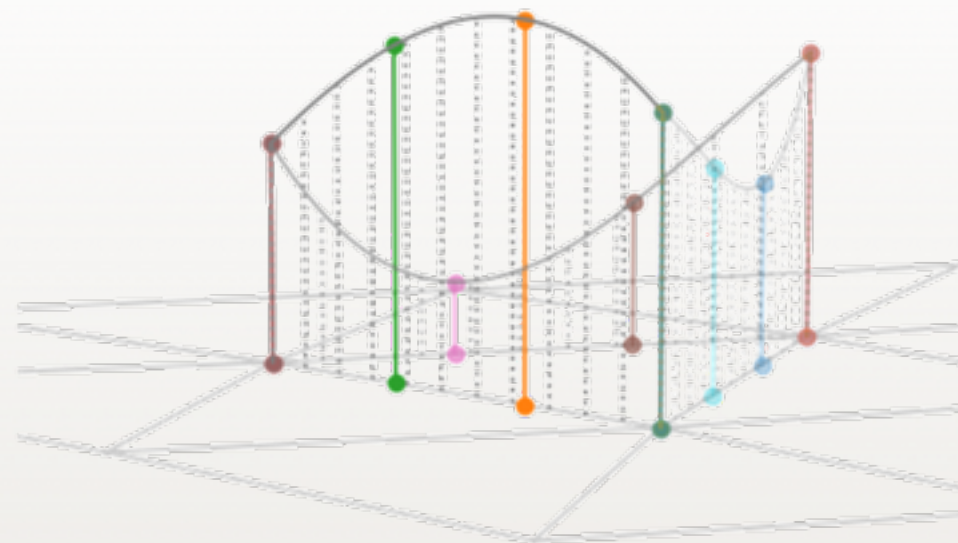
- 王文嵩等, CVM'19
- 该尖时要尖, 该连时要连; 几何正确, 拓扑正确; 精度要比较高
- 不能增加网格的复杂度 (增加也不能彻底解决问题)
- 不要有大量额外的计算

我们的算法

- 每条边上加两个Steiner points, 在计算距离场时, 也要同时报告出这些点的距离值



Linear Interpolation



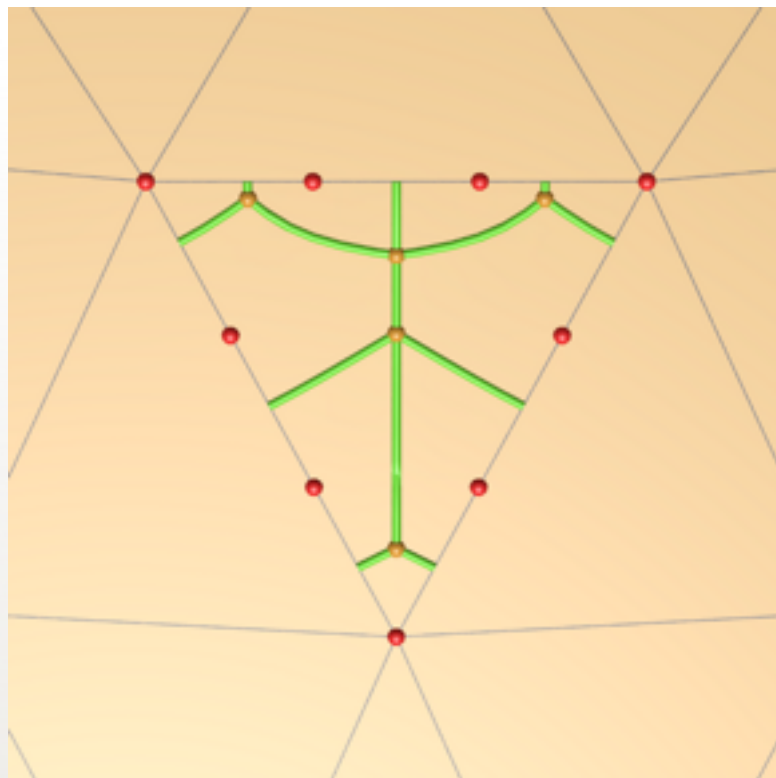
Cubic Interpolation

我们的算法

- 在每个三角形中，计算Apollonius图

$$d_g(s, q_1) + \|q_1 r\| = d_g(s, q_2) + \|q_2 r\|$$

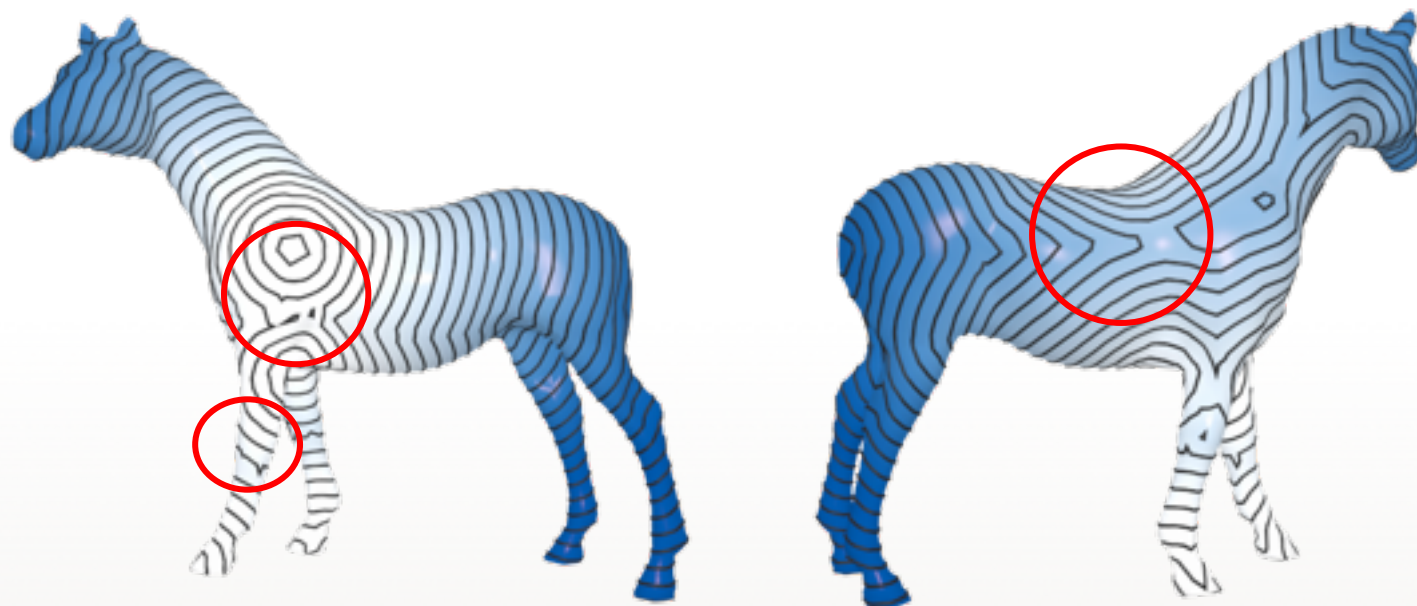
- 所有三角形中的Apollonius图是脊线的超集



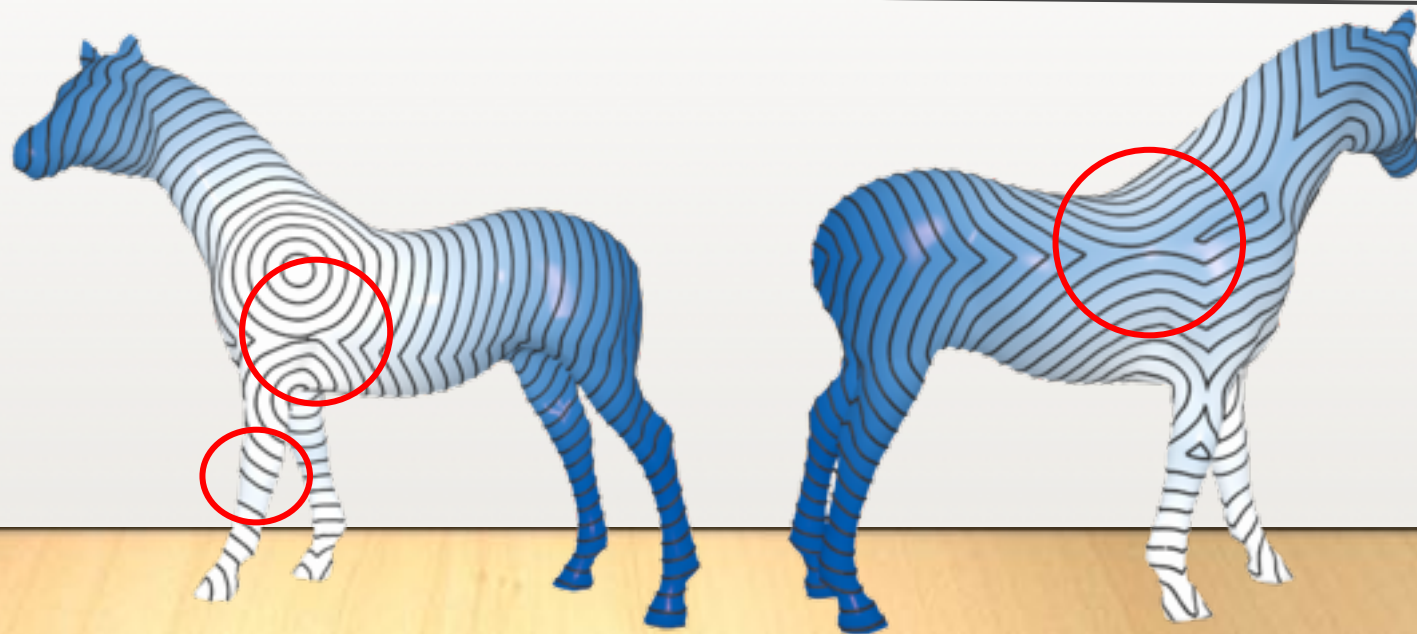
35

实验结果

Linear interpolation



Our method



应用之二：数值积分

数值积分的定义

- The definite integral of a real-valued function $f(x)$ over the domain Ω is written as

$$\int_{\Omega} f(x) d\sigma$$

数值积分的定义

- The definite integral of a real-valued function $f(x)$ over the domain Ω is written as

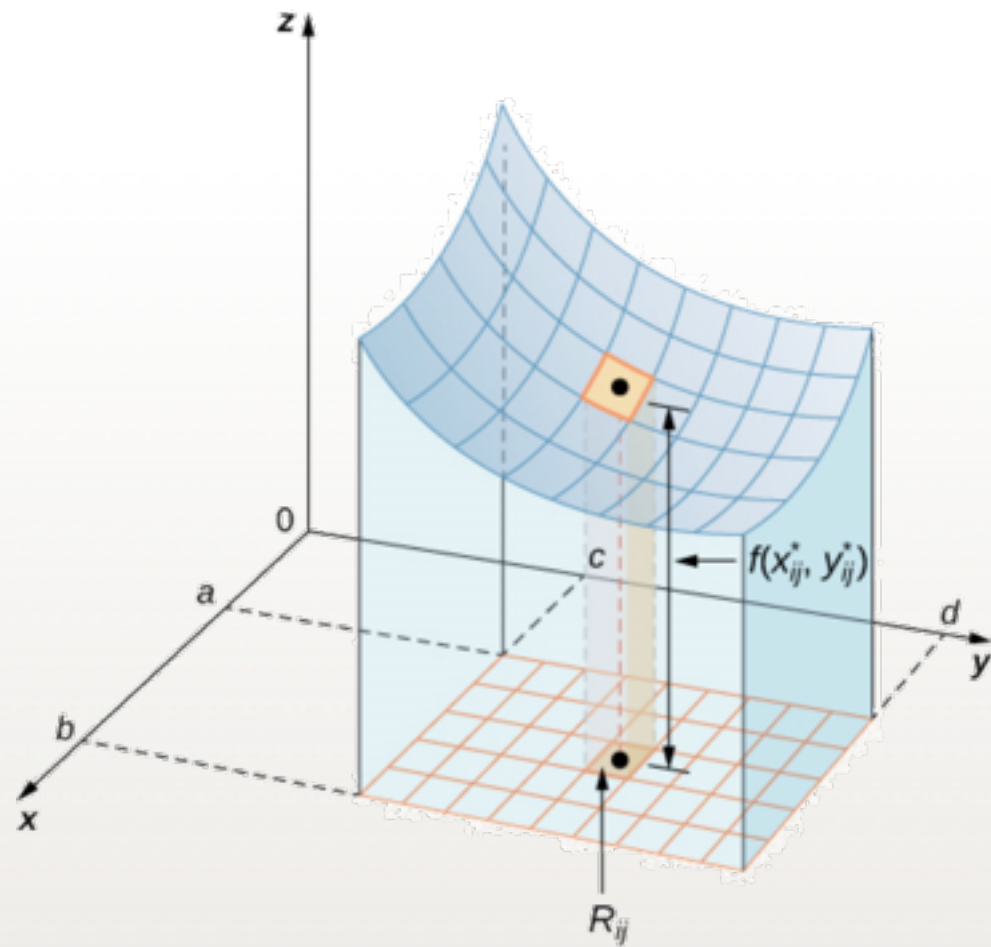
$$\int_{\Omega} f(x) d\sigma$$

- When $F(x) = \int f(x) dx$ is not available, evaluation of the definite integral is not easy.

数值积分

- 对定义域进行离散化

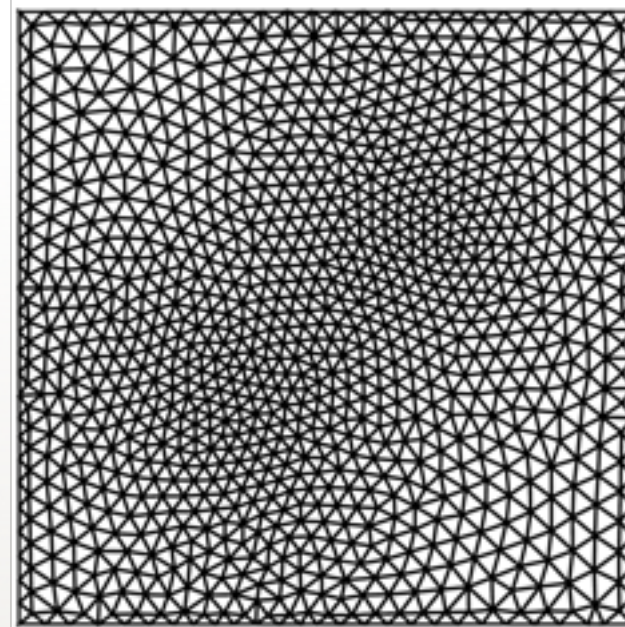
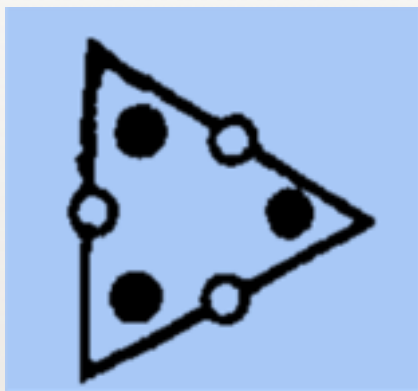
$$\int_{\Omega} f(x) d\sigma \approx \sum_{i=1}^n f(x_i) |\Omega_i|$$



数值积分

- 对定义域进行离散化

$$\int_{\Omega} f(x) d\sigma \approx \sum_{i=1}^n f(x_i) |\Omega_i|$$



六点积分格式

我们的目标

- 曹路明等, ACDDE 2019
- 研究剖分与积分精度的关系：什么样的剖分才是最好的？
- 特别地，给定一族满足Lipschitz continuous条件的函数；
- 以少量的剖分单元换取较高的精度
- 给定一个误差，生成相应的剖分，保证给定的精度

$$\begin{aligned} |\epsilon| &= \left| \int_{\Omega} f(x) dx - \sum_{i=1}^n f(x_i) |\Omega_i| \right| \\ &= \left| \sum_{i=1}^n \int_{\Omega_i} f(x) dx - \sum_{i=1}^n \int_{\Omega_i} f(x_i) dx \right| \\ &= \left| \sum_{i=1}^n \int_{\Omega_i} f(x) - f(x_i) dx \right| \end{aligned}$$

误差界

$$\begin{aligned} |\epsilon| &= \left| \sum_{i=1}^n \int_{\Omega_i} f(x) - f(x_i) dx \right| \\ &\leq \sum_{i=1}^n \left| \int_{\Omega_i} f(x) - f(x_i) dx \right| \\ &\leq \sum_{i=1}^n \int_{\Omega_i} |f(x) - f(x_i)| dx \\ &\leq l \sum_{i=1}^n \int_{\Omega_i} \|x - x_i\| dx \end{aligned}$$

$$|a + b| \leq |a| + |b|$$

Lipschitz continuous

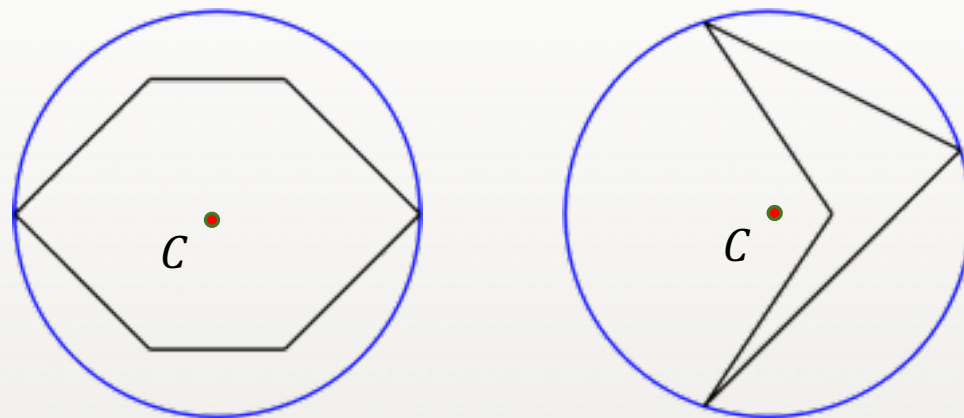
$$|f(x) - f(y)| \leq l \|x - y\|, \forall x, y \in \Omega$$

误差界

$$|\epsilon| \leq l \sum_{i=1}^n \int_{\Omega_i} \|x - x_i\| dx$$

$$\leq l \sum_{i=1}^n R_i |\Omega_i|$$

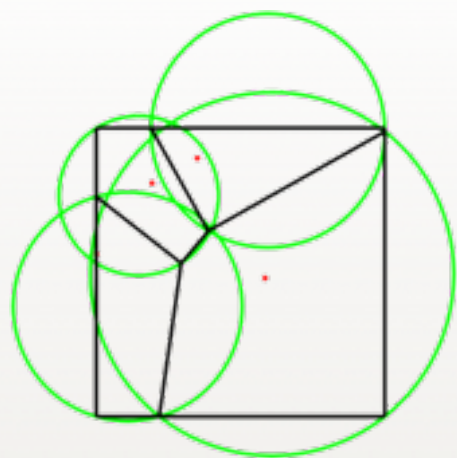
$$\leq lR |\Omega|, \quad R = \max\{R_i\}_{i=1}^n$$



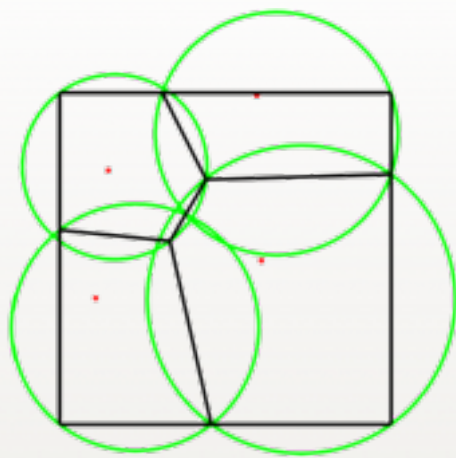
minimum enclosing circle

45

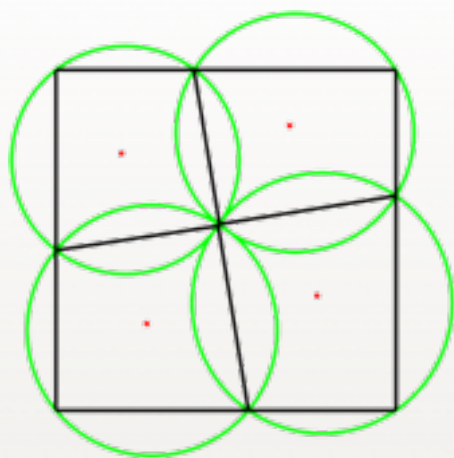
不断优化剖分



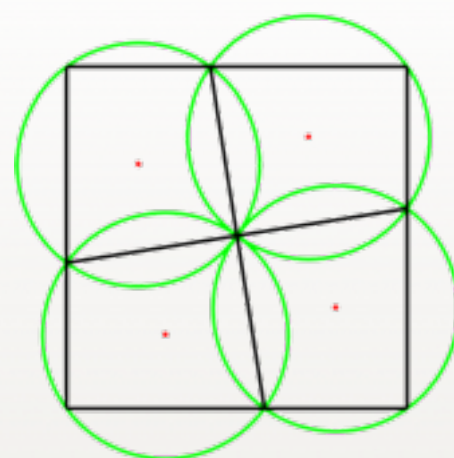
Initial



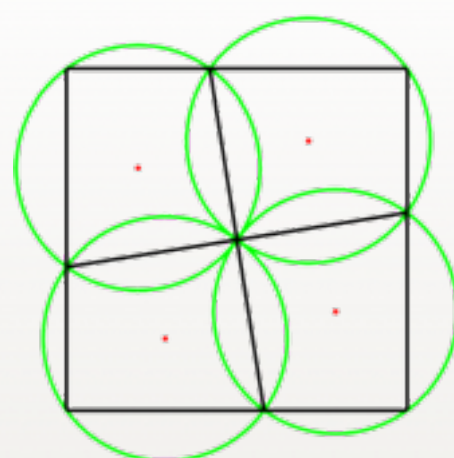
Iter=1



Iter=3



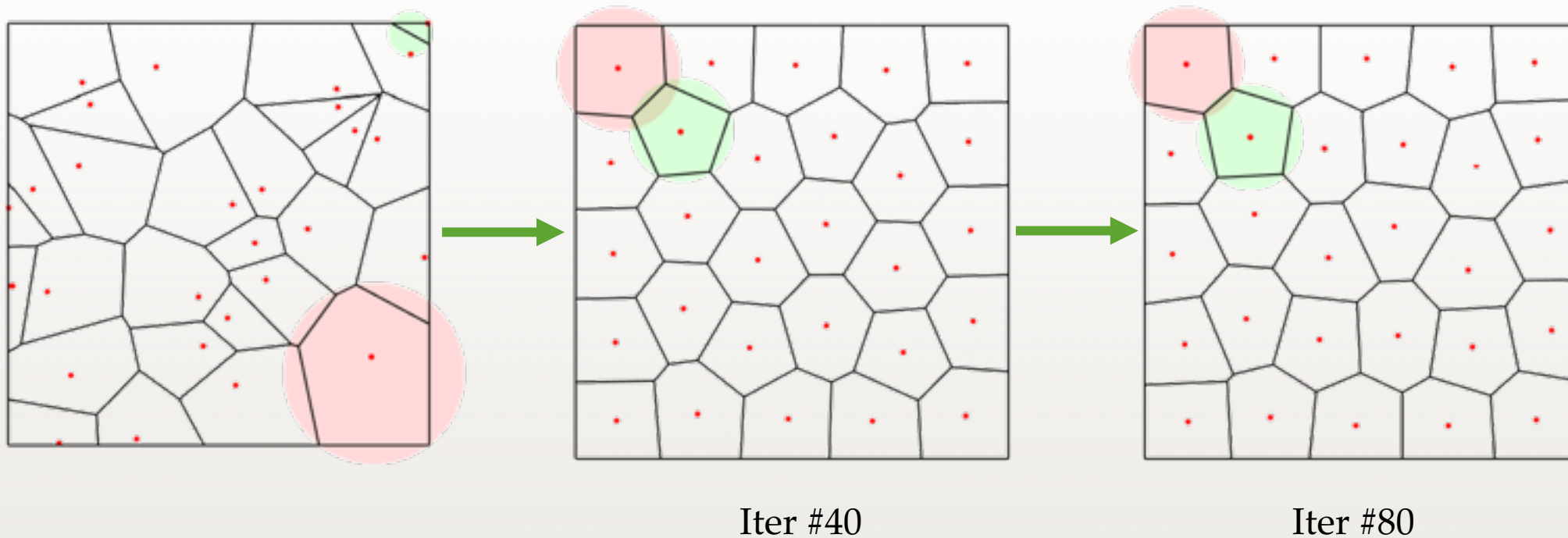
Iter=7



Iter=10

一般的优化过程

- 最大半径越来越小；每个cell的形状的近圆性越来越好



剖分单元数目的估计

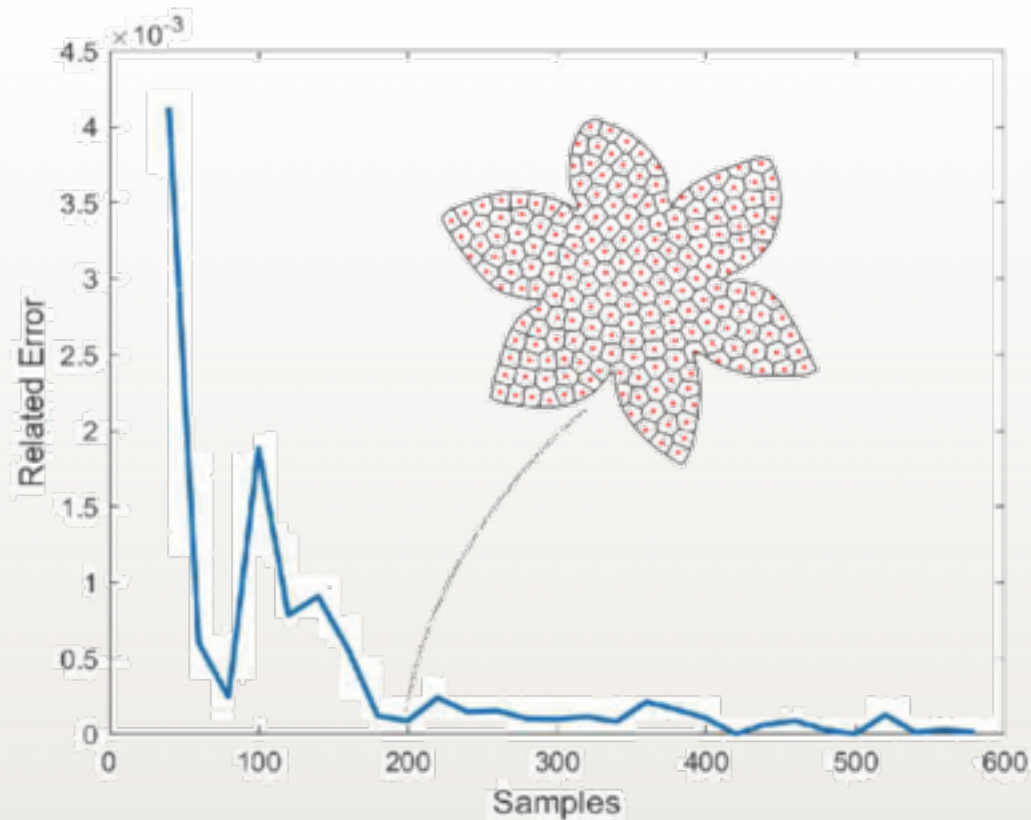
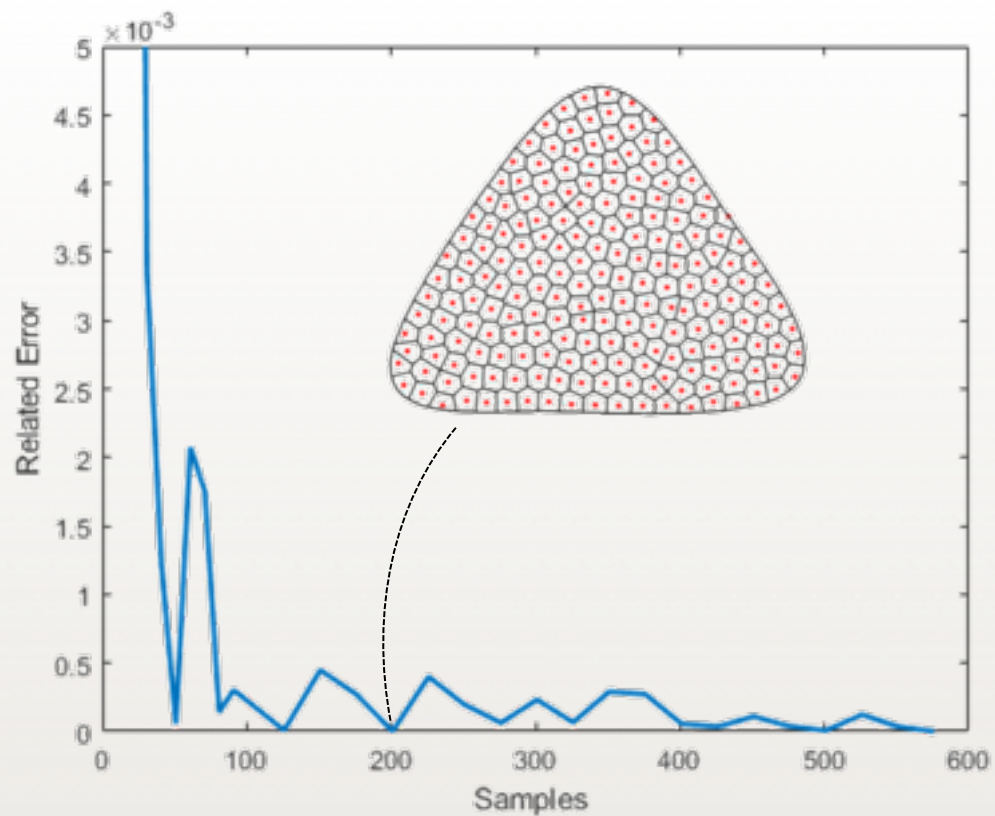
$$|\Omega| = \sum_{i=1}^n |\Omega_i^{\text{vor}}| \leq \sum_{i=1}^n |C_i| \leq n\pi R_{\max}^2$$

The number of seed points required to acquire the specified accuracy ϵ is

$$n \geq \frac{l^2 |\Omega|^3}{\pi \epsilon^2}.$$

精度与点数

$$\text{Relative Error} = \frac{|E_i - E_{i-1}|}{|E_i|}$$



定量比较

Functions $f(x,y)$	CDT	CVT	Ours
x^2y^2	4.30e-04	2.73e-04	2.11e-05
$x^2(3 + xye^{xy})$	4.24e-03	2.57e-03	2.53e-04
$x^3 + y^3$	1.87e-03	1.29e-03	3.74e-05
$\cos(30x) + \sin(30y)$	2.03e-03	1.97e-03	1.77e-03
$x^2\cos(20x) + y\sin(40x)$	8.47e-03	3.87e-03	2.12e-04
$\frac{2y(x - 1 - y^2)}{(1 + x + y^2)^3}$	2.41e-04	1.77e-04	2.41e-05
$\frac{2\log(1 + xy) + 2xy}{(1 + xy)^2}$	9.53e-04	8.27e-04	3.20e-04
$-40\sin(40xy)$ $- 1700xycos(40xy)$	4.49823	1.08727	3.49495

结论

- 三角网格曲面上，人们往往假设某个场在一个三角形内是线性变化的。这种假设在某些场合是不对的。
- 在标量场的极小点, 极大点, 鞍点, 脊点, 不连续点, 脊, 谷处尤其需要注意。
- 为了更好的适应给定的标量场，有时候需要修正定义域的离散化表达（未必是稠密就好）。

合作者

- 教师

- 王文平（港大），屠长河（山大），周元峰（山大），舒振宇（浙大宁波理工学院），贺英（NTU），傅红波，陈中贵（厦大），Bruno Levy（Inria）

- 学生

- 王文嵩（大三），曹路明（研一），刘邦权（宁波财经学院教师），孟文龙（博一）

谢谢！