Progressive Embedding

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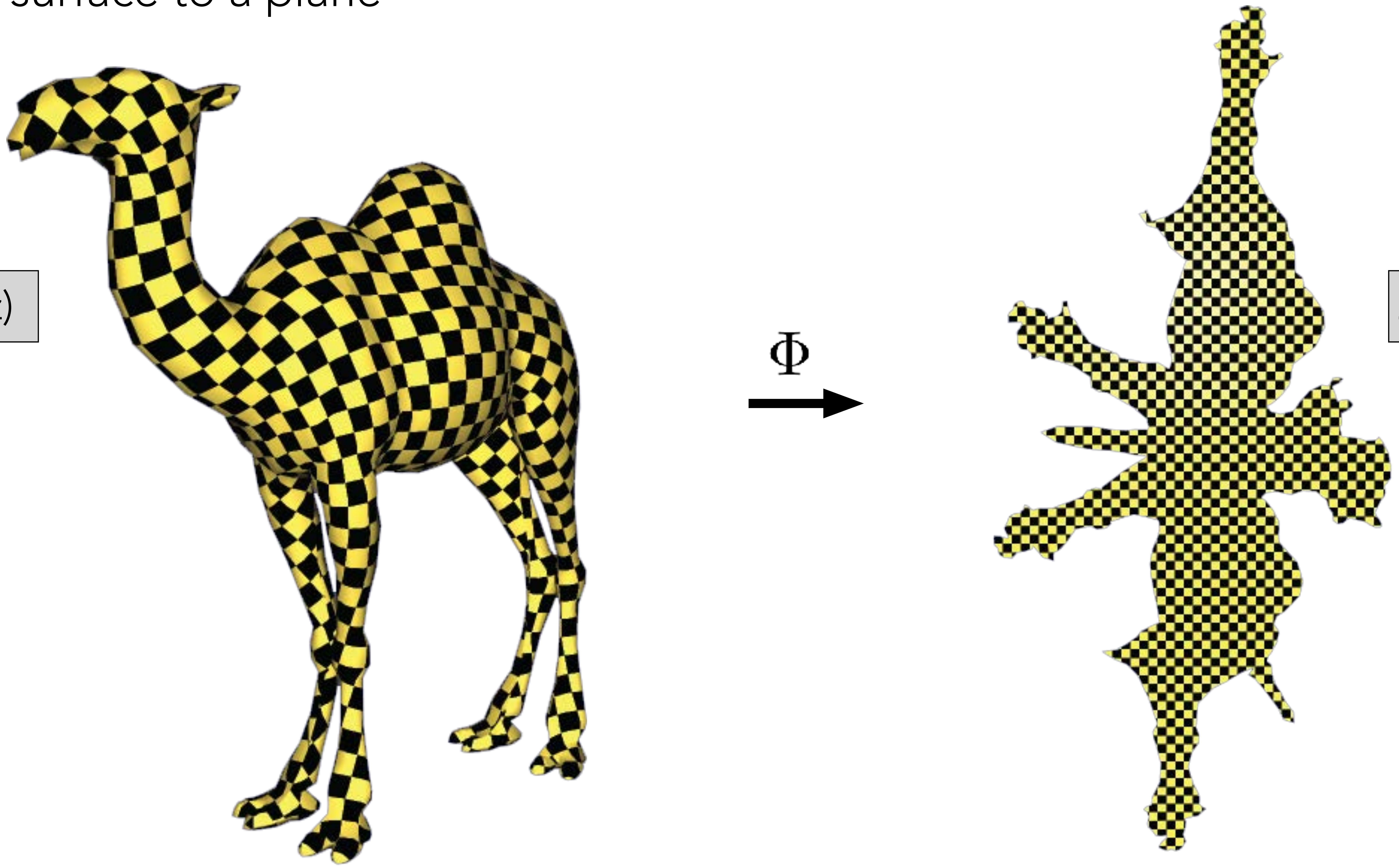
Surface Parametrization

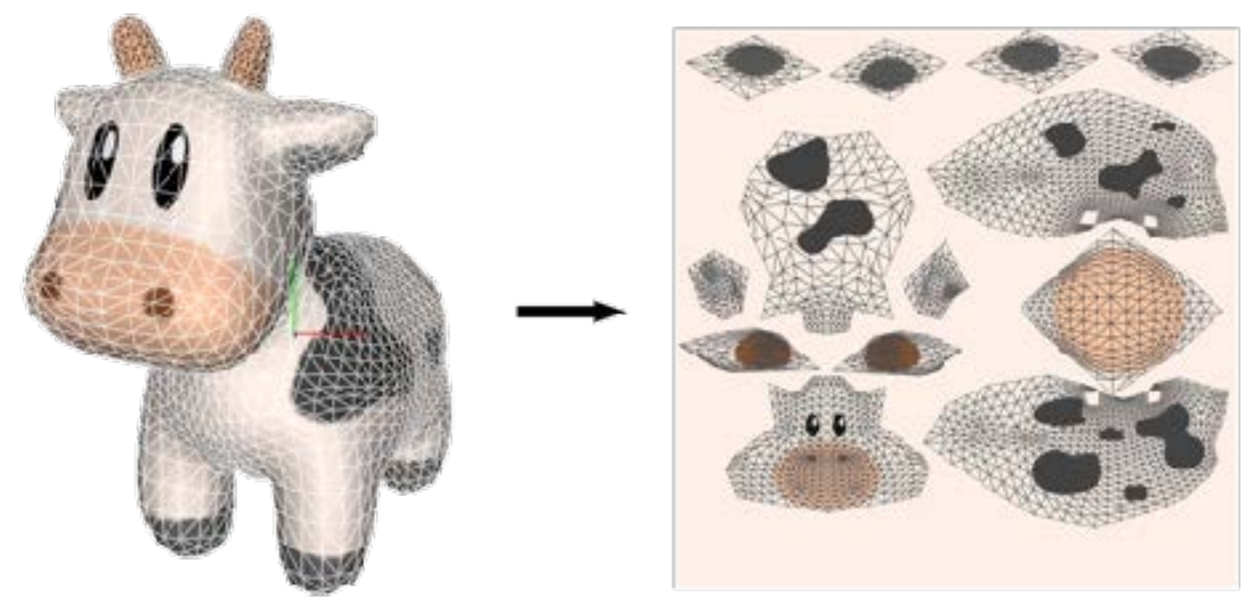
Flatten a surface to a plane

3D Mesh (x,y,z)

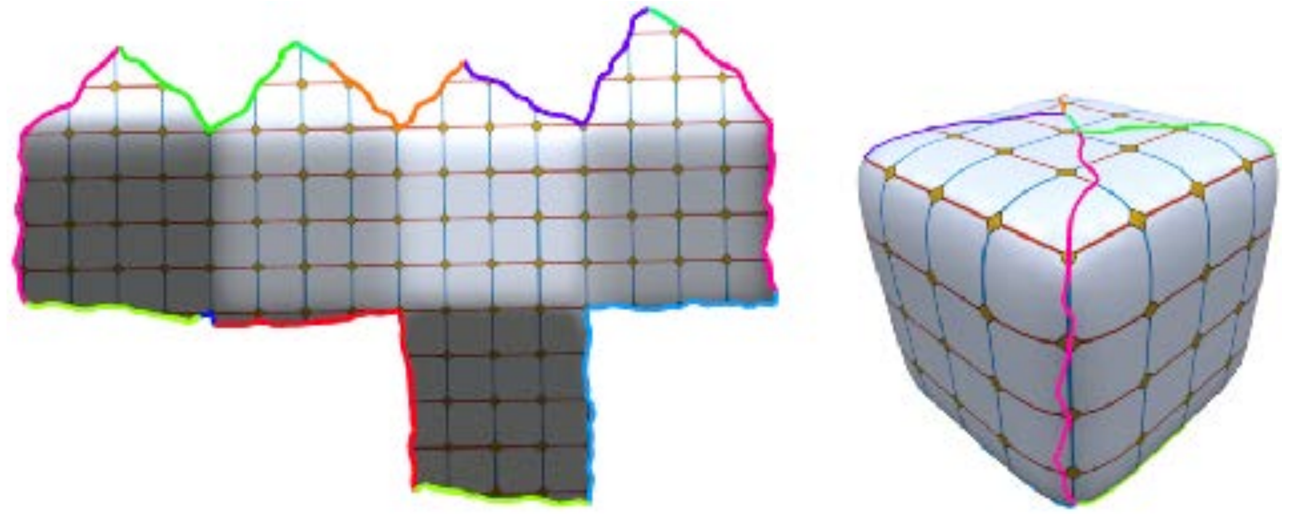


2D Parametric domain (u,v)

Applications

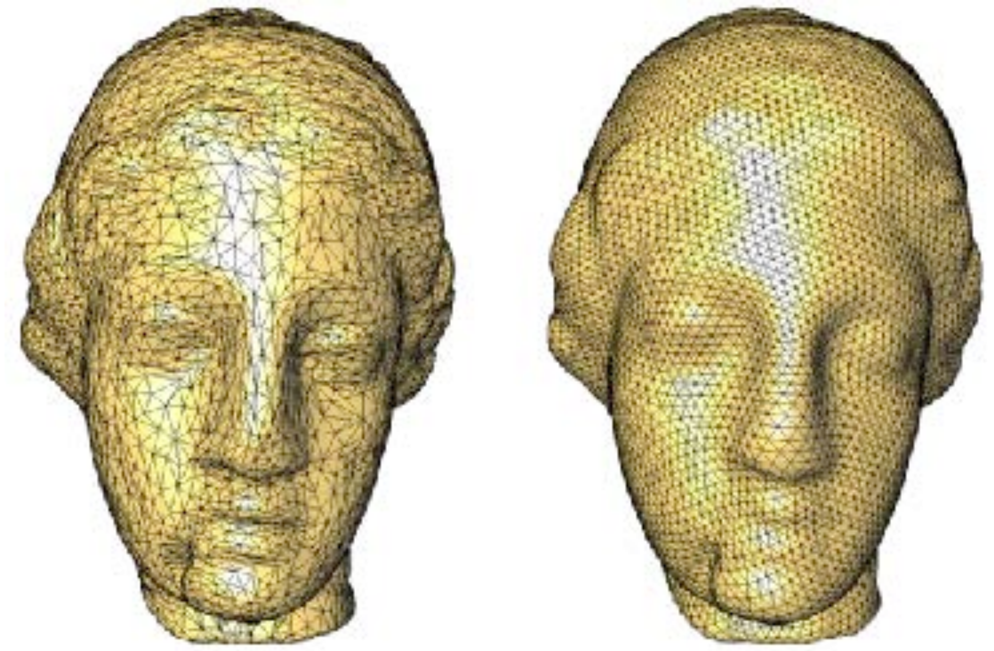


Texture mapping



[Bommes et al. 2012]

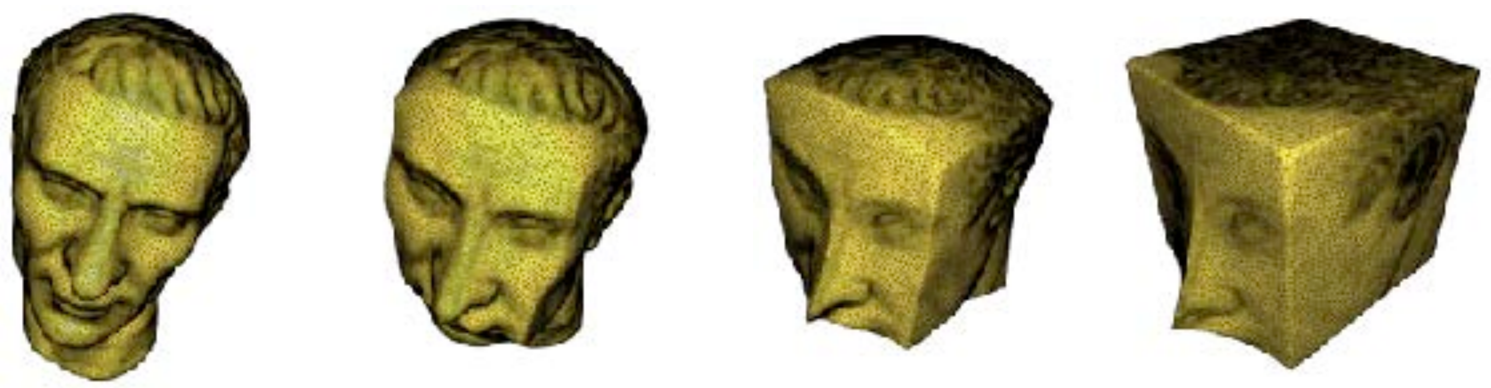
Quadrangulation



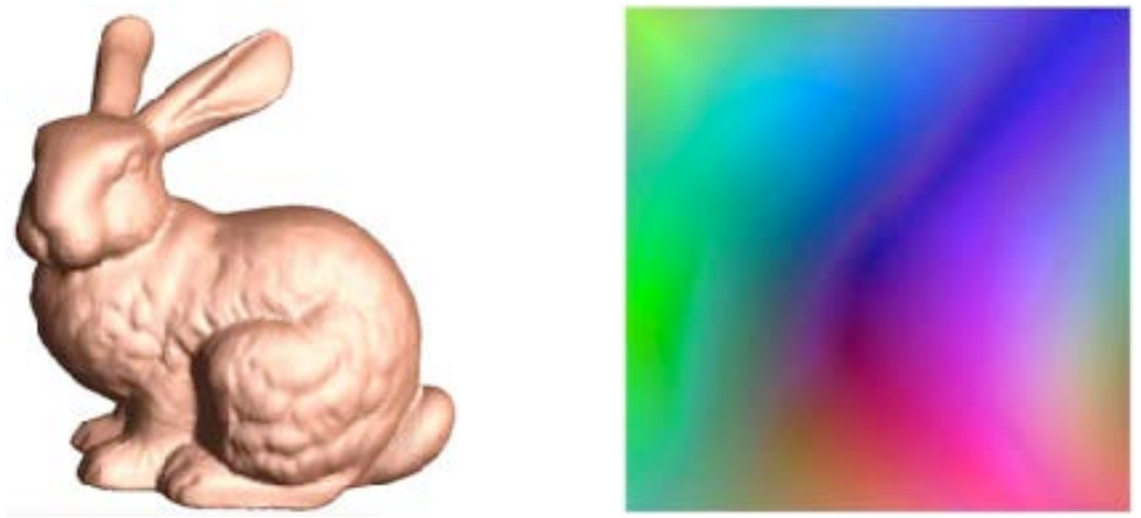
[Botsch et.al 2010]

Remeshing

Cartography



Shape Interpolation

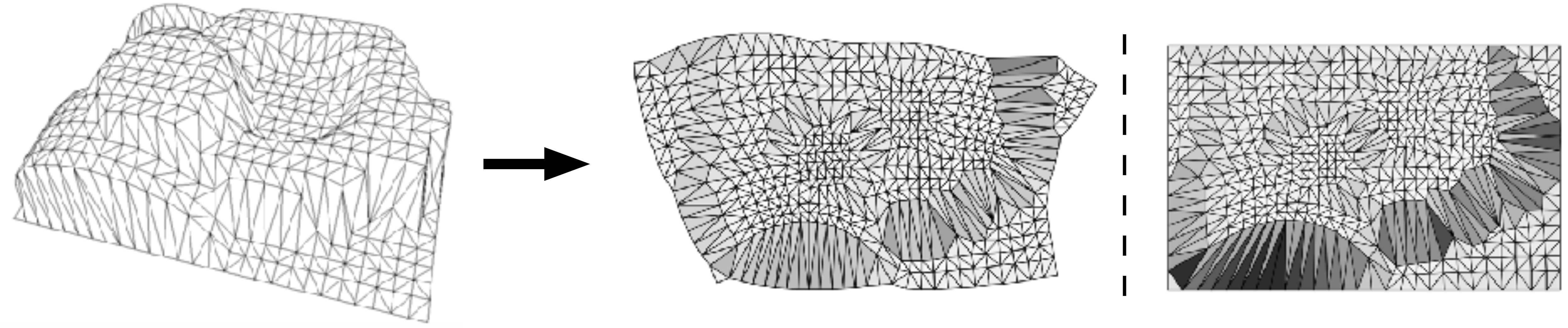


[Gu et al. 2002]

Compression

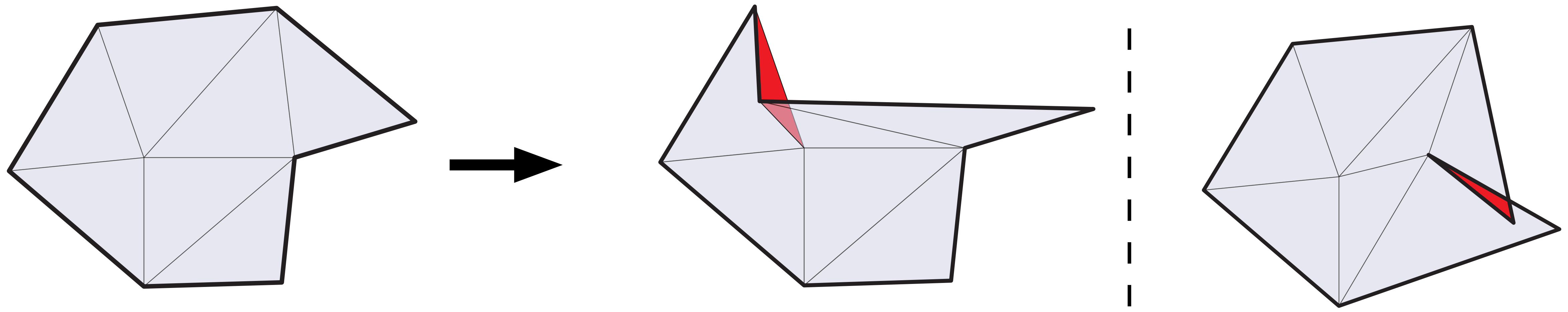
Desirable Properties

Low Geometric Distortion

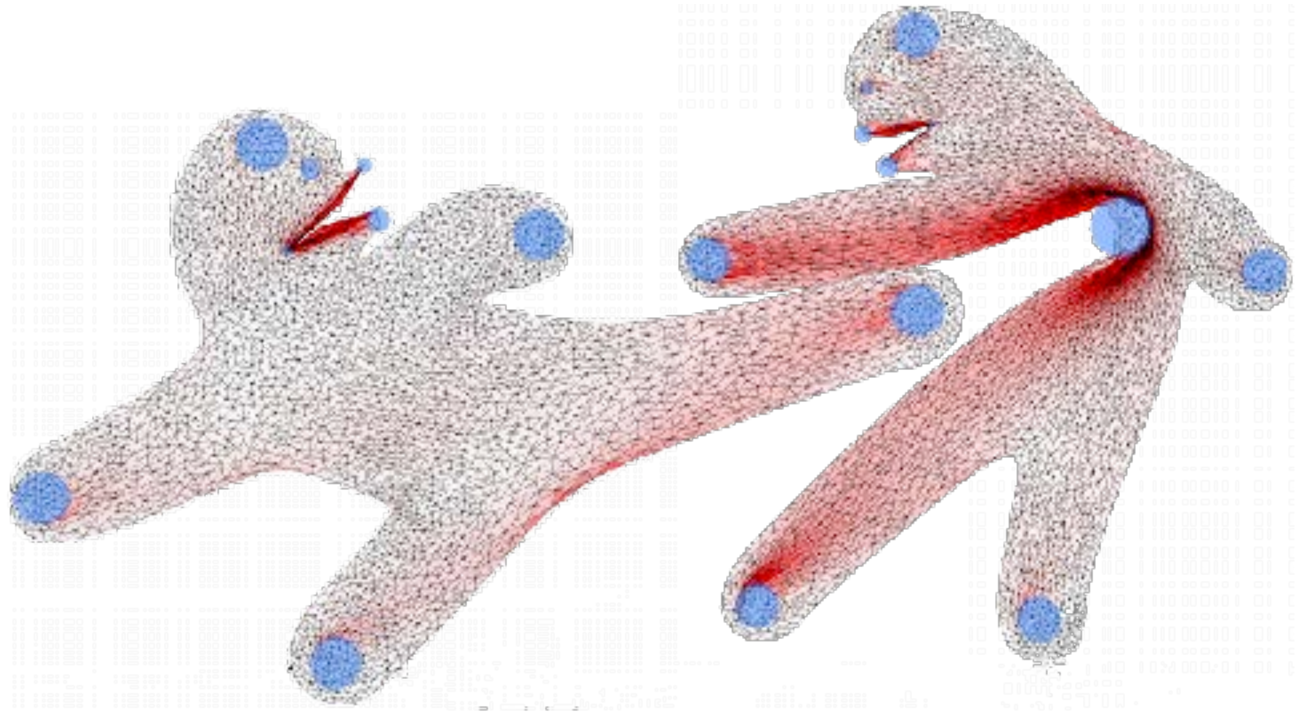


[Hormann & Greiner et al. 2000]

Local/Global Bijectivity



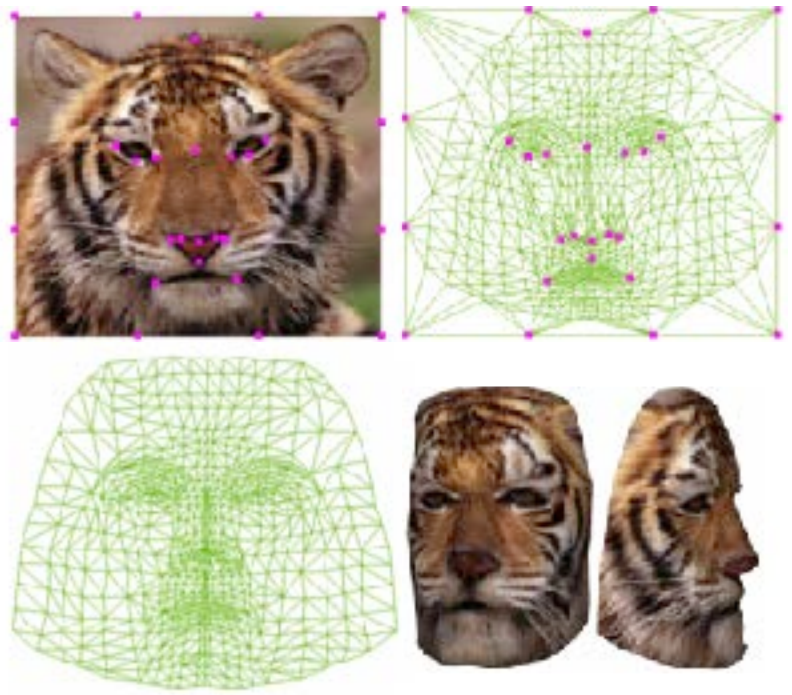
Control with Positional Constraints



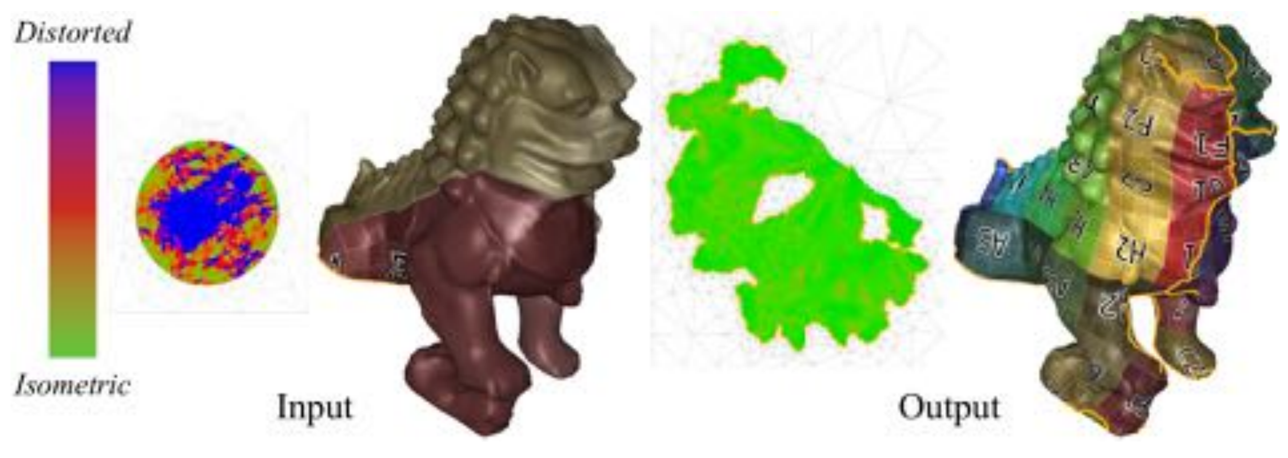
[Schüller et al. 2013]

Efficiency & Scalability

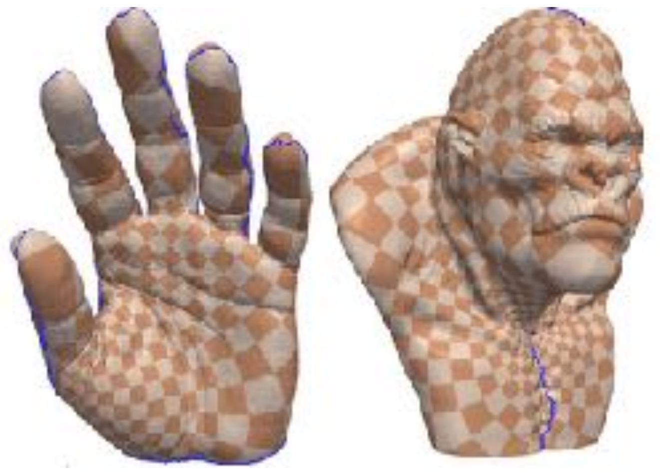
Distortion-Minimizing Mappings



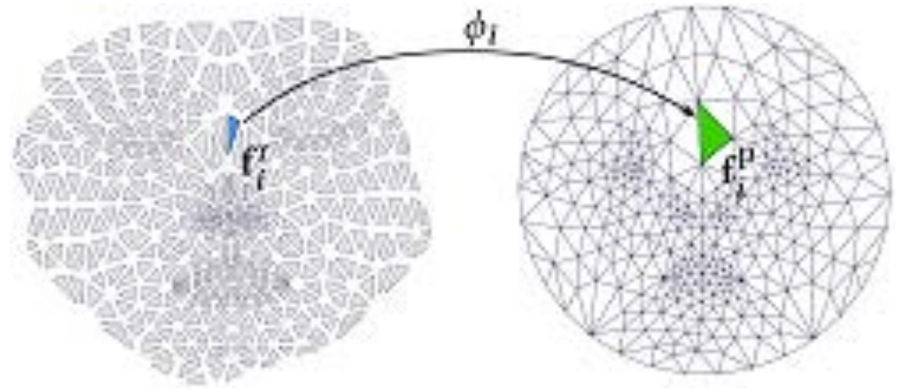
[Kraevoy et al. 2003]



[Smith and Schaefer 2015]

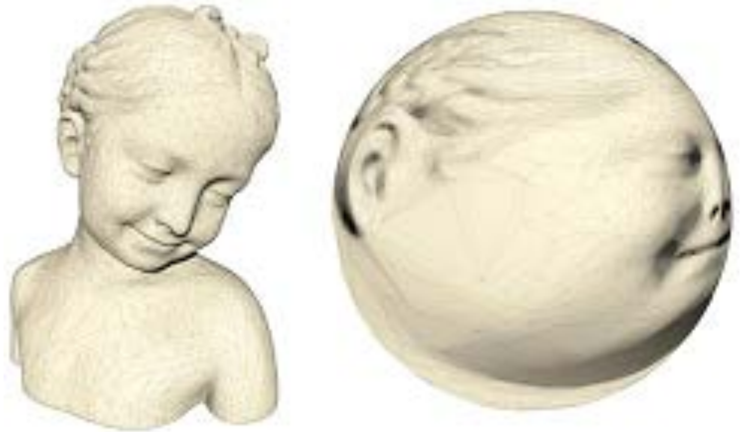


[Claici et al. 2017]



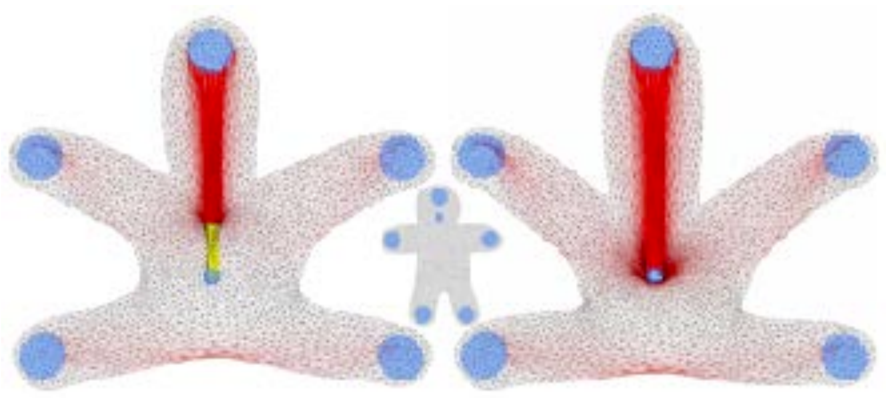
[Liu et al. 2018]

Tutte Embedding

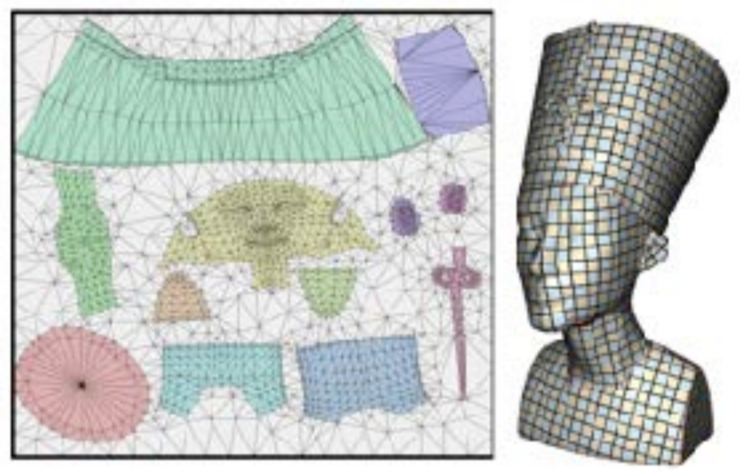


[Wang et al. 2016]

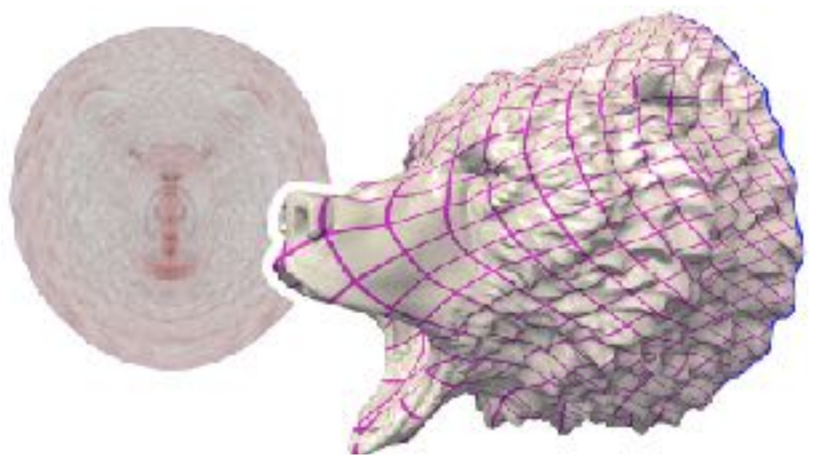
[Rabinovich et al. 2016]



[Schüller et al. 2013]



[Jiang et al. 2017]

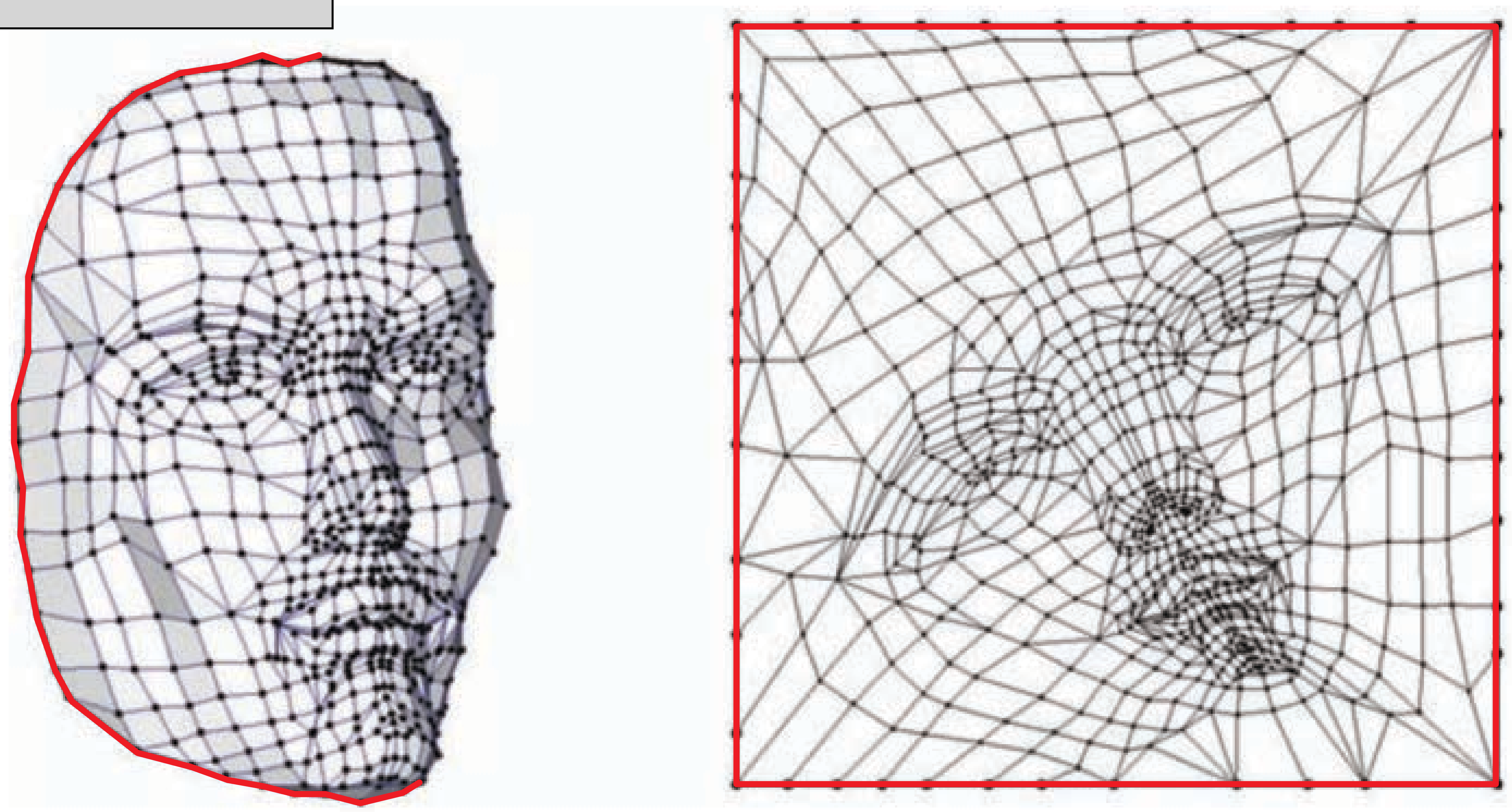


[Shtengel et al. 2017]

[Fu et al. 2015]

Tutte Embedding

How to draw a graph [Tutte. 1963]



Map a disk mesh to the interior of **convex** boundary

Guarantee for bijectivity

How about floating point coordinates?

(Meshes are assumed to be 3-connected)

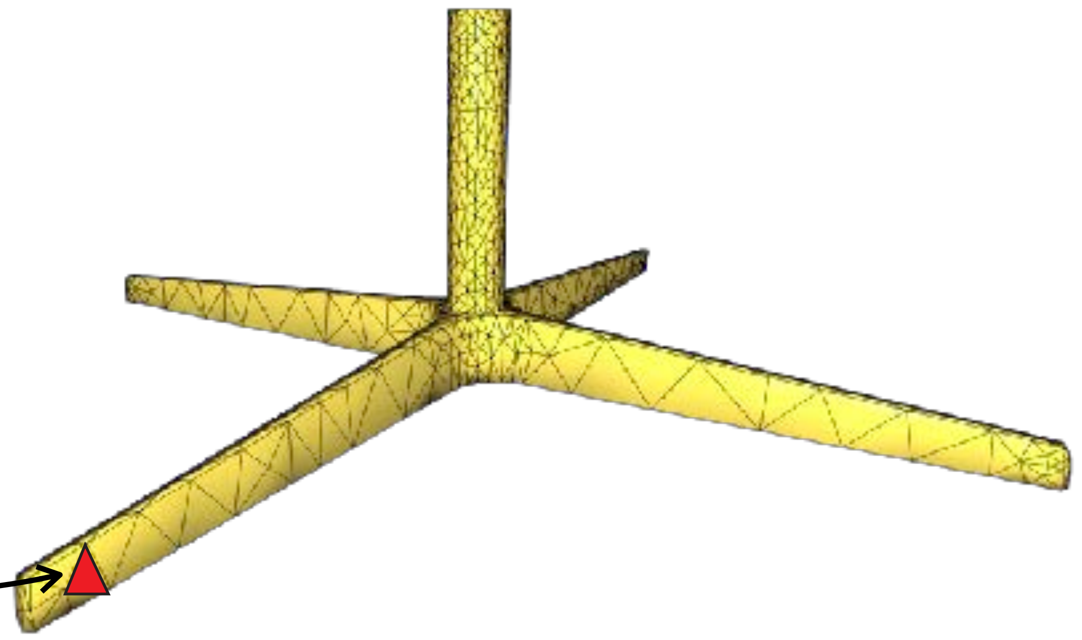
Robustness Test

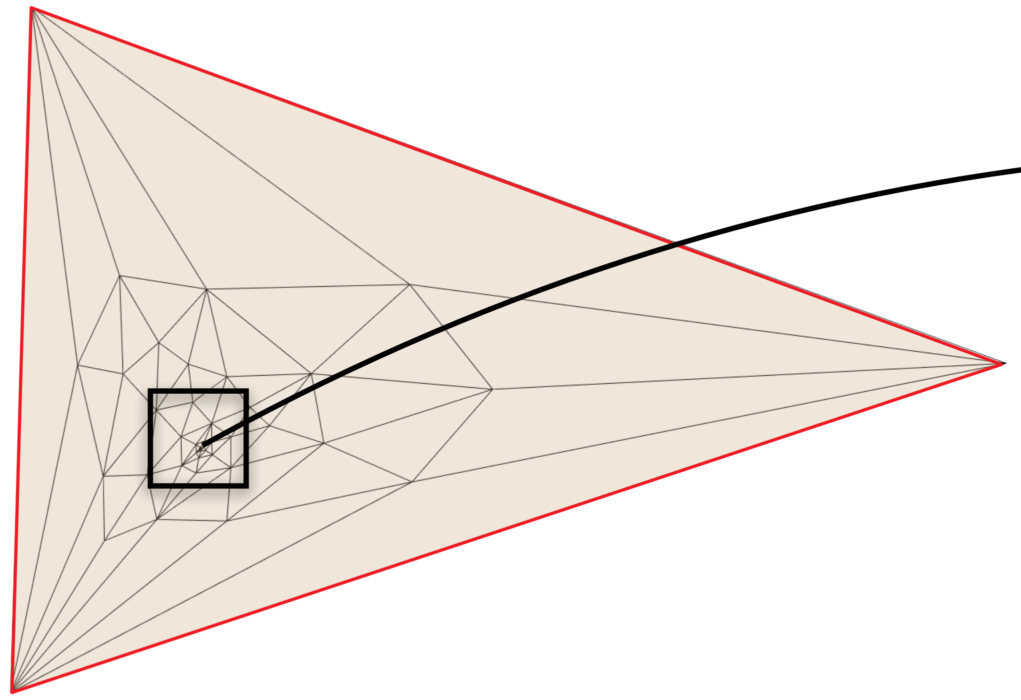
The genus 0 models from **Thingi10k** dataset [Zhou & Jacobson 2016]

Floating Point Implementation of Tutte Embedding from **libigl** [Jacobson et al. 2016]

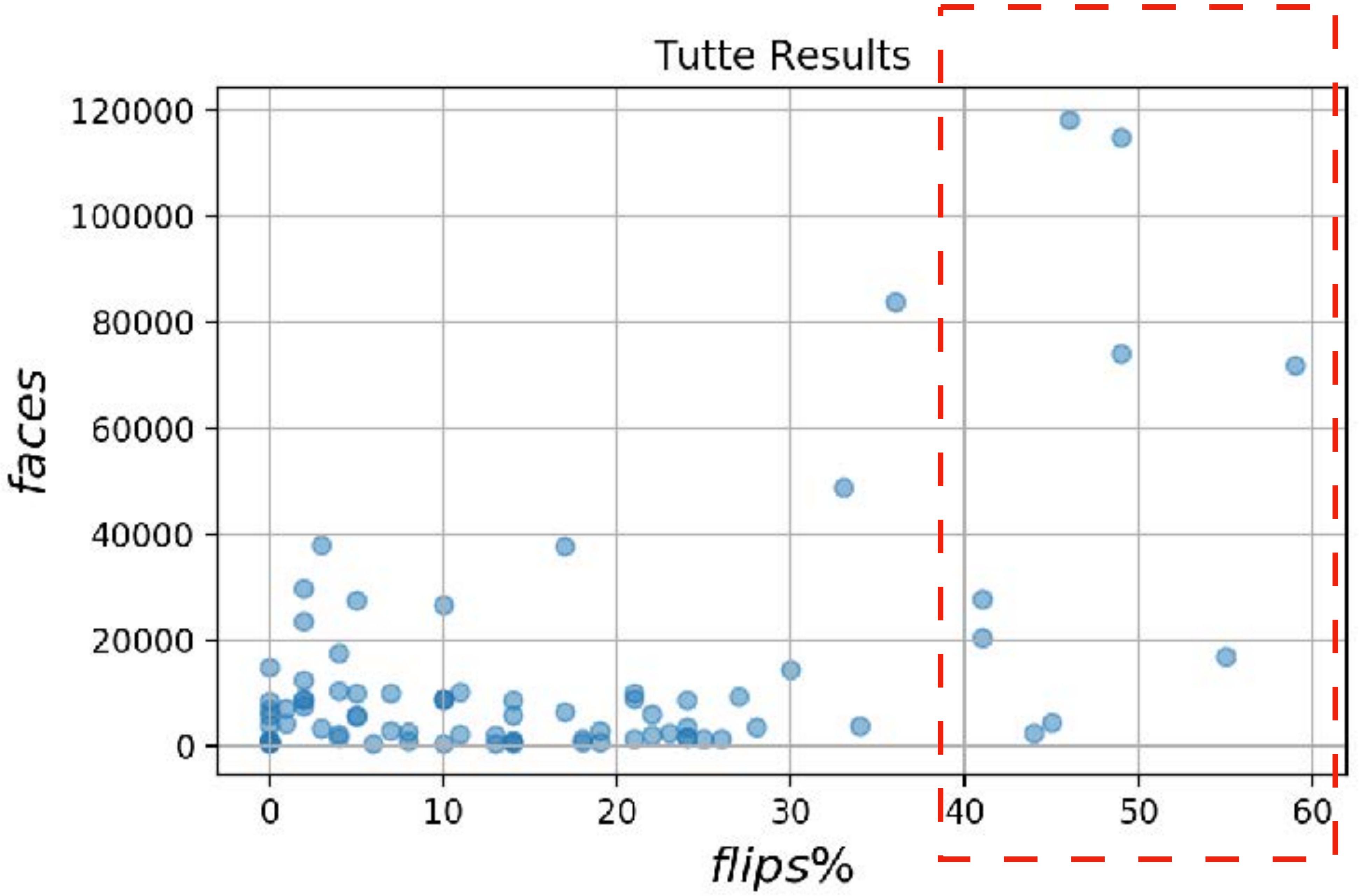
Robustness Test

Randomly drop one triangle





70 flipped elements



80 / 2718 models have flipped elements

Fixing Tutte Embedding

Obvious Ways to Try

Multi-precision

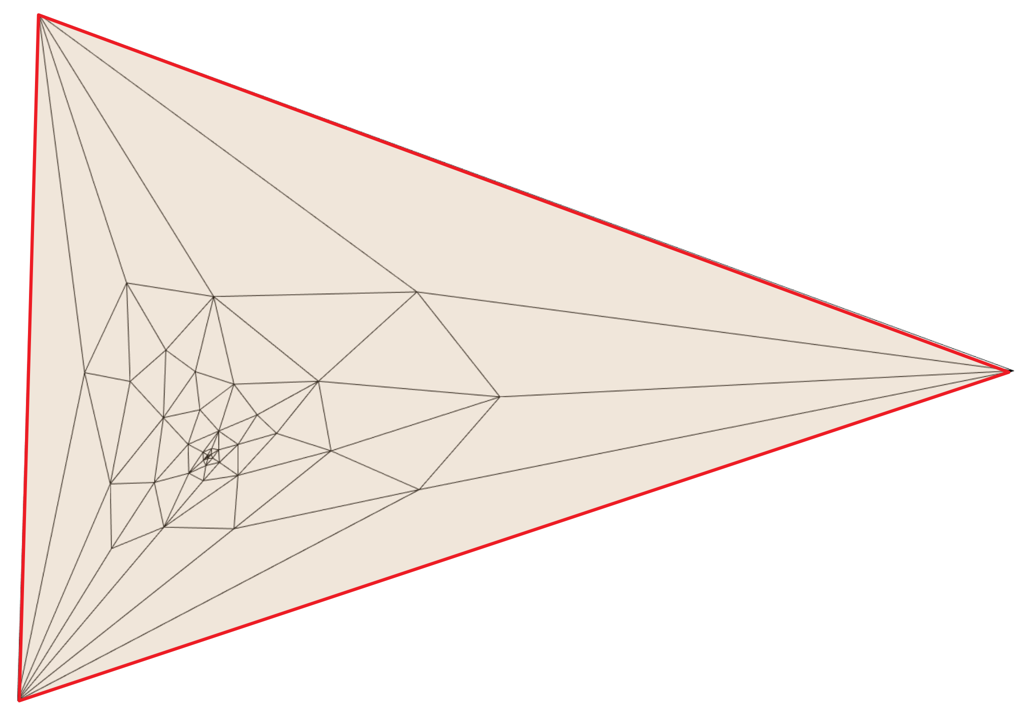
Snap Rounding

Fixing Tutte Embedding

Obvious Ways to Try: Multi-precision

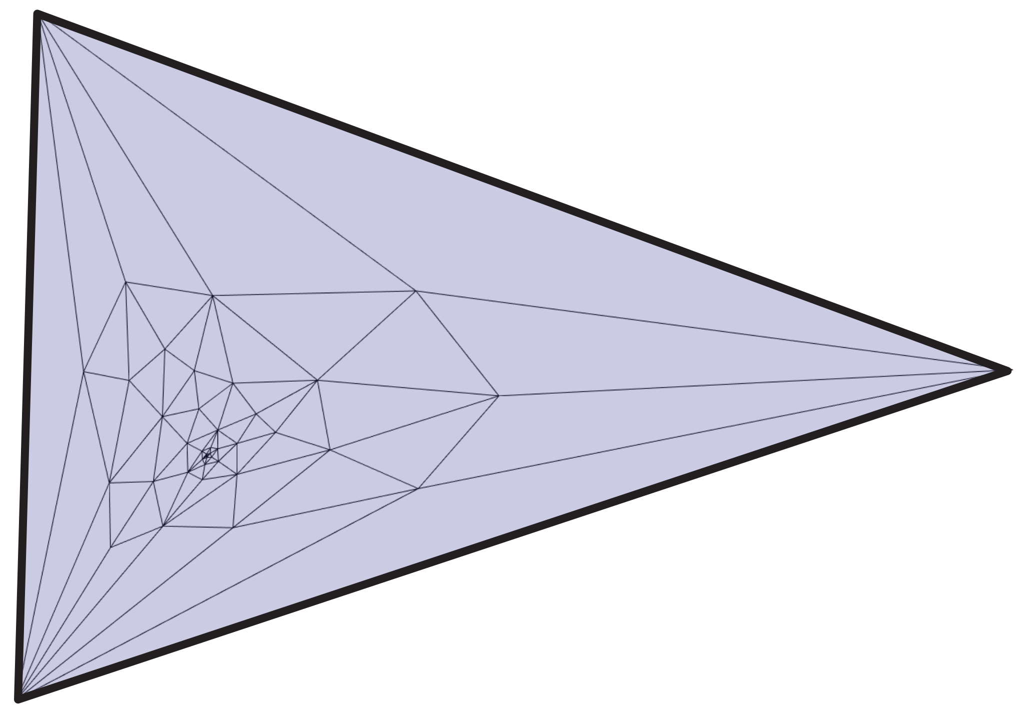
Floating Point Arithmetic

MPFR The *MPFR* library [Fousse et al. 2007]



64 bits

70 flipped elements



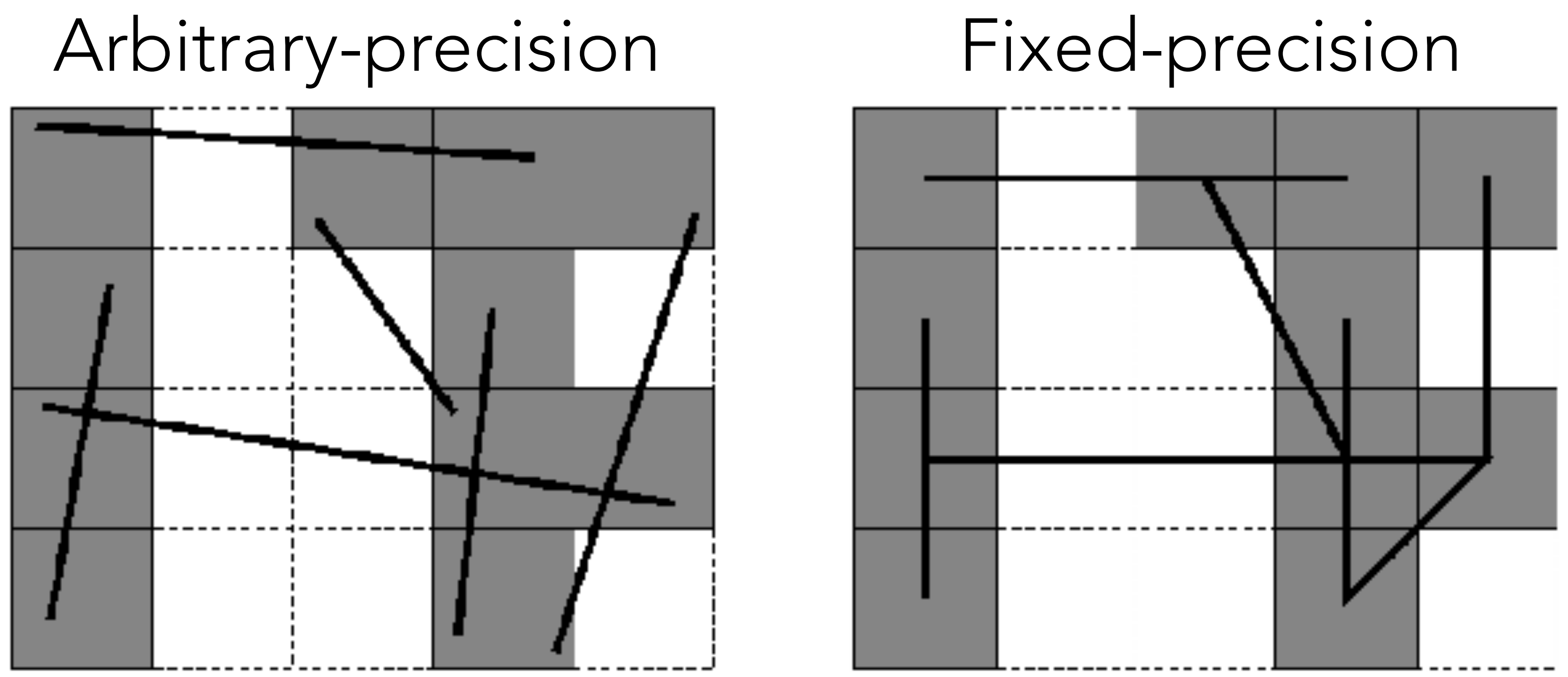
128 bits

0 flipped elements

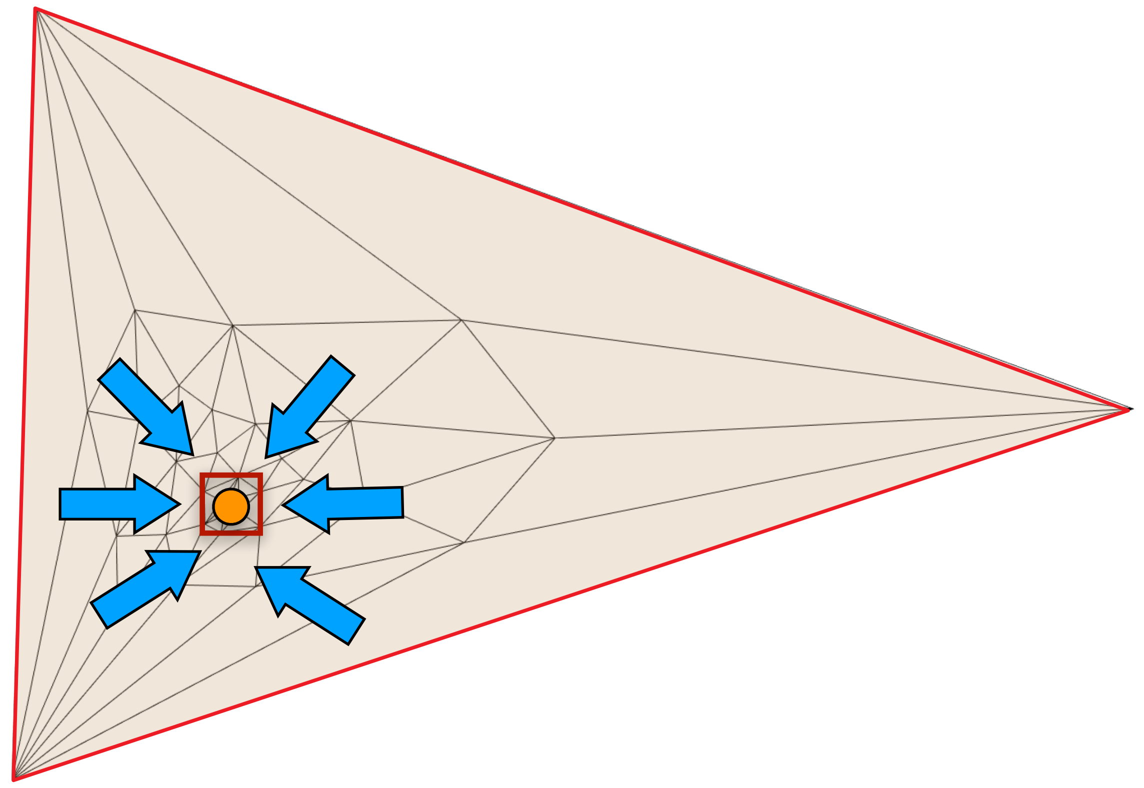
- Not usable for downstream applications.
- Naive rounding strategy may lead to flips!

Fixing Tutte Embedding

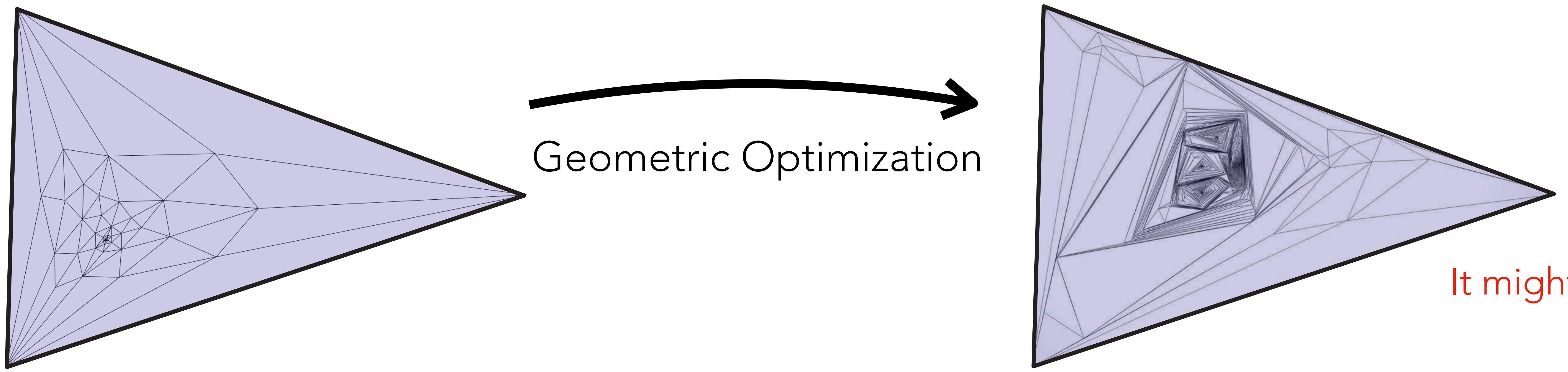
Obvious Ways to Try: Snap Rounding



Snap Rounding [Packer 2018]



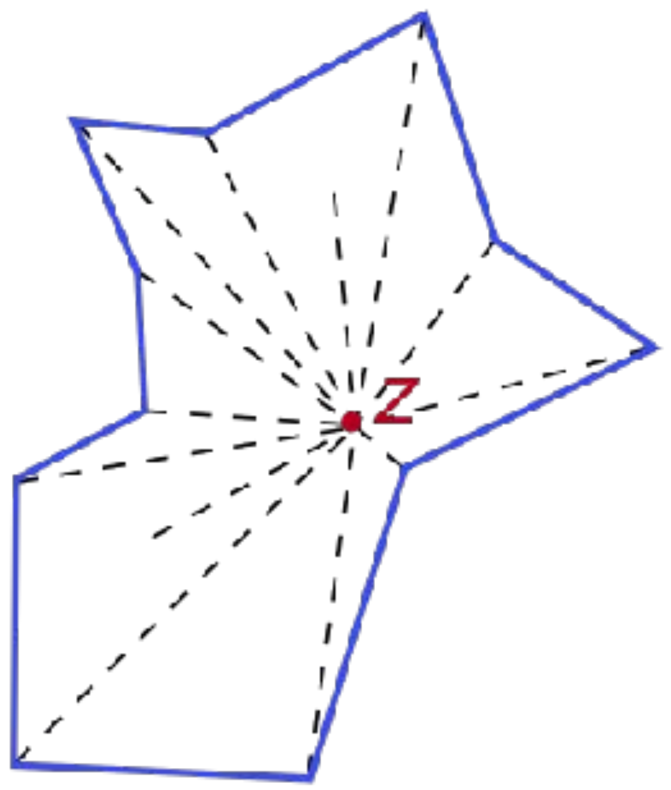
A large part of mesh is snapped to a single point!

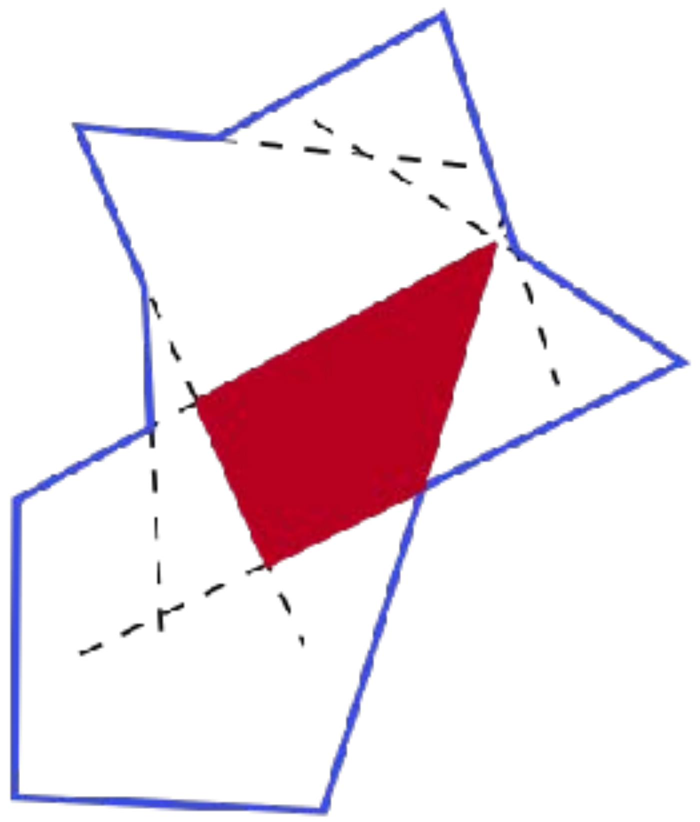


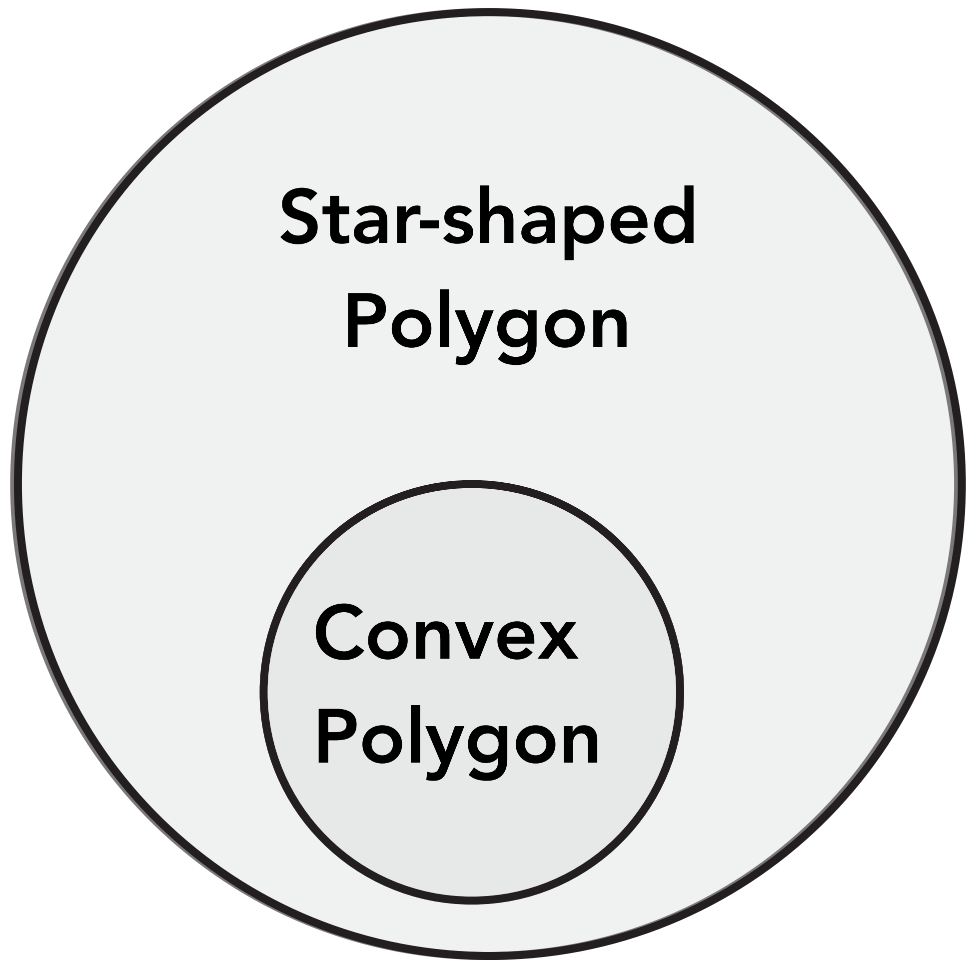
It might stuck during update!

Problem Formulation

Given a possibly invalid embedding with convex boundary for a disk-like surface mesh as input, how do we generate an inversion-free embedding using floating point coordinates?



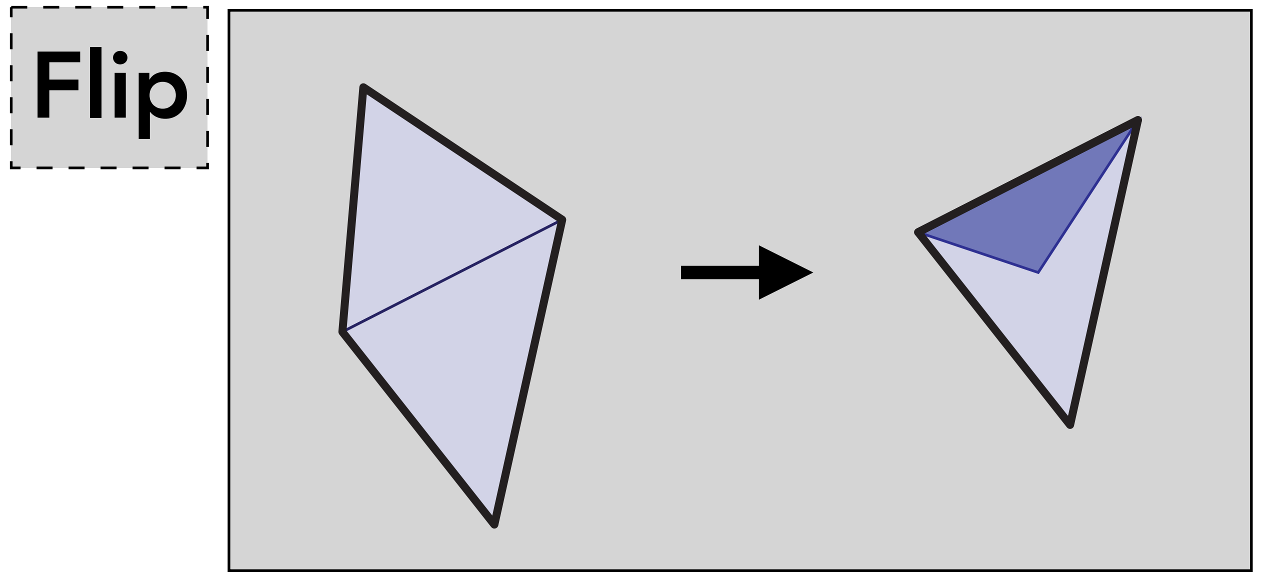




Invalid Element

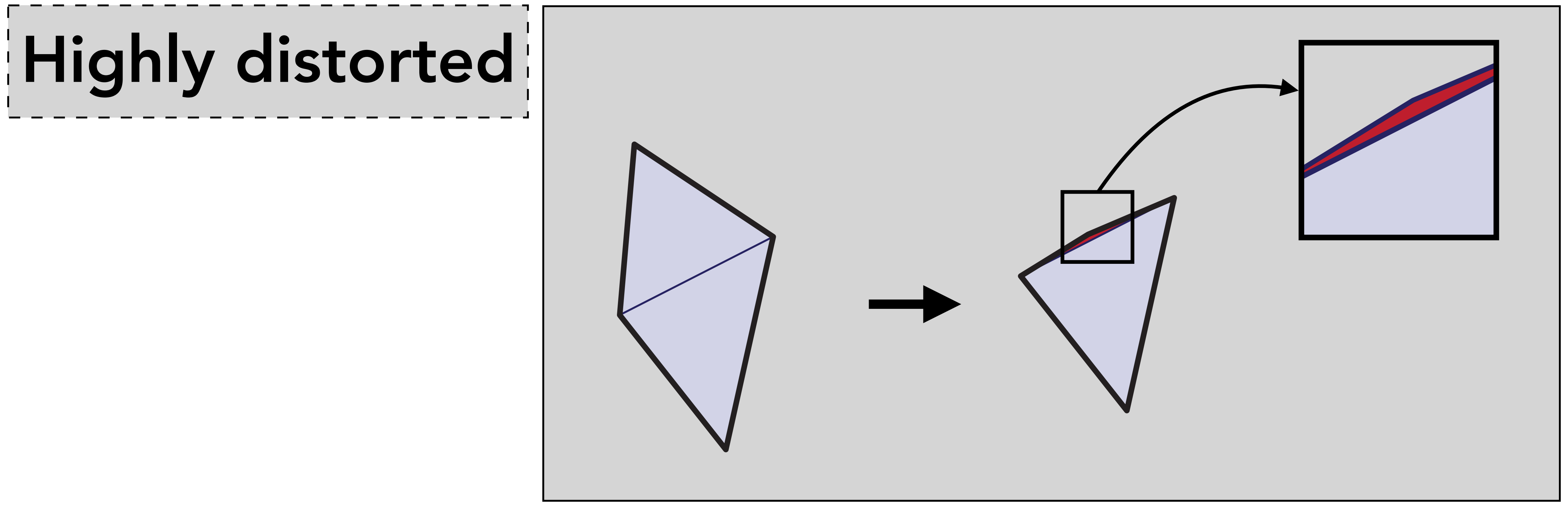
A triangle is said to be invalid if

1. Its signed area is negative



* Exact floating point predicates from CGAL [Brönnimann et al. 2018]

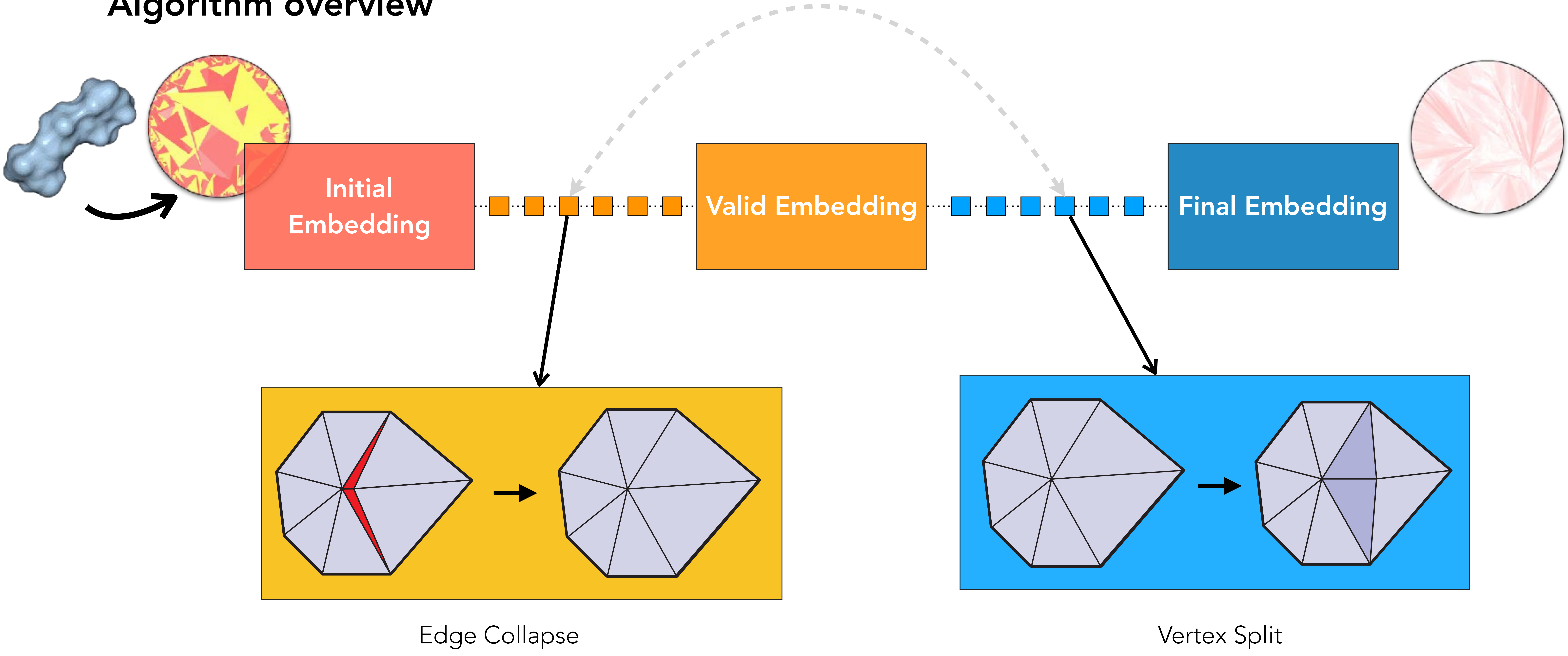
2. Its quality measure is below a threshold



* Symmetric Dirichlet Energy [Smith & Schaefer 2015]

Progressive Embedding

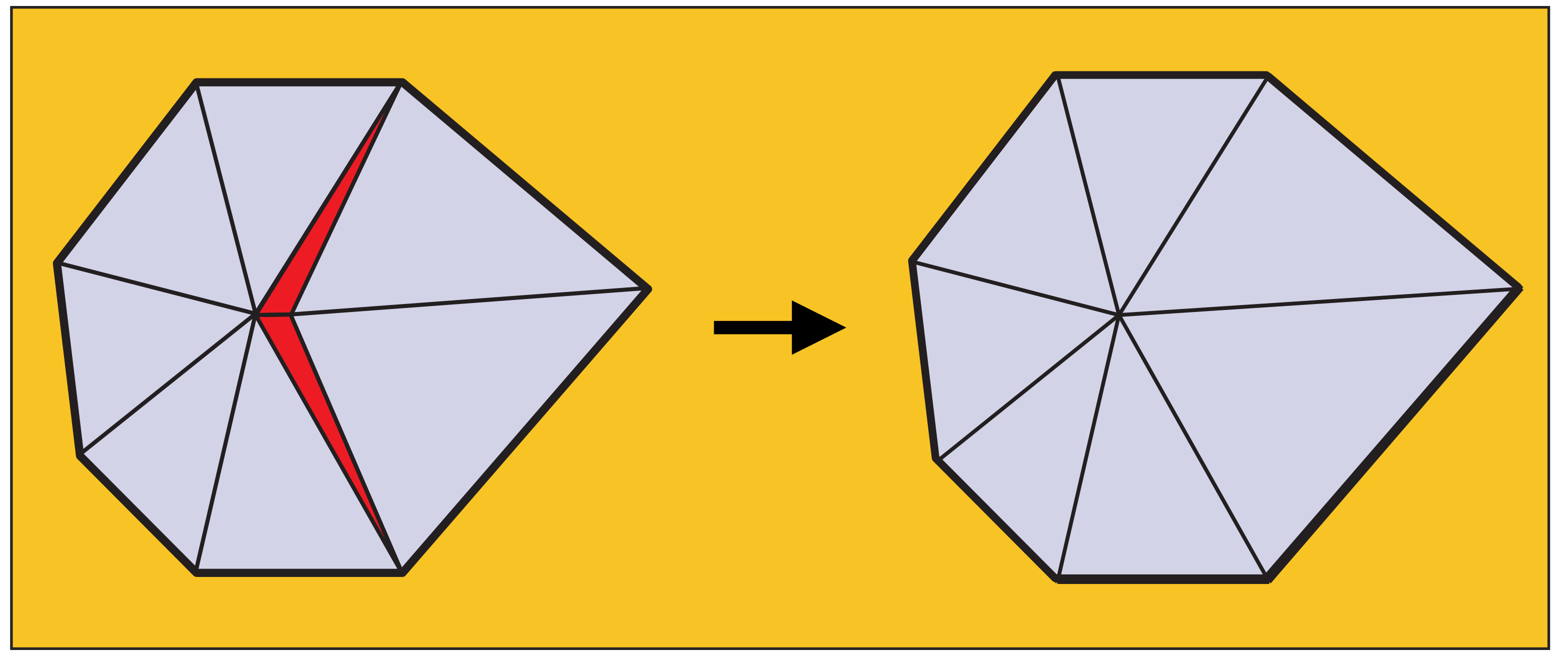
Algorithm overview



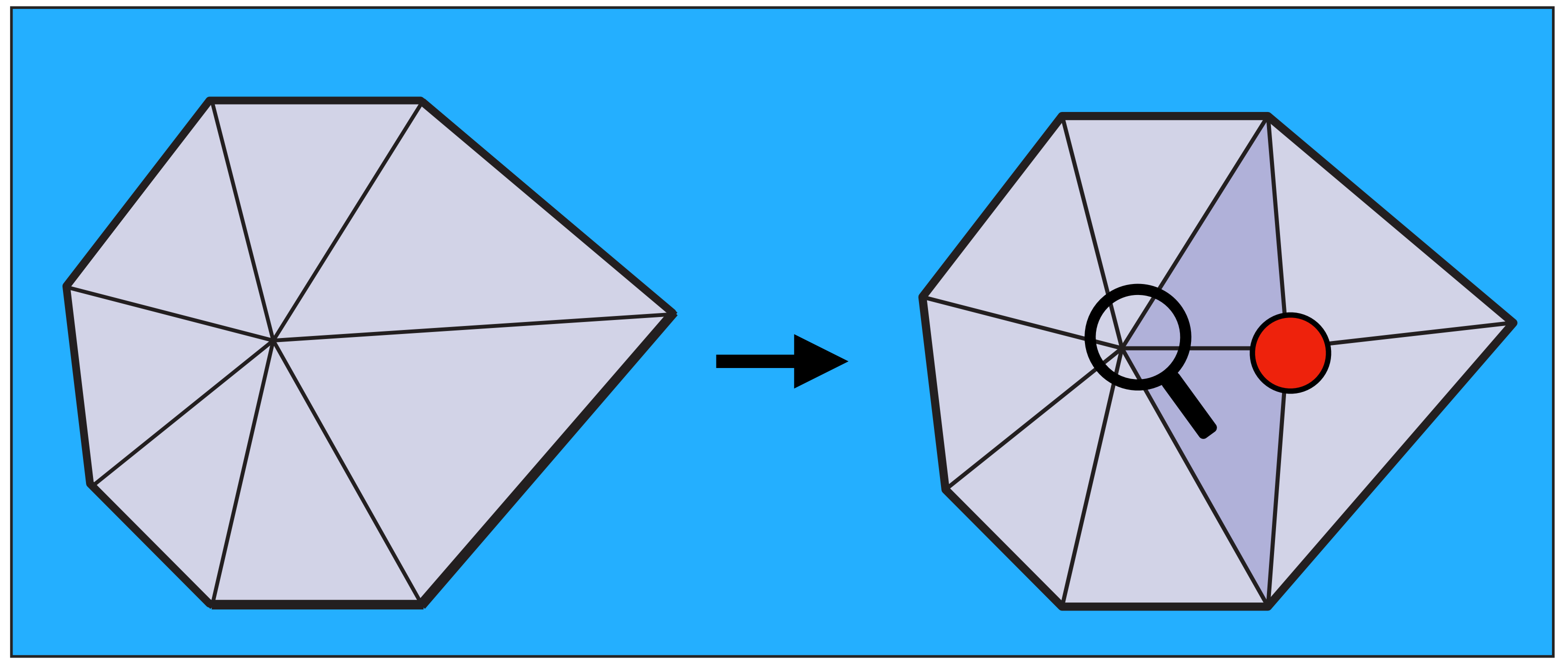
Basic Operations

Progressive Meshes
[Hoppe 1996]

- **Edge Collapse**: Takes two connected vertices and replaces them with a single vertex.

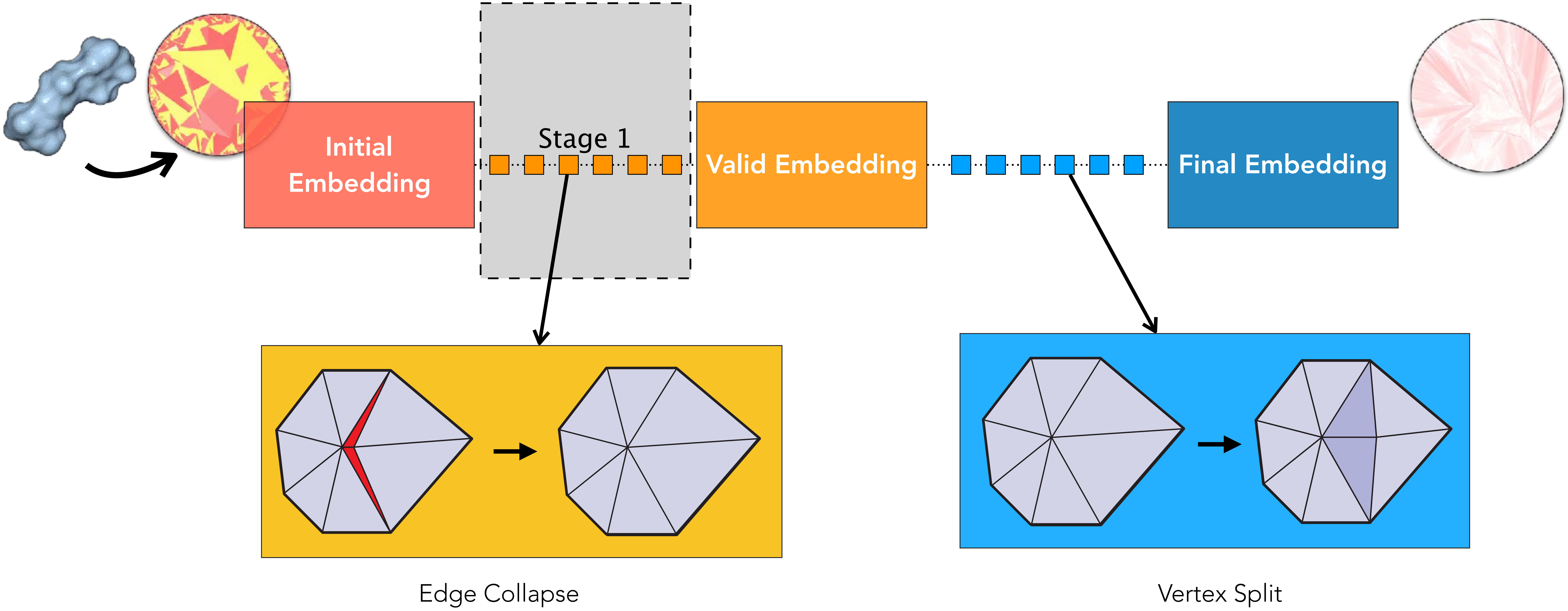


- **Vertex Split**: Divides the vertex into two new vertices, creates two new triangles



Progressive Embedding

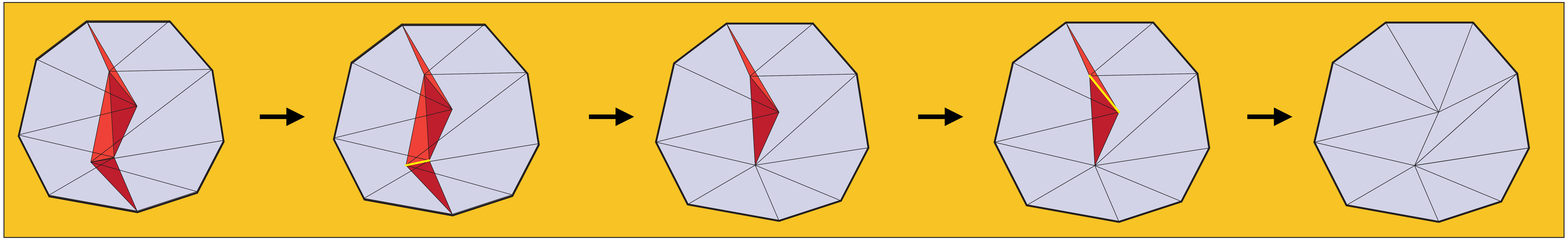
Algorithm overview



Stage 1: Collapse

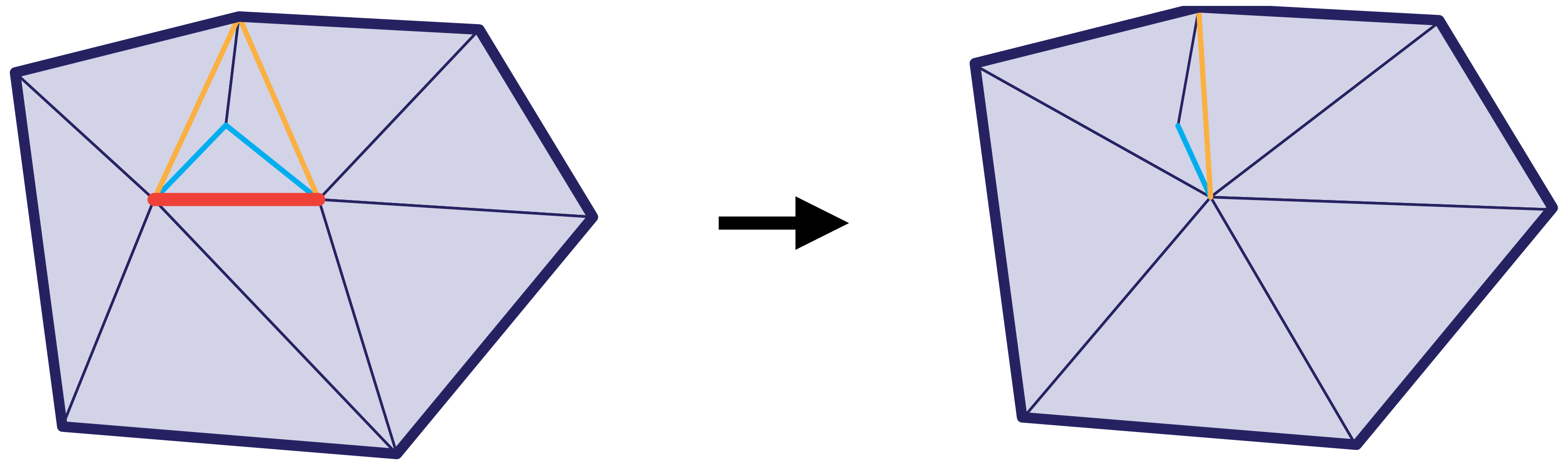
Initial embedding, with invalid elements

No invalid elements are left



An edge collapse operation is illegal if the edge violates link condition

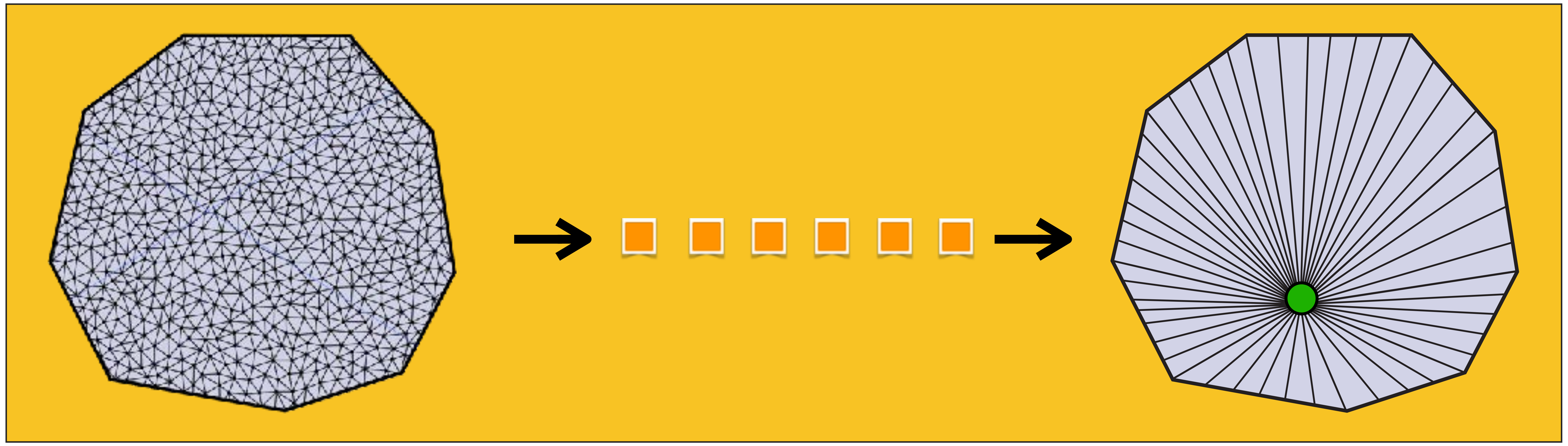
Non-manifold!



* Iteratively collapse edges
* Stop when no invalid elements remain

Existence of Collapsing Sequence

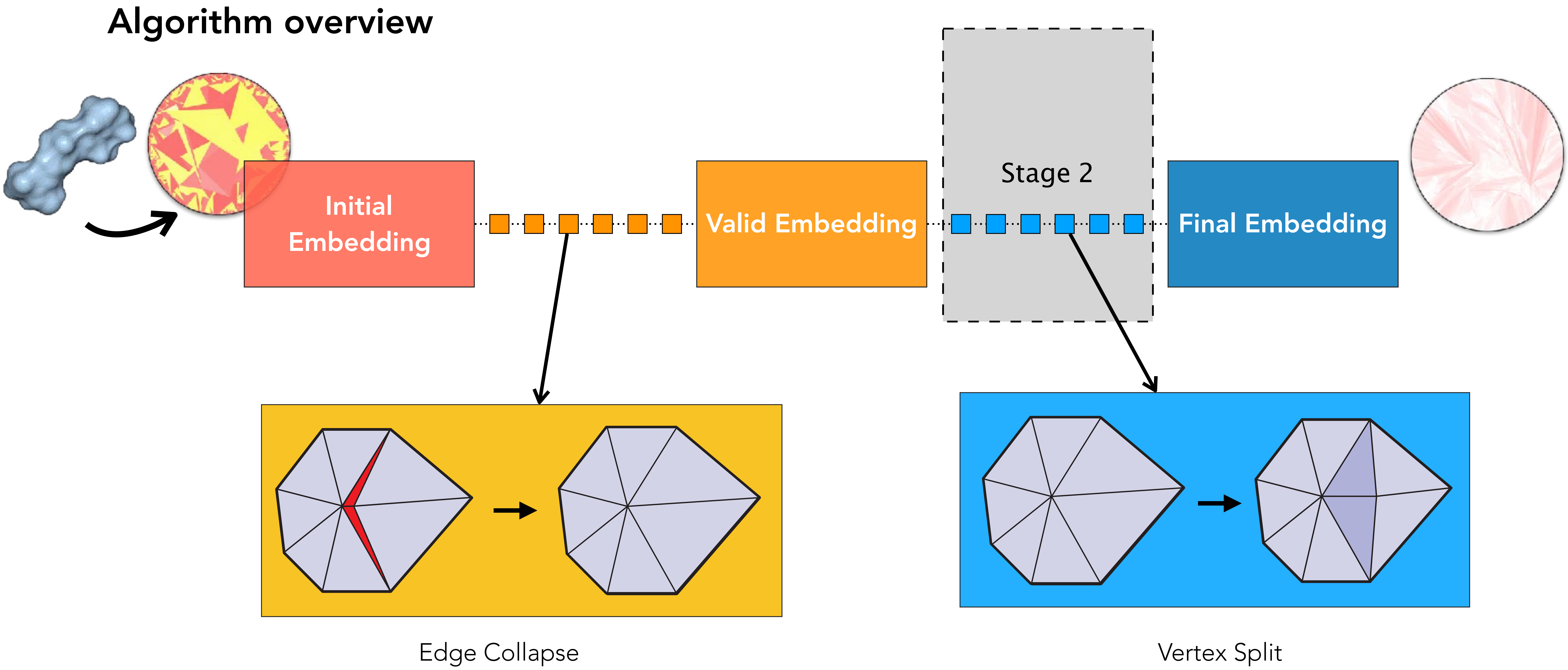
Theorem: We can always find an edge to collapse until only one interior vertex left.



0 flips!

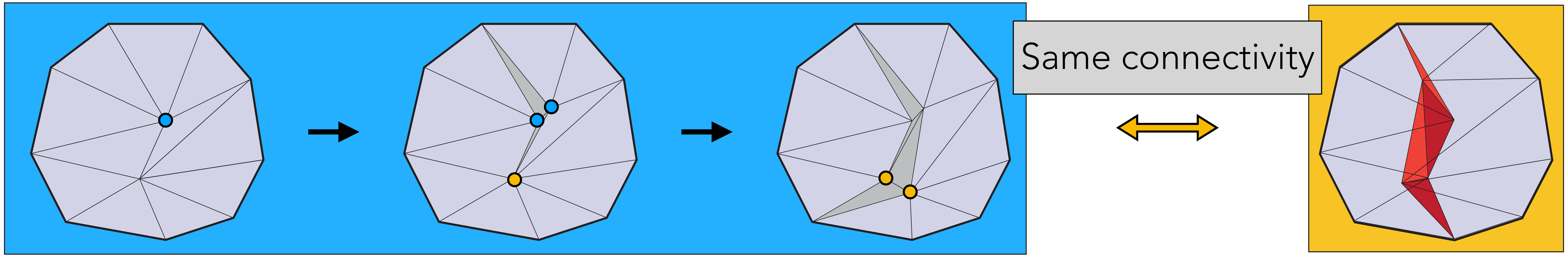
(For a planar and 3-connected mesh)

Progressive Embedding



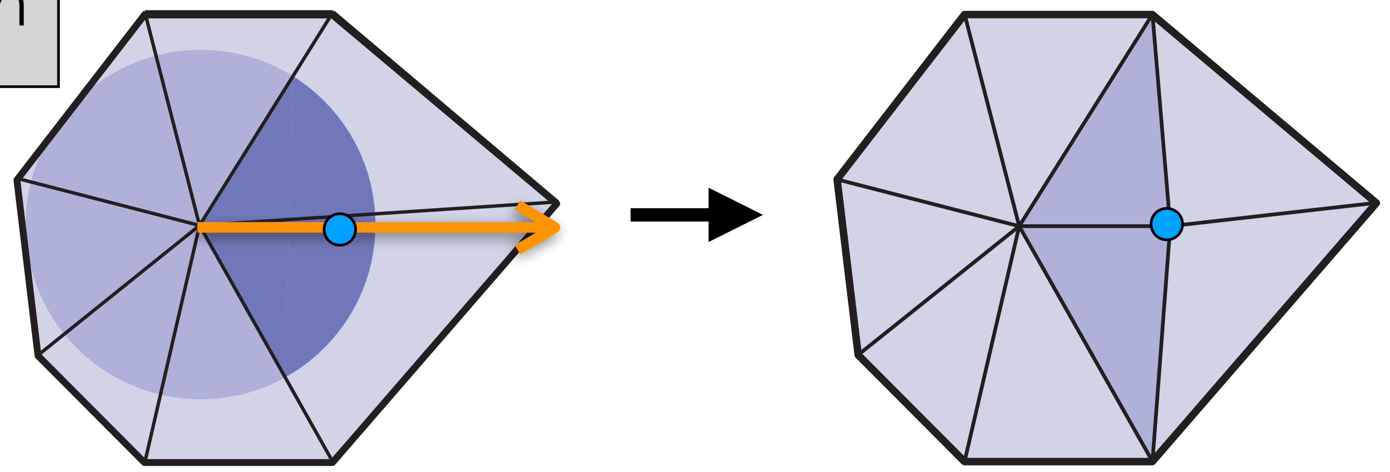
Stage 2: Insertion

Result of stage 1



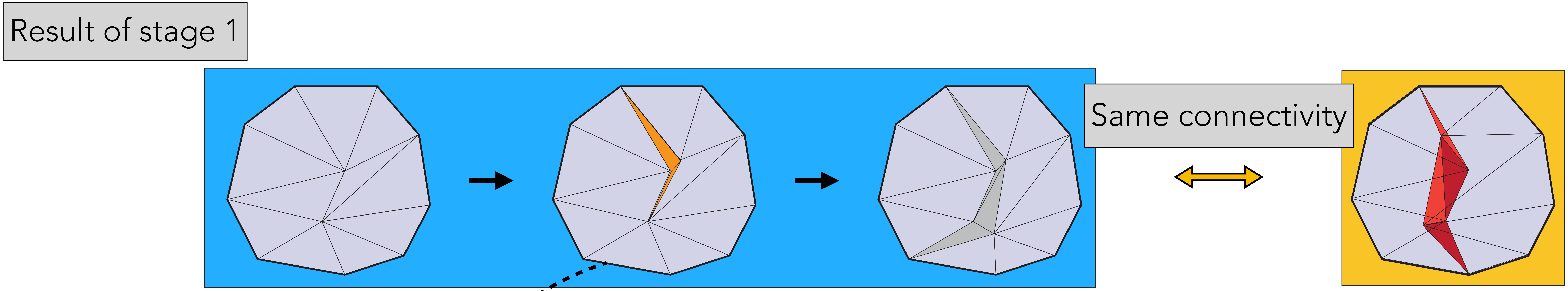
Theorem: A vertex split reversing any collapse can always be done.

Standard Line Search

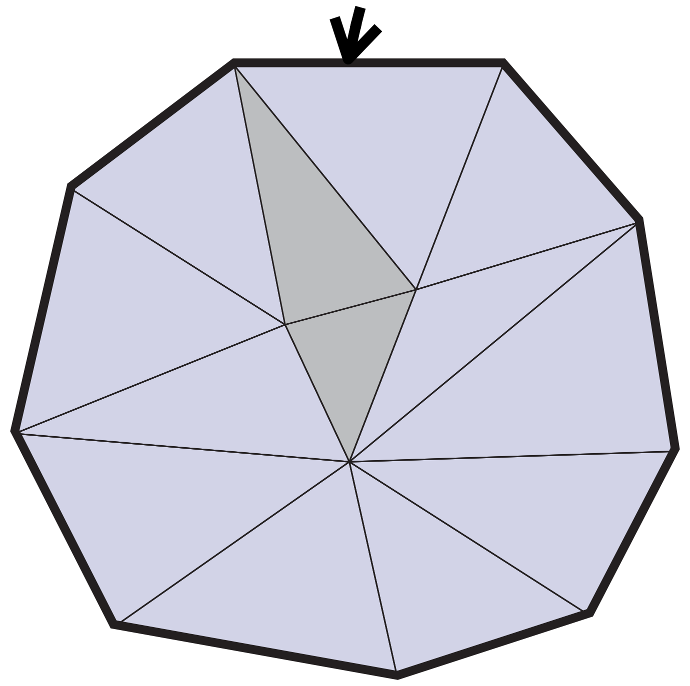


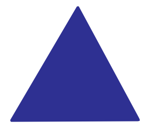
For implementation simplicity: we consider the inscribed cycle of the 1-ring neighbor of vertex to be split as valid region.

Stage 2: Insertion



Local Smoothing: improve quality after each insertion



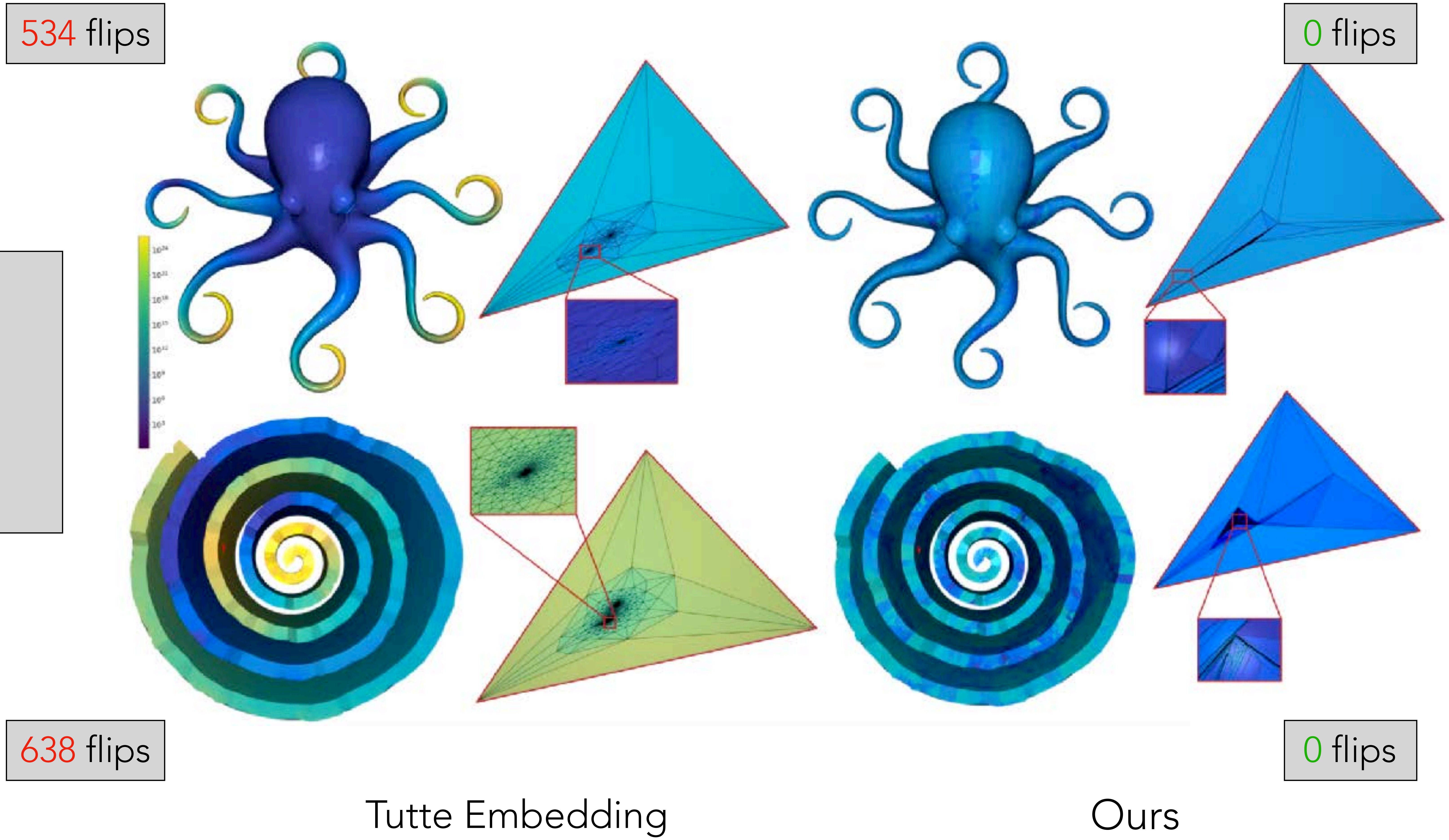


Reference Shape: Equilateral triangle with average area of the whole mesh in the parametric domain

- Symmetric Dirichlet Energy [Smith & Schaefer 2015]
- One vertex at a time using Newton iterations [Fu et al. 2015; Hormann & Greiner 2000; Labsik et al. 2000]
- Updated in parallel independently.

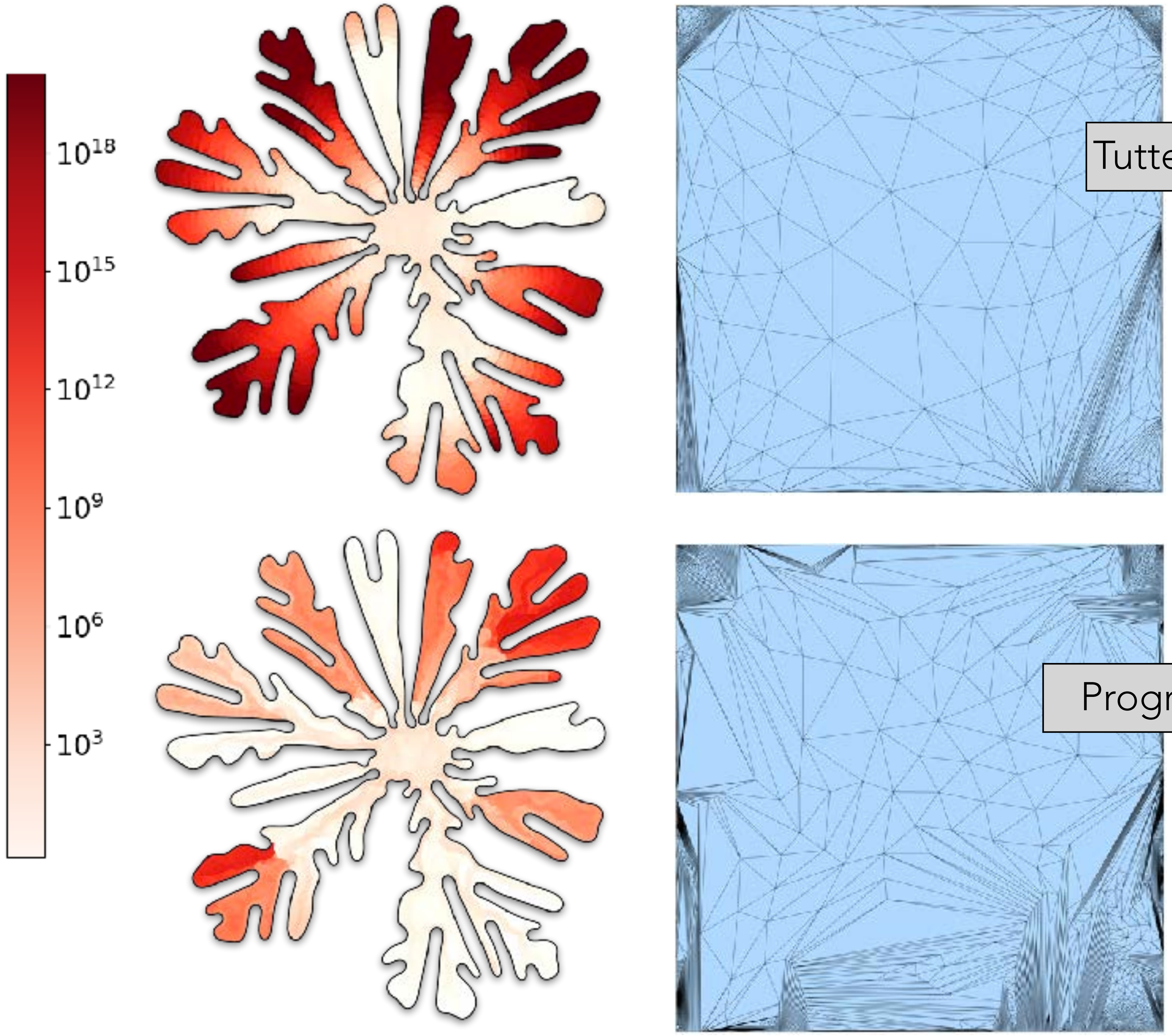
Robustness Test

0 / 2718 model have flipped elements



Results

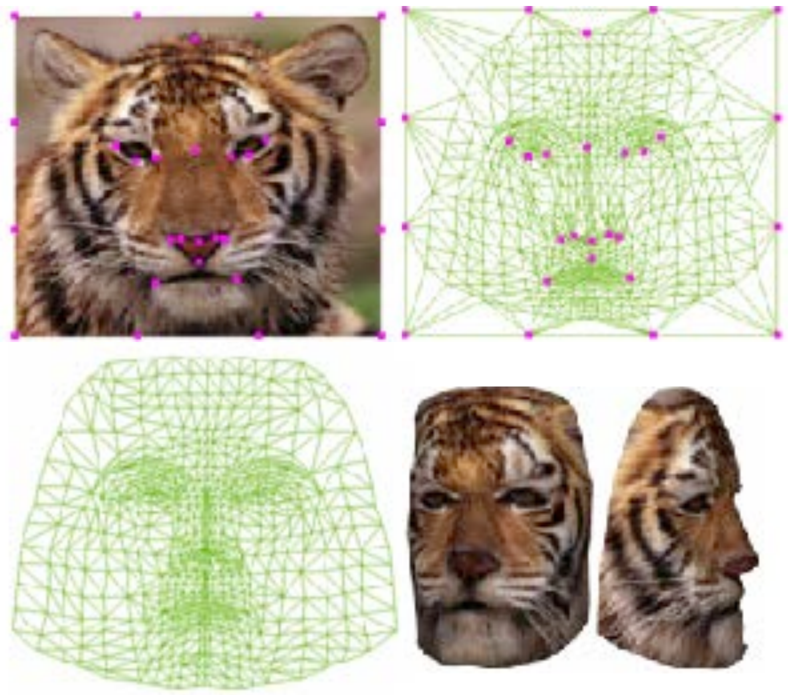
Mapping triangulated Hele-Shaw polygon [Segall et al. 2016] to the interior of a square



Tutte Embedding: 46 flips!

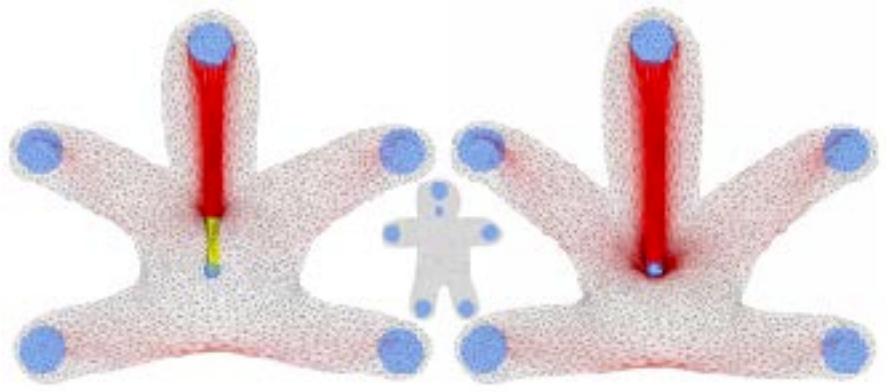
Progressive Embedding: 0 flips

Distortion-Minimizing Mappings



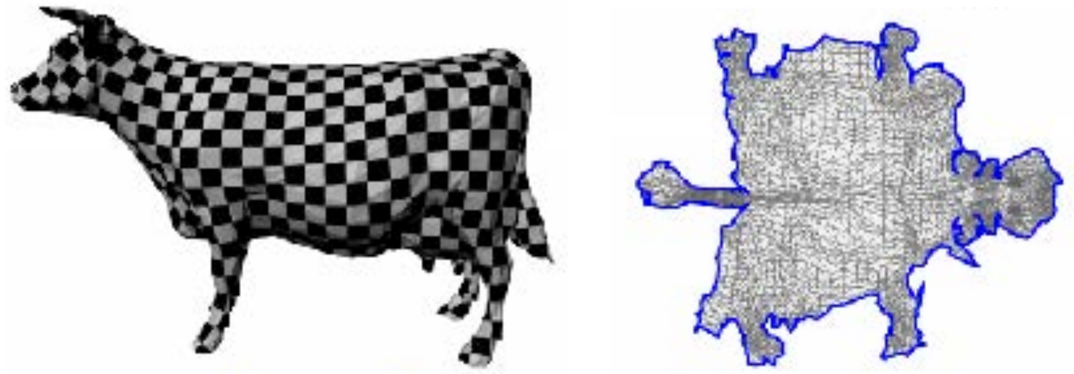
[Kraevoy et al. 2003]

[Rabinovich et al. 2016]

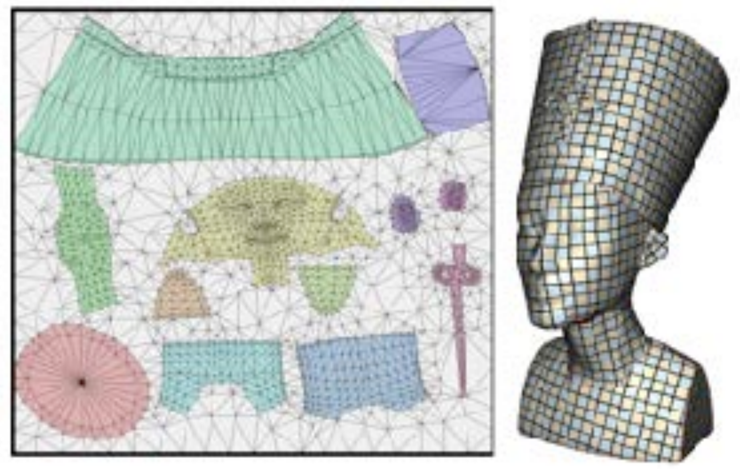


[Schüller et al. 2013]

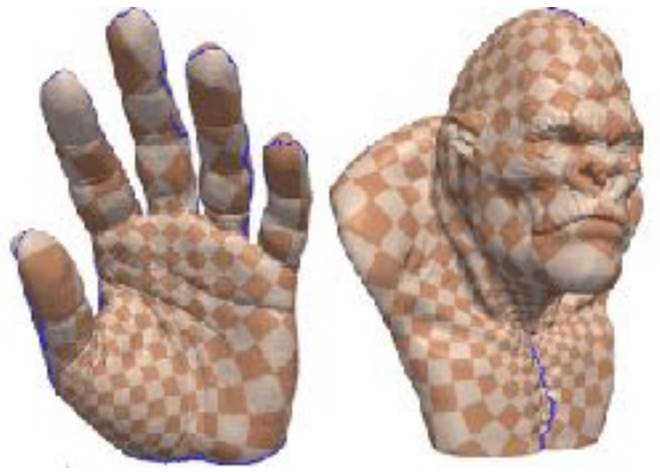
[Li et al. 2018]



[Smith and Schaefer 2015]

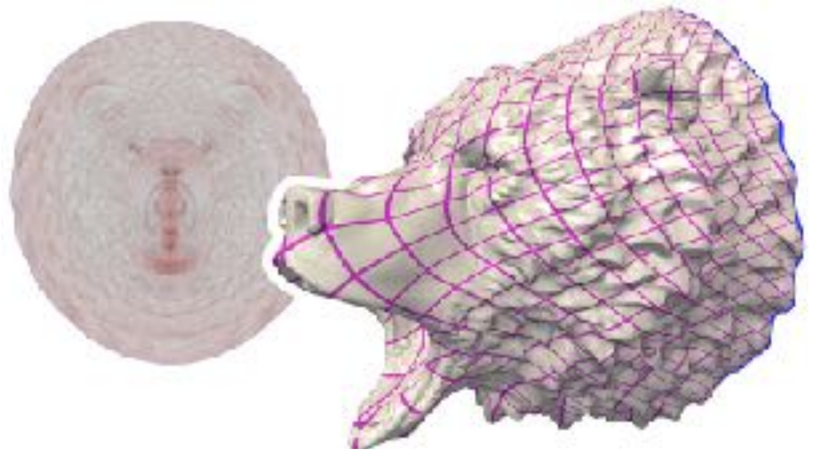


[Jiang et al. 2017]

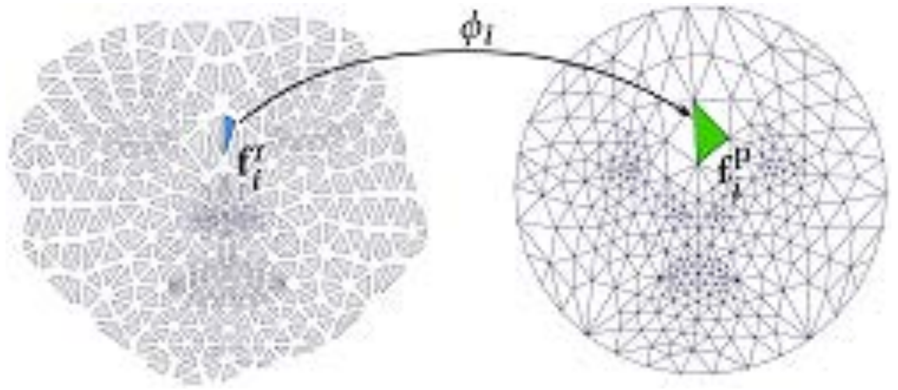


[Kovalsky et al. 2016]

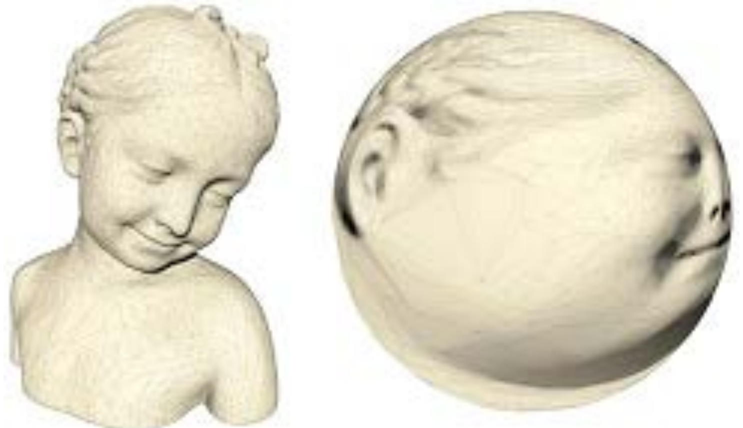
[Claici et al. 2017]



[Shtengel et al. 2017]



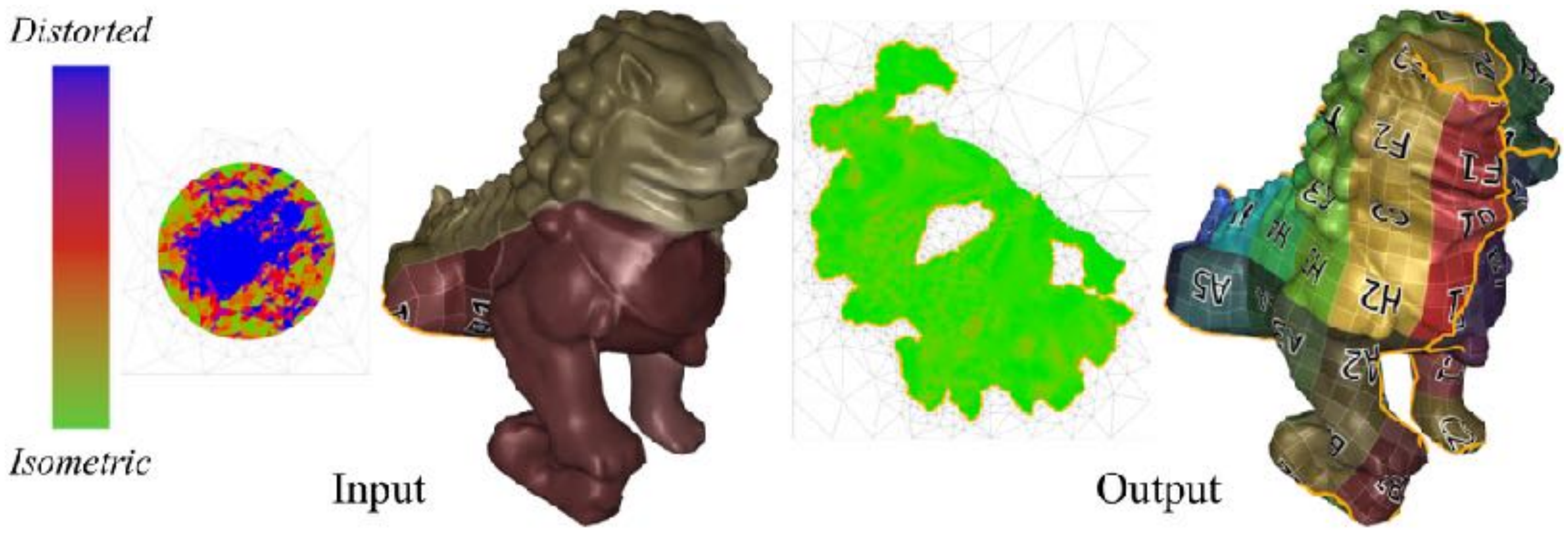
[Liu et al. 2018]



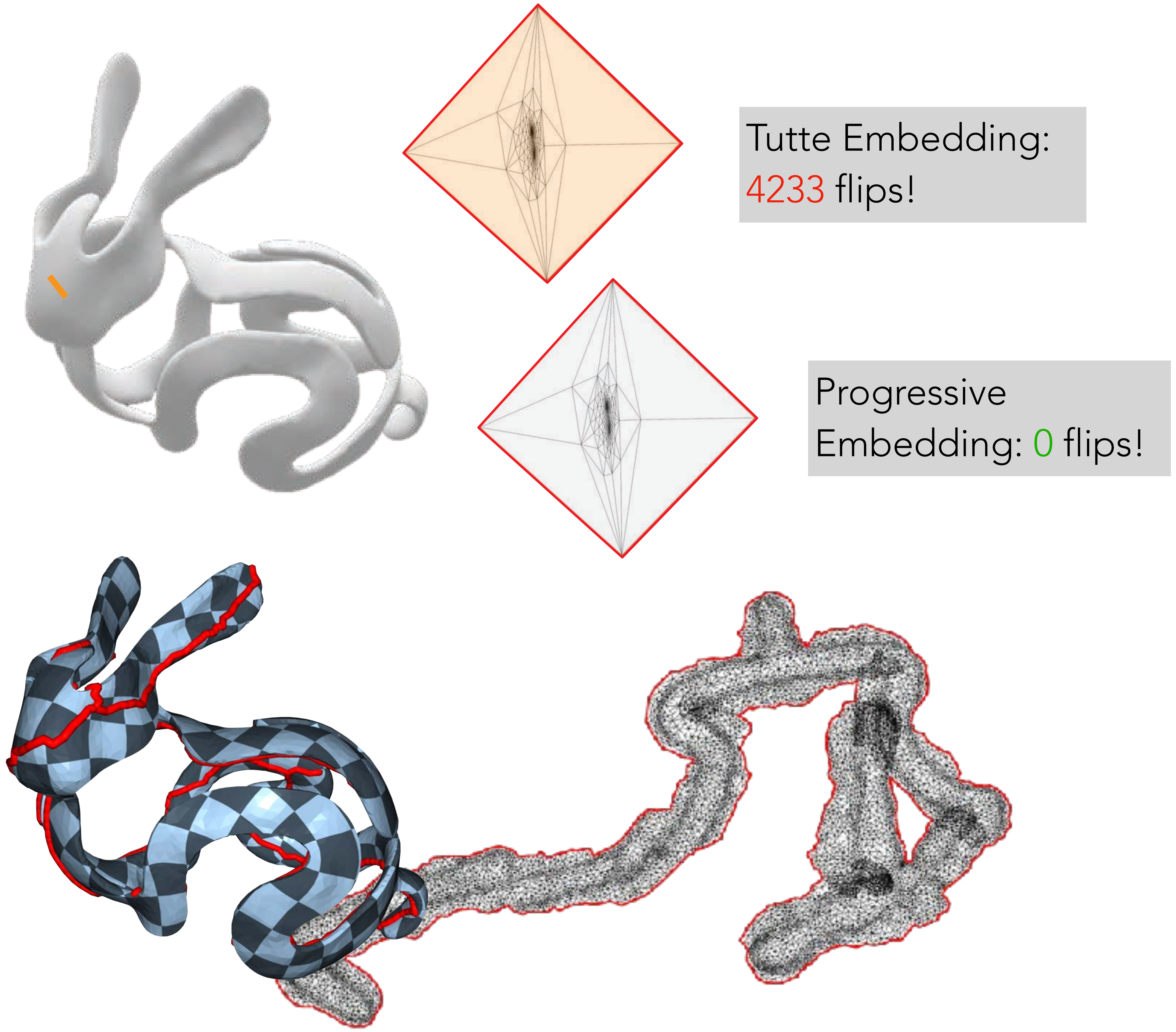
[Wang et al. 2016]

[Fu et al. 2015]

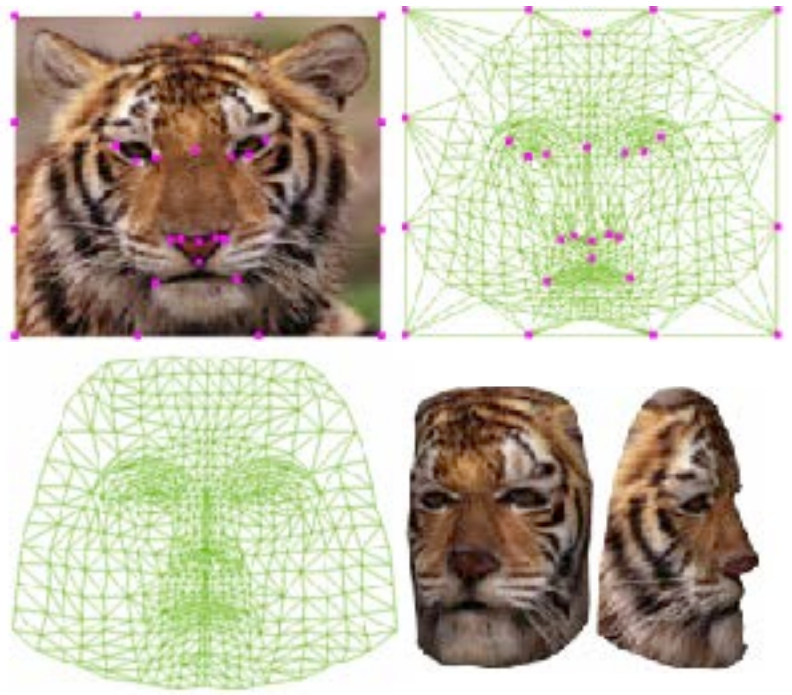
OptCuts



OptCuts [Li et al. 2018] Joint Optimization of Surface Cuts and Parameterization



Distortion-Minimizing Mappings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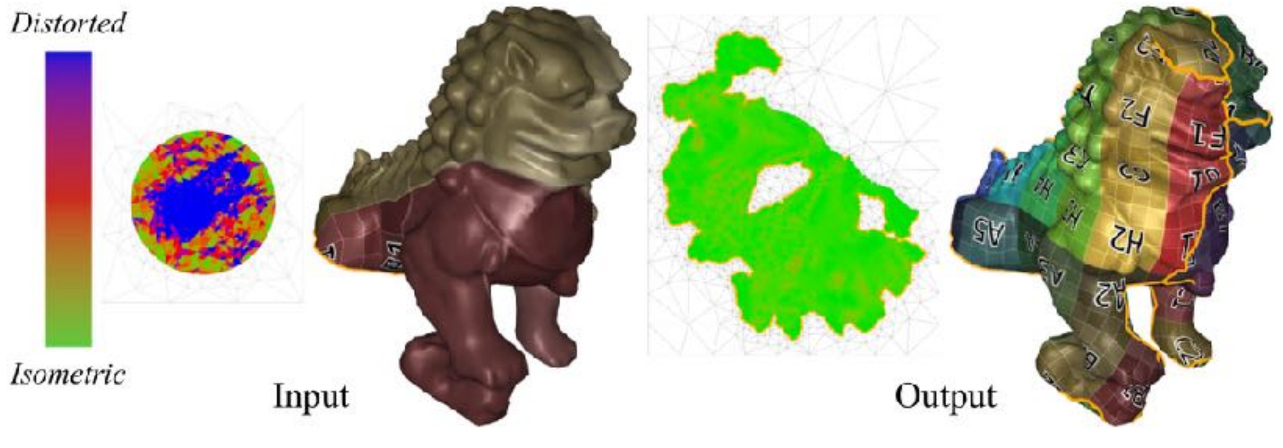

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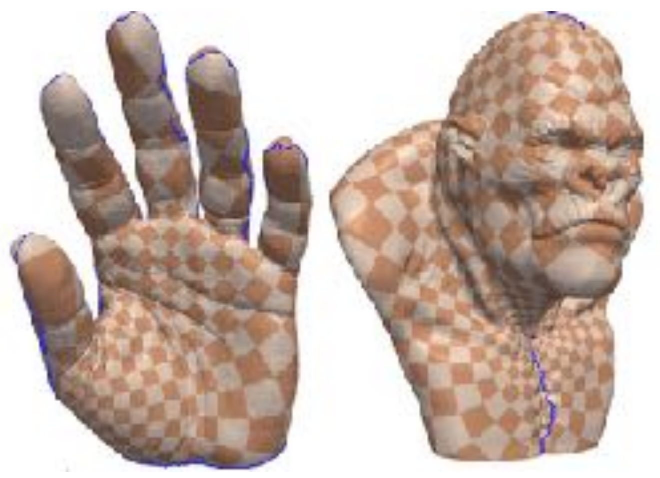
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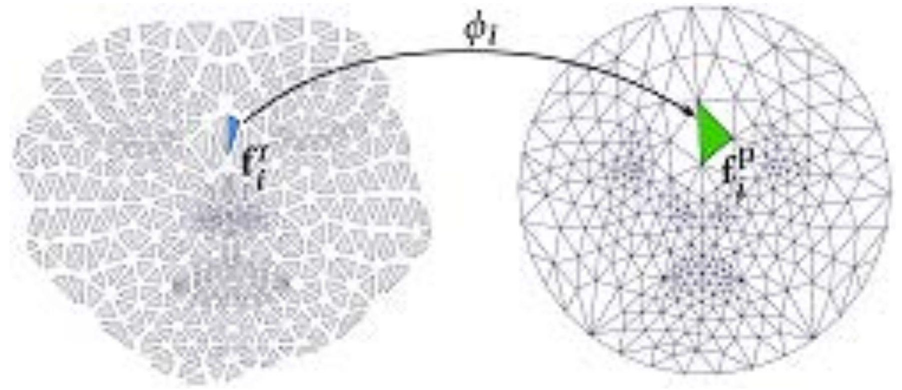
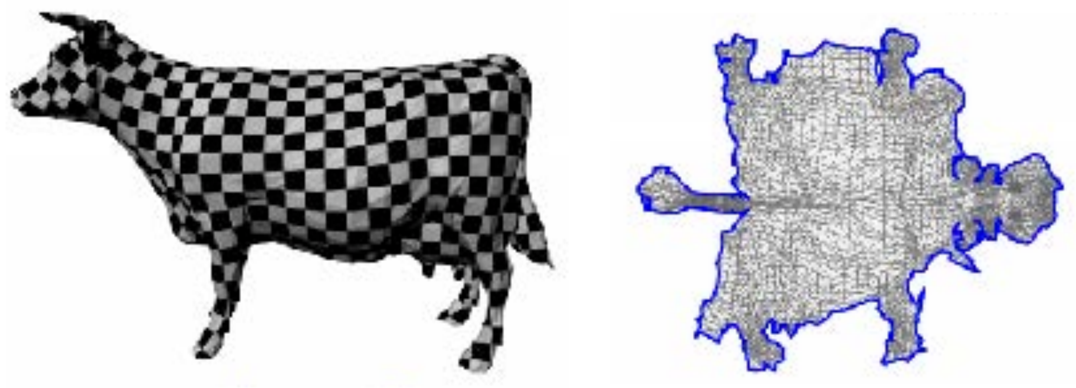

[Liu et al. 2018]


[Rabinovich et al. 2016]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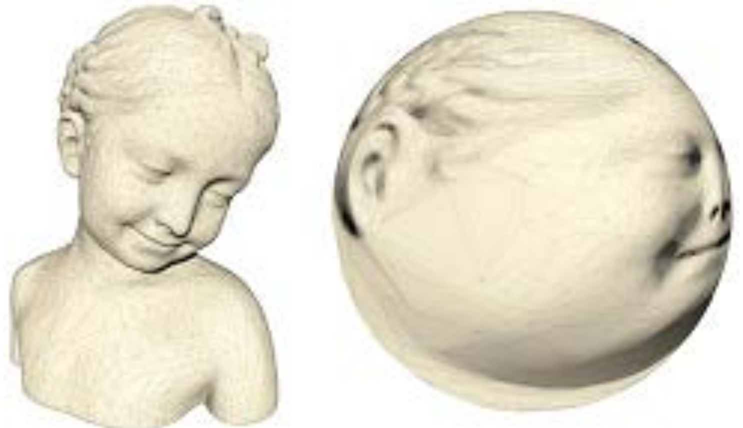
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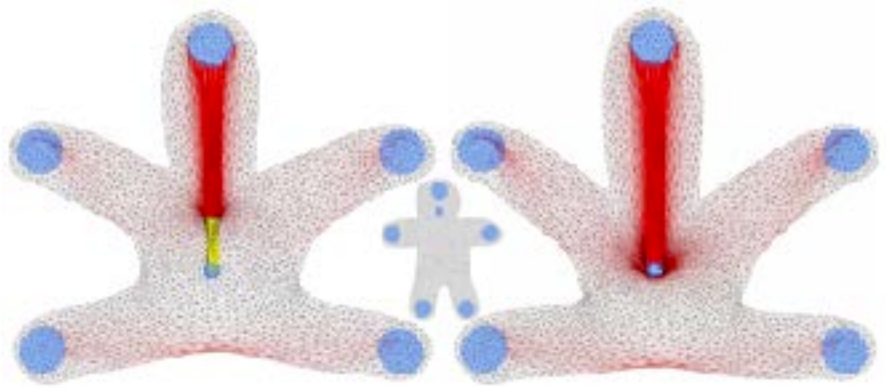
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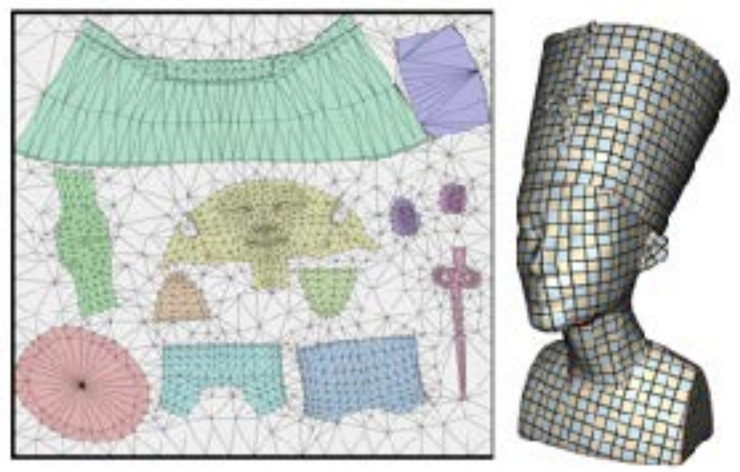
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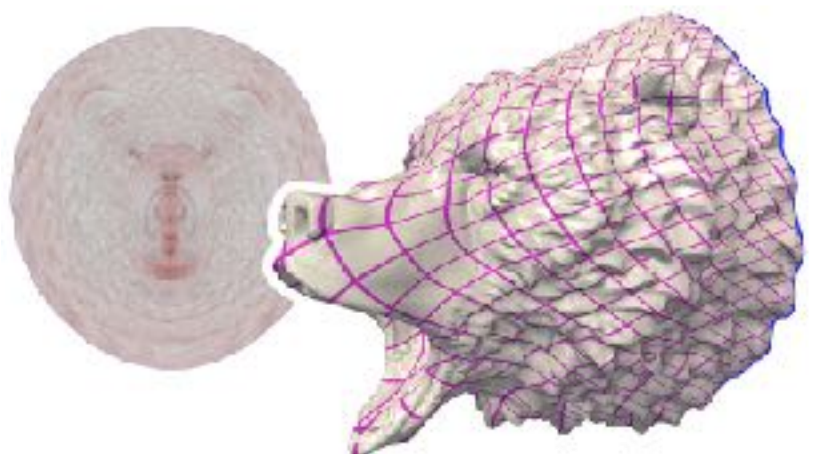
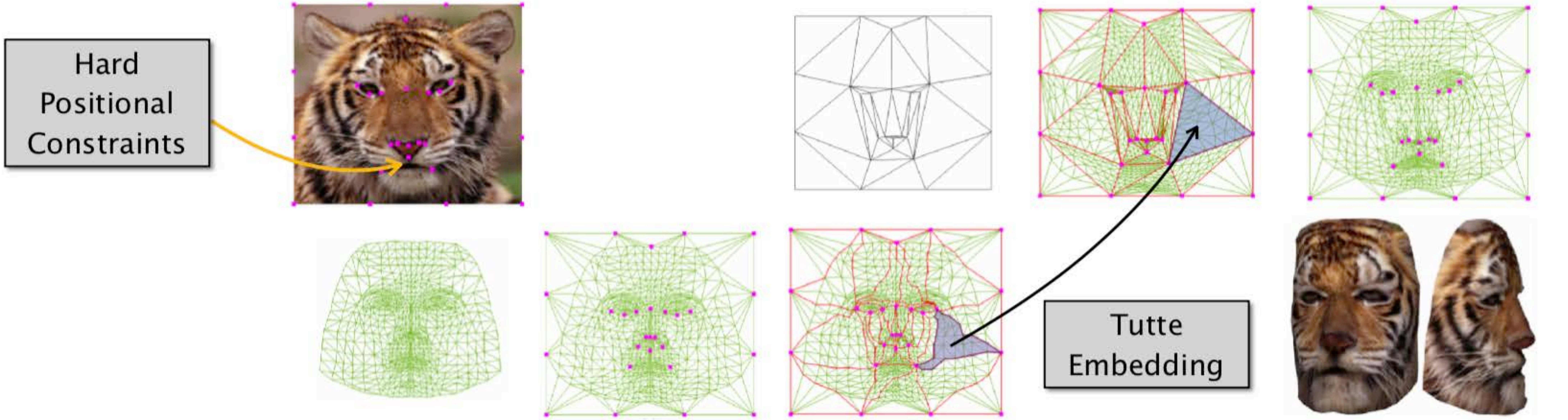

[Shtengel et al. 2017]


[Fu et al. 2015]

Matchmaker

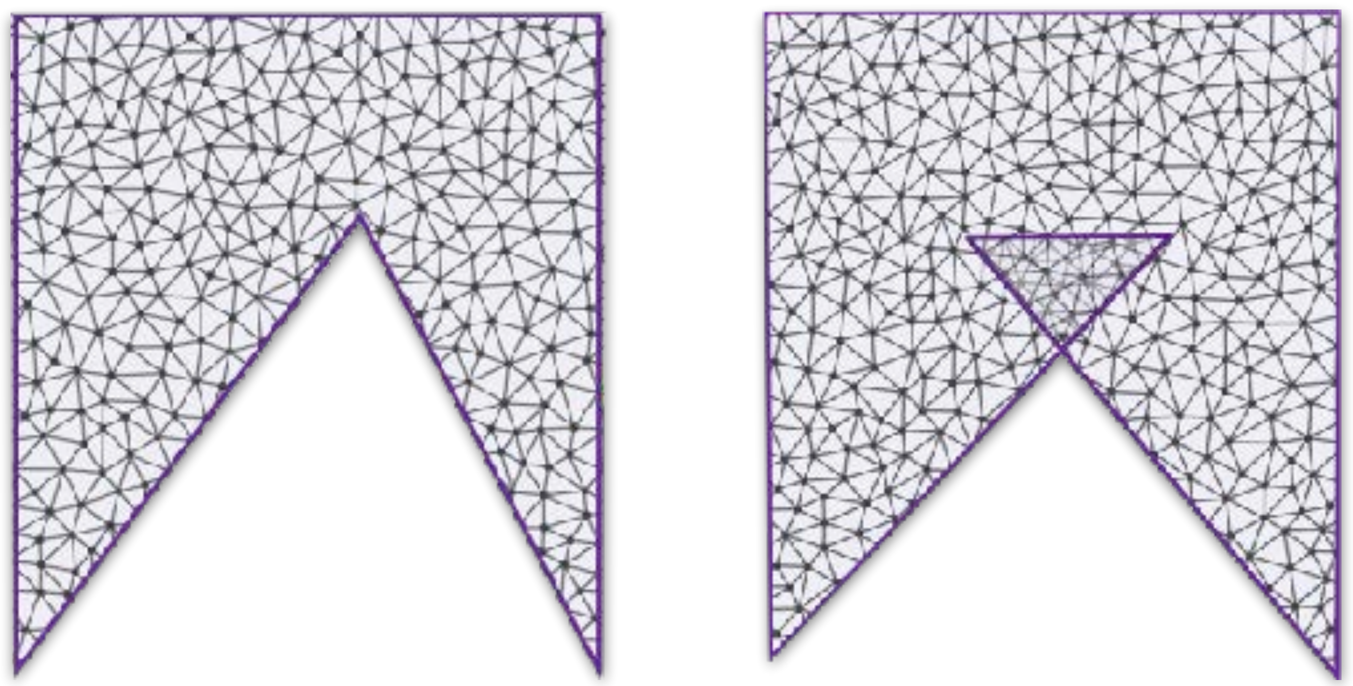
[Kraevoy et al. 2003]



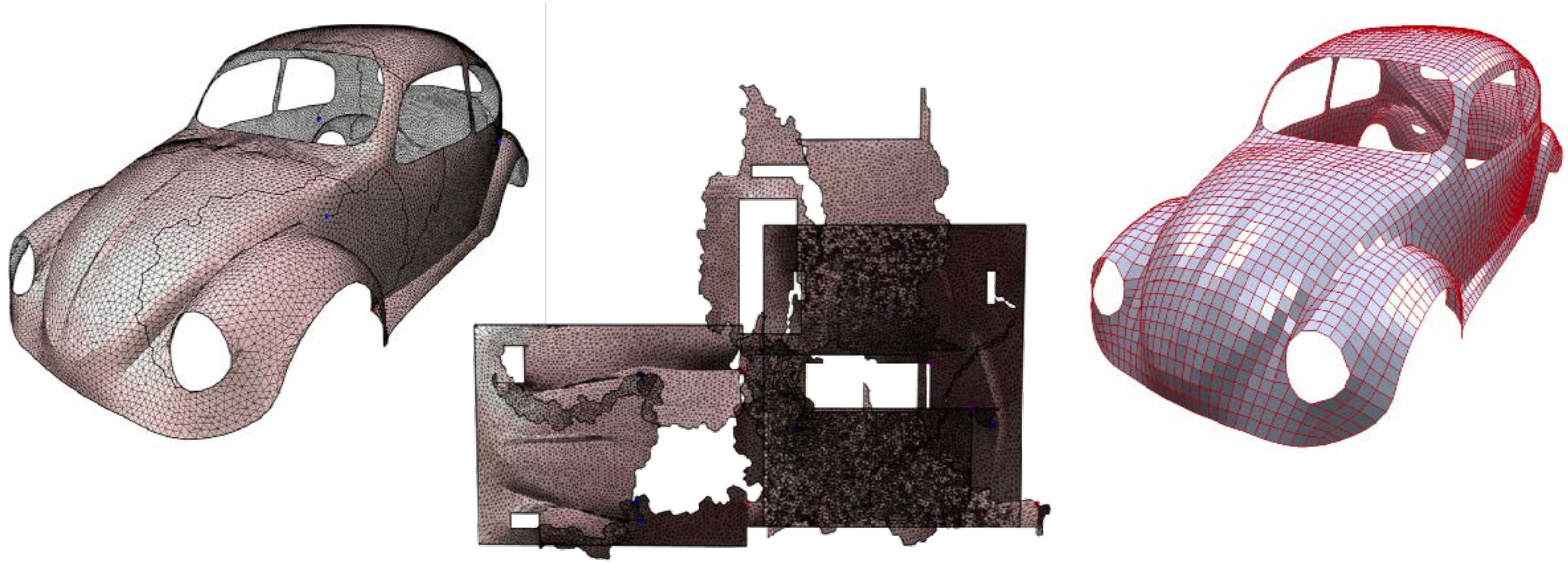
Issues:

* Tutte Embedding as a building block may introduce flips.
* Target domain must not have self-intersection.

Self-overlapping Domains

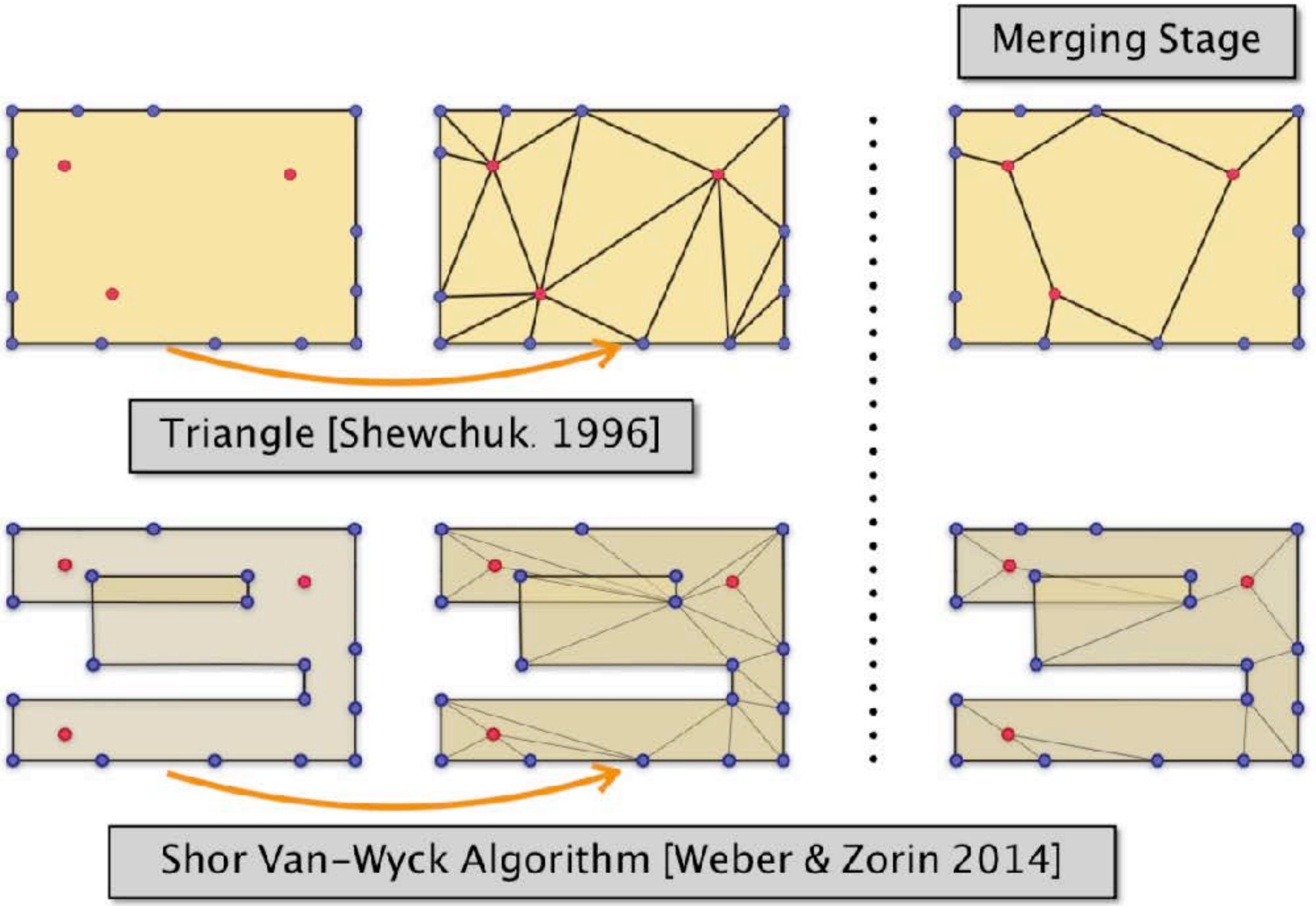


Self-overlapping boundary



[Weber & Zorin 2014, Bommes et al. 2009]

Domain Decomposition



Goal: divide the domain into multiple ~~triangles.~~

convex polygons

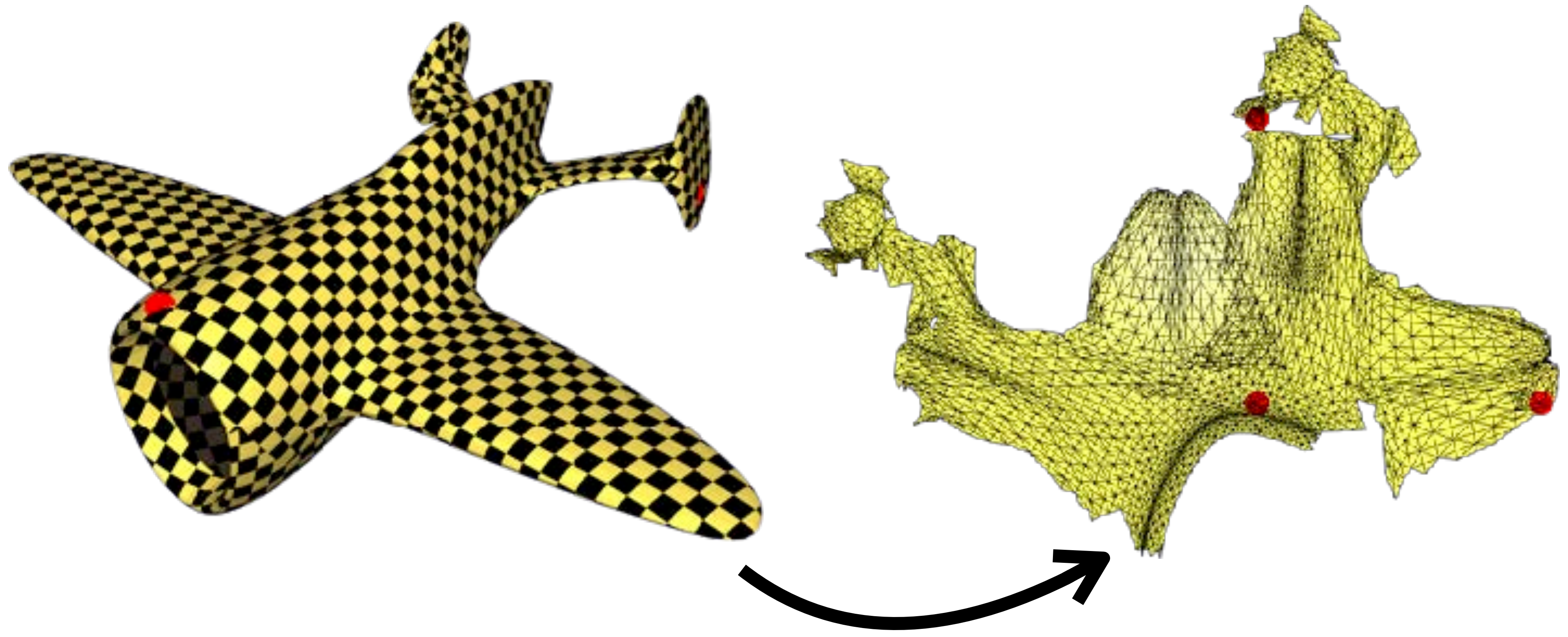
1. Triangulate the domain
2. Embed the hard constraints
3. Greedily merge adjacent triangles

Matchmaker++

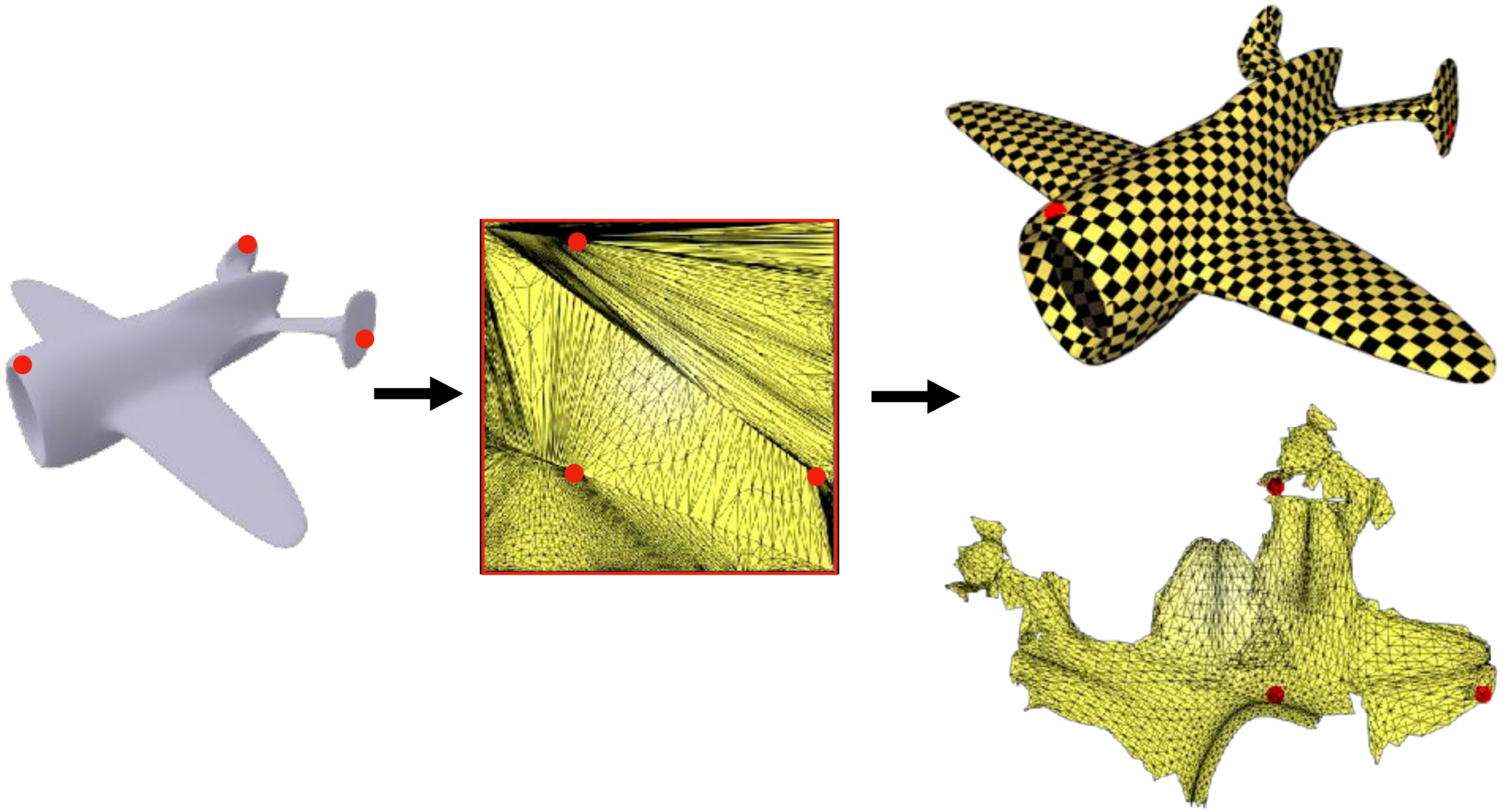
[Weber & Zorin 2014]

MatchMaker + Shor-Van Wyck Triangulation + Progressive Embedding

Robust surface parametrization method for multiply-connected domains with arbitrary point constraints

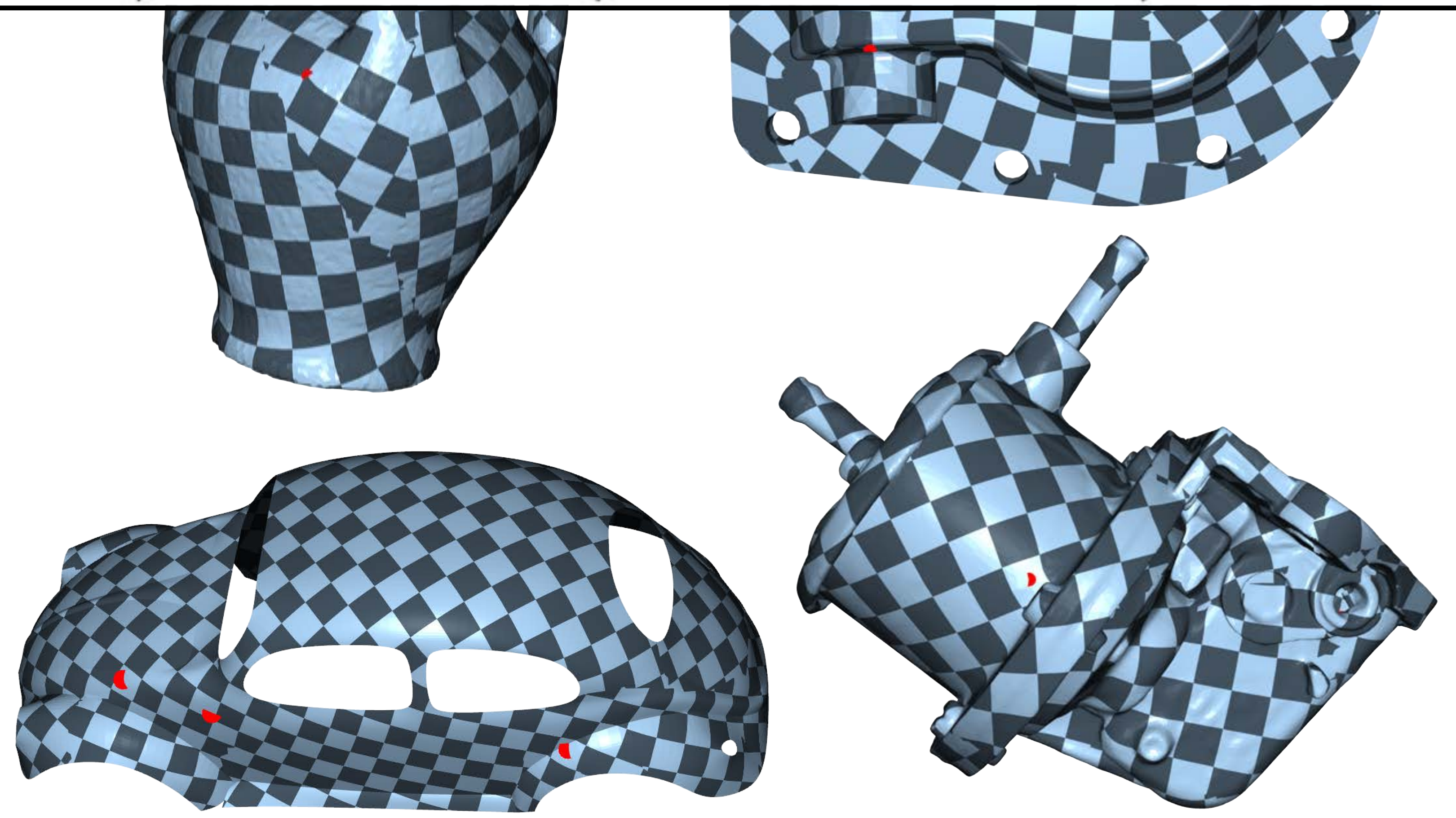


Random Hard Constraints



Dataset: 102 models from [Myles & Zorin 2014]

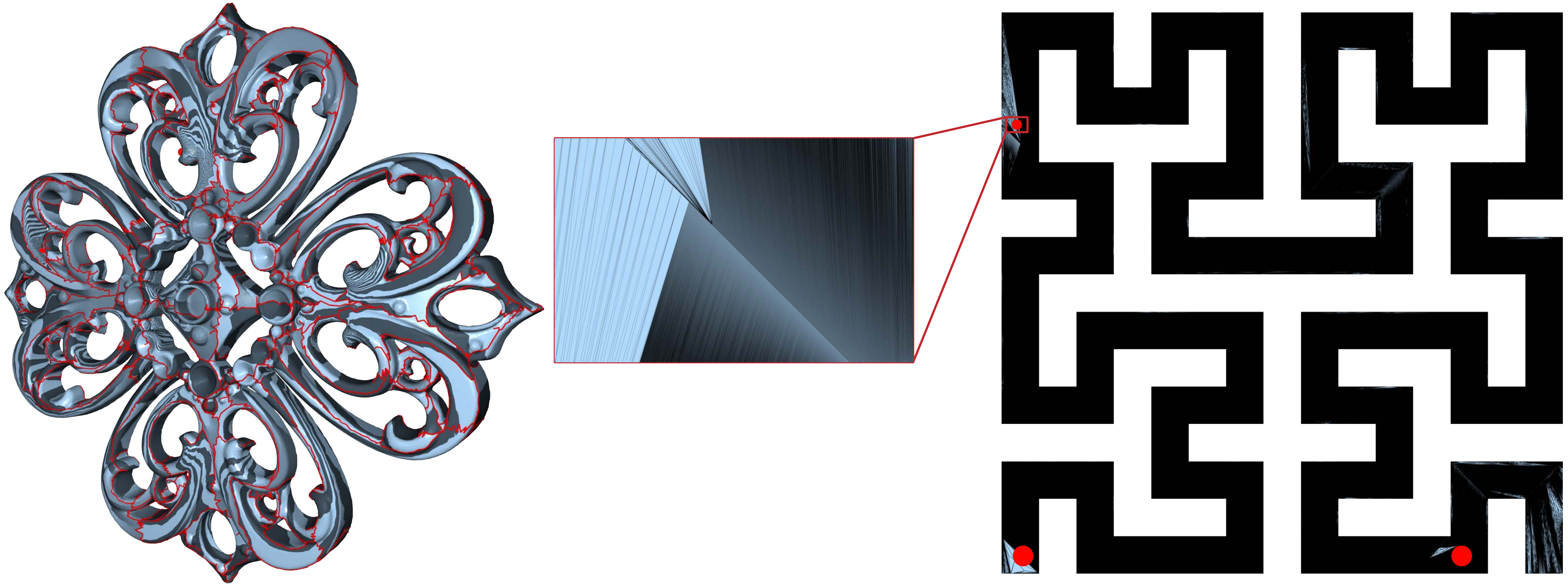
Name	#V	#F	#invalid	#flipped	PE(s)	MM++(s)
Botijo	43786	83788	0	0	NA	3.9
Beetle	20619	39276	0	0	NA	1.1
Casting	21236	39438	67	40	27.4	1.3
Oil pump	54135	103778	5	0	2.3	4.8



(Seams are part of the dataset)

Challenging Stress Test

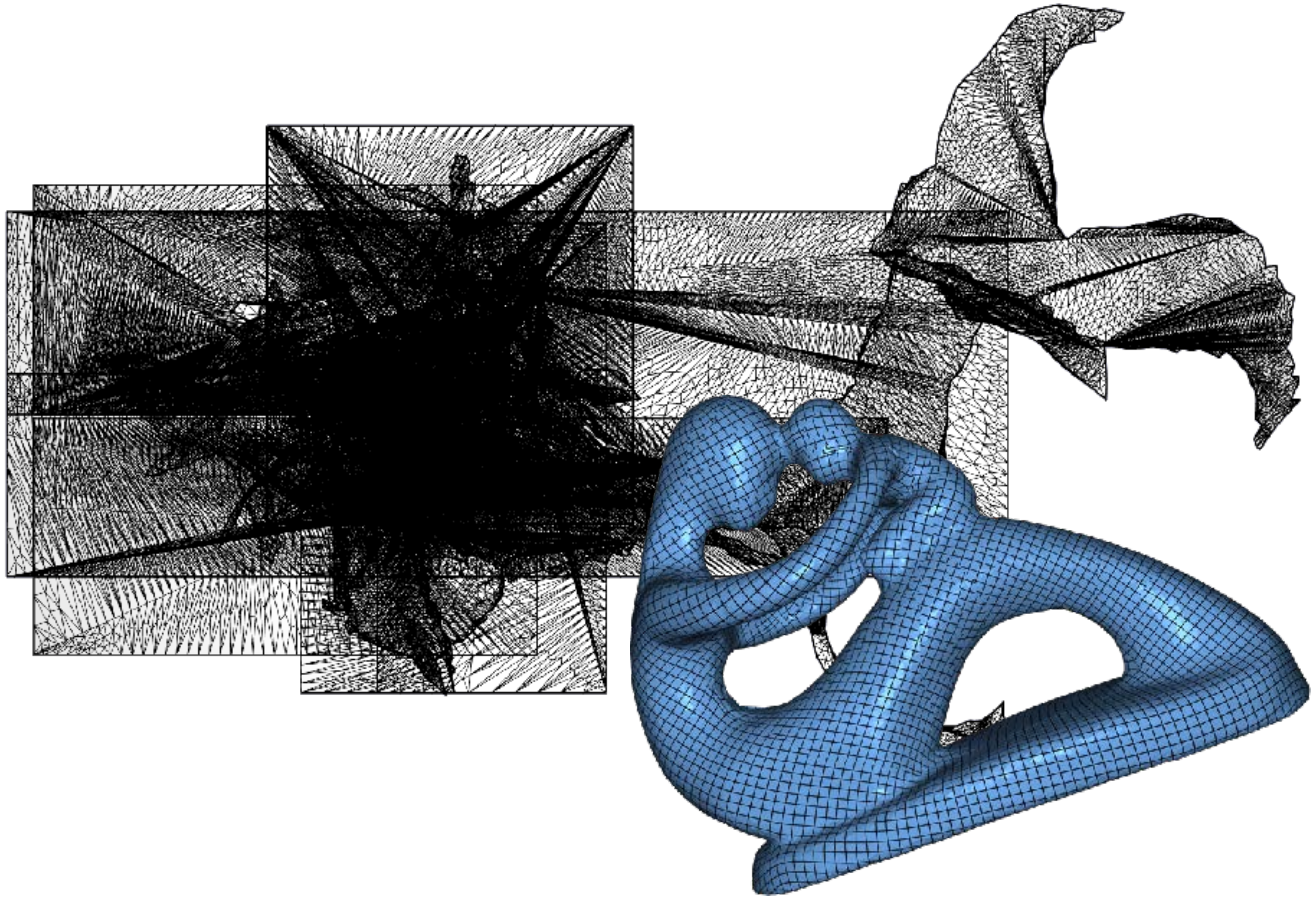
- 102 models from [Myles & Zorin 2014]
- 3 random point constraints

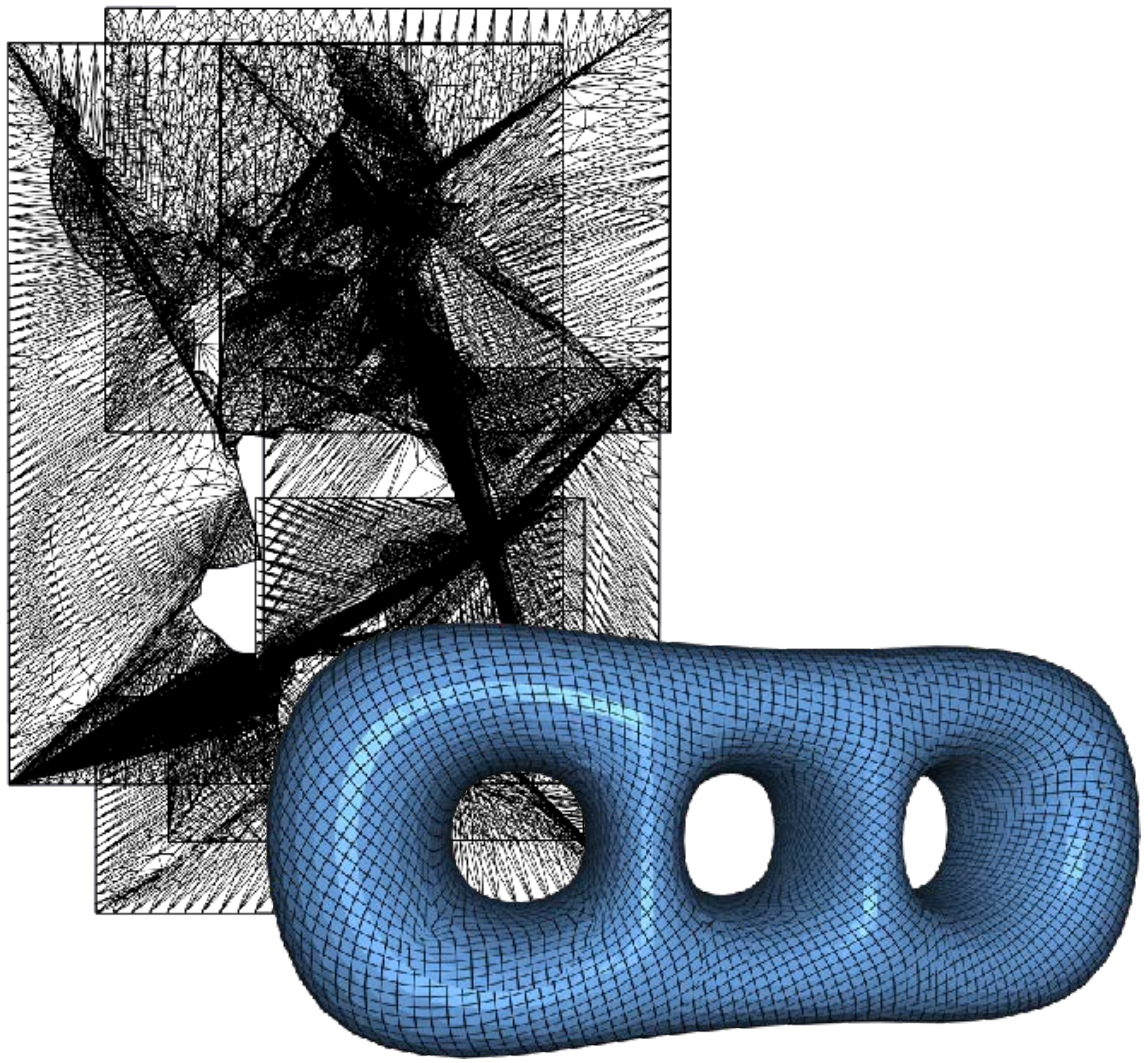


Name	#V	#F	#invalid	#flipped	PE(s)	MM++(s)
Filigree	49872	100000	32	0	72.4	30.8

Self-overlapping Boundary

Name	#V	#F	#invalid	#flipped	PE(s)	MM++(s)
Fertility	16508	33028	0	0	NA	582.4
3 holes	7440	14886	0	0	NA	107.4





Summary

- A robust algorithm to compute planar embeddings, which has the same guarantees as the Tutte Embedding, but works robustly in floating point coordinates.
- Paired with Matchmaker, our algorithm enables the robust generation of constrained locally injective maps with hard constraints.

Thank you!

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Reference Implementation

https://github.com/hankstag/progressive_embedding