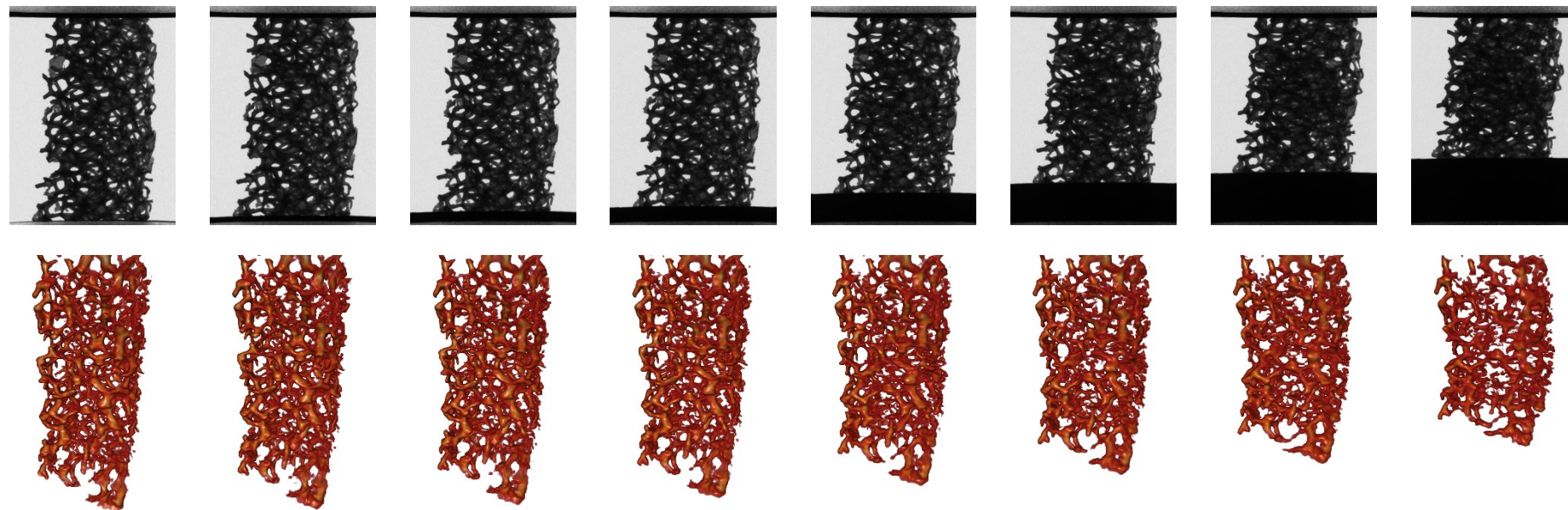
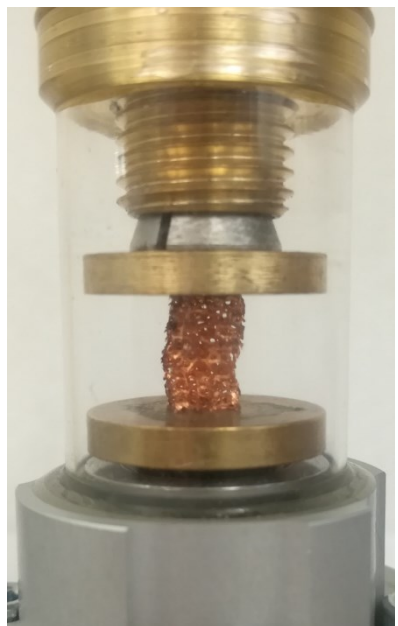




# Warp-and-Project Tomography for Rapidly Deforming Objects

Guangming Zang, Ramzi Idoughi, Ran Tao, Gilles Lubineau, Peter Wonka, Wolfgang Heidrich,  
King Abdullah University of Science And Technology



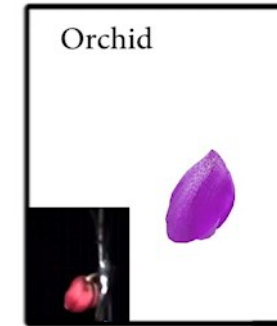
# Dynamic scene reconstruction



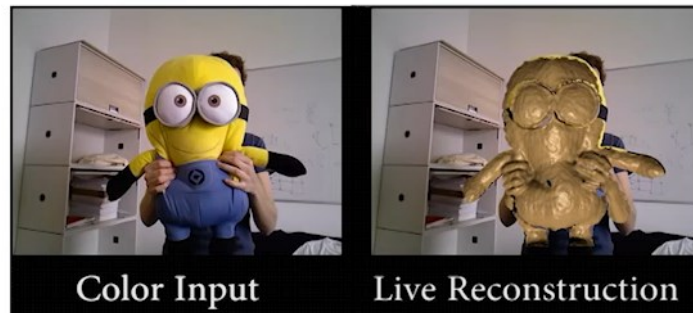
[Wang et al. 2009]



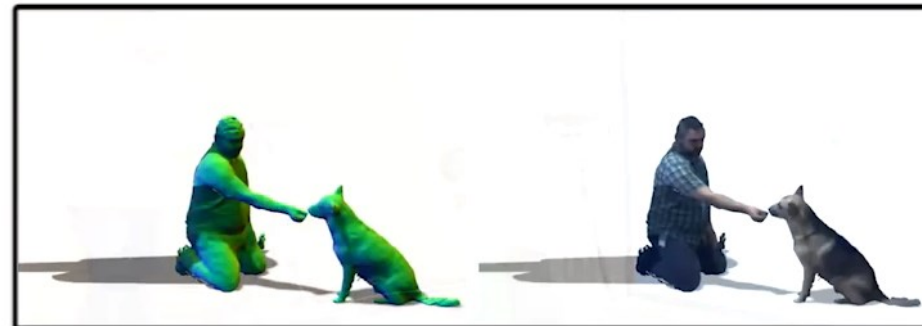
[Li et al. 2013]



[Zheng et al. 2017]



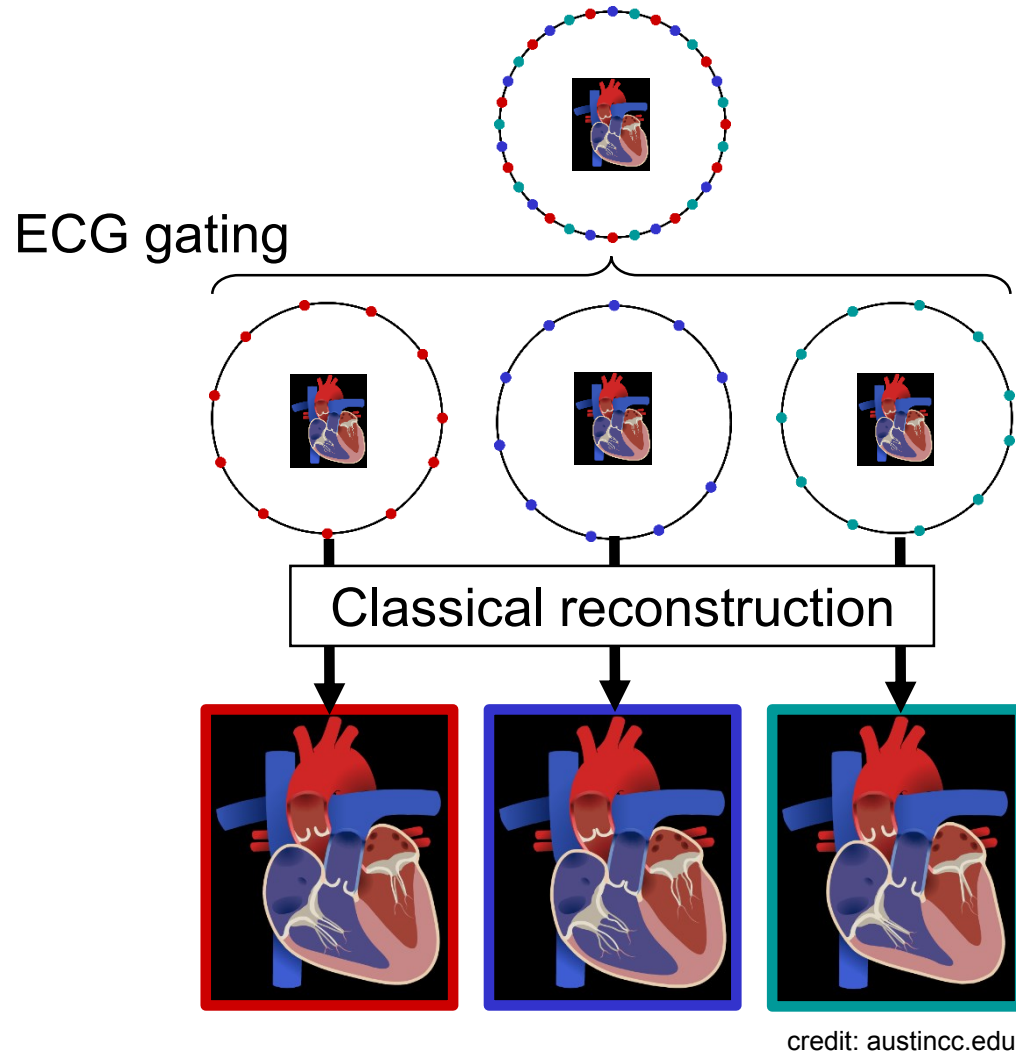
[Innmann et al. 2016]



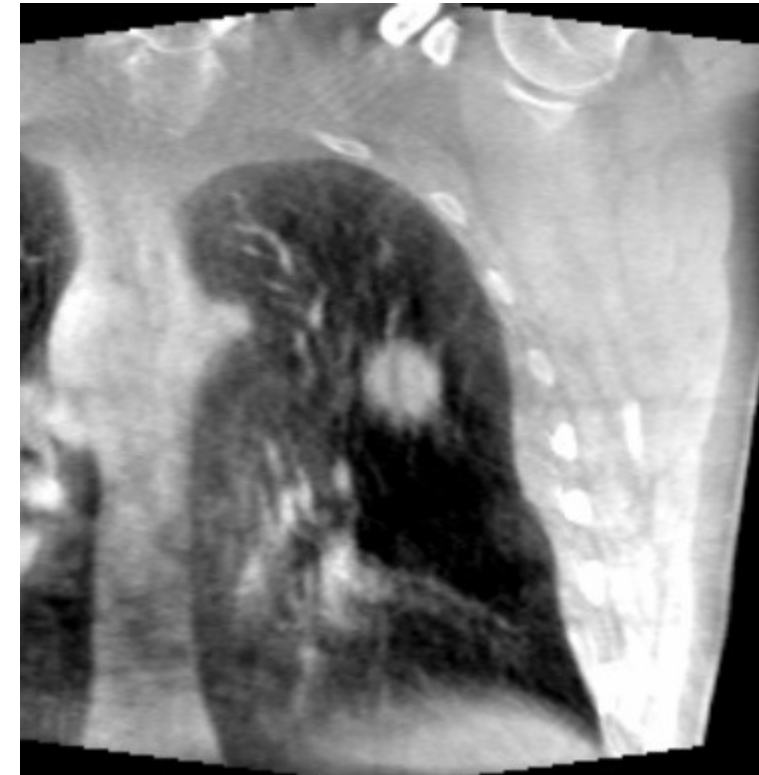
[Dou et al. 2016]



# Dynamic X-ray tomography



[Chen et al. 2012]



credit: RTK

[Mory et al. 2016]



# Dynamic scene reconstruction

	Optical means	X-ray tomography
<b>Cameras/ sensors</b>	One or more	<b>One</b>
<b>Resolution</b>	Low	<b>High</b>
<b>Reconstruction</b>	Surface	<b>Surface + internal structures</b>
<b>Capture Speed</b>	<b>Fast</b>	Slow
<b>Deformation type</b>	<b>General</b>	Periodic or with Pattern
<b>Application fields</b>	<b>General</b>	Medical, Security, Industry



# Dynamic scene reconstruction

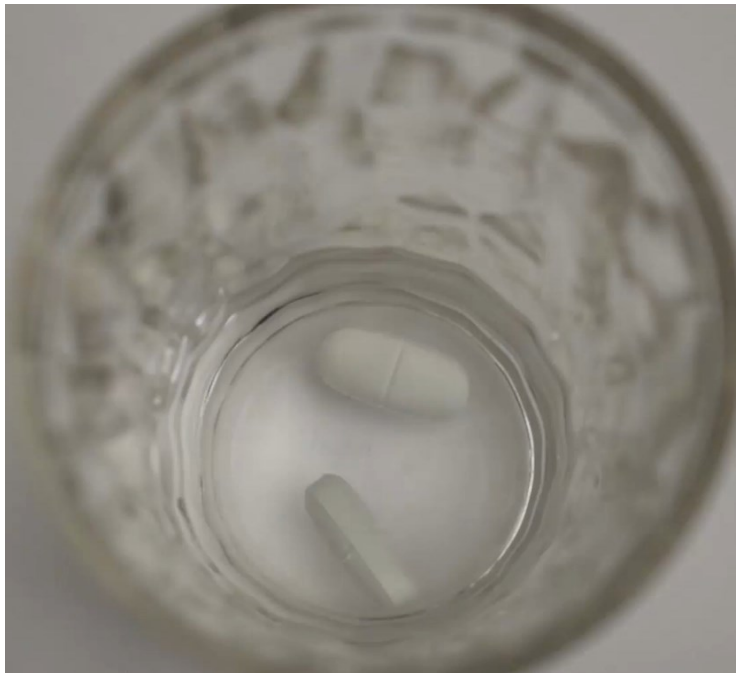
	Optical means	X-ray tomography
<b>Cameras/ sensors</b>	One or more	<b>One</b>
<b>Resolution</b>	Low	<b>High</b>
<b>Reconstruction</b>	Surface	<b>Surface + internal structures</b>
<b>Capture Speed</b>	<b>Fast</b>	Slow
<b>Deformation type</b>	<b>General</b>	Periodic or with Pattern
<b>Application fields</b>	<b>General</b>	Medical, Security, Industry

High quality surface and internal reconstruction for fast deforming objects with general motion



# Motivation

Is it possible to scan **rapidly** deforming objects with **internal details**?



Pills dissolving



Hydro-gel balls



# Motivation



Frame 01



Frame 02



Frame 46



Frame 91



Frame 92



Space time tomography  
[Zang et al. SIGGRAPH18]

## Shortcomings ?

- **Assumption:** slow and smooth motion fields
- **Trade-off:** spatial VS. temporal reconstruction quality
- **Sampling:** costly uniform temporal sampling

# Motivation



**SART-ROF**  
[Getreuer 2012]



**ST-Tomography**  
[Zang et al. 2018]



**Warp-and-Project**  
[Ours]

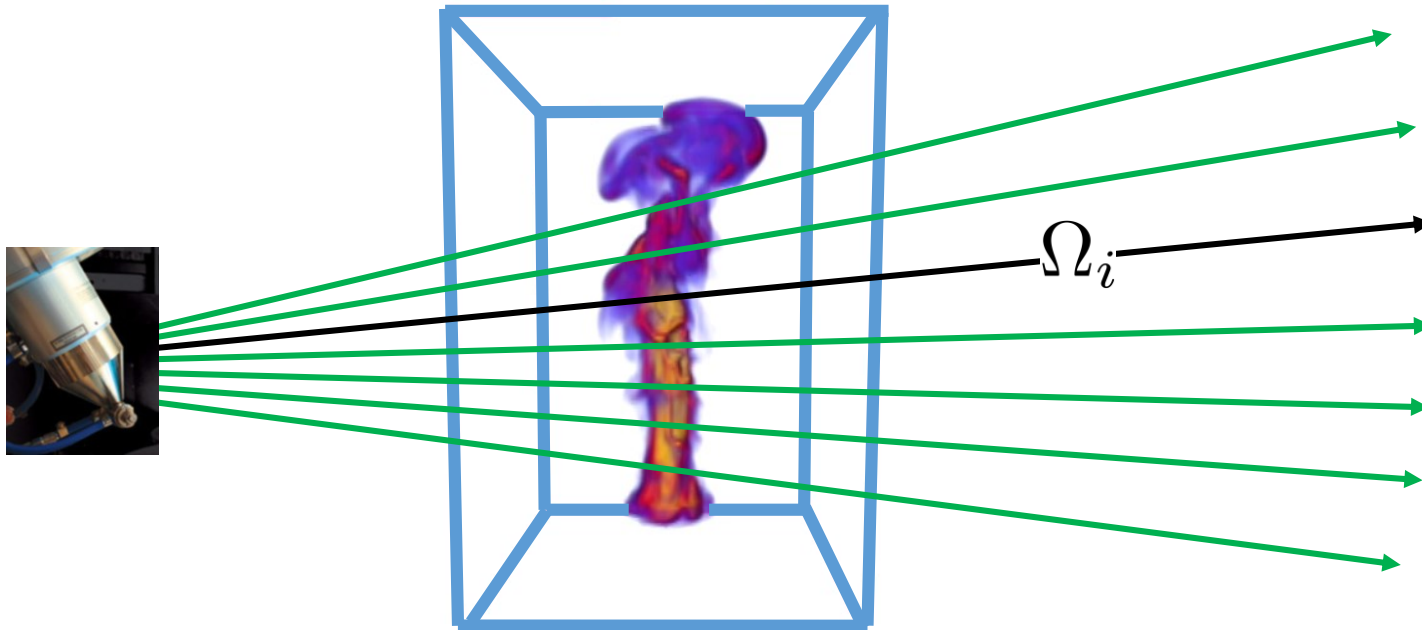






# Image formation model

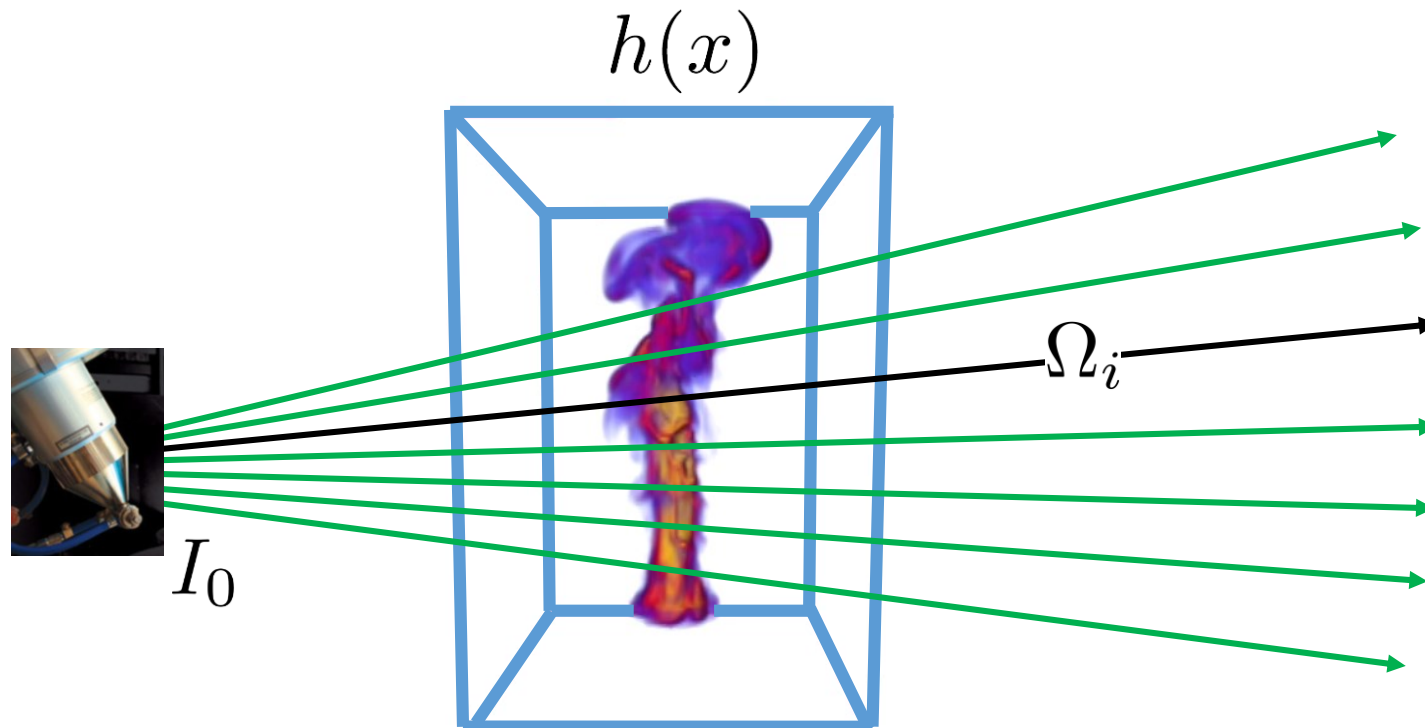
$\Omega_i$ : X-ray path





# Image formation model

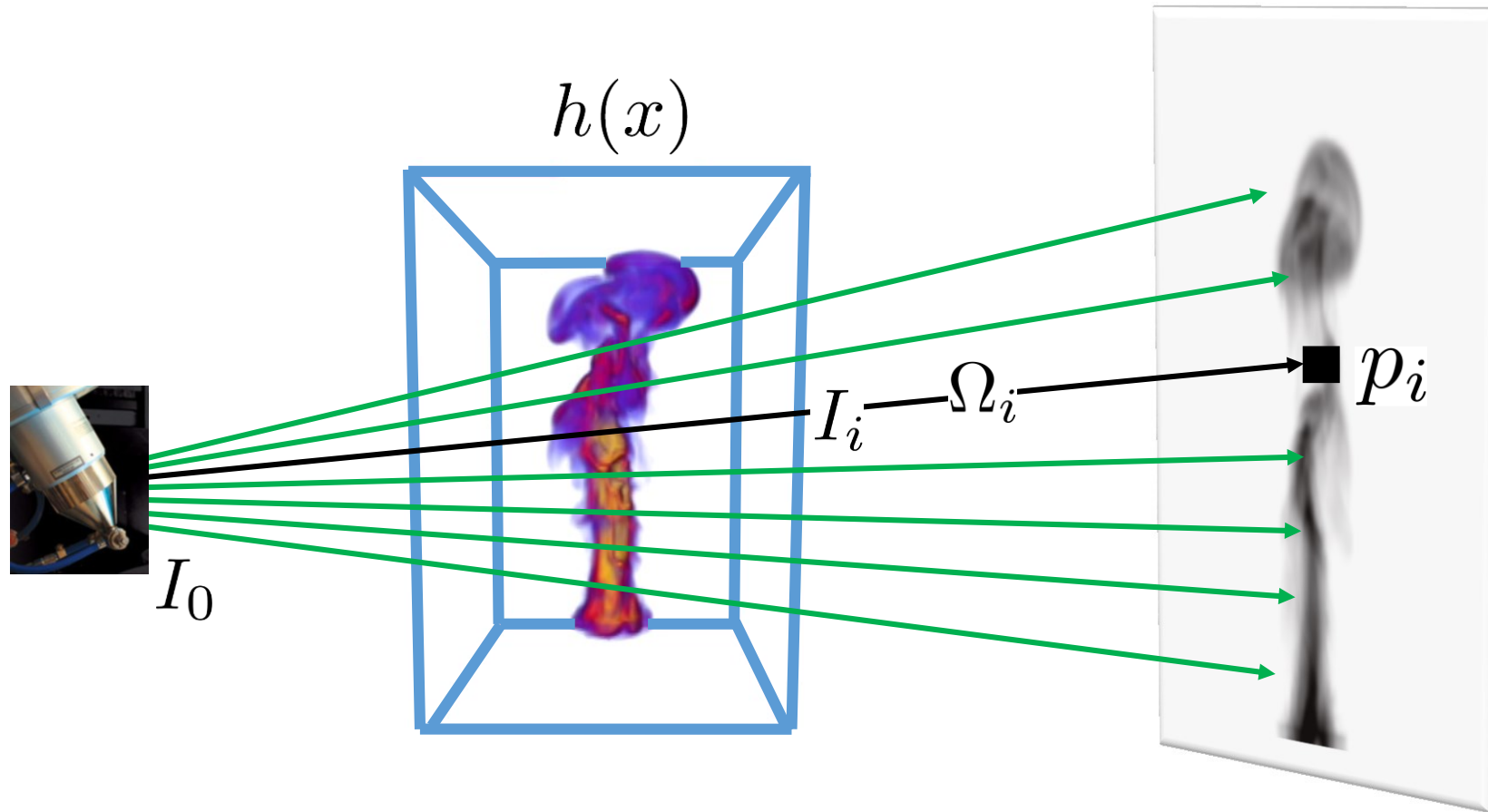
$\Omega_i$ : X-ray path     $h(x)$ : Unknown field





# Image formation model

$\Omega_i$ : X-ray path     $h(x)$ : Unknown field     $p_i$ : Measurement

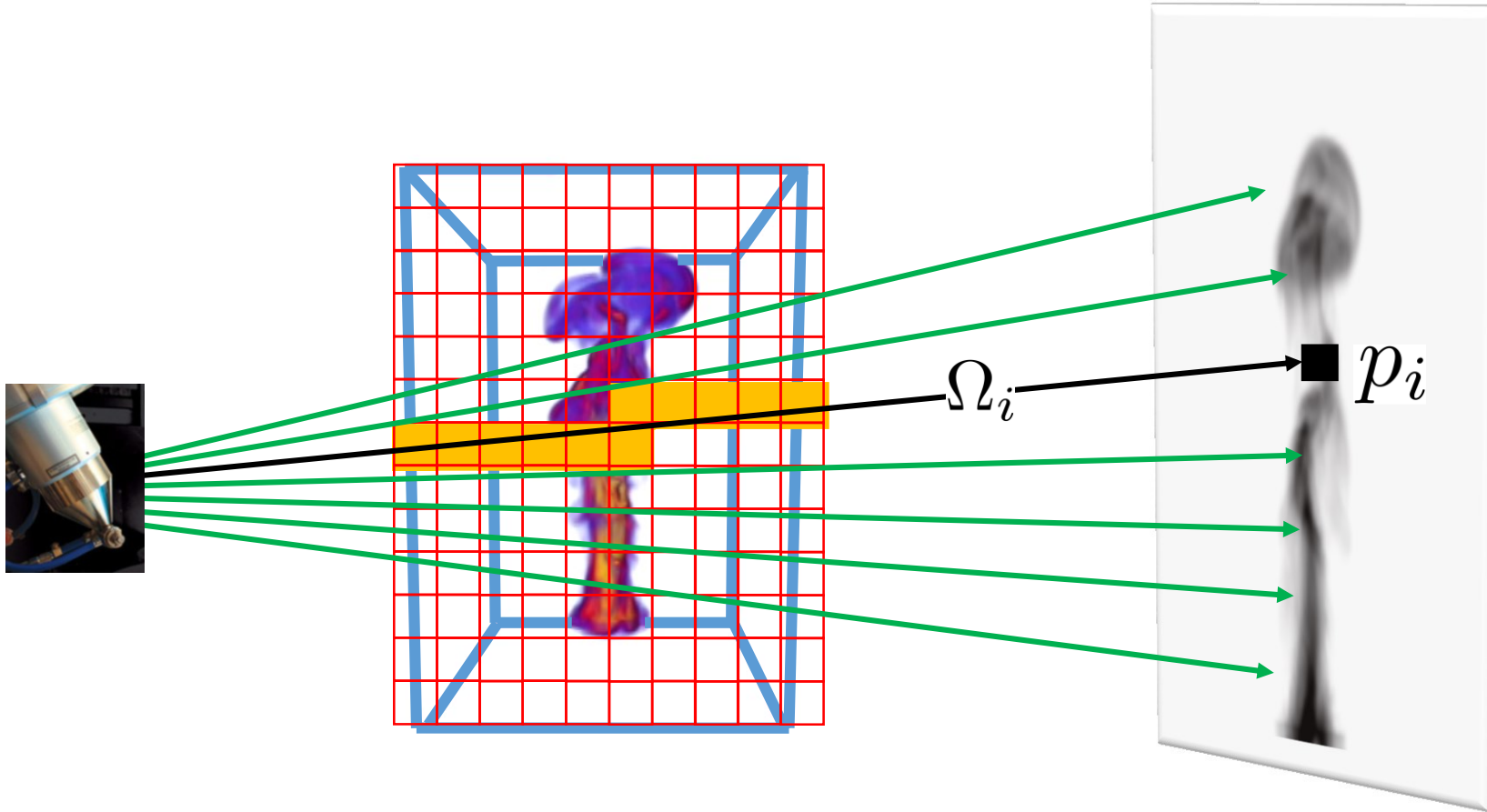


$$\int_{\Omega_i} h(x) d\Omega_i = \underbrace{-\log(I_i/I_0)}_{p_i}$$



# Image formation model

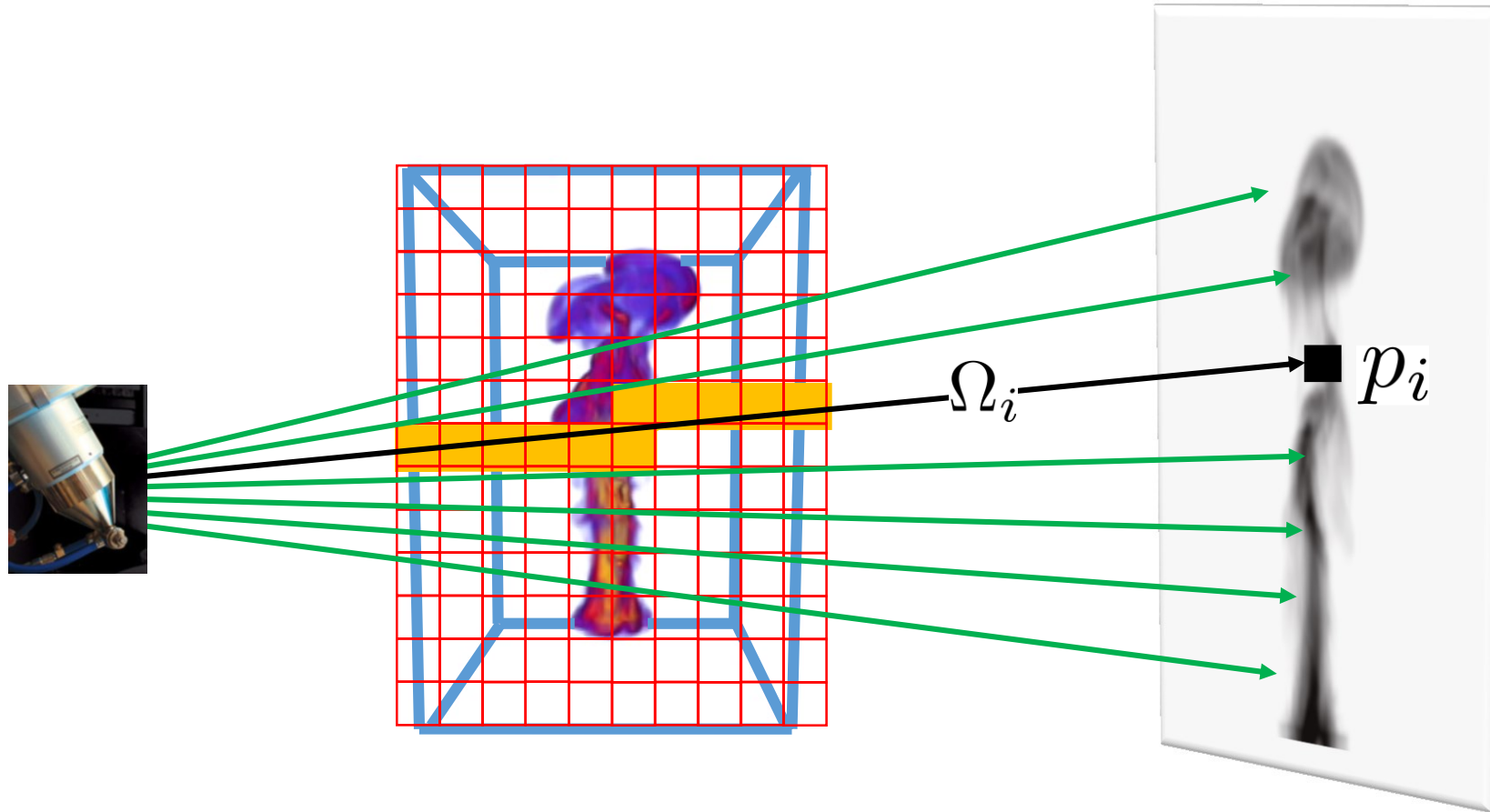
$\Omega_i$ : X-ray path     $h(x)$ : Unknown field     $p_i$ : Measurement



$$\sum_j h_j A_{ij} = p_i$$

# Image formation model

$\Omega_i$ : X-ray path     $h(x)$ : Unknown field     $p_i$ : Measurement



$$\sum_j h_j A_{ij} = p_i$$

$$A_t h_t = p_t$$

Each projection image has its own time stamp!

# Linear system

- Sparse system
- Memory consuming
- Ill-posed problem

$$\underbrace{\begin{pmatrix} A_1 & & & \\ & \ddots & & \\ & & A_j & \\ & & & \ddots \\ & & & & A_{N_p} \end{pmatrix}}_A \cdot \underbrace{\begin{pmatrix} h_1 \\ \vdots \\ h_j \\ \vdots \\ h_{N_p} \end{pmatrix}}_h = \underbrace{\begin{pmatrix} p_1 \\ \vdots \\ p_j \\ \vdots \\ p_{N_p} \end{pmatrix}}_p,$$

Radon Transform

Frames Measurements

# Warp and project tomography

- Non-parametric and matrix-free
- No assumption of the motion
- A non-uniform temporal up-sampling



# Objective function

$$\begin{aligned}
 \min_{\mathbf{f}, \mathbf{u}} & \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^f(\mathbf{f}_{j^-}) - \mathbf{p}_j \right\|_2^2 + \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^b(\mathbf{f}_{j^+}) - \mathbf{p}_j \right\|_2^2 \\
 & + \kappa_1 \sum_{k=1}^{N_k-1} \left\| \nabla_T \mathbf{f}_k + \nabla_S \mathbf{f}_k \cdot \mathbf{u}_k \right\|_1 \\
 & + \sum_{k=1}^{N_k} \left[ \kappa_2 \left\| \nabla_S \mathbf{f}_k \right\|_{\mathbf{H}_\epsilon} + \kappa_3 \left\| \nabla_T \mathbf{f}_k \right\|_2^2 \right] \\
 & + \sum_{k=1}^{N_k-1} \sum_{i=x,y,z} \left[ \kappa_4 \left\| \nabla_S \mathbf{u}_{k,i} \right\|_{\mathbf{H}_\tau} + \kappa_5 \left\| \nabla_T \mathbf{u}_{k,i} \right\|_2^2 \right]
 \end{aligned}$$





# Objective function

$$\min_{\mathbf{f}, \mathbf{u}} \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^f(\mathbf{f}_{j^-}) - \mathbf{p}_j \right\|_2^2 + \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^b(\mathbf{f}_{j^+}) - \mathbf{p}_j \right\|_2^2$$

$$+ \kappa_1 \sum_{k=1}^{N_k-1} \left\| \nabla_T \mathbf{f}_k + \nabla_S \mathbf{f}_k \cdot \mathbf{u}_k \right\|_1$$

$$+ \sum_{k=1}^{N_k} \left[ \kappa_2 \left\| \nabla_S \mathbf{f}_k \right\|_{\mathbf{H}_\epsilon} + \kappa_3 \left\| \nabla_T \mathbf{f}_k \right\|_2^2 \right]$$

$$+ \sum_{k=1}^{N_k-1} \sum_{i=x,y,z} \left[ \kappa_4 \left\| \nabla_S \mathbf{u}_{k,i} \right\|_{\mathbf{H}_\tau} + \kappa_5 \left\| \nabla_T \mathbf{u}_{k,i} \right\|_2^2 \right]$$

**Data fitting**  
**(Forward / backward warping)**



# Objective function

$$\min_{\mathbf{f}, \mathbf{u}} \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^f(\mathbf{f}_{j^-}) - \mathbf{p}_j \right\|_2^2 + \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^b(\mathbf{f}_{j^+}) - \mathbf{p}_j \right\|_2^2$$

$$+ \kappa_1 \sum_{k=1}^{N_k-1} \left\| \nabla_T \mathbf{f}_k + \nabla_S \mathbf{f}_k \cdot \mathbf{u}_k \right\|_1$$

$$+ \sum_{k=1}^{N_k} \left[ \kappa_2 \left\| \nabla_S \mathbf{f}_k \right\|_{H_\epsilon} + \kappa_3 \left\| \nabla_T \mathbf{f}_k \right\|_2^2 \right]$$

$$+ \sum_{k=1}^{N_k-1} \sum_{i=x,y,z} \left[ \kappa_4 \left\| \nabla_S \mathbf{u}_{k,i} \right\|_{H_\tau} + \kappa_5 \left\| \nabla_T \mathbf{u}_{k,i} \right\|_2^2 \right]$$

**Data fitting**  
(Forward / backward warping)

**Volume correlation**



# Objective function

$$\begin{aligned} \min_{\mathbf{f}, \mathbf{u}} & \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^f(\mathbf{f}_{j^-}) - \mathbf{p}_j \right\|_2^2 + \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^b(\mathbf{f}_{j^+}) - \mathbf{p}_j \right\|_2^2 \\ & + \kappa_1 \sum_{k=1}^{N_k-1} \left\| \nabla_T \mathbf{f}_k + \nabla_S \mathbf{f}_k \cdot \mathbf{u}_k \right\|_1 \\ & + \sum_{k=1}^{N_k} \left[ \kappa_2 \left\| \nabla_S \mathbf{f}_k \right\|_{H_\epsilon} + \kappa_3 \left\| \nabla_T \mathbf{f}_k \right\|_2^2 \right] \\ & + \sum_{k=1}^{N_k-1} \sum_{i=x,y,z} \left[ \kappa_4 \left\| \nabla_S \mathbf{u}_{k,i} \right\|_{H_\tau} + \kappa_5 \left\| \nabla_T \mathbf{u}_{k,i} \right\|_2^2 \right] \end{aligned}$$

Data fitting  
(Forward / backward warping)

Volume correlation

Volume smoothness



# Objective function

$$\begin{aligned}
 \min_{\mathbf{f}, \mathbf{u}} & \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^f(\mathbf{f}_{j^-}) - \mathbf{p}_j \right\|_2^2 + \sum_{j=1}^{N_p} \left\| \mathbf{A}_j \mathbf{W}_j^b(\mathbf{f}_{j^+}) - \mathbf{p}_j \right\|_2^2 \\
 & + \kappa_1 \sum_{k=1}^{N_k-1} \left\| \nabla_T \mathbf{f}_k + \nabla_S \mathbf{f}_k \cdot \mathbf{u}_k \right\|_1 \\
 & + \sum_{k=1}^{N_k} \left[ \kappa_2 \left\| \nabla_S \mathbf{f}_k \right\|_{H_\epsilon} + \kappa_3 \left\| \nabla_T \mathbf{f}_k \right\|_2^2 \right] \\
 & + \sum_{k=1}^{N_k-1} \sum_{i=x,y,z} \left[ \kappa_4 \left\| \nabla_S \mathbf{u}_{k,i} \right\|_{H_\tau} + \kappa_5 \left\| \nabla_T \mathbf{u}_{k,i} \right\|_2^2 \right]
 \end{aligned}$$

**Data fitting**  
(Forward / backward warping)

**Volume correlation**

**Volume smoothness**

**Deformation field smoothness**

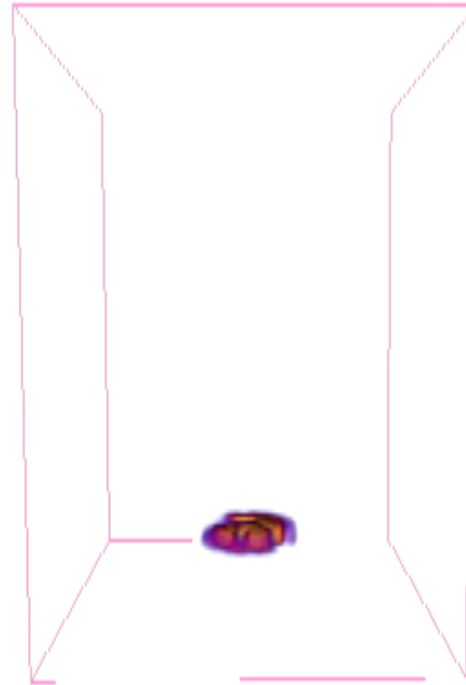


# Optimization framework

Simulated plume data

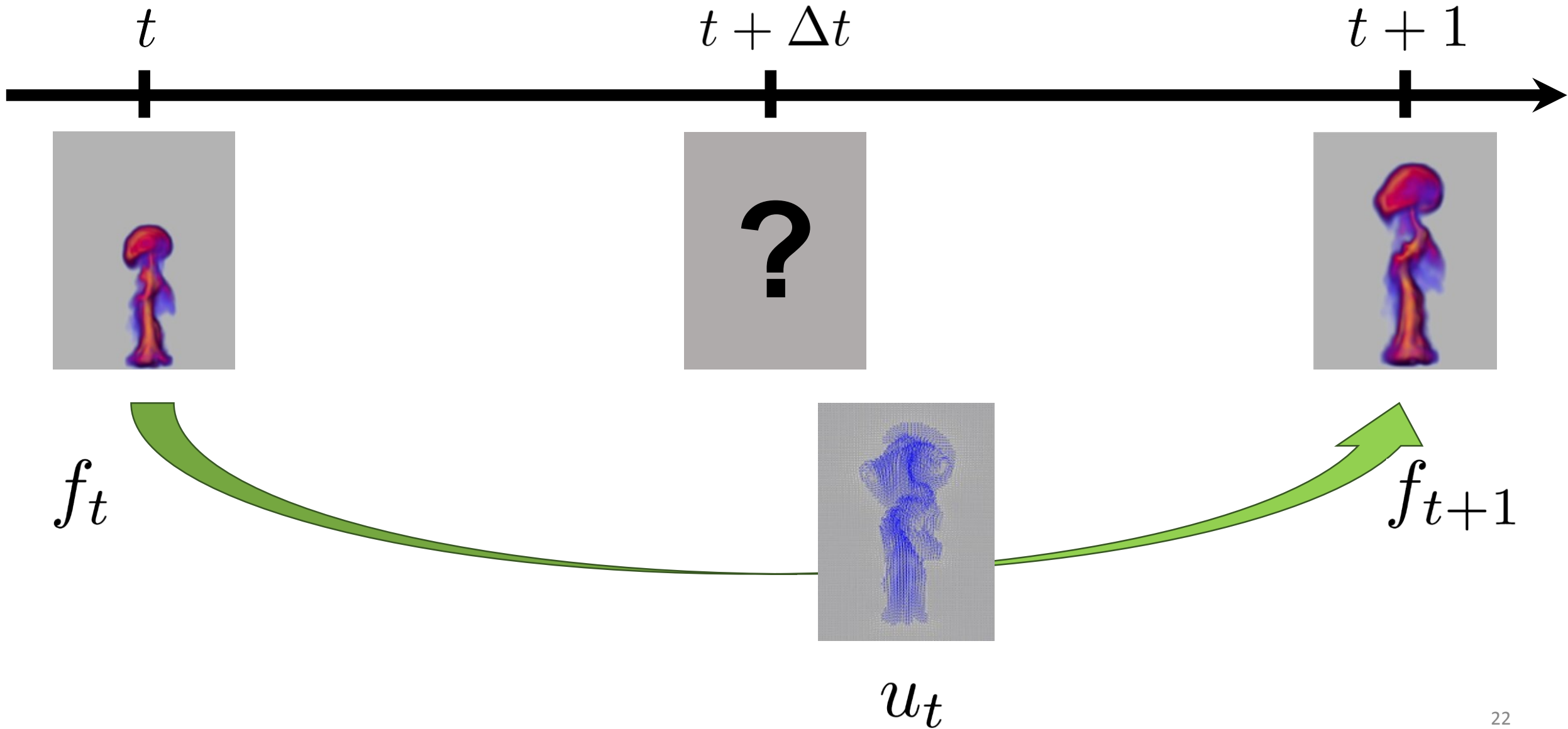
Volume size: 100x150x100

Time frames: 300



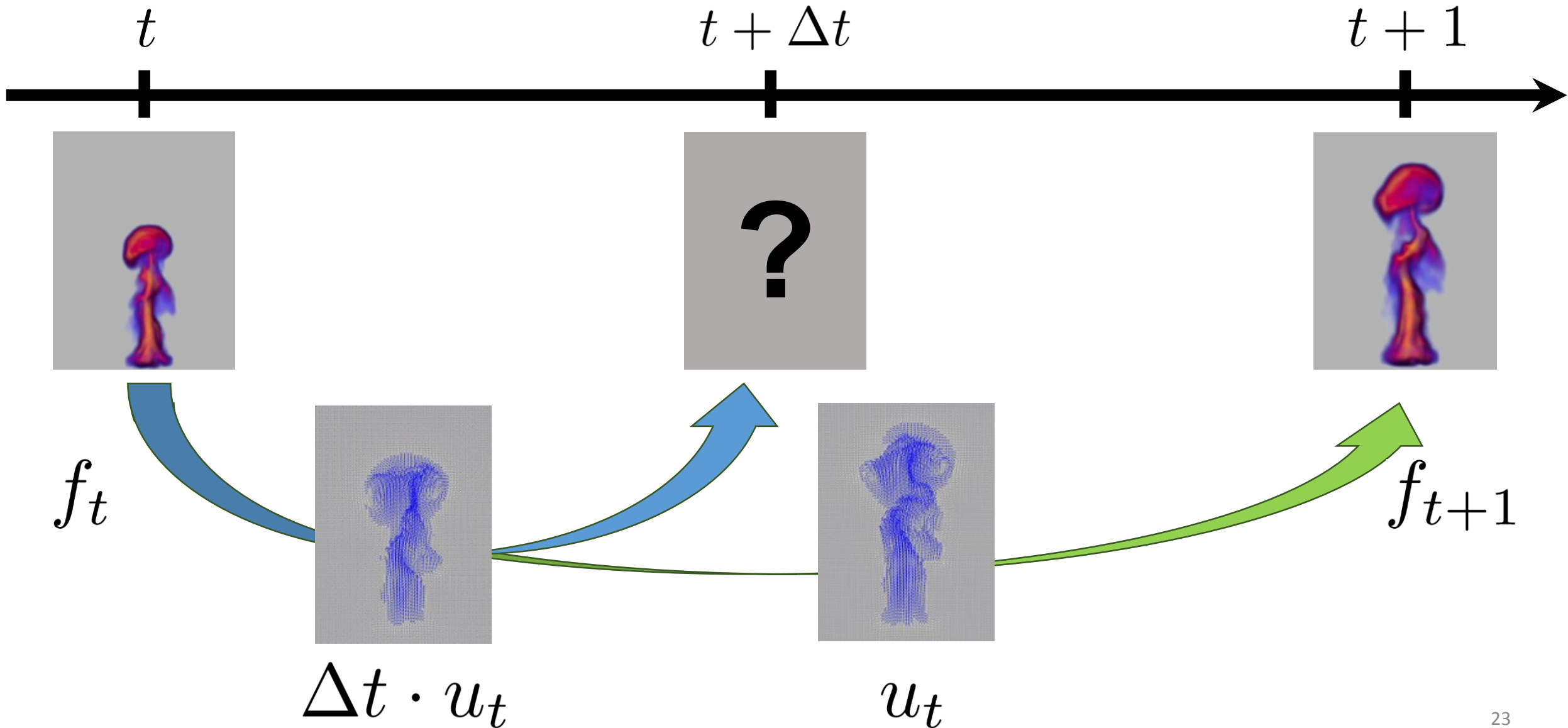


# Warping operator



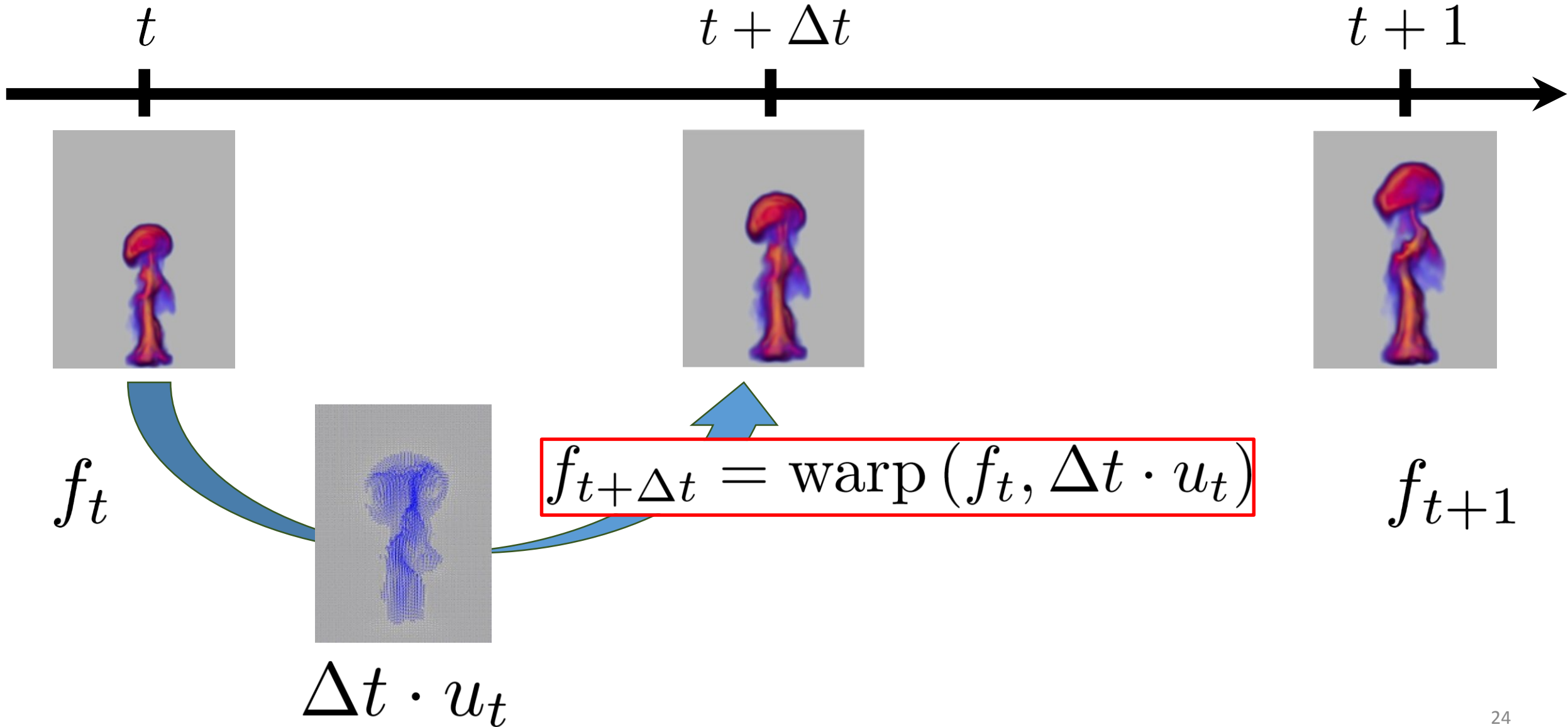


# Warping operator



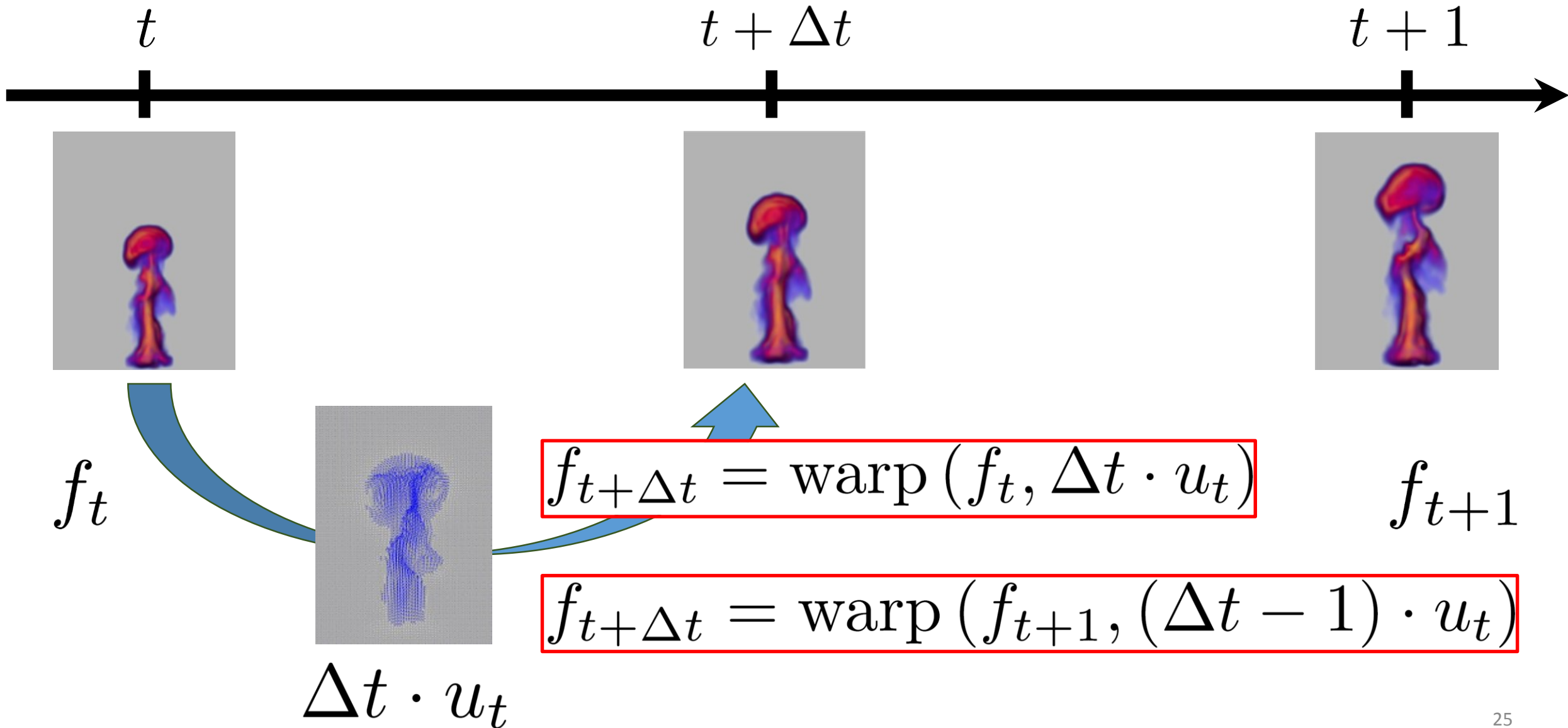


# Warping operator





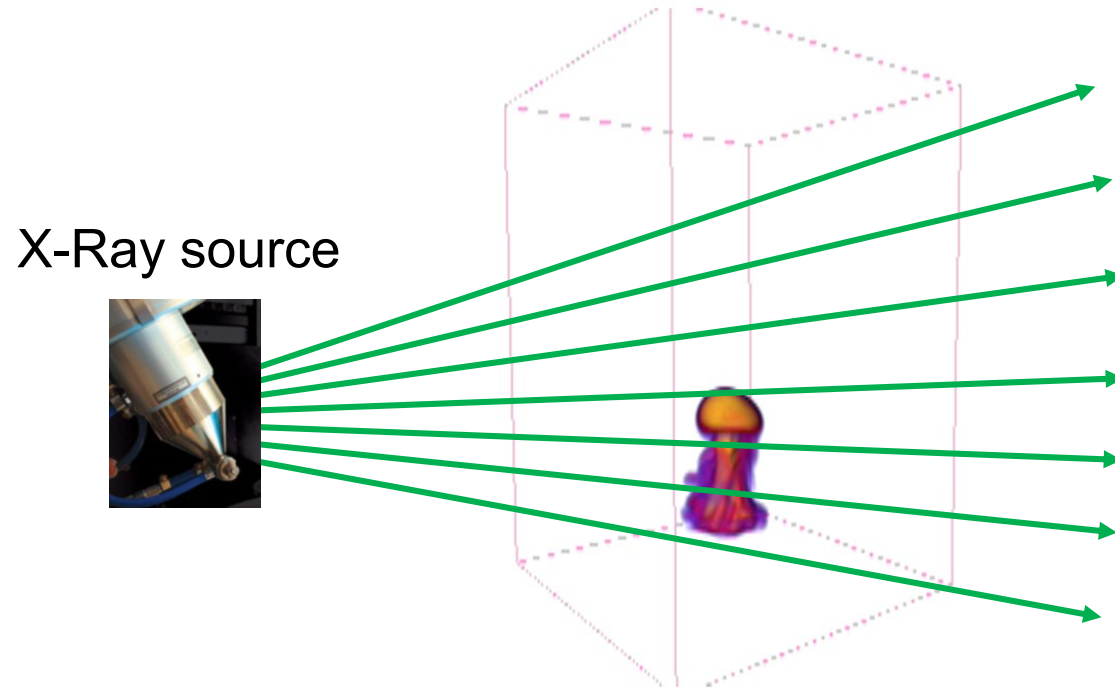
# Warping operator





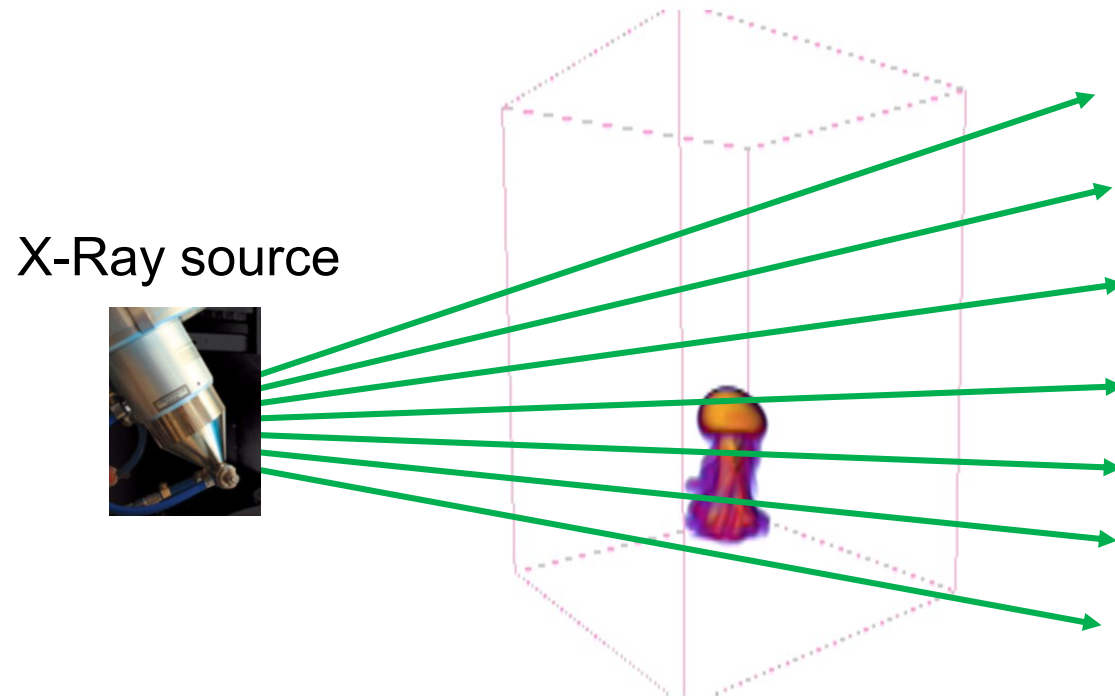
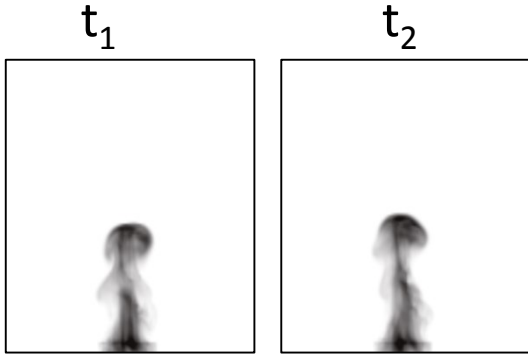
# Optimization framework

$t_1$



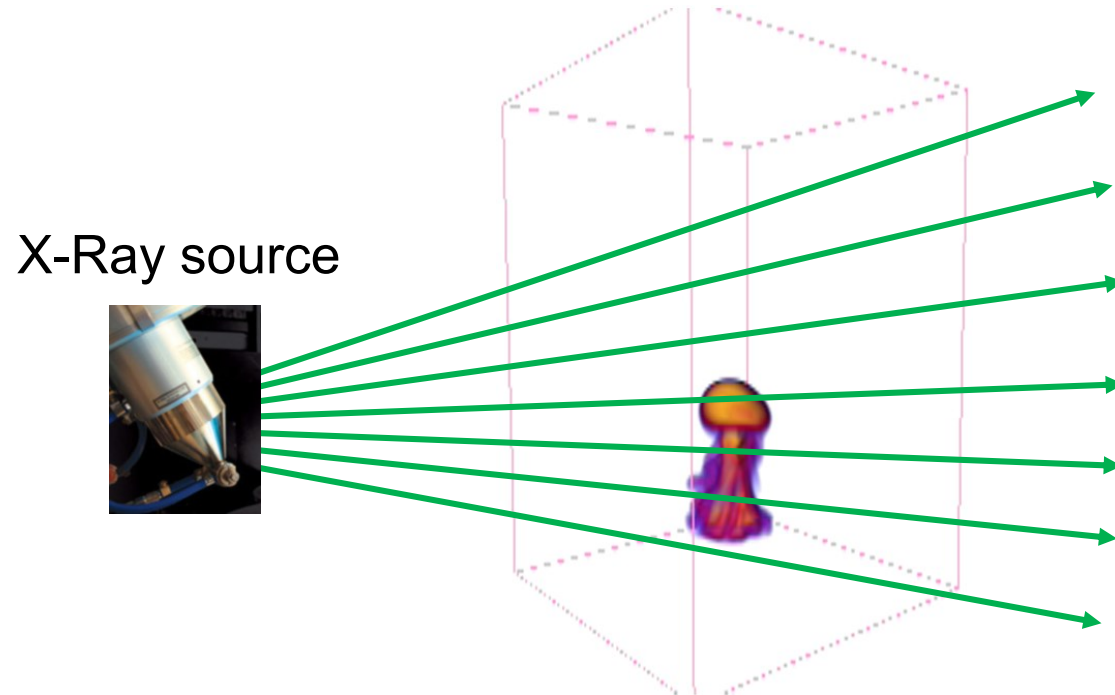
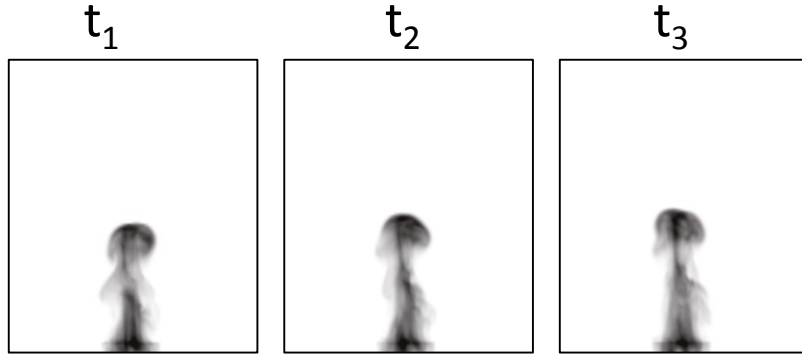


# Optimization framework



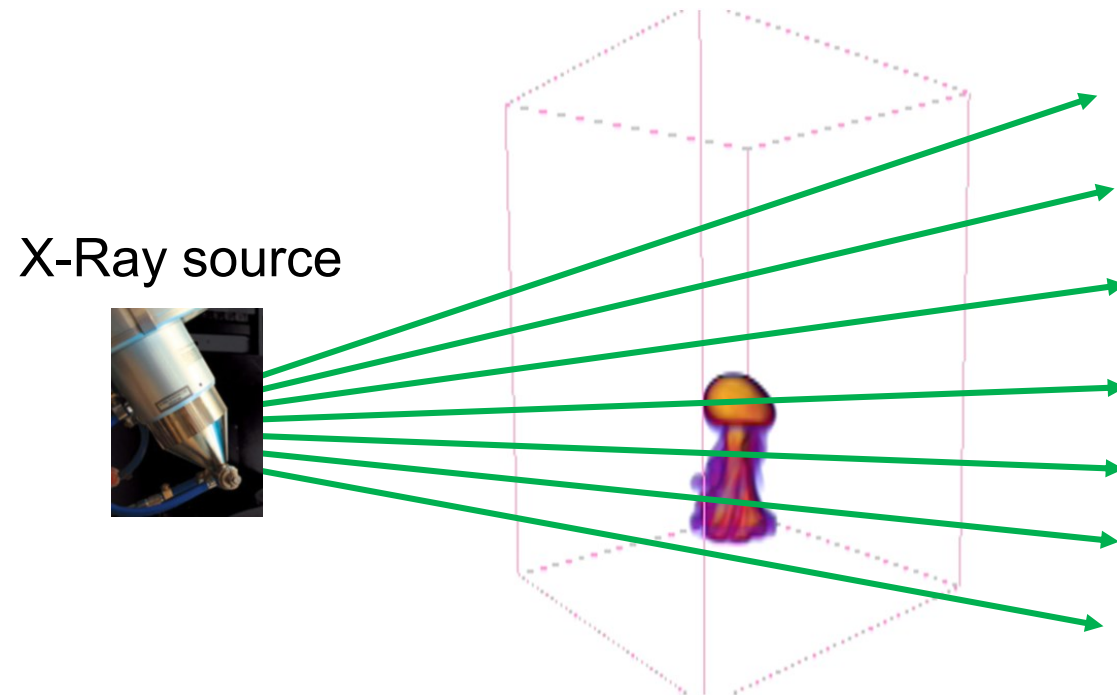
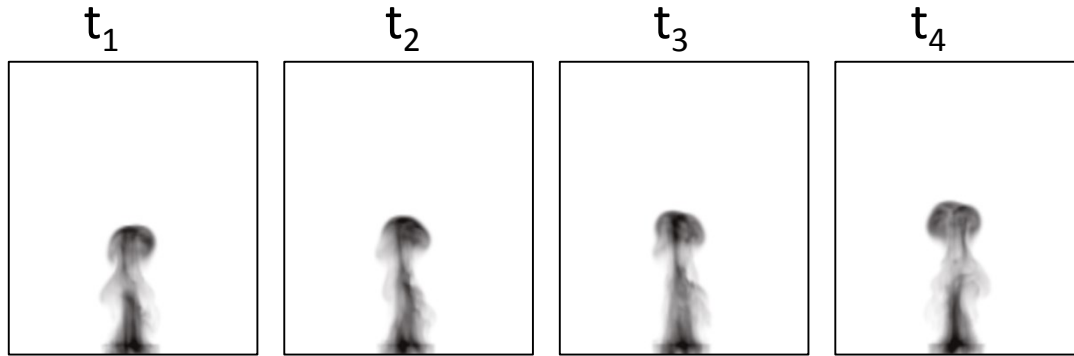


# Optimization framework



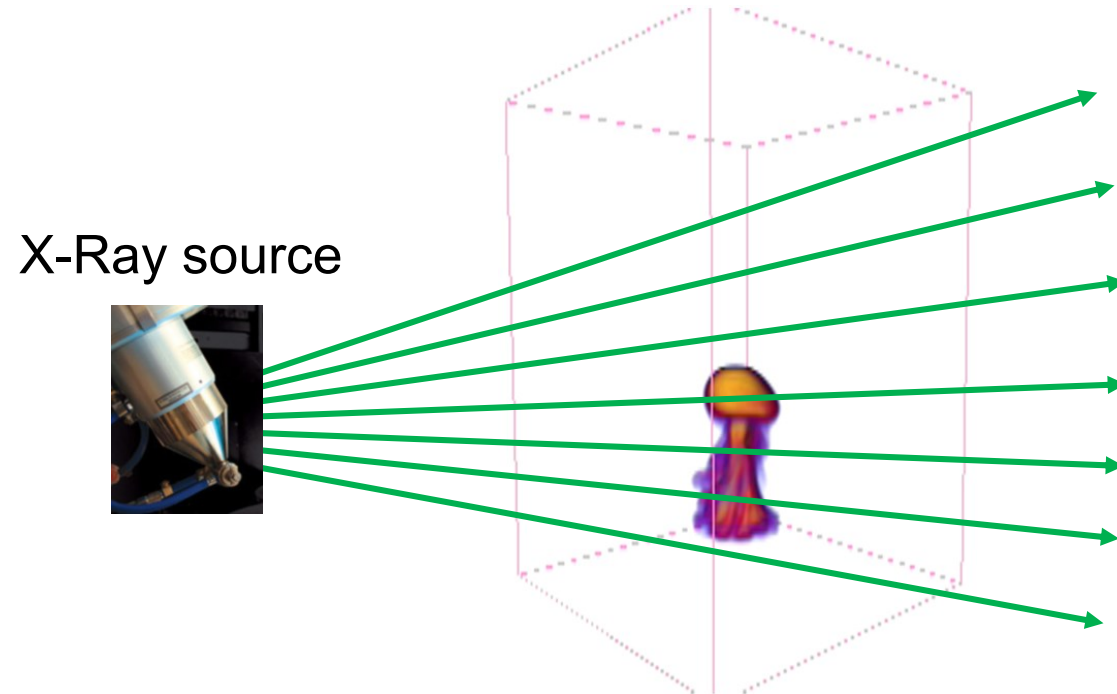
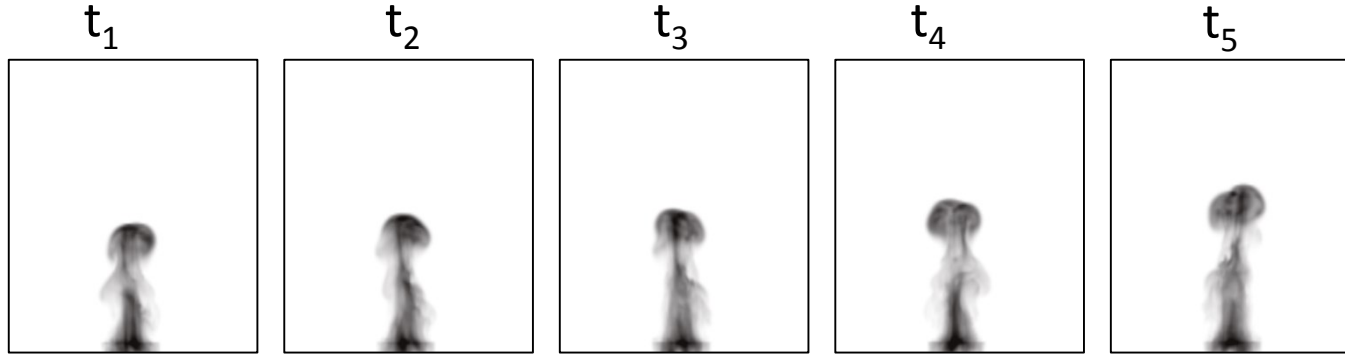


# Optimization framework



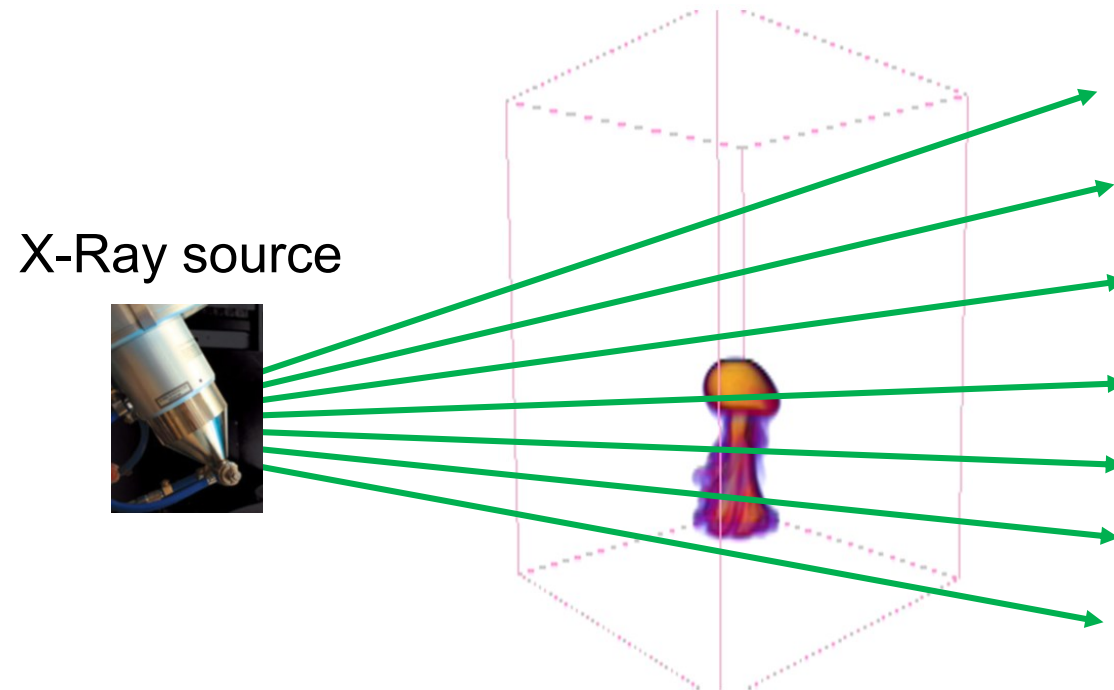
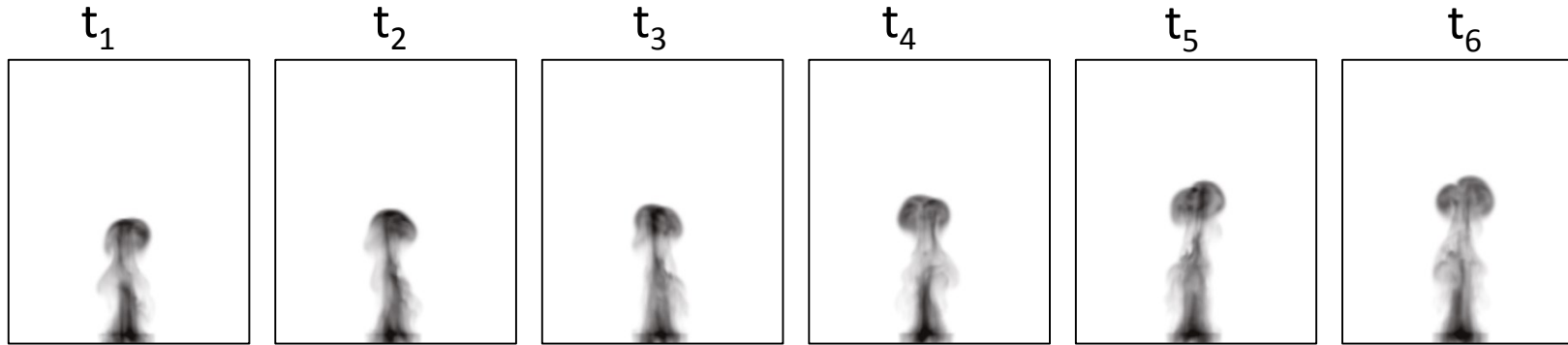


# Optimization framework



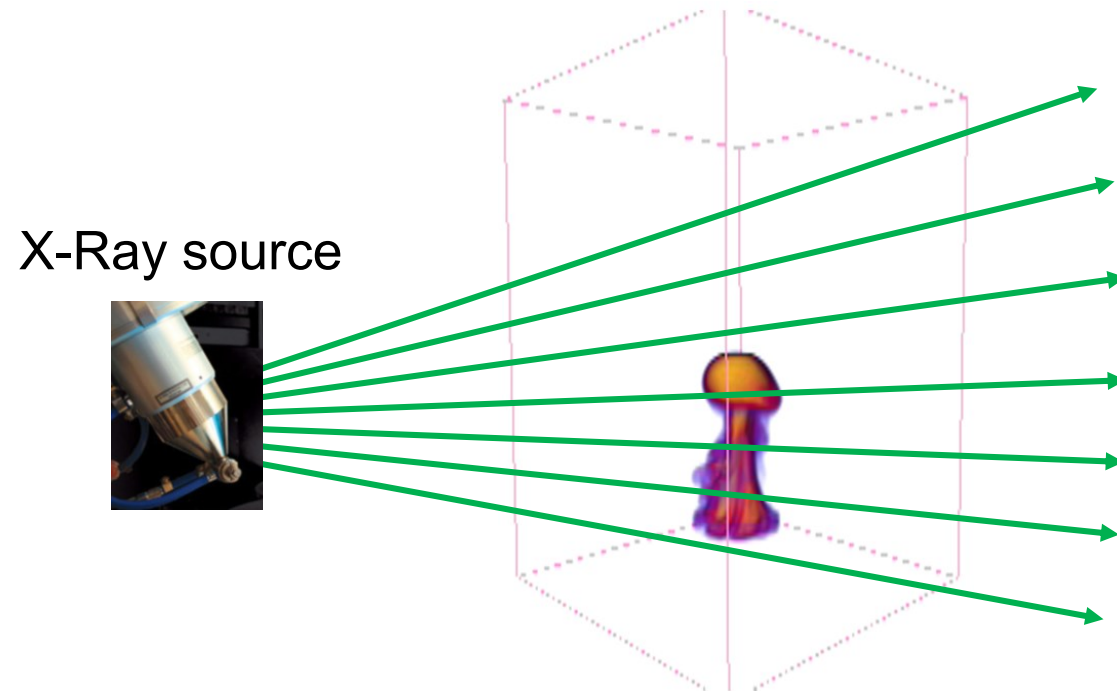
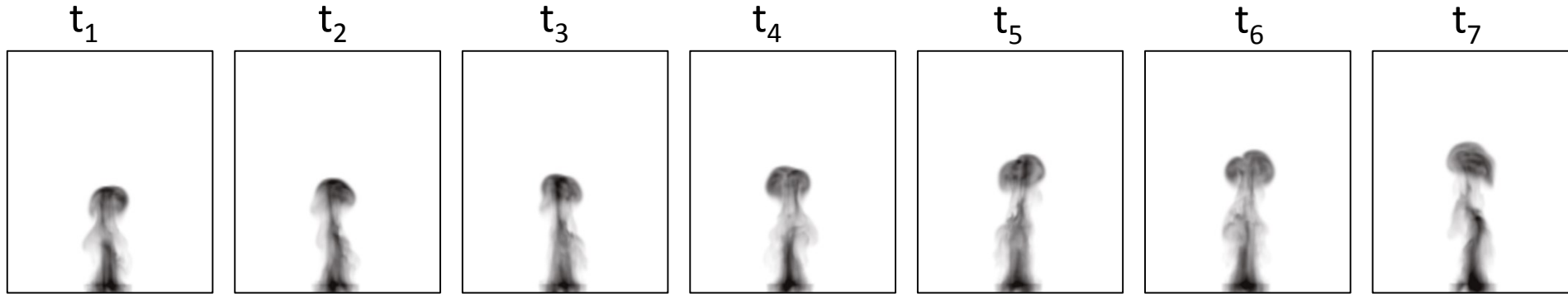


# Optimization framework





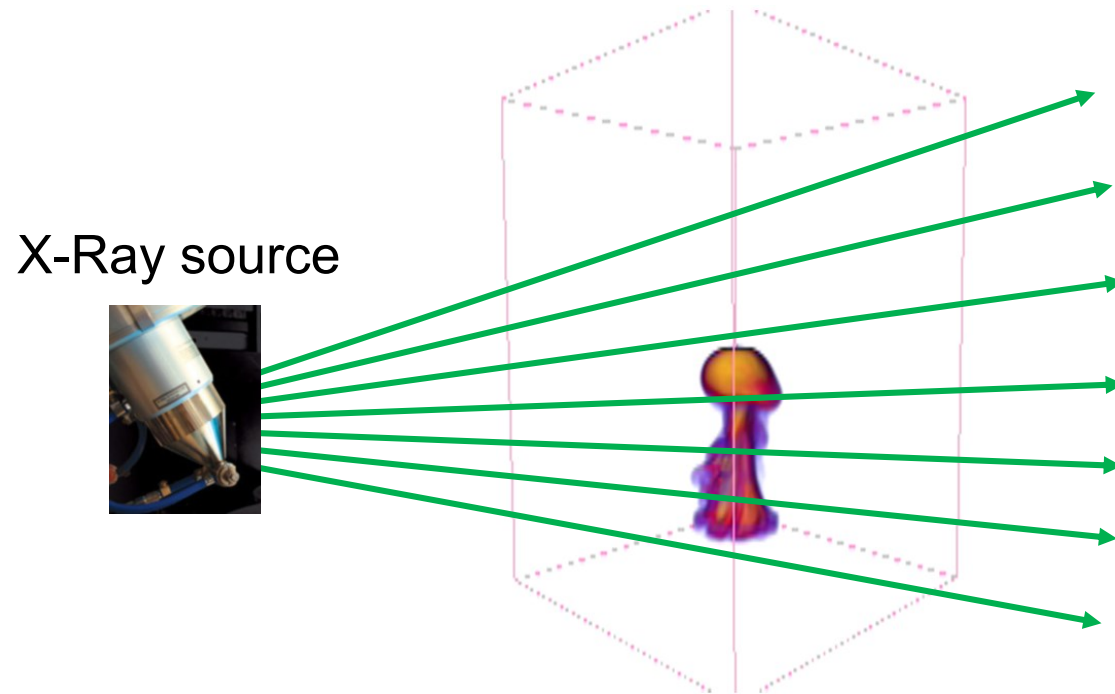
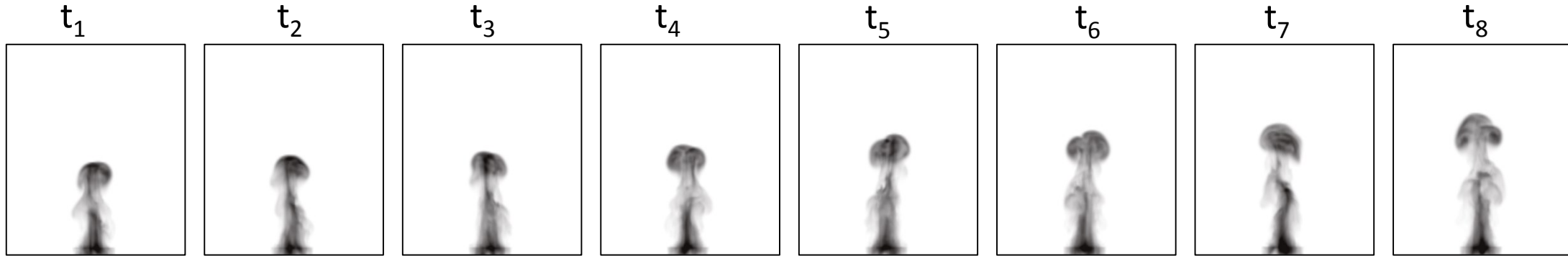
# Optimization framework





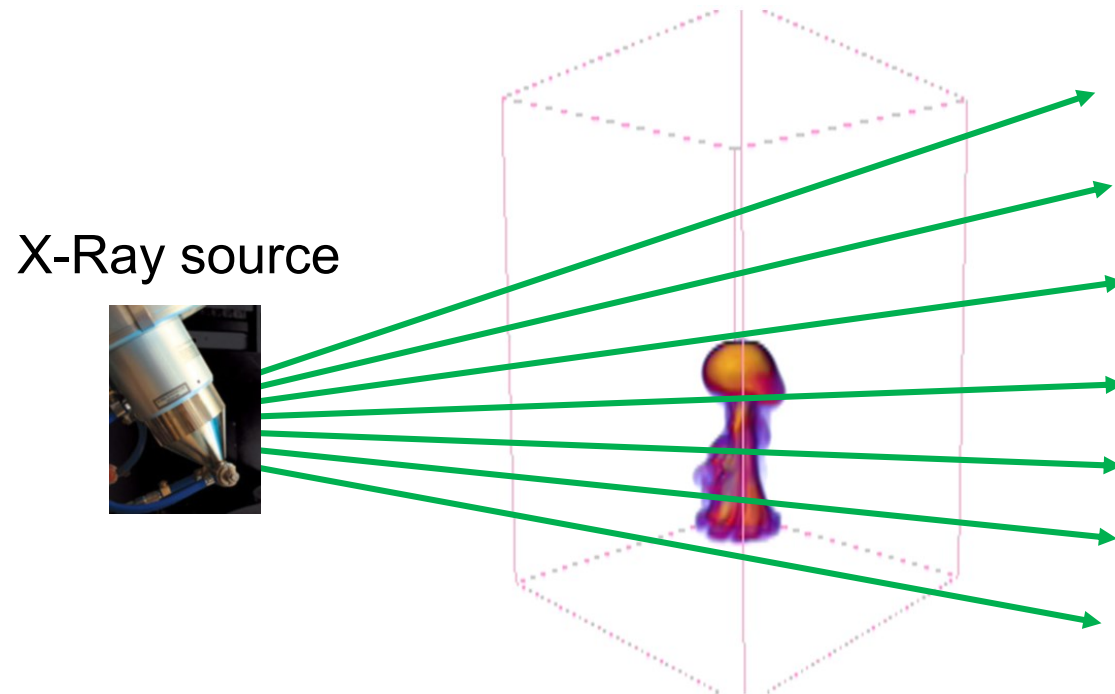
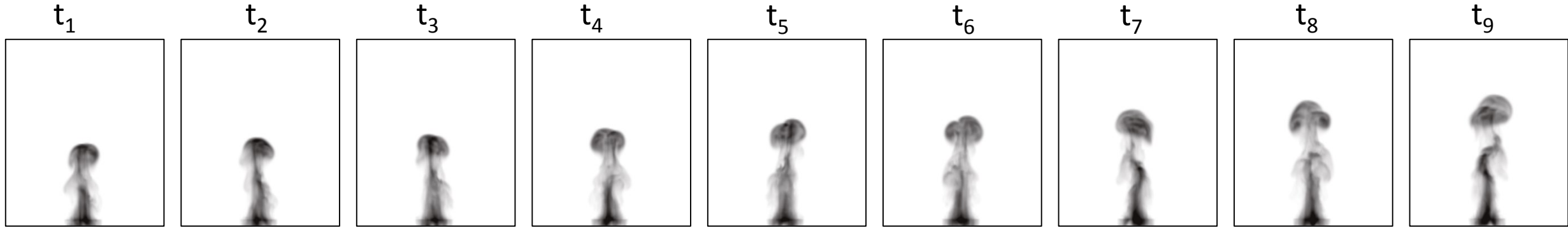


# Optimization framework



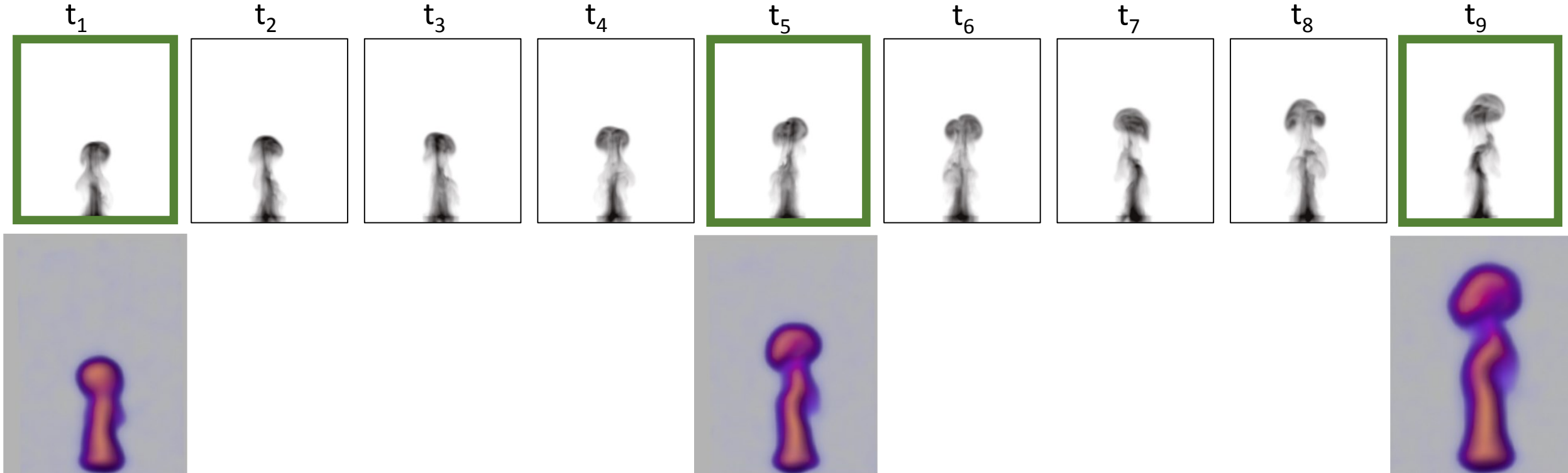


# Optimization framework



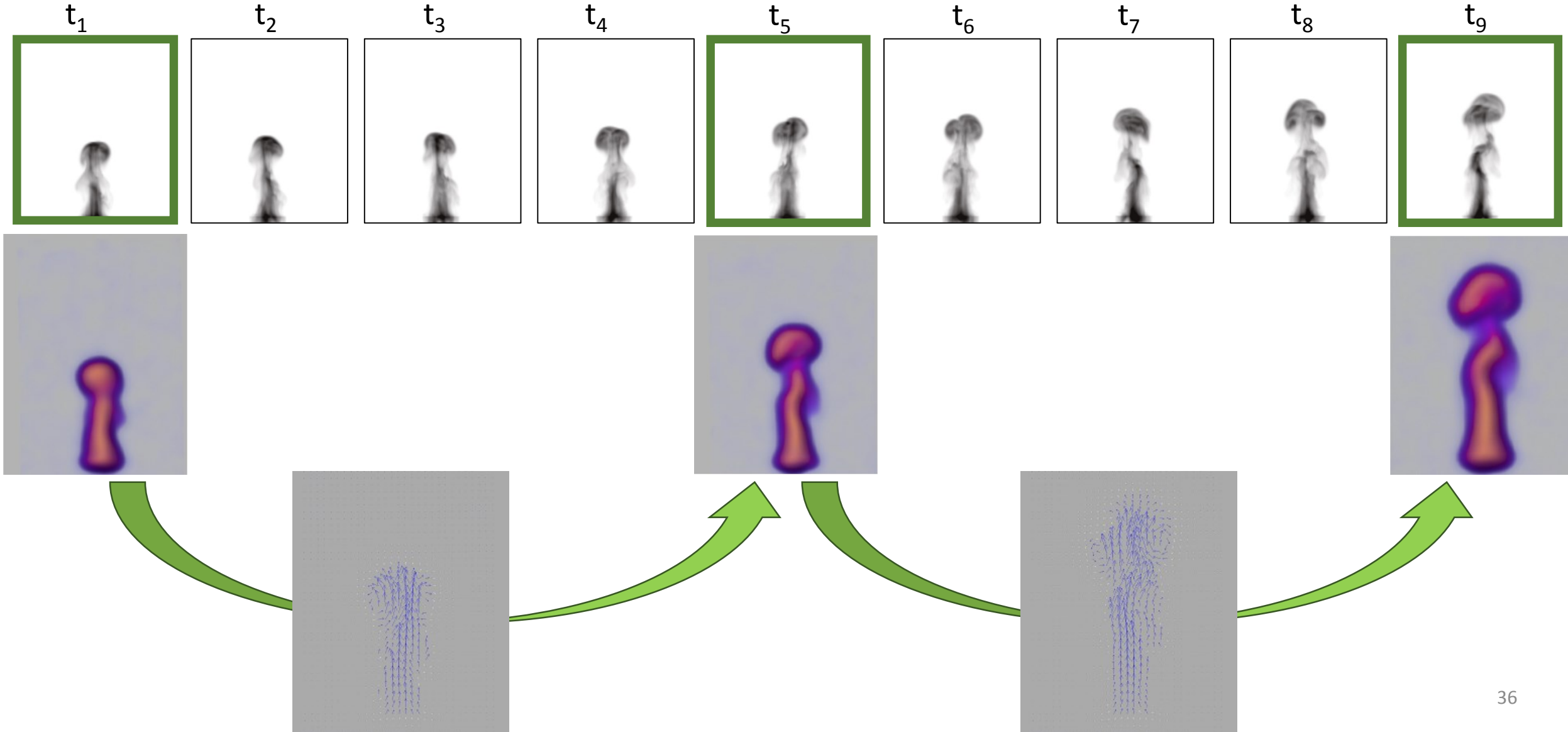


# Optimization framework



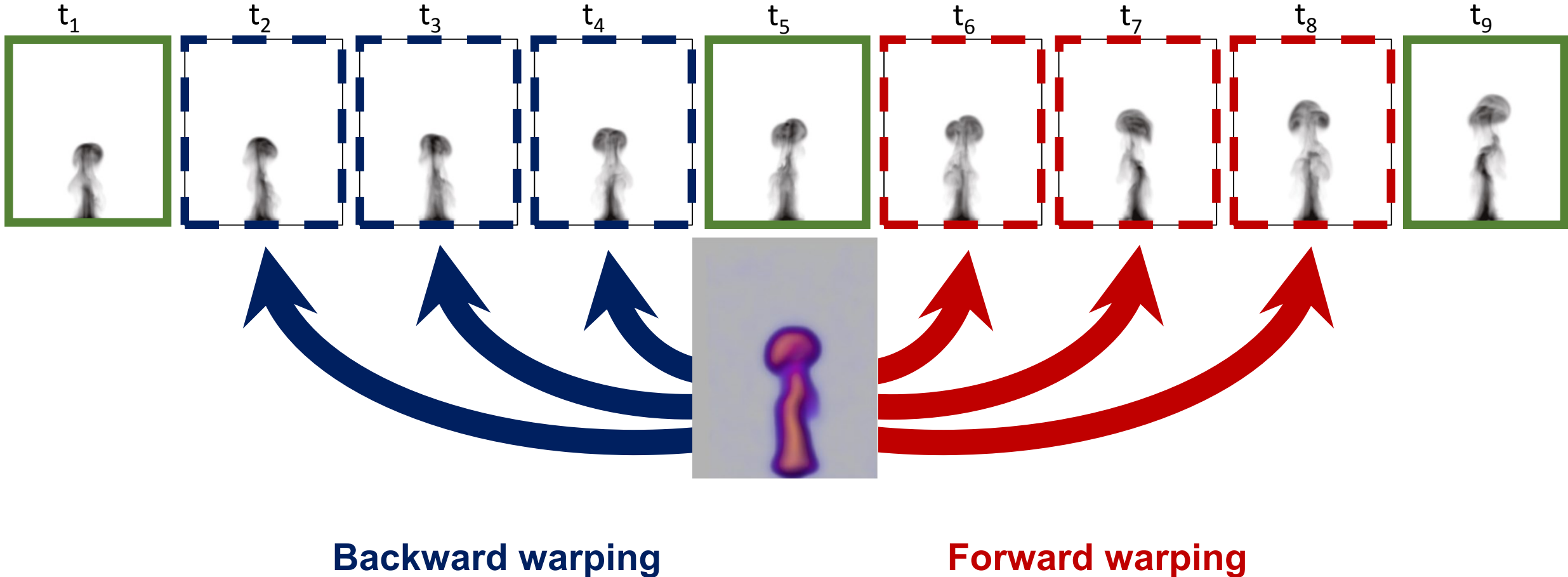


# Optimization framework



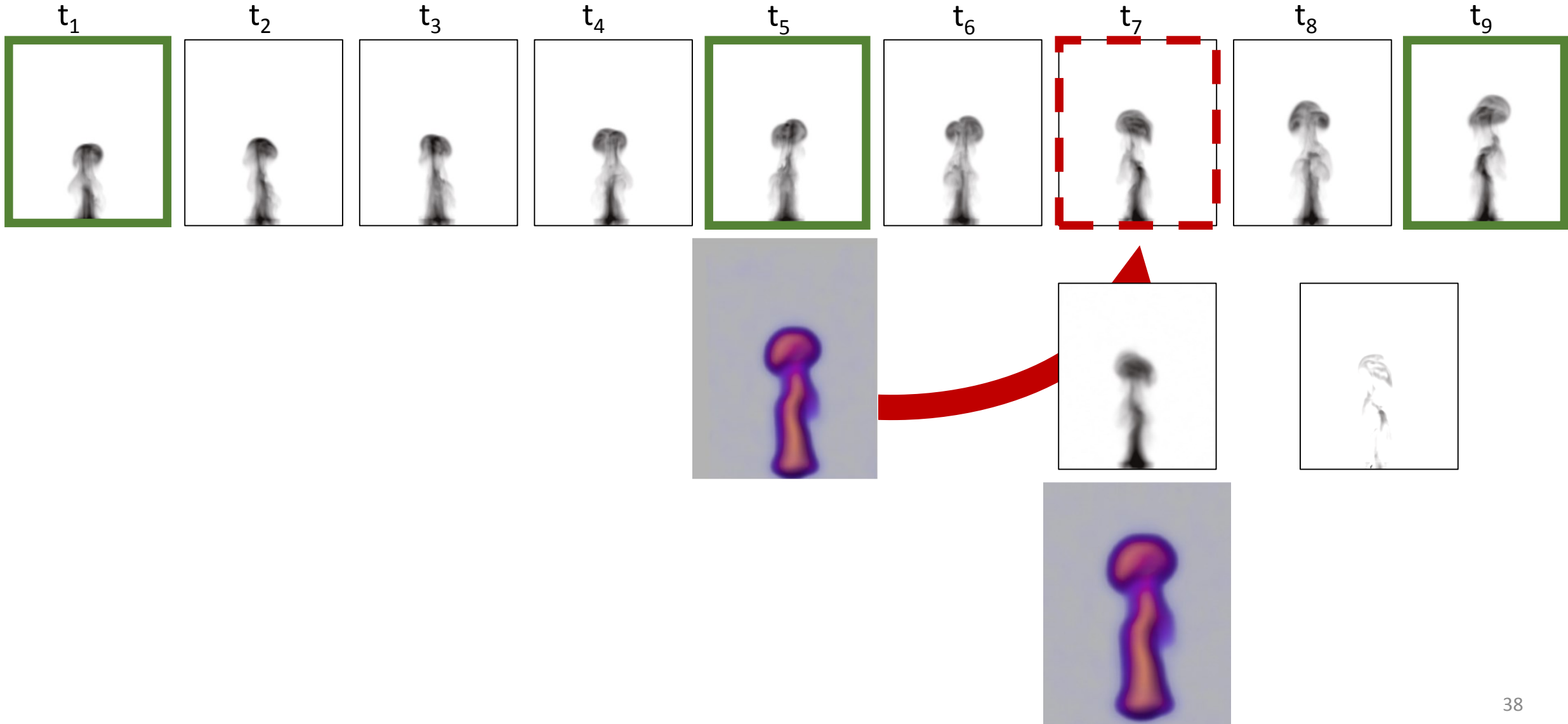


# Optimization framework



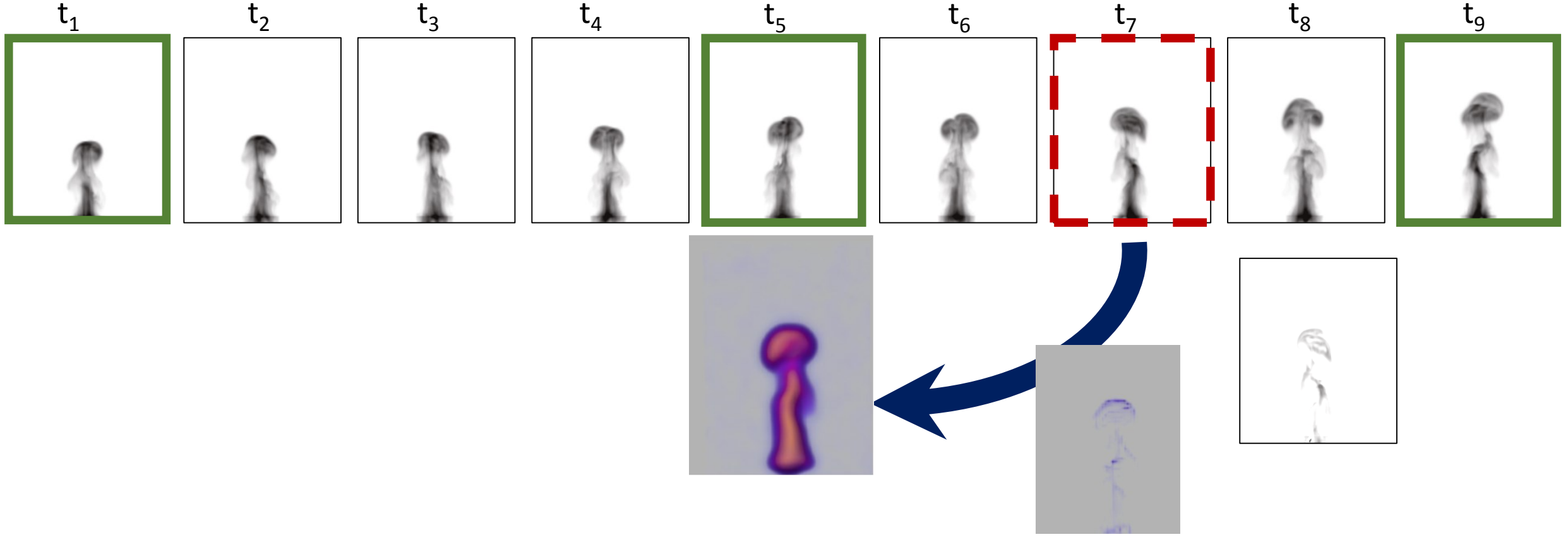


# Optimization framework



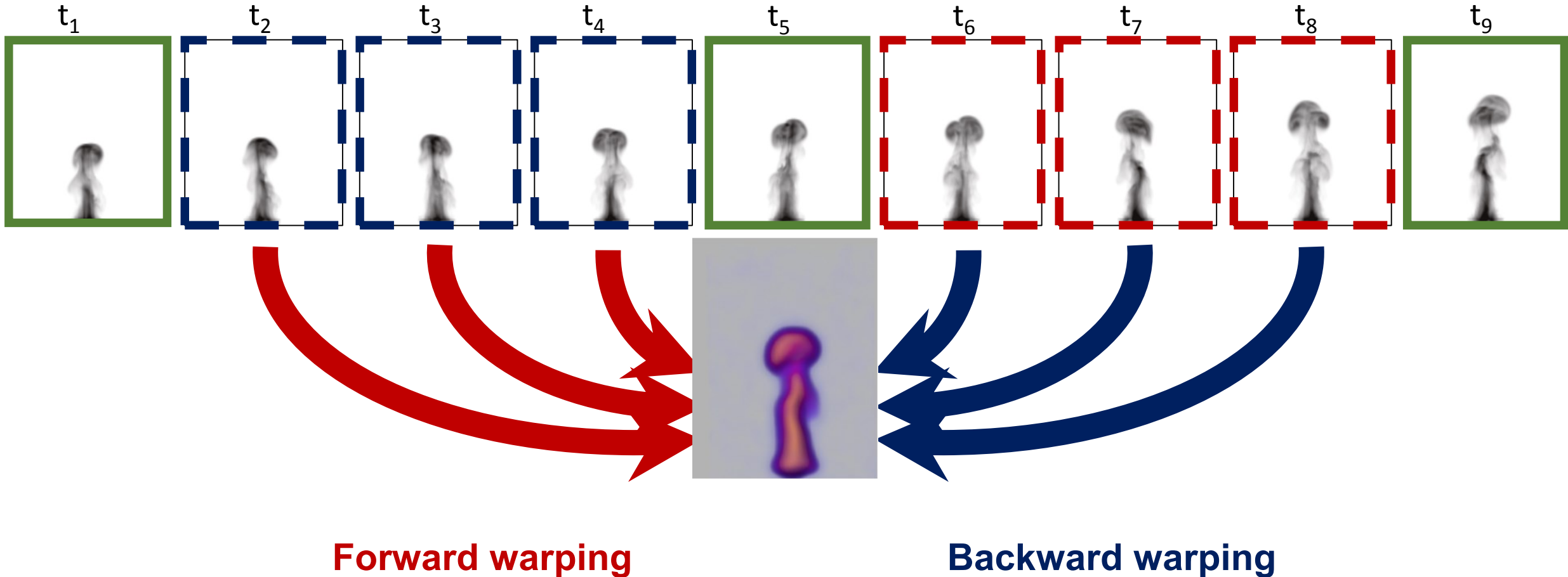


# Optimization framework





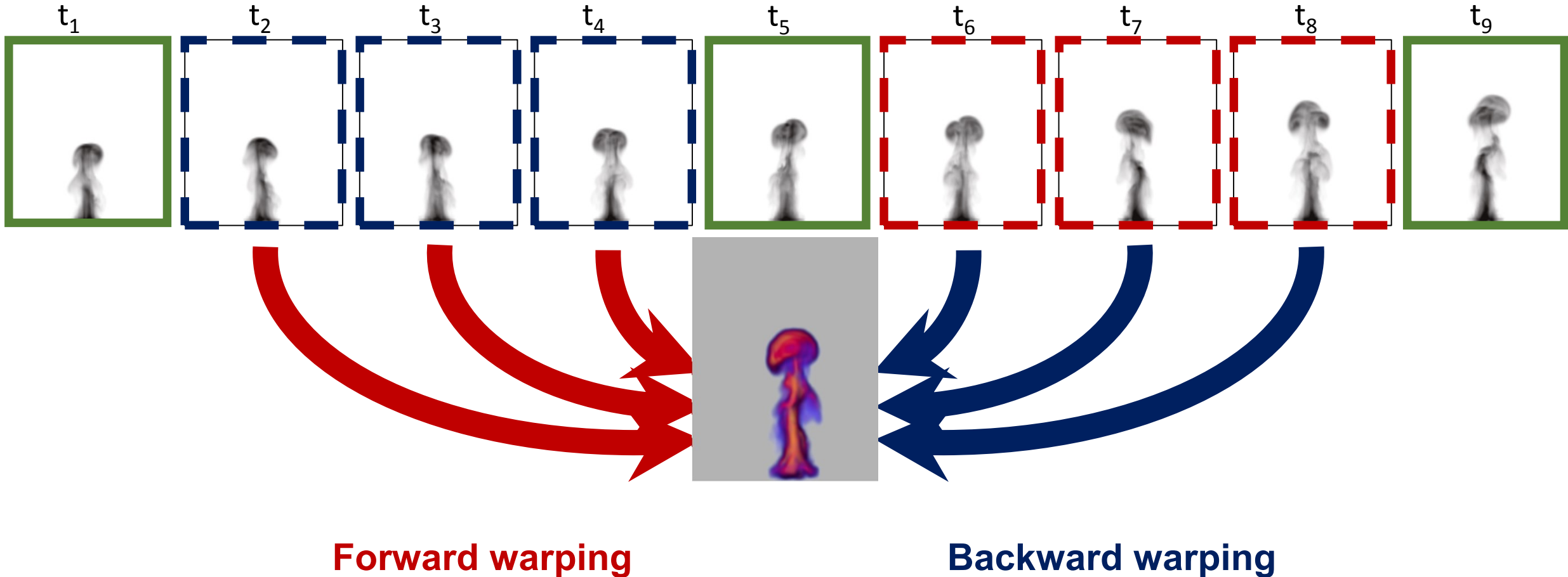
# Optimization framework





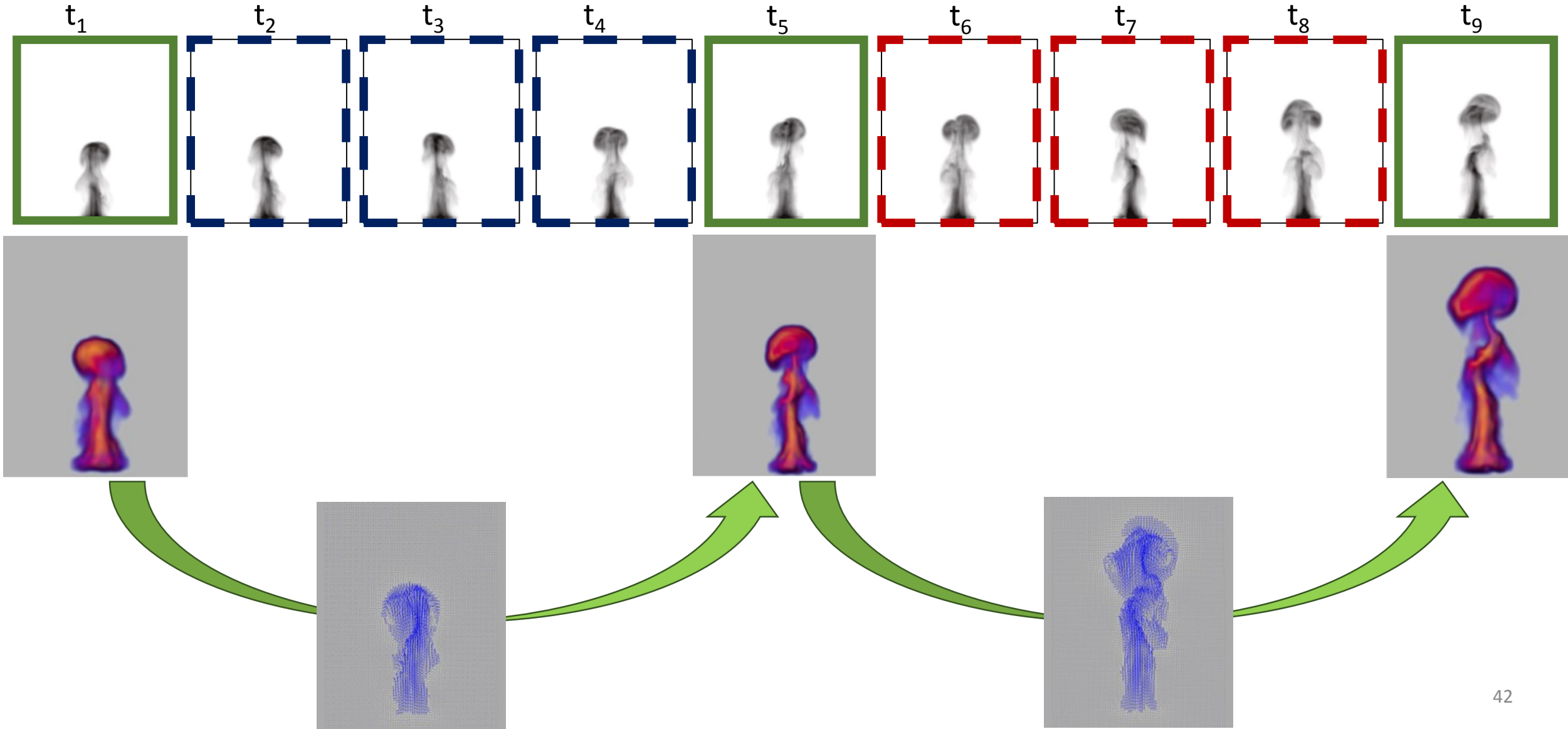


# Optimization framework



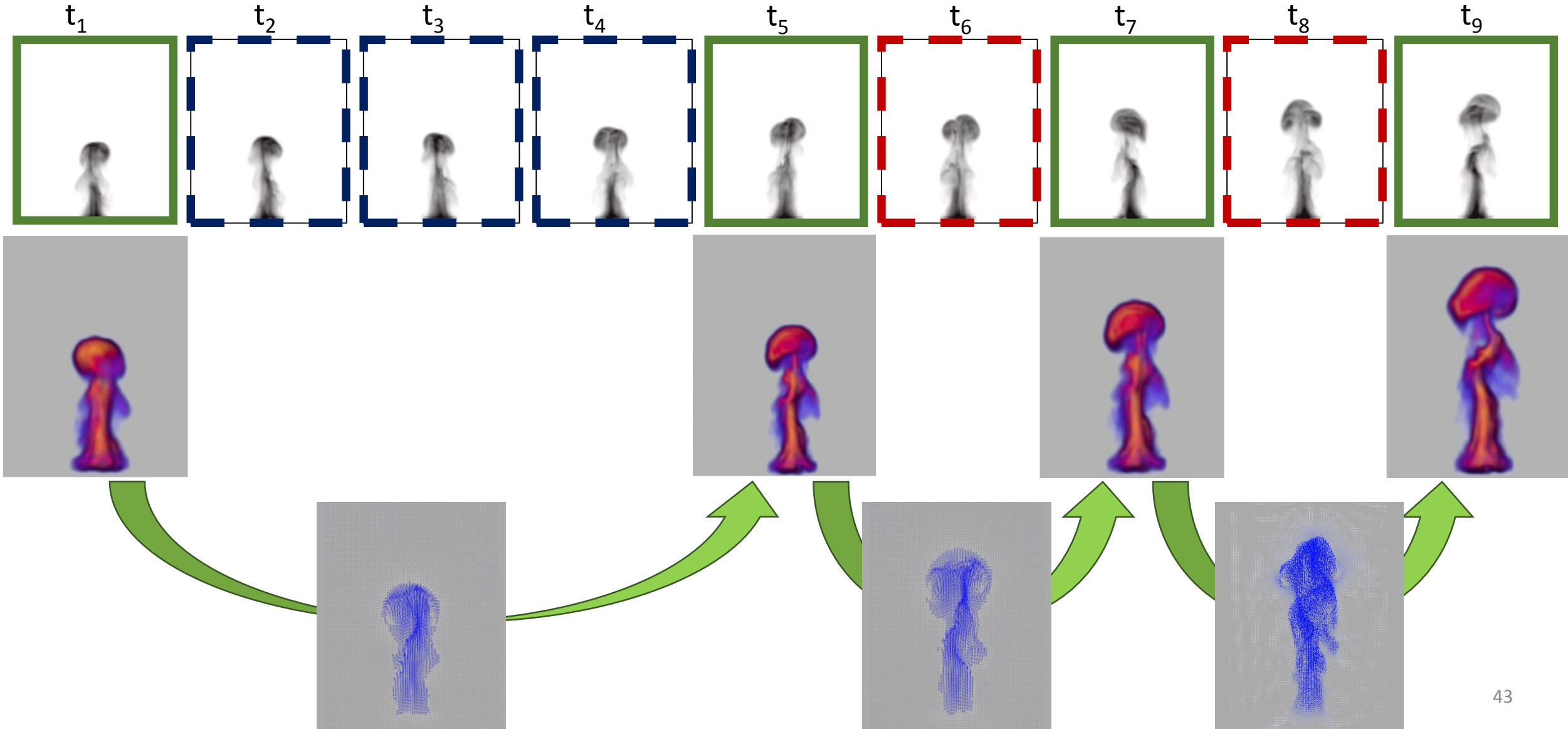


# Optimization framework





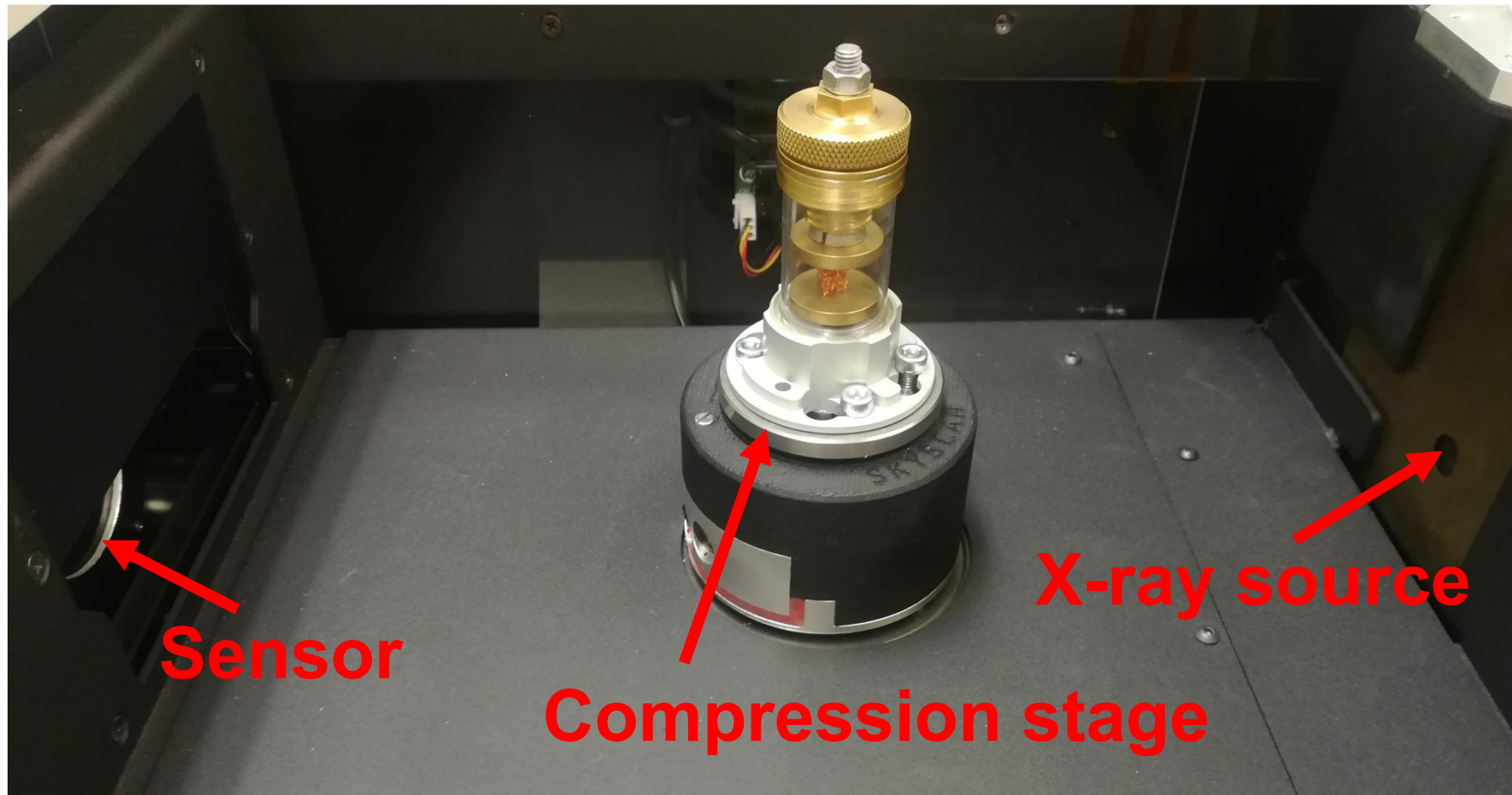
# Optimization framework





# Material deformation analysis

## Controlled compression of a copper foam



# Material deformation analysis



## Stop-motion capture

- Controlled compression
- 192 intermediate states
- 60 projections for each state

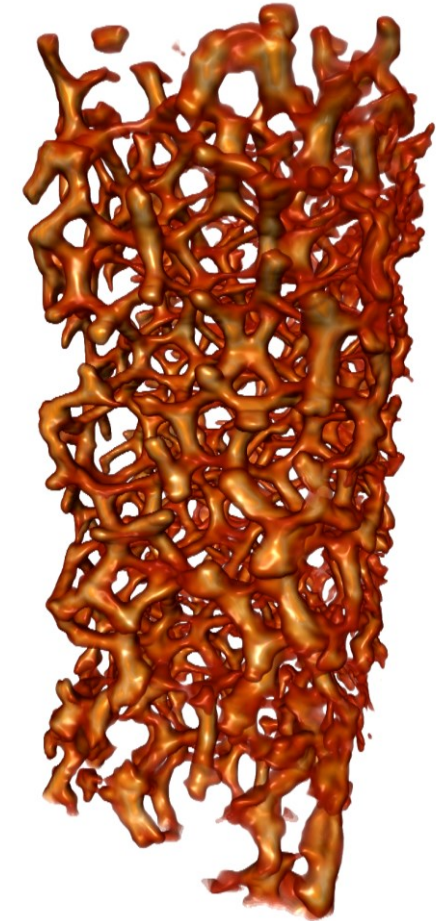


## Reconstruction

- Ground truth: 192\*60 projections
- Other reconstructions: only 192 projections



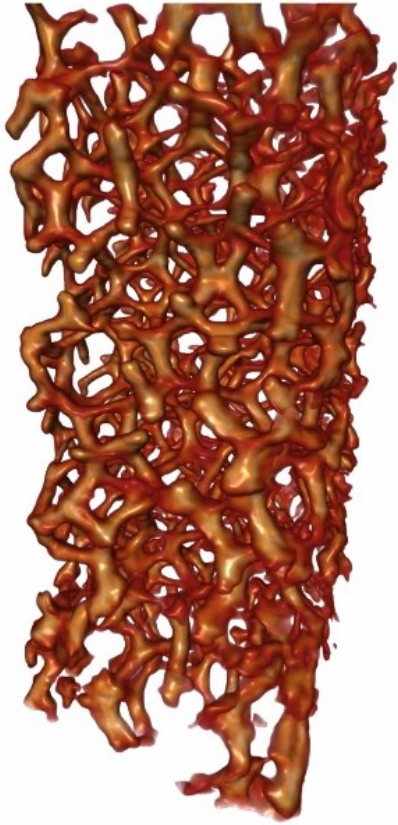
## Ground truth reconstruction



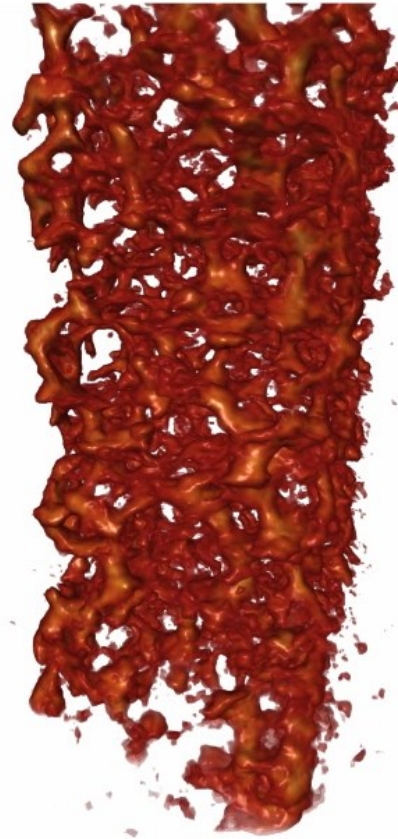


# Material deformation analysis

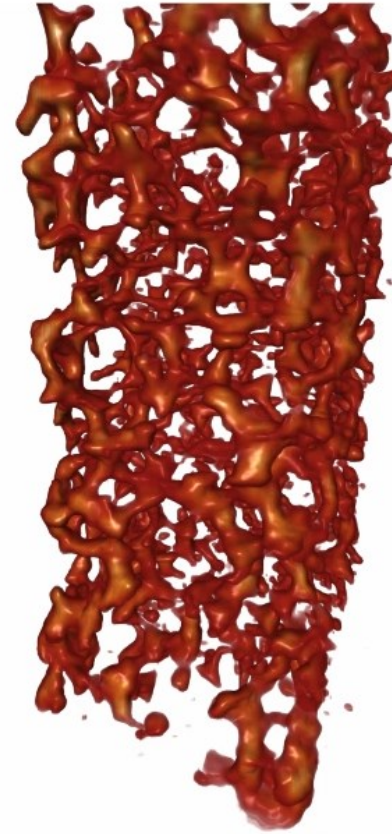
Ground truth



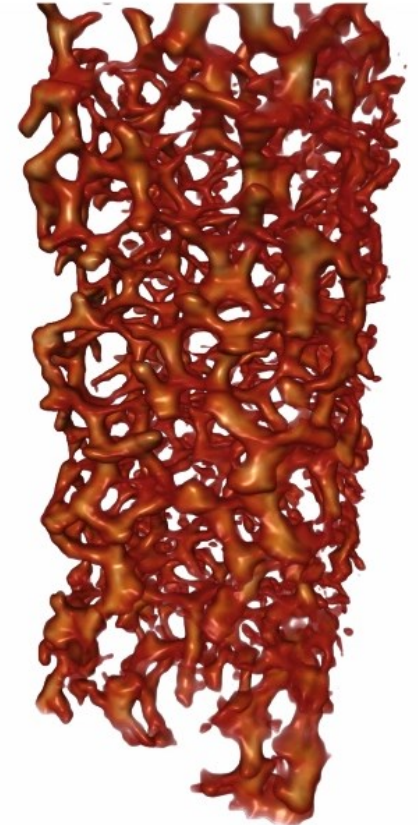
SART-ROF  
[Getreuer 2012]



ST-Tomography  
[Zang et al. 2018]



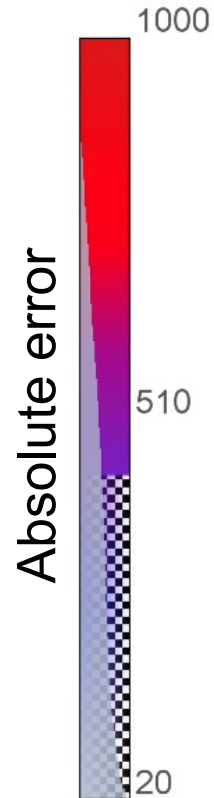
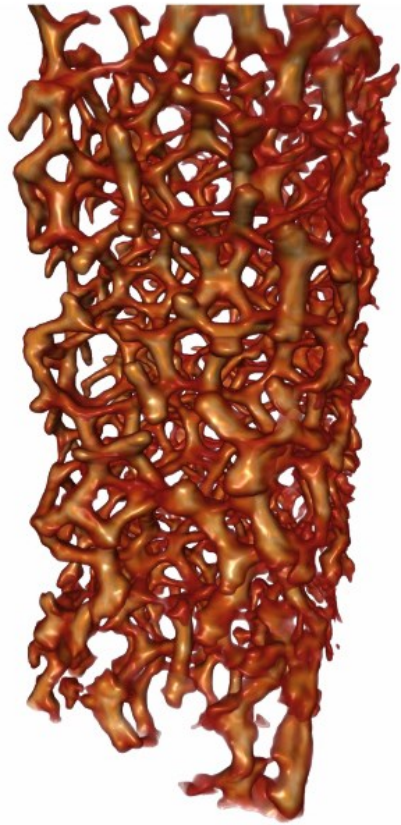
Warp-and-Project  
[Ours]





# Material deformation analysis

Ground truth



SART-ROF  
[Getreuer 2012]



ST-Tomography  
[Zang et al. 2018]



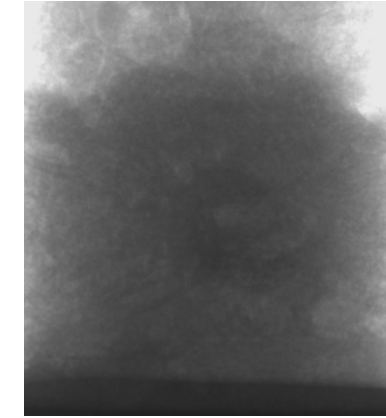
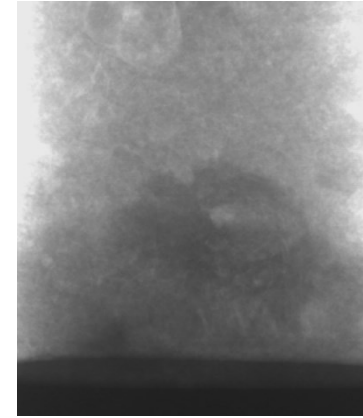
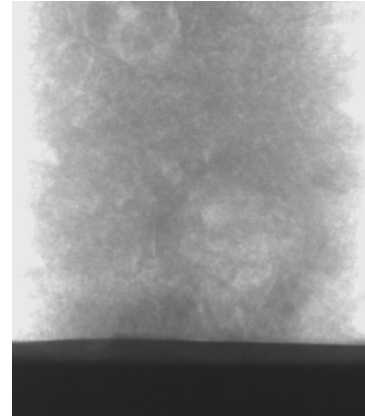
Warp-and-Project  
[Ours]



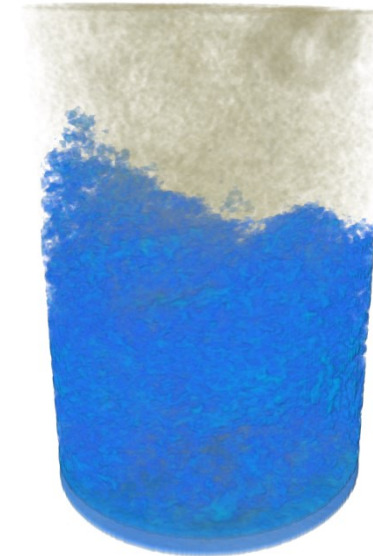
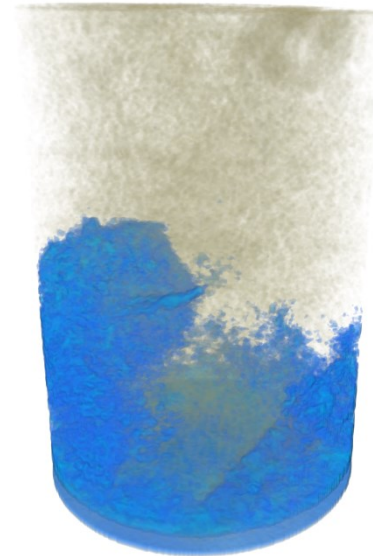
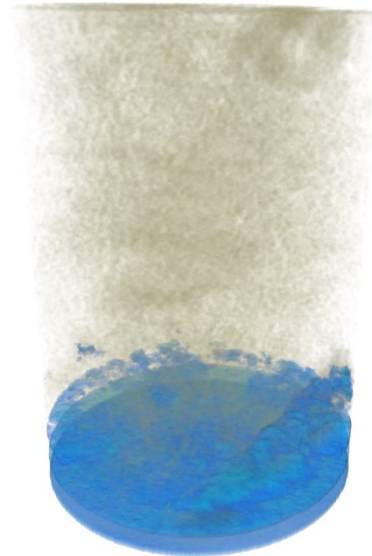
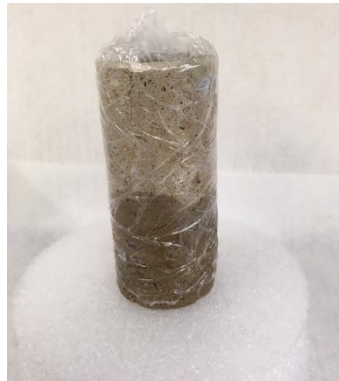


# Rock porosity characterization

**Before**

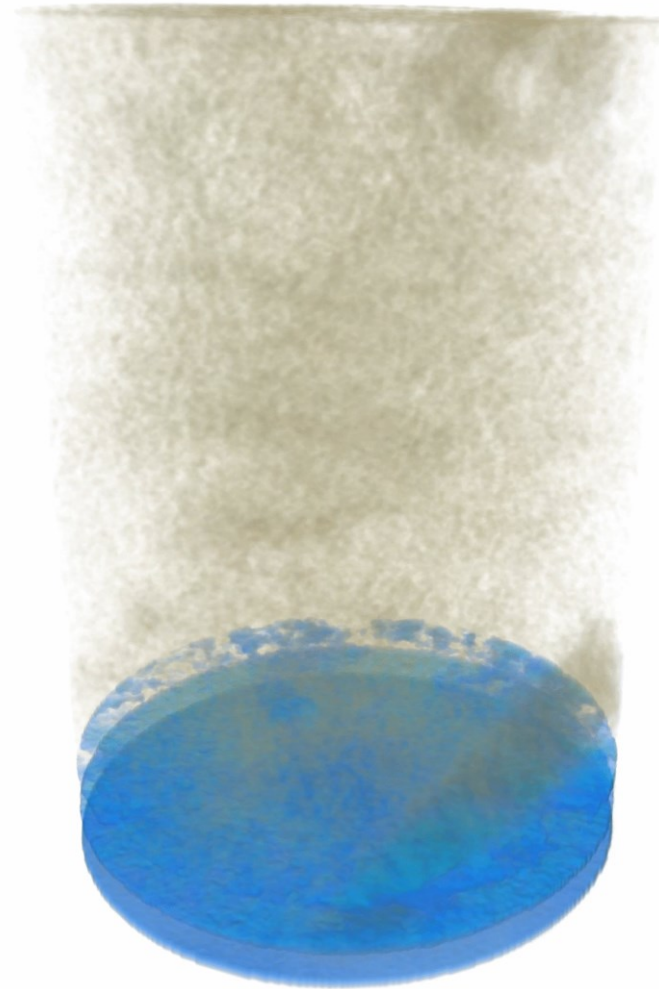


**After**





# Rock porosity characterization





# Fungus re-hydration

**Before**



**After**



Capture duration: 38min  
# projections: 600



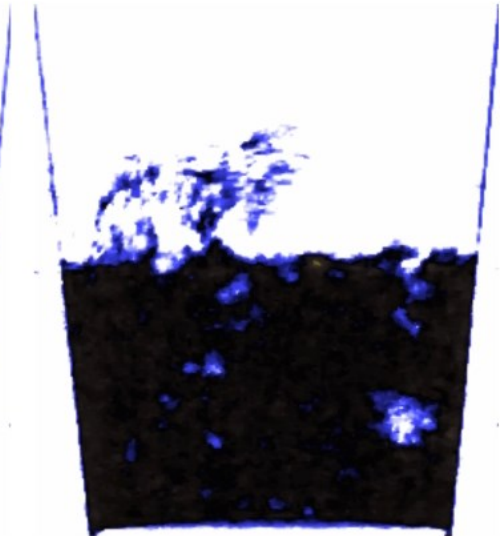
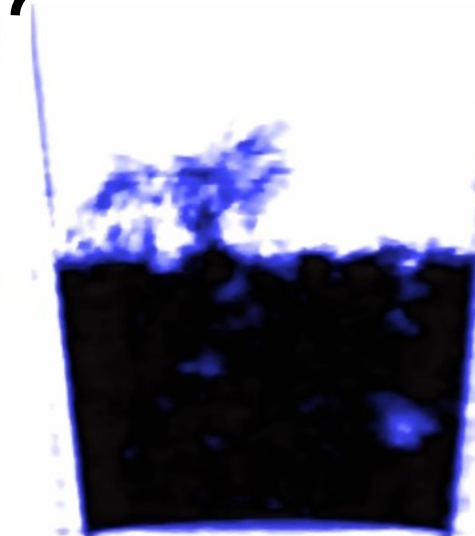
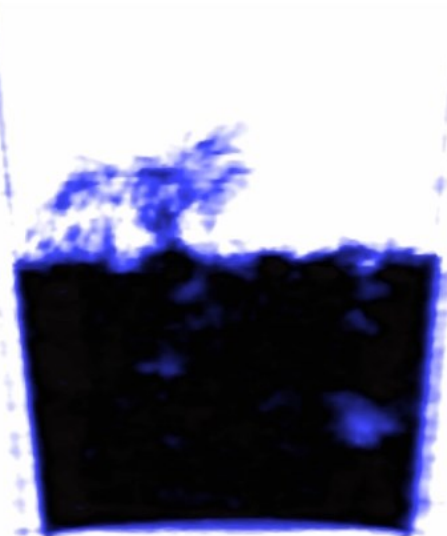
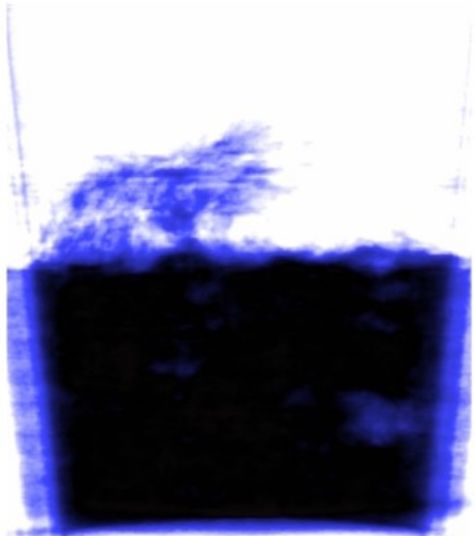
# Fungus re-hydration

## Temporal sampling

SART-ROF  
[Getreuer 2012]

ST-Tomography  
[Zang et al. 2018]

Warp-and-Project  
[Ours]



30 key frames

30 key frames

30 key frames

64 key frames

128 key frames



# Hydro-gel balls

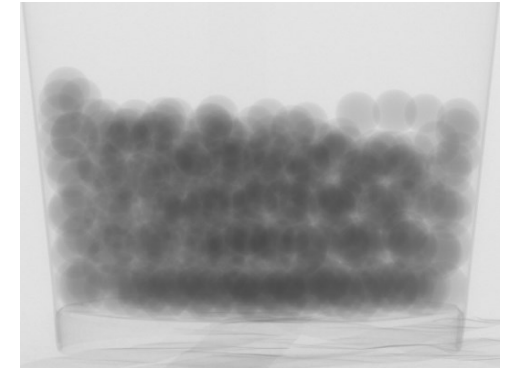
**Before**



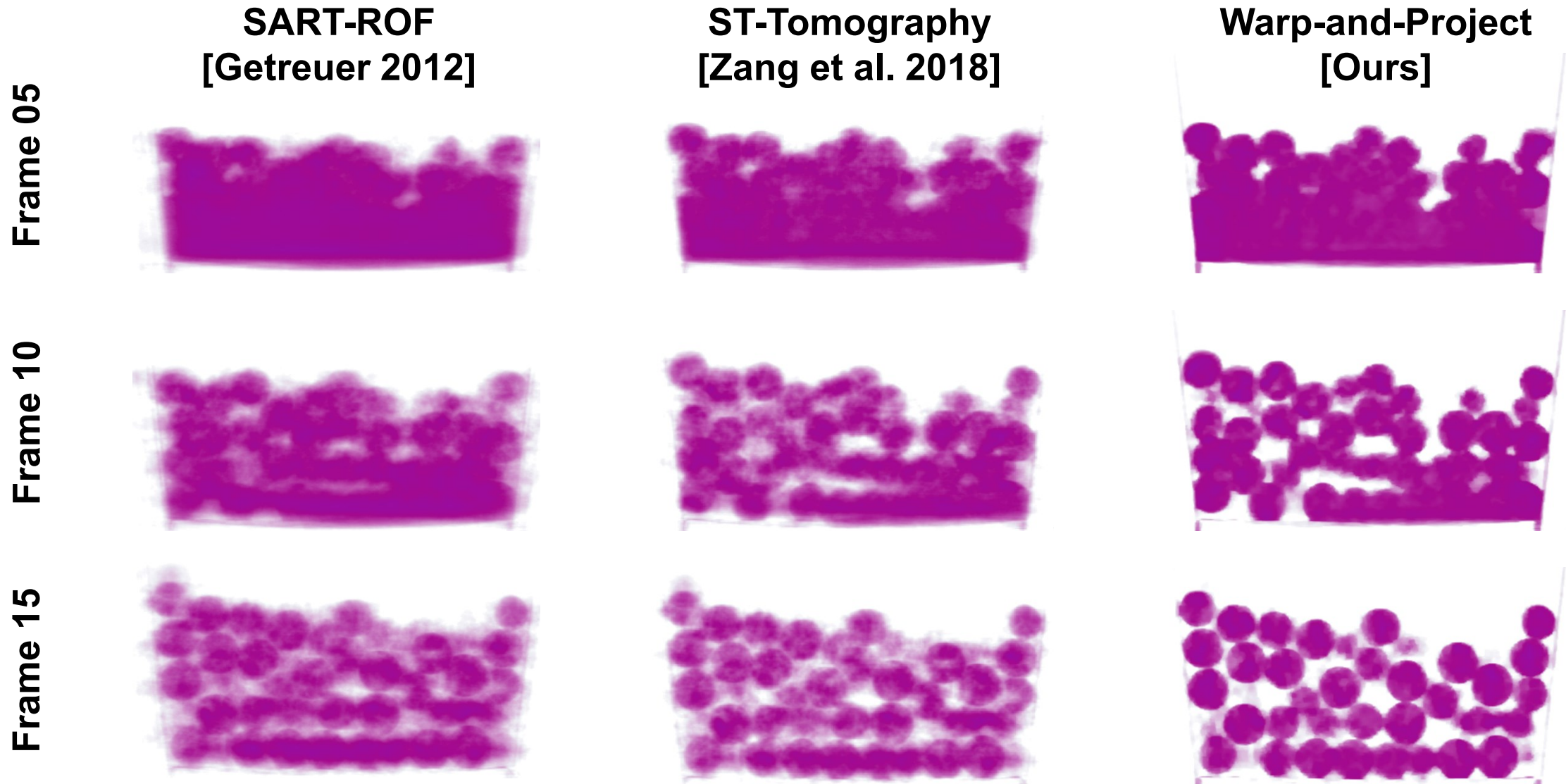
**After**



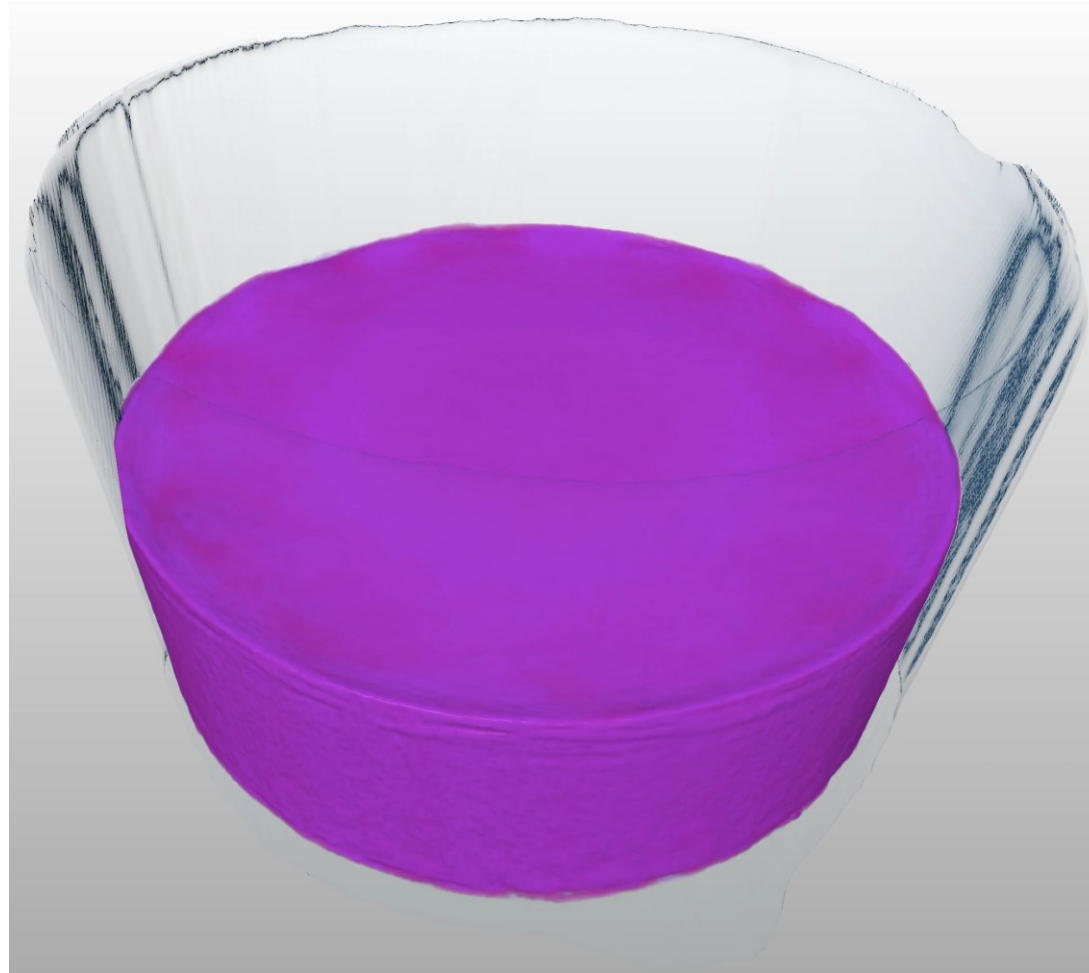
Capture duration: 43min  
# projections: 640



# Hydro-gel balls



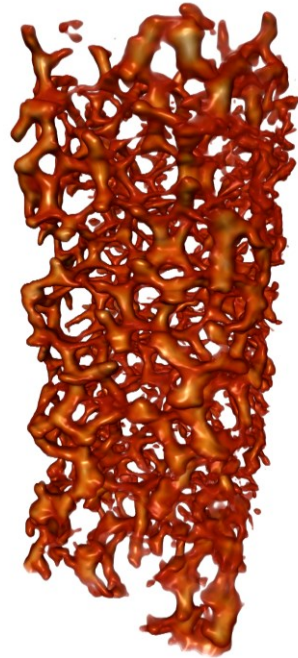
# Hydro-gel balls





# Summary

- A new 4D tomographic reconstruction for rapidly deforming objects
- Well suited to graphics, and several scientific applications



Material deformation analysis



Porosity characterization



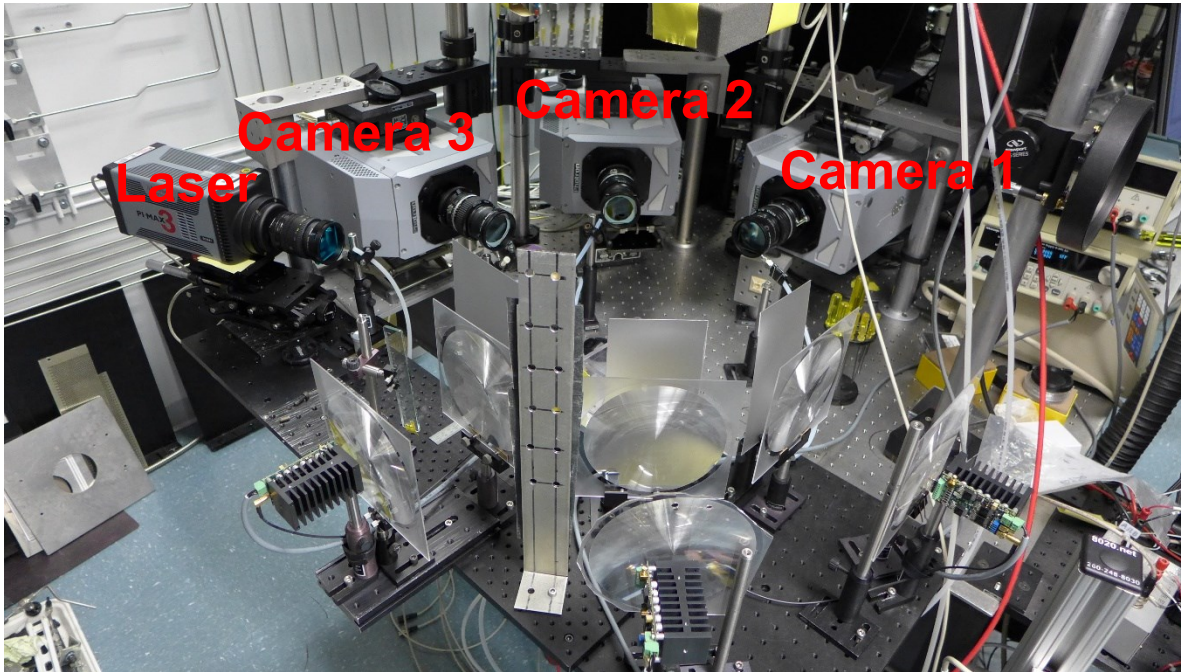
# What is next ?

- Software engineering (memory, computation speed, etc.)
- Combination with other tomography techniques (e.g. phase contrast tomography)
- Extension to other imaging modalities (e.g. electron microscopy)

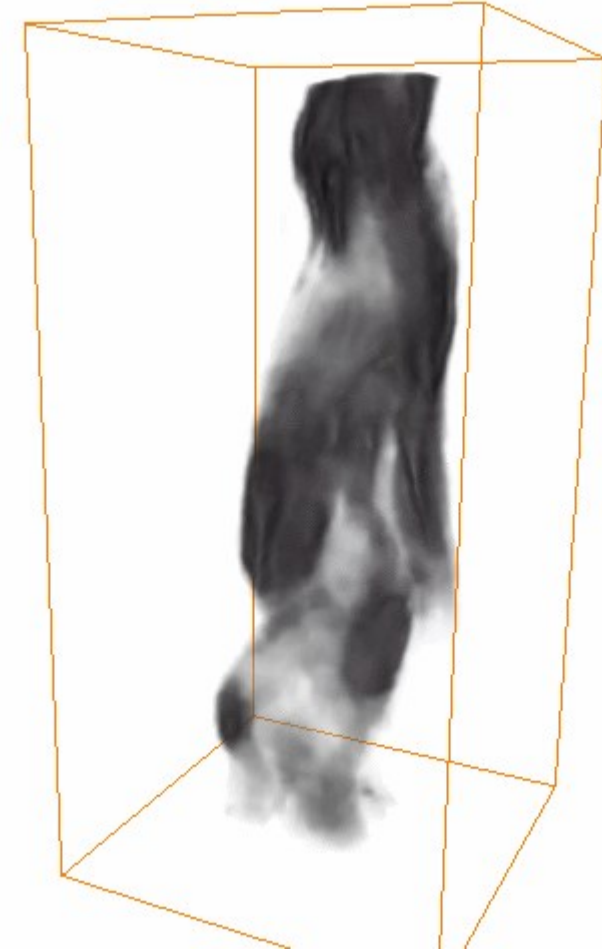


# What is next ?

## Applications: High speed 3D soot imaging



Camera 2



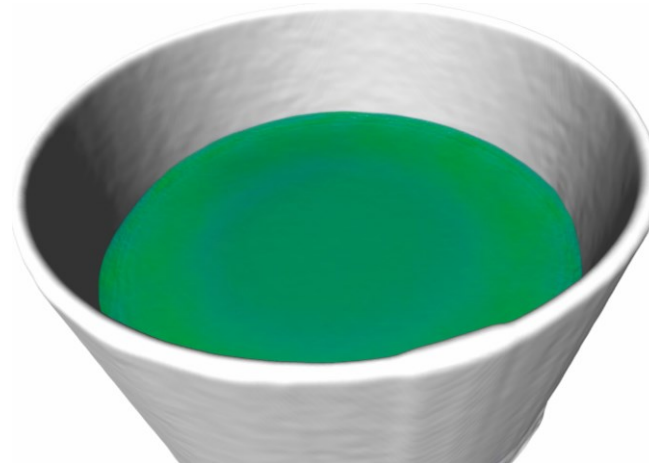
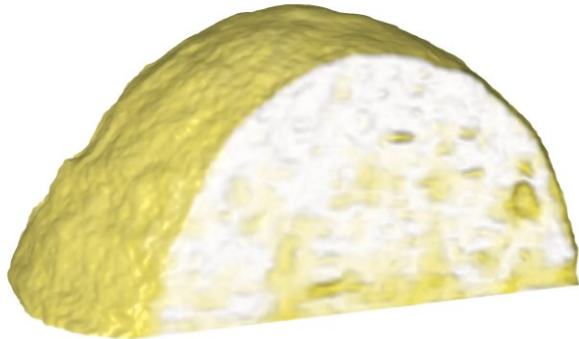
# Thank you !



**Project webpage:**



Code and data  
available soon



Sponsorship: KAUST as part of VCC Center Competitive Funding.