A BRIEF INTRODUCTION TO FLUID SIMULATION IN COMPUTER GRAPHICS

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OVERVIEW

- Simulation: what and why
- For Newton's sake
- Between continuous and discrete worlds
- Fine arts make difference

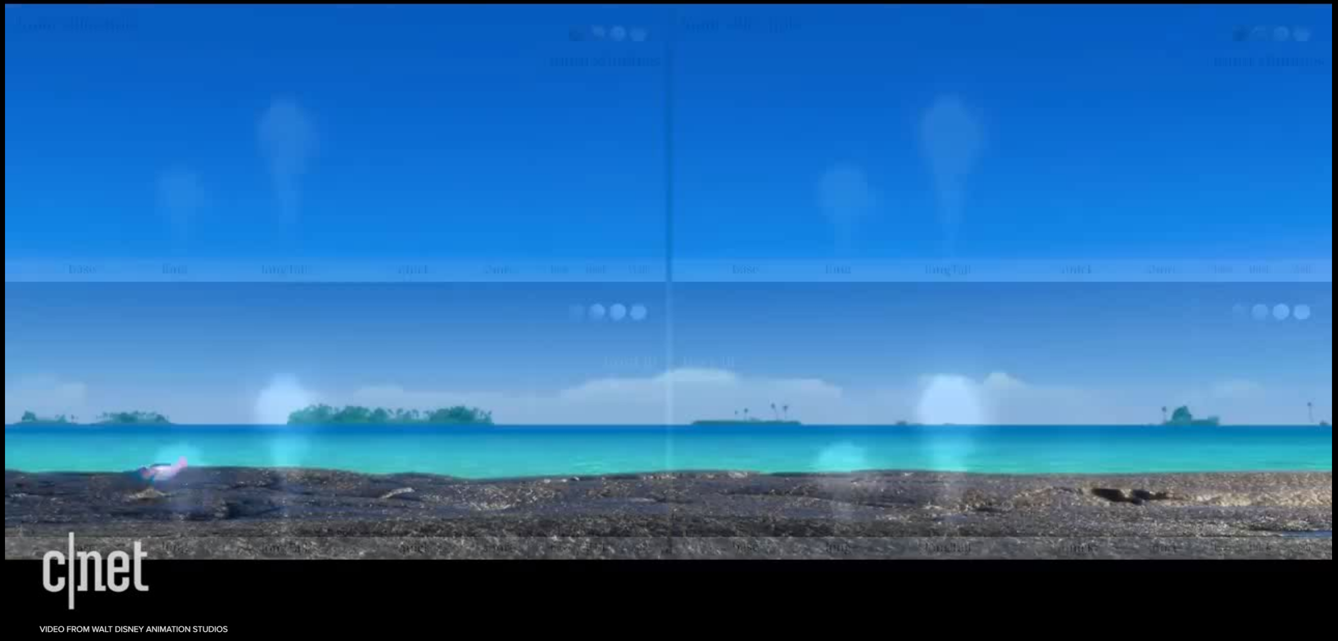
SIMULATION: WHAT AND WHY

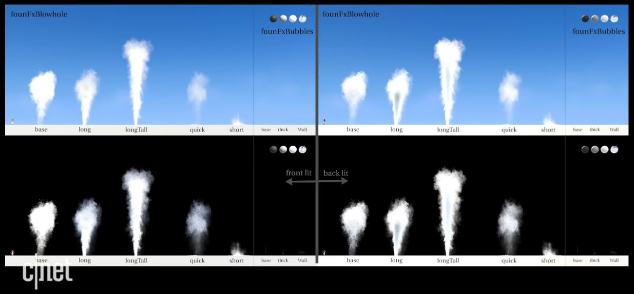
- Duplicate of the real world
  - Reality

Clothes are simulated by physics engine in *Bao*, Pixar 2018

SIMULATION: WHAT AND WHY

- Duplicate of the real world
  - Reality
- Prediction and design
  - Flexibility



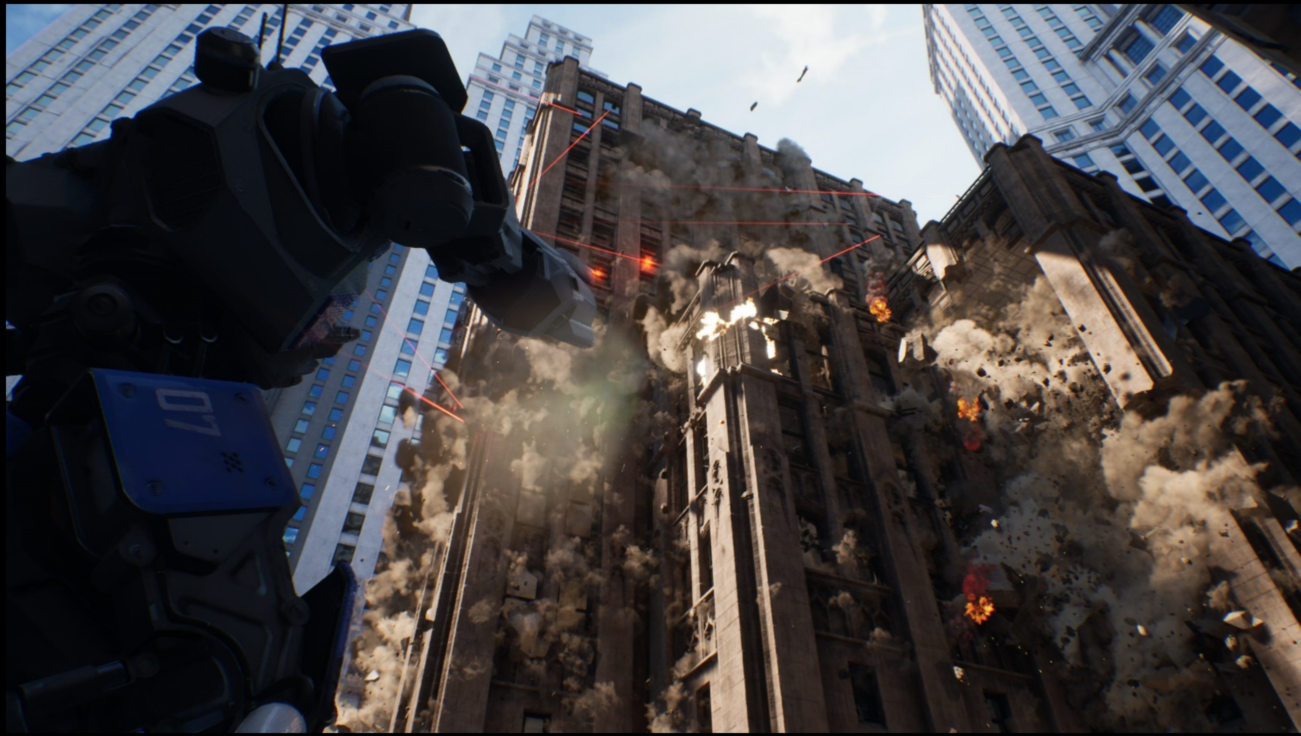


Foundational simulation results become elements in a more complex scenario in *Moana*, Disney 2016

Video credit from Youtube.com

SIMULATION: WHAT AND WHY

- Duplicate of the real world
  - Reality
- Prediction and design
  - Flexibility
- Immersion in the virtual environment
  - Interactivity



Interactive distruction in “Chaos” physics system by Epic, 2019

SIMULATION: WHAT AND WHY

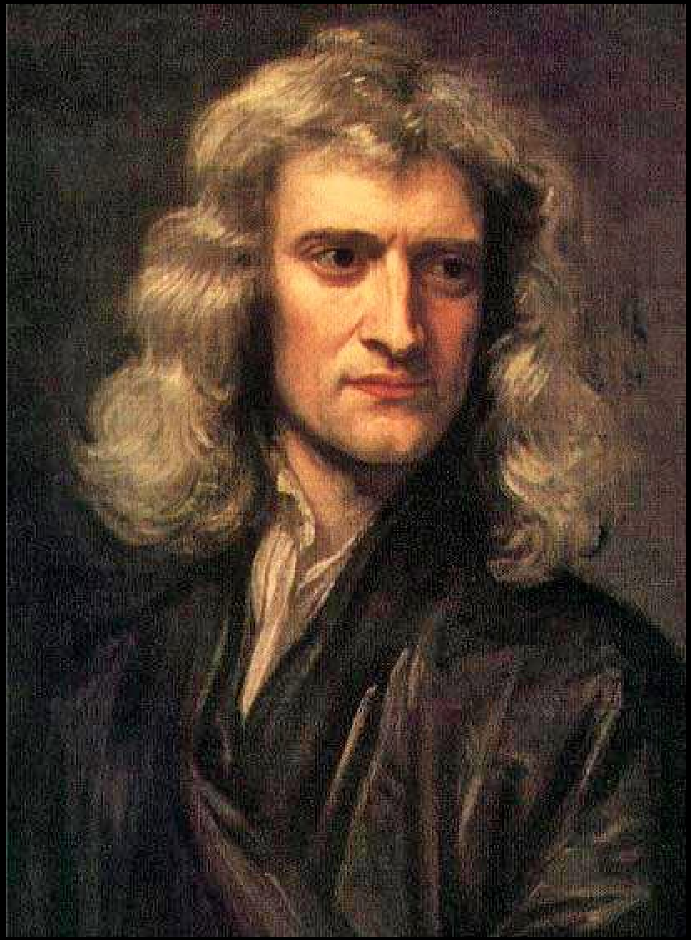
- It profits




VFX in *The Wandering Earth*, 2019

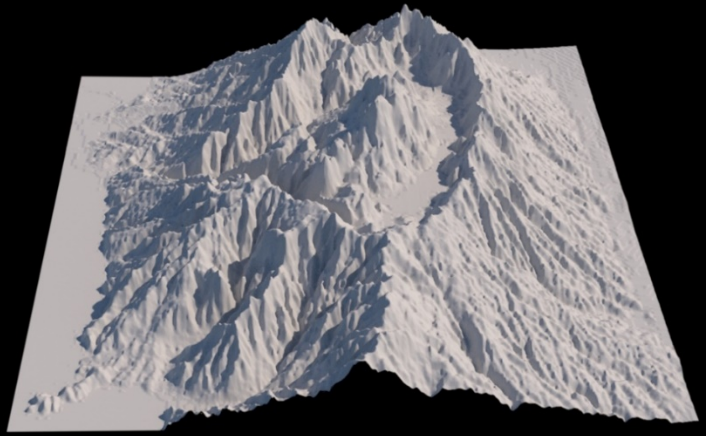
FOR NEWTON'S SAKE

- Newton's Laws of Motion
  - Esp. the second law $\mathbf{F} = m\frac{d\mathbf{u}}{dt} = m\mathbf{a}$
- Newton-Leibniz formula
  - $\int_a^b f(x)dx = F(b) - F(a)$

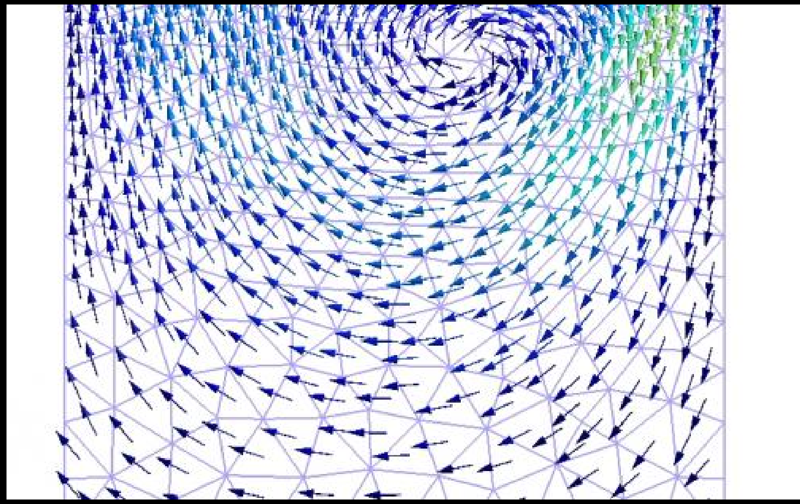


FOR NEWTON'S SAKE

- Scalar and Vector fields
  - $f = f(\vec{x}, t)$
  - $\vec{x}$ is d-dimensional vector
  - f can be scalar or vector



- A height field



- A velocity field

FOR NEWTON'S SAKE

- The Navier-Stokes Equations (incompressible fluid, simple form)

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + g + \nu \nabla \cdot \nabla u \quad \Leftarrow \quad a = \frac{F}{m}$$

$$\nabla \cdot u = 0 \quad \Leftarrow \quad \text{Mass conservation}$$

- Unknowns: u (d unknowns), p (1 unknown), total d+1
- Equations: d momentum equations, 1 mass equation, total d+1

- We can solve it … theoretically

FOR NEWTON'S SAKE

- The material derivative

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + g + \nu \nabla \cdot \nabla u$$

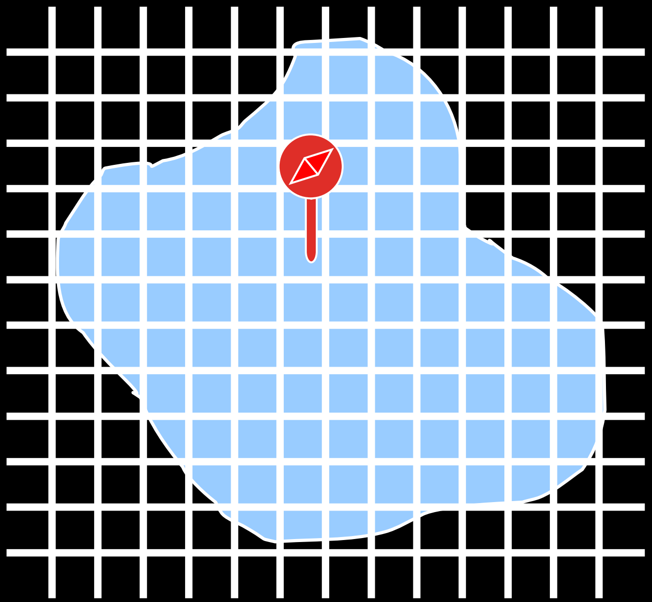
- Math view: total derivative
- $a = \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + u \cdot \nabla u$

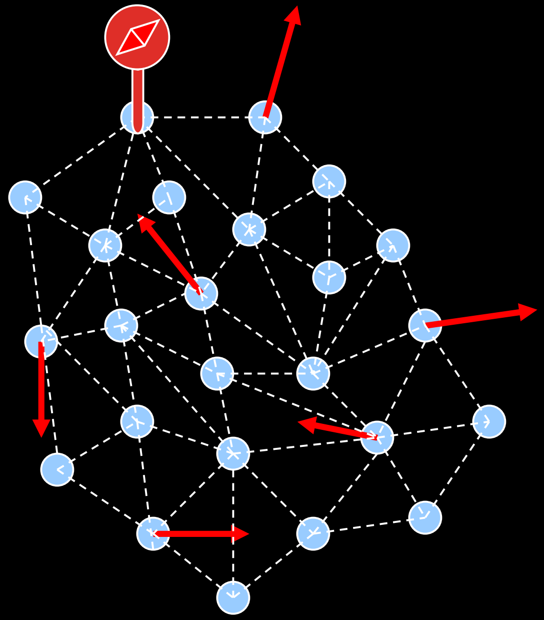
$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + u \cdot \nabla q$$ for any $q(\vec{x}, t)$

- Physics view:
- Change on a material particle $\frac{dq}{dt}$ vs. change on a fixed space point $\frac{\partial q}{\partial t}$

BETWEEN CONTINUOUS AND DISCRETE WORLDS

- Descretization
  - Eulerian – Using a grid structure
    - Calculate $\frac{\partial q}{\partial t}$ on each grid point
  - Lagrangian – Using particles
    - Calculate $\frac{dq}{dt}$ on each particle





Some figure credit in this section from Michael Su, Christopher Batty

BETWEEN CONTINUOUS AND DISCRETE WORLDS

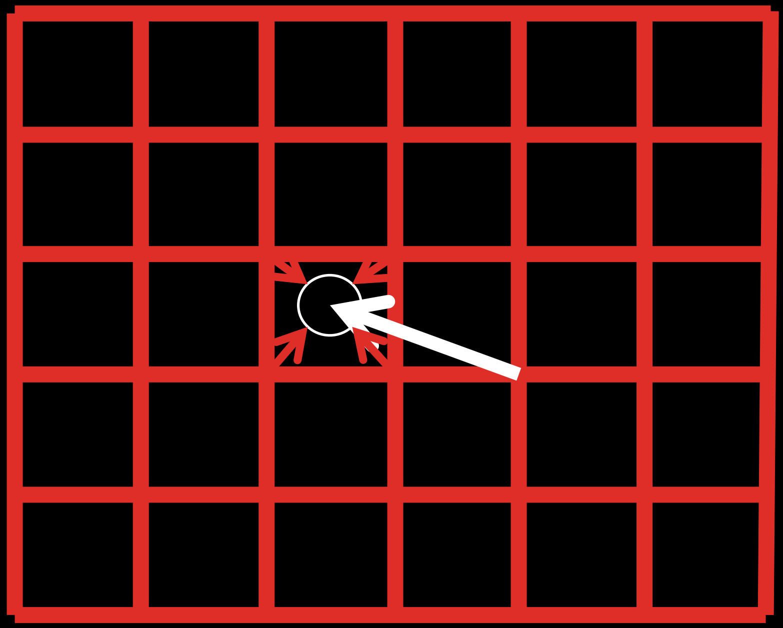
- How to solve the Navier-Stokes equations?
  - $\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + g + \nu \nabla \cdot \nabla u$
  - For now we use Eulerian discretization
- The first thing: operator splitting
  - Advection: $\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla \boldsymbol{u})$
  - Pressure: $\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla p$
  - Viscosity: $\frac{\partial \boldsymbol{u}}{\partial t} = \nu \nabla \cdot \nabla u$
  - External: $\frac{\partial \boldsymbol{u}}{\partial t} = g$

In a "time step" Δt, the change to u is approximately equal to the sum of all changes of four separate equations.

BETWEEN CONTINUOUS AND DISCRETE WORLDS

- The discrete version - 1D
  - $\frac{\partial u}{\partial t} \sim \frac{u_i^{t+1} - u_i^t}{\Delta t}$, $\nabla u \sim \frac{u_{i+1}^t - u_{i-1}^t}{2\Delta x}$
  - 1D-Advection:
  - $\frac{{u_i}^{n+1} - {u_i}^n}{\Delta t} + u_i \frac{{u_{i+1}}^n - {u_{i-1}}^n}{2\Delta x} = 0$
    - But unstable
  - Better method: Semi-Lagrangian
    - Look backwards in time from grid points
    - Interpolate data at previous time

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla \boldsymbol{u})$$



BETWEEN CONTINUOUS AND DISCRETE WORLDS

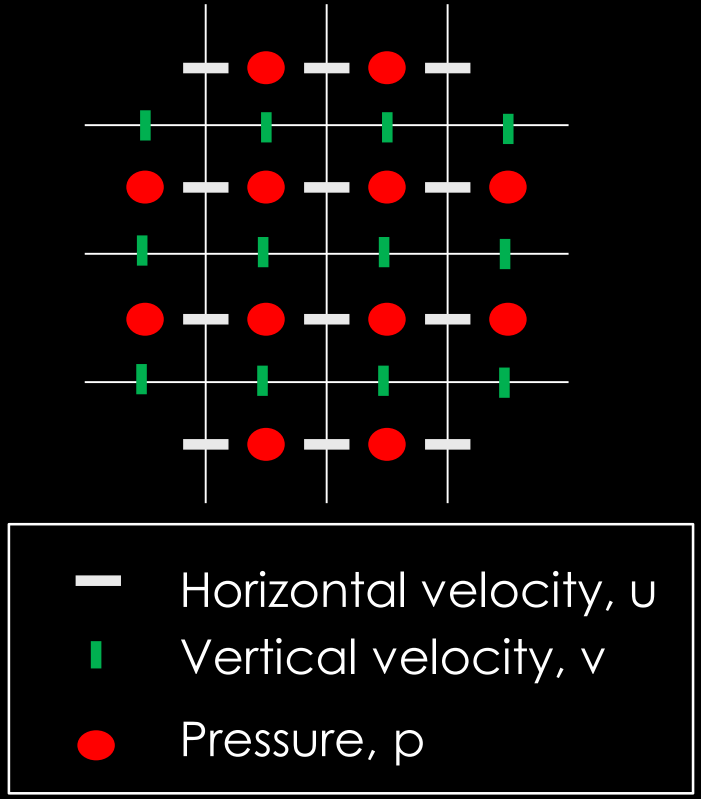
$$\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{1}{\rho}\nabla p \longrightarrow \boldsymbol{u}_{new} = \boldsymbol{u}_{old} - \frac{\Delta t}{\rho}\nabla p$$

- Pressure
  - 1 more unknown p
  - Mass conservation helps: $\nabla \cdot u = 0$
- The problem is usually changed to "find p so that $\nabla \cdot u = 0$"
  - $\nabla \cdot \left(\boldsymbol{u}_{old} - \frac{\Delta t}{\rho}\nabla p\right) = 0 \longrightarrow \frac{\Delta t}{\rho}\nabla \cdot \nabla p = \nabla \cdot \boldsymbol{u}_{old}$
- 2D discretization

$$\frac{\Delta t}{\rho}\frac{(p_{i+1,j}-p_{i,j}) - (p_{i,j}-p_{i-1,j})}{(\Delta x)^2} + \frac{\Delta t}{\rho}\frac{(p_{i,j+1}-p_{i,j}) - (p_{i,j}-p_{i,j-1})}{(\Delta x)^2} = \frac{u^{old}_{i+\frac{1}{2},j} - u^{old}_{i-\frac{1}{2},j}}{\Delta x} + \frac{v^{old}_{i,j+\frac{1}{2}} - v^{old}_{i,j-\frac{1}{2}}}{\Delta x}$$

BETWEEN CONTINUOUS AND DISCRETE WORLDS

- $\frac{\Delta t}{\rho}\frac{(p_{i+1,j}-p_{i,j})-(p_{i,j}-p_{i-1,j})}{(\Delta x)^2}+\frac{\Delta t}{\rho}\frac{(p_{i,j+1}-p_{i,j})-(p_{i,j}-p_{i,j-1})}{(\Delta x)^2}=\frac{u^{old}_{i+\frac{1}{2},j}-u^{old}_{i-\frac{1}{2},j}}{\Delta x}+\frac{v^{old}_{i,j+\frac{1}{2}}-v^{old}_{i,j-\frac{1}{2}}}{\Delta x}$
- Staggered grid
- One above equation for each red dot
- i*j unknowns $p_{i,j}$, i*j linear equations
  - $Ap = b$
- Lots of the simulation works are about changing equations into $Ax = b$ and solve it
- Viscosity and other forces handled similarly



BETWEEN CONTINUOUS AND DISCRETE WORLDS

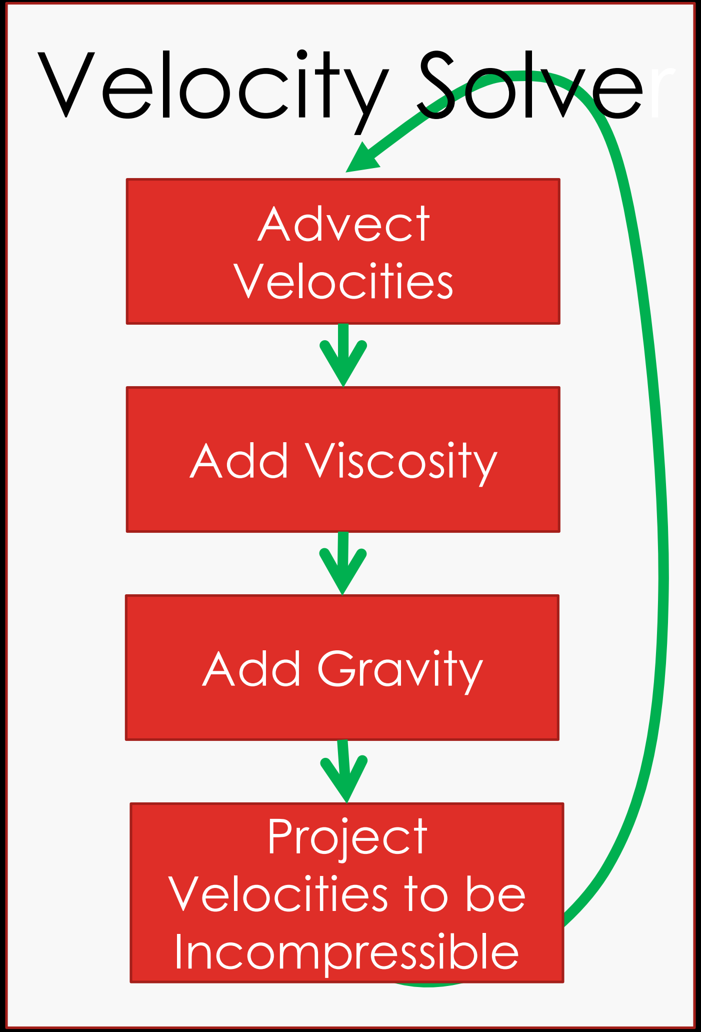
- Lagragian discretization
  - ~~Advection: $\frac{d\boldsymbol{u}}{dt} = \boldsymbol{0}$~~
  - Pressure: $\frac{d\boldsymbol{u}}{dt} = -\frac{1}{\rho}\nabla p$
  - Viscosity: $\frac{d\boldsymbol{u}}{dt} = \nu\nabla \cdot \nabla u$
  - External: $\frac{d\boldsymbol{u}}{dt} = g$
- But it is a bit tricky to calculate derivatives from a random group of particles
  - Most popular approach: Smoothed Particle Hydrodynamics (SPH)

BETWEEN CONTINUOUS AND DISCRETE WORLDS

- SPH interpolation – a weighted interpolation in the neighborhood
- $A(\vec{x}) = \sum_j m_j \frac{A_j}{\rho_j} W(\vec{x} - \vec{x_j}, h)$
  - $\nabla A(\vec{x}) = \sum_j m_j \frac{A_j}{\rho_j} \nabla W(\vec{x} - \vec{x_j}, h)$
- Weakly compressible fluid
  - $\rho = \sum_j m_j W(\vec{x} - \vec{x_j}, h)$
  - $p = p(\rho) = \kappa(\rho - \rho_0)$

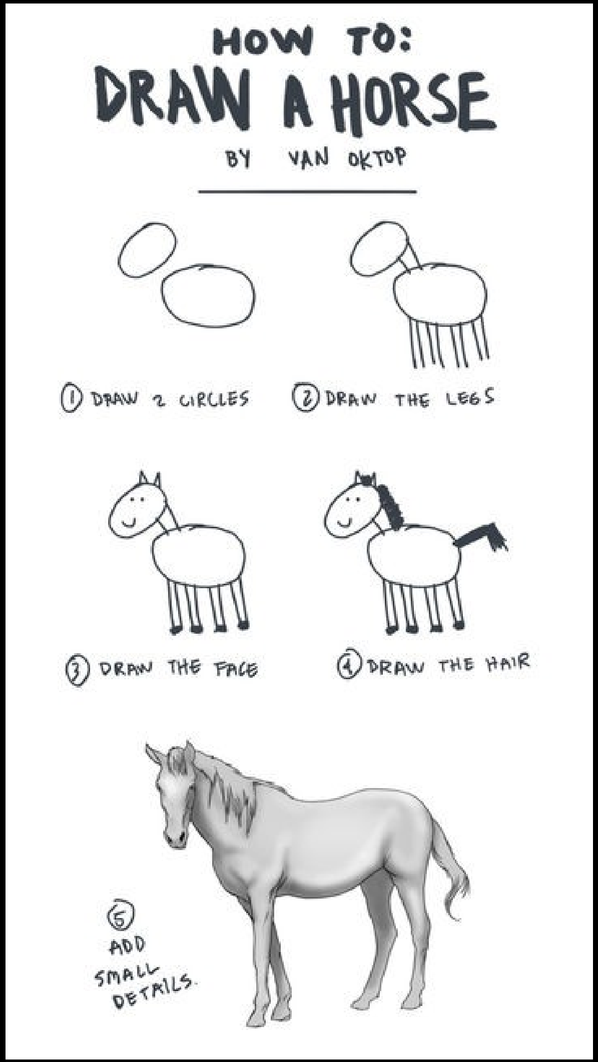
BETWEEN CONTINUOUS AND DISCRETE WORLDS

- The basic structure of a simulator
  - Solve velocity and we know all
  - One loop for each time step
  - The state of material (position, velocity, etc.) is updated time-step by time-step
  - Output a data sequence (and render it into a video)



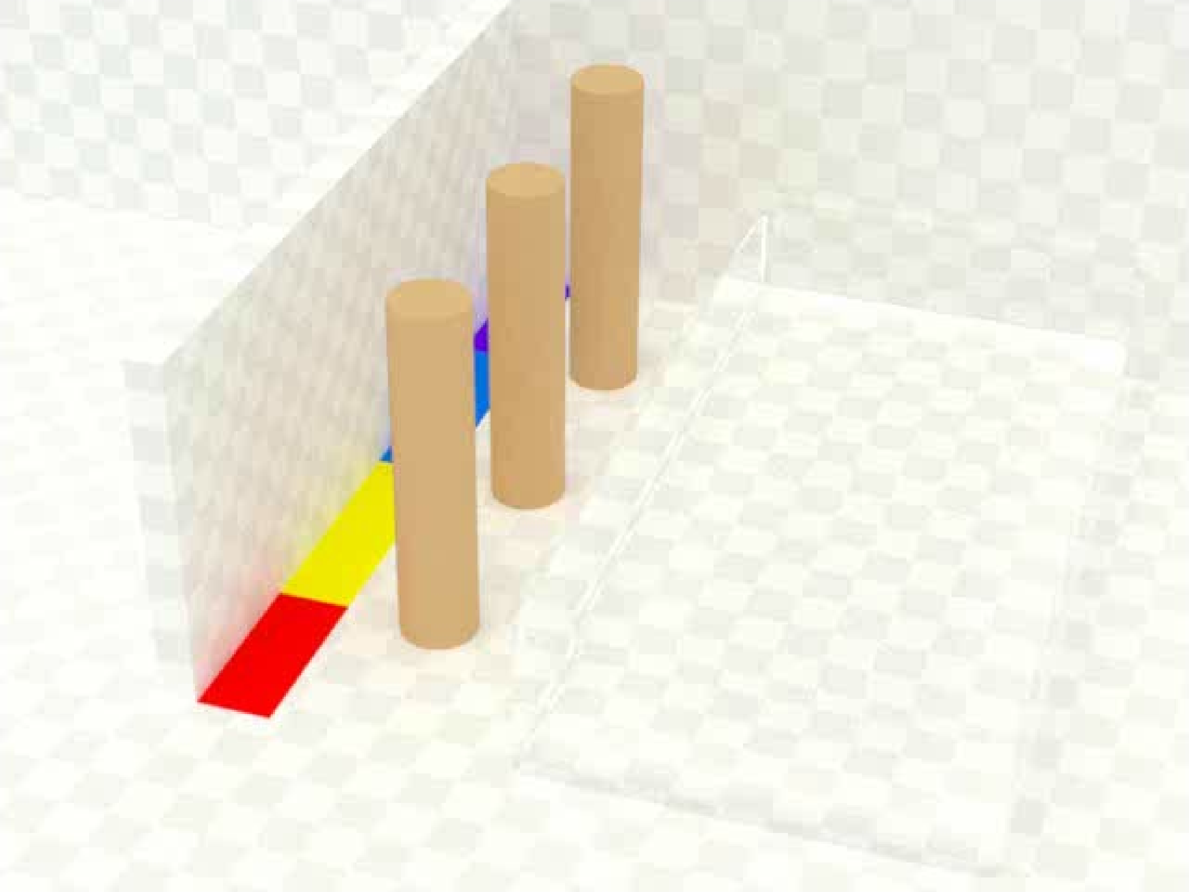
FINE ARTS MAKE DIFFERENCE

- Time to show the horse?
  - Much more to learn
    - [1] R. Bridson and M. Müller-Fischer. Fluid Simulation. *SIGGRAPH 07 Course Notes*
    - [2] R. Bridson. Fluid Simulation for Computer Graphics. *A K Peters, 2008*
    - [3] J. Stam. Real-Time Fluid Dynamics for Games. *GDC 2003*
    - [4] B. Kim, Y. Liu, I. Llamas, and J. Rossignac. FlowFixer: Using BFECC for Fluid Simulation. *EGWNP 05*
    - [5] R. Fedkiw, J. Stam, and H.W. Jenson. Visual Simulation of Smoke. *SIGGRAPH 01*
    - [6] N. Foster, R. Fedkiw, Practical Animation of Liquids. *SIGGRAPH 01*
    - [7] D. Enright, S. Marschner, R. Fedkiw. Animation and Rendering of Complex Water Surfaces. *SIGGRAPH 02*
    - [8] M. Carlson, P. J. Mucha, G. Turk. Rigid Fluid: Animating the Interplay Between Rigid Bodies and Fluid. *SIGGRAPH 04*



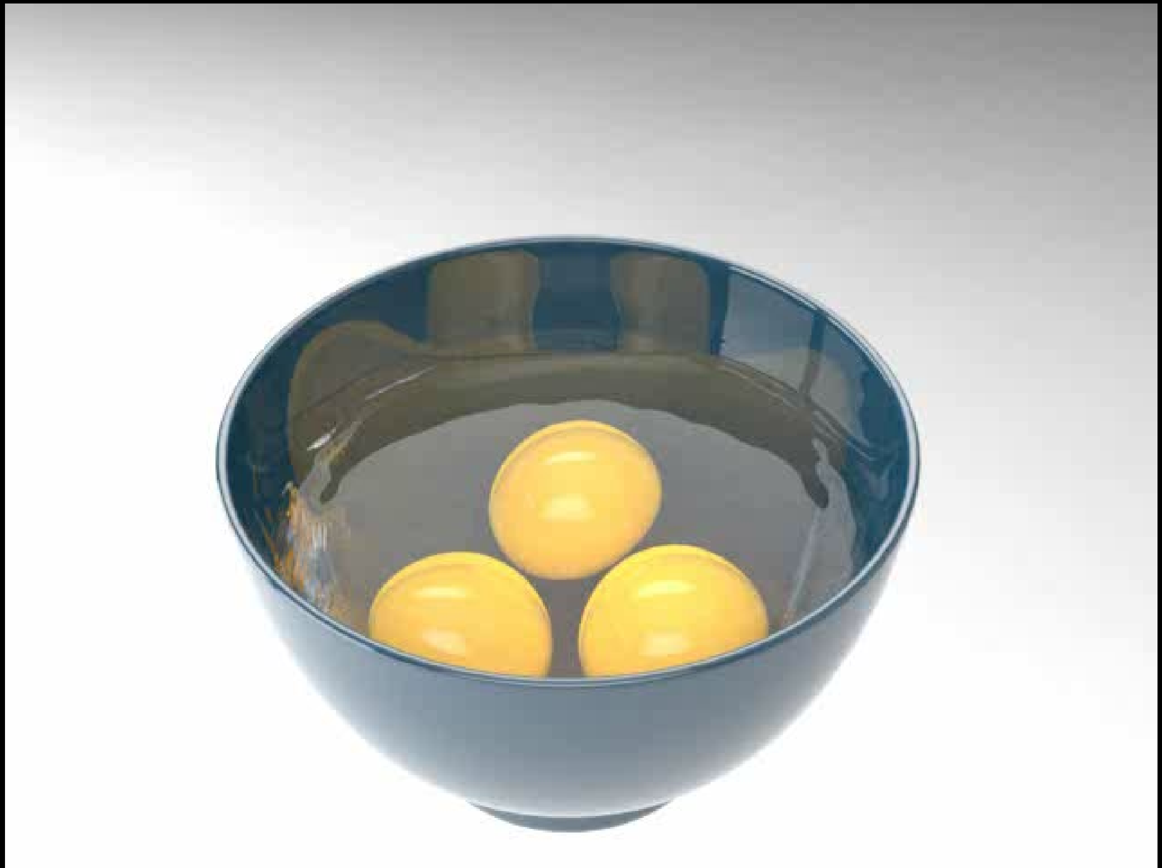
FINE ARTS MAKE DIFFERENCE

- What we are keeping on



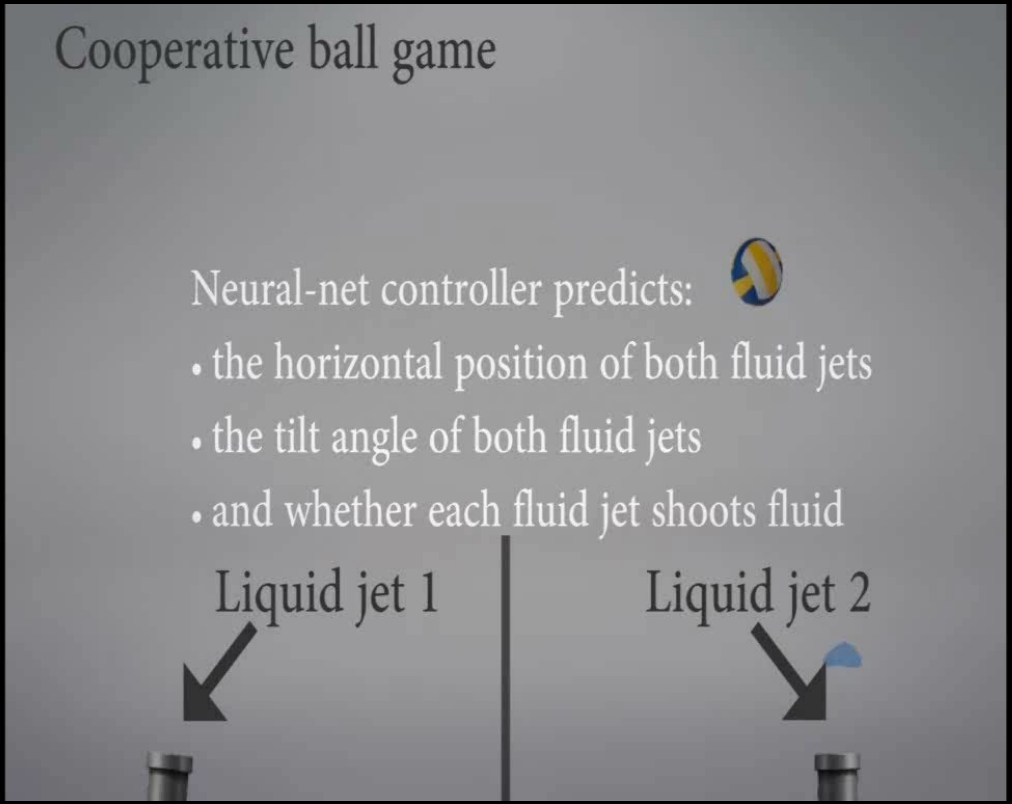
FINE ARTS MAKE DIFFERENCE

- is make the virtual become real



FINE ARTS MAKE DIFFERENCE

- and let things be more interesting



THANK YOU!