Atlas Refinement with Bounded Packing Efficiency

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Texture

3D modeling  UV Unwrapping  Texture painting
Packing Efficiency (PE)

PE = 86.1%
High pixel usage rate

PE = 45.6%
Low pixel usage rate
Packing Efficiency (PE)

Maximizing atlas packing efficiency is NP-hard!

[Garey and Johnson 1979; Milenkovic 1999]
Other Requirements

- Low distortion
Other Requirements

- Low distortion
  - [Golla et al. 2018; Liu et al. 2018; Shtengel et al. 2017; Zhu et al. 2018]
- Consistent orientation
  - [Floater 2003; Tutte 1963; Claici et al. 2017; Hormann and Greiner 2000; Rabinovich et al. 2017; Schüller et al. 2013]
- Bijection
  - [Jiang et al. 2017; Smith and Schaefer 2015]
- Low boundary length
  - [Li et al. 2018; Poranne et al. 2017; Sorkine et al. 2002]

These methods do not consider PE!
Atlas Refinement

Input

Bijective
High PE
Previous Work

Box Cutter [Limper et al. 2018]

- Cut and repack

No guarantee for a high PE result!
Motivation
Packing Problems

Irregular shapes
Hard to achieve high PE

Rectangles
Simple to achieve high PE
Widely used in practice
Axis-Aligned Structure

Axis-aligned structure  Rectangle decomposition  High PE (87.6%)!
General Cases

Not axis-aligned

Axis-aligned deformation

Axis-aligned
Higher distortion
Distortion Reduction

- Axis-aligned
  High distortion

- Bijective & High PE
  High distortion

- Bijective & High PE
  Low distortion
  Bounded PE

Scaffold-based method
[Jiang et al. 2017]
Axis-aligned deformation

Pipeline

Rectangle decomposition and packing

Distortion reduction
Axis-Aligned Deformation

• Input

Single chart
Not bijective

10 charts
Bijective
Axis-Aligned Deformation

- Targets of boundary edges
- Smoothing
- Labeling
- Deformation
Axis-Aligned Deformation

Direction vector
Ambiguous rotating directions

Fail!
Axis-Aligned Deformation

Polar angle
Clear rotating direction

Success!
Polar Angle

\[ \theta = \text{atan2}(y, x) + 2k\pi \]

\[ d\theta = \frac{x\, dy - y\, dx}{x^2 + y^2} \]
Polar Angle

Discrete boundary curvature

\[ \theta_{i+1} = \theta_i + (\pi - \alpha_i) \]

Gauss–Bonnet formula

\[ \sum_i (\pi - \alpha_i) = 2\pi \]
Target Calculation

- Boundary smoothing
- Gaussian smooth

\[ G_\sigma(s_i^k) = \sum_{b_j} l_j \exp \left( -\frac{\text{dist}(b_i, b_j)^2}{2\sigma^2} \right) s_j^k \]

\[ \hat{s}_i^k = \frac{G_\sigma(s_i^k)}{\|G_\sigma(s_i^k)\|} \]

- Accept \( \hat{s}_i^k \) if \( \hat{s}_i^k \cdot s_i^k \geq 0 \)

- Update interior angles

\[ \hat{\alpha}_i^{k+1} = \hat{\alpha}_i^k + \angle(s_i^k, s_i^{k+1}) - \angle(s_i+1^k, s_i+1^{k+1}) \]

- Global rotation

- Polar angle axis-alignment
Axis-Aligned Deformation

Target polar angle $\theta_i$

Corners
Axis-Aligned Deformation

- Energy of boundary alignment

\[ E_{\text{edge}}(b_i) = \frac{1}{2} (1 - \gamma) \left( \theta_i - \frac{\pi}{2} \Theta_i \right)^2 + \frac{1}{2} \gamma \left( \frac{l_i}{l_i^0} - 1 \right)^2 \]

\[ E_{\text{align}}(c) = \sum_{i=1}^{N_b} \frac{l_i^0}{l_i} E_{\text{edge}}(b_i) \]
Axis-Aligned Deformation

• Energy of isometric distortion (symmetric Dirichlet)

\[ E_d(c) = \frac{1}{4} \sum_{f_i \in \mathcal{F}^c} \frac{\text{Area}(f_i)}{\text{Area}(M^c)} \left( \|J_i\|_F^2 + \|J_i^{-1}\|_F^2 \right) \]

Keep low distortion and orientation consistency.
Axis-Aligned Deformation

$$\min_c \quad E_d(c) + \lambda E_{\text{align}}(c)$$

s.t. $\det J_i > 0, \forall i$
The faces are all rectangles. But the number is too many.
Rectangle Decomposition and Packing

- Motorcycle graph algorithm

\[
\text{Score} = \text{PE} - \omega \frac{BL_1}{BL_0}
\]

<table>
<thead>
<tr>
<th>PE</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.0%</td>
<td>0.688</td>
</tr>
<tr>
<td>83.6%</td>
<td>0.659</td>
</tr>
<tr>
<td>84.4%</td>
<td>0.658</td>
</tr>
</tbody>
</table>
Distortion Reduction

\[
\min_C E_{\text{reduction}} = E_d(C) + E_{\text{PE}}(C)
\]

s.t. \( \Phi \) is bijective

Scaffold-based method

[Jiang et al. 2017]
Distortion reduction
Experiments
PE Bound

Input
\( E_d = 1.039 \)

\( PE = 80\% \)
\( E_d = 1.037 \)

\( PE = 85\% \)
\( E_d = 1.041 \)

\( PE = 90\% \)
\( E_d = 1.049 \)
Collection of Models

Input

$E_d = 1.022$

$PE = 80\%$

$E_d = 1.026$
Comparison to Box Cutter [Limper et al. 2018]

Input
#F=4,656

Box Cutter
PE=81.1%
$E_d=1.149$
179.8s

Ours
PE=88.9%
$E_d=1.087$
1.69s
Comparison to Box Cutter [Limper et al. 2018]

Input
#F=100,000

Box Cutter
PE=75.8%
$E_d=1.114$
247.8s

Ours
PE=91.3%
$E_d=1.066$
43.84s
Benchmark (5,588)

PE = 86.2%
$E_d = 1.020$

PE = 86.7%
$E_d = 1.024$
Benchmark (5,588)

PE=90.5%
$E_d=1.011$

PE=91.0%
$E_d=1.001$
Texture

PE=80.4%
Ed=1.119

PE=92.6%
Ed=1.018
Single-source Geodesics [Prada et al. 2018]

PE=89.1%
$E_d=1.041$
Conclusion
Conclusions

• Our method provides a novel technique to refine input atlases with bounded packing efficiency.

• Key idea: converting polygon packing problems to a rectangle packing problems

• High and bounded packing efficiency

• Good performance and quality

• Practical robustness
Limitation & Future Work

• Modification of the input atlas may not meet the original intention.
• Boundary length elongation is not explicitly bounded.
• There is no theoretical guarantee, especially for the axis-aligned deformation process.
Thank you!