Frame Field-Driven Quad- and Hex-Remeshing

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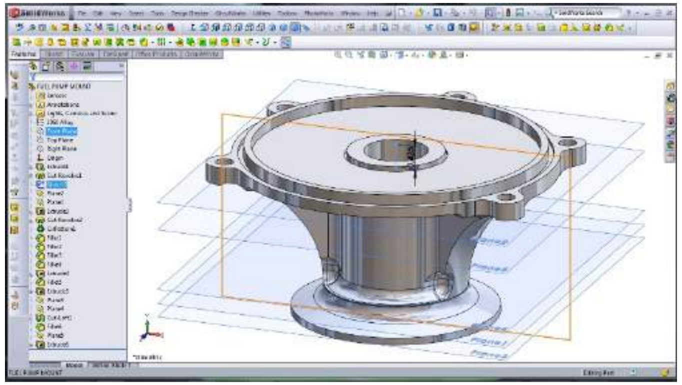
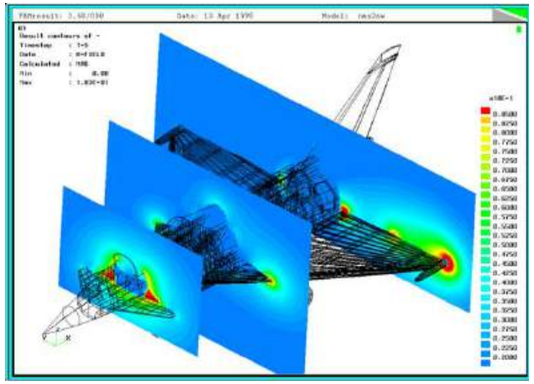
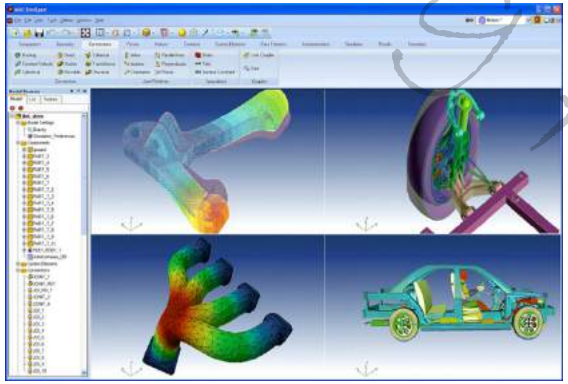
Background

- CAD & CAE & CAM ……
- 3D models are widely used
- Use Mesh to represent models
- Remeshing

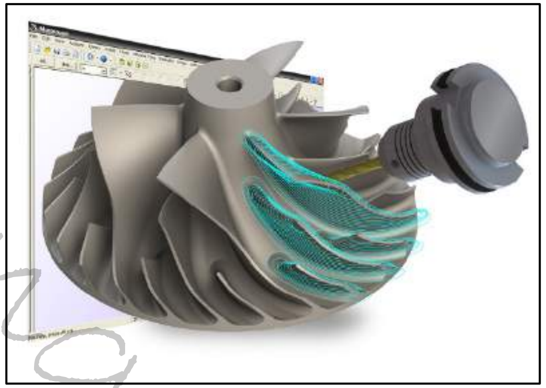


CAD



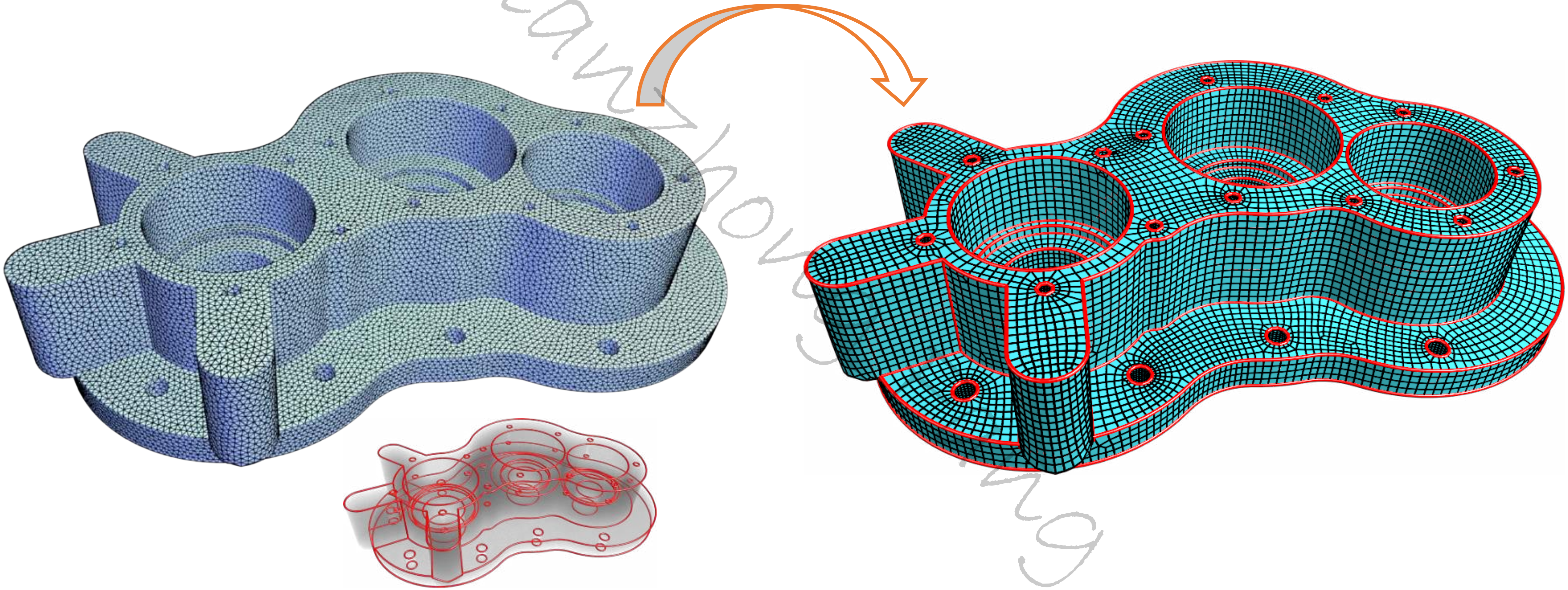


CAE



CAM

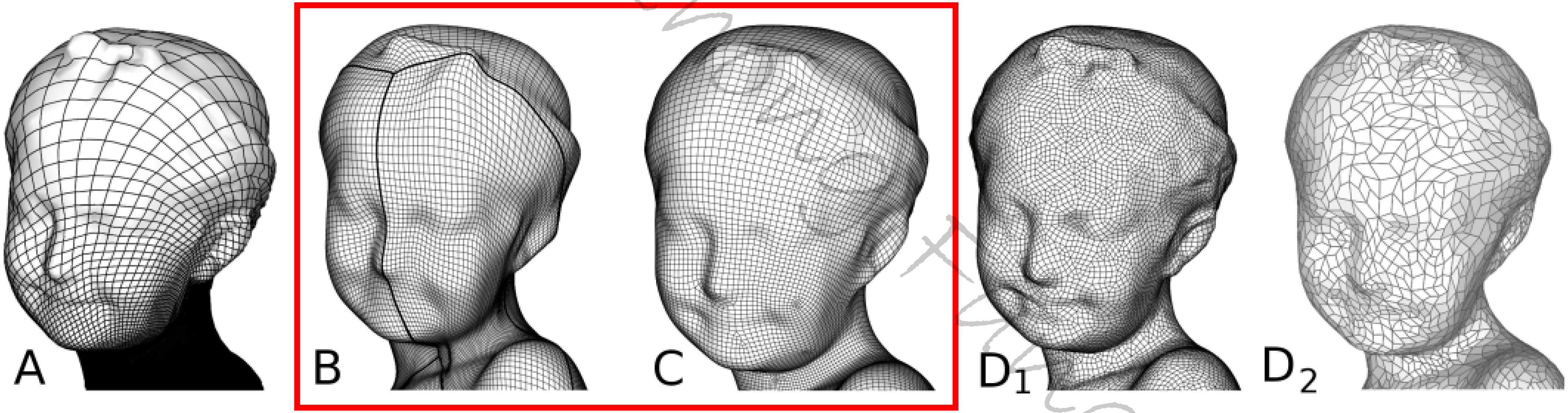
Remeshing



Frame field-driven quad- and hex-remeshing

Semi-Regular Mesh

- High computational accuracy
- Less elements



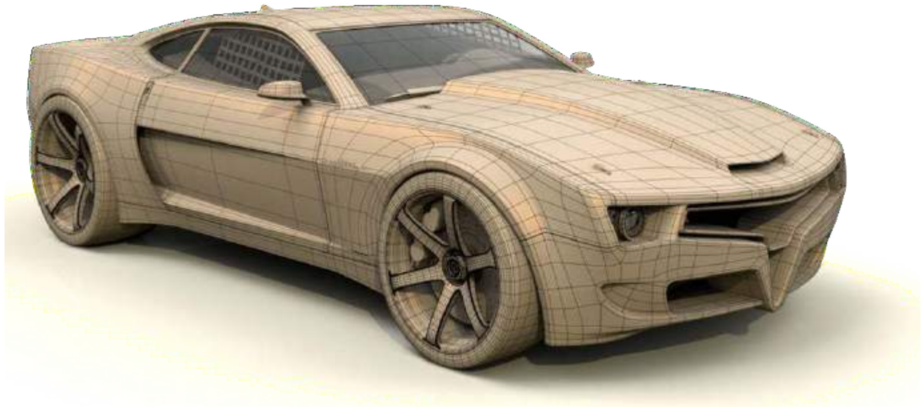
Regular quad-mesh — Semi-regular quad-mesh — Unstructured quad-mesh

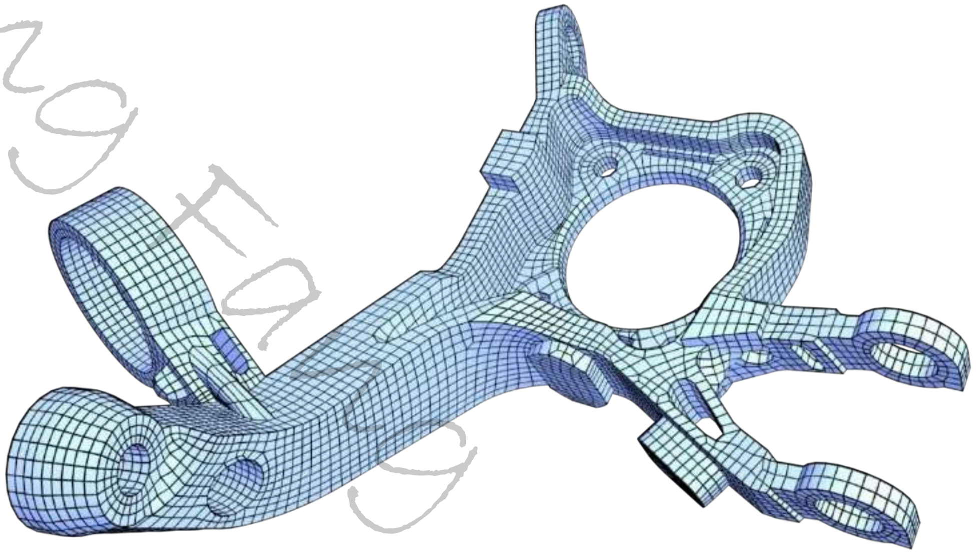
[Bommes et al. CGF 2013]

Applications

- B-spline fitting
- Simulation
- Reverse Engineering

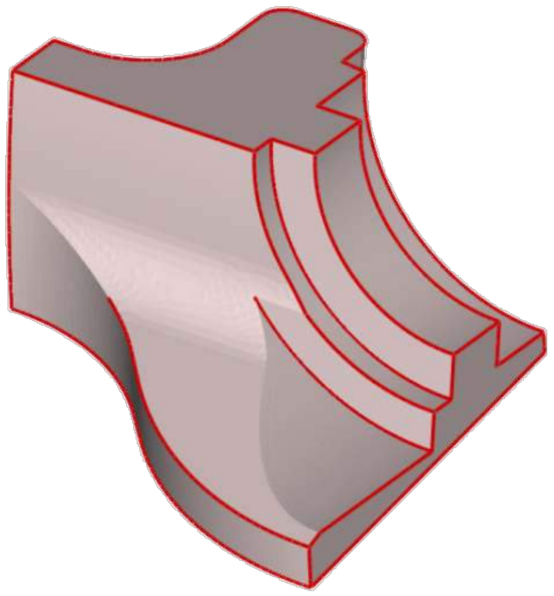
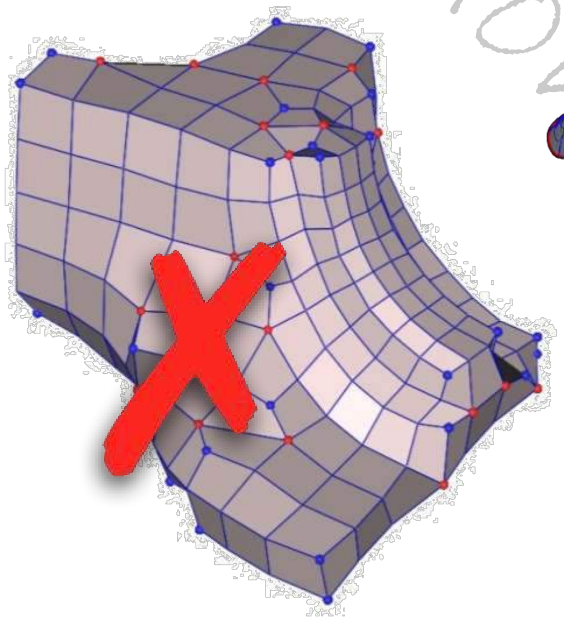
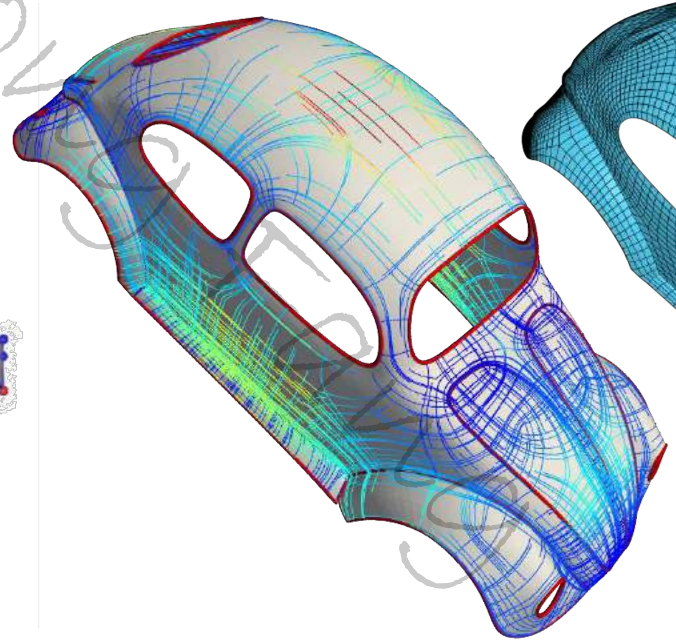
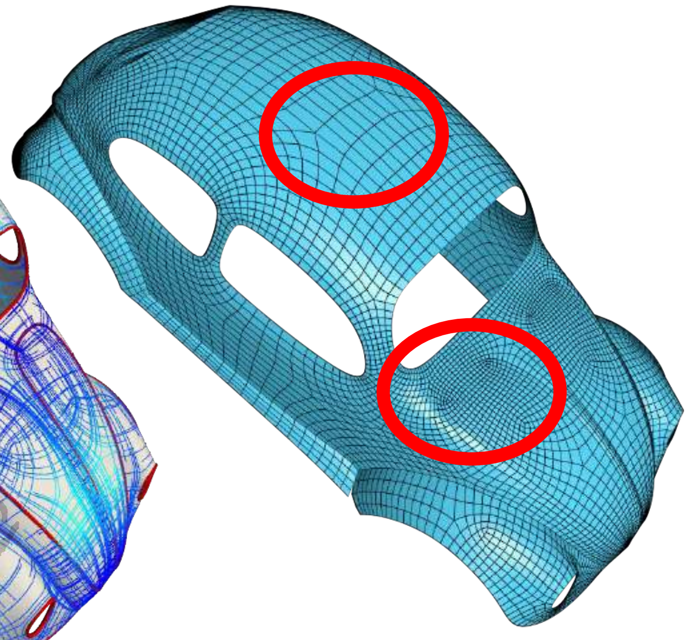
- Finite Element Method
- Engineering Analysis
- …





Requirements of Remeshing

- Low Hausdorff distance
- Feature preservation
- Good element shape
- Controllable: direction & size
- Robustness & efficiency

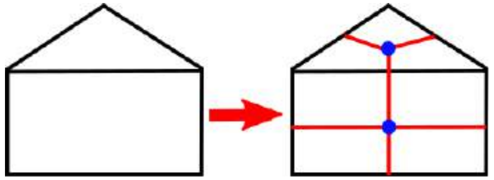
Methods of Remeshing

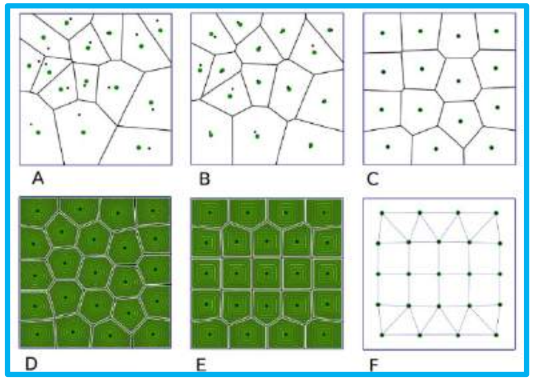
- Computational geometry
- Differential geometry

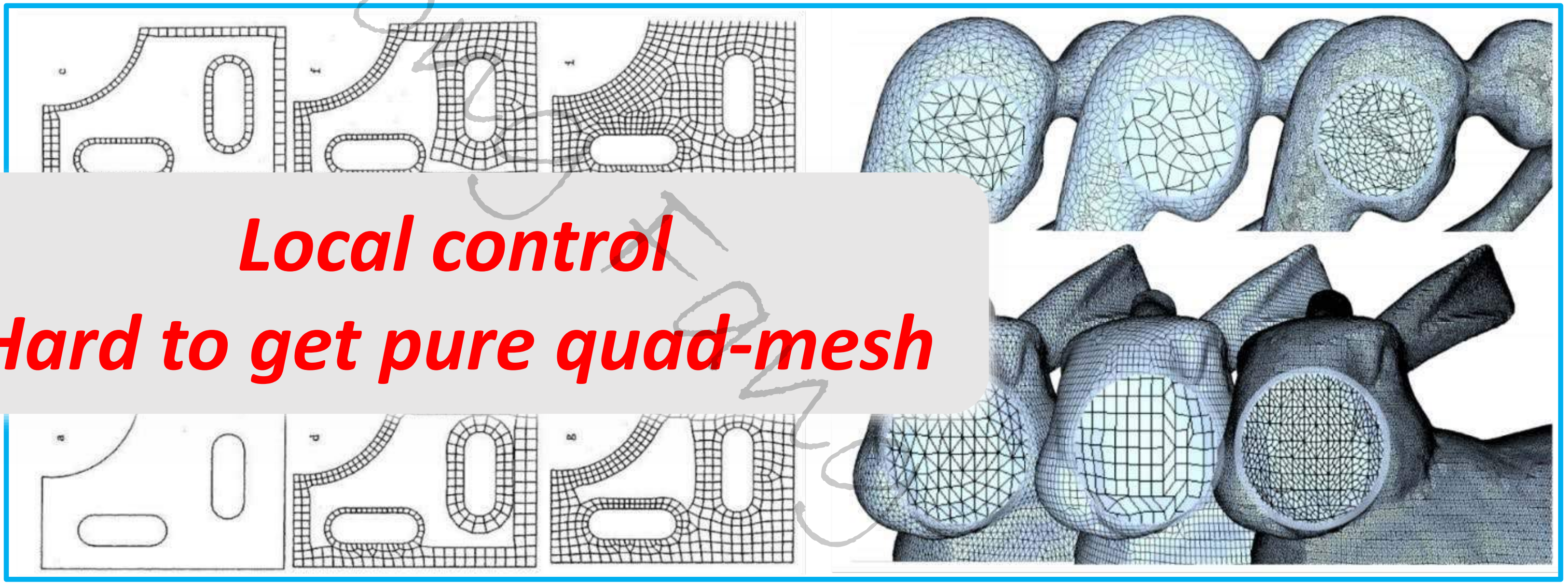
Computational geometry methods

- Catmull-Clark subdivision [Catmull and Clark '78]
- Advance front [Owen et al. '99]
- ……

- Tri-to-quad conversion [Gurung et al. '11, Remacle et al. '11]
- Voronoi diagram [Lévy and Liu '10]
- ……



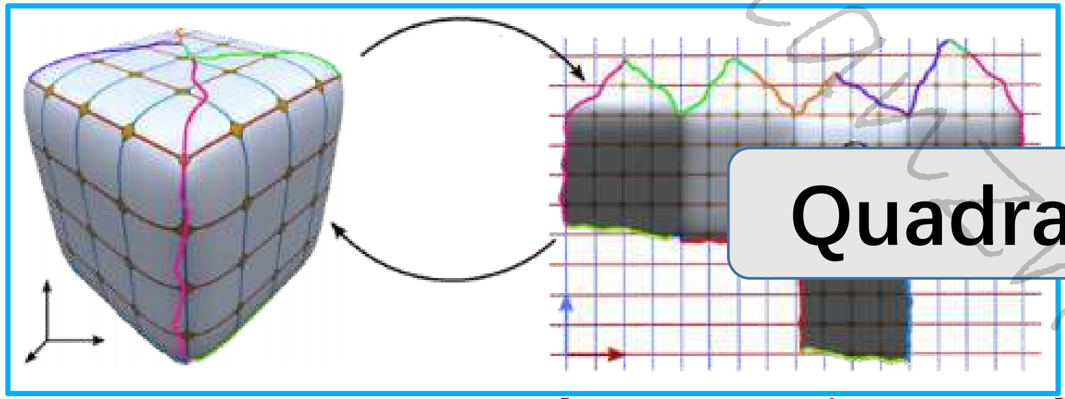




Local control

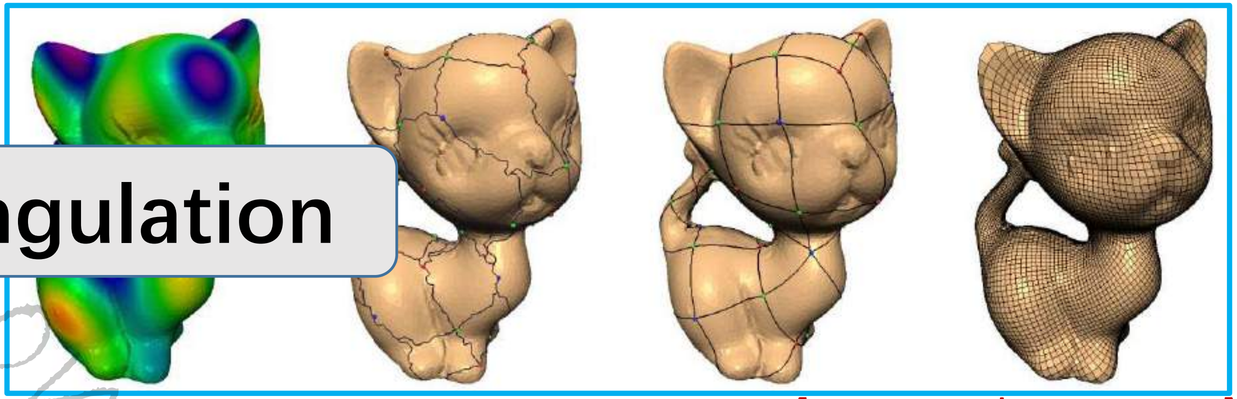
Hard to get pure quad-mesh

Differential geometry methods



[Bommes et al. TOG 2012]

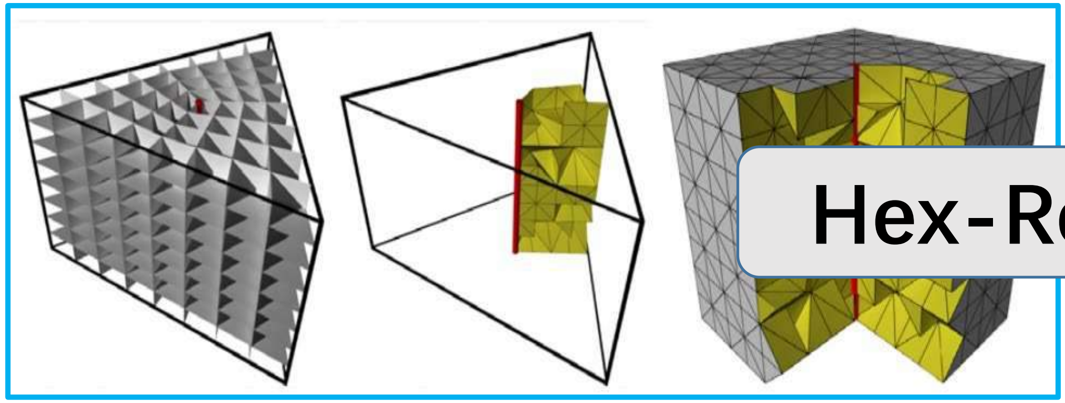
Surface parametrization



[Dong et al. TOG 2006]

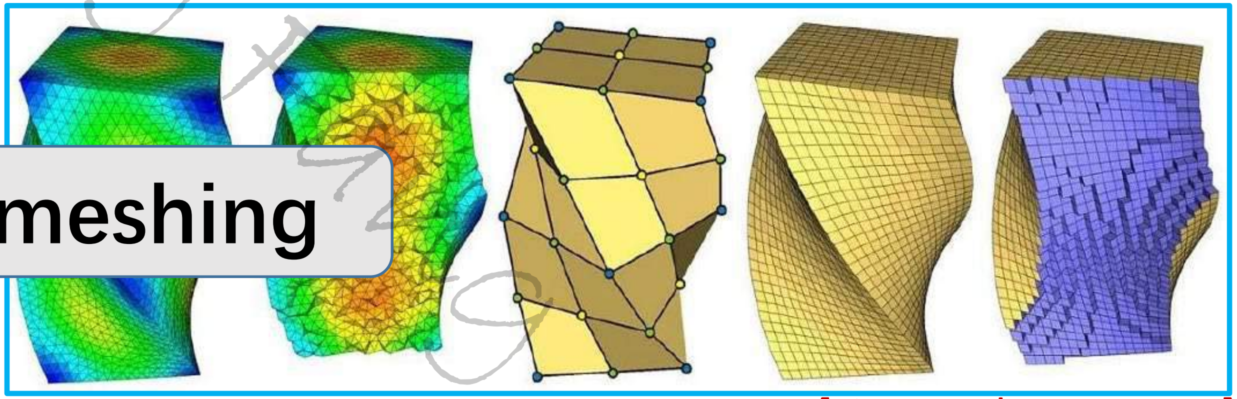
2D Morse-Smale complex

Quadrangulation



[Nieser et al. CGF 2011]

Volume parametrization



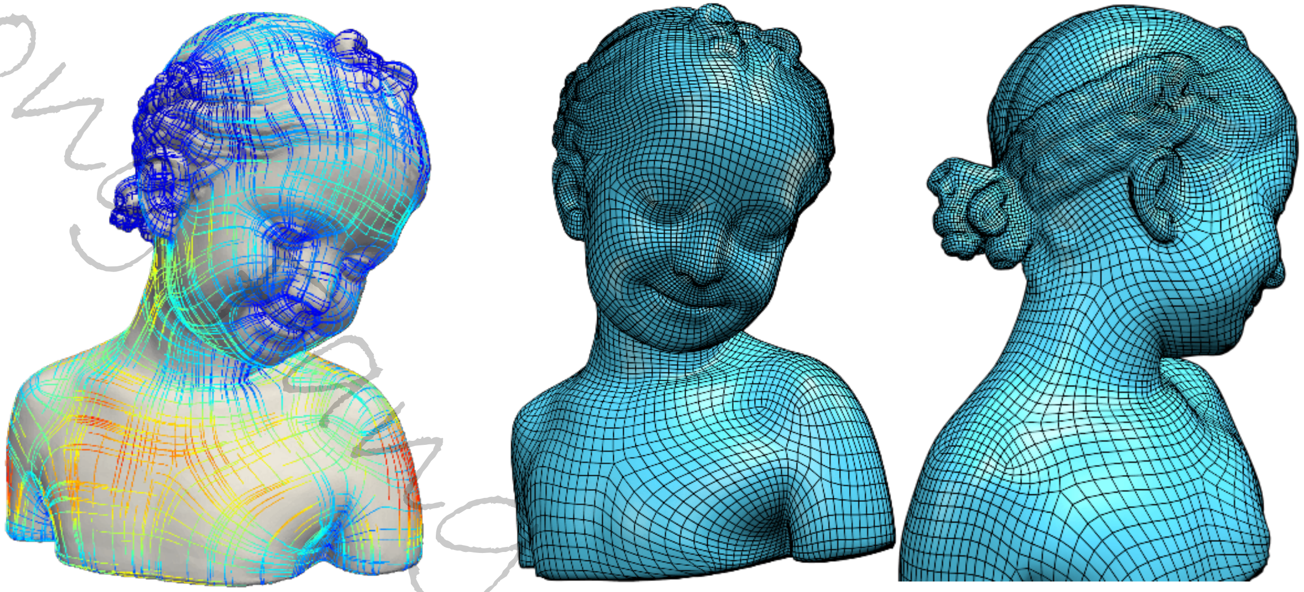
[Ling et al. TOG 2014]

3D Morse-Smale complex

Hex-Remeshing

Frame Field-Driven Remeshing

- Advantages
  - Feature & boundary alignment
  - Global topology control
  - Size & direction control

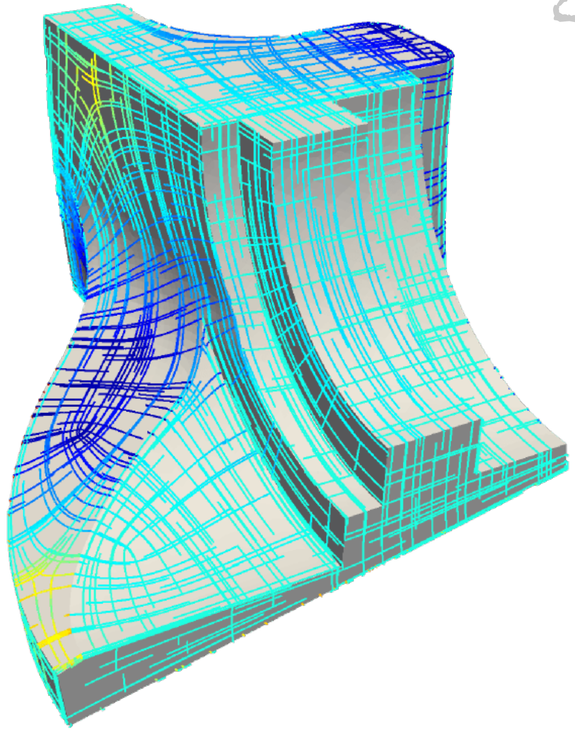


2D frame field

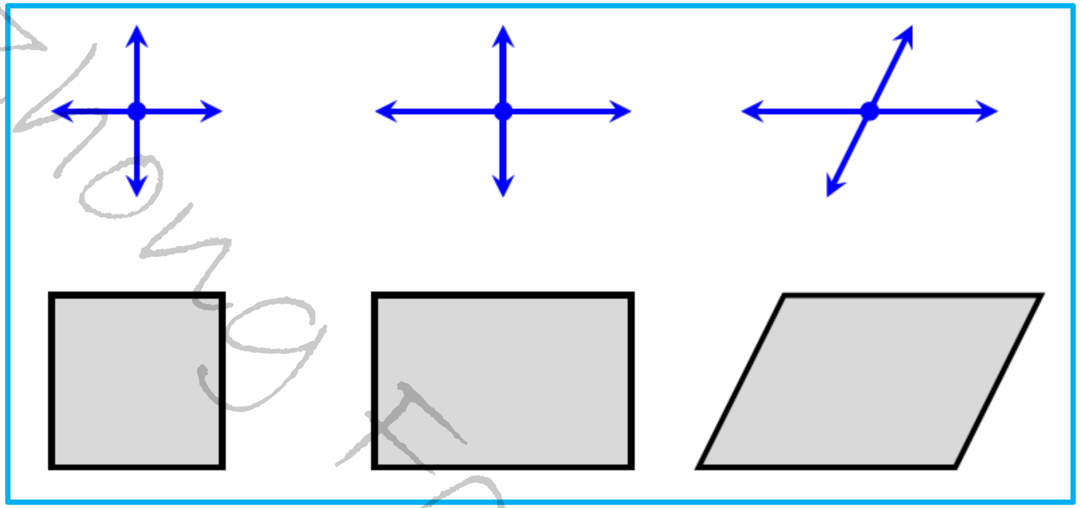
Quad-mesh

Frame field-driven quad- and hex-remeshing

2D Frame Field

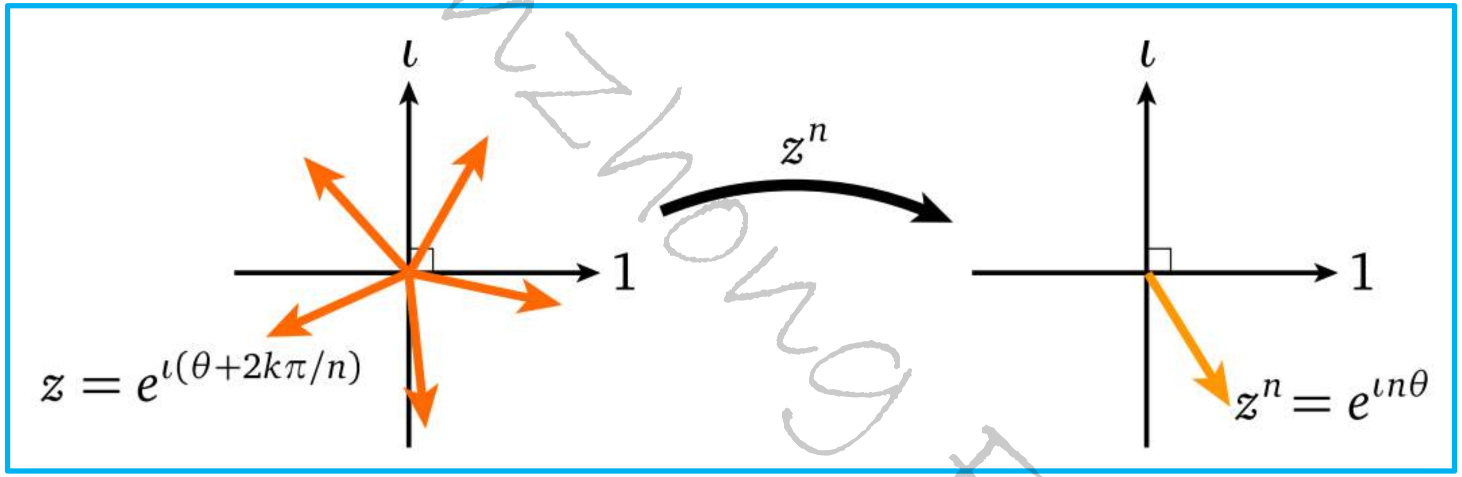


Frame field on surface



Symmetric Representation

- N-Rotational Symmetric field



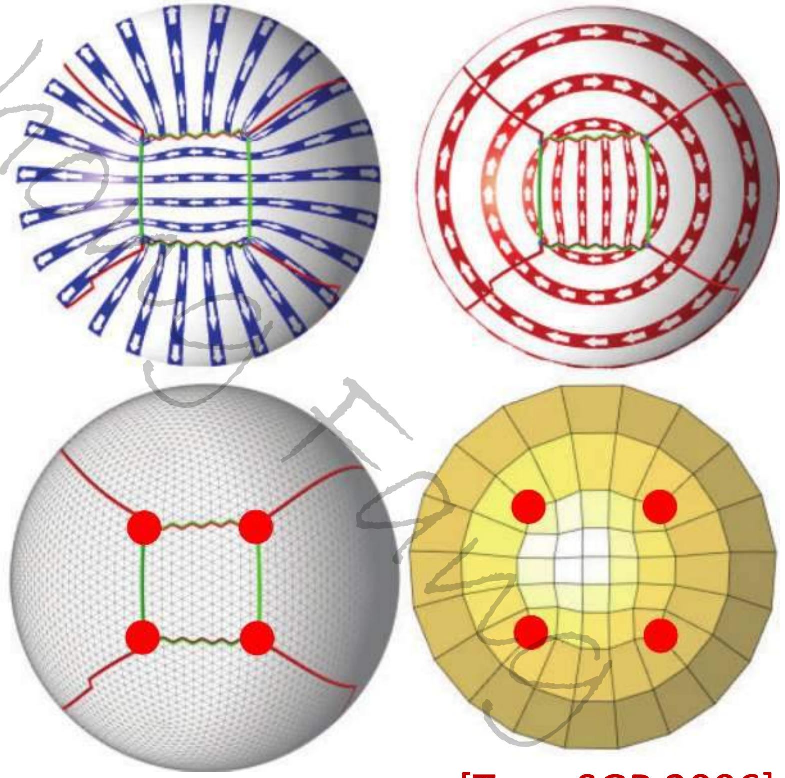
[Knöppel et al. '13]

- 4-Rotational Symmetric field (2D cross frame field)

$$(\cos(\theta + \frac{\pi}{2}k), \sin(\theta + \frac{\pi}{2}k)) \Leftrightarrow (\cos 4\theta, \sin 4\theta)$$

Frame Field and Quad Mesh

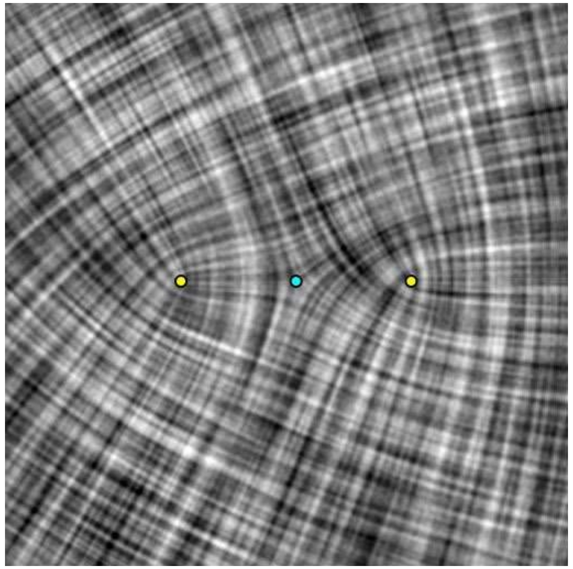
- Two directions for quadrangulation
- Smooth change
- Align to features
- With size control



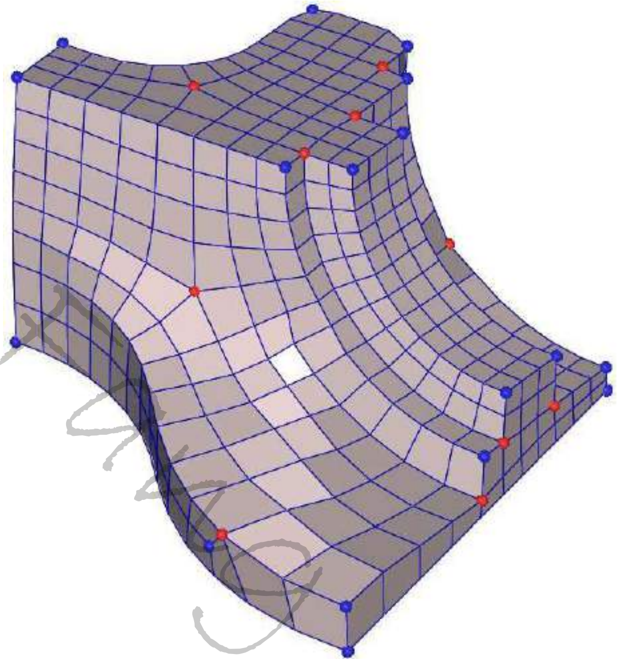
[Tong SGP 2006]

Singularity

- Non-smooth places of frame field
- Non-regular valence in quad mesh

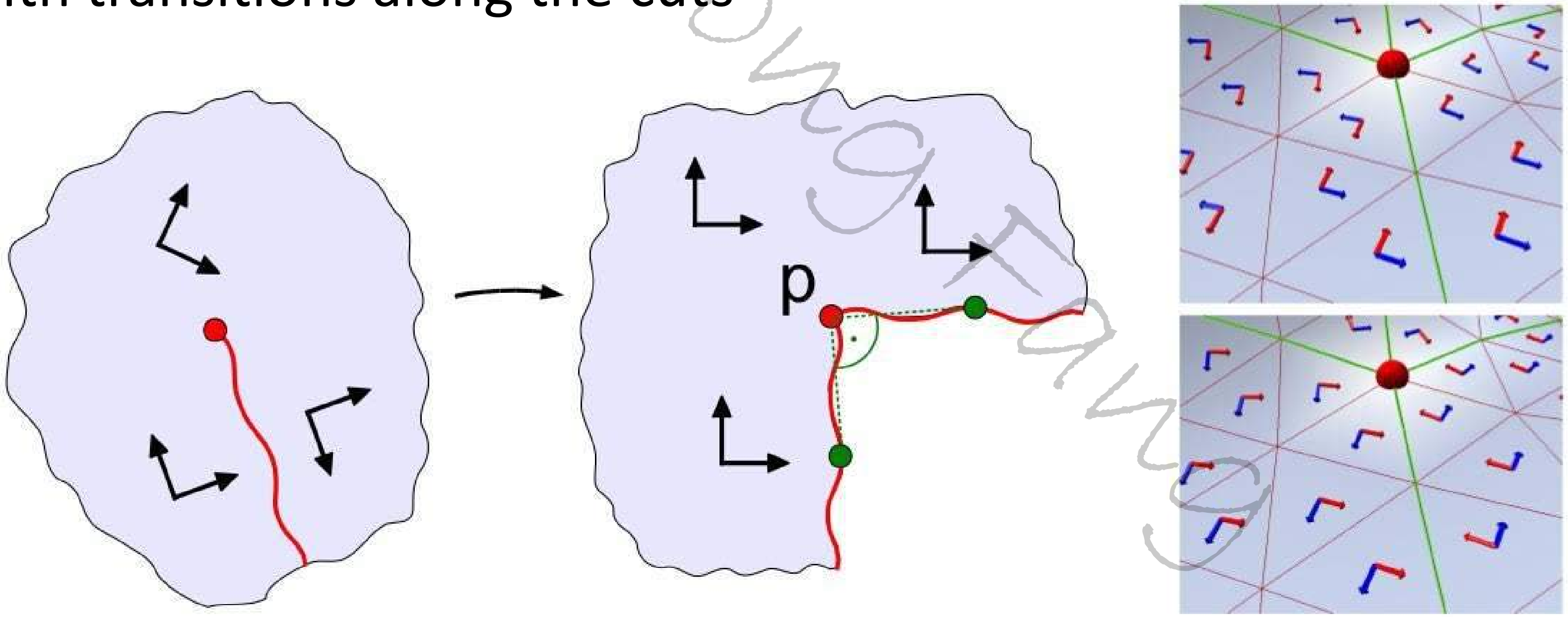


[Palacios et al. TOG ‘07]



Frame Field-Driven Parametrization

- Given frame field on surface
- Locate the singularities, and make cuts
- A single chart with aligned frames
  - But with transitions along the cuts



Frame field-driven quad- and hex-remeshing

Frame Field-Driven Parametrization

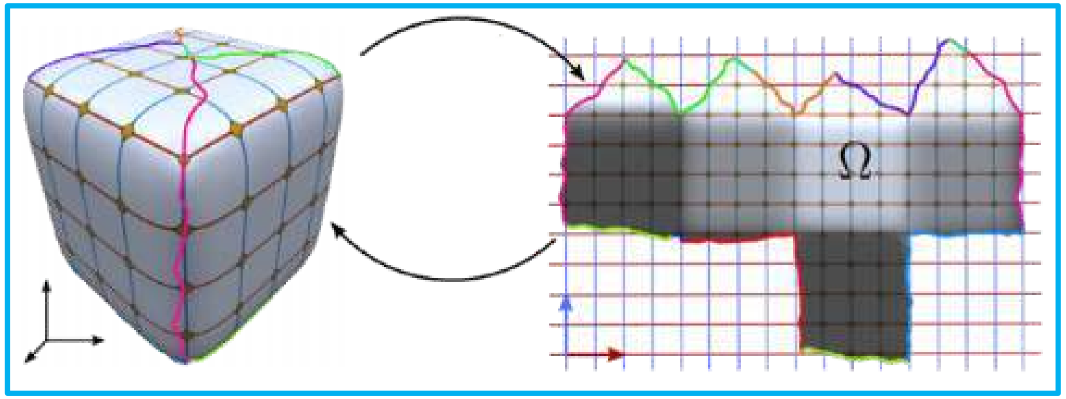
- Frame field V drives parameterization φ

$$\varphi : S \rightarrow \binom{u}{v} \in \mathbb{R}^2$$

Poisson reconstruction
$$\min_{\varphi} \int_S \|\nabla\varphi - V\|^2$$

Decomposition
$$V = \nabla\varphi + \nabla \times \psi$$

Integrable (Curl-free)



[Bommes et al. TOG 2013]

Parametrization-based Quadrangulation

$$\min_{\varphi} \int_S \|\nabla\varphi - V\|^2$$

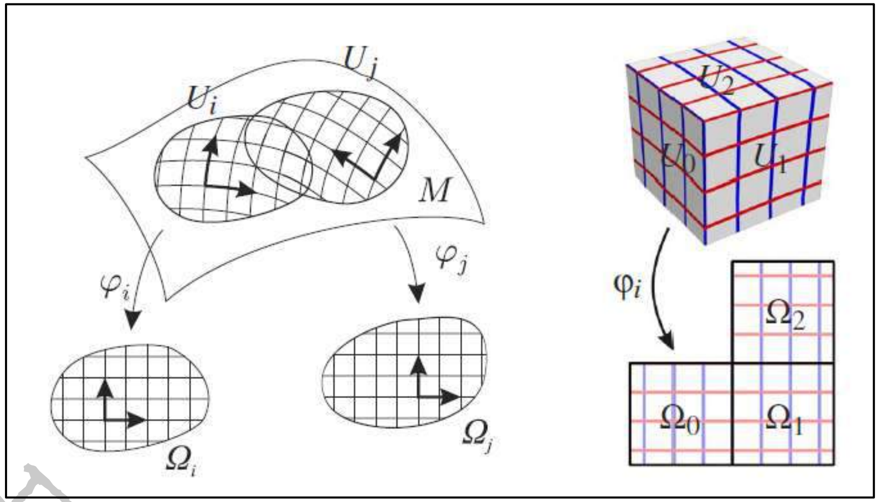
Seamless & non-degenerated

$$s.t.\ \ \varphi_j(p) = R^{k_{ij}\frac{\pi}{2}}\varphi_i(p) + T_{ij}$$

$$\det(\nabla\varphi) > 0$$

Integer constraints

$$k_{ij} \in \mathbb{Z},\ T_{ij} \in \mathbb{Z}^2$$



[Kälbere et al. '07]

Multi-chart parametrization

Parametrization-based Quadrangulation

$$\min_{\varphi} \int_S \|\nabla\varphi - V\|^2$$

Seamless & non-degenerated

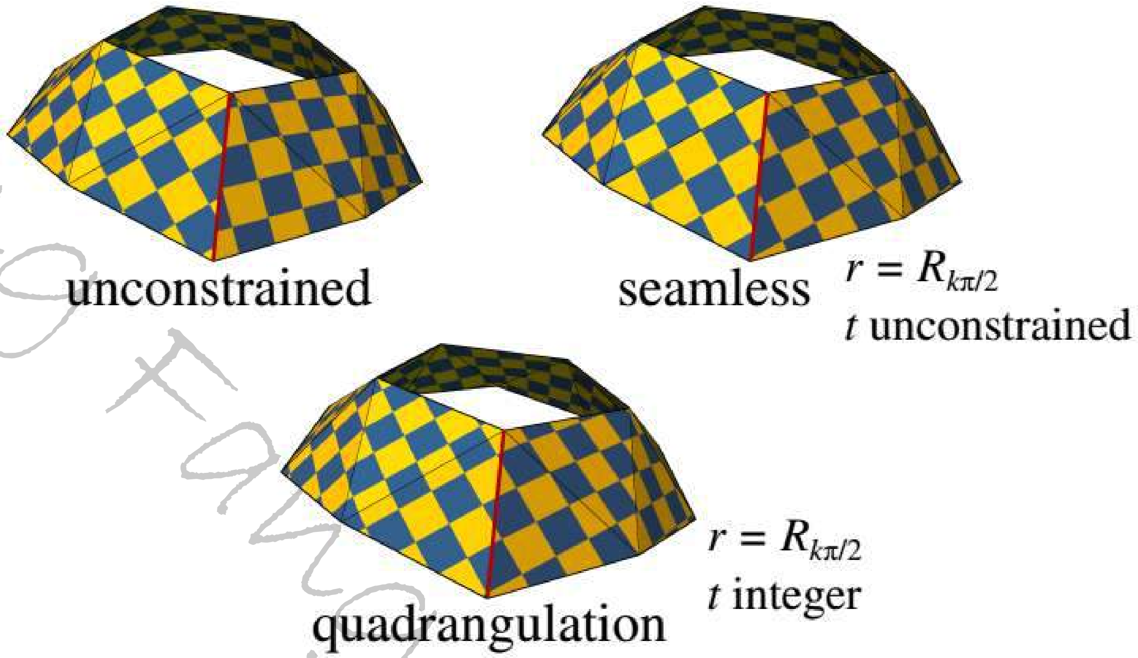
$$s.t.\ \ \varphi_j(p) = R^{k_{ij}\frac{\pi}{2}}\varphi_i(p) + T_{ij}$$

$$\det(\nabla\varphi) > 0$$

Integer constraints

$$k_{ij} \in \mathbb{Z},\ T_{ij} \in \mathbb{Z}^2$$

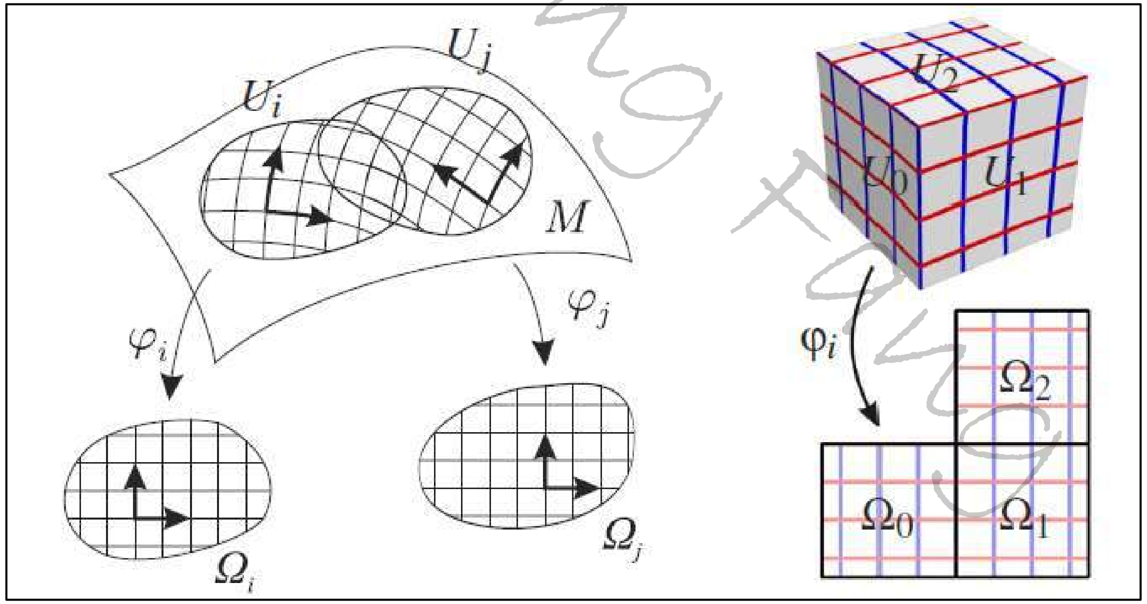
Boundary condition



[Myles and Zorin '13]

Parametrization-based Quadrangulation

- Given the parametrization of the input
- ***Extract Integer points***
- ***Re-map integer points onto the input surface***
- ***These integer points construct the quad mesh***



[Kälberer et al. ‘07]

Frame field-driven quad- and hex-remeshing

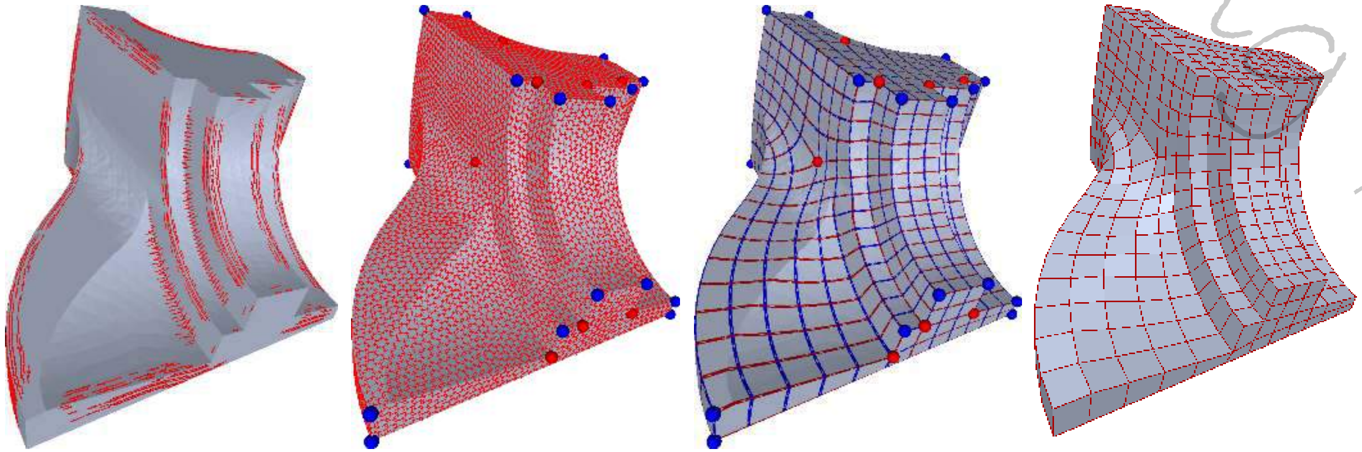
Invalid Parameterization

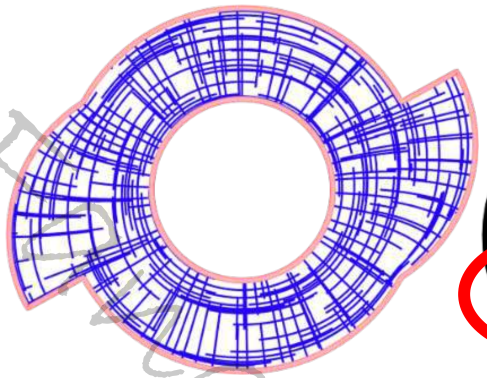
- The solution may even not exist!
  - Feasible region is a null set.
- **Conflict** among
  - Boundary & feature alignment
  - Non-degenerated
  - Transition at cut

Related works: Parametrization-based Quadrangulation

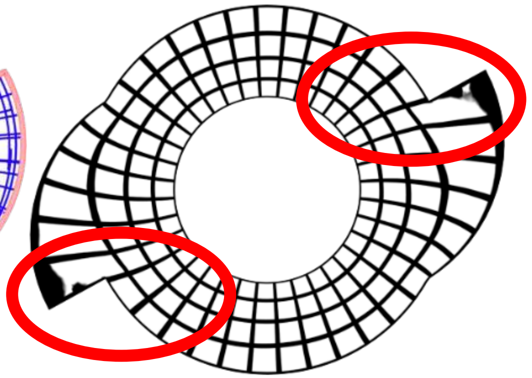
- QuadCover: using branched coverings [Kälberer et al. ‘07]
- Mixed-Integer Quadrangulation [Bommes et al. ‘09]
- Integer-Grid Maps for Reliable Quad Meshing [Bommes et al. ‘13]
- ……

No guarantee





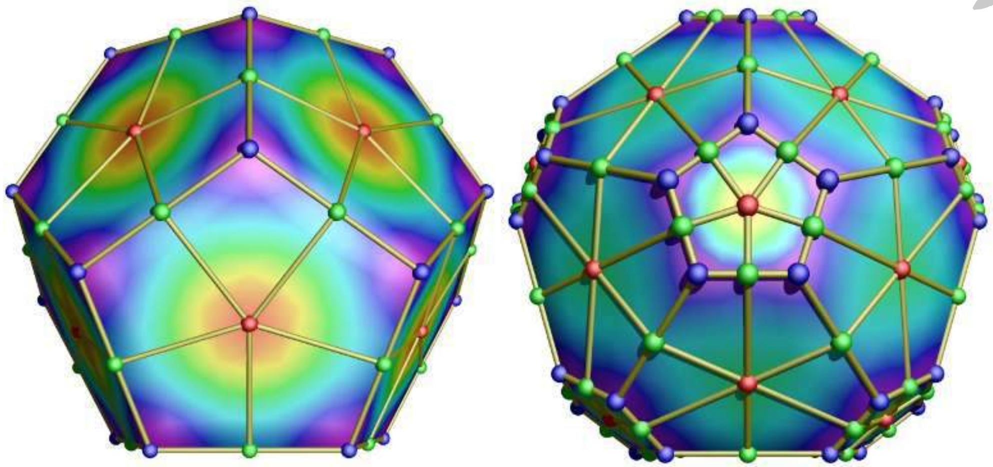
2D cross frame field



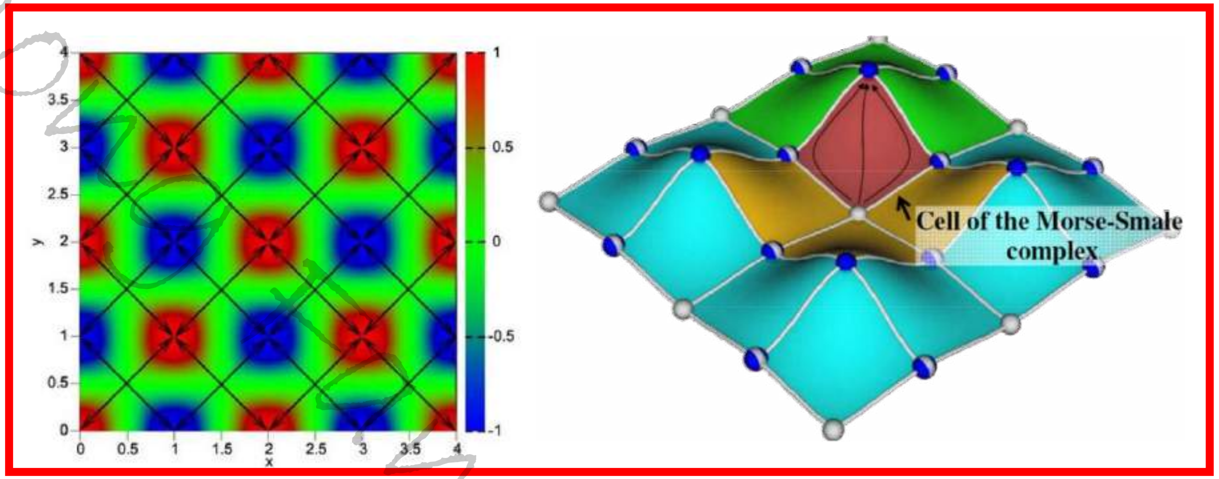
Parametrization (Texture)

Related works:
MSC-based quadrangulation

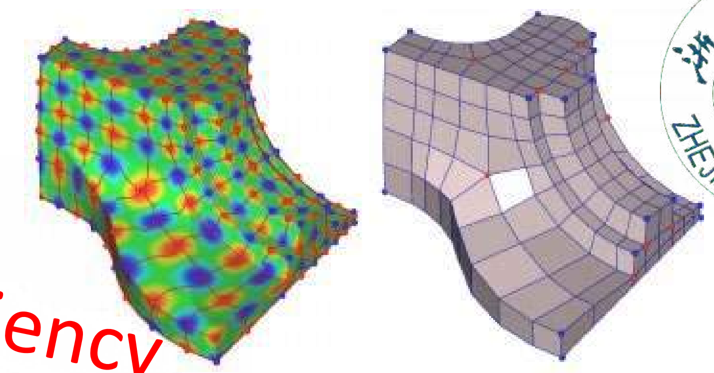
- Morse function and Morse-Smale complex (MSC)



[Dong et al. TOG 2006]

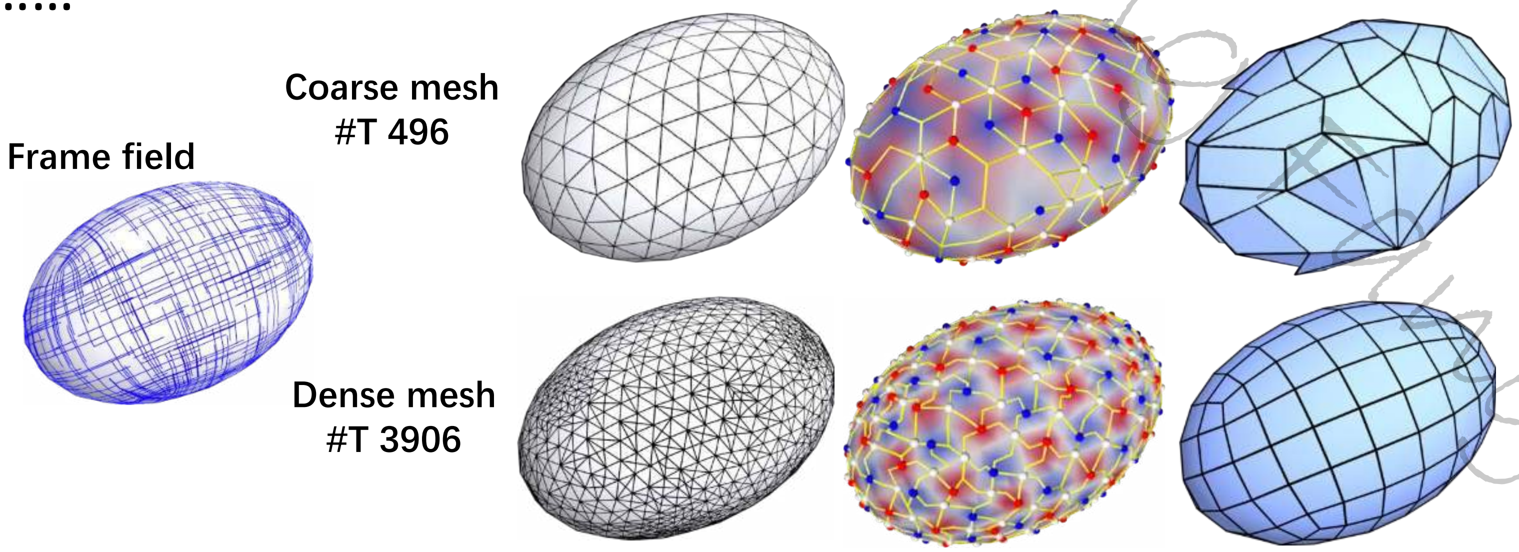


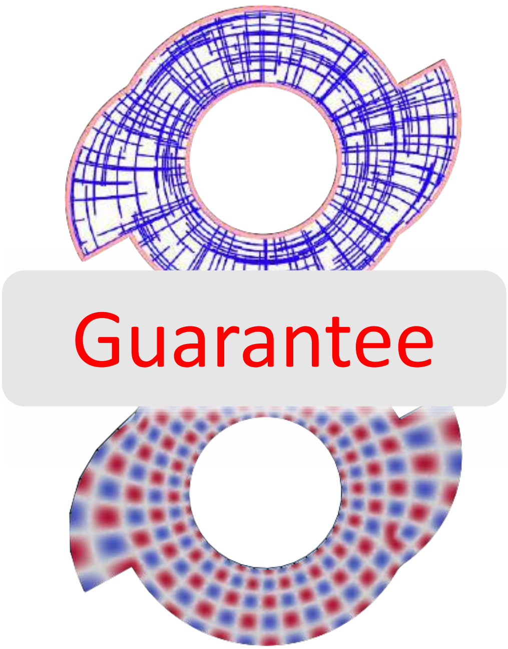
Related works: (MSC-based methods)



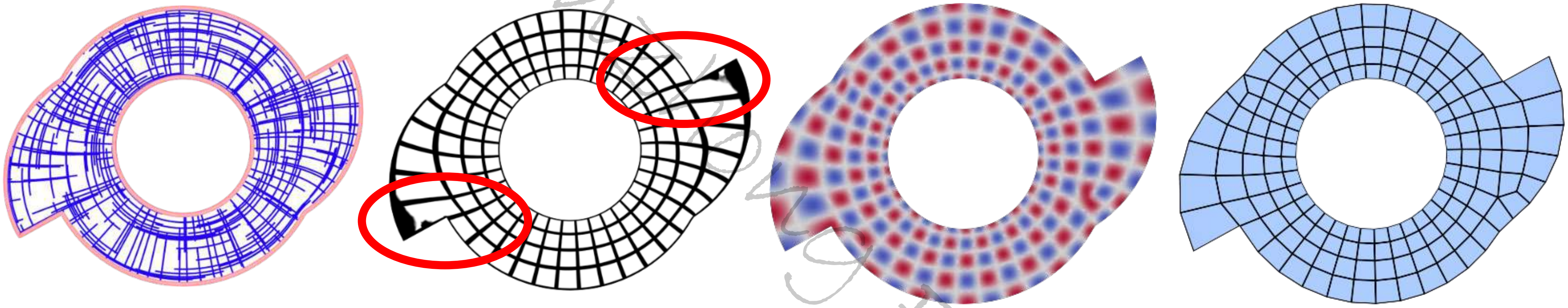
Low efficiency

- Spectral surface quadrangulation [Dong et al. ‘06]
  - (Dual) Morse-Smale complex to guarantee quads [Edelsbrunner et al. ‘03]
  - Orientation, alignment & size control [Huang et al. ‘08, Ling et al. ‘14]
- Wave-based anisotropic quadrangulation [Zhang et al. ‘10]
- ……





Topological conflicts



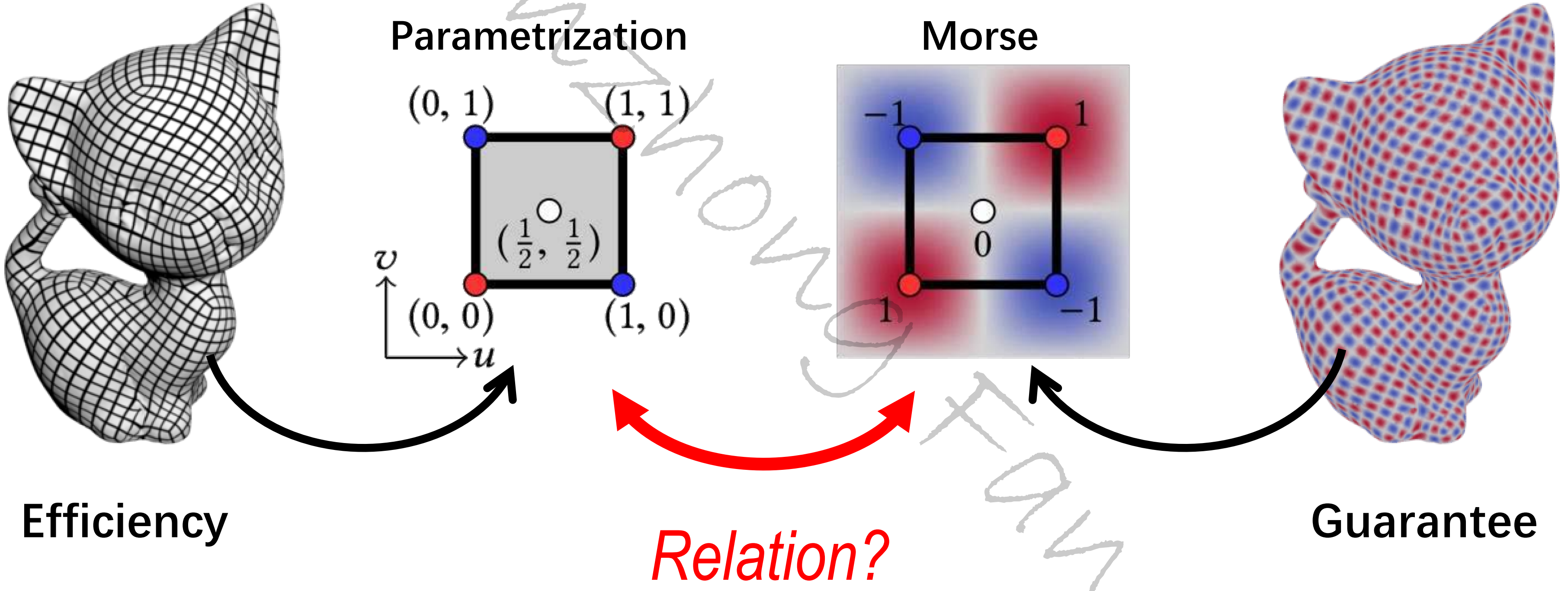
Cross frame field

Degenerated Parametrization

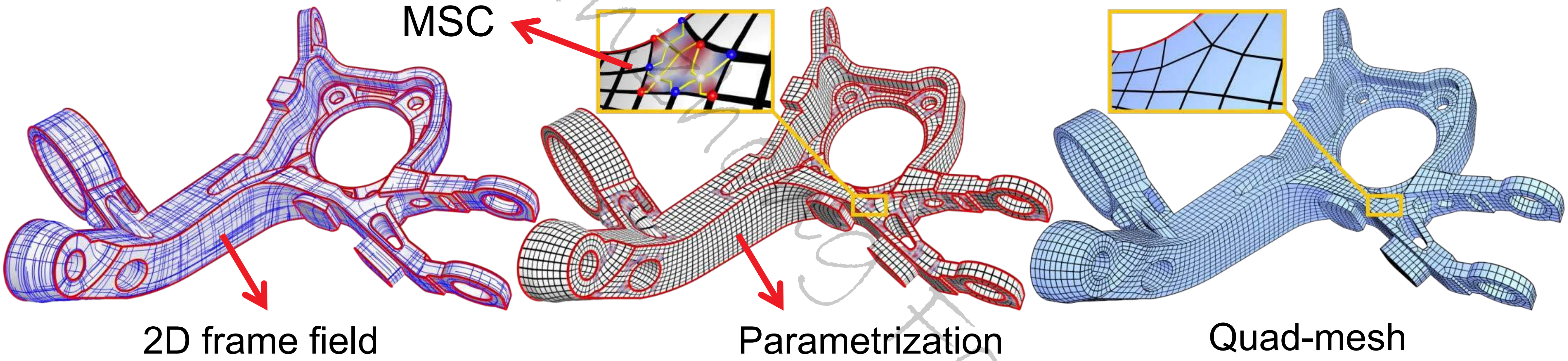
Holonomy condition

MSC: guarantee, but low efficiency. Need dense enough mesh as input

Parametrization & Morse function



Parametrization & MSC -- Quadrangulation



[Fang et al. TOG '18]

MSC automatically inserts singularities.
Resolve topological conflicts.

Periodic vector field $\boldsymbol{\Psi}$: From Morse function to parametrization

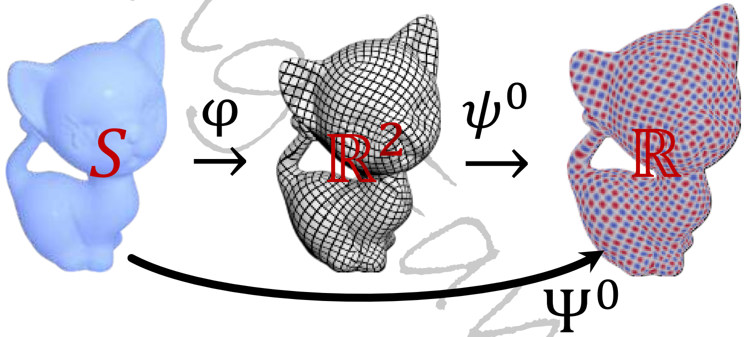
$$\boldsymbol{\Psi} = \boldsymbol{\psi} \circ \boldsymbol{\varphi} \quad \text{with}$$

Morse function

$$\boldsymbol{\psi}(\boldsymbol{u},\boldsymbol{v}) = (\boldsymbol{cc}(\boldsymbol{u},\boldsymbol{v}), \boldsymbol{sc}(\boldsymbol{u},\boldsymbol{v}), \boldsymbol{cs}(\boldsymbol{u},\boldsymbol{v}), \boldsymbol{ss}(\boldsymbol{u},\boldsymbol{v}))^{T}$$

$$\varphi = \psi^{-1} + (T_1, T_2)^T \qquad T_1 \pm T_2 \in 2\mathbb{Z}$$

$$\begin{cases} cc(u,v) = \cos(\pi u)\cos(\pi v) \\ sc(u,v) = \sin(\pi u)\cos(\pi v) \\ cs(u,v) = \cos(\pi u)\sin(\pi v) \\ ss(u,v) = \sin(\pi u)\sin(\pi v) \end{cases}$$



Conditions

$$\|\Psi\|_2 = 1$$

$$\Psi^0\Psi^3 = \Psi^1\Psi^2$$

$$\hat{\nabla}_{\mathbf{v}}\Psi = \frac{\partial \Psi}{\partial p} - \pi W(F^{-1}\mathbf{v})\Psi$$

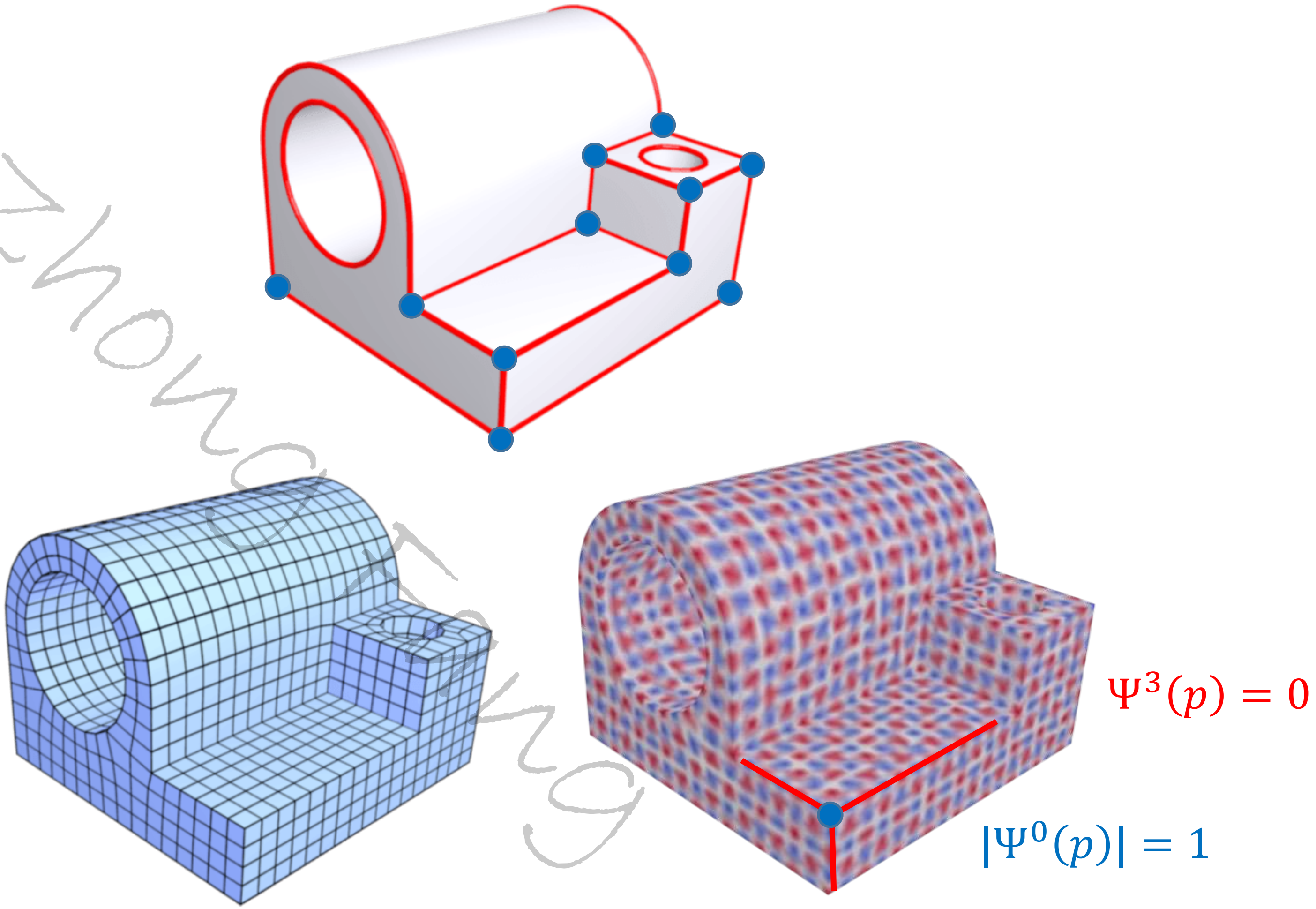
Idea: 1) Compute a good Ψ
2) Reconstruct φ where it's easy
3) Revert to Morse if needed

Compatible

Frame field-driven quad- and hex-remeshing

Boundary & feature alignment

- Vertices on boundary & feature
  - u **or** $v \in \mathbb{Z}$ ...
  - $\Psi^3(p) = ss\big(u(p), v(p)\big) = 0$
- Corners
  - u **and** $v \in \mathbb{Z}$ ...
  - $\Psi^1(p) = \Psi^2(p) = \Psi^3(p) = 0$

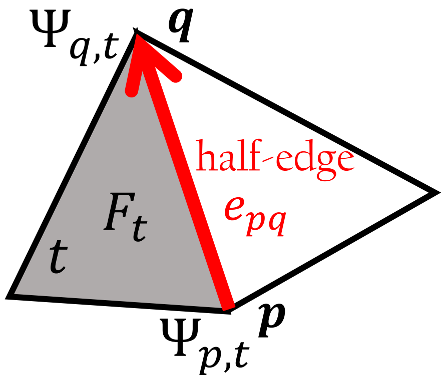


Periodic 4D Vector Field Optimization

- **Frame field driven periodic vector field generation**

$$\min_{\Psi} \int_S \left\|\hat{\nabla}\Psi\right\|^2 \quad s.t. \begin{cases} \|\Psi\|_2 = 1 \\ \Psi^0\Psi^3 = \Psi^1\Psi^2 \\ \text{corners \& features} \end{cases}$$

$$\mathcal{E}(\Psi) = \sum_{(p,q)} \mathcal{E}_{pq} \quad \text{with} \quad \mathcal{E}_{pq} = \frac{|t|}{2|e_{pq}|^2} \left\| \Psi_{q,t} - \exp\left(\pi W\left(F_t^{-1} e_{pq}\right)\right) \Psi_{p,t} \right\|^2$$

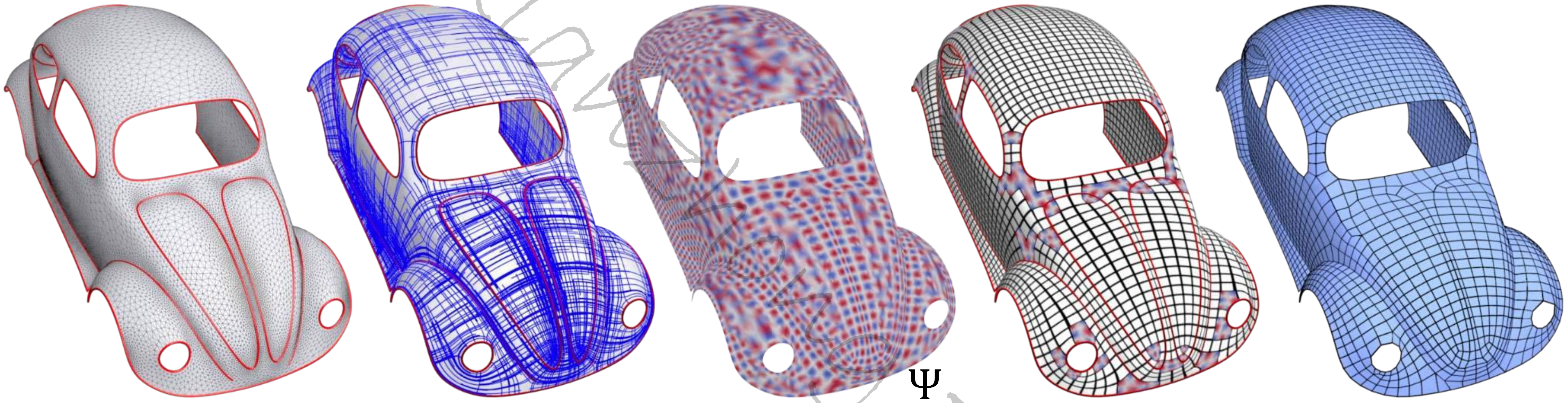


- **Constrained nonlinear optimization**

$$\min_{\Psi} \mathcal{E}(\Psi) \quad s.t. \begin{cases} \left\|\Psi_p\right\|^2 = 1, & p \in V \\ \Psi_p^0\Psi_p^3 = \Psi_p^1\Psi_p^2, & p \in V \\ \Psi_p^3 = 0, & p \in V_b \\ \Psi_p^1 = \Psi_p^2 = 0, & p \in V_c \end{cases}$$

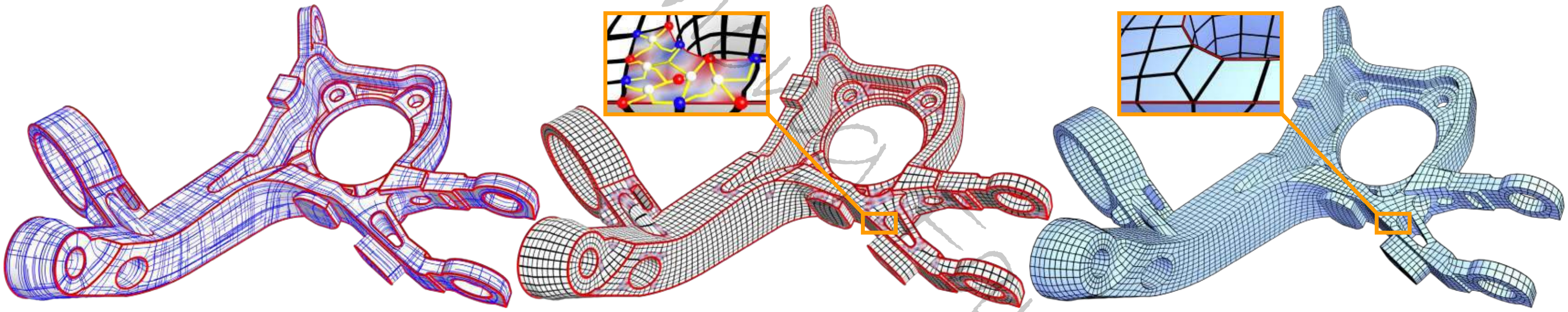
Frame field-driven quad- and hex-remeshing

Pipeline



- **Extract quad-mesh from periodic vector function**
  - Construct global parametrization
  - Extract quads by two steps: **Regular region & Singular region**

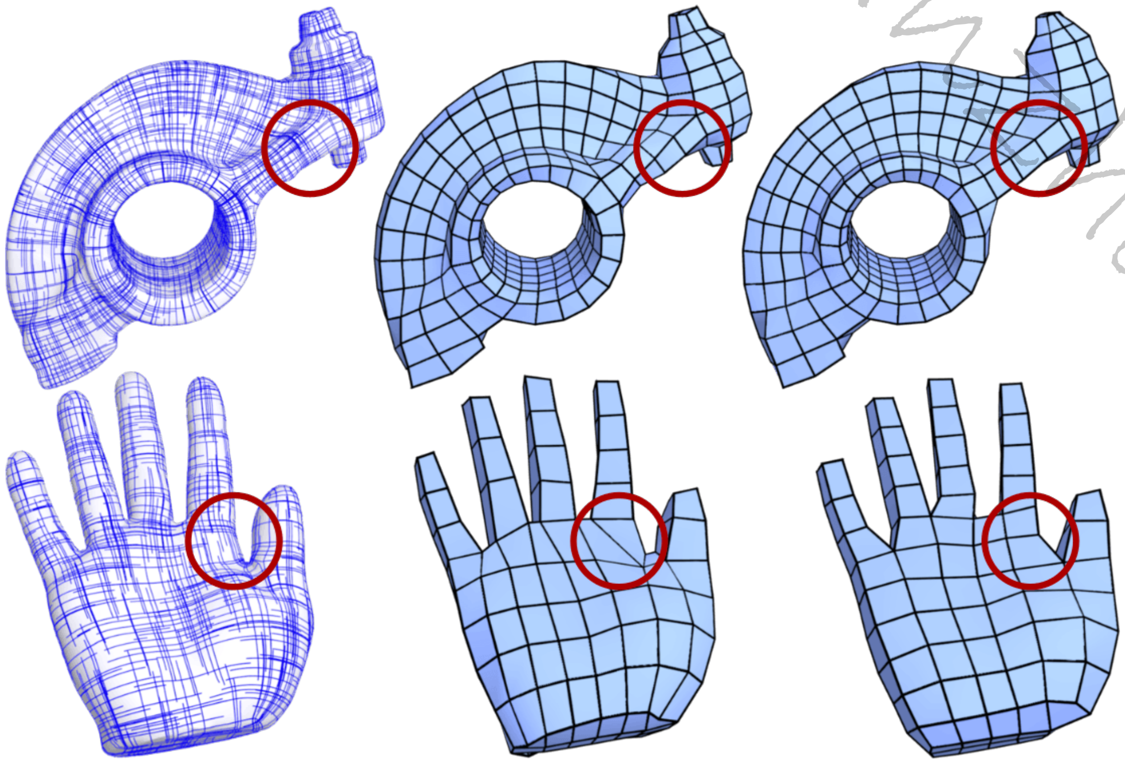
Results



Frame field-driven quad- and hex-remeshing

V.S. IGM

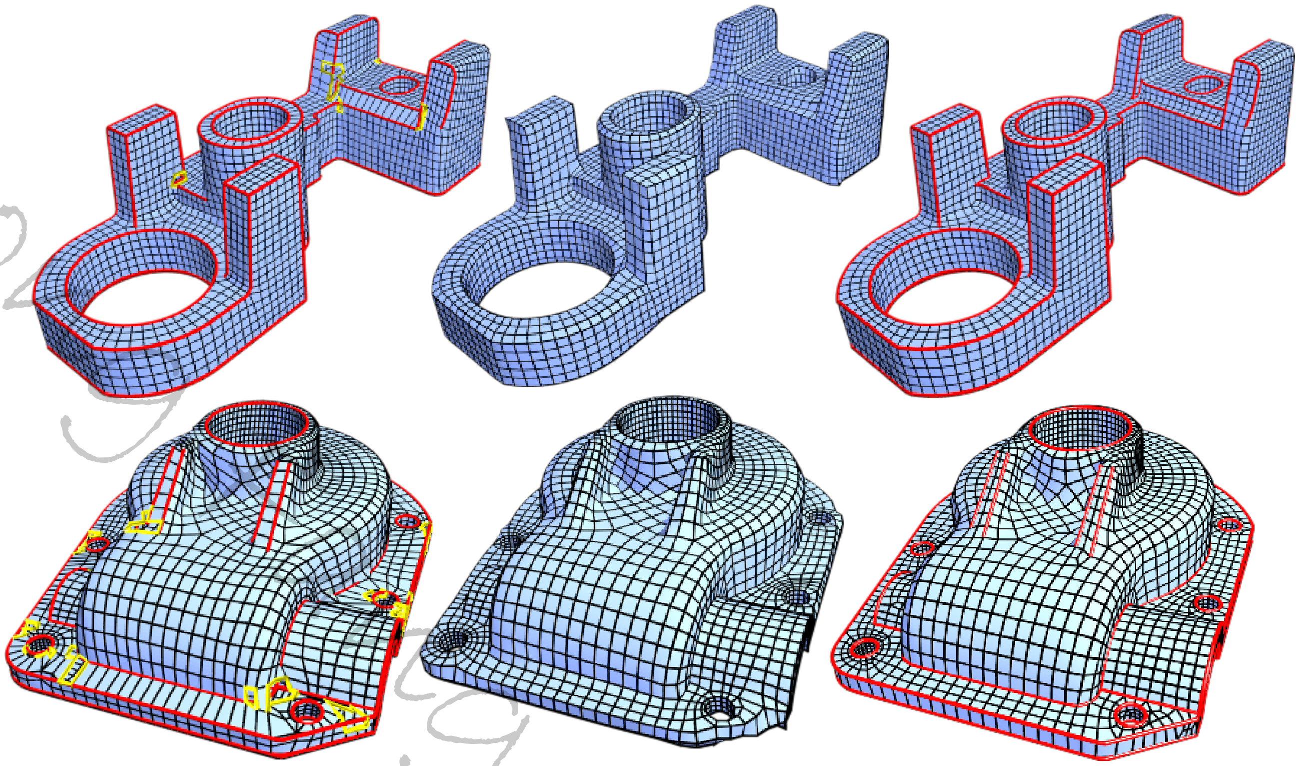
- Better frame alignment



Frame field IGM Ours

V.S. MIQ

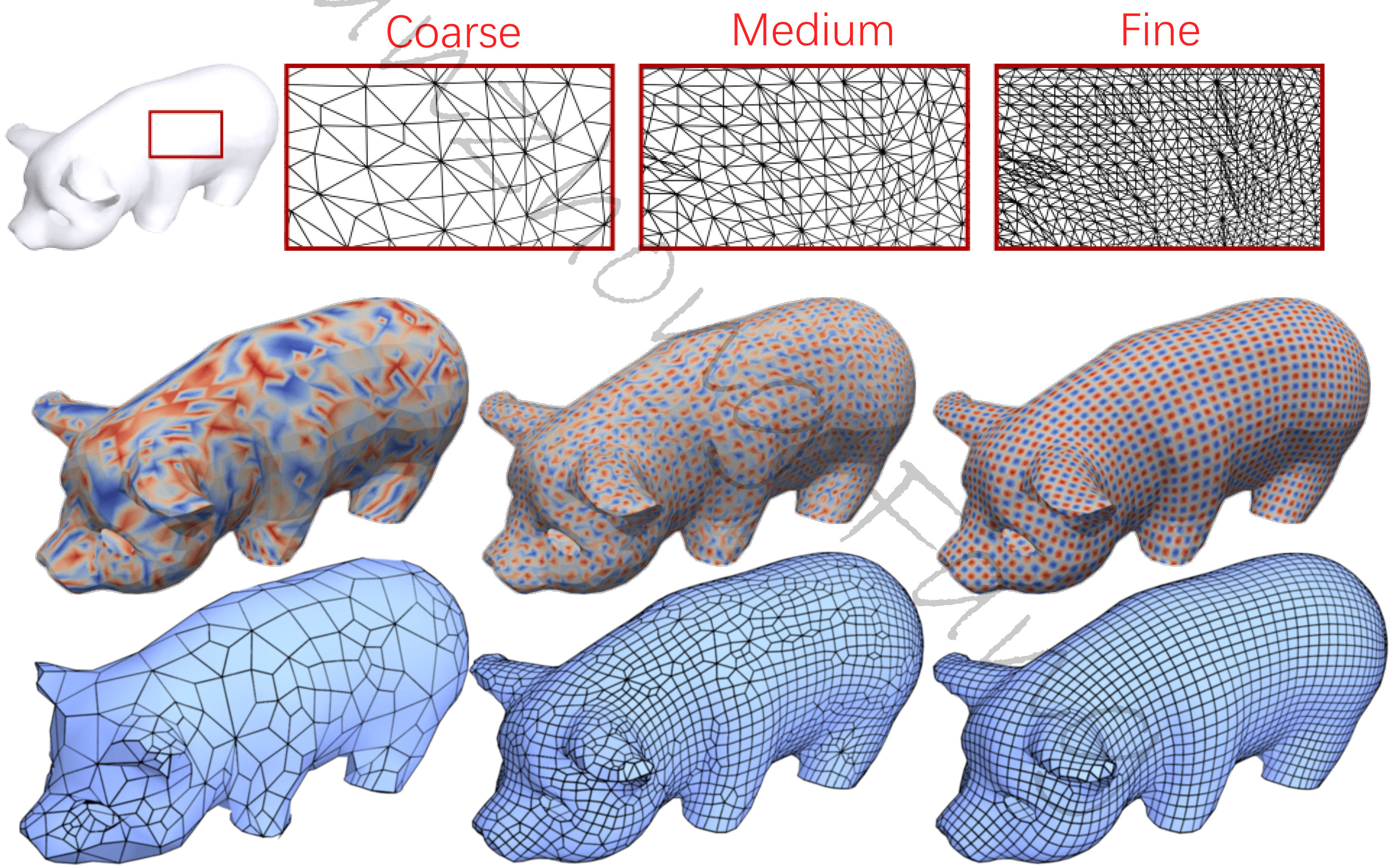
- Guarantee



MIQ (Feature) MIQ (No feature) Ours (Feature)

V.S. Wave-based method

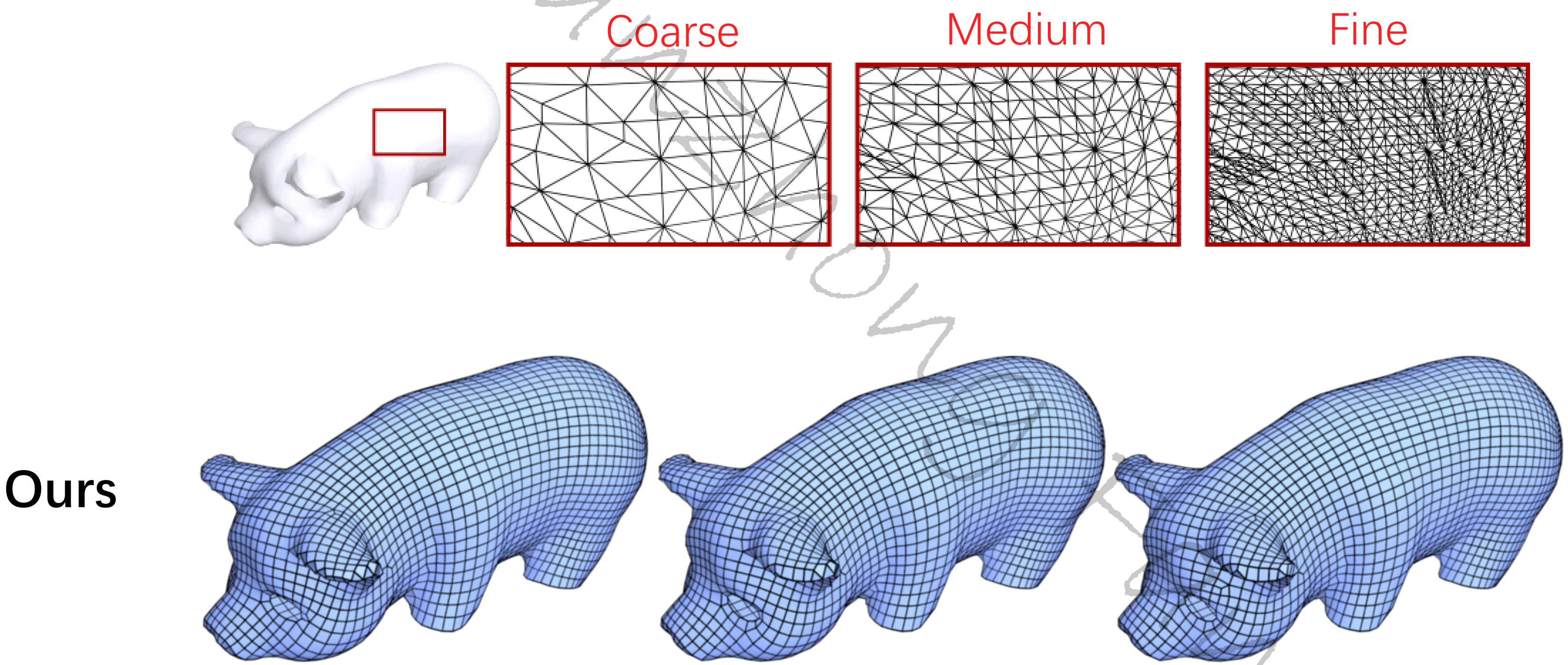
- More efficient



Wave-based method

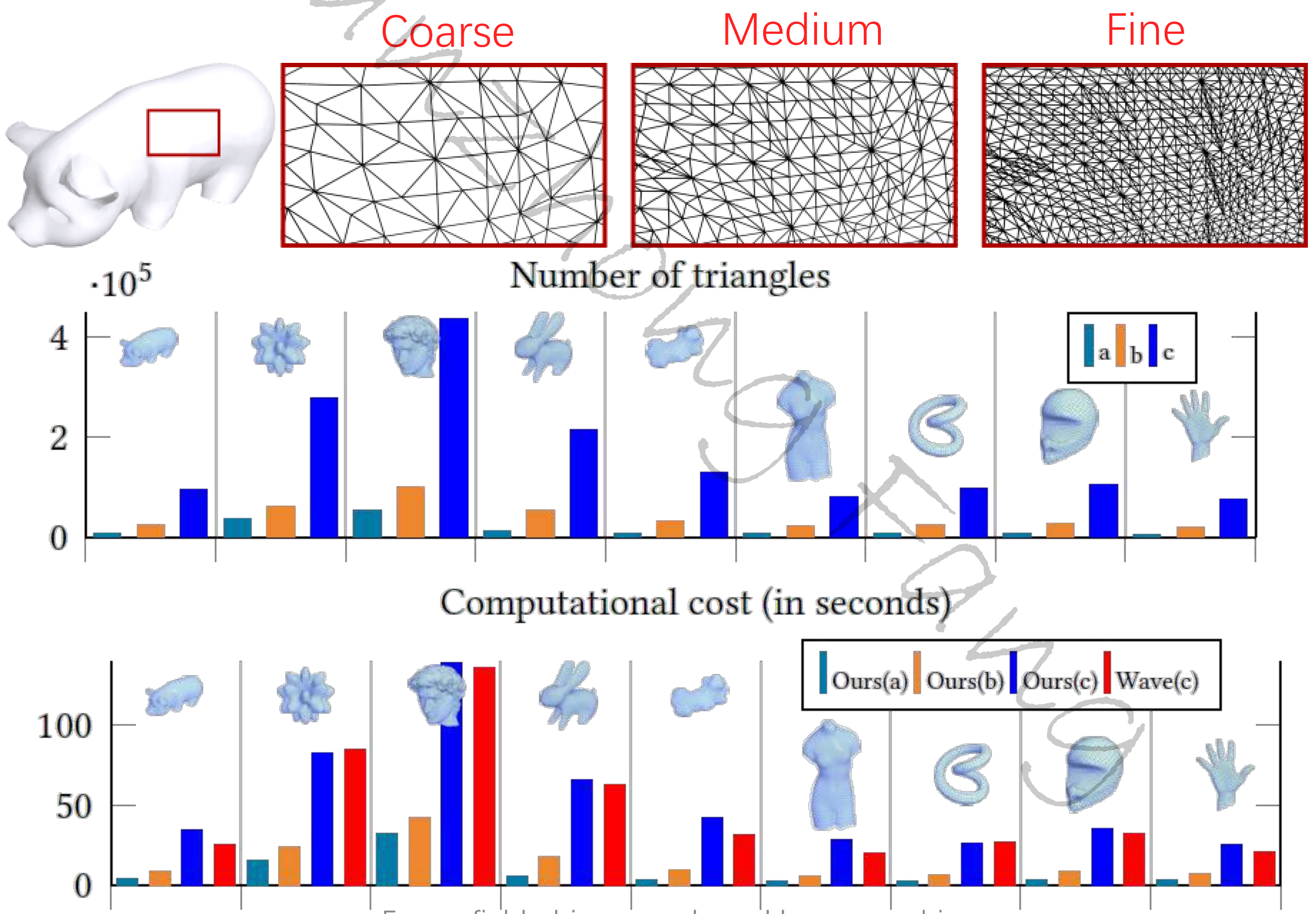
V.S. Wave-based method

- More efficient



V.S. Wave-based method

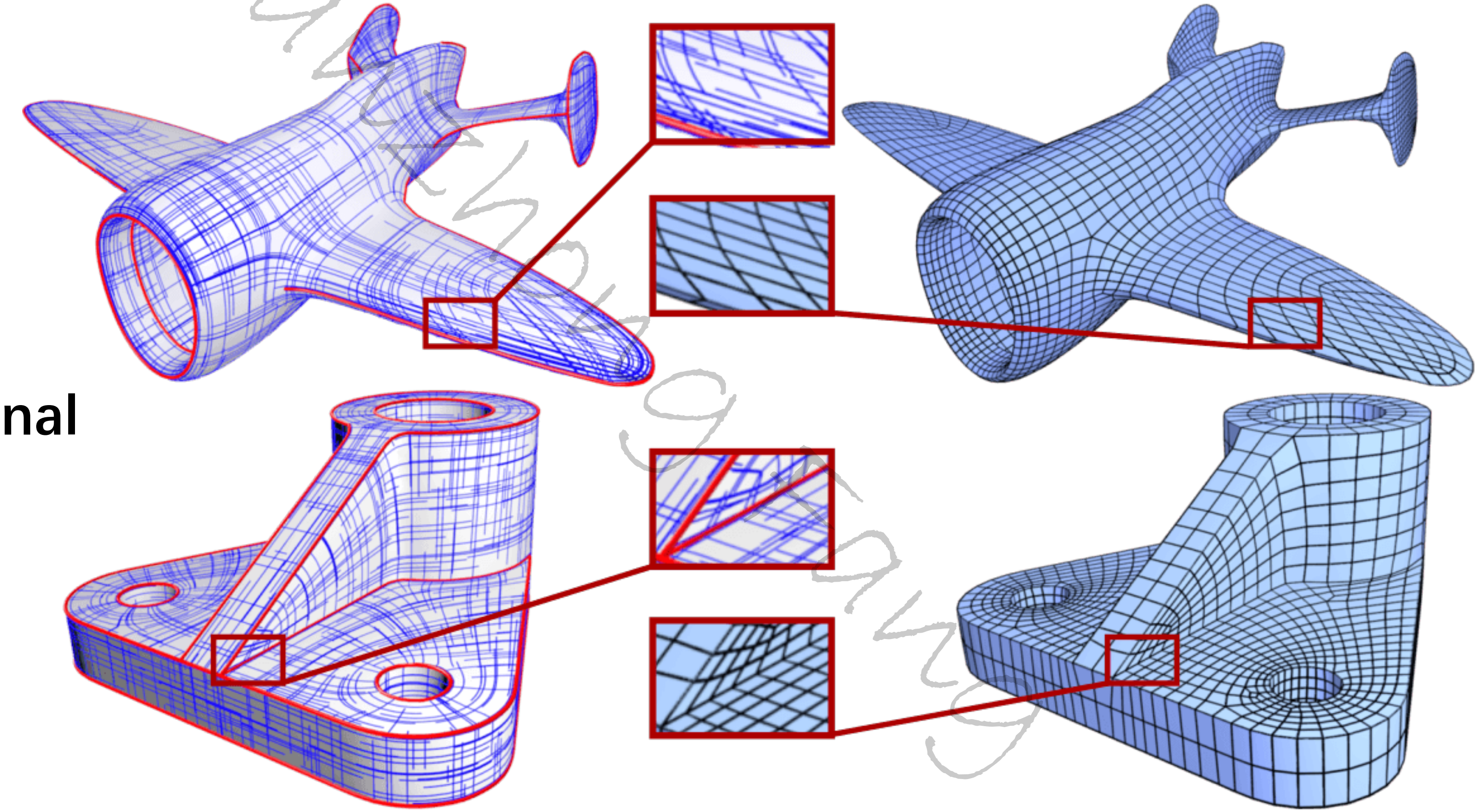
- More efficient



Frame field-driven quad- and hex-remeshing

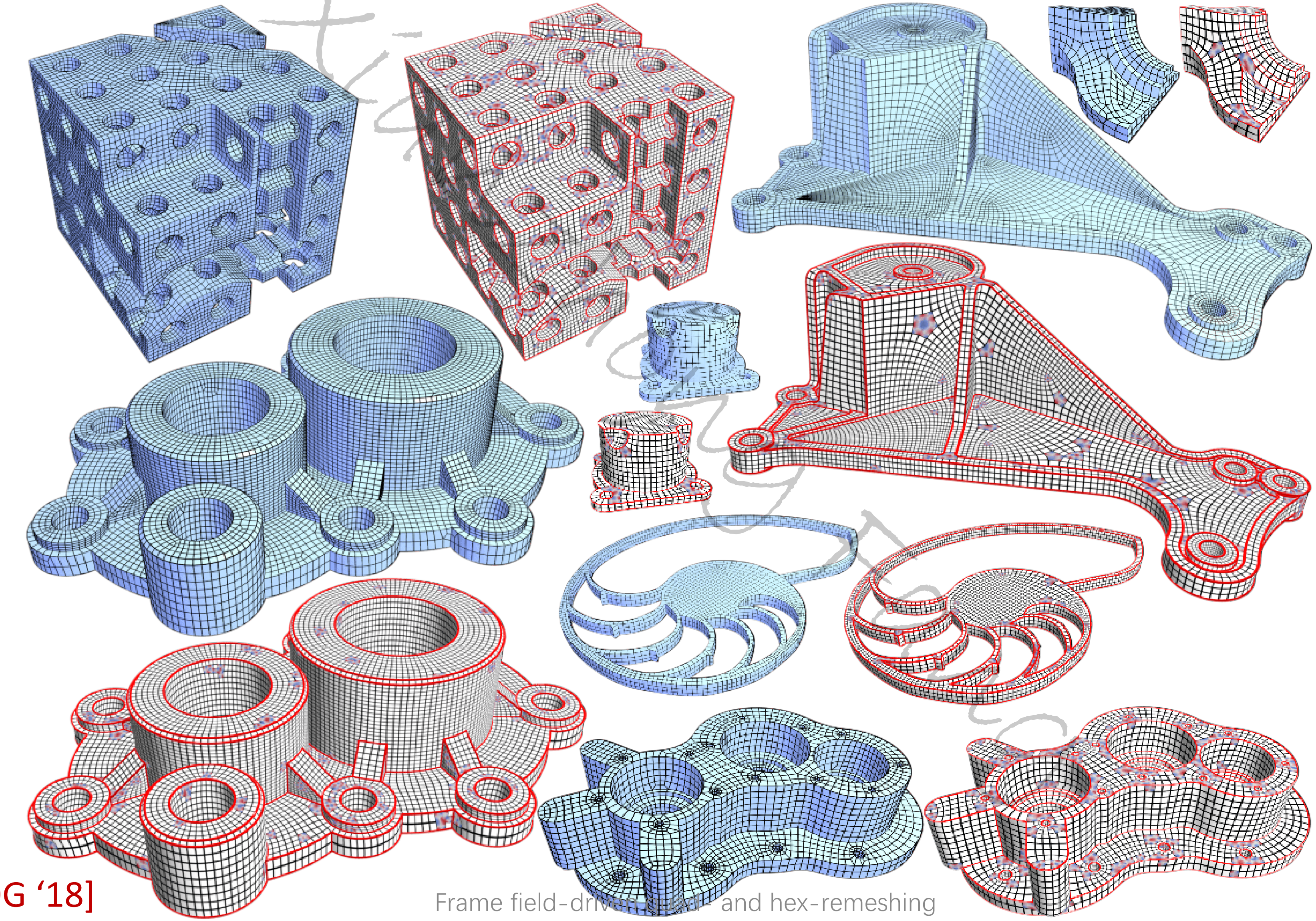
Frame Field Control

- Anisotropic
- Non-orthogonal



Frame field-driven quad- and hex-remeshing

Results on models with complicated features



[Fang et al. TOG '18]

Can these QUAD techs extend to HEX ?

There are several big challenges!

- Algebra representation of 3D frame field: **Non-commutative**
- Global topology: **No Gauss-Bonnet theorem** in 3D
- **No conformal structure** in 3D

Can these quad techs extend to Hex?

Frame field driven parametrization

Hard to get hex-meshable frame field

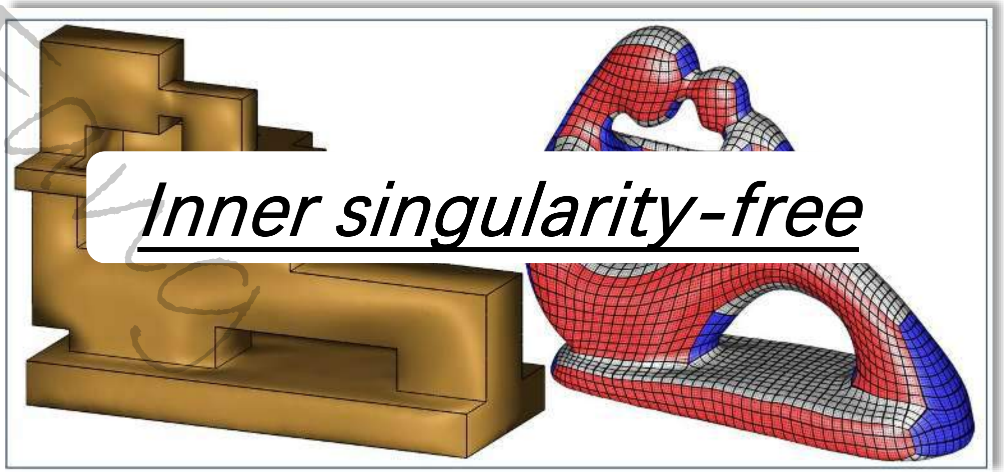
Hex-Remeshing

3D Morse-Smale complex

No guarantee

<u>Hard to control the inner singularity structure</u>

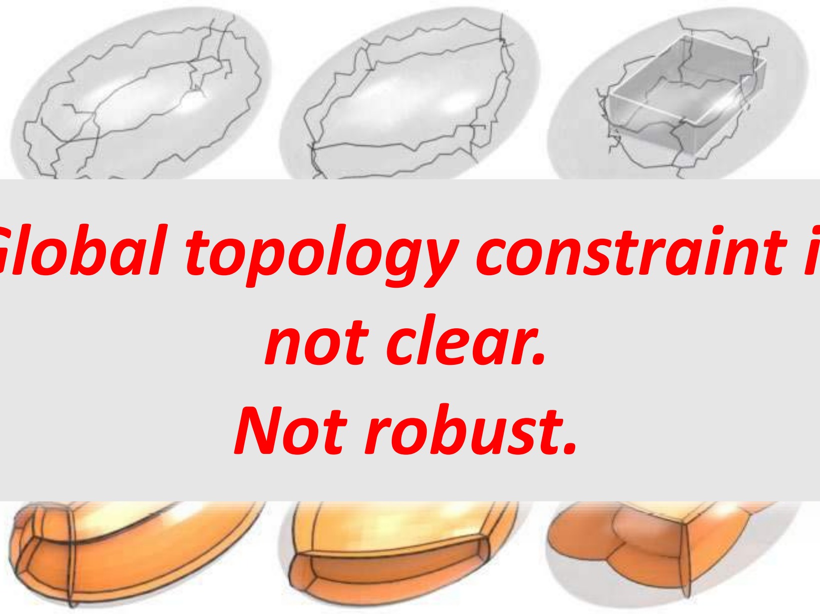
Polycube



[Livesu et al. TOG '13]

Parametrization-based Hex-Remeshing

Frame field driven parametrization



Global topology constraint is not clear.
Not robust.

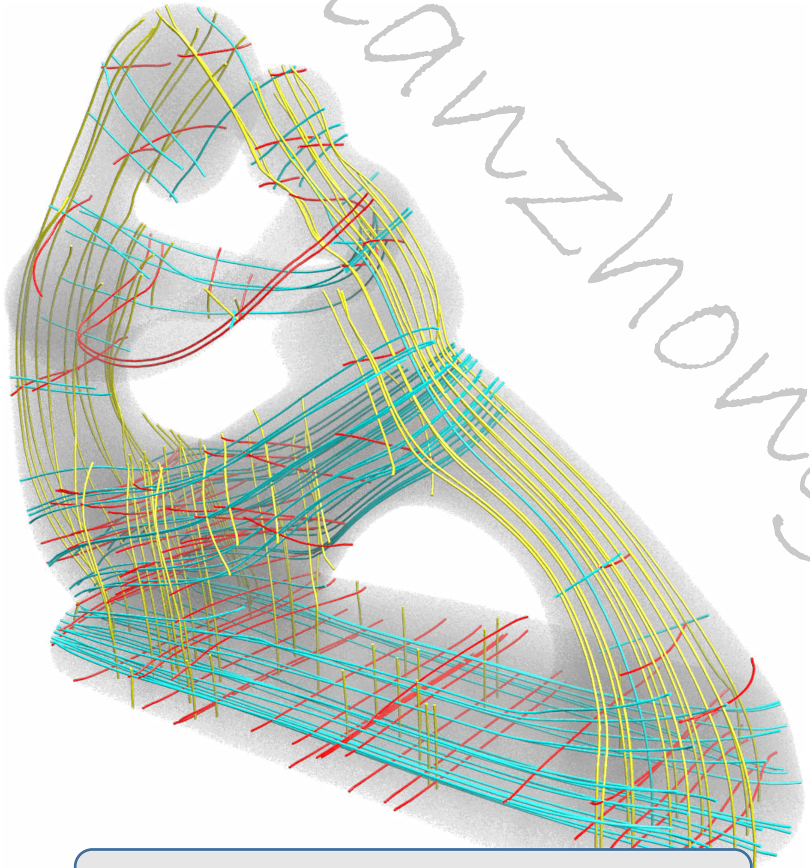
[Li et al. TOG '12]

Polycube

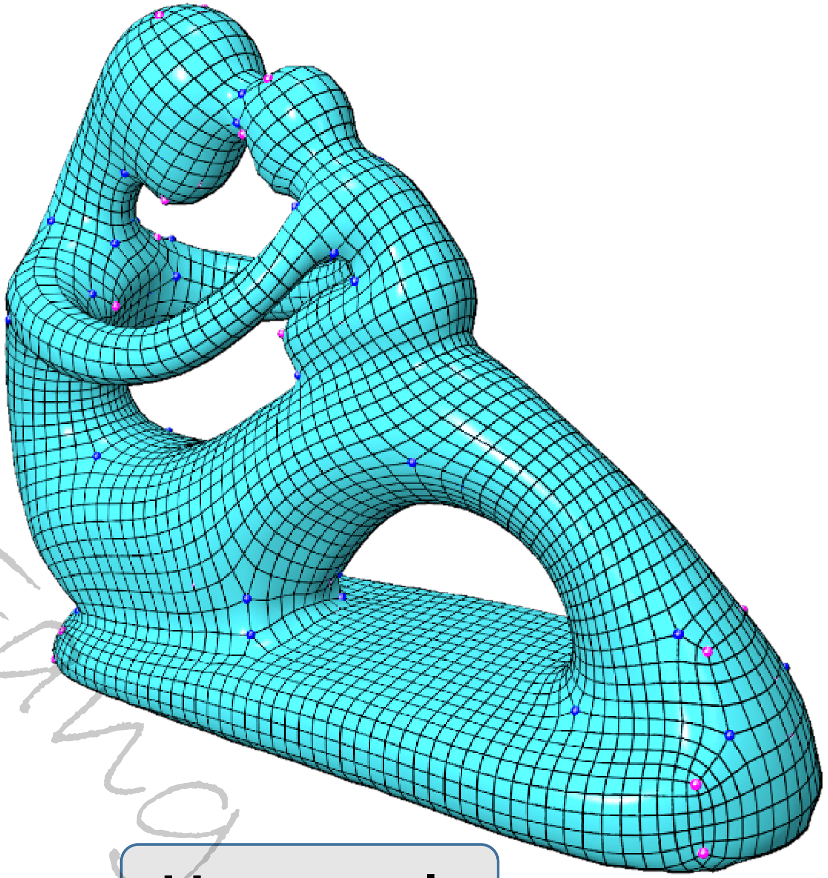
Simple global topology structure.
Robust.
Bad element quality.

[Livesu et al. TOG '13]

3D Frame Field



3D cross frame field

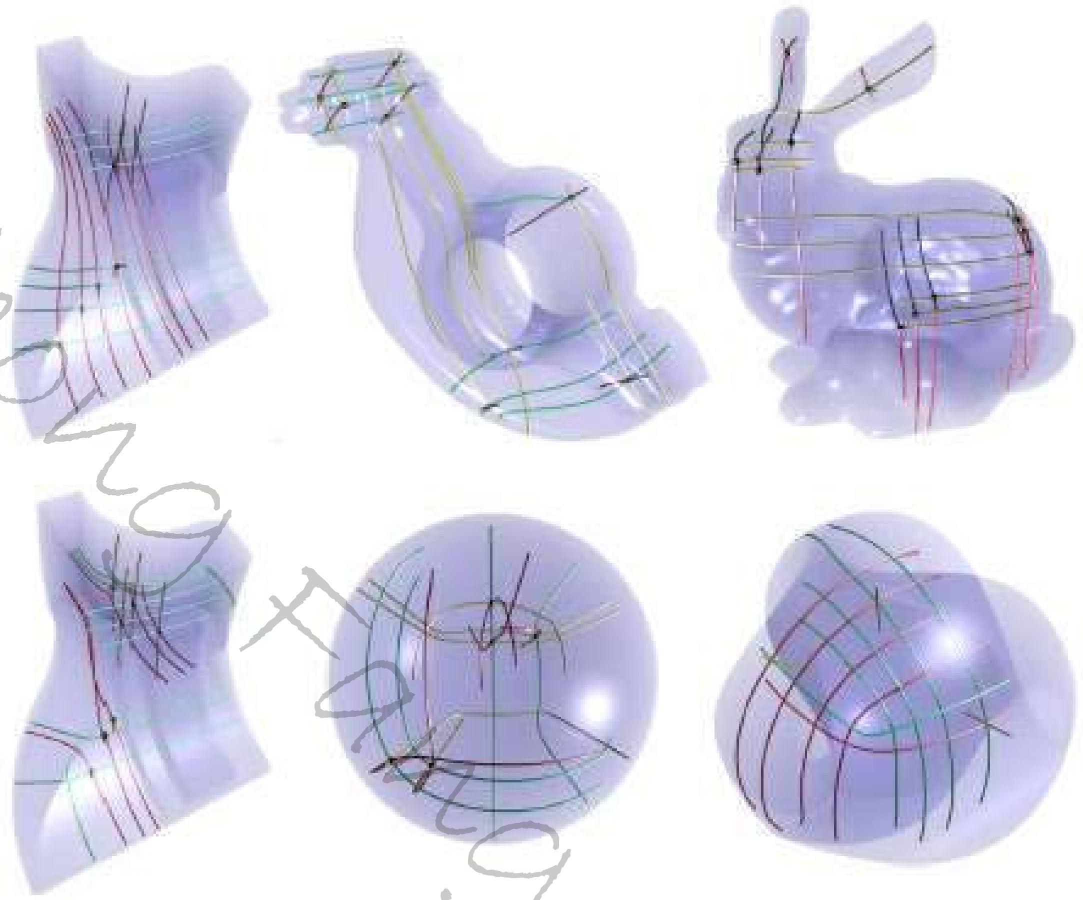


Hex-mesh

Frame field-driven quad- and hex-remeshing

3D Frame Field Generation

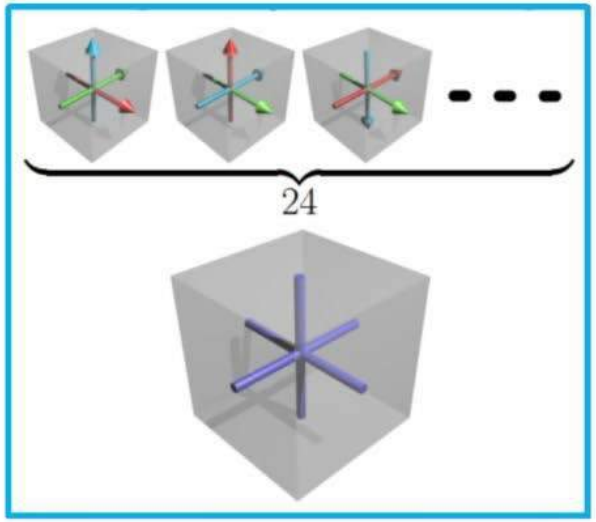
- Cubic symmetry
- Smoothness
- Boundary alignment

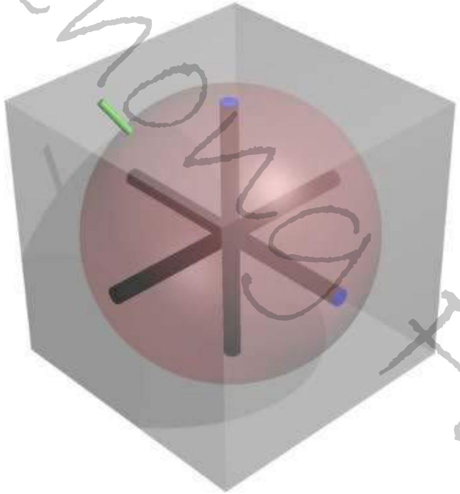


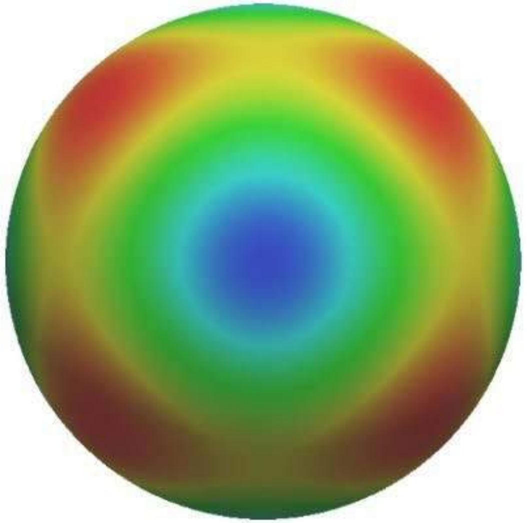
[Huang TOG 2011]

SH for Symmetric Field

- Symmetric spherical function



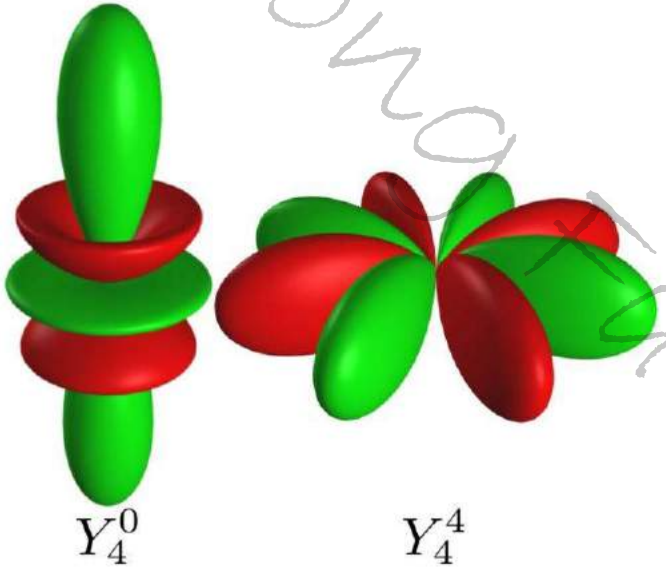


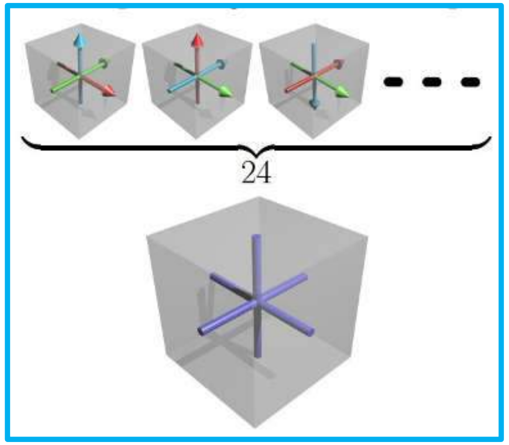


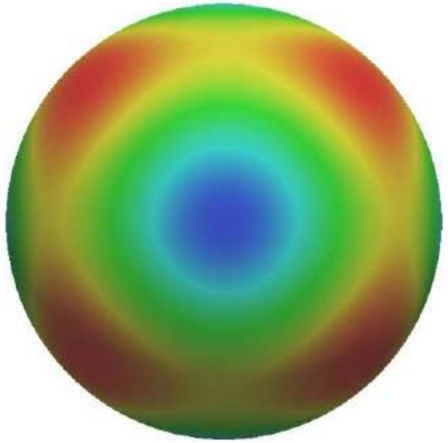
SH for Symmetric Field

- Symmetric spherical function
- Vector space of spherical harmonics

$$f = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{7} \\ 0 \\ 0 \\ 0 \\ \sqrt{5} \end{pmatrix}$$





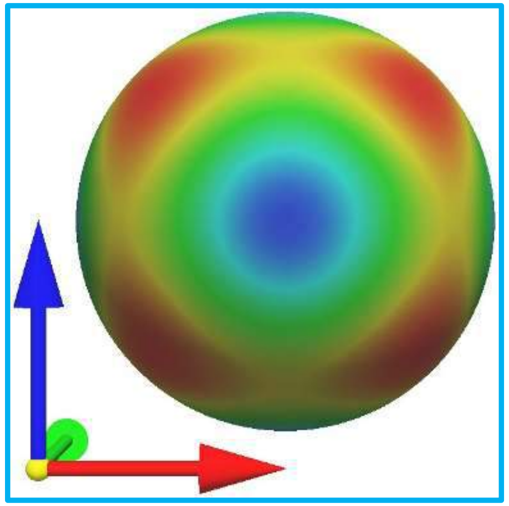


Advantages

- Simple math in a vector space
  - A single vector represents all 24 symmetries.
  - Smoothness is measured by Dirichlet energy of vectors

$$\int_\Omega ||\nabla f||^2$$

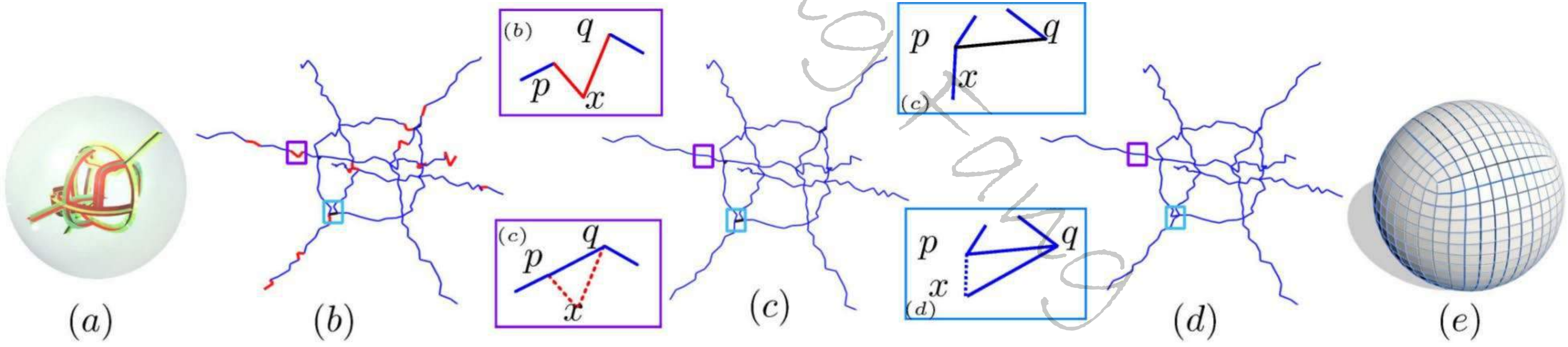
- Normal alignment is a linear constraint
  - If and only if $(R_{n\to z}f)[4] = \sqrt{7}$
- Linear initialization for non-linear optimization



Spherical harmonical function

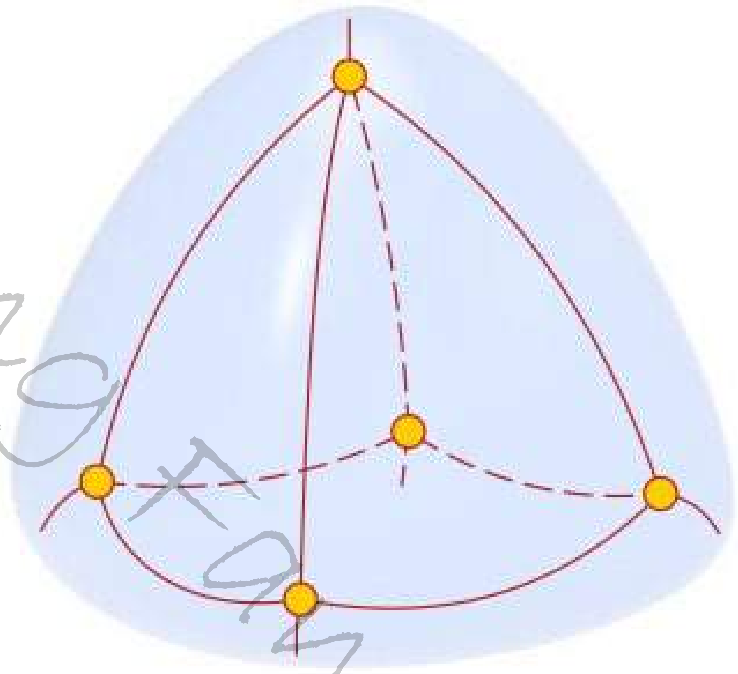
Topology Conflict

- Local conflict [Jiang TVCG 2014]
  - Defines all (inner & boundary) local conflicts
  - Detects all and fix all
  - All can be removed with proof



Topology Conflict

- BUT: there is global conflict!
  - No clear definition of it
  - Cannot be detected and fixed



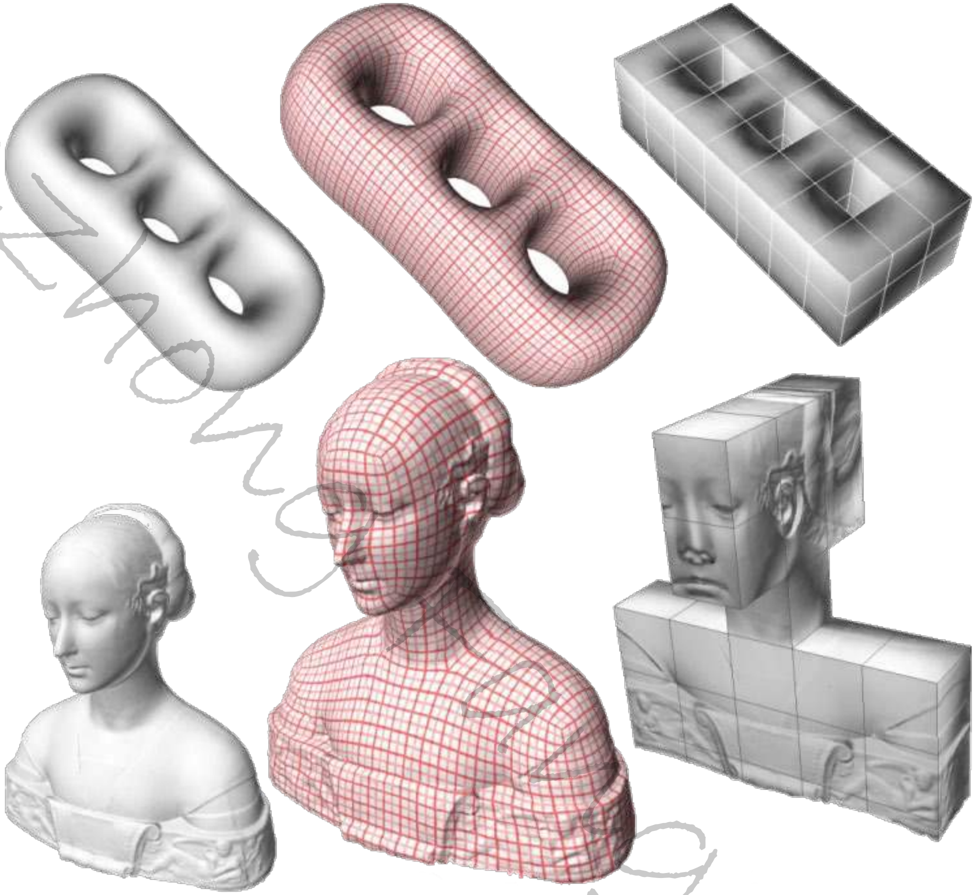
[Li et al. TOG 2012]

Frame Field-Driven Method

- High DOFs and quality if the feasible region is not empty
- But global conflict is hard to be resolved
  - Begin with ***simple and robust structure***
  - Gradually introducing more DOFs for better quality

Polycube-Based Method

No Inner Singularity

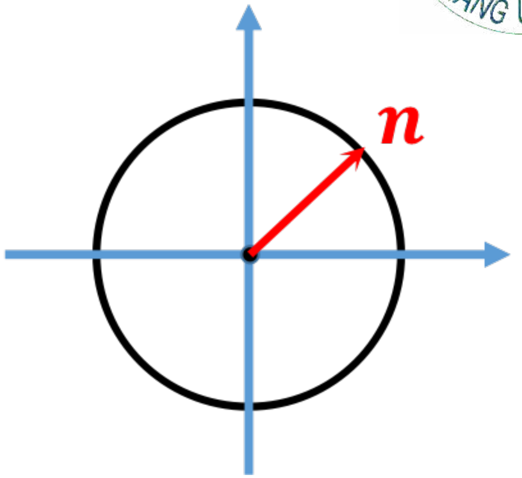


[Tarini et al. TOG ’04]

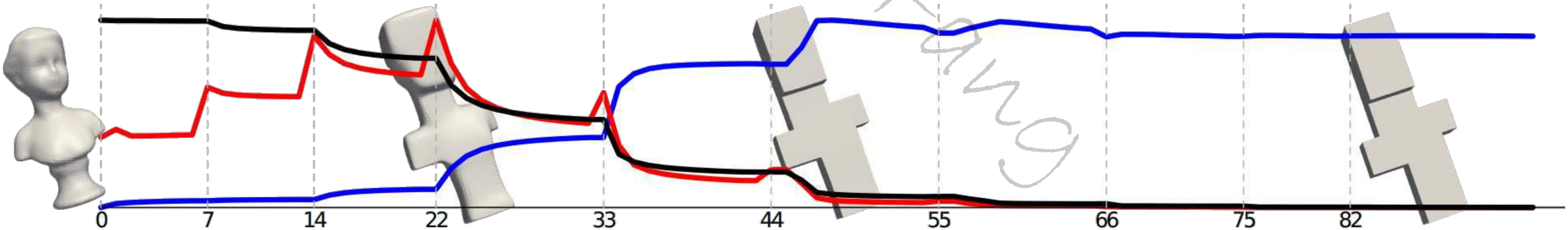
Polycube-Based Method

- Polycube: Global structure is simple
- L1 Polycube [Huang TOG 2014]

$$\|\boldsymbol{n}\|_1$$

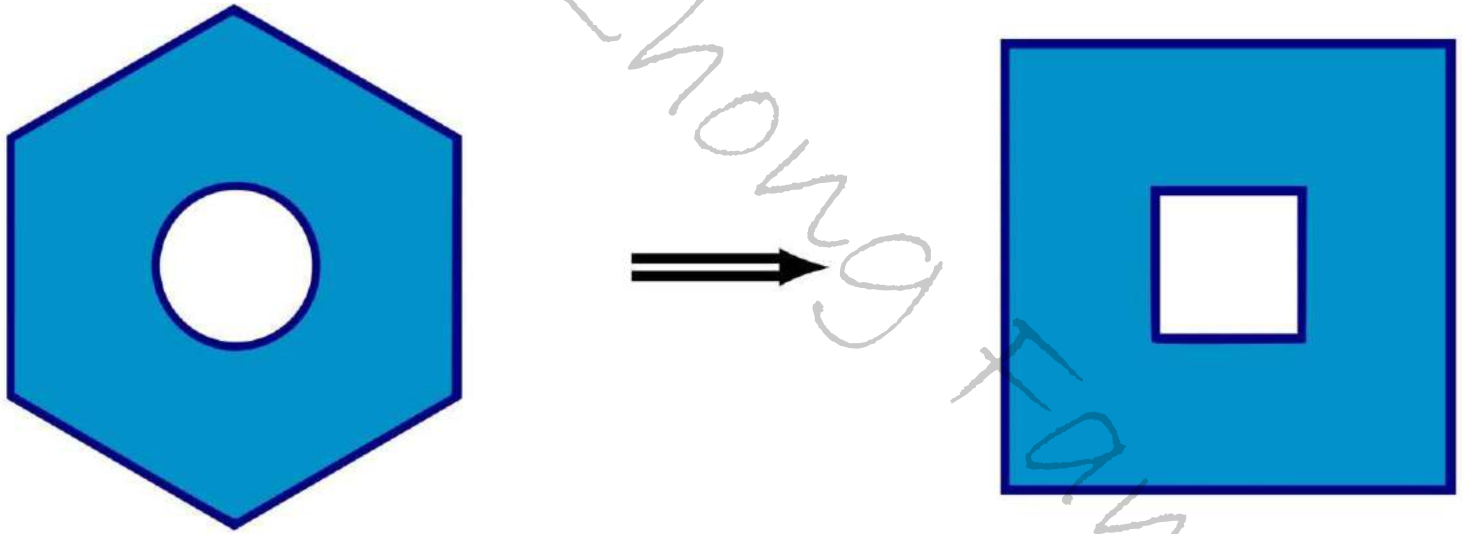


$$\min \int_{\partial\Omega} \|\mathbf{n}\|_1 + E_{ARAP}$$



Polycube-Based Method

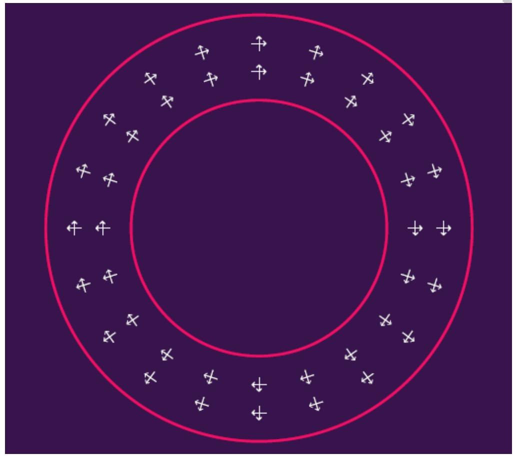
- Robust, but low quality

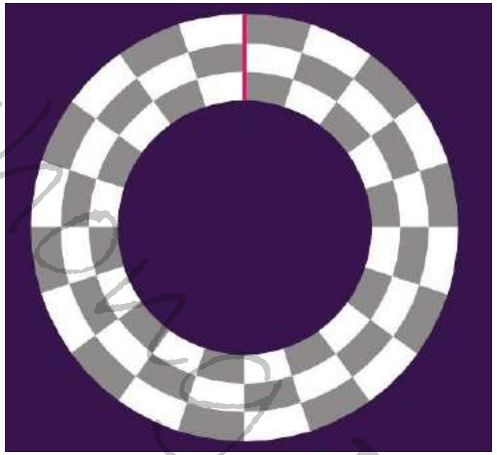


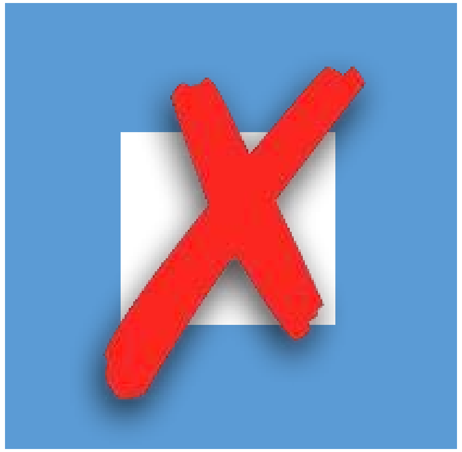
Another Simple Case

No singularity

Parametrization domain is not polycube







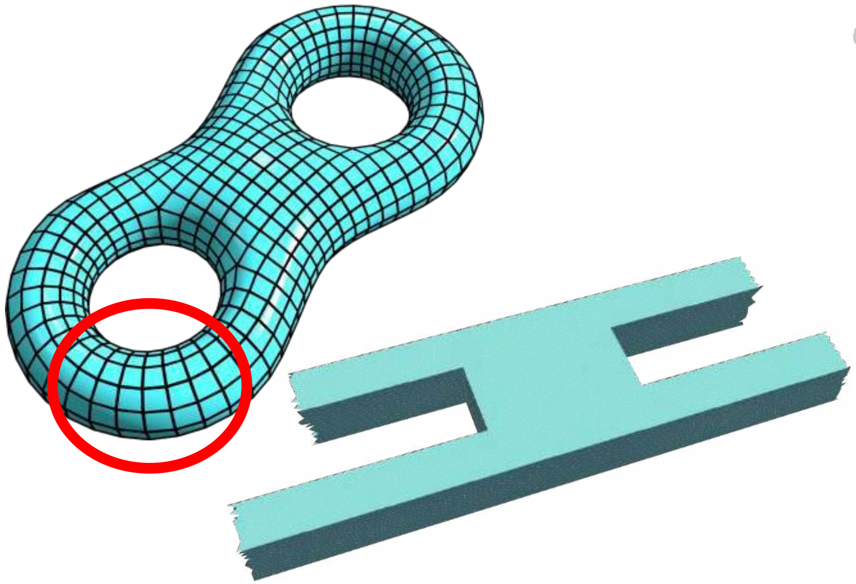
$$\boxed{\omega} = \boxed{d\alpha} + \boxed{h}$$

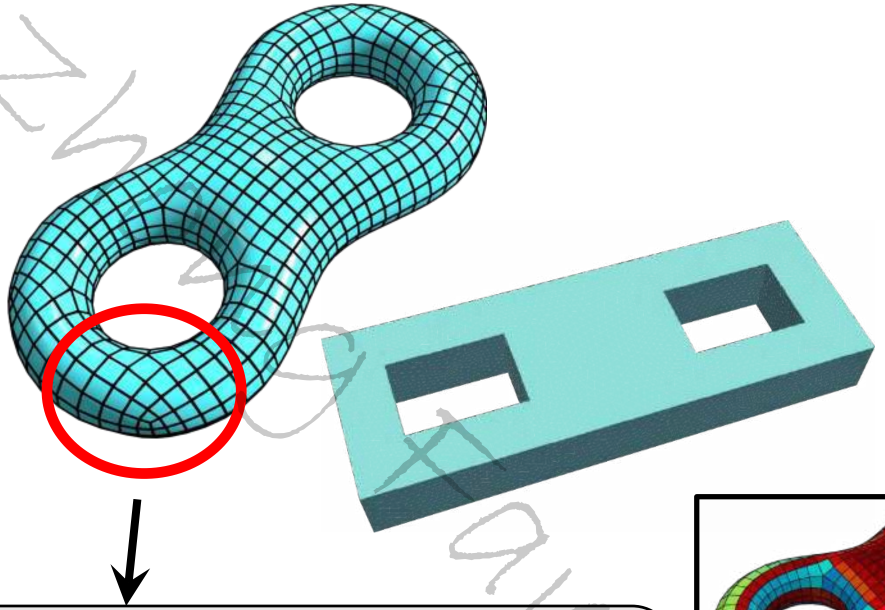
Closed-form **Exact-form** **Harmonic form**

Related to global topology

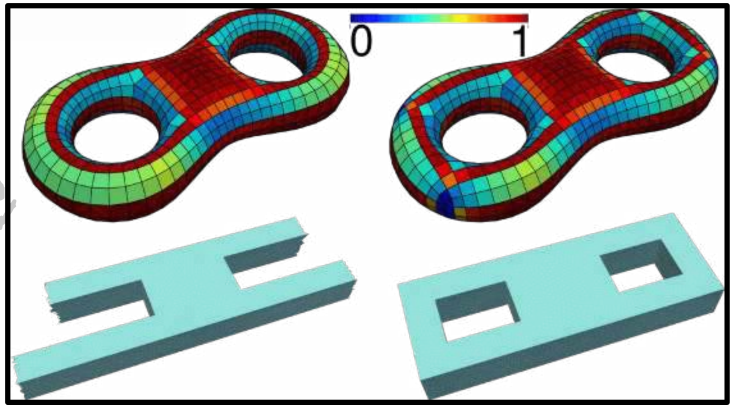
Global Topology Introduces DoF

- Genus, non-contractible loops, first homology group



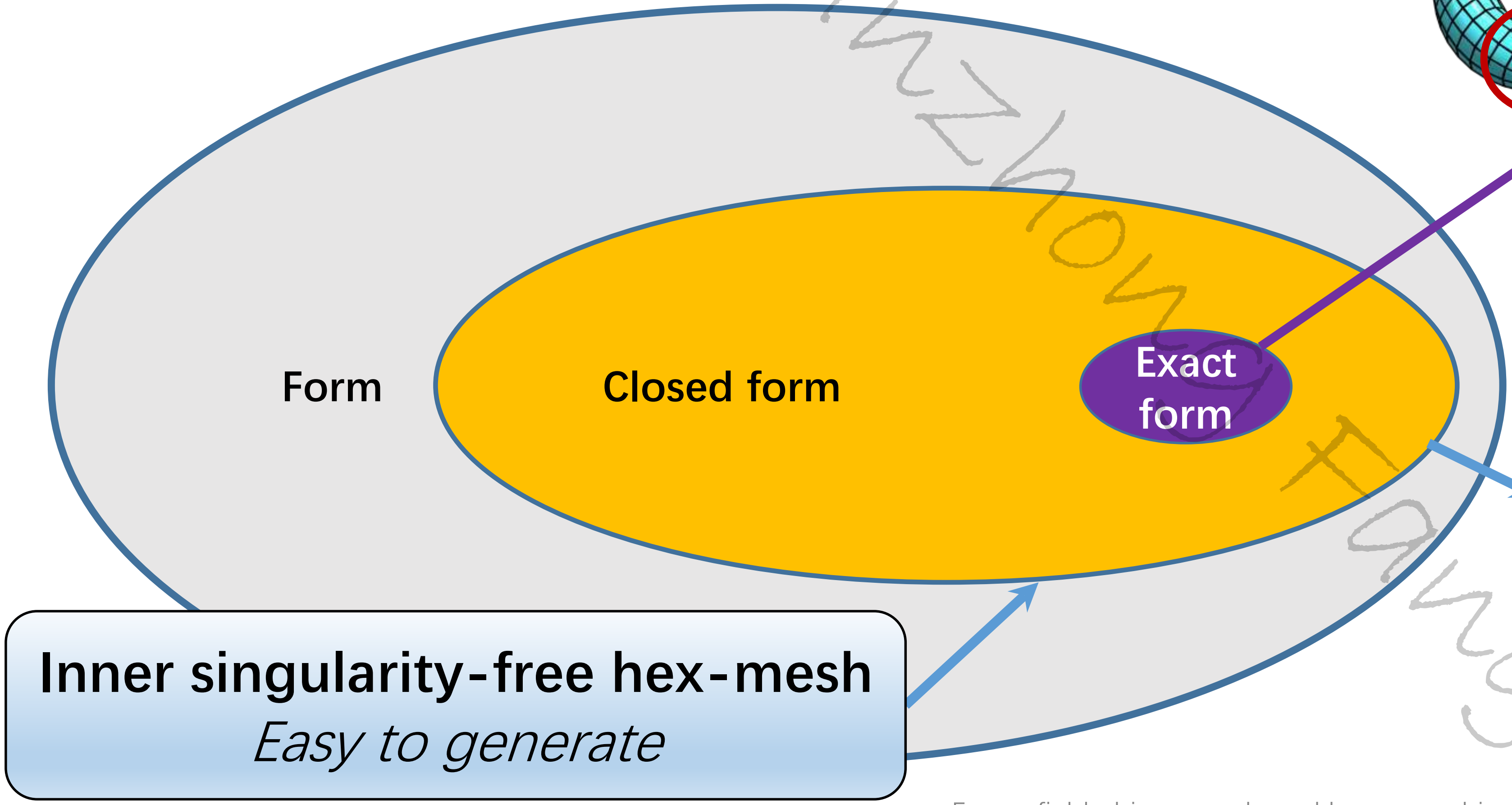


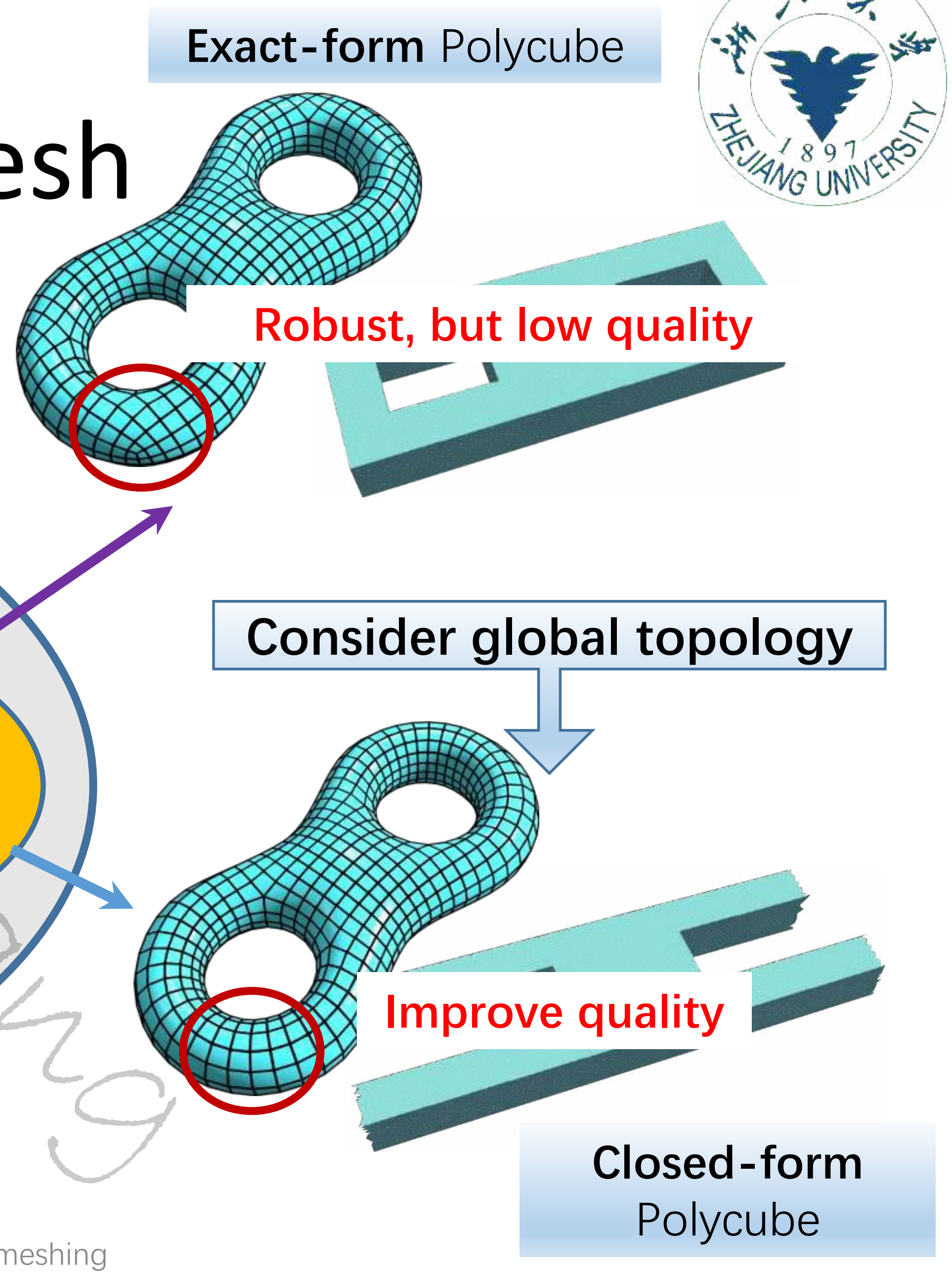
Do not consider global topology.
The quality of hex elements is low.



Frame field-driven quad- and hex-remeshing

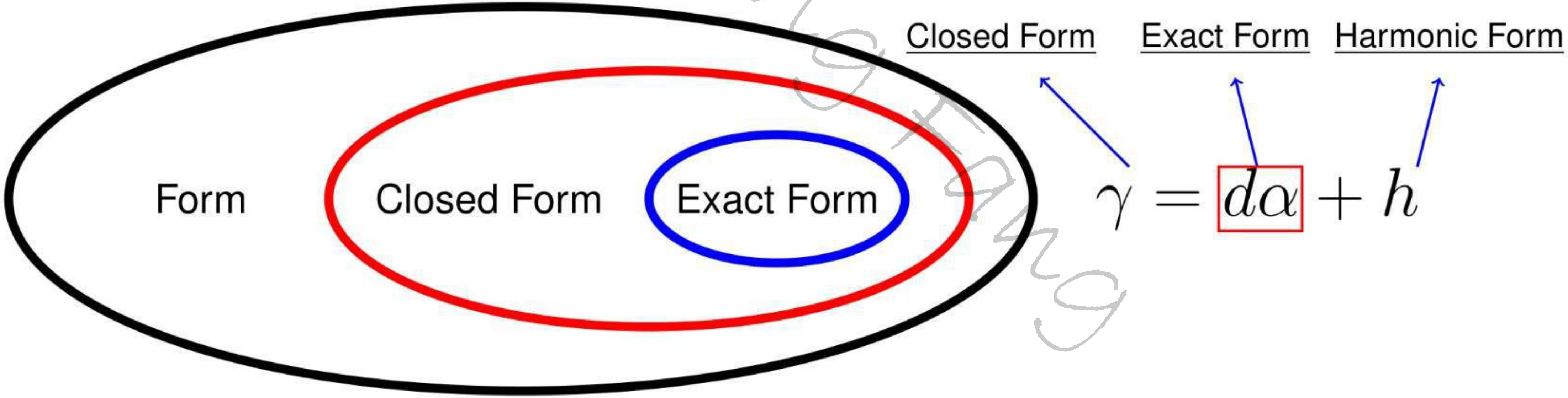
Inner Singularity-Free Hex-mesh



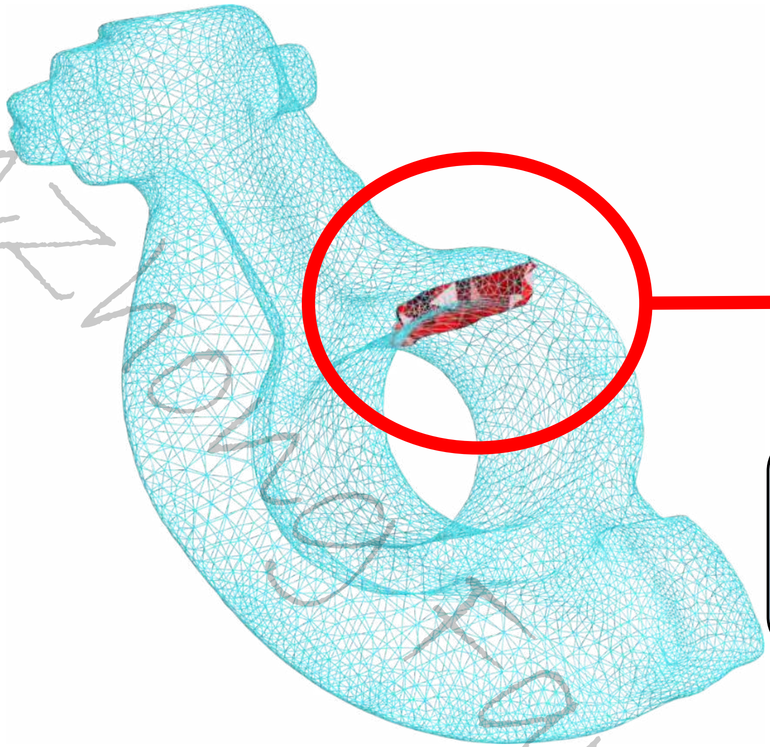


Inner Singularity-Free Structure

- Hex-meshes ⇒ Differential form (**Non-robust**, **High DOFs & High quality**)
- Polycube-based ⇒ Exact-form (**Robust**, but **Low DOFs & Low quality**)
- Keep robustness with more DOFs:
  - Inner singularity-free ⇒ Closed-form



Topology Cutting: Introduce DoF



Cut

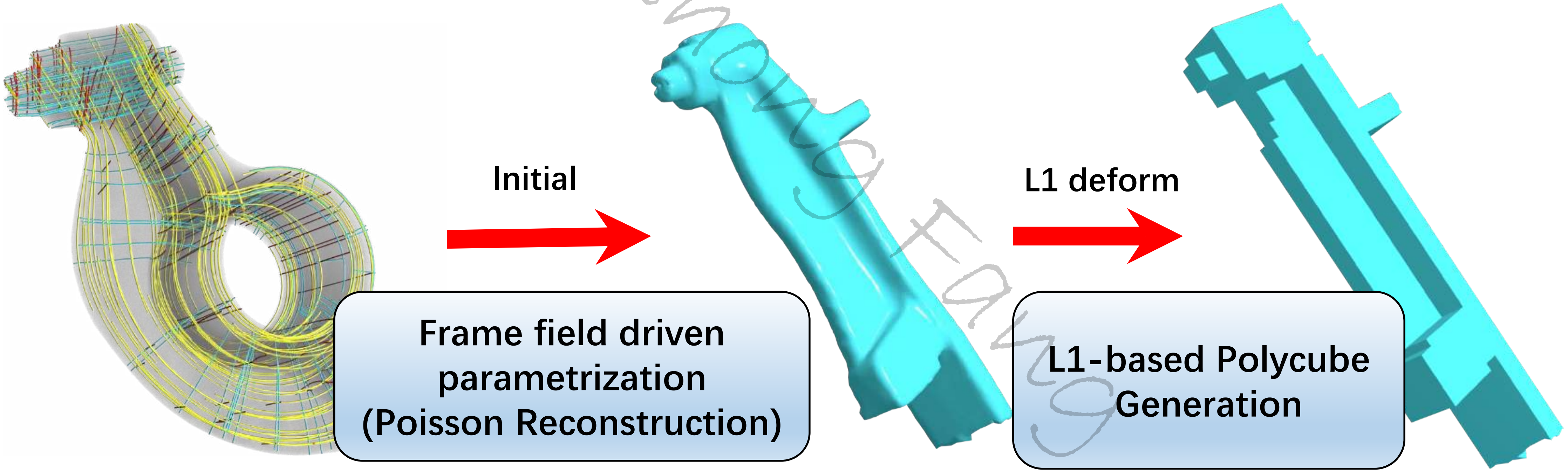
Introduce DoF
for parametrization

Input tetrahedral mesh
(Genus 1)

Cutted mesh
(Genus 0)

Closed-Form induced Polycube

- Inner singularity-free frame field
- L1 Polycube



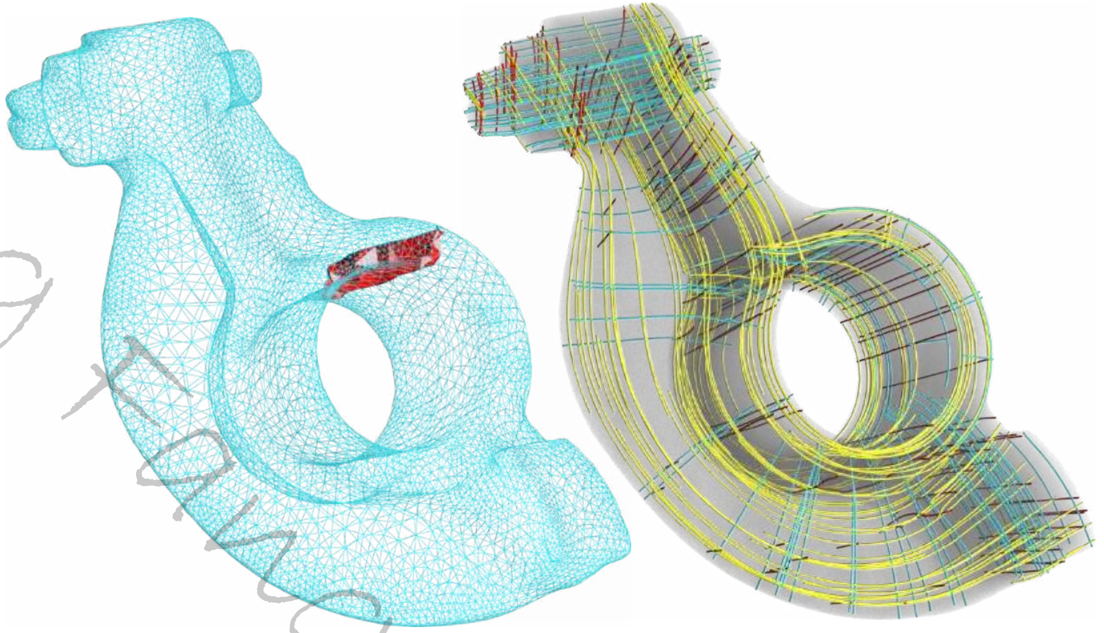
Inner Singularity-Free Frame Field

- Smoothness: As smooth as possible about **R**
- Boundary alignment: Align the boundary normals

Smoothness *Smoothness on cuts*

$$\min_{\mathbf{R}} \quad w_f \int_{\mathcal{F}\setminus\mathcal{C}} \|\nabla \mathbf{R}\|^2 + w_c \int_{\mathcal{C}} \widetilde{\nabla} \mathbf{R}$$
$$+ w_a \int_{\partial\mathcal{M}} \|\mathbf{R}n\|_1 + w_d \int_{\partial\mathcal{M}\setminus\partial\mathcal{C}} \|\nabla(\mathbf{R}n)\|^2$$
$$+ w_R \int_{\mathcal{M}} \|\mathbf{R}^T\mathbf{R} - I\|^2.$$

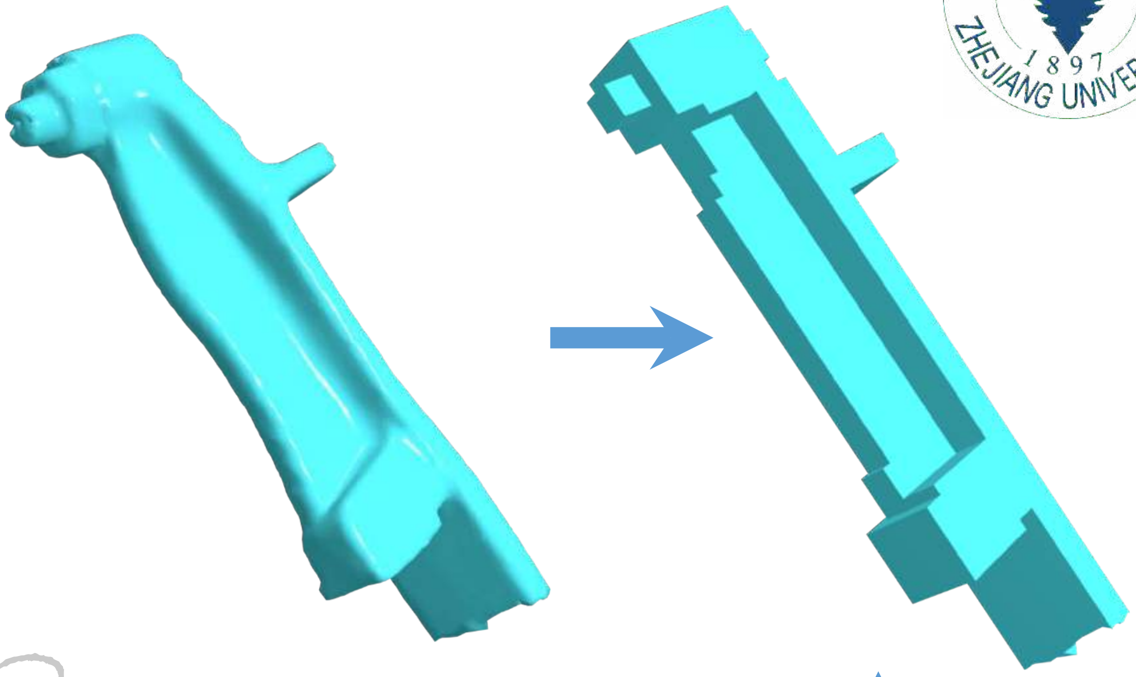
Alignment



Boundary-aligned Smooth Frame field as Initial → *After above optimization* → Inner singularity-free

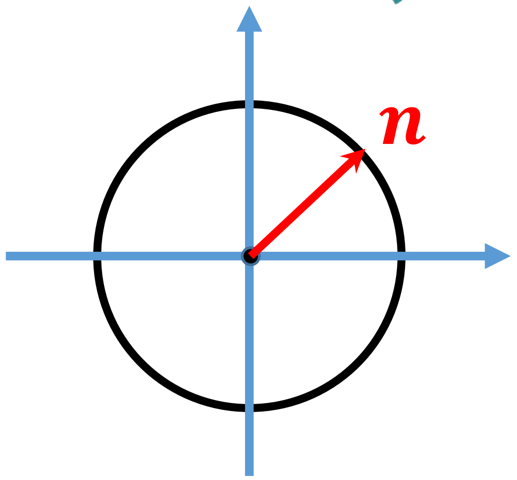
Polycube Generation

- Based on L1 deformation
- Use Poisson Reconstruction as Initial value
- ARAP + Alignment
- Transition functions on cuts
- Nonlinear optimizaton



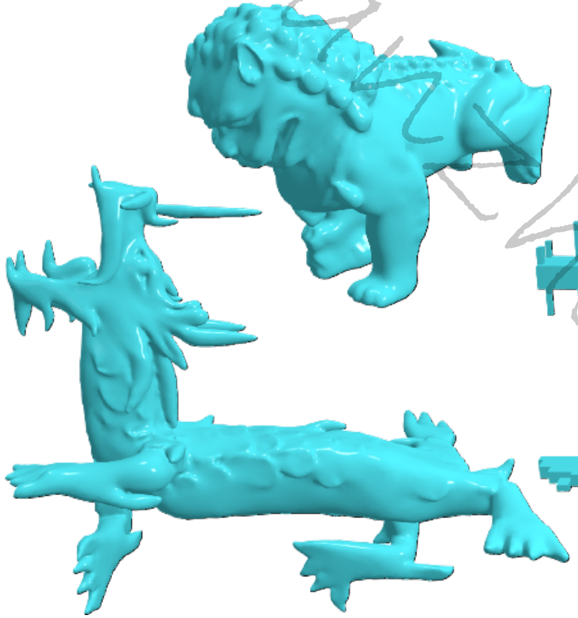
$$
\begin{aligned}
\min_{\overline{\mathcal{X}}} \quad & E_{arap} + w_{align} E_{align} + w_{diff} E_{diff} \\
\text{s.t.} \quad & \overline{A}(\overline{\mathcal{X}}(\mathcal{M})) = A_{\partial \mathcal{M}} \\
& \Pi_{a,b} \overline{\mathcal{X}}_a(e) = \overline{\mathcal{X}}_b(e), \quad \forall e \in t_a \cap t_b \in \mathcal{C}.
\end{aligned}
$$

$\|\boldsymbol{n}\|_1$

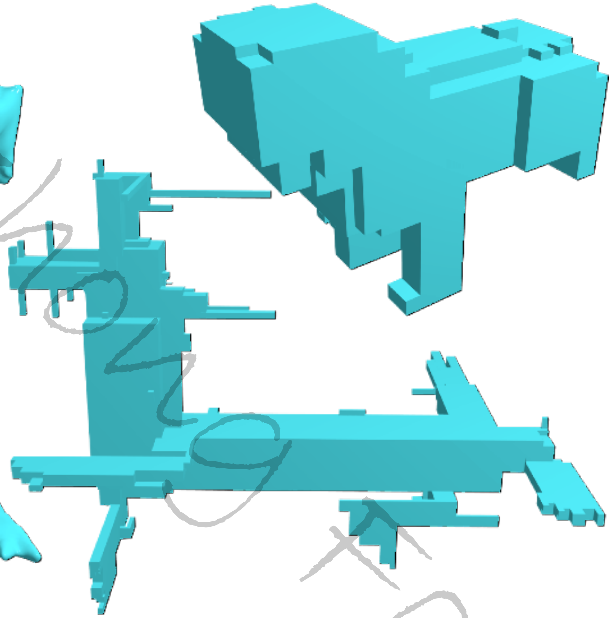


Better Alignment

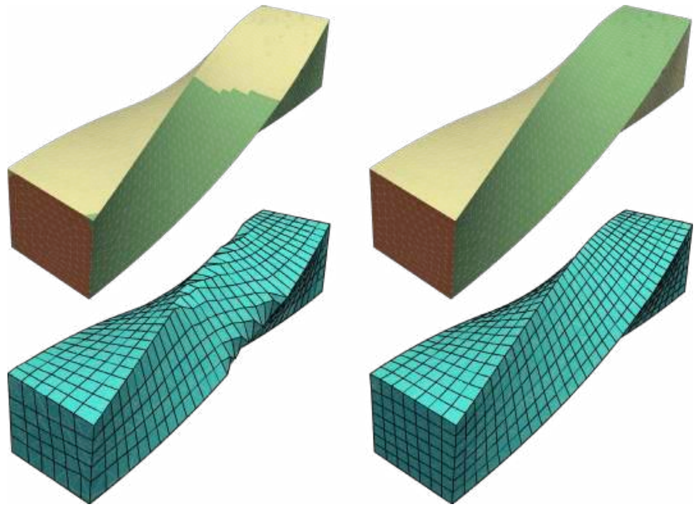
Input mesh



Poisson Reconstruction

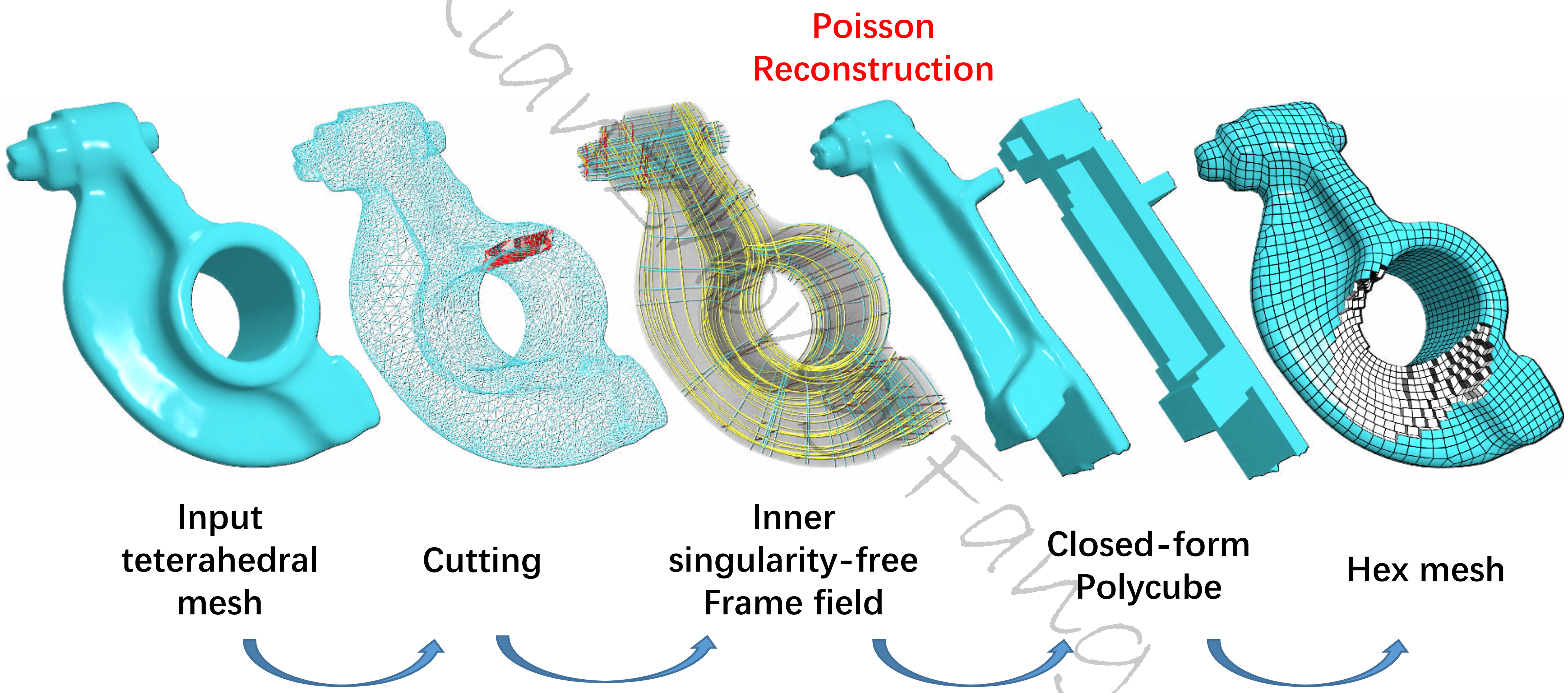


Closed-form polycube



Better Feature alignment

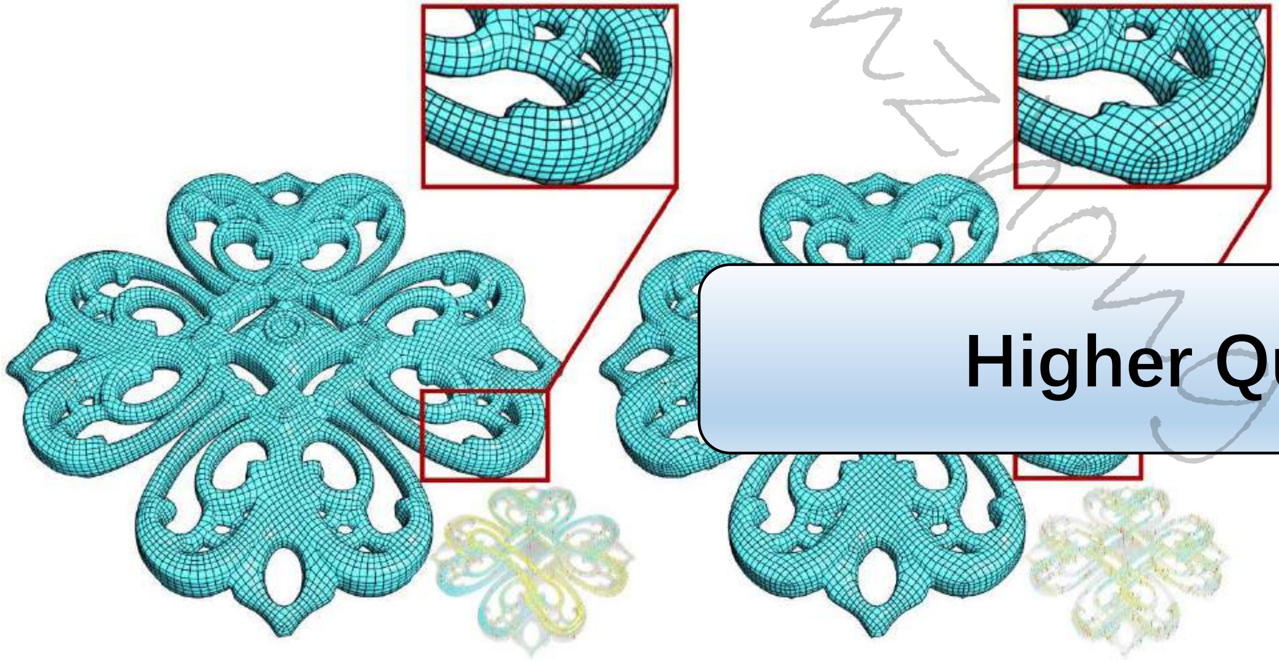
Pipeline



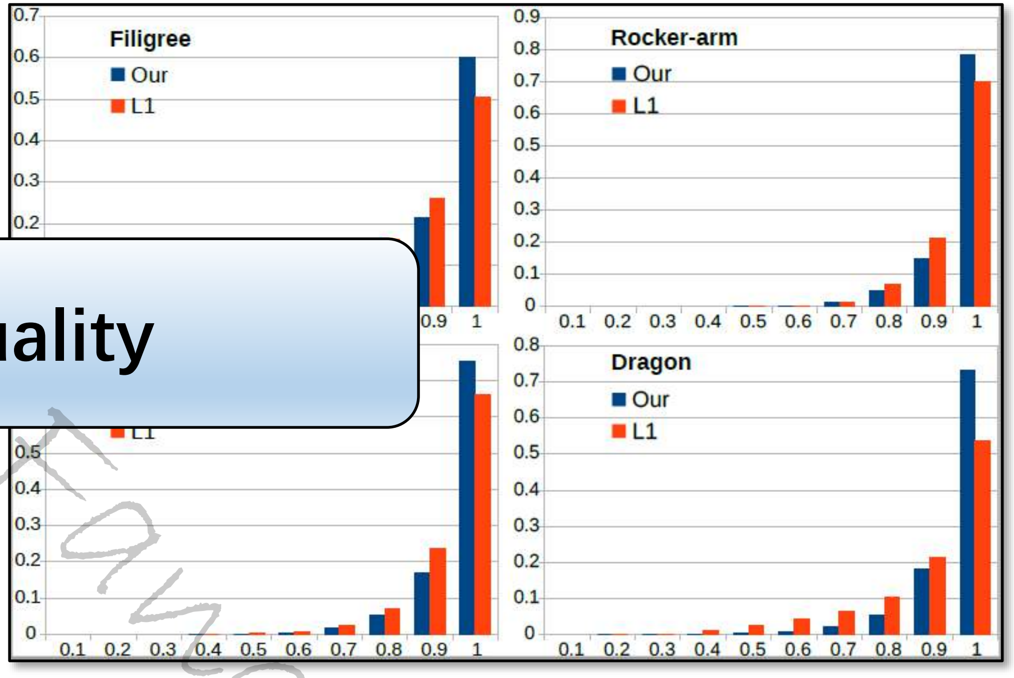
[Fang et al. TOG '16]

Results

- Closed-form Polycube *VS* Exact-form Polycube



Higher Quality

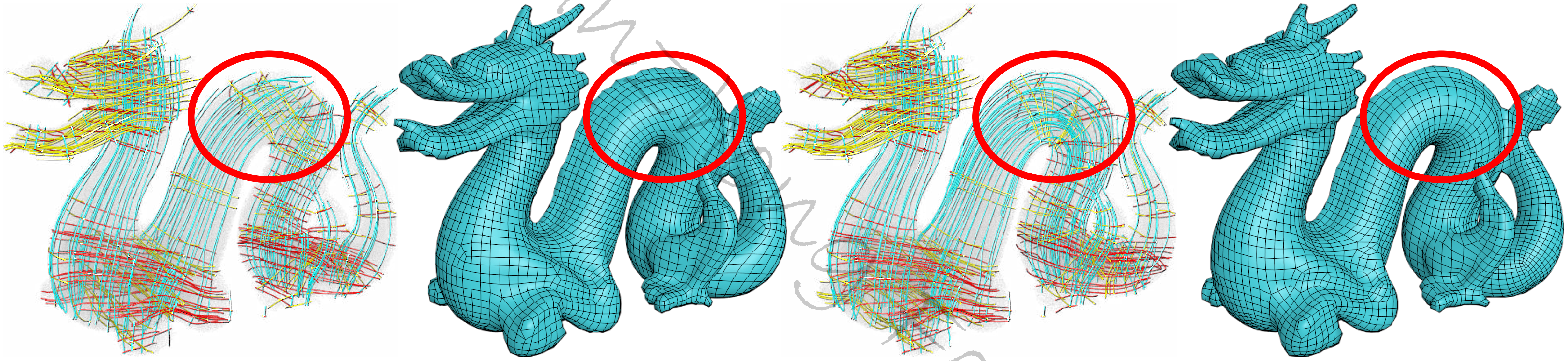


Scaled Jacobian Distribution of Results

Consider topology

Constrained by global topology

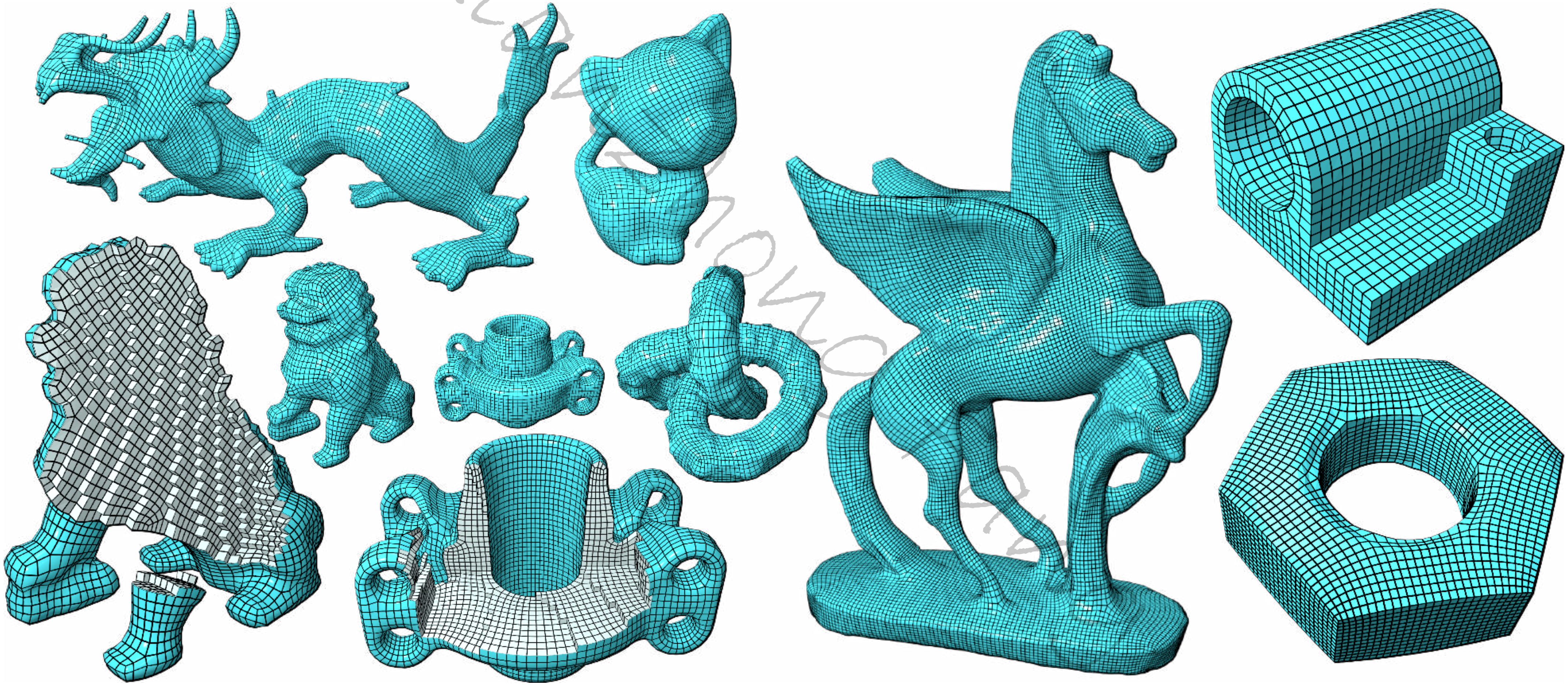
Frame Field Control



Frame field　　Hex mesh　　Frame field　　Hex mesh

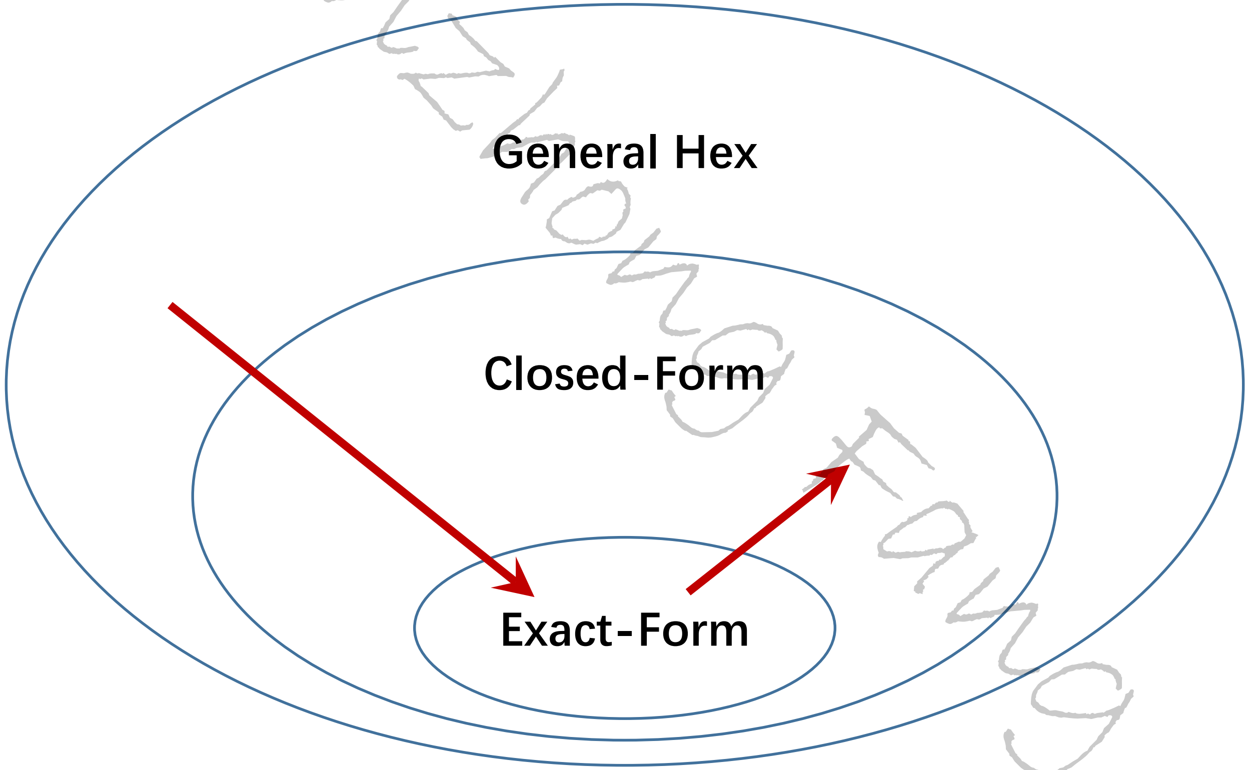
Frame field-driven quad- and hex-remeshing

Results



[Fang et al. TOG ‘16]

The Big Picture of Hex-Remeshing



Conclusion

- Internal singularity-free hexahedral mesh can be reliably generated
- Future works:
  - Consider internal singularities
  - Flexible control: arbitrary size/direction

Reference

- [Fang et al. TOG ‘16] All-hex meshing using closed-form induced polycube
- [Huang et al. TOG ‘14] ℓ1-based construction of polycube maps from complex shapes
- [Huang et al. TOG ‘11] Boundary aligned smooth 3d cross-frame field
- [Jiang et al. TVCG ‘14] Frame field singularity correction for automatic hexahedralization
- [Li et al. TOG ‘12] All-hex meshing using singularity-restricted field
- [Fang et al. TOG ‘18] Quadrangulation through Morse-parameterization hybridization
- [Zhang et al. TOG ‘10] A wave-based anisotropic quadrangulation method
- [Bommes et al. TOG ‘12] Mixed-integer quadrangulation
- [Bommes et al. TOG ‘13] Integer-grid maps for reliable quad meshing
- [Myles and Zorin TOG ‘13] Controlled-distortion constrained global parametrization
- [Bommes et al. CGF ‘13] Quad-mesh generation and processing: A survey
- [Livesu et al. TOG ‘13] Polycut: Monotone graph-cuts for polycube base-complex construction
- [Nieser et al. CGF ‘11] Cubecover-parameterization of 3d volumes
- [Palacios et al. TOG ‘07] Rotational symmetry field design on surfaces
- [Kälbere et al. CGF ‘07] Quadcover: Surface parameterization using branched coverings
- [Dong et al. TOG ‘06] Spectral surface quadrangulation
- [Tarini et al. TOG ’04] Polycube-maps

Thanks!

Q & A