Differentiable Rendering Theory and Applications

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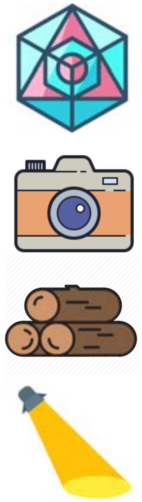
Outline

- Introduction
  - Definition
  - Motivations
- Related work
- Our work
  - *A Differential Theory of Radiative Transfer* (SIGGRAPH ASIA 2019)
- Future work

What is diff. rendering?



Geometry

Camera

Material

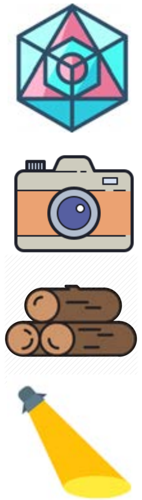
Light

$$F(\boldsymbol{\pi})$$

Scene Parameter $\boldsymbol{\pi}$

Rendering Image $\boldsymbol{I}$

What is diff. rendering?



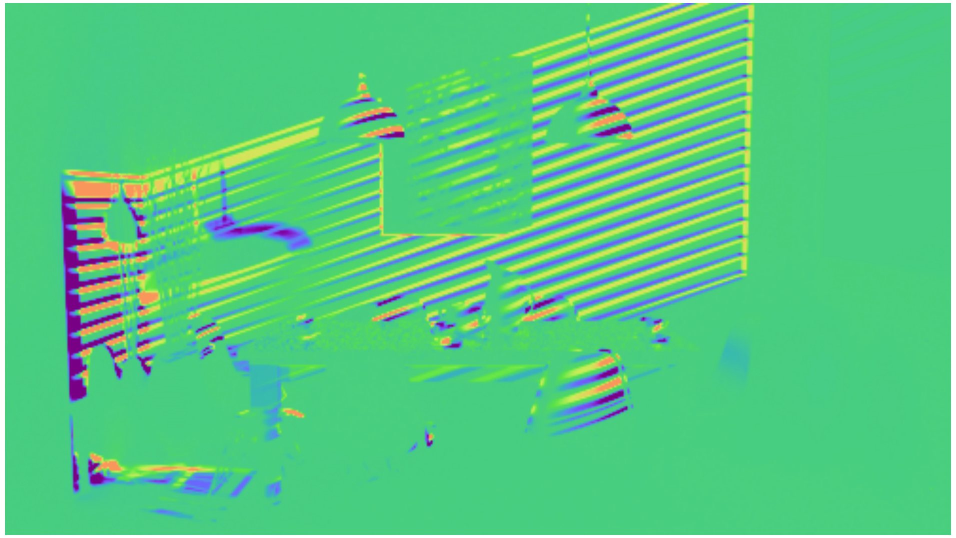
Geometry

Camera

Material

Light

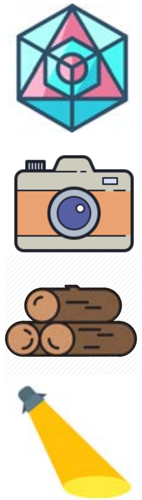
$$\partial_\pi F(\boldsymbol{\pi})$$



Scene Parameter $\boldsymbol{\pi}$

Derivative Image $\boldsymbol{I}'$

Why is diff. rendering important?



Geometry

Camera

Material

Light

Inverse

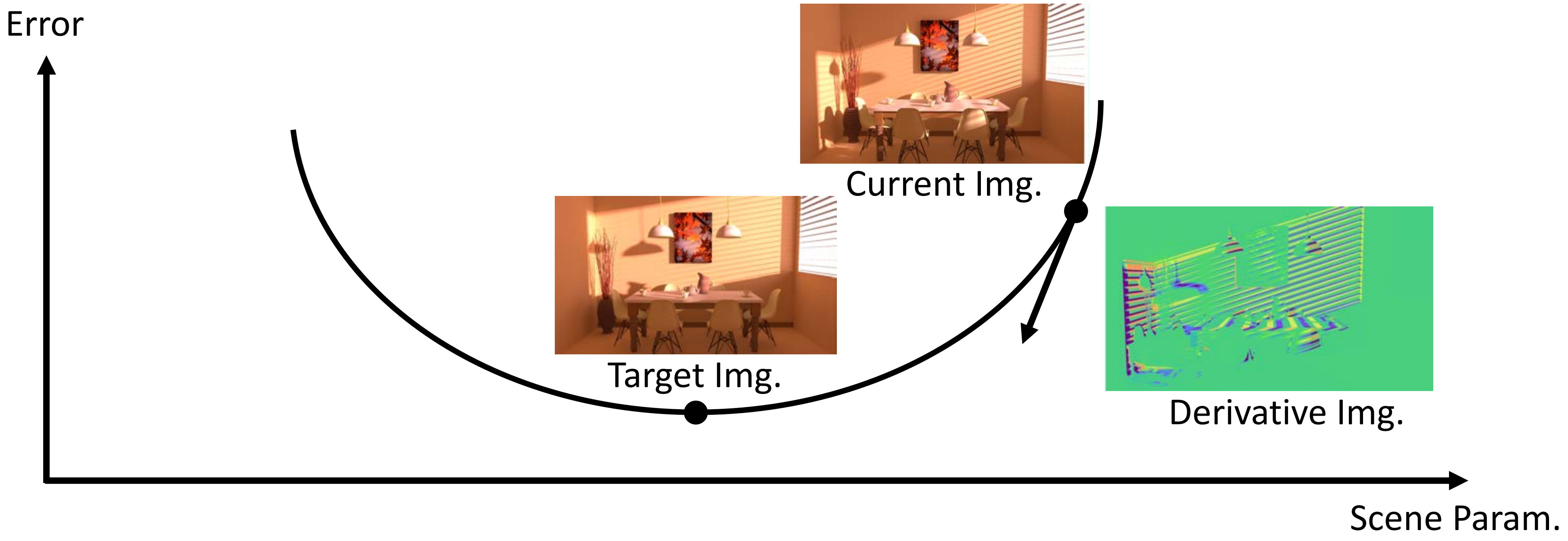
Rendering

Scene Parameter $\boldsymbol{\pi}$

Rendering Image $\boldsymbol{I}$

Why is diff. rendering important?

- Inverse rendering
  - Enable gradient-based optimization
  - Backpropagation through rendering (machine learning)



Why is diff. rendering important?

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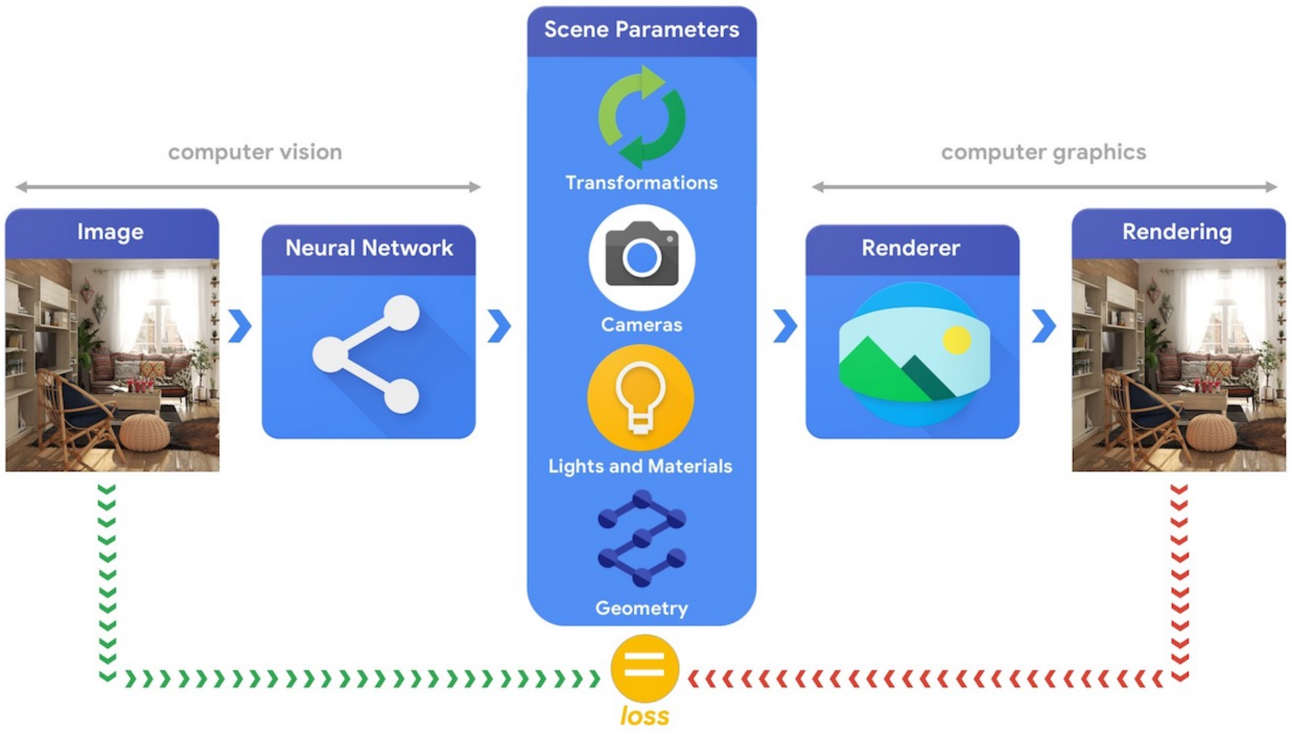
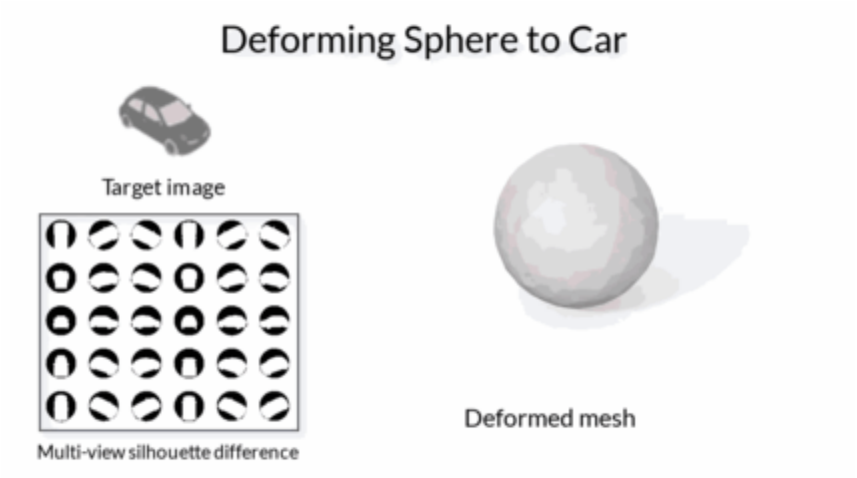
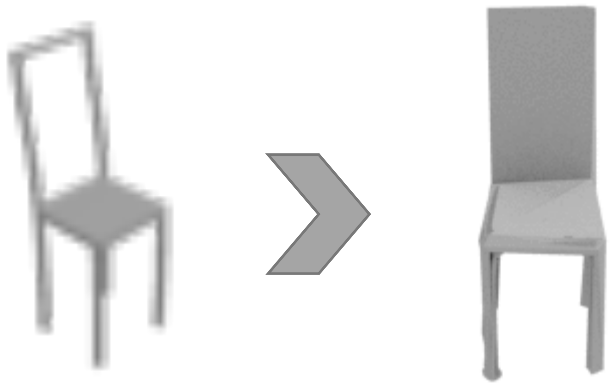


Related work

- Rasterization rendering
  - *Soft Rasterizer: A Differentiable Renderer for Image-based 3D Reasoning (ICCV 2019)*
  - *Neural 3D Mesh Renderer (CVPR 2018)*
  - *TensorFlow, pytorch3D etc.*



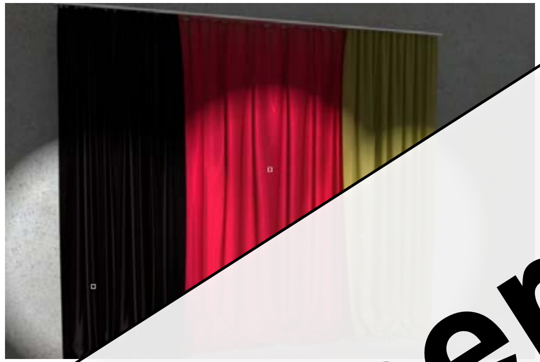
Soft Rasterizer



Neural 3D Mesh Renderer

Related work

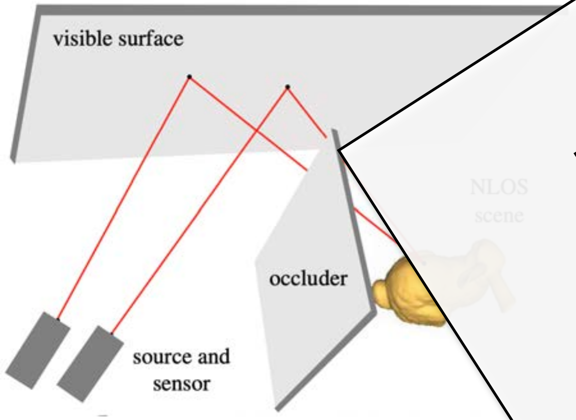
Volume Scattering
Gkioulekas et al. 2013, 2016



Cloth Rendering
Khungurn et al. 2015

Human Teeth
Velinov et al. 2018

Not General



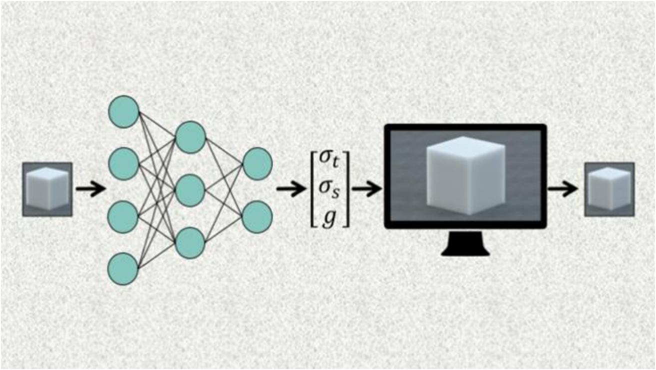
NLOS 3D Reconstruction
Tsai et al. 2019

Fabrication
Sumin et al. 2019

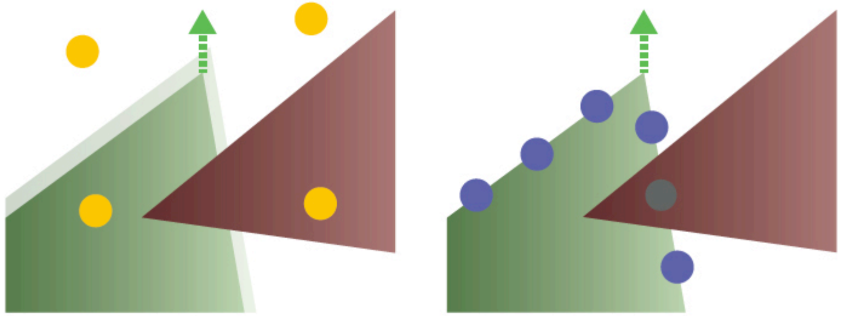
Reflectance & Lighting Estimation
Azinovic et al. 2019

Related work

- ***Inverse transport networks,*** Che et at. [2018]
  - Volumetric scattering ✓
  - Geometry X



- ***Differentiable Monte Carlo ray tracing through edge sampling,*** Li et at. [2018]
  - Volumetric scattering X
  - Geometry ✓



(a) area sampling (b) edge sampling

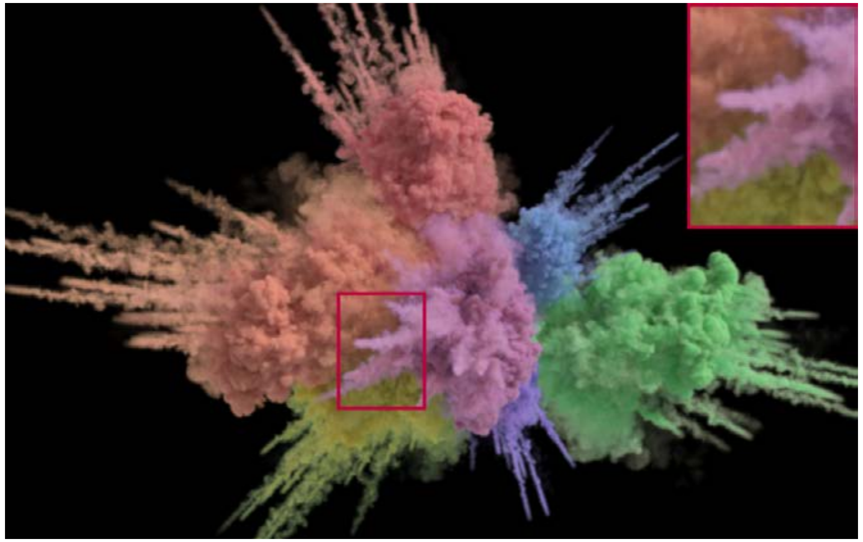
Our work

- ***A Differential Theory of Radiative Transfer*** (SIGGRAPH ASIA 2019)
  - Differential theory of radiative transfer
    - Captures **all** surface and volumetric light transport effects
    - Supports derivative computation with respect to **any** parameters
  - Monte Carlo estimator
    - Unbiased estimation
    - Analogous to volumetric path tracing

Radiative Transfer

- Radiative Transfer

  a mathematical model describing how light interacts within participating media (e.g. smoke) and translucent materials (e.g. marble and skin)



Kutz et al. 2017

Gkioulekas et al. 2013

Radiative Transfer

- Radiative Transfer

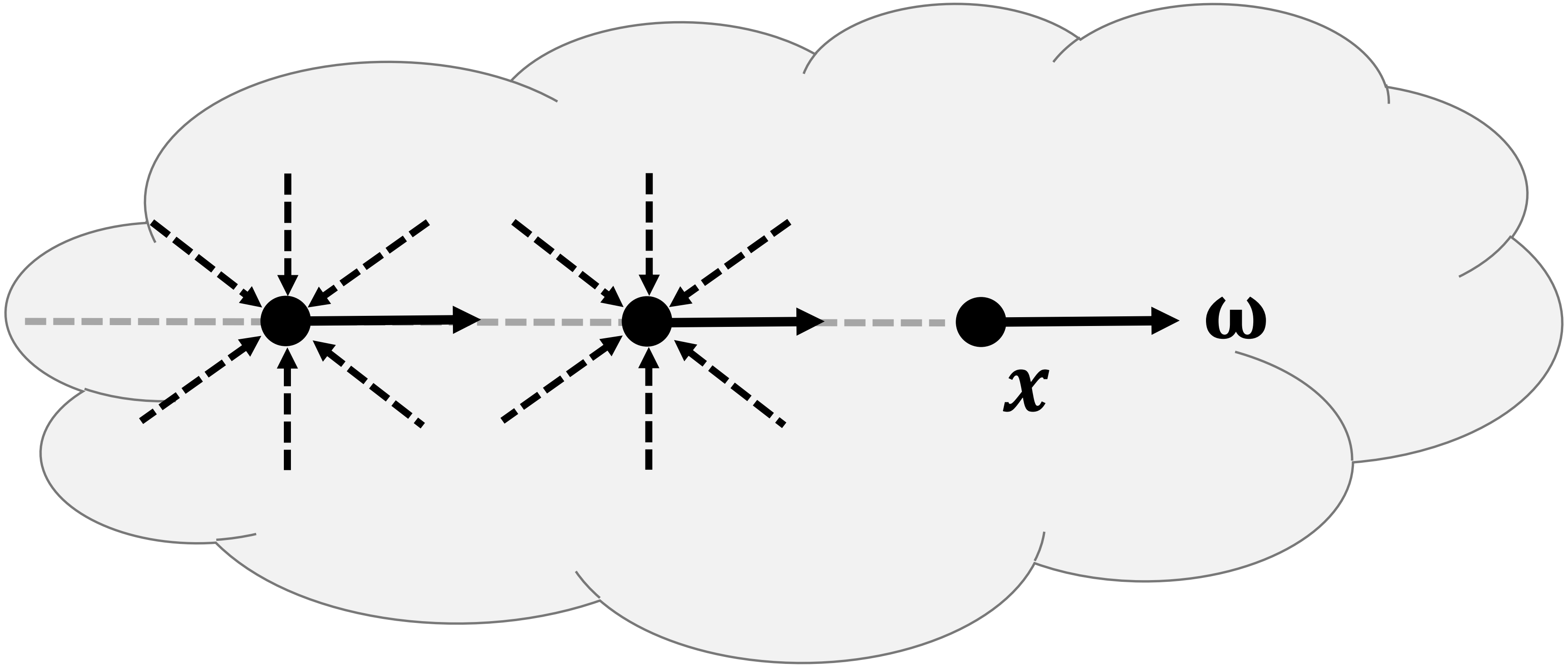
  a mathematical model describing how light interacts within participating media (e.g. smoke) and translucent materials (e.g. marble and skin)

  - Now used in many areas
    - Astrophysics (light transport in space)
    - Biomedicine (light transport in human tissue)
    - Nuclear science & engineering (neutron transport)
    - Remote sensing
    - …

Radiative Transfer

- Radiative Transfer

  a mathematical model describing how light interacts within participating media (e.g. smoke) and translucent materials (e.g. marble and skin)



Radiative Transfer Equation (RTE)

Transport operator

Collision operator

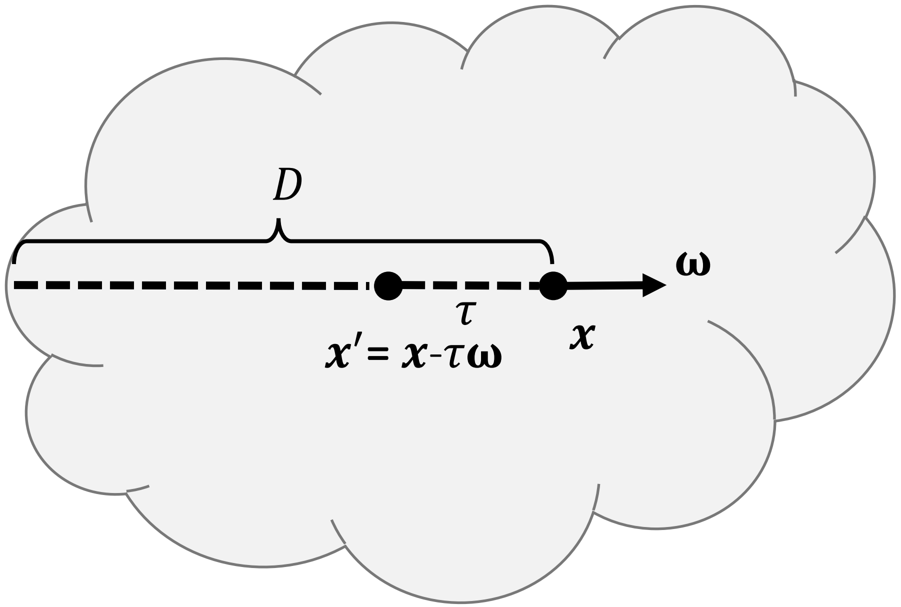
Source

$$L = K_T K_C L + Q$$

Radiative Transfer Equation

(Operator Form)

Transport Operator K_T



Transmittance

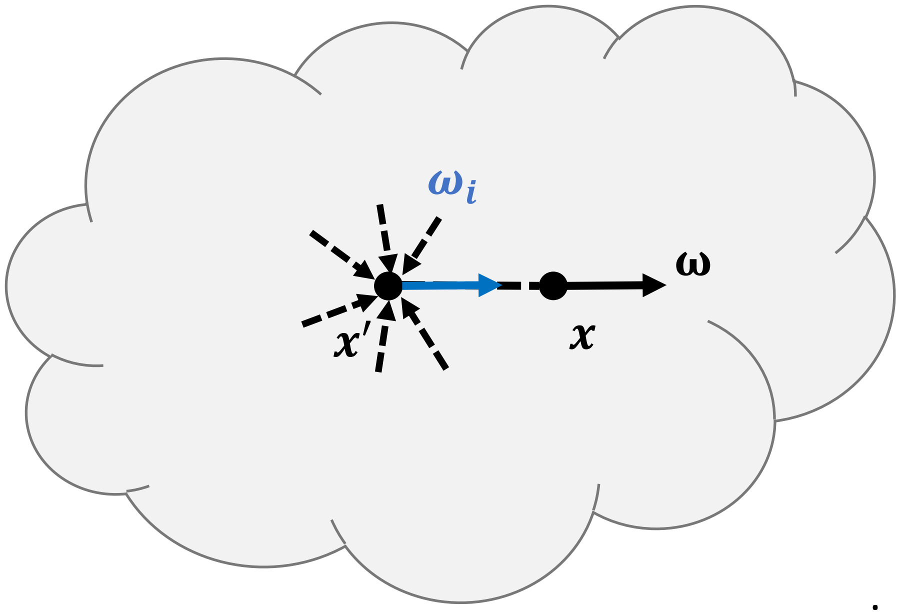
$$T(x', x) = \exp\left(-\int_0^{\tau} \sigma_t(\boldsymbol{x} - \tau'\boldsymbol{\omega})d\,\tau'\right)$$

Extinction coefficient $\sigma_t(\boldsymbol{x})$

controls how frequently light scatters and is also known as optical density

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \int_0^D T(\boldsymbol{x}', \boldsymbol{x})\,(K_c L)(\boldsymbol{x}', \boldsymbol{\omega})d\tau + Q$$

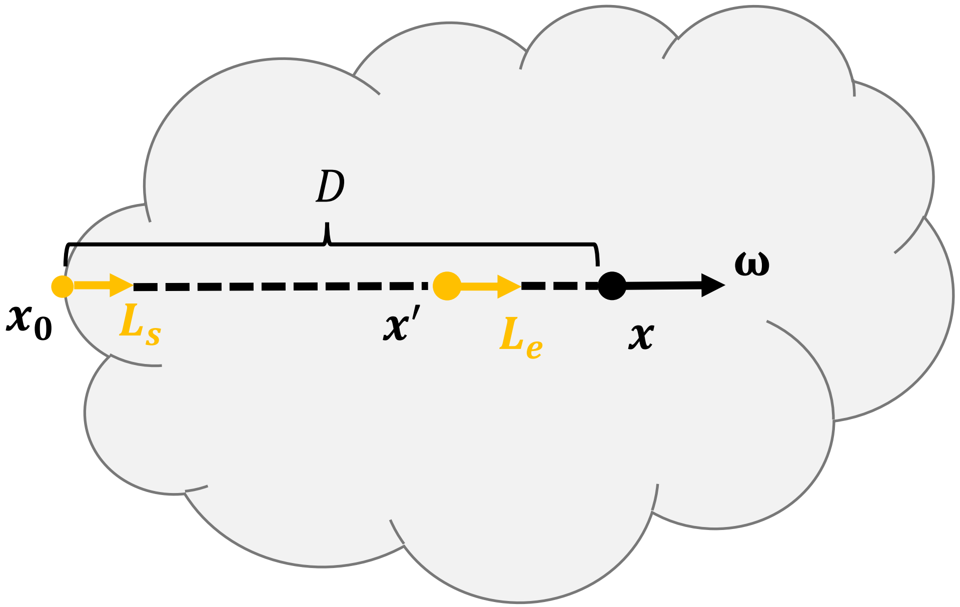
Collision Operator K_c



- phase function $f_p(\boldsymbol{x}, \boldsymbol{\omega_i}, \boldsymbol{\omega})$
  A probability density over $\mathbb{S}^2$ given $\boldsymbol{x}$ and $\boldsymbol{\omega_i}$
- scattering coefficient $\sigma_s(\boldsymbol{x})$

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = K_T \sigma_s(\boldsymbol{x}) \underbrace{\int_{\mathbb{S}^2} f_p(\boldsymbol{x}', \boldsymbol{\omega_i}, \boldsymbol{\omega}) L(\boldsymbol{x}', \boldsymbol{\omega_i}) d\boldsymbol{\omega_i}}_{\substack{=: L^{\text{ins}}(x, \omega) \\ \text{in-scattered radiance}}} + Q$$

Source Q



- Absorption coefficient $\sigma_a(\boldsymbol{x})$
- Interfacial radiance L_s

  Boundary condition

Attenuation

$$L(\boldsymbol{x},\boldsymbol{\omega}) = K_T K_c L + \int_0^D T(\boldsymbol{x}',\boldsymbol{x})\,\sigma_a(\boldsymbol{x}) L_e(\boldsymbol{x}',\boldsymbol{\omega})\,d\tau + T(\boldsymbol{x}_0,\boldsymbol{x})\,L_s(\boldsymbol{x}_0,\boldsymbol{\omega})$$

radiant emission *Interfacial radiance*

$\boldsymbol{K_T}$

Transport Operator

$\boldsymbol{K_c}$

Collision Operator

$$L(\boldsymbol{x},\boldsymbol{\omega}) = \underbrace{\int_0^D T(\boldsymbol{x}',\boldsymbol{x})\,\sigma_s(\boldsymbol{x}) \underbrace{\int_{\mathbb{S}^2} f_p(\boldsymbol{x}',\boldsymbol{\omega_i},\boldsymbol{\omega})L(\boldsymbol{x}',\boldsymbol{\omega_i})\boldsymbol{d\omega_i}}_{K_c}\,d\tau}_{K_T}$$

$$+ \underbrace{\int_0^D T(\boldsymbol{x}',\boldsymbol{x})\,\sigma_a(\boldsymbol{x}')L_e(\boldsymbol{x}',\boldsymbol{\omega})d\tau + T(\boldsymbol{x_0},\boldsymbol{x})L_s(\boldsymbol{x_0},\boldsymbol{\omega})}_{Q}$$

$\boldsymbol{Q}$

Source

Radiative Transfer Equation

(Integral Form)

Differentiating the RTE

$$L = K_T K_c L + Q$$

Differentiating both sides

$$\partial_\pi L = \partial_\pi (K_T K_c L) + \partial_\pi Q$$

Differentiating individual operators

Differentiating the Collision Operator

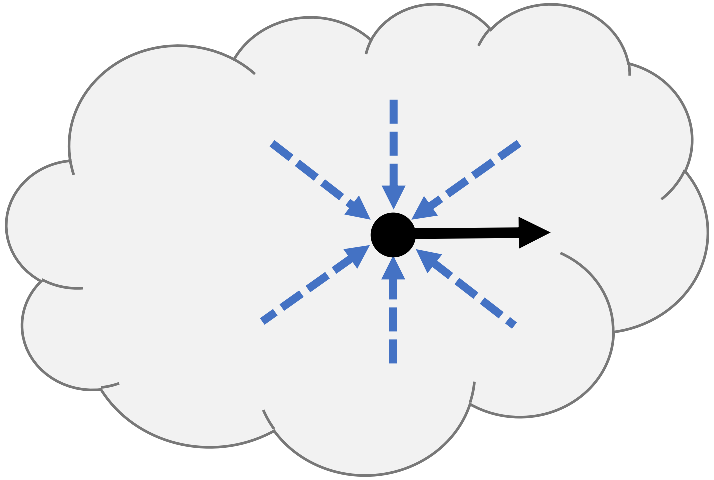
RTE: $L = K_T K_c L + Q$

(x omitted for notational simplicity)

$$(K_c L)(\boldsymbol{\omega}) = \sigma_s \int_{\mathbb{S}^2} \overbrace{f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega})\, L(\boldsymbol{\omega_i})}^{f(\boldsymbol{\omega_i})} \mathrm{d}\boldsymbol{\omega_i}$$

Scattering coefficient

Phase function



$$\partial_\pi \int_{\mathbb{S}^2} f(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i} = \ ?$$

Requires differentiating a (spherical) integral

Differentiating the Collision Operator

$$(K_c L)(\boldsymbol{\omega}) = \sigma_s \int_{\mathbb{S}^2} \overbrace{f_p(\boldsymbol{\omega}_i, \boldsymbol{\omega})\, L(\boldsymbol{\omega}_i)}^{f(\boldsymbol{\omega}_i)} \mathrm{d}\boldsymbol{\omega}_i$$

$$\partial_\pi \int_{\mathbb{S}^2} f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i \neq \int_{\mathbb{S}^2} \partial_\pi f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i$$

$$\partial_\pi \int_{\mathbb{S}^2} f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i = \int_{\mathbb{S}^2} \partial_\pi f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i + \int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega}_i}{\partial \pi} \right\rangle \Delta f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i$$

when f has discontinuities that depend π

Boundary term

$$\partial_\pi \int_{\mathbb{S}^2} f(\boldsymbol{\omega}_i)\mathrm{d}\boldsymbol{\omega}_i$$

(according to Reynolds transport theorem)

Boundary term

$$(\color{blue}{K_c}\color{black} L)(\boldsymbol{\omega}) = \sigma_s \int_{\mathbb{S}^2} \overbrace{f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega})\, L(\boldsymbol{\omega_i})}^{f(\boldsymbol{\omega_i})} \mathrm{d}\boldsymbol{\omega_i}$$

$$\Delta f(\boldsymbol{\omega_i}) = f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i})$$

when f_p is continuous

$$\int_{\boxed{\mathbb{S}}} \underbrace{\left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle}_{\text{change rate of discontinuity}} \boxed{\Delta f(\boldsymbol{\omega_i})} \mathrm{d}\boldsymbol{\omega_i}$$

discontinuities of integrand f

change rate of discontinuity

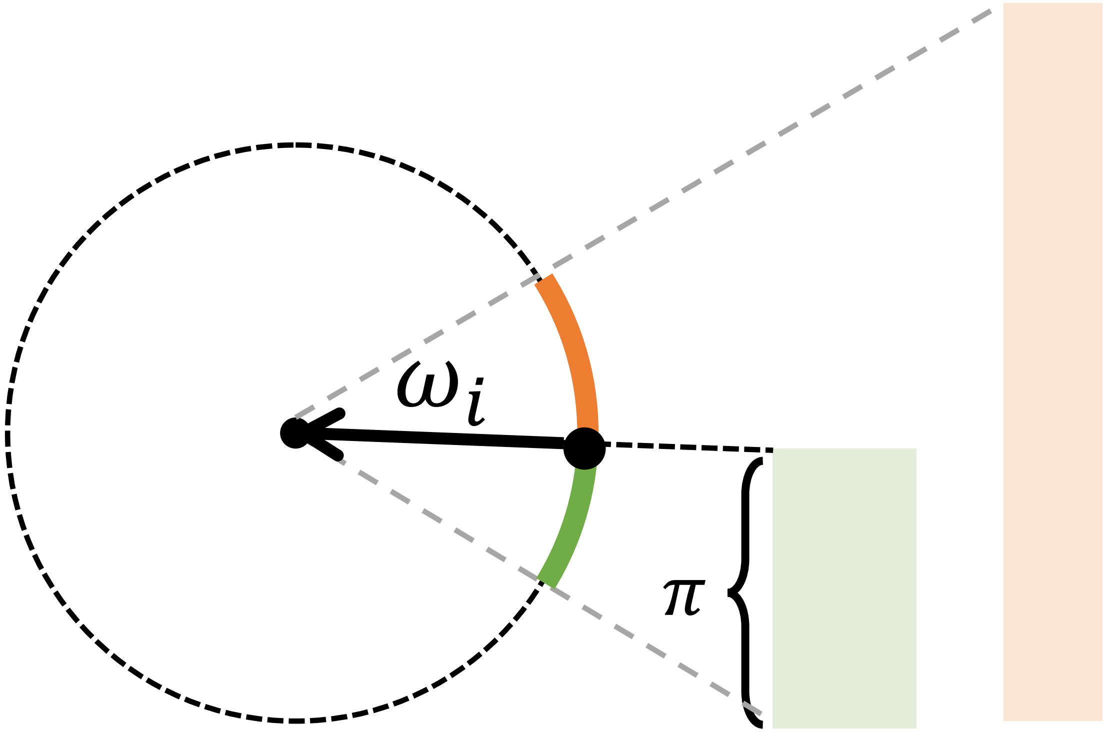
(in the normal direction)

Δf is the difference of integrand f across the discontinuity $\boldsymbol{\omega_i}$

Sources of Discontinuity

The boundary term:

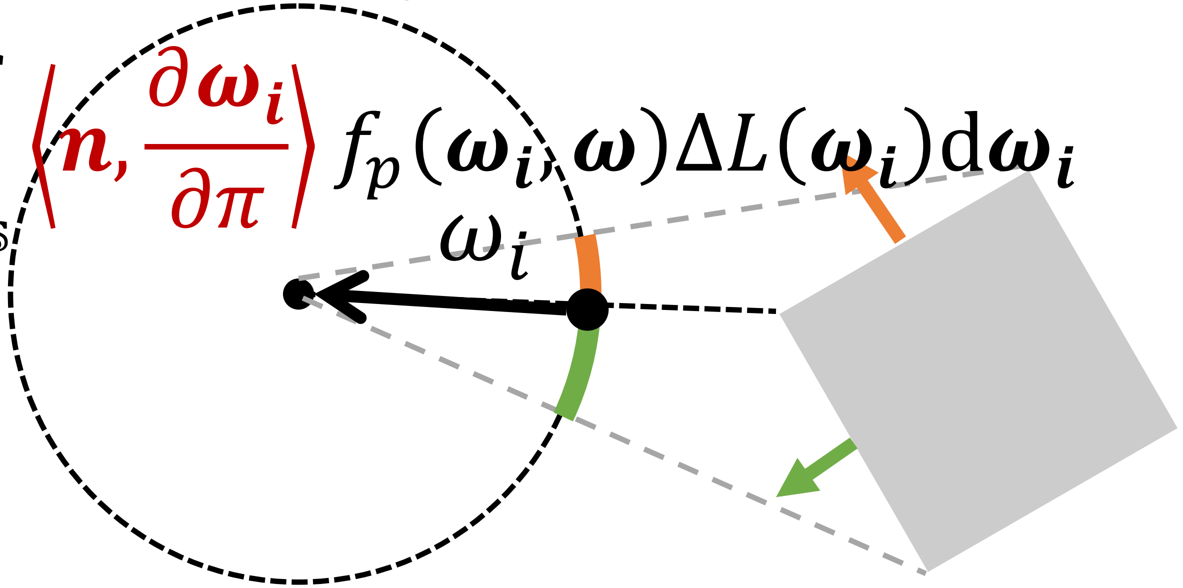
$$\int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$



Visibility

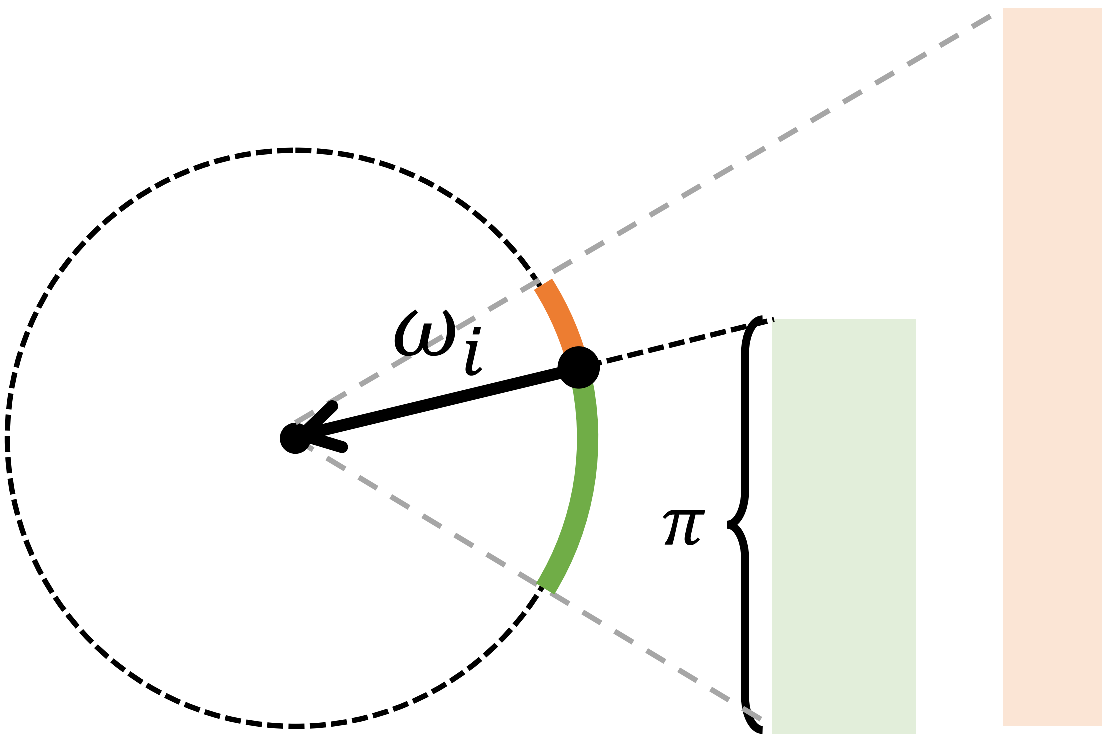
The boundary term:

$$\int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$



Normal

Sources of Discontinuity

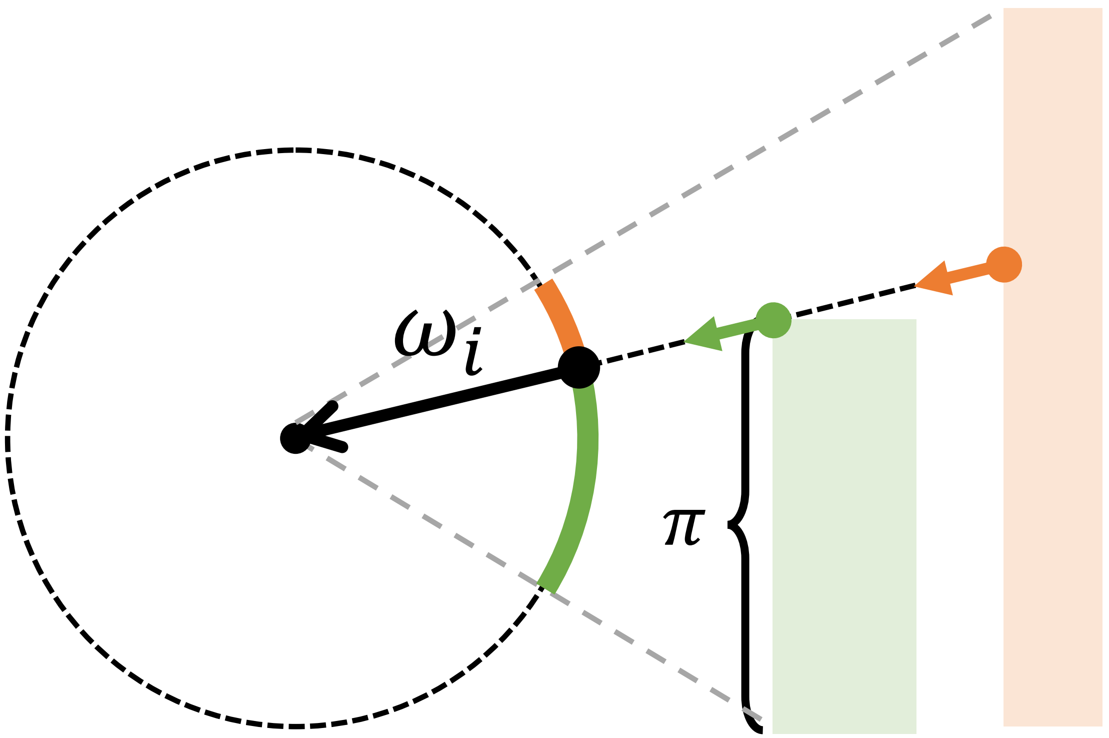


The boundary term:

$$\int_{\mathbb{S}} \underbrace{\left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle}_{} f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$

Reduces to the change rate of $\boldsymbol{\omega_i}$ (as an angle)

Sources of Discontinuity



The boundary term:

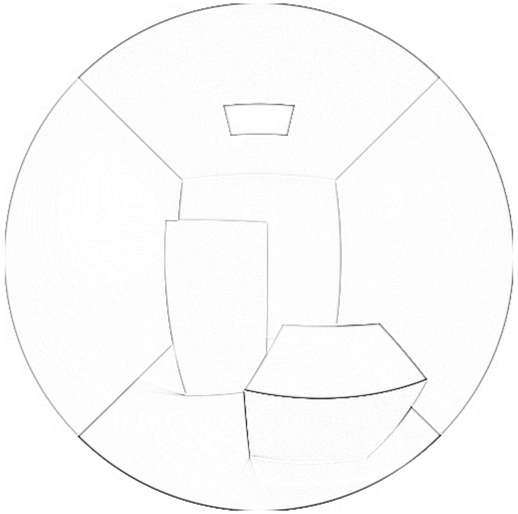
$$\int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$

$$\Delta L(\omega_i) = L(\quad) - L(\quad)$$

(with the absence of attenuation)

Discontinuities in 3D

visualization of L

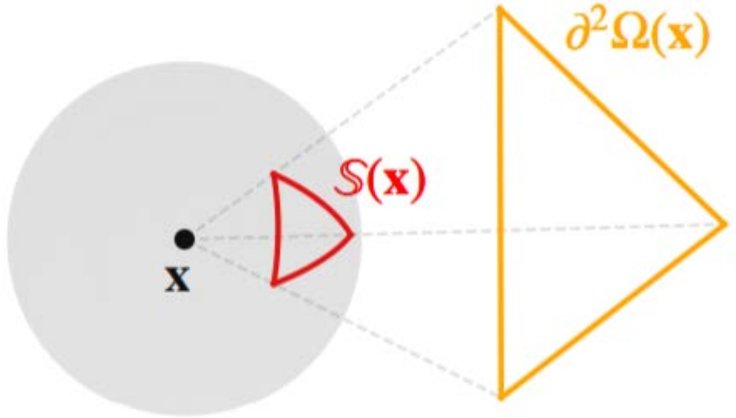


visualization of discontinuity curves $\mathbb{S}$

line integral

$$\boxed{\int_{\mathbb{S}}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$

Discontinuities in 3D

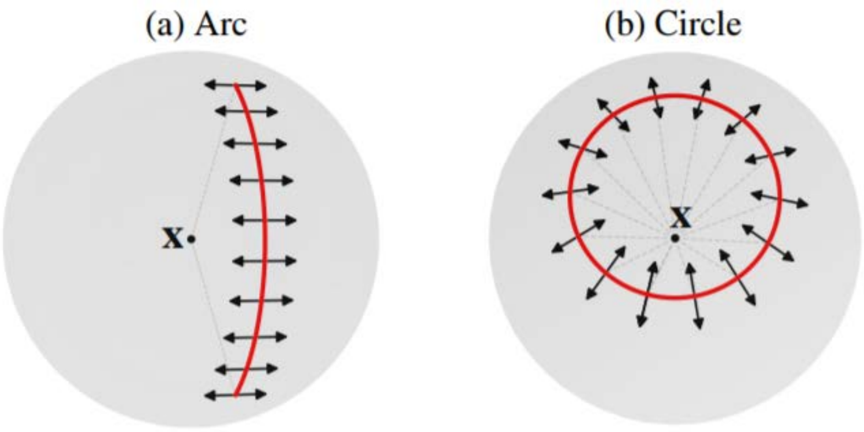


Discontinuities curves:

Projection of moving geometric edges onto the sphere

$$\int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$

Discontinuities in 3D



Edge normal

- in the tangent space of the sphere
- perpendicular to the discontinuity curve at direction $\boldsymbol{\omega_i}$

$$\int_{\mathbb{S}} \left\langle \boldsymbol{n}, \frac{\partial \boldsymbol{\omega_i}}{\partial \pi} \right\rangle f_p(\boldsymbol{\omega_i}, \boldsymbol{\omega}) \Delta L(\boldsymbol{\omega_i}) \mathrm{d}\boldsymbol{\omega_i}$$

Other Terms in the RTE

$$L = K_T K_c L + Q$$

Transport operator

$$(K_T K_c L)(x, \omega) = \int_0^D T(x', x)\,(K_c L)(x', \omega)\,d\tau$$

Transmittance

Source

$$Q = T(\boldsymbol{x_0}, \boldsymbol{x}) L_s(\boldsymbol{x_0}, \boldsymbol{\omega})$$

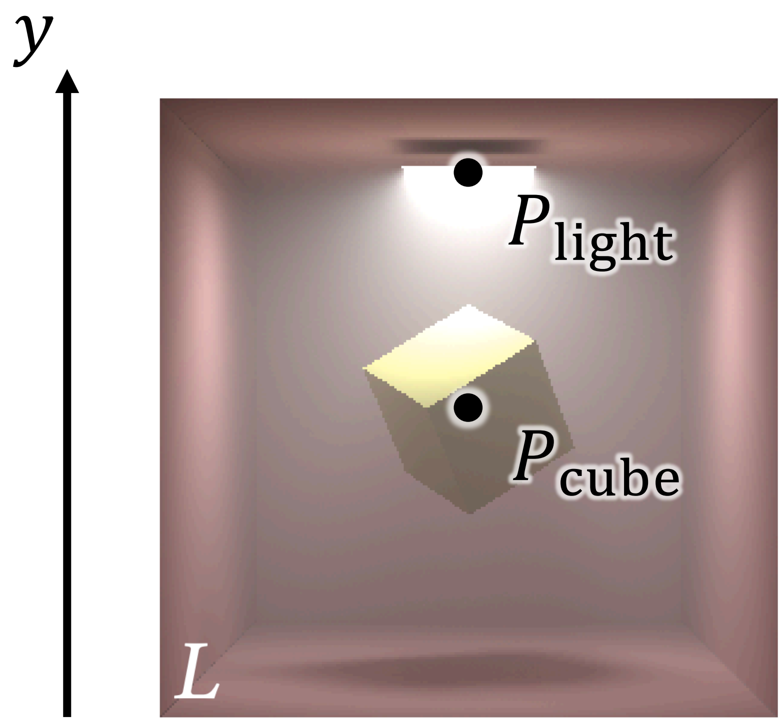
Full Radiance Derivative

$$\dot{L}(\boldsymbol{x}, \boldsymbol{\omega}) = \int_0^D T(\boldsymbol{x}', \boldsymbol{x}) \Big[\sigma_{\mathrm{s}}(\boldsymbol{x}')\, \dot{L}^{\mathrm{ins}}(\boldsymbol{x}', \boldsymbol{\omega}) + \big(\dot{\sigma}_{\mathrm{s}}(\boldsymbol{x}') - \Sigma_{\mathrm{t}}(\boldsymbol{x}, \boldsymbol{\omega}, \tau)\, \sigma_{\mathrm{s}}(\boldsymbol{x}')\big)\, L^{\mathrm{ins}}(\boldsymbol{x}', \boldsymbol{\omega}) \Big]\, \mathrm{d}\tau$$
$$+\, T(\boldsymbol{x}_0, \boldsymbol{x}) \Big[-\big(\Sigma_{\mathrm{t}}(\boldsymbol{x}, \boldsymbol{\omega}, D) + \dot{D}\, \sigma_{\mathrm{t}}(\boldsymbol{x}_0)\big)\, L_{\mathrm{s}}(\boldsymbol{x}_0, \boldsymbol{\omega}) + \dot{L}_{\mathrm{s}}(\boldsymbol{x}_0, \boldsymbol{\omega}) + \dot{D}\, \sigma_{\mathrm{s}}(\boldsymbol{x}_0)\, L^{\mathrm{ins}}(\boldsymbol{x}_0, \boldsymbol{\omega}) \Big],$$

where Σ_{t} is defined in Eq. (17), $\dot{L}^{\mathrm{ins}}$ follows Eq. (22), and $\dot{L}_{\mathrm{s}} = \dot{L}_{\mathrm{s}}^{\mathrm{r}} + \dot{L}_{\mathrm{s}}^{\mathrm{e}}$ with $\dot{L}_{\mathrm{s}}^{\mathrm{r}}$ given by Eq. (29)

This is Eq. (32) of the paper

Significance of the Boundary Terms



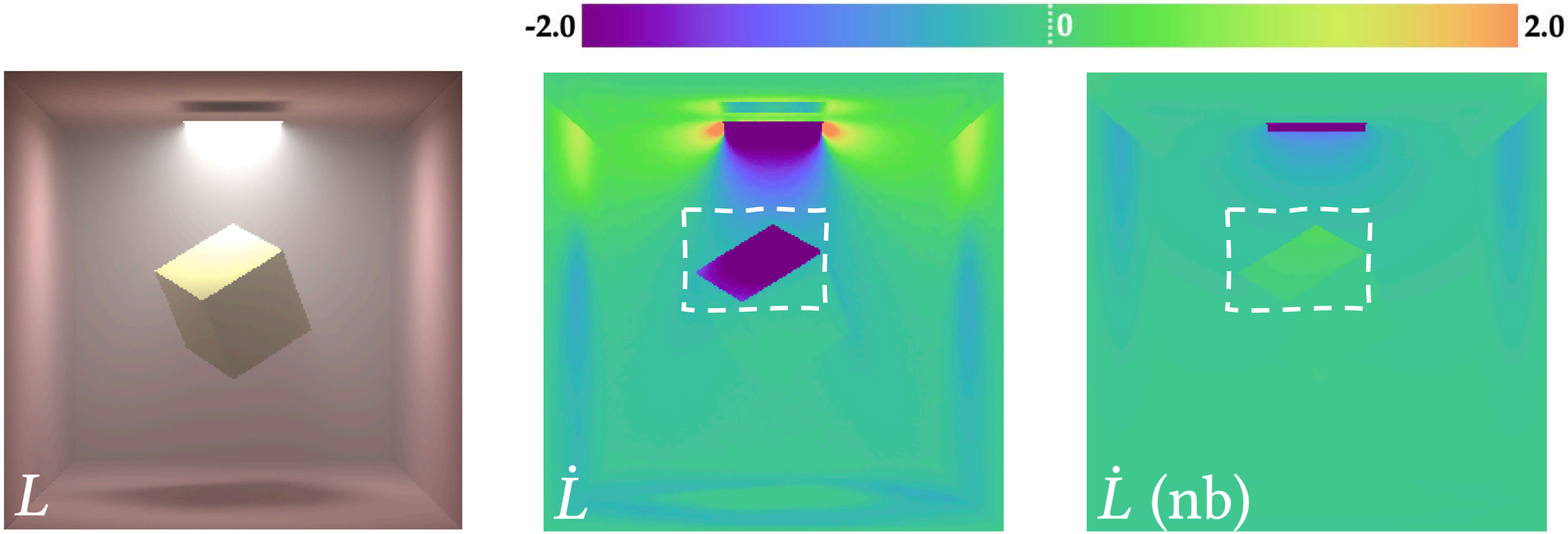
Orig. Image

$$P_{\text{light}} = \boxed{P_0} + \begin{pmatrix} 0 \\ \pi \\ 0 \end{pmatrix}$$

$$P_{\text{cube}} = \boxed{P_1} + \begin{pmatrix} 0 \\ \pi \\ 0 \end{pmatrix}$$

Initial position
(constant)

Significance of the Boundary Terms



Orig. Image

Deriv. Image

Deriv. Image
(no boundary term)

Differentiating the RTE: Summary

$$L = K_T K_c L + Q$$

Key:

- Tracking discontinuities of integrands
- Establishing boundary terms accordingly

$$\partial_\pi L = \partial_\pi (K_T K_c L) + \partial_\pi Q$$

Differential RTE

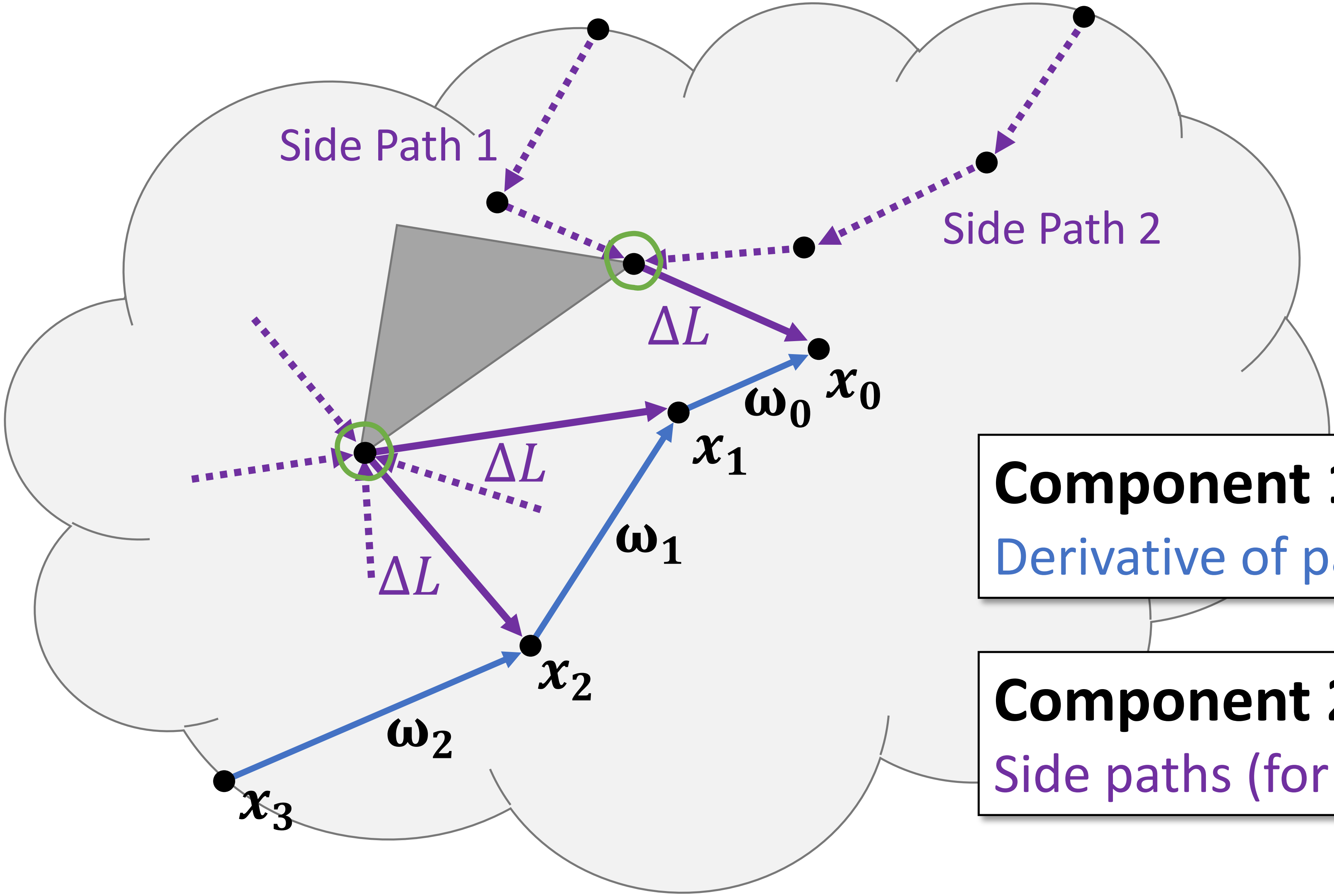
$$L = K_T K_c L + Q$$

$$\partial_\pi L = \partial_\pi (K_T K_c L) + \partial_\pi Q$$

Boundary terms are included

$$\begin{pmatrix} \partial_\pi L \\ L \end{pmatrix} = \begin{pmatrix} K_T K_c & K_* \\ 0 & K_T K_c \end{pmatrix} \begin{pmatrix} \partial_\pi L \\ L \end{pmatrix} + \begin{pmatrix} \partial_\pi Q \\ Q \end{pmatrix}$$

Differentiable Volumetric Path Tracing

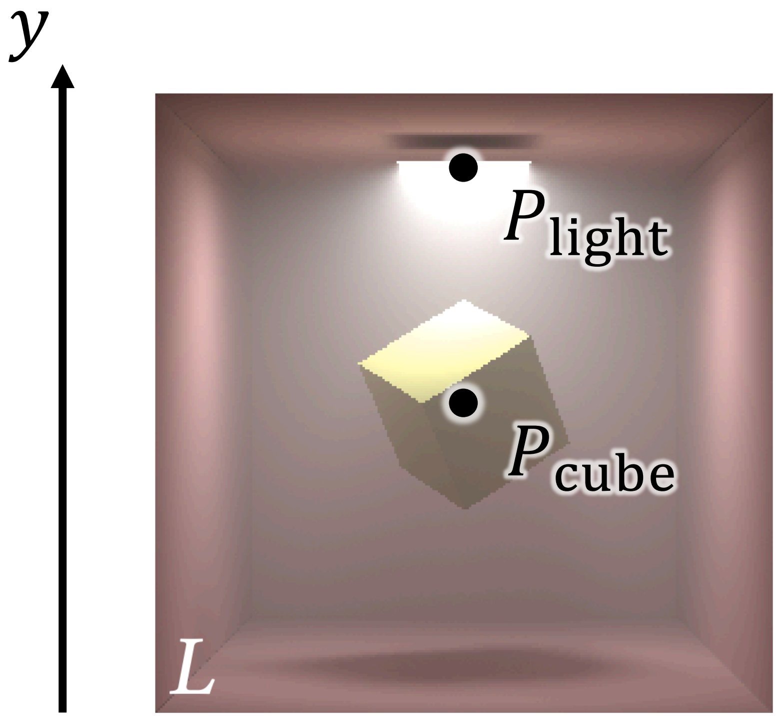


Component 1:
Derivative of path throughput

Component 2:
Side paths (for estimating ΔL)

Results

Results: Validation



Orig. Image

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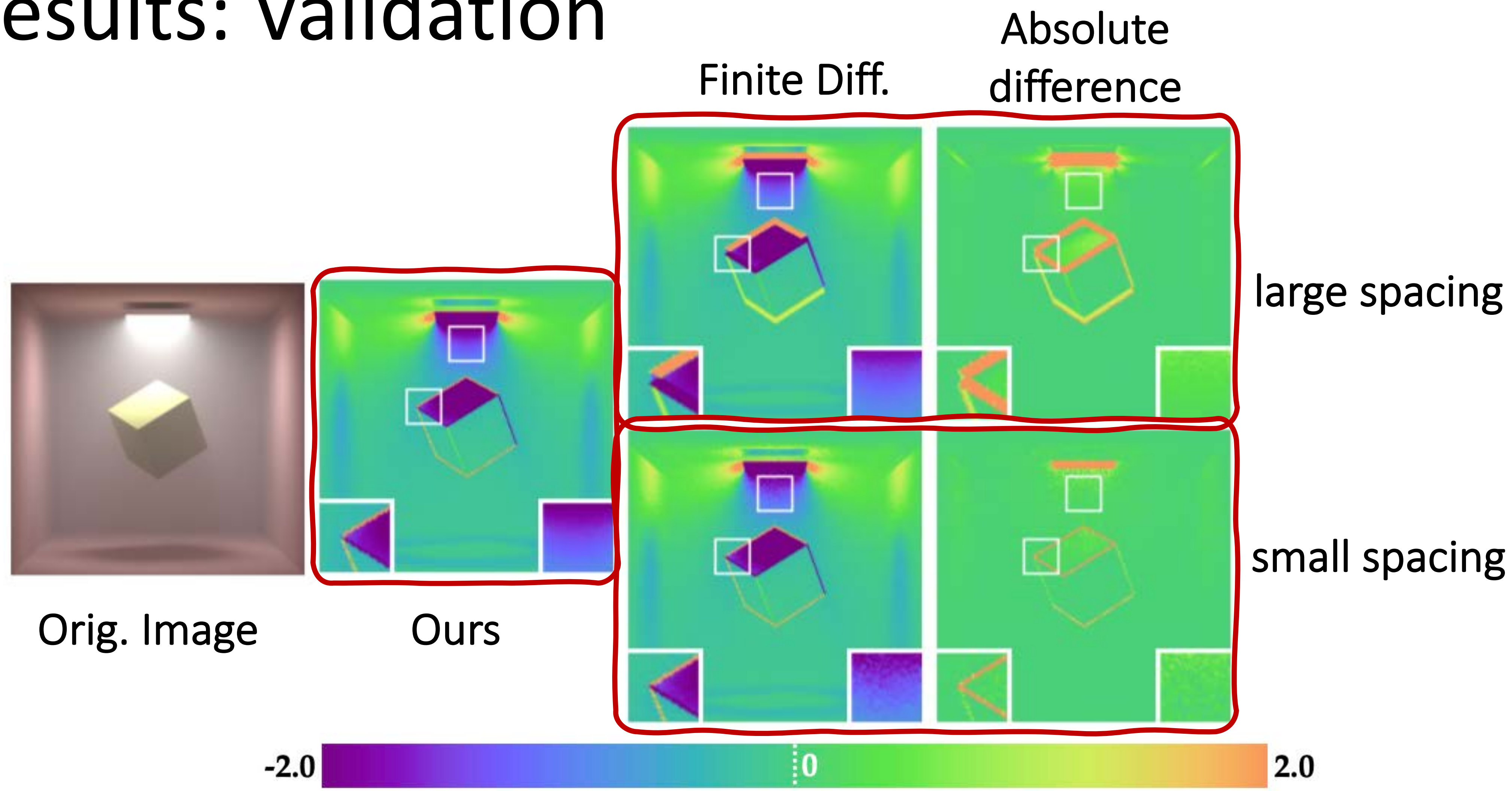
Initial position
(constant)

Results: Validation



(equal-time comparison)

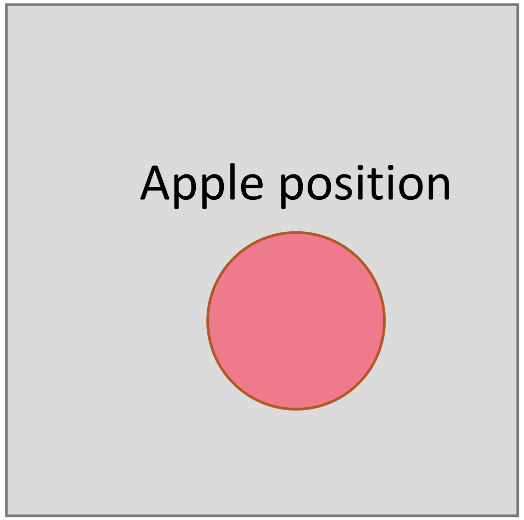
Results: Inverse Rendering

- Scene configurations
  - participating media
  - changing geometry

- Optimization
  - L2 loss for its simplicity
  - Any differentiable metric can be used with our method

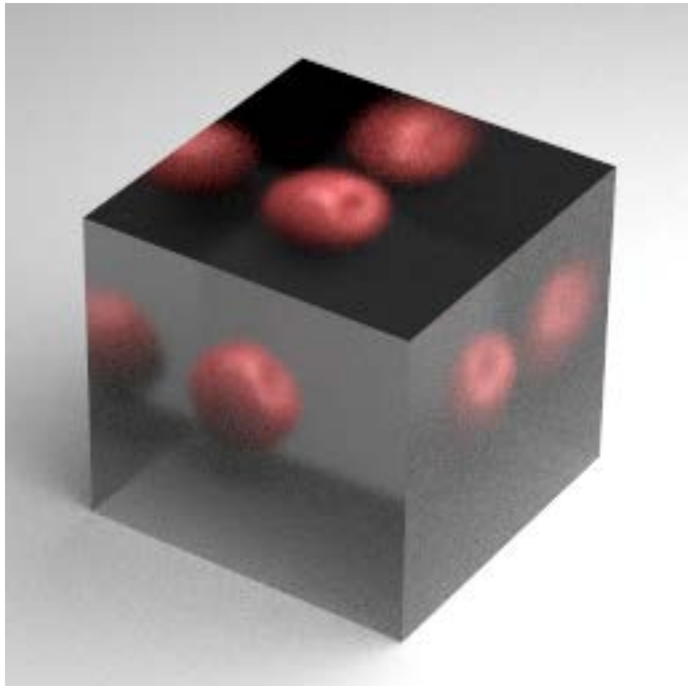
Apple in a Box

Parameters

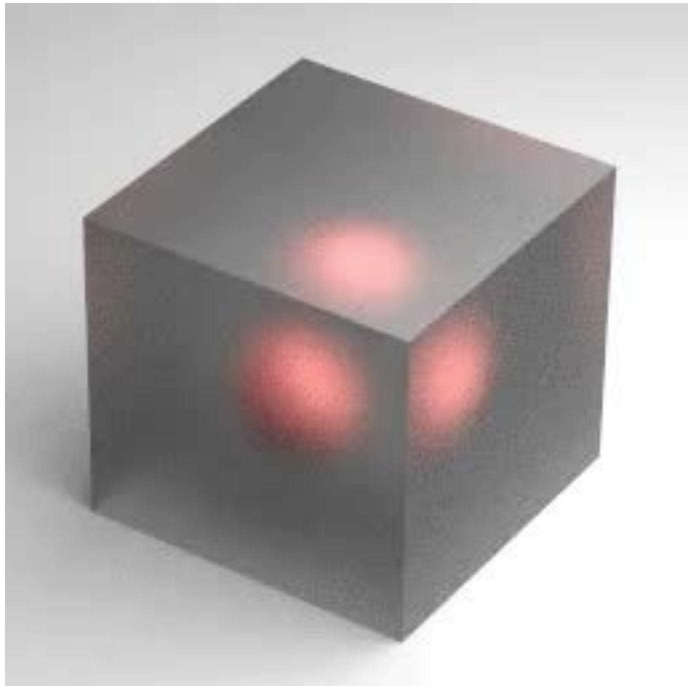


Cube roughness

Target



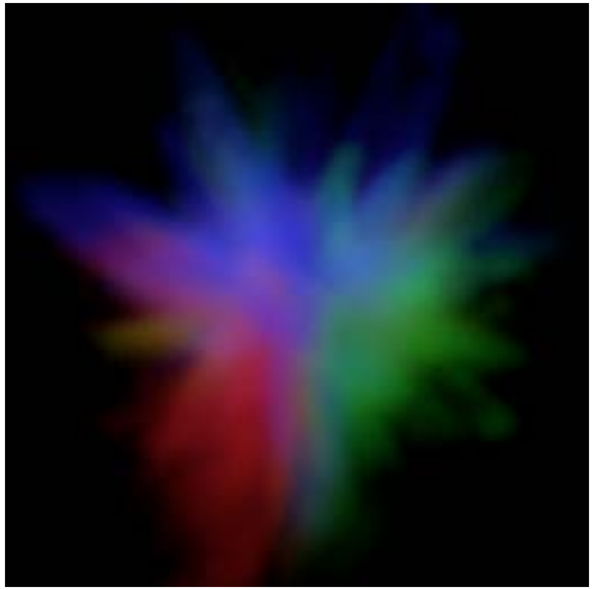
Optimization process



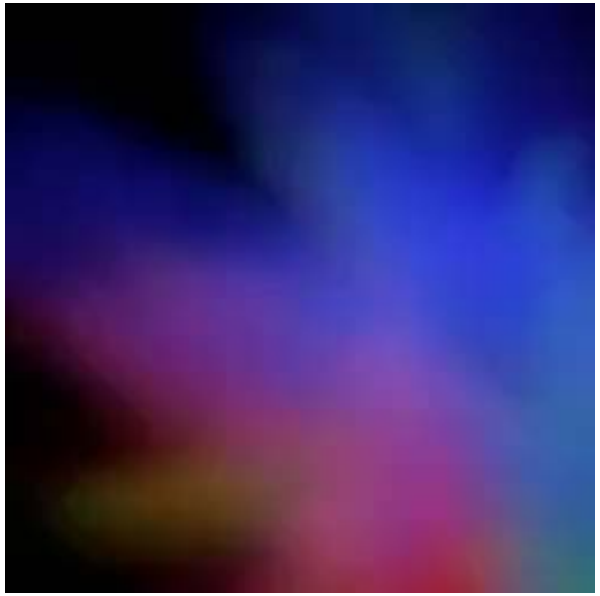
#iterations	Time (CPU core minute per iteration)
80	12.2

Camera Pose

Target



Optimization Process



#iterations	Time (CPU core minute per iteration)
220	9.3

None-Line-of-Sight

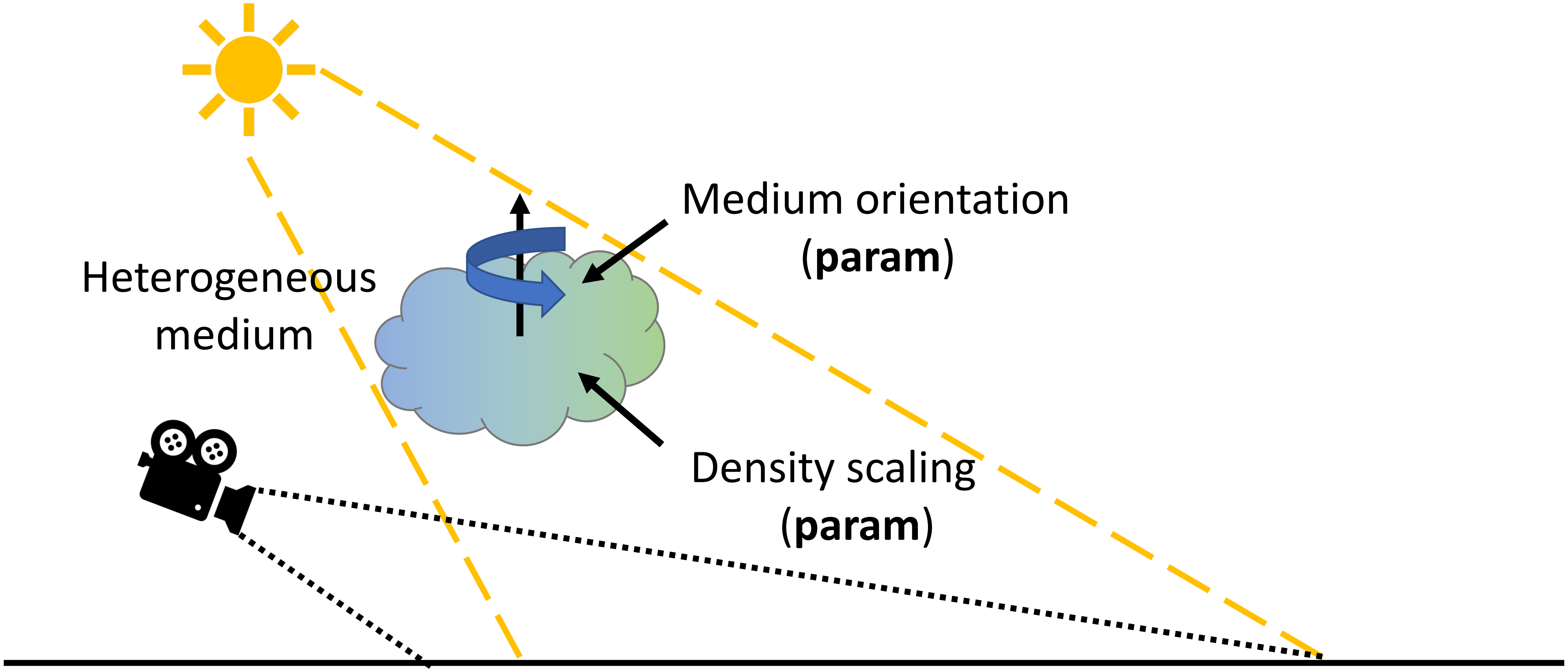
Target

Optimization Process

Diff. View
(not used in optimization)

#iterations	Time (CPU core minute per iteration)
100	9

None-Line-of-Sight



None-Line-of-Sight

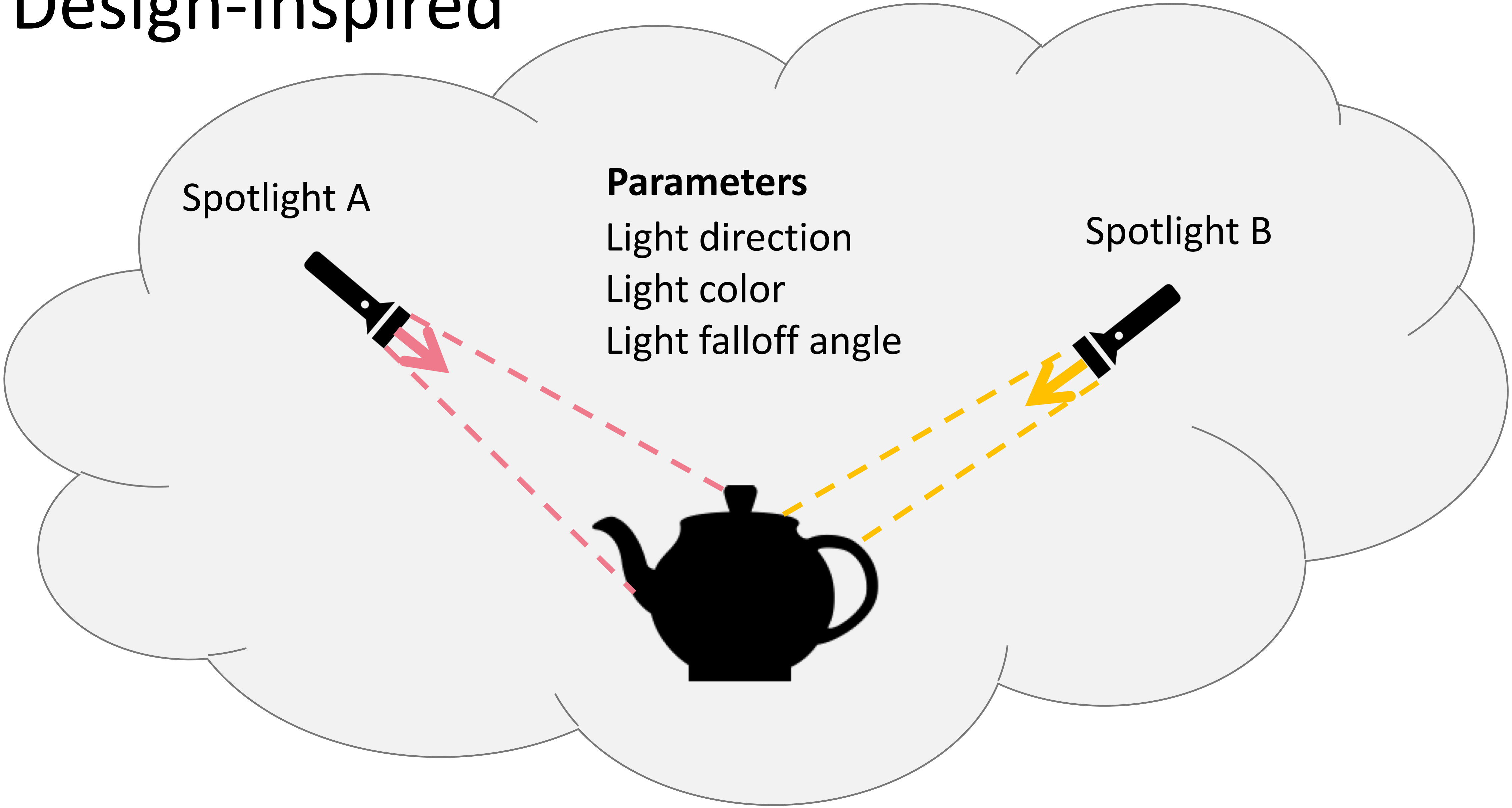
Target

Optimization process

Diff. View

#iterations	Time (CPU core minute per iteration)
60	7.6

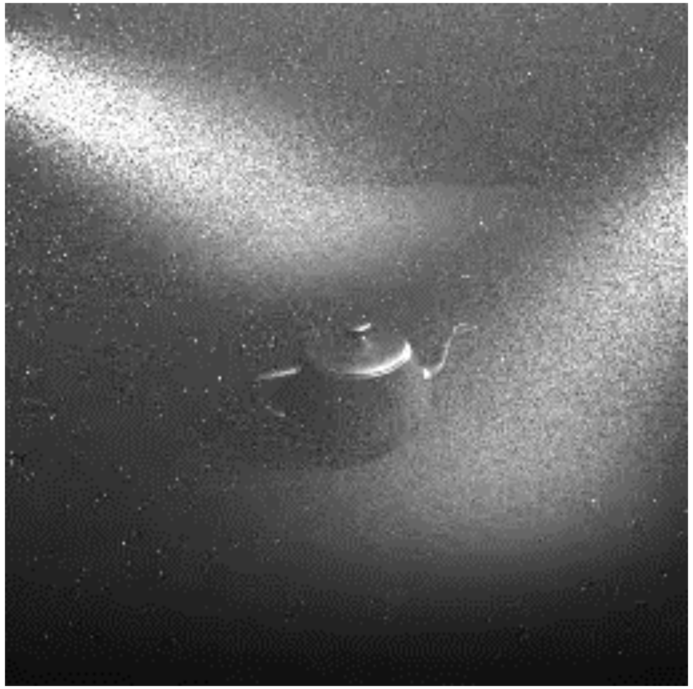
Design-Inspired



Design-Inspired

Target

Optimization process

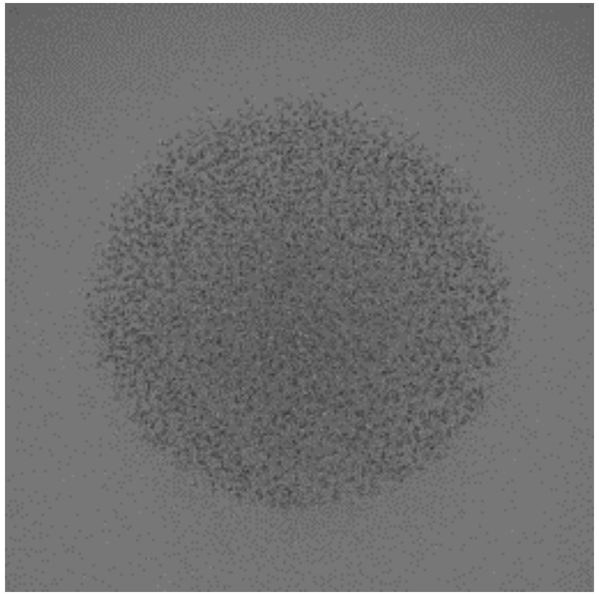


#iterations	Time (CPU core minute per iteration)
110	11.2

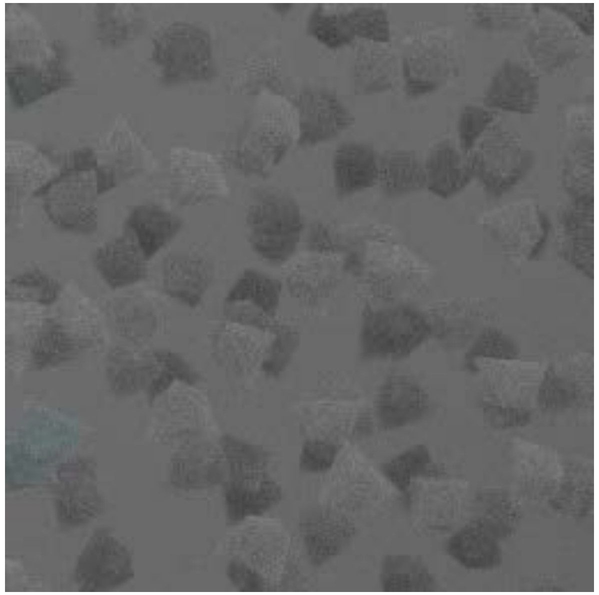
Design-inspired

Target

Optimization Process



Diff. View



#iterations	Time (CPU core minute per iteration)
100	27.2

Future work

- Differentiable rendering is slow due to
  - Main term
    - Needs better sampling methods
  - Boundary term
    - Detecting visibility changes (e.g., object silhouettes)
    - Tracing side paths

Thank you