A Hybrid Material Point Method for Frictional Contact with Diverse Materials

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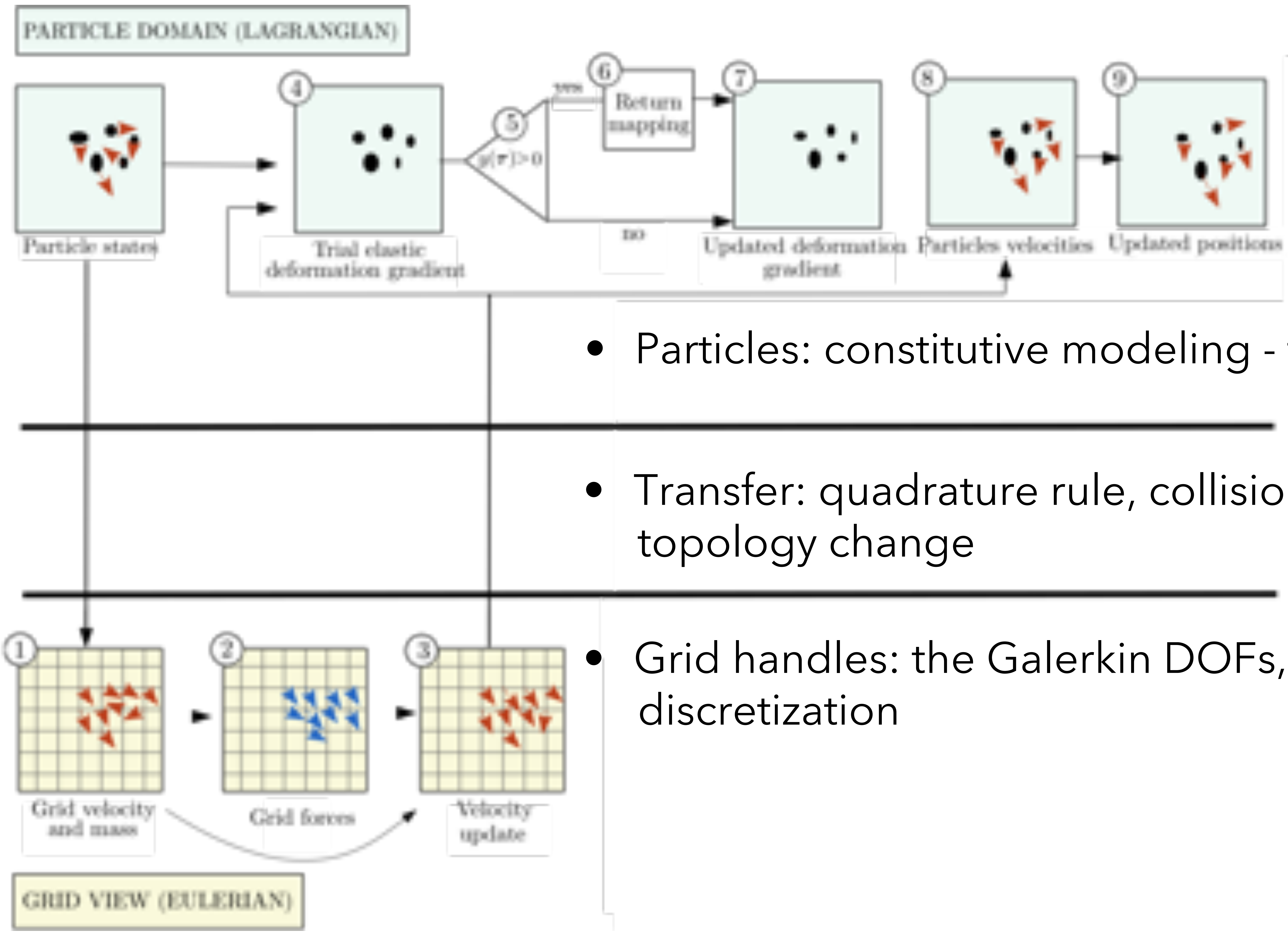
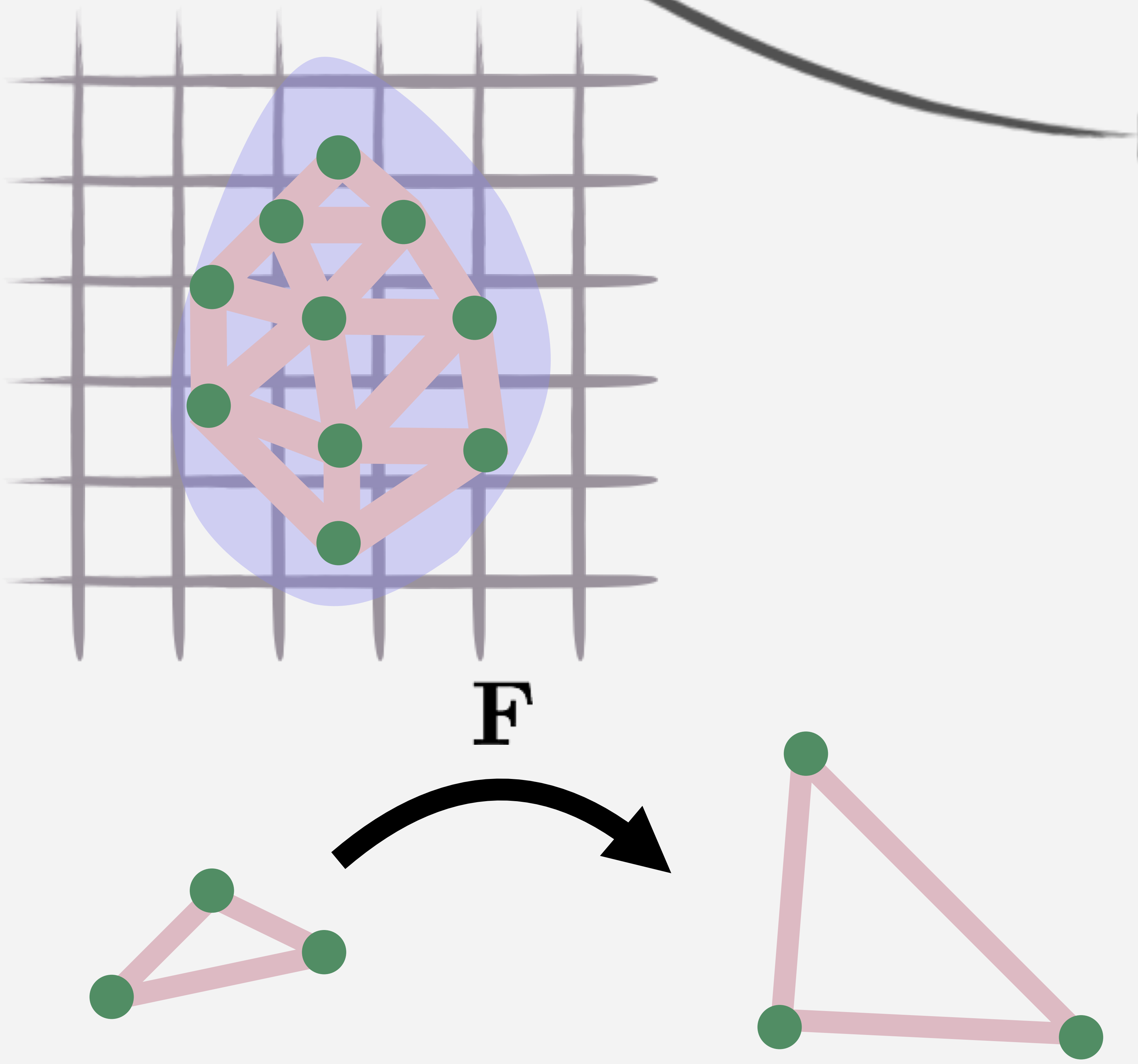
MPM is hybrid Lagrangian/Eulerian



- Particles: constitutive modeling - the physics
- Transfer: quadrature rule, collision detect, topology change
- Grid handles: the Galerkin DOFs, BC, discretization

Stomakhin et al. 2013
Gast et al. 2015
Ram et al. 2015
Yue et al. 2015
Klar et al. 2016
Daviet et al. 2016
Stomakhin et al. 2015
Gao et al. 2018
Pradhana et al. 2018
Wolper et al. 2019

MPM for meshed objects



Elastic potential

Element wise energy density

Deformation gradient

$$\Phi = \sum_e V_e^0 \Psi(\mathbf{F}_e)$$

Grid node force

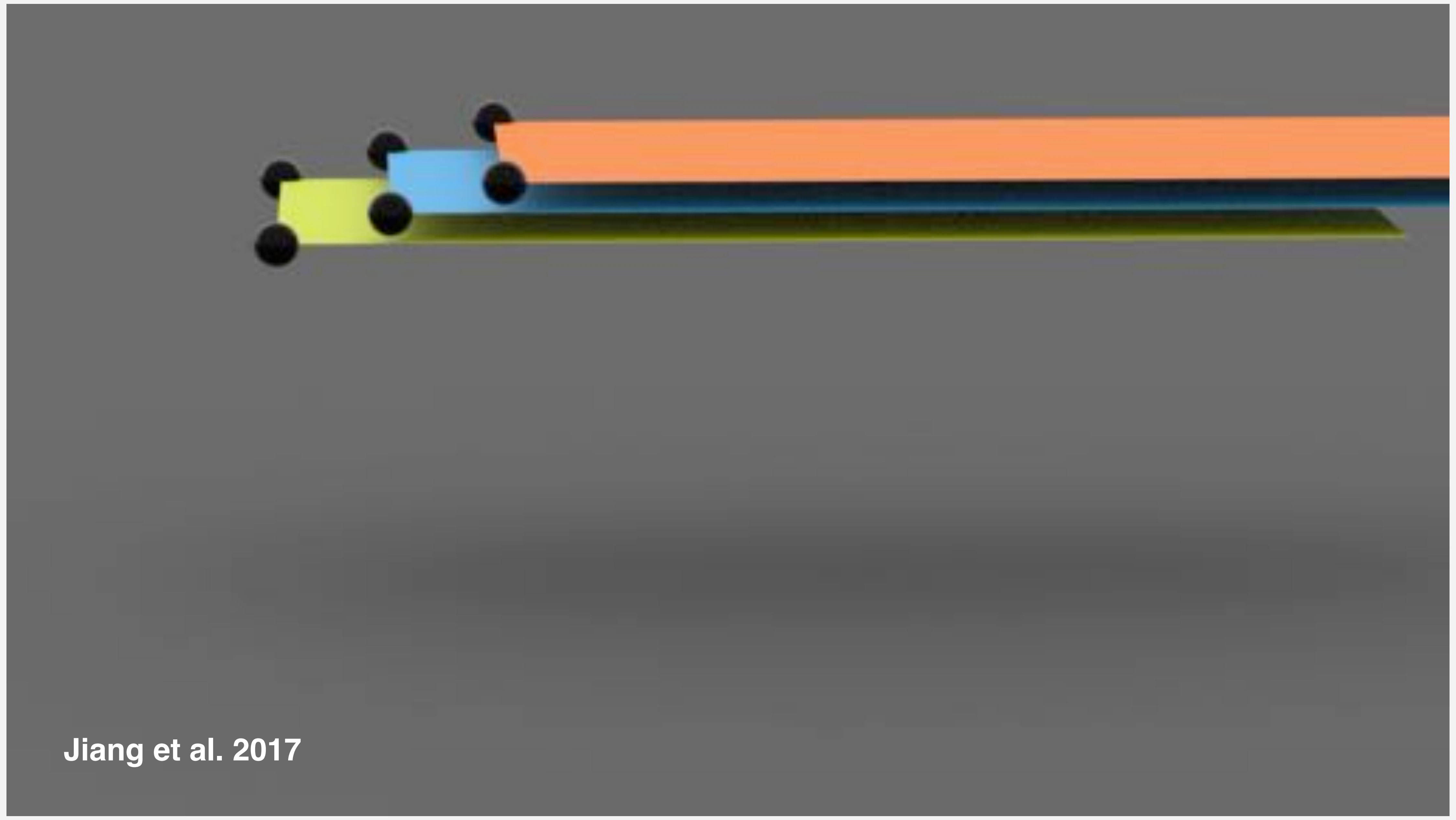
Grid node position

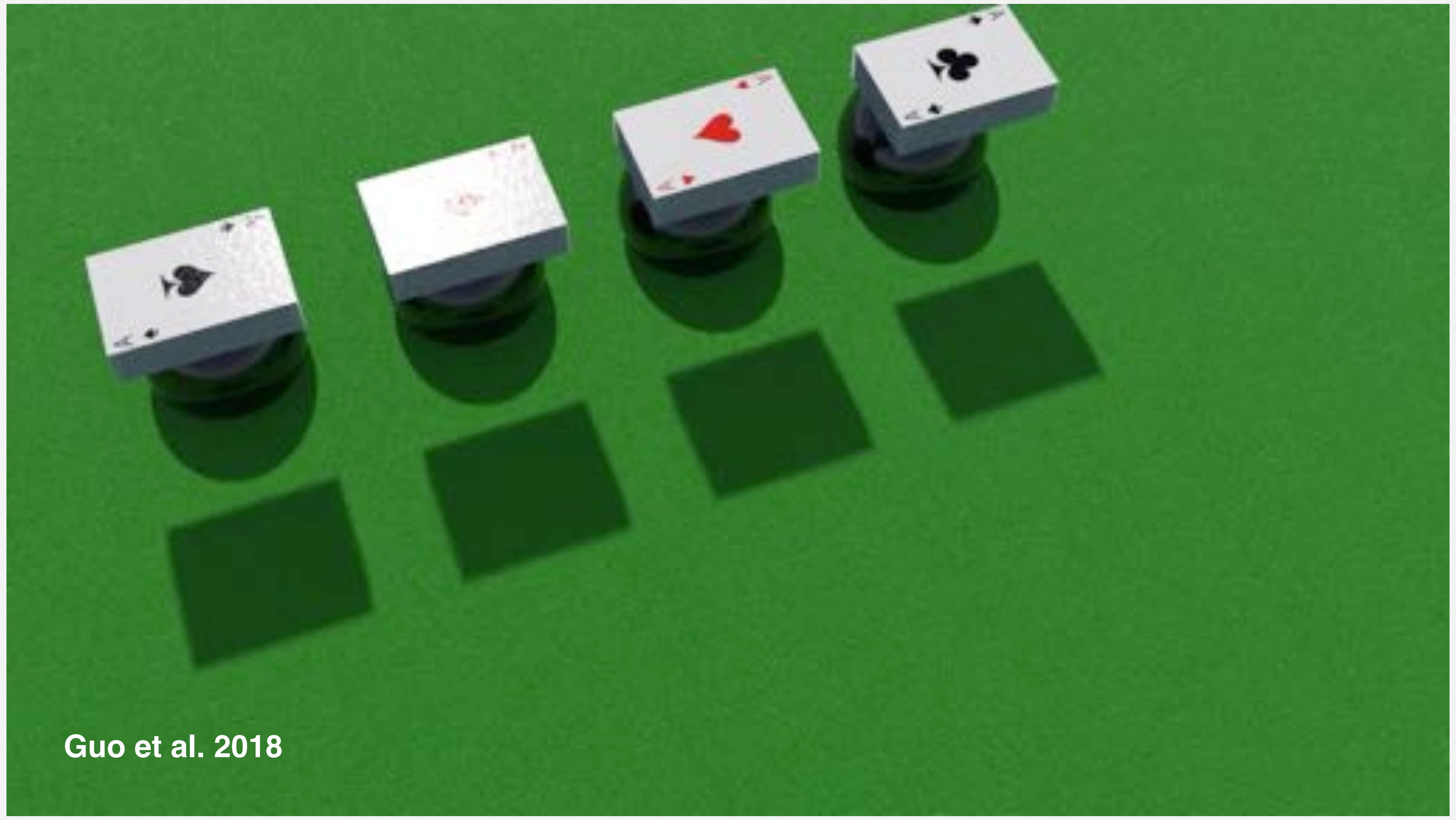
$$\mathbf{f}_i = -\frac{\partial \Phi}{\partial \mathbf{x}_i}$$

$$\Psi = \Psi(\mathbf{F}_e(\mathbf{x}_p(\mathbf{x}_i)))$$

Particle position


Jiang et al. 2017

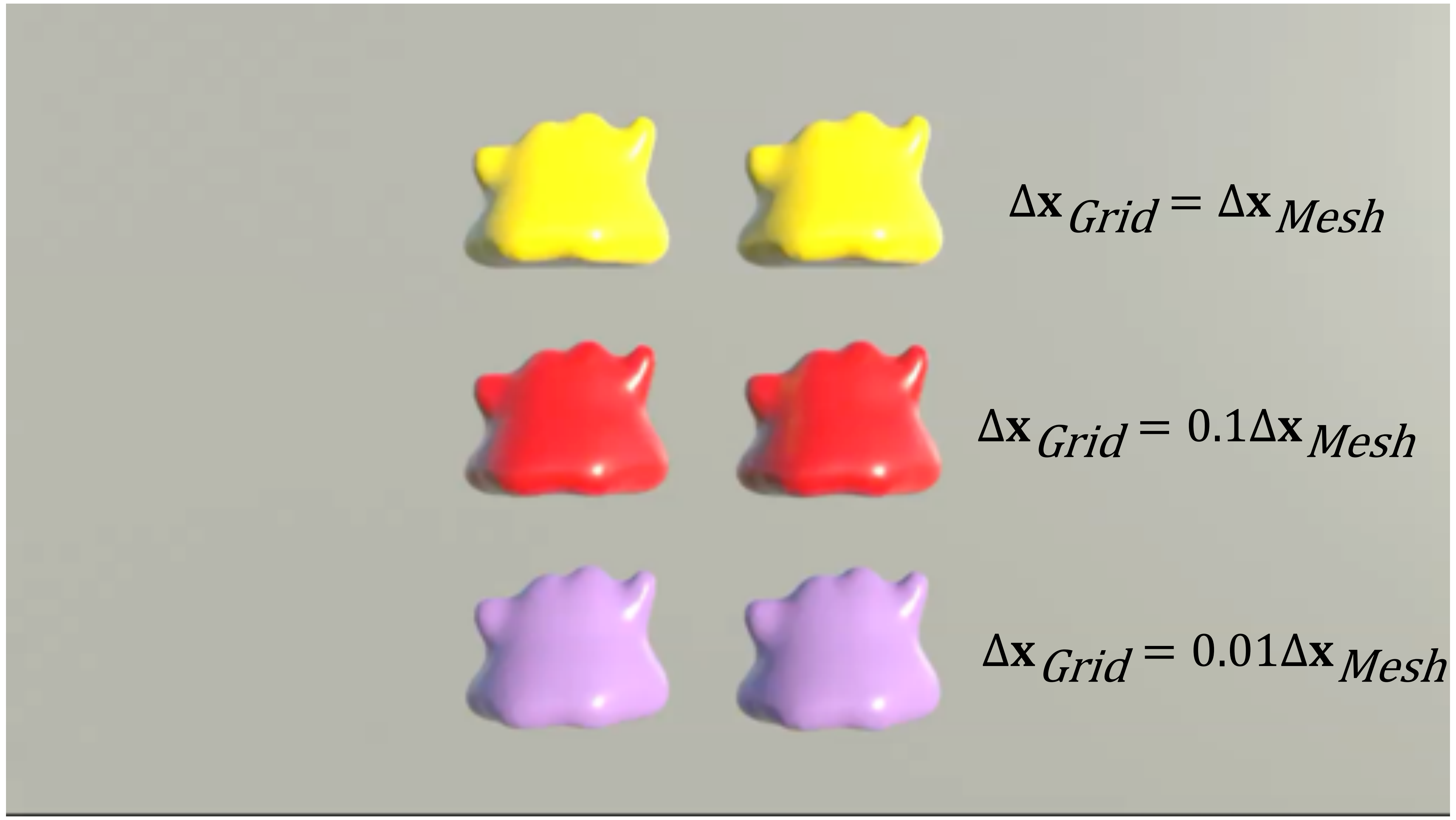

Jiang et al. 2017


Guo et al. 2018


Guo et al. 2018

Drawbacks of MPM

Numerical Friction

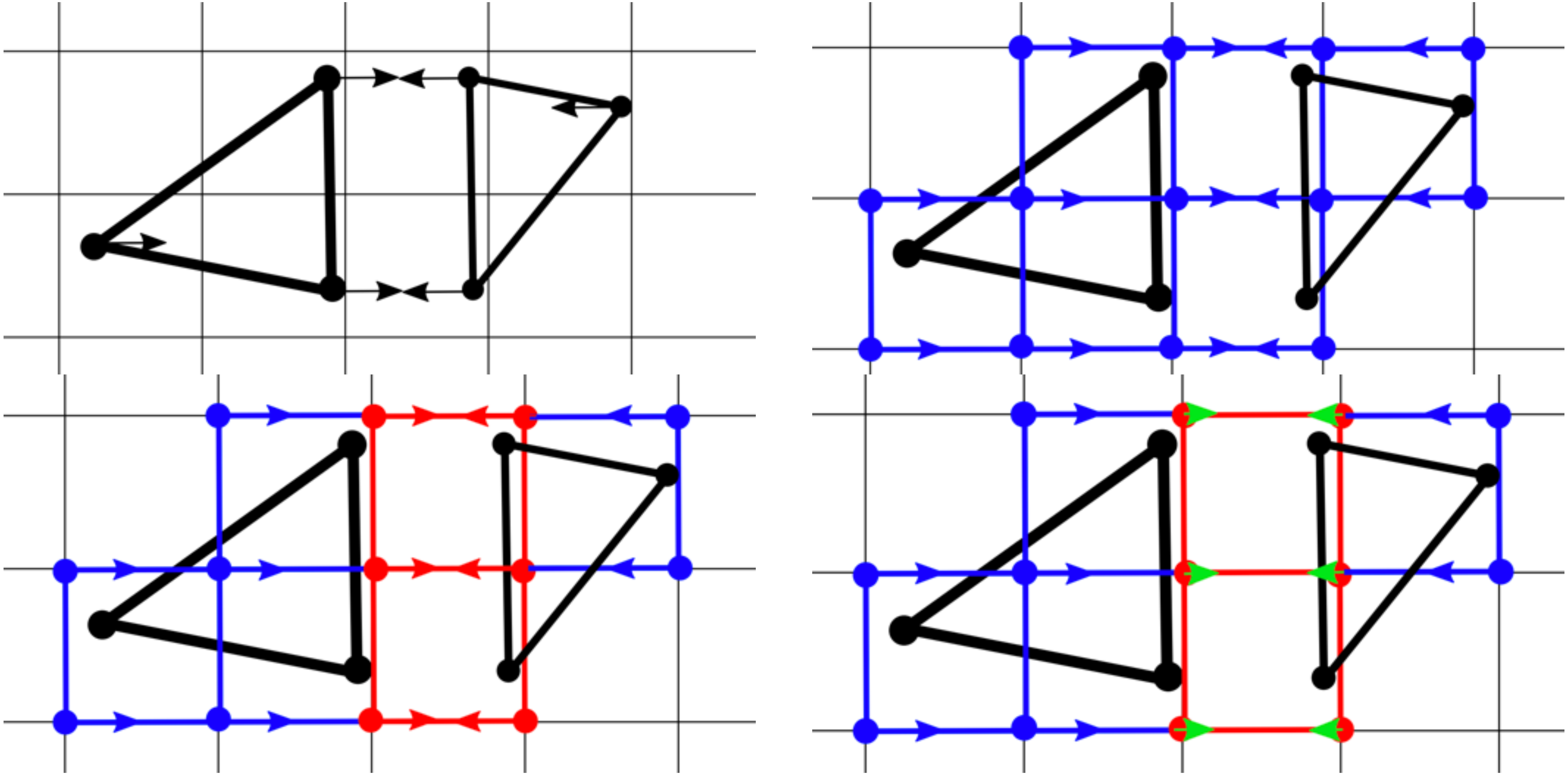


Grid Dependency

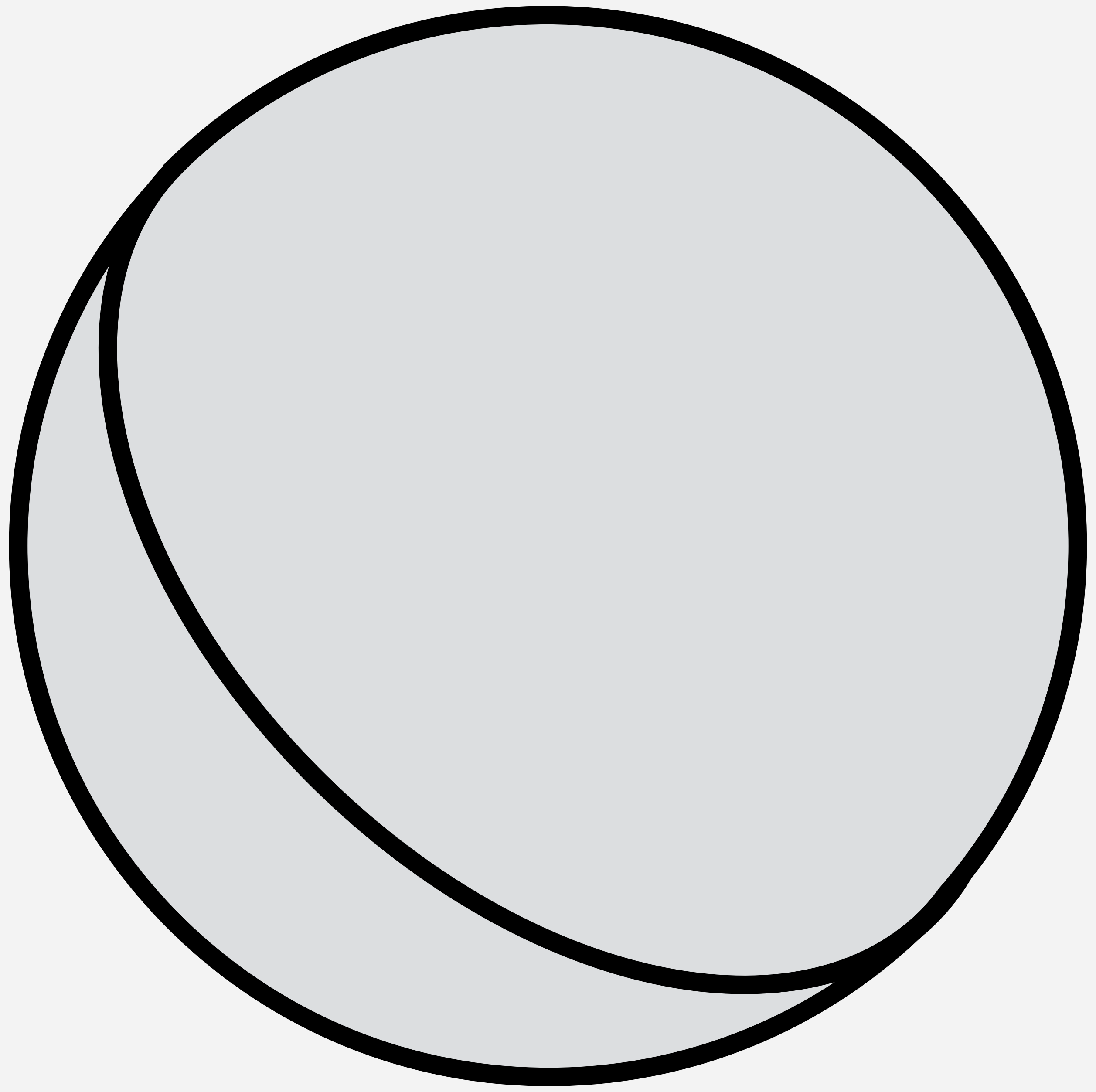
MPM Collision Prevention

- **Type I:** Collision modes penalized via potential energy

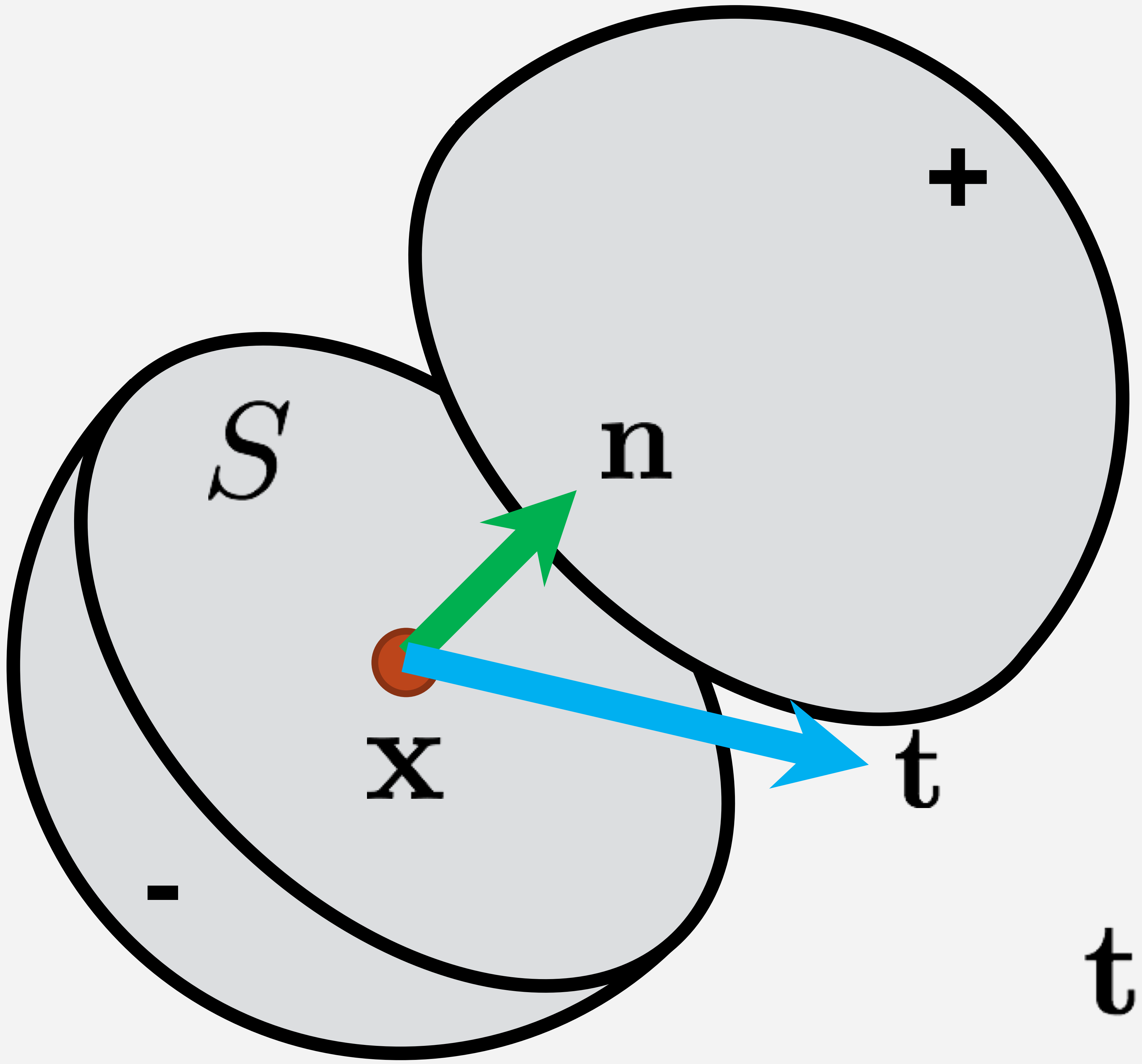
Type I Collision Resolution



Traction in a continuum



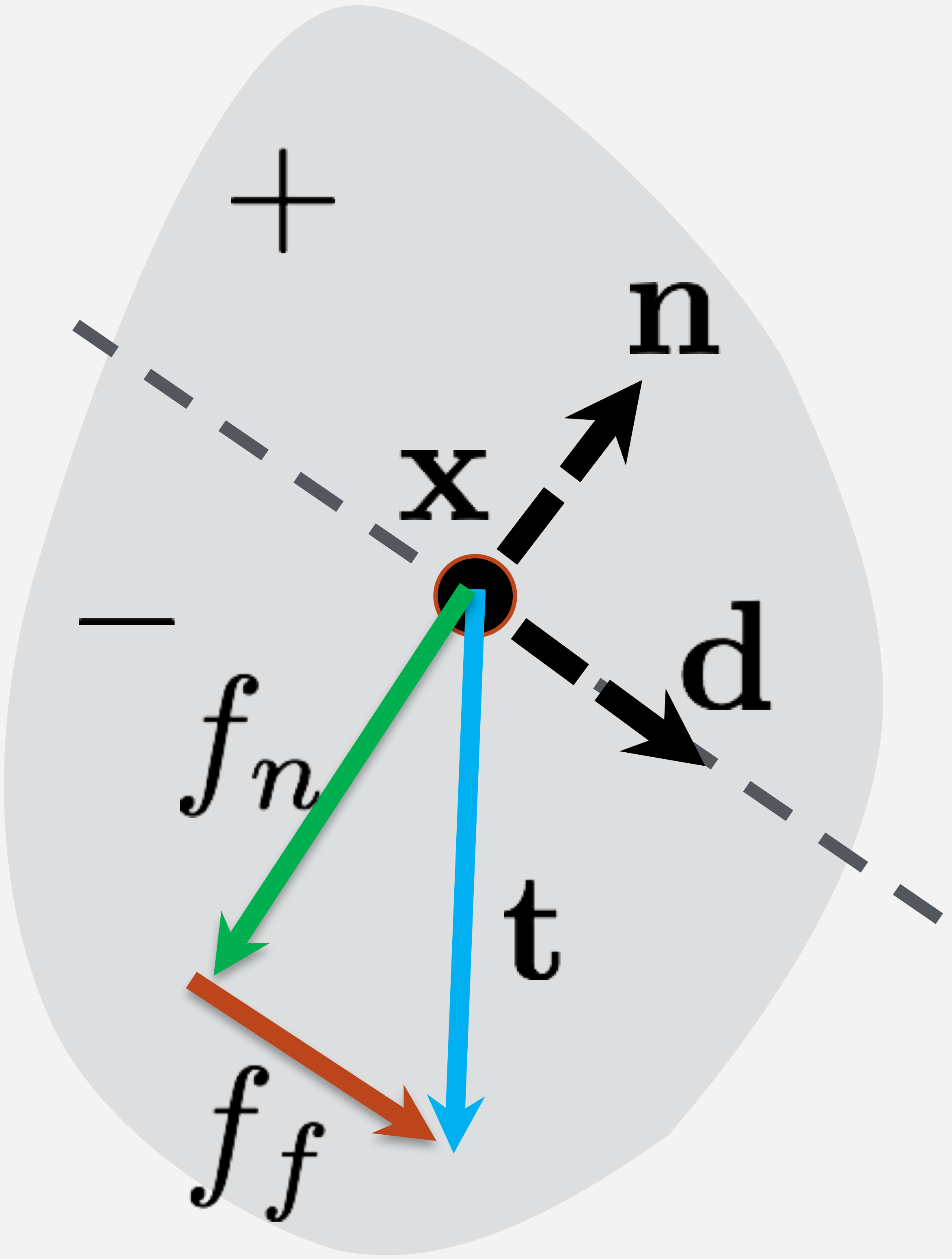
Traction in a continuum



Cauchy stress

$$\mathbf{t}(\mathbf{x}, \mathbf{n}) = \boldsymbol{\sigma}(\mathbf{x})\mathbf{n}$$

Friction and plasticity



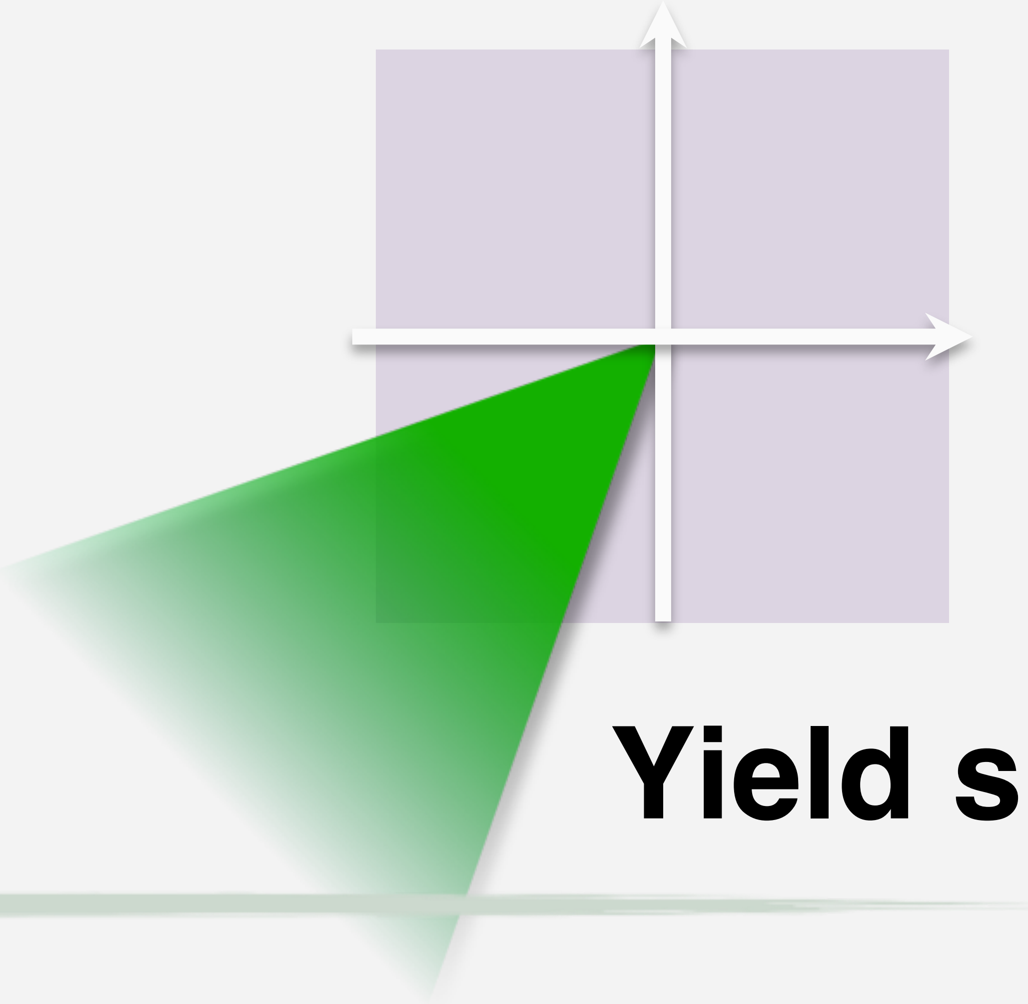
Traction	$\mathbf{t} = \boldsymbol{\sigma}\mathbf{n}$
Normal force	$f_n = -\mathbf{n}^T\mathbf{t}$
Frictional force	$f_f = \mathbf{d}^T\mathbf{t}$
Coulomb friction	$f_f \leq c_F f_n$

$$\mathbf{d}^T\boldsymbol{\sigma}\mathbf{n} + c_F\mathbf{n}^T\boldsymbol{\sigma}\mathbf{n} \leq 0$$

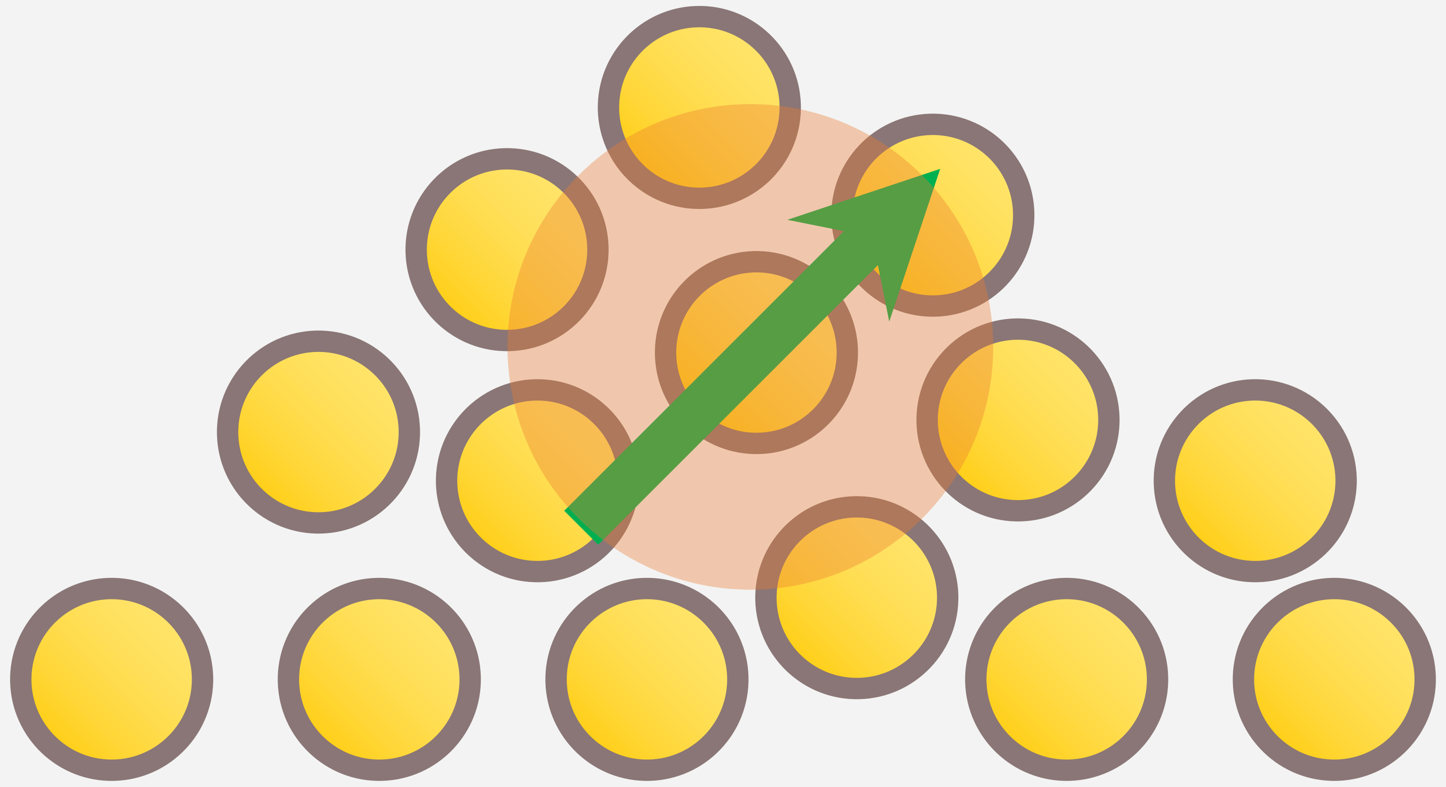
Friction and plasticity

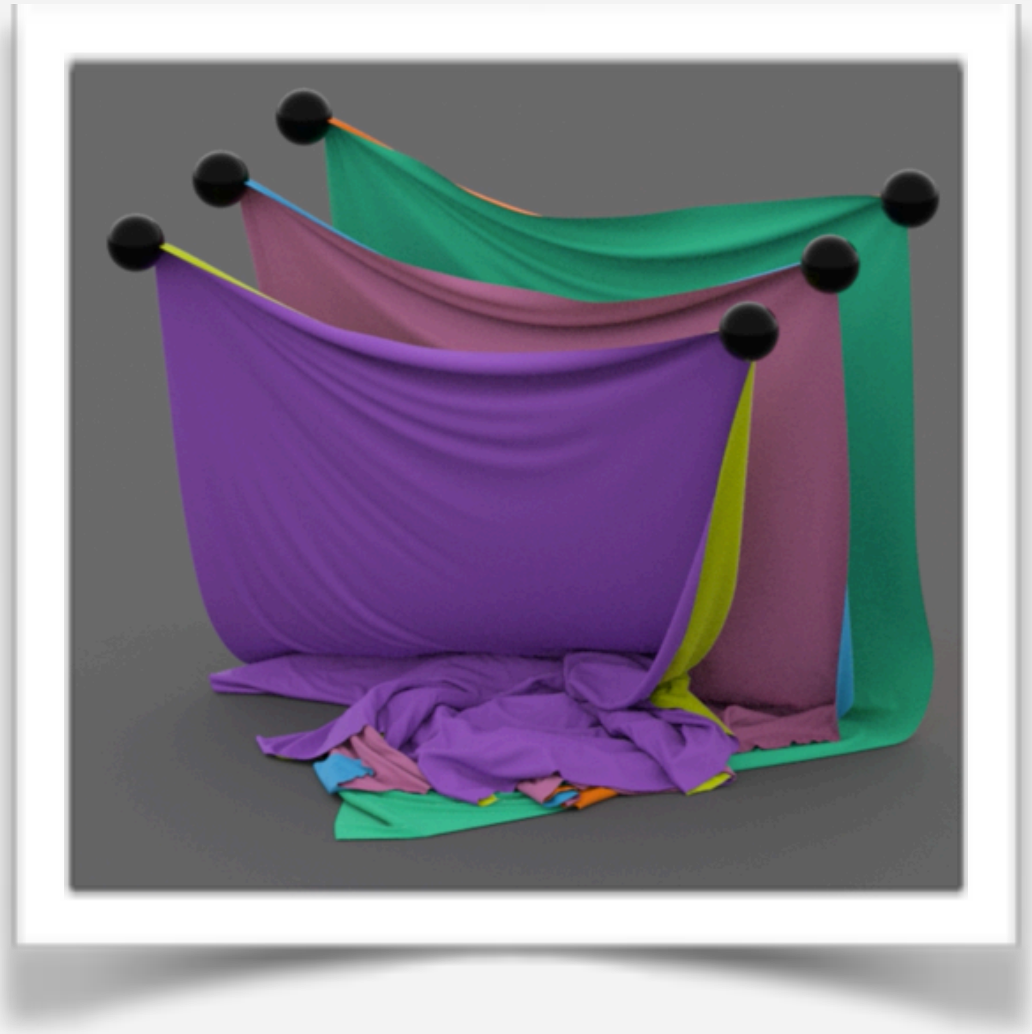
$$\mathbf{d}^T \boldsymbol{\sigma} \mathbf{n} + c_F \mathbf{n}^T \boldsymbol{\sigma} \mathbf{n} \leq 0$$

all $\mathbf{n}$



Yield surface





$\mathbf{n} \perp$ surface


MPM
Our method

Our method

MPM

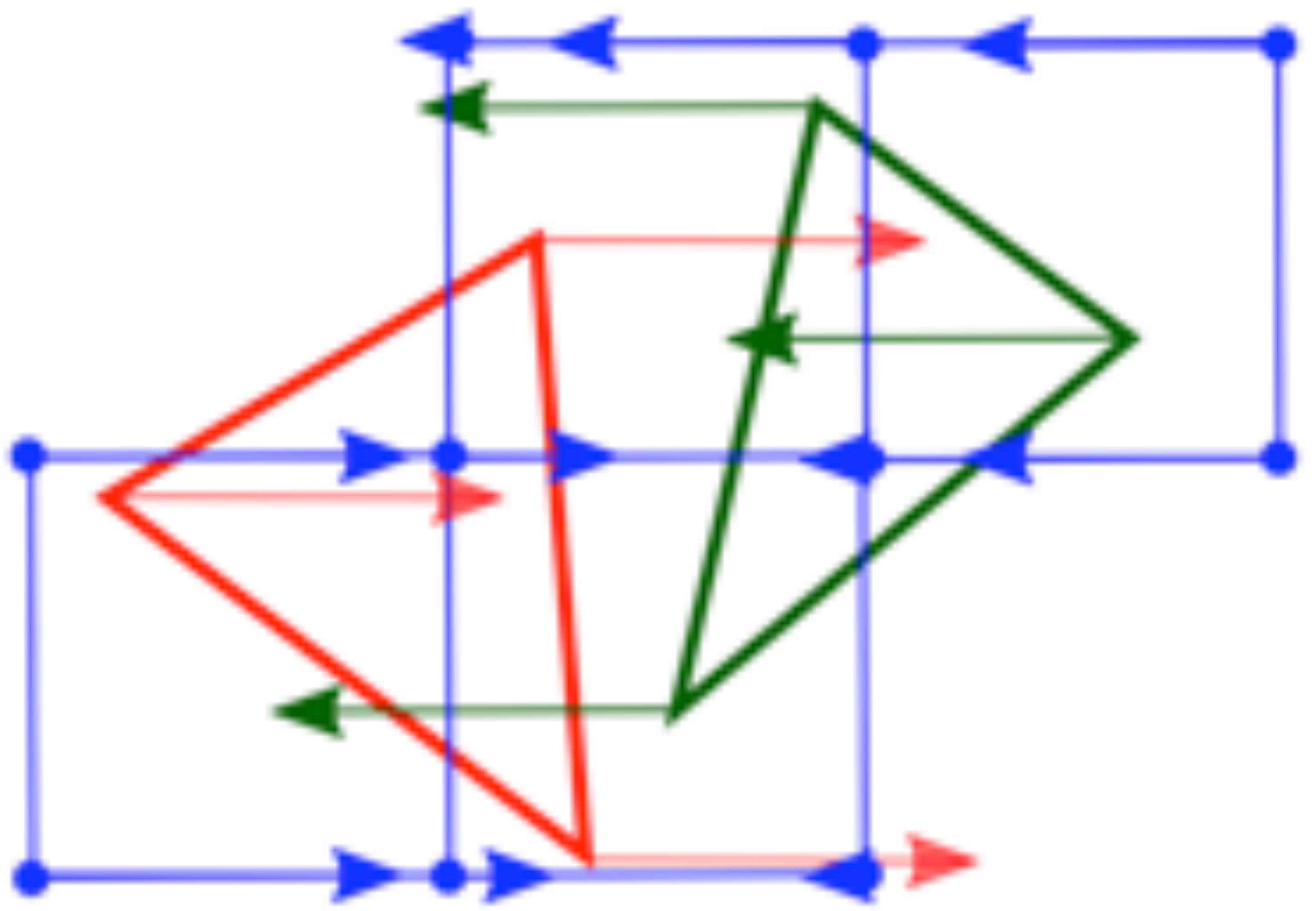
MPM Collision Prevention

- ~~**Type I:** Collision modes penalized via potential energy~~
- Move the DOFs to the Lagrangian Mesh

MPM Collision Prevention

- ~~**Type I:** Collision modes penalized via potential energy~~
- **Type II:** Smooth interpolation

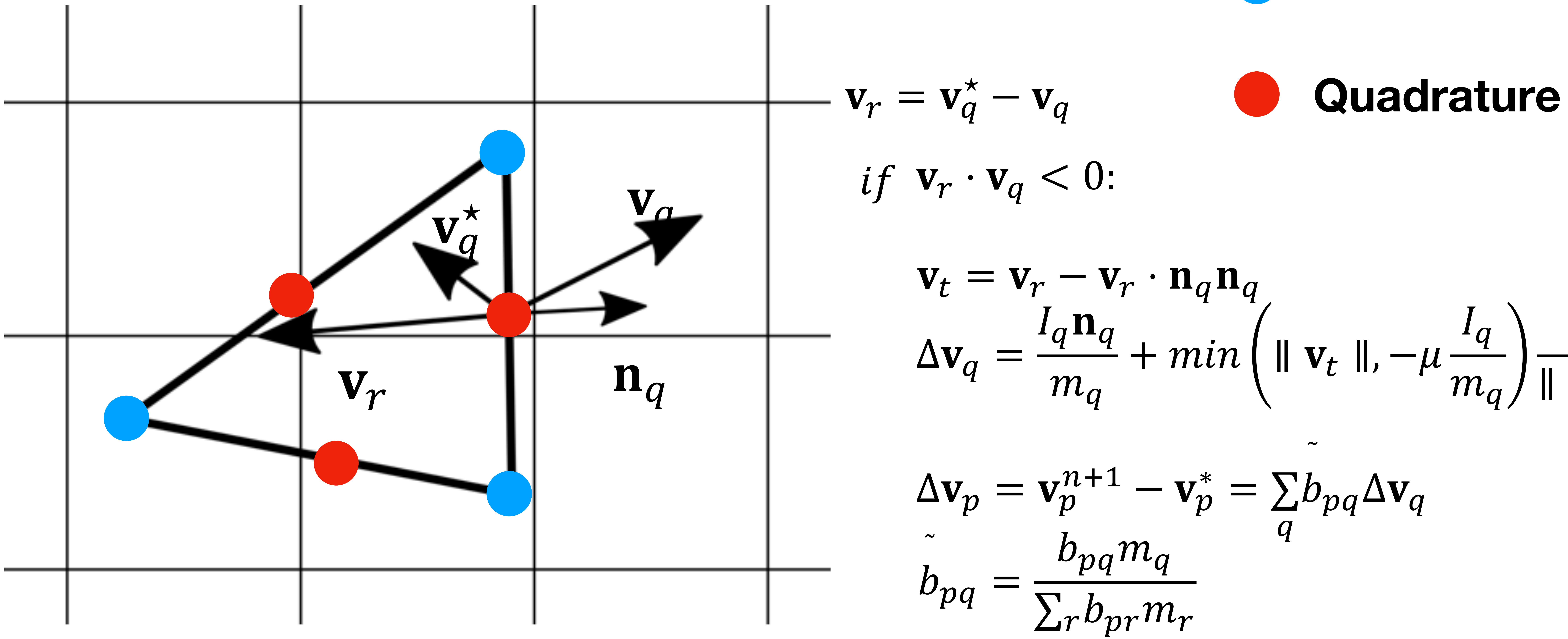
Type II Collision Resolution



MPM Collision Prevention

- ~~**Type I:** Collision modes penalized via potential energy~~
- **Type II:** Smooth interpolation ⟶ Modify

Hybrid MPM



$$\mathbf{v}_r = \mathbf{v}_q^\star - \mathbf{v}_q$$

$$if \;\; \mathbf{v}_r \cdot \mathbf{v}_q < 0:$$

$$\mathbf{v}_t = \mathbf{v}_r - \mathbf{v}_r \cdot \mathbf{n}_q \mathbf{n}_q$$

$$\Delta \mathbf{v}_q = \frac{I_q \mathbf{n}_q}{m_q} + min\left(\| \mathbf{v}_t \|, -\mu \frac{I_q}{m_q} \right) \frac{\mathbf{v}_t}{\| \mathbf{v}_t \|}$$

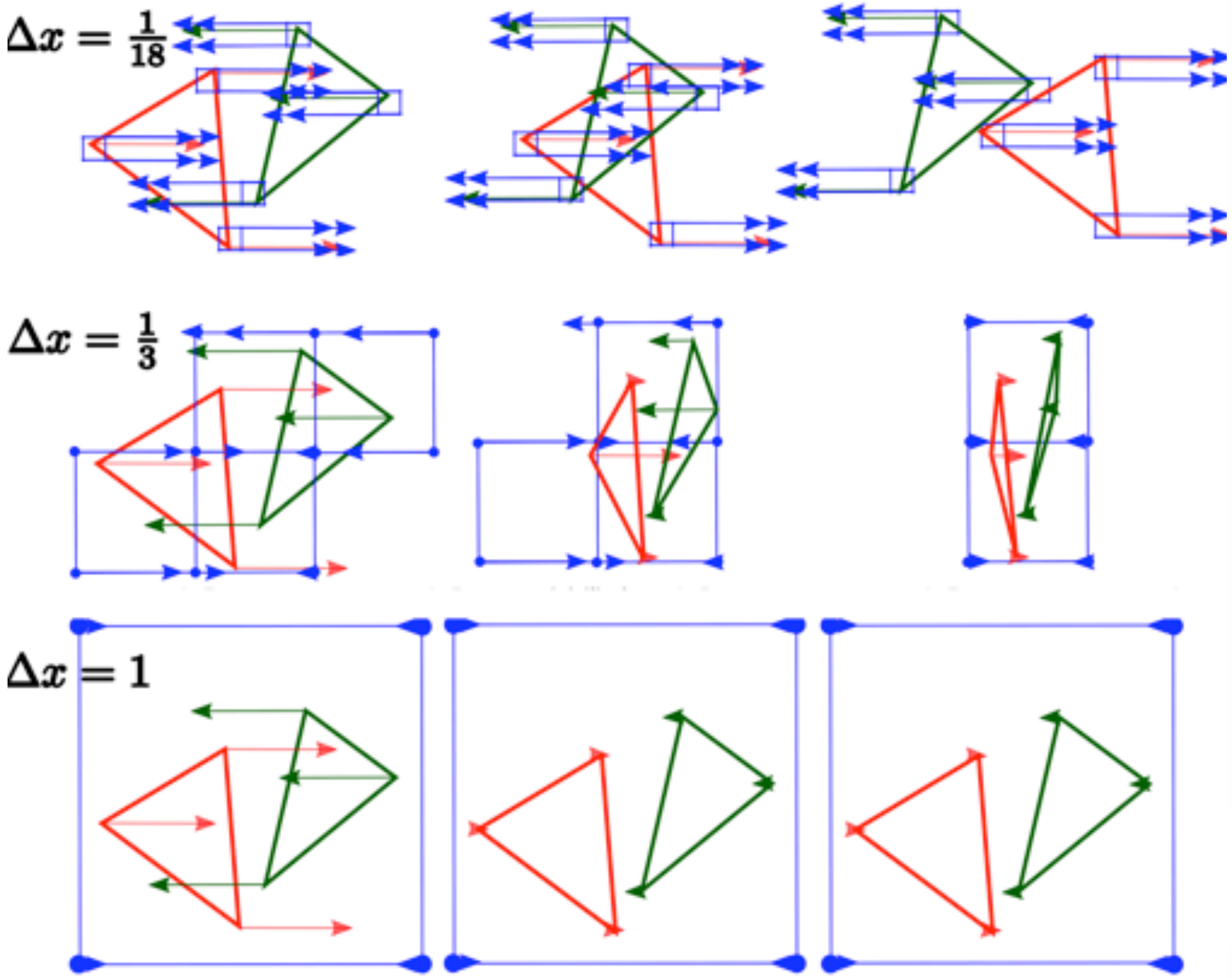
$$\Delta \mathbf{v}_p = \mathbf{v}_p^{n+1} - \mathbf{v}_p^* = \sum_q \tilde{b}_{pq} \Delta \mathbf{v}_q$$

$$\tilde{b}_{pq} = \frac{b_{pq} m_q}{\sum_r b_{pr} m_r}$$

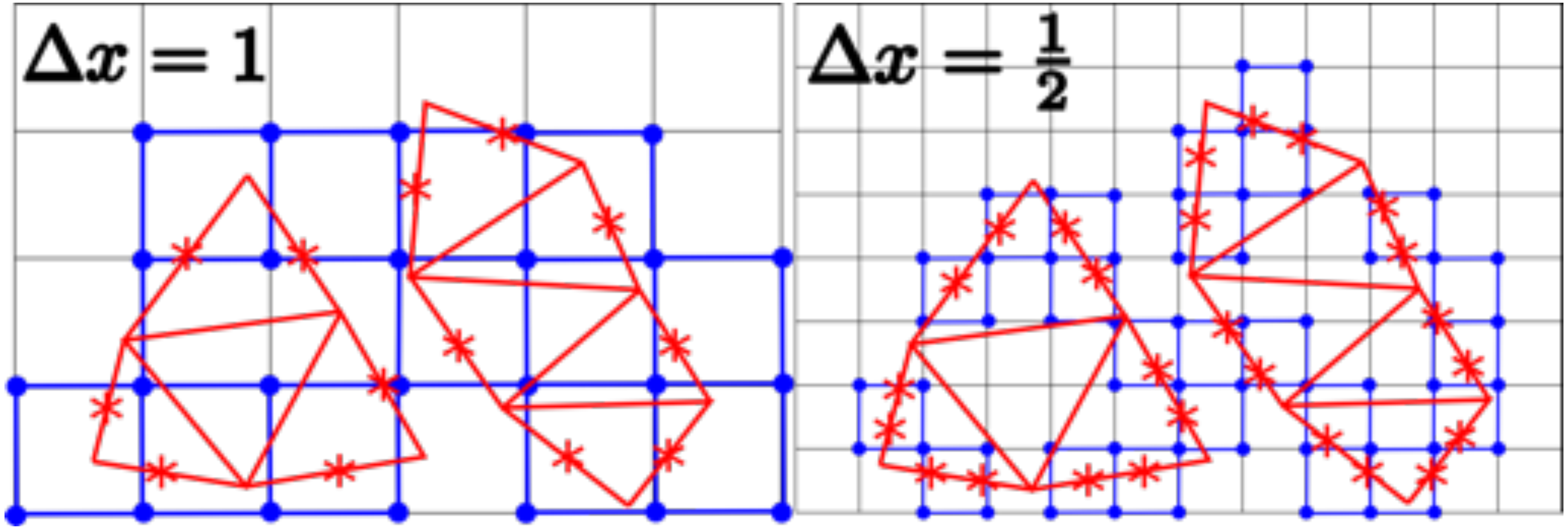
Hybrid MPM

- 1) Lagrangian Update
- 2) Transfer to grid
- 3) Transfer to collision particles
- 4) Apply impulses
- 5) Update positions

Type II Collision Resolution

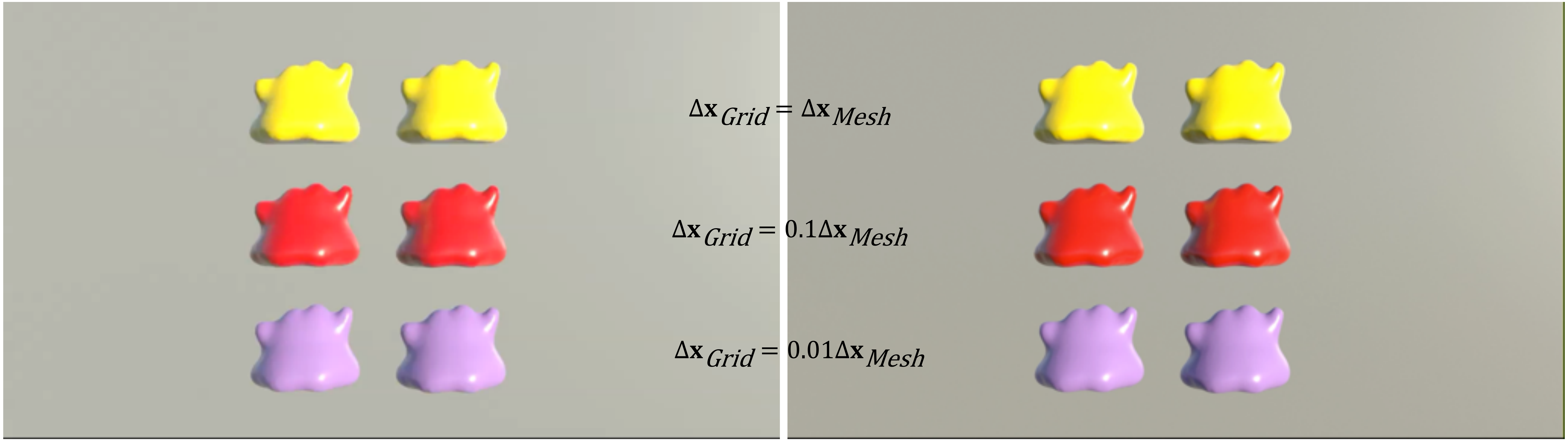


Modify Type II Collision



Sampling density based on Eulerian grid Δx

Modify Type II Collision

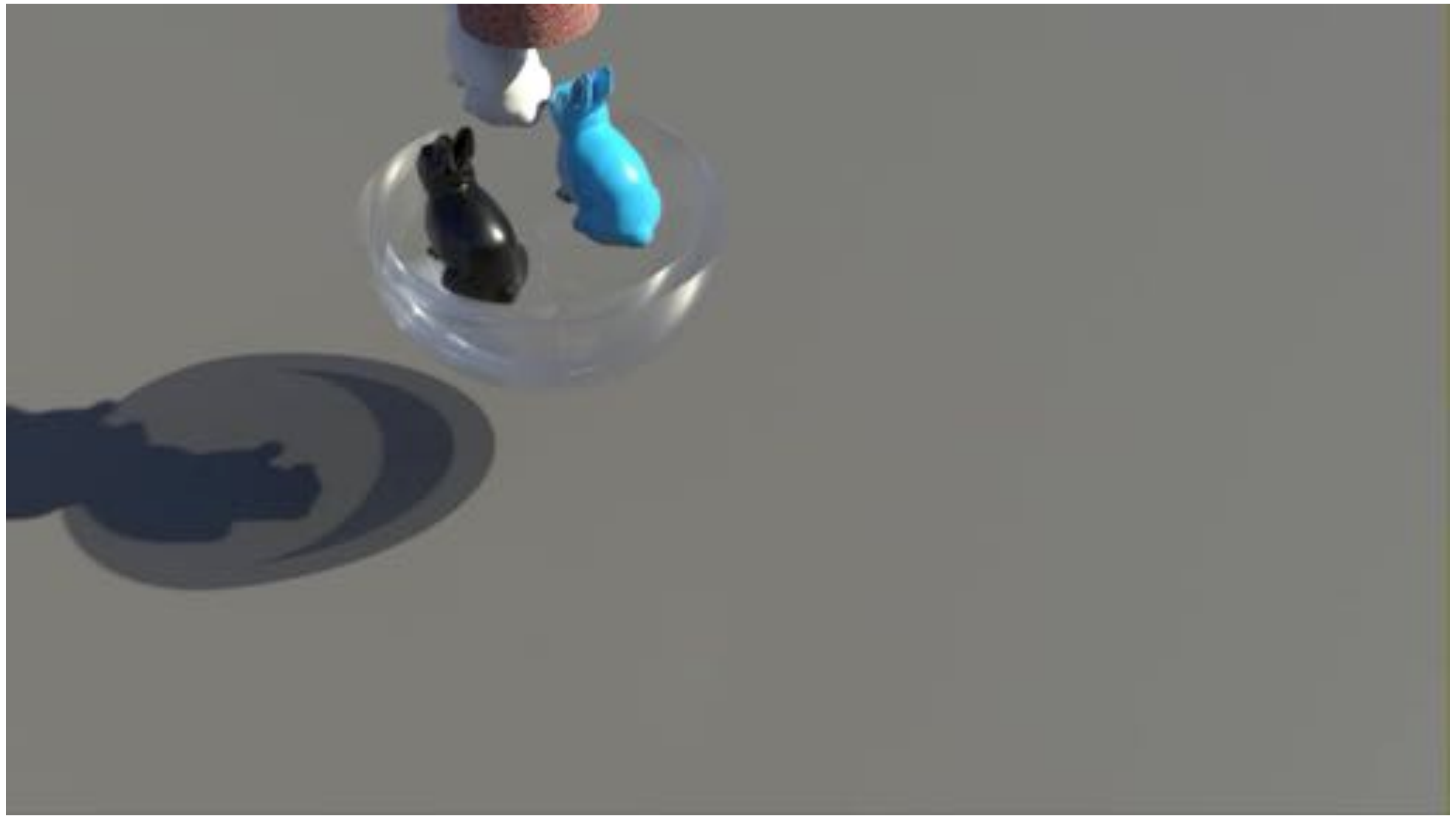


MPM

Our Method

Coupling with granular Material

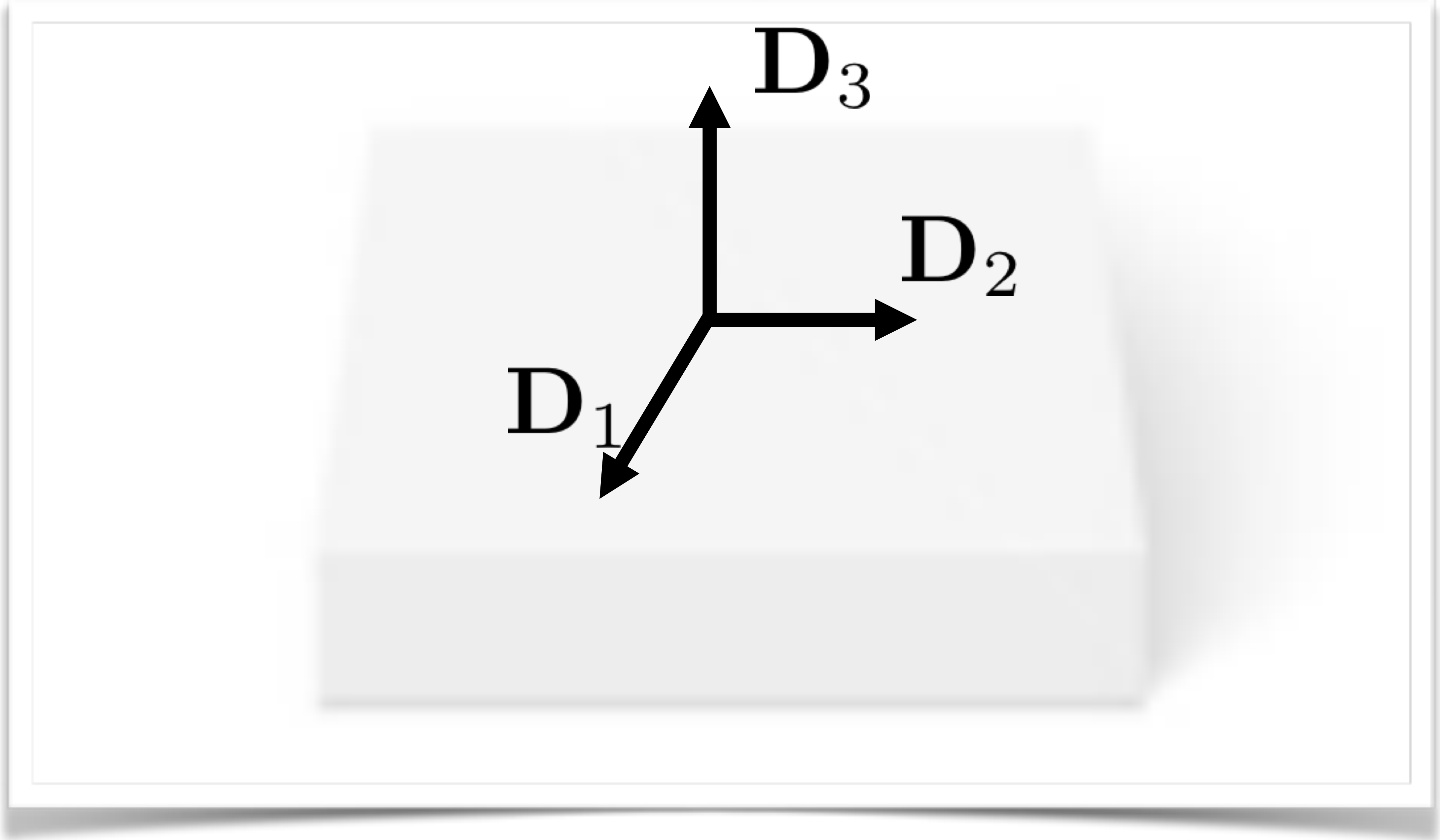
MPM (186s/frame)



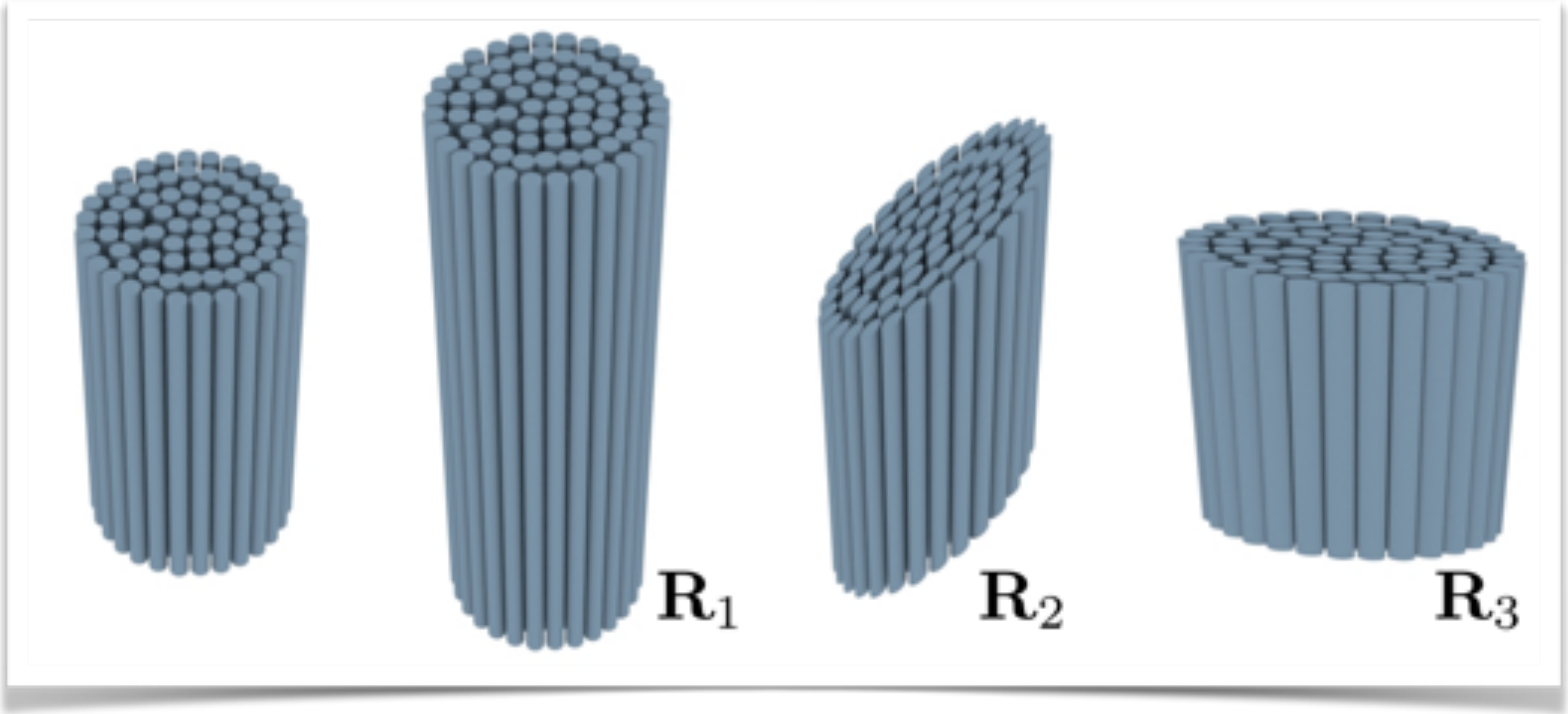
Our Method (66s/frame)


220s/frame
MPM
73s/frame
Our Method

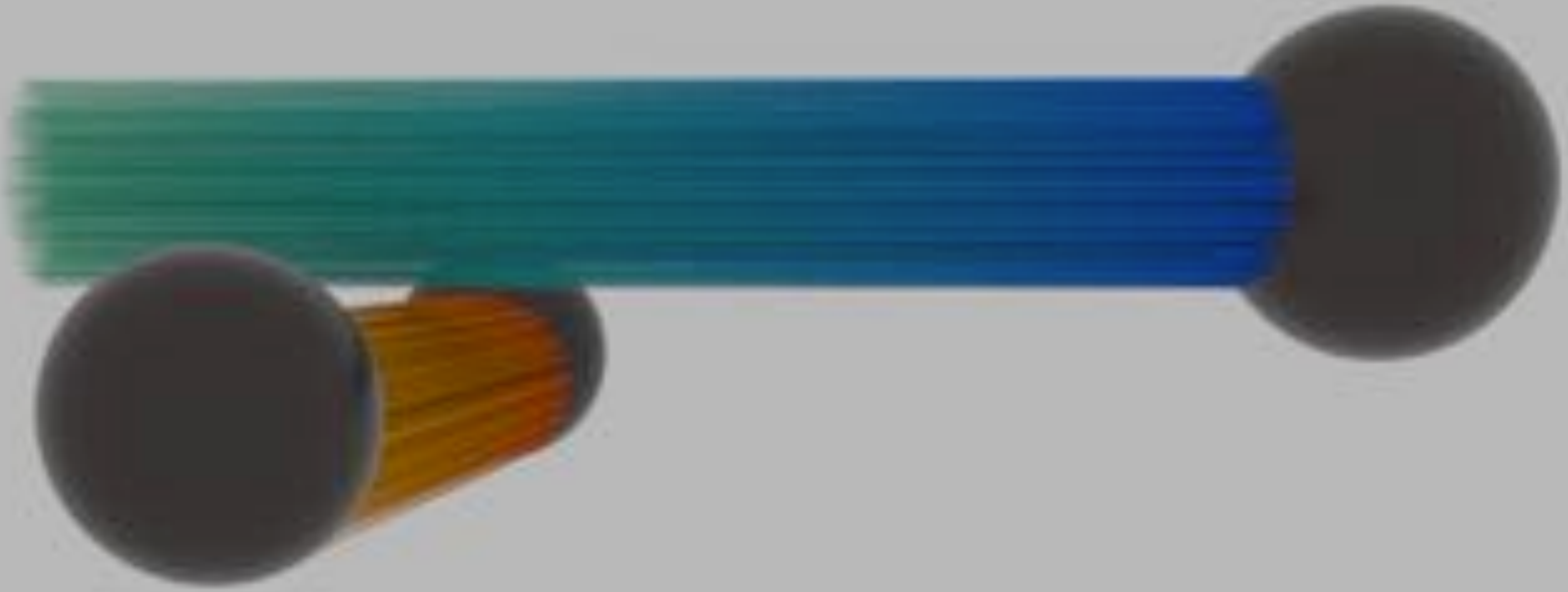
Strands



Surface



Curve



Strands

Jiang et al. (2017)

Our Method

Strands: Method (Continuous)

- Decompose motion into $\boldsymbol{\phi} = \boldsymbol{\phi}^d \circ \boldsymbol{\phi}^s$
- $\mathbf{F} = \mathbf{F}^d \mathbf{F}^s$ and $\mathbf{F}^d = \mathbf{F}^{d,E} \mathbf{F}^{d,P}$
- Strand energy: $\Psi = \Psi^s(\mathbf{F}^{d,E}) + \Psi^{DER}(\mathbf{F}^s)$
- $\Psi^{DER}(\mathbf{F}^s)$ consists of stretching, twisting, and bending potentials, see [Bergou et al. 2010]
- $\Psi^s(\mathbf{F}^{d,E})$ is the St.Venant-Kirchhoff Hencky energy, chosen for the ease of plasticity return mapping

Strands: Method (Discrete)

- Strand energy: $\Psi = \Psi^{S}(\mathbf{F}^{d,E}) + \Psi^{DER}(\mathbf{F}^{S})$

- $\Psi^{DER}(\mathbf{F}^{S})$ Lagrangian

$$\mathbf{v}_p^* = \mathbf{v}_p^n + \Delta t \frac{\mathbf{f}_p^{DER}}{m_p}$$

- $\Psi^{S}(\mathbf{F}^{d,E})$ MPM

$$\mathbf{v}_{\mathbf{i}}^{\star} = \mathbf{v}_{\mathbf{i}}^{*} - \frac{dt}{m_{\mathbf{i}}^{n}} \sum_{p} \frac{\partial \psi^{S}}{\partial \mathbf{F}^{E}} (\tilde{\mathbf{F}}_p^E(\tilde{\mathbf{x}}^{n+\alpha}))(\mathbf{F}_p^{E,n})^T \nabla w_{\mathbf{i}p}^n V_P^0 + \Delta t \mathbf{g}.$$

$$\mathbf{v}_p^{\star} = (1-\xi)\left(\mathbf{v}_p^* + \sum_{\mathbf{i}} \left(\mathbf{v}_{\mathbf{i}}^{\star} - \mathbf{v}_{\mathbf{i}}^{*}\right) w_{\mathbf{i}p}^n\right) + \xi \sum_{\mathbf{i}} \mathbf{v}_{\mathbf{i}}^{\star} w_{\mathbf{i}p}^n.$$

- Clean up with geometric collision algorithm

Similar to Bridson et al. [2002]

Strands Comparison

McAdams et al. (2009)

More than 500 thousand missed collisions

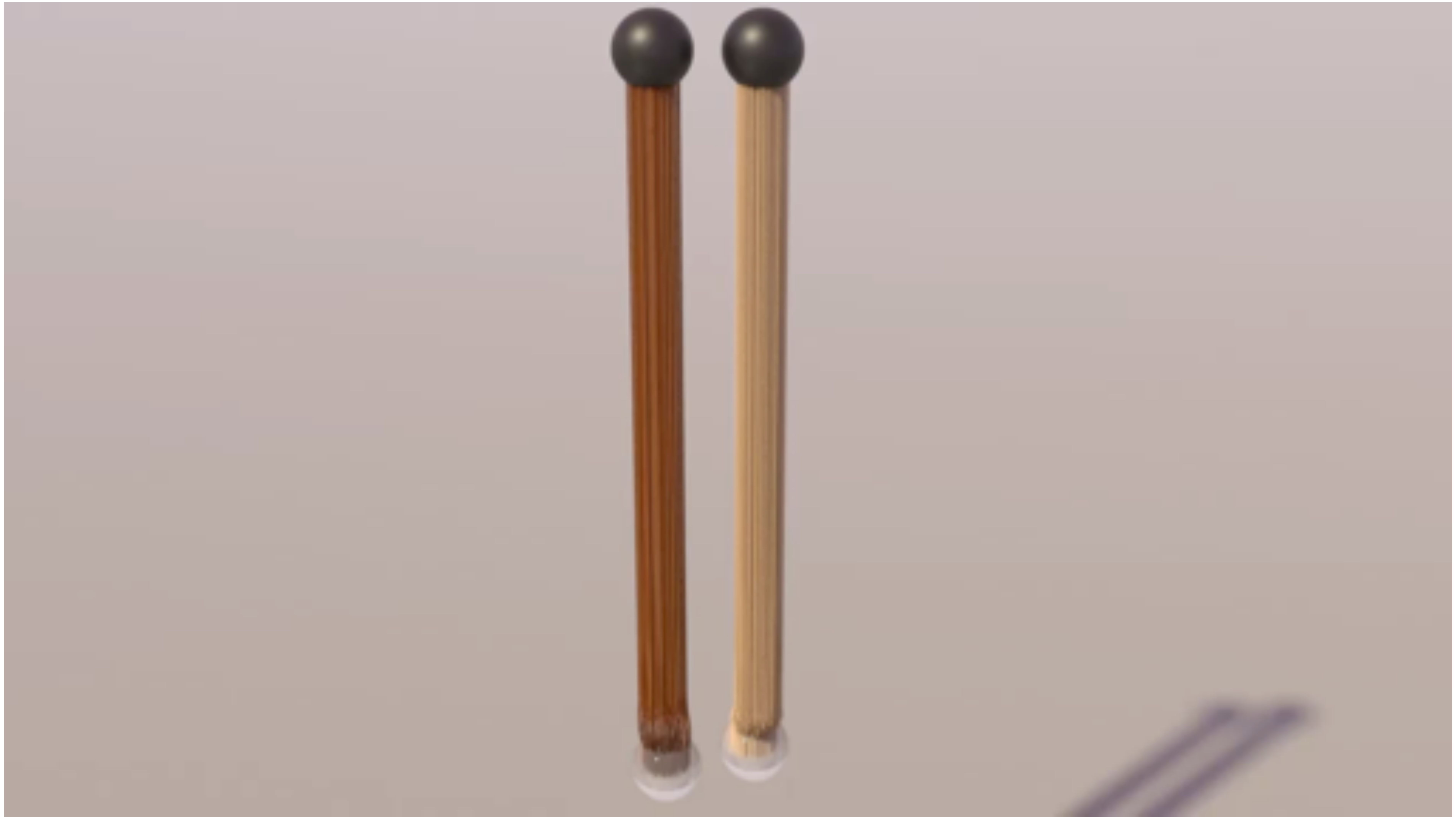
156 seconds/frame

Our Method

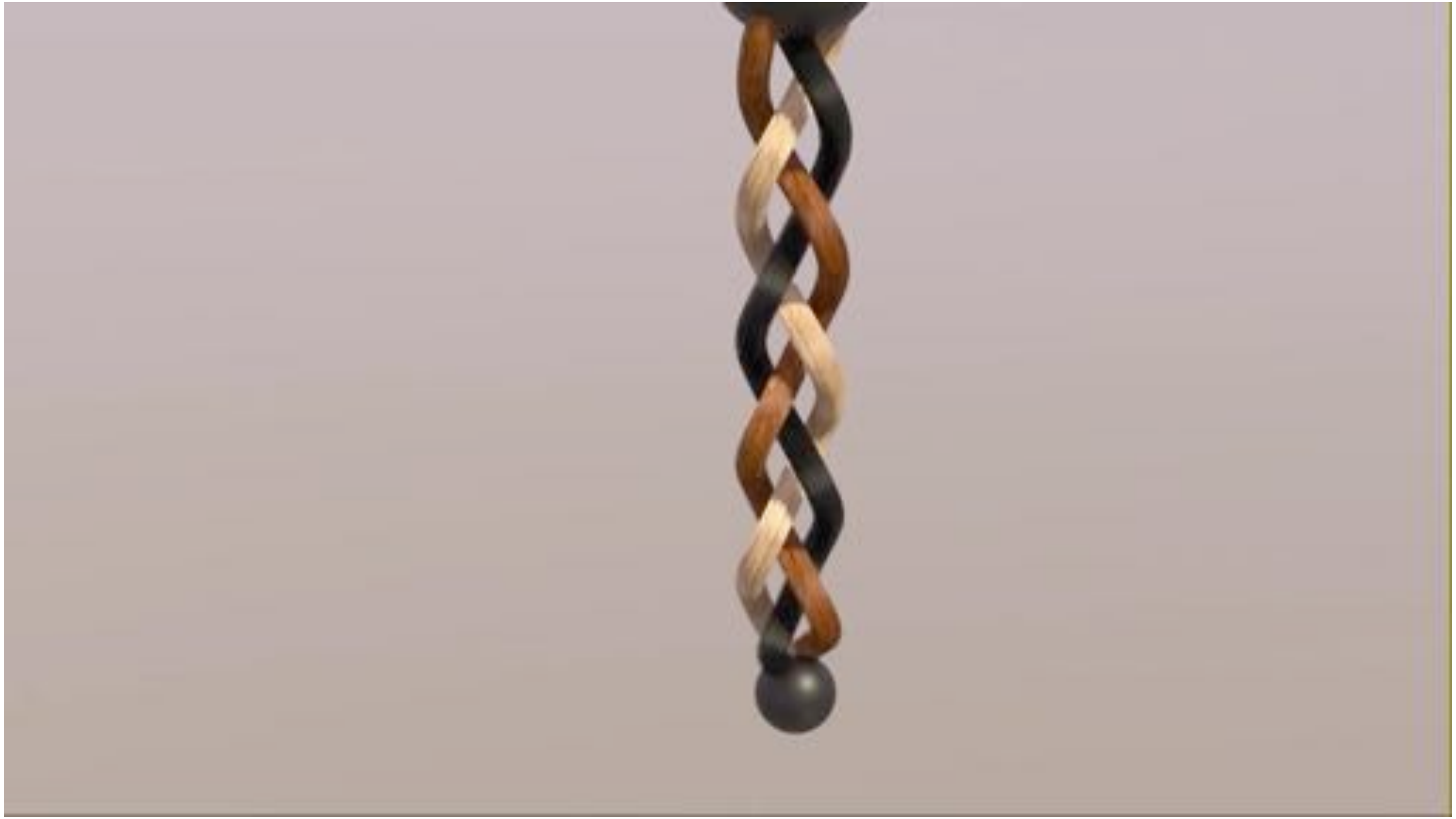
120 missed collisions

55 seconds/frame

Results



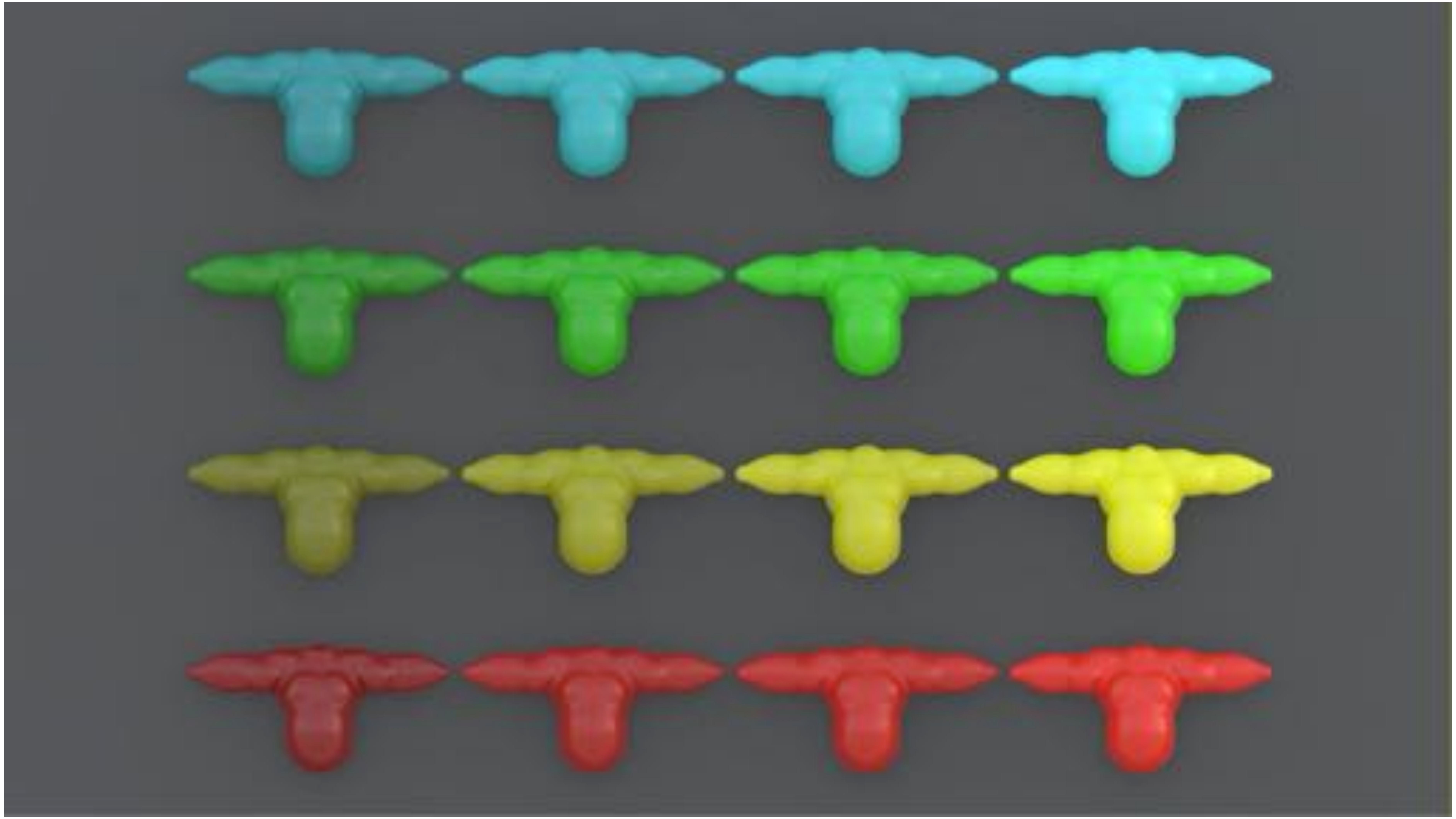
Results



Results

Results

Results



Results