Geometric Modeling from Flat Sheet Material

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KAUST
Aug. 27, 2020
GAMES Webinar

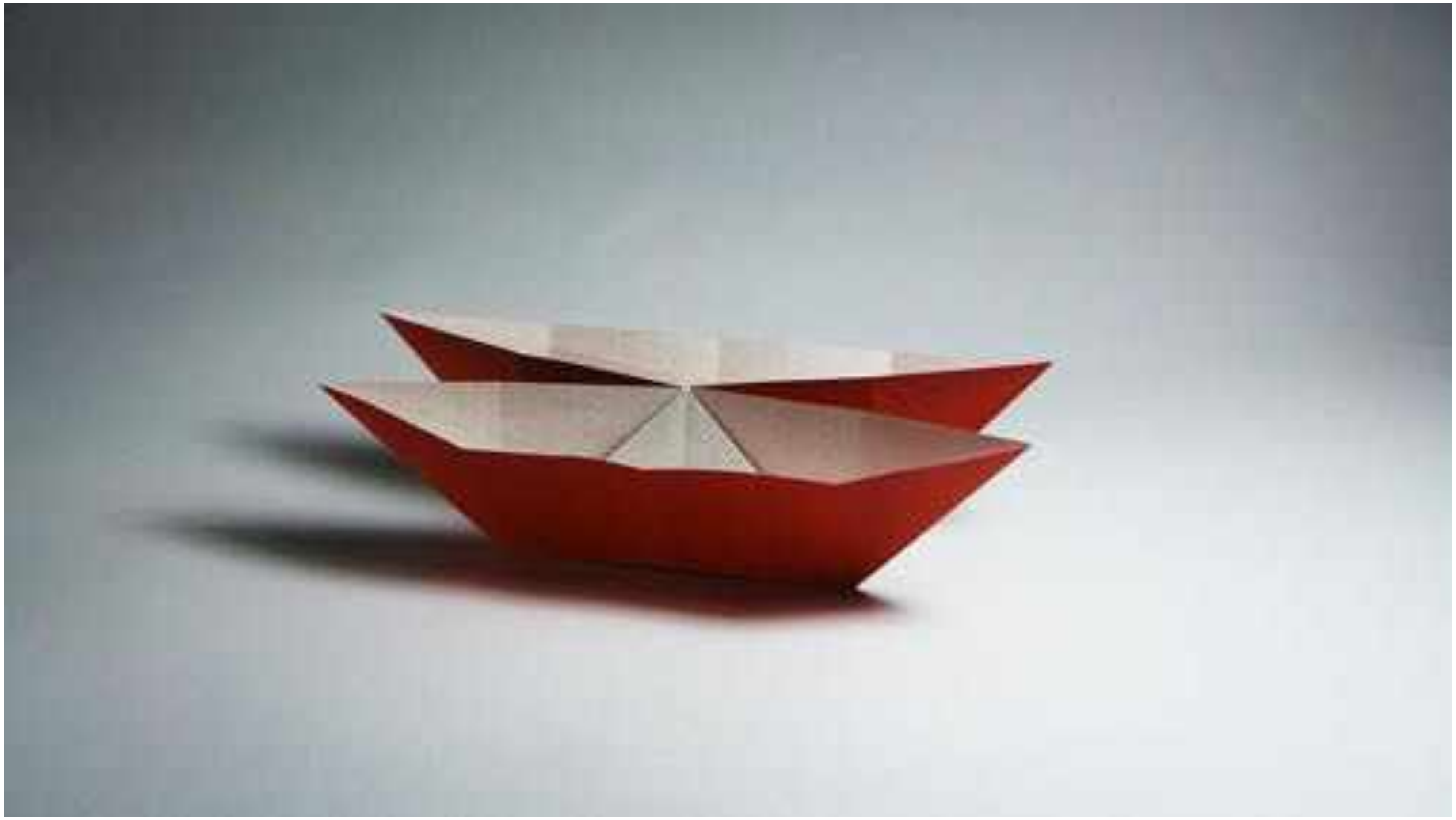
Outline

- Research background
- Curved-pleated structures (SIGGRAPH Asia 2019)
- Checkerboard patterns with Black Rectangles (SIGGRAPH Asia 2019)
- Quad-Mesh Based Isometric Mappings and Developable Surfaces (SIGGRAPH 2020)
- Freeform Quad-based Kirigami (SIGGRAPH Asia 2020)

Background

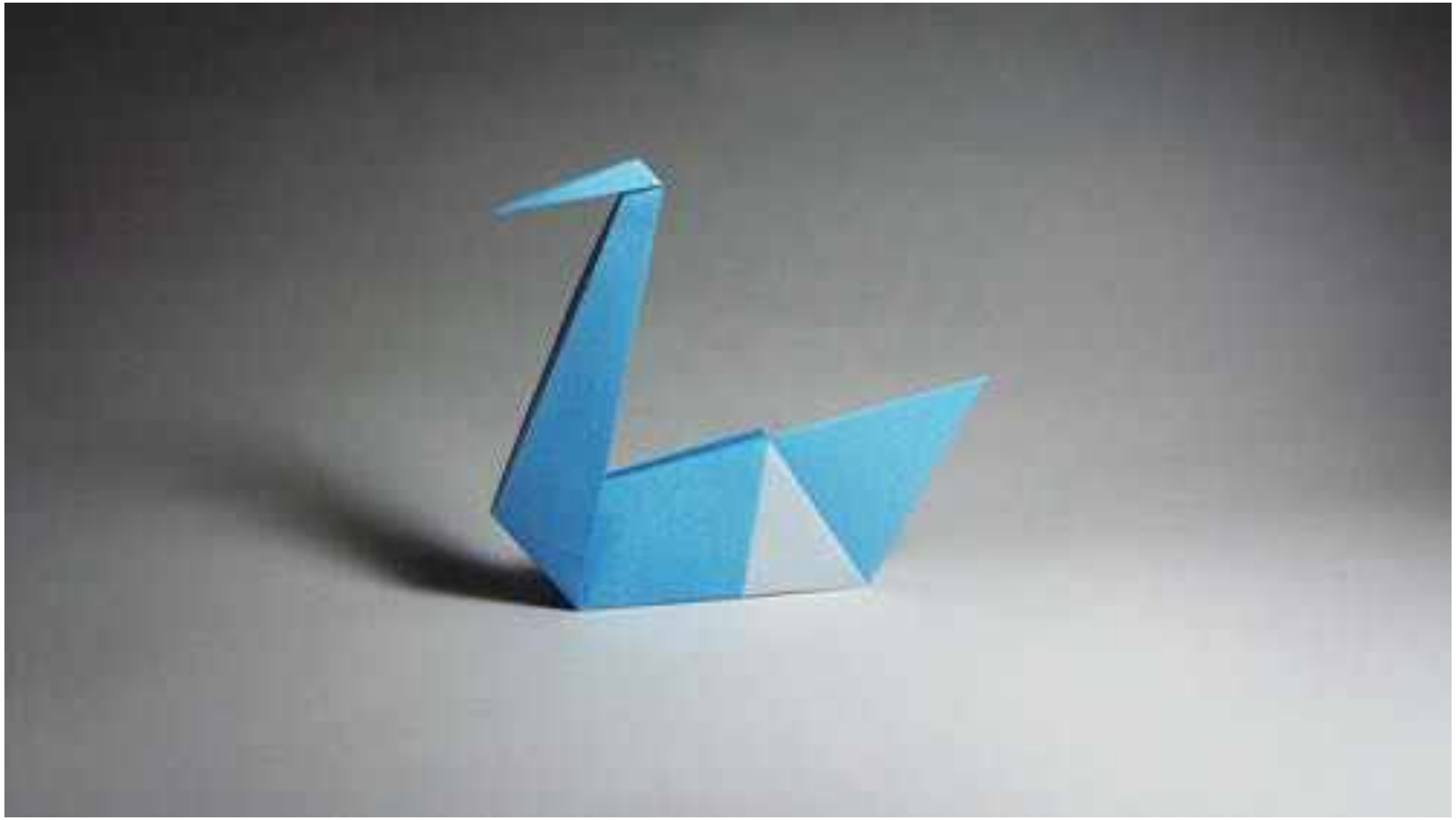
- Origami (折纸)
- Kirigami (剪纸)
- Developable surfaces（可展曲面）

Origami (折纸)



origami.me

Origami (折纸)



origami.me

Origami (折纸)

origami.me

Origami (折纸)

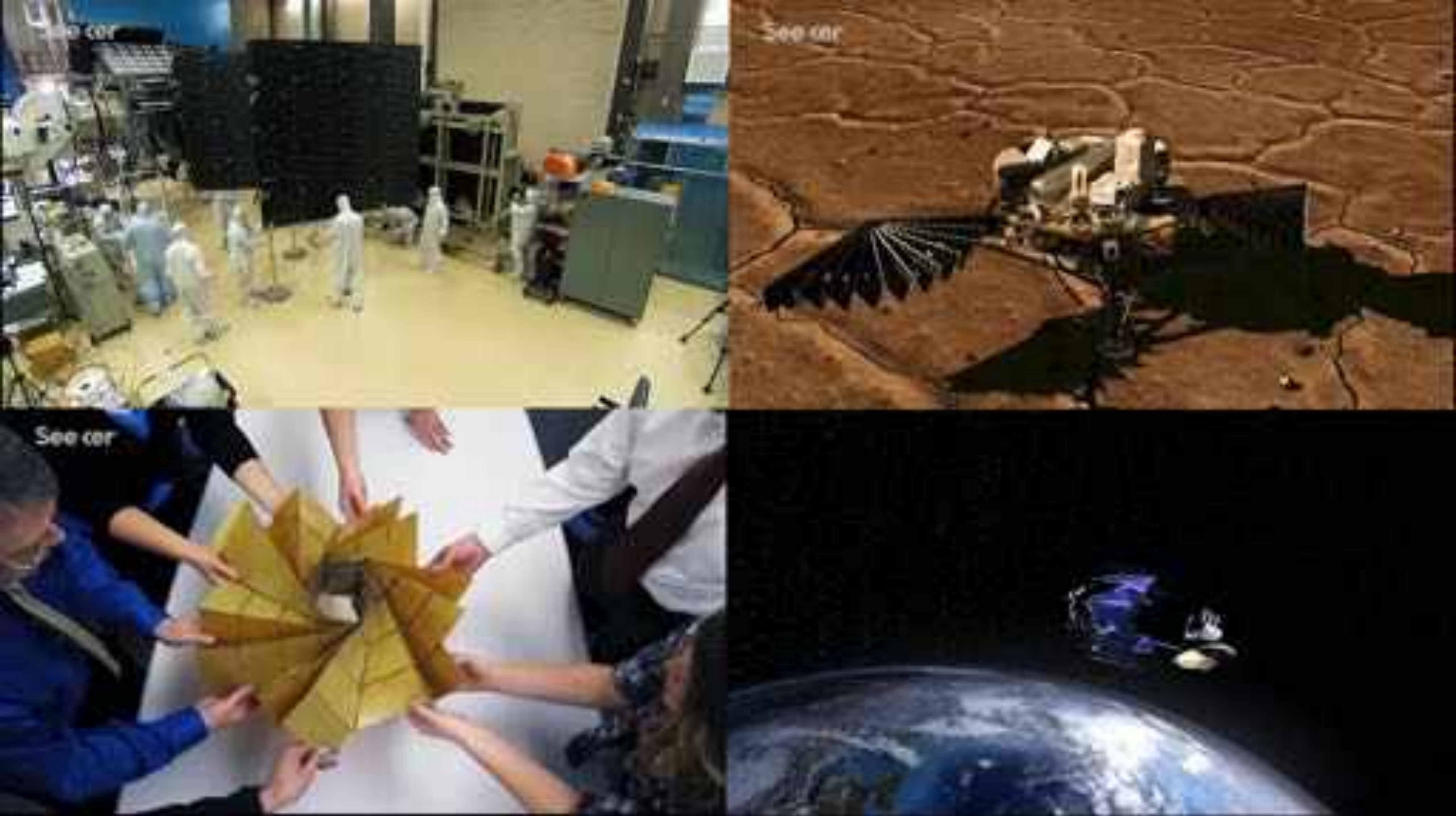
Designed by Shuki Kato

Designed by Jason Ku

Origami (折纸)

- An art as old as paper

From the first known book on origami, *Hiden senbazuru orikata*, published in Japan in 1797 (wikipedia)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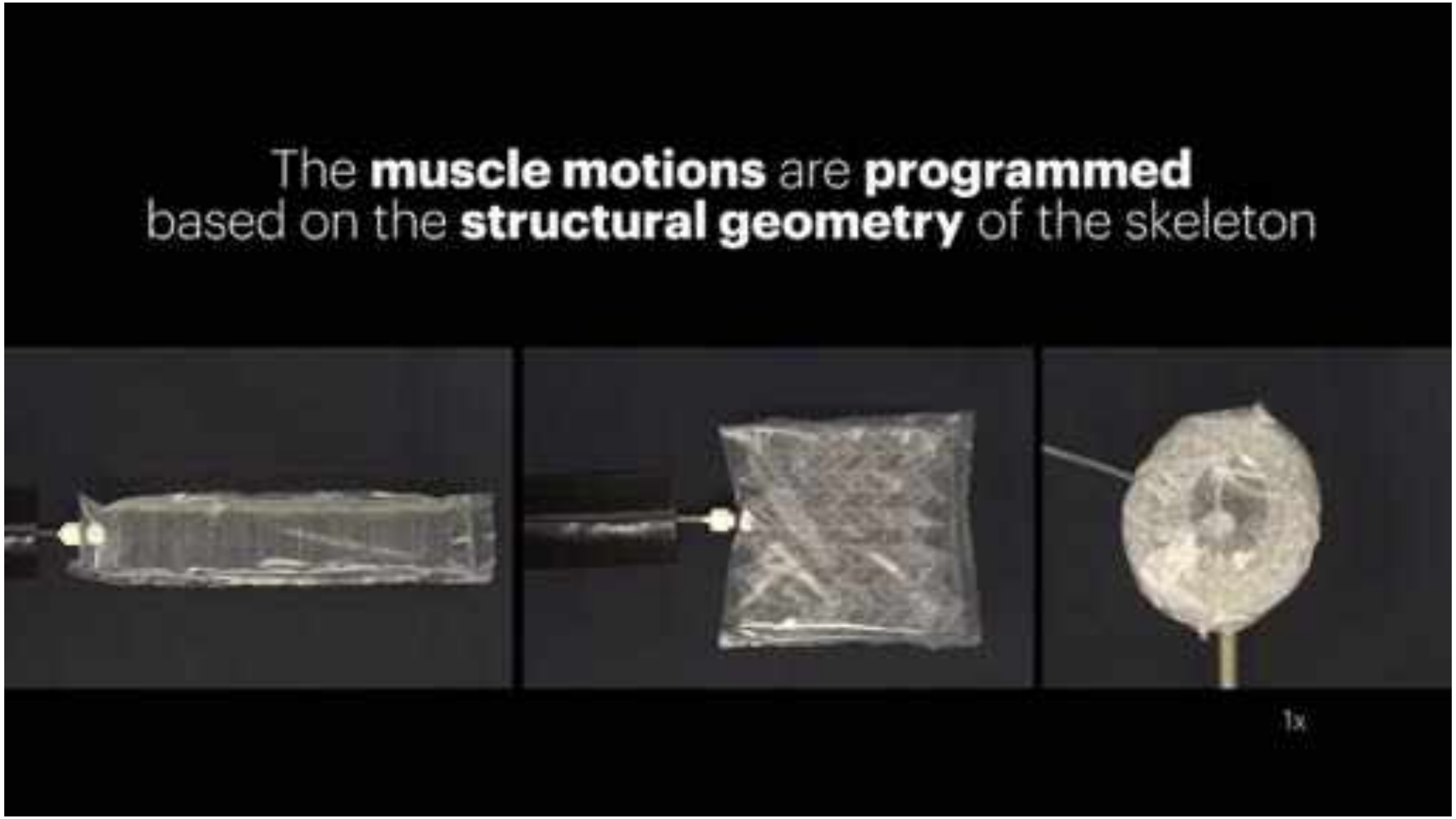
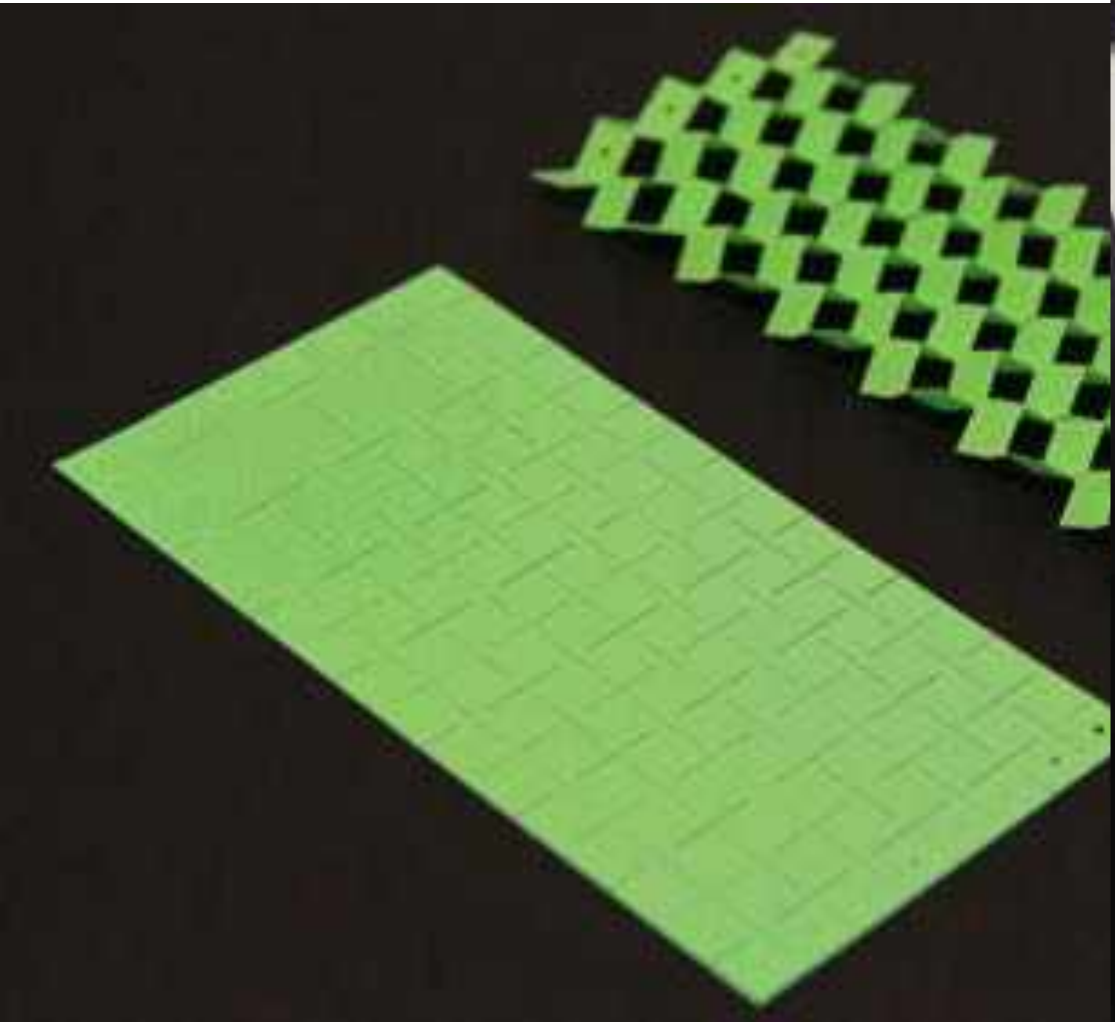
Origami



Credit: Wyss Institute at Harvard University

Kirigami(剪纸)

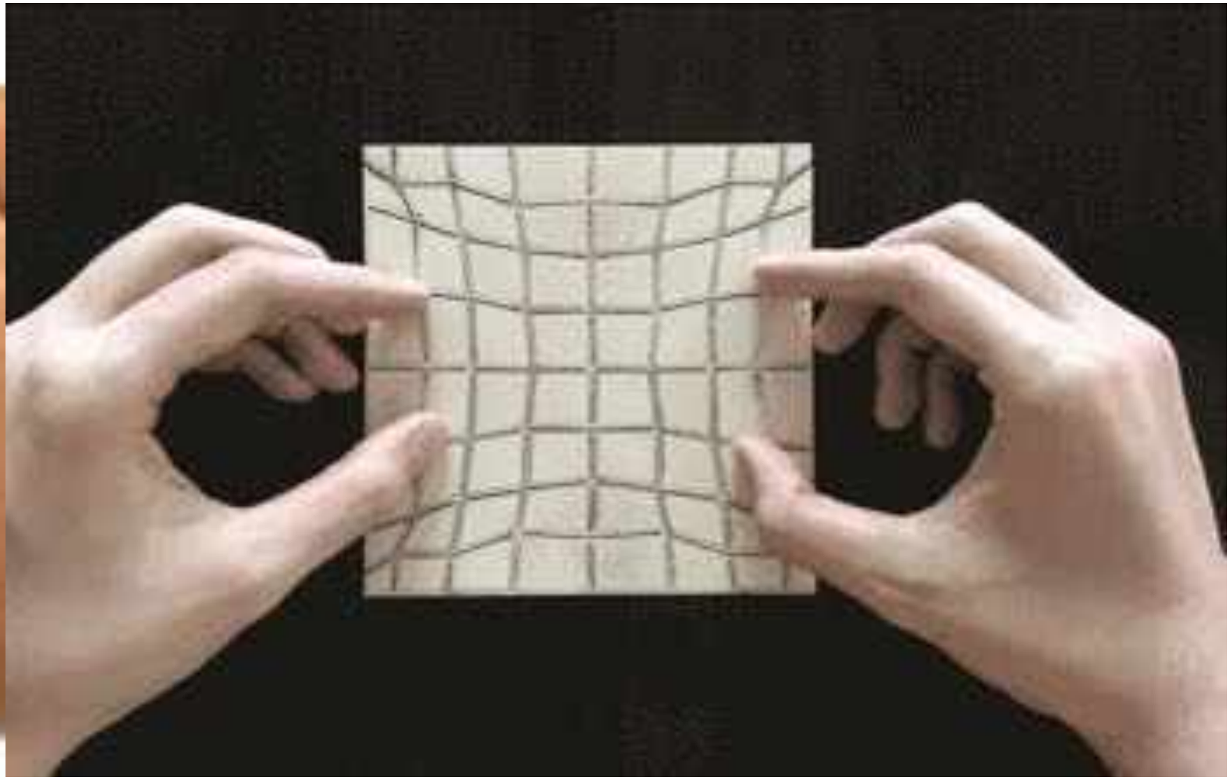


Credit: Ahmad Rafsanjani/Harvard SEAS

Credit: Paper Dandy

Kirigami(剪纸)

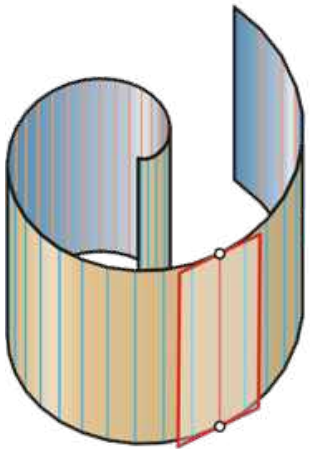
Credit: Ahmad Rafsanjani/Harvard SEAS



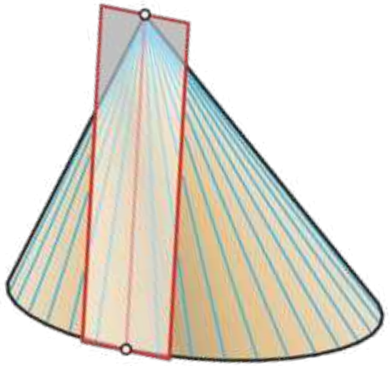
Credit: Gary P. T. Choi

Developable surfaces(可展曲面)

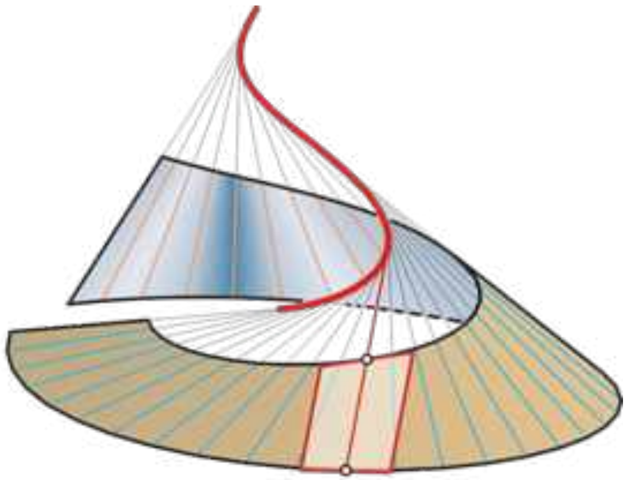
- smooth surface with zero Gaussian curvature.
- can be flattened onto a plane without distortion.



general cylinder



general cone



tangent surface

Developable surfaces(可展曲面)

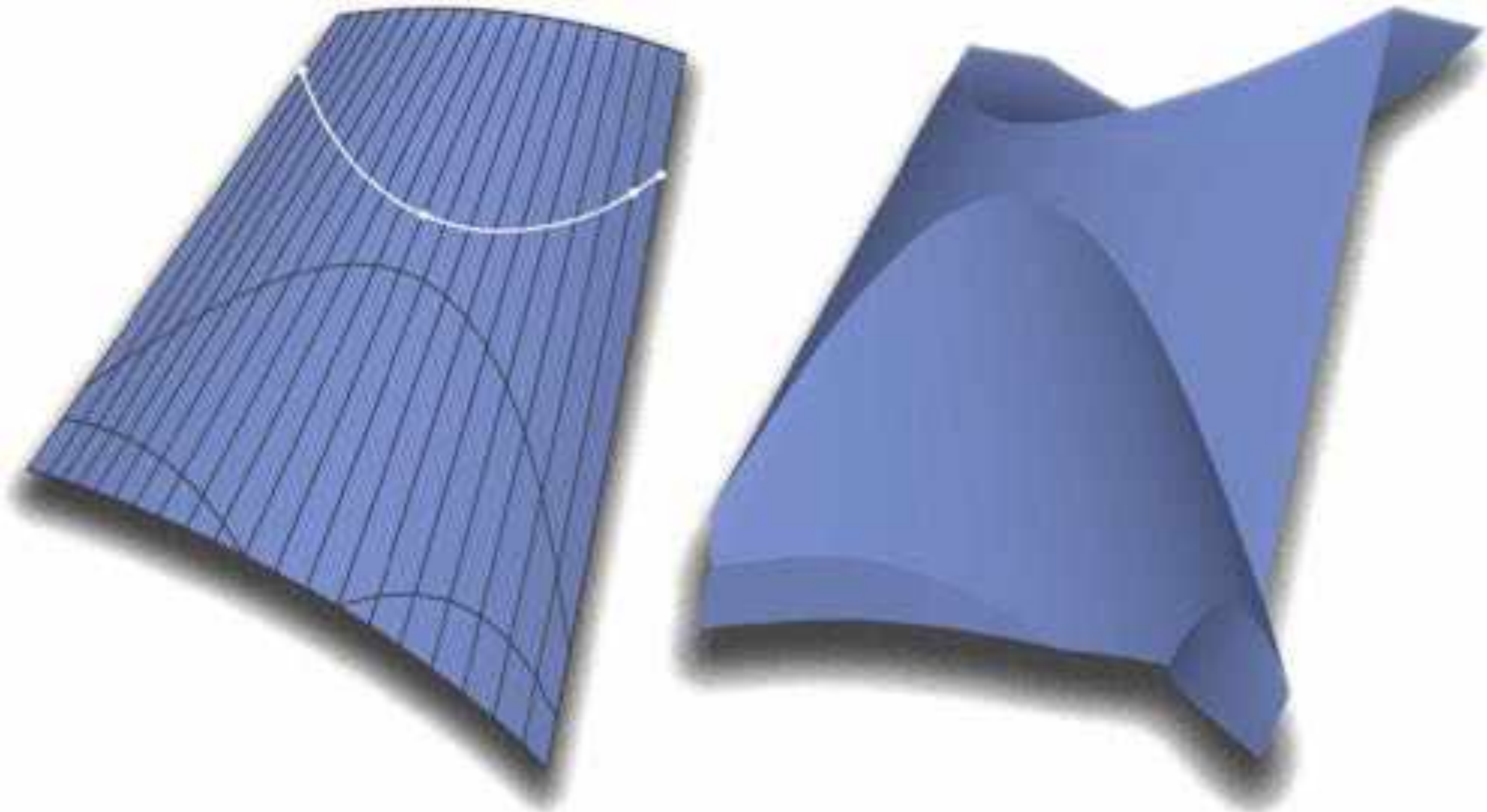
Frank Gehry, Guggenheim Museum Bilbao

Curved-pleated structures

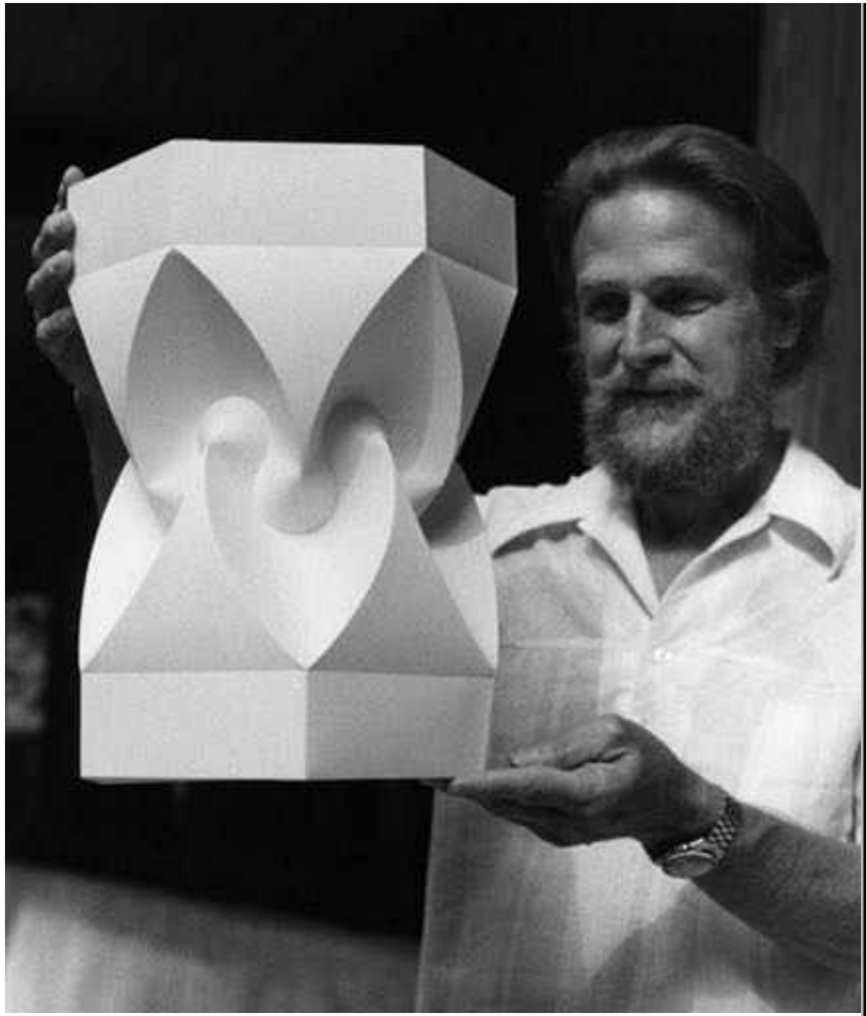
(SIGGRAPH Asia 2019)
with Klara Mundilova, Florian Rist, Johannes Wallner, Helmut Pottmann

Erik and Martin Demaine

What is a curved fold?



Previous work

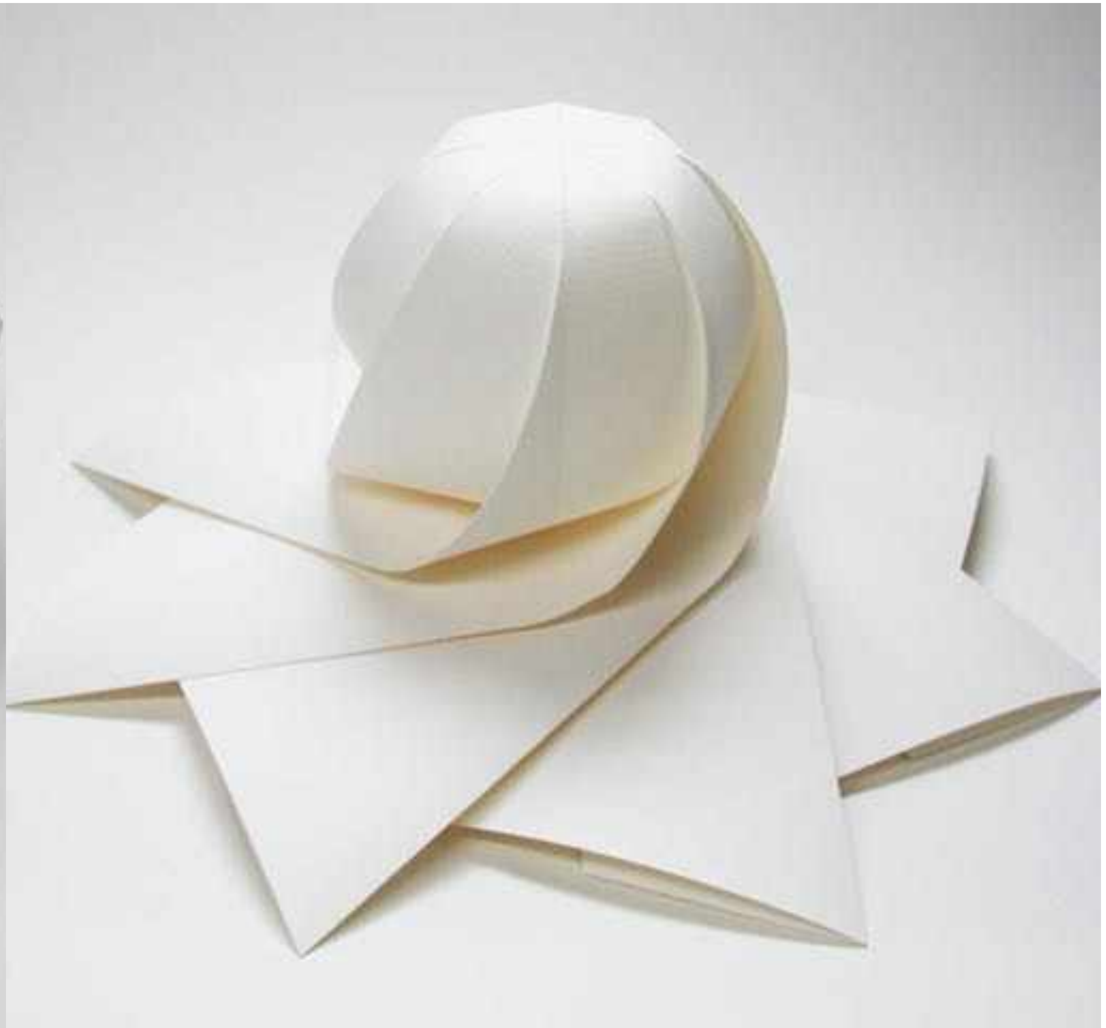


David Huffman

Previous work

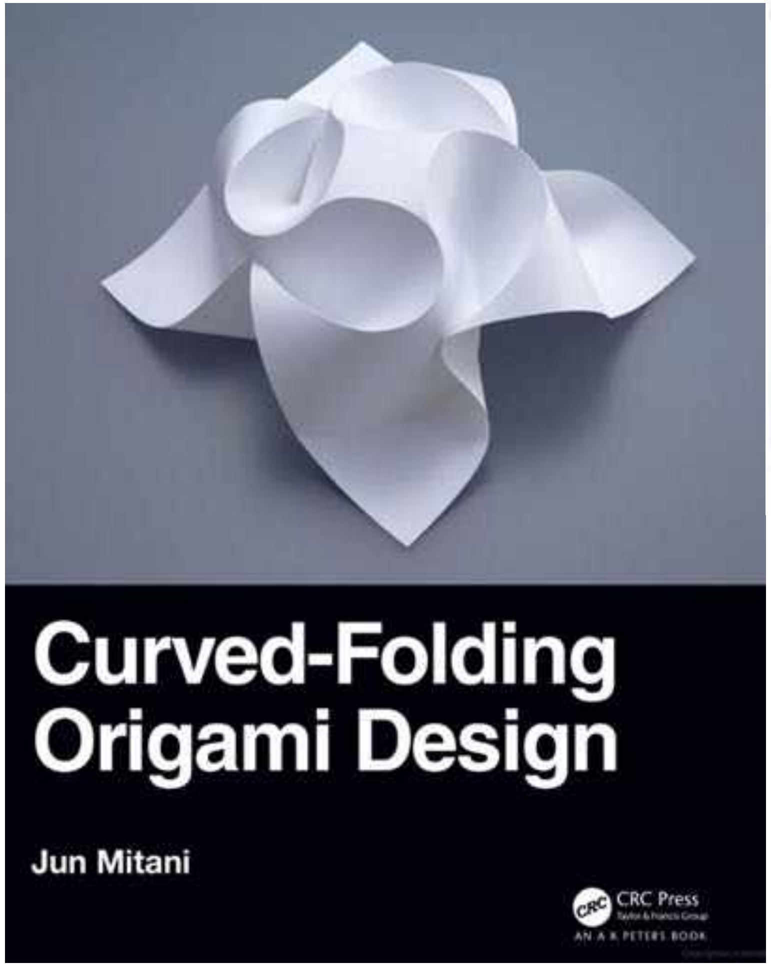
Demaine et al.

Previous work

Jun Mitani
三谷 純

Previous work



Jun Mitani
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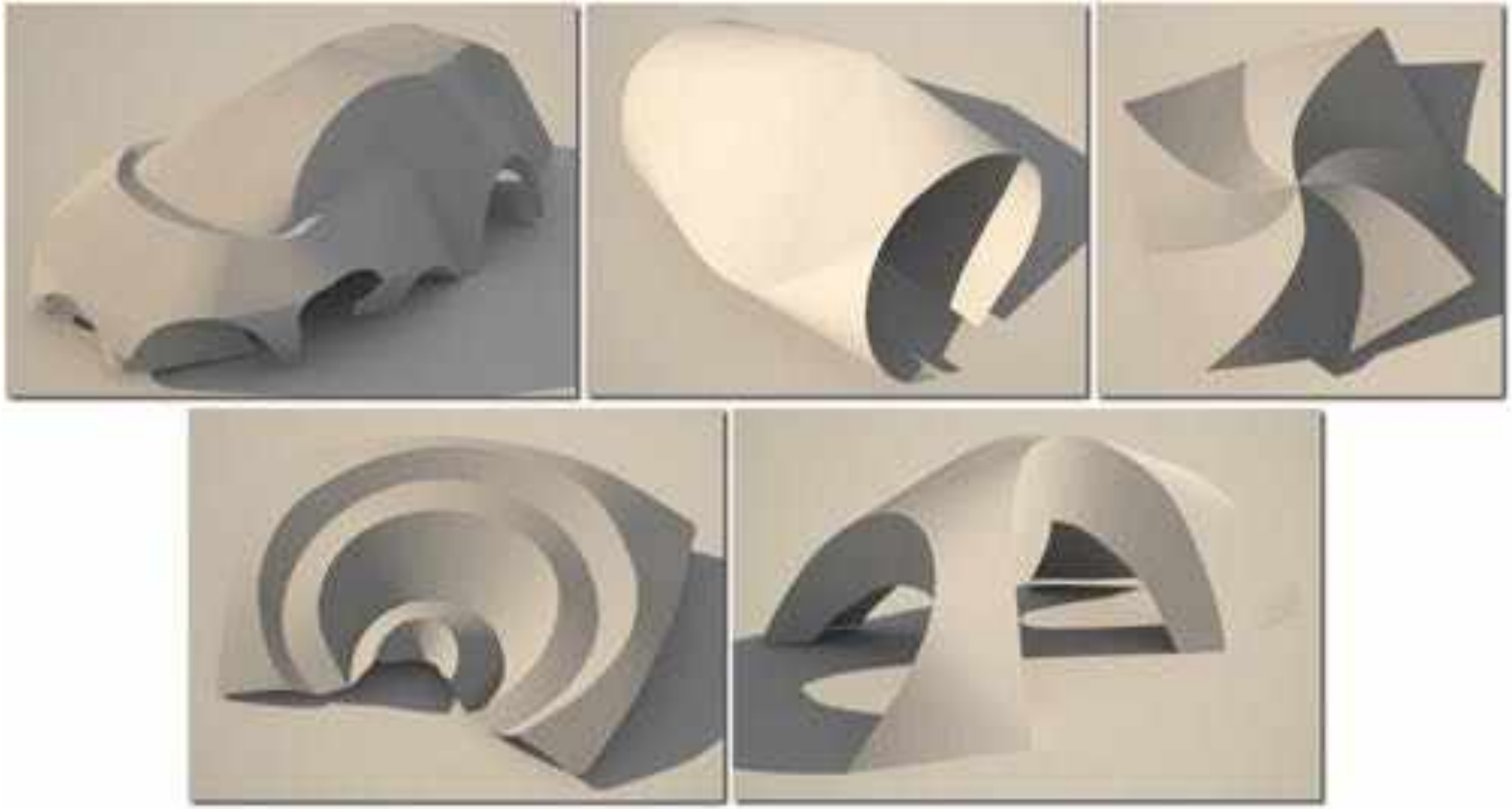


Kilian et al.
Siggraph 2008

Previous work

Rabinovich et al.

Face shied design

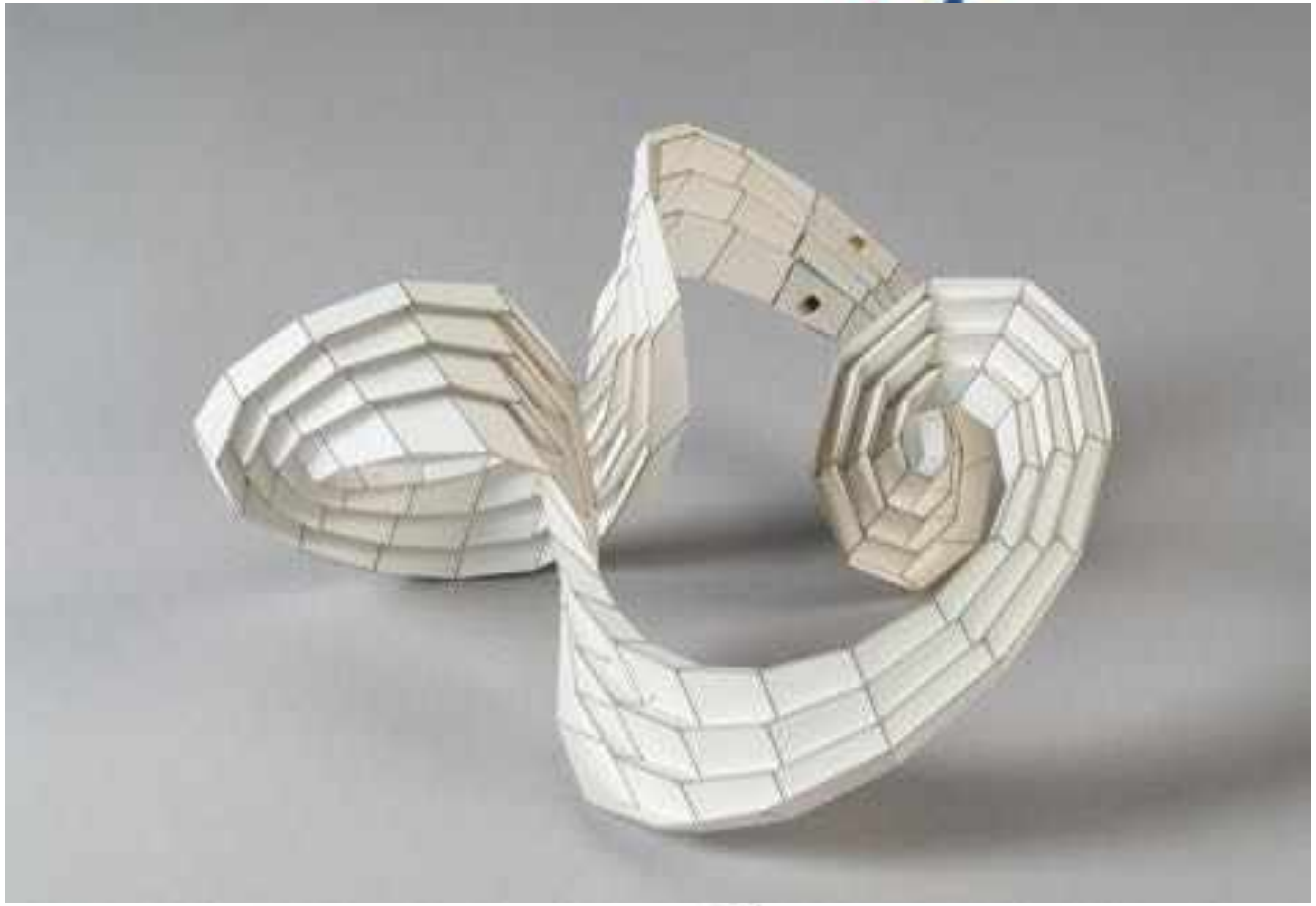


Designed by the University of Cambridge's Centre for Natural Material Innovation and University of Queensland's Folded Structures Lab

https://happyshield.github.io/en/

Our contributions

- Design of pleated structures
- Approximation of a given shape by a pleated structure
- Introduce principal pleated structures and a discrete model for them
- Design of flexible mechanisms in form of quad meshes



Geometry background

Meshes from planar quads

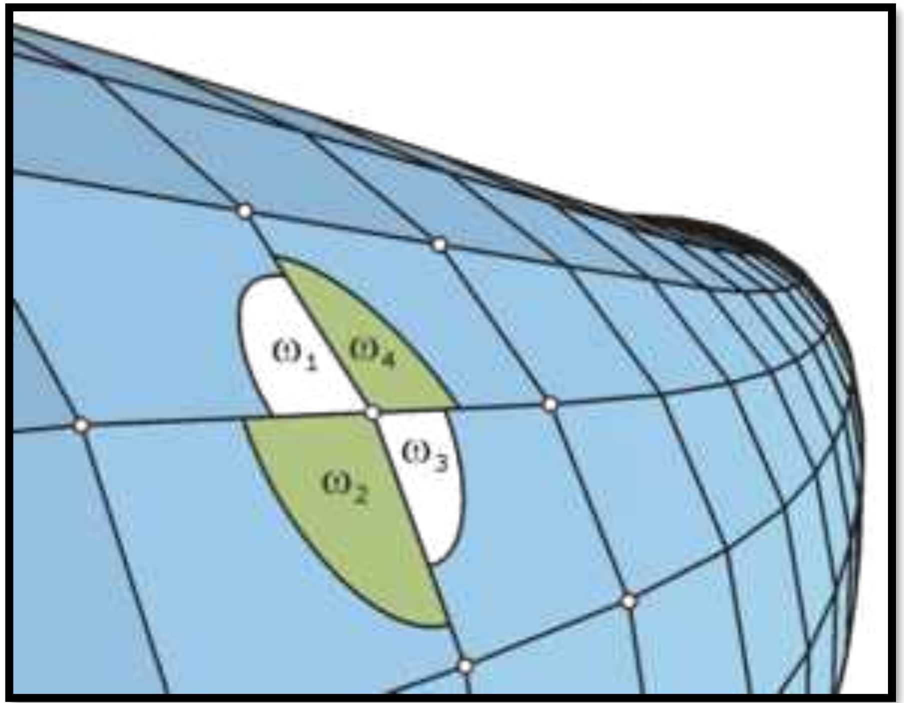
Chadstone Shopping Center, Melbourne: Global Architectural Practice Callison, aterlier one, Seele

- Application in architecture: structures from *flat quadrilateral panels*
- *PQ meshes*

Conical meshes

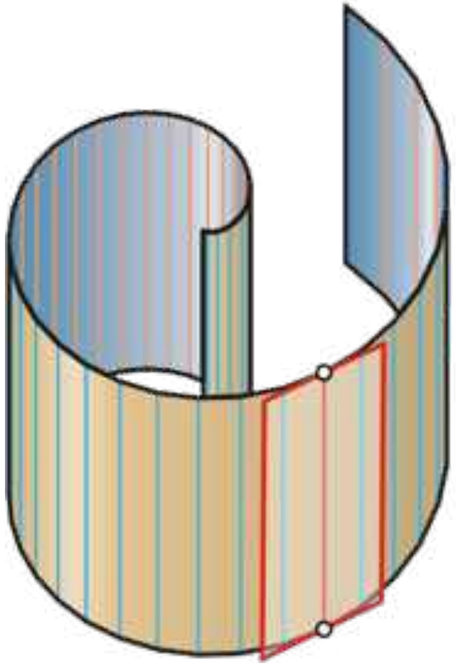
- PQ meshes with nearly rectangular panels follow principal curvature lines of a reference surface.
- One type of principal mesh: *conical mesh*
- PQ mesh is conical if at each vertex the incident face planes are tangent to a right circular cone
- Equal sum of opposite angles at each vertex

$$\omega_1 + \omega_3 = \omega_2 + \omega_4$$



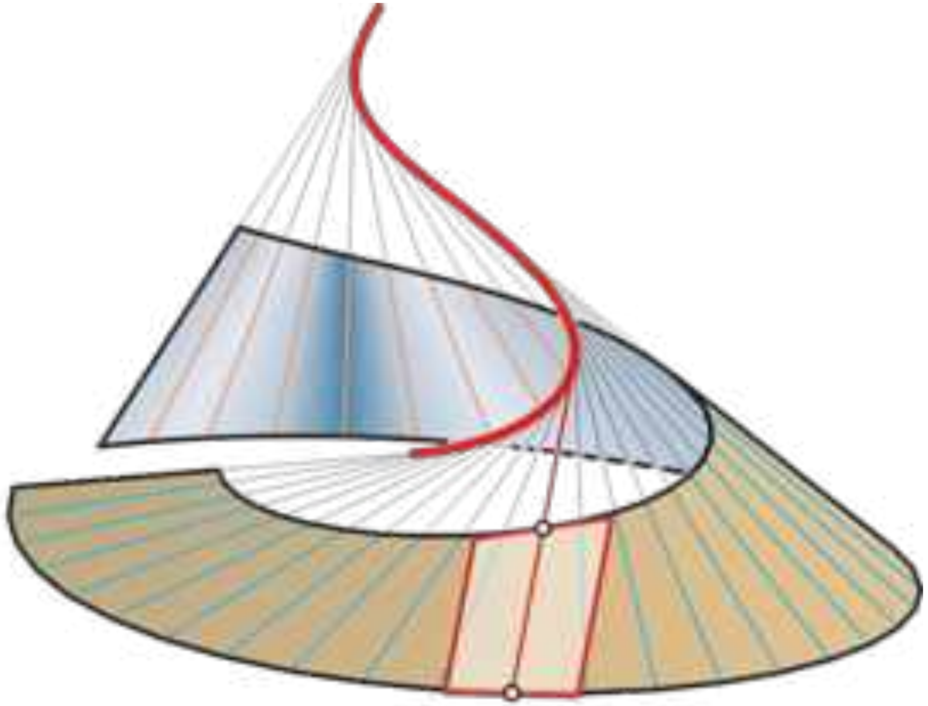
Developable surfaces

- Curved folded objects consist of smooth developable surfaces



general cylinder

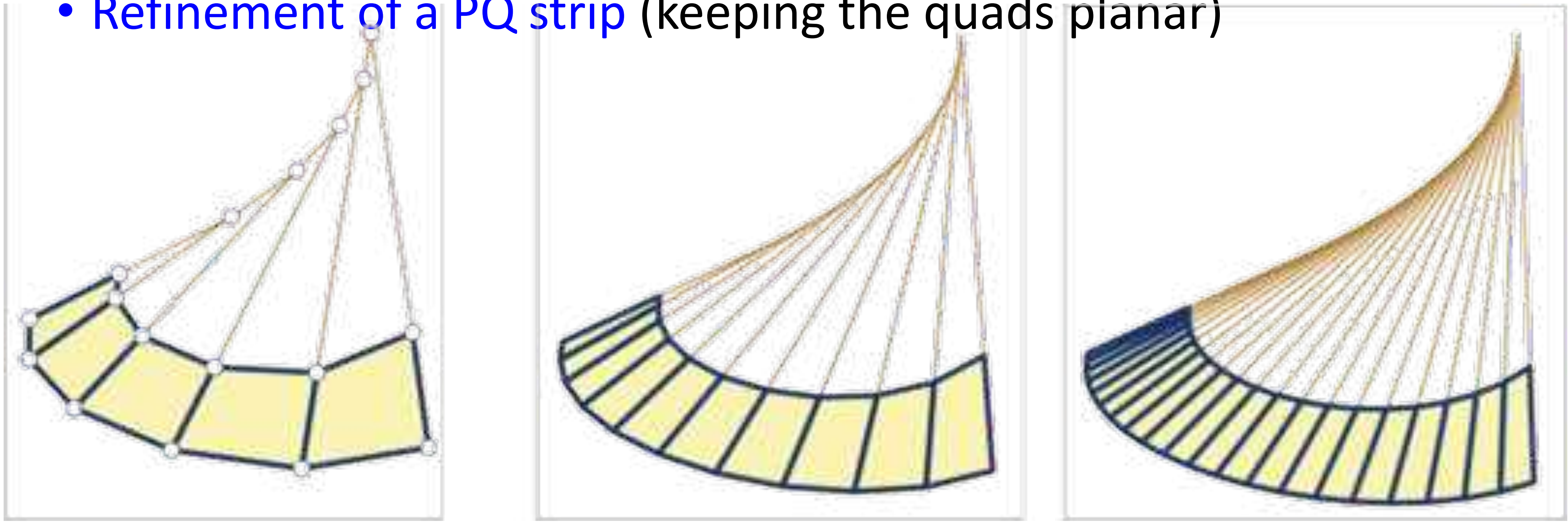
general cone



tangent surface

Discrete model

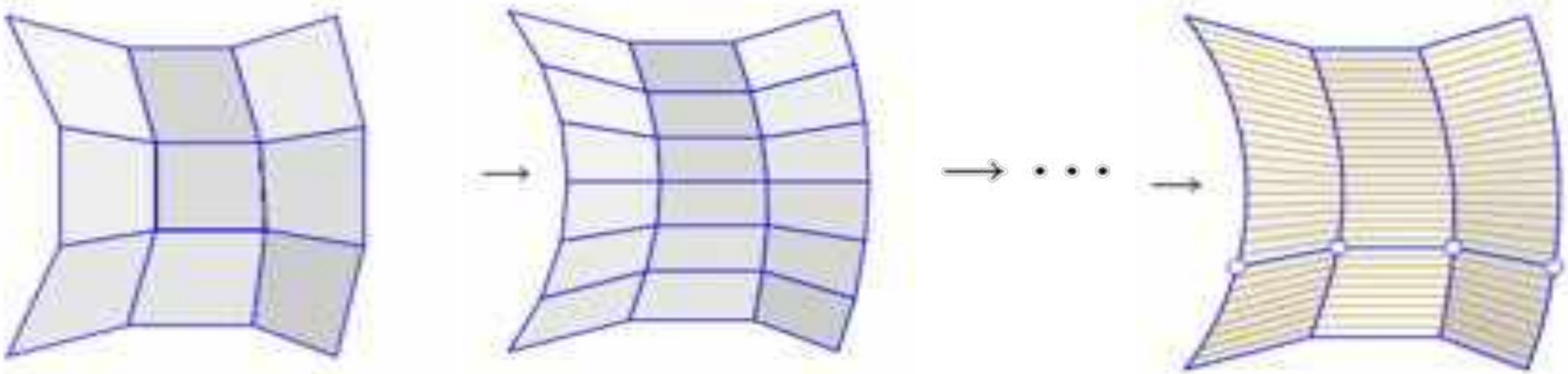
- Refinement of a PQ strip (keeping the quads planar)



Limit: developable surface strip

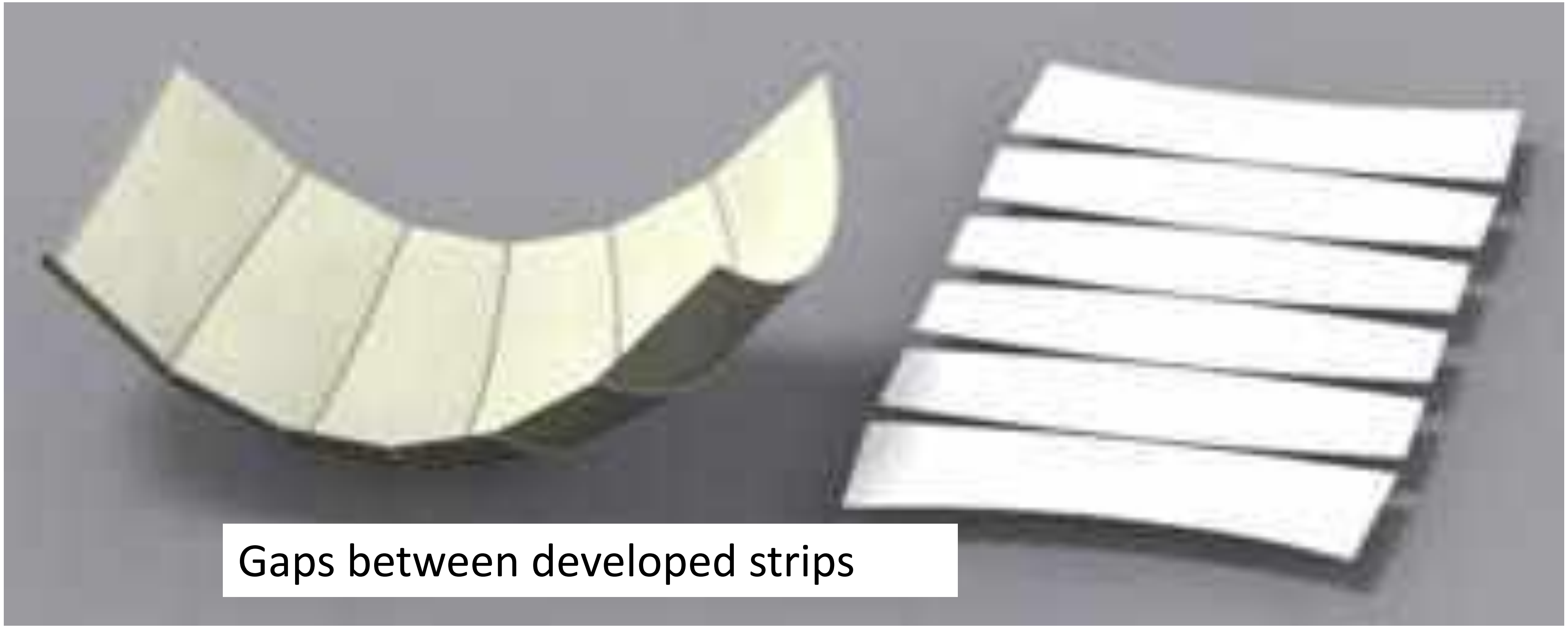
Developable strip models

- One-directional limit of a PQ mesh:



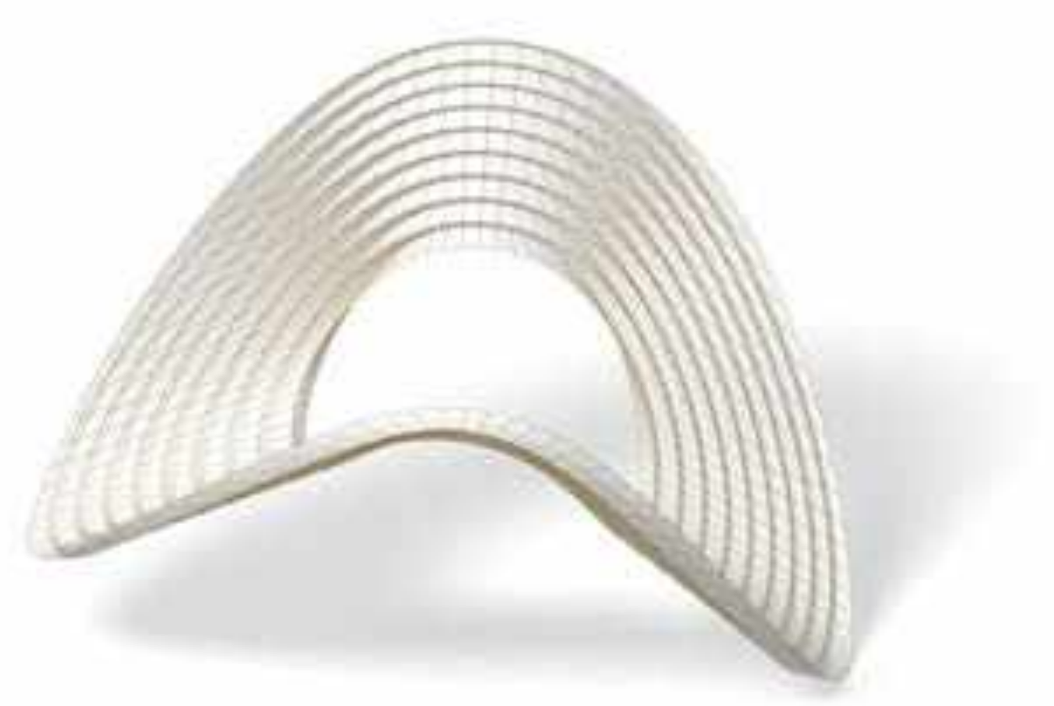
developable strip model

Planar unfolding of a developable strip model



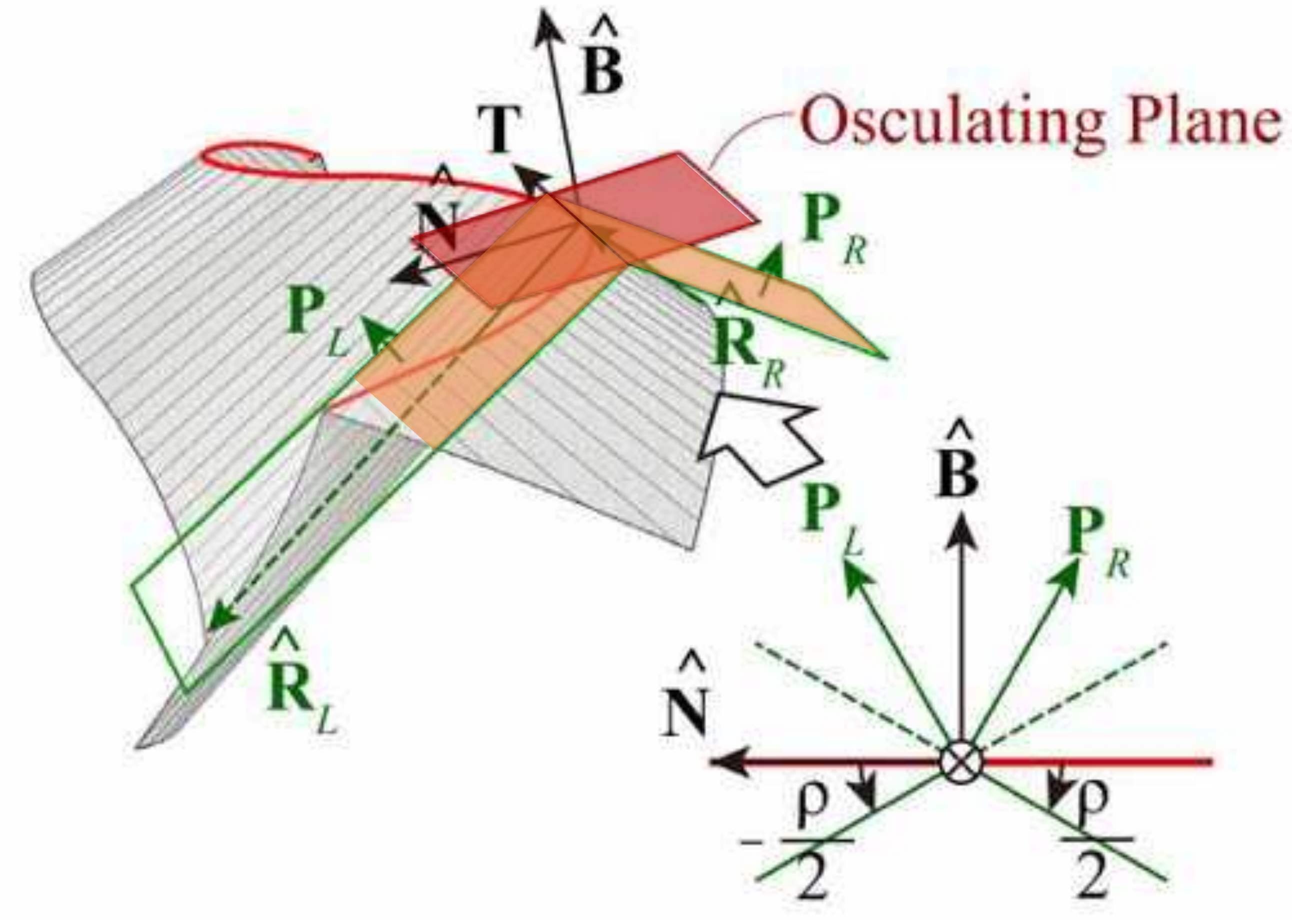
Gaps between developed strips

Unfolding of a pleated structure: no gaps



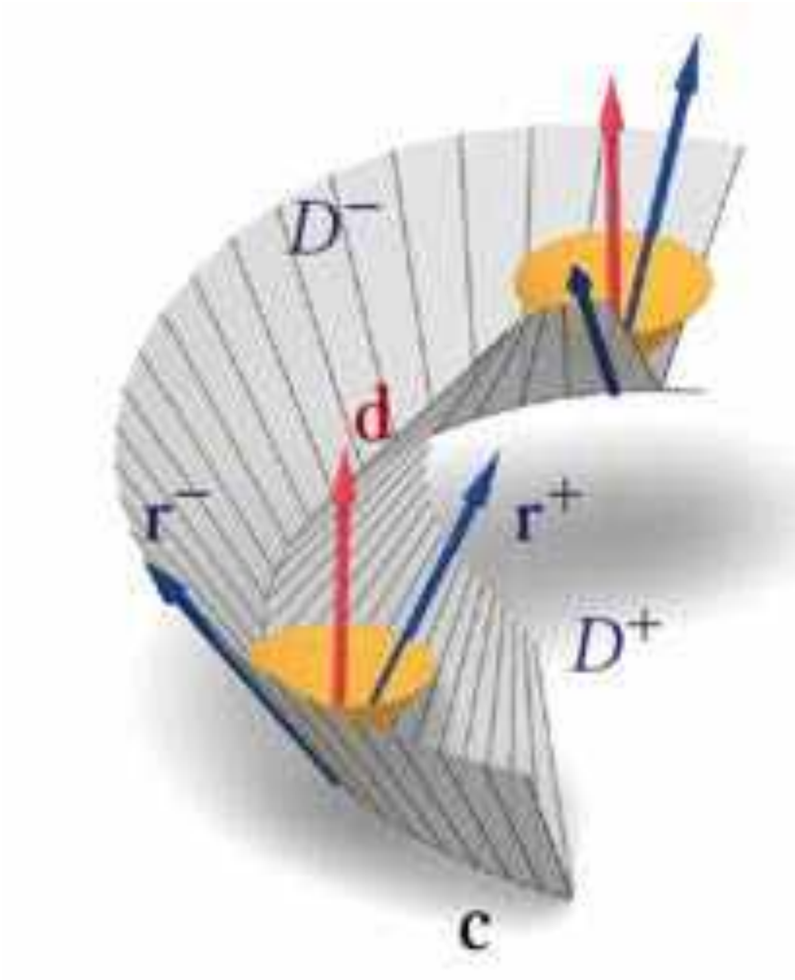
Geometry of curved folds

- Osculating plane of the crease curve bisects the tangent planes on either side.



Geometry of curved folds

- Constant fold angle along a crease:
  - rulings are symmetric with respect to the fold curve.
  - ruling preserving isometric mapping to the plane
- We call these structures principal pleated structures (PPLS)

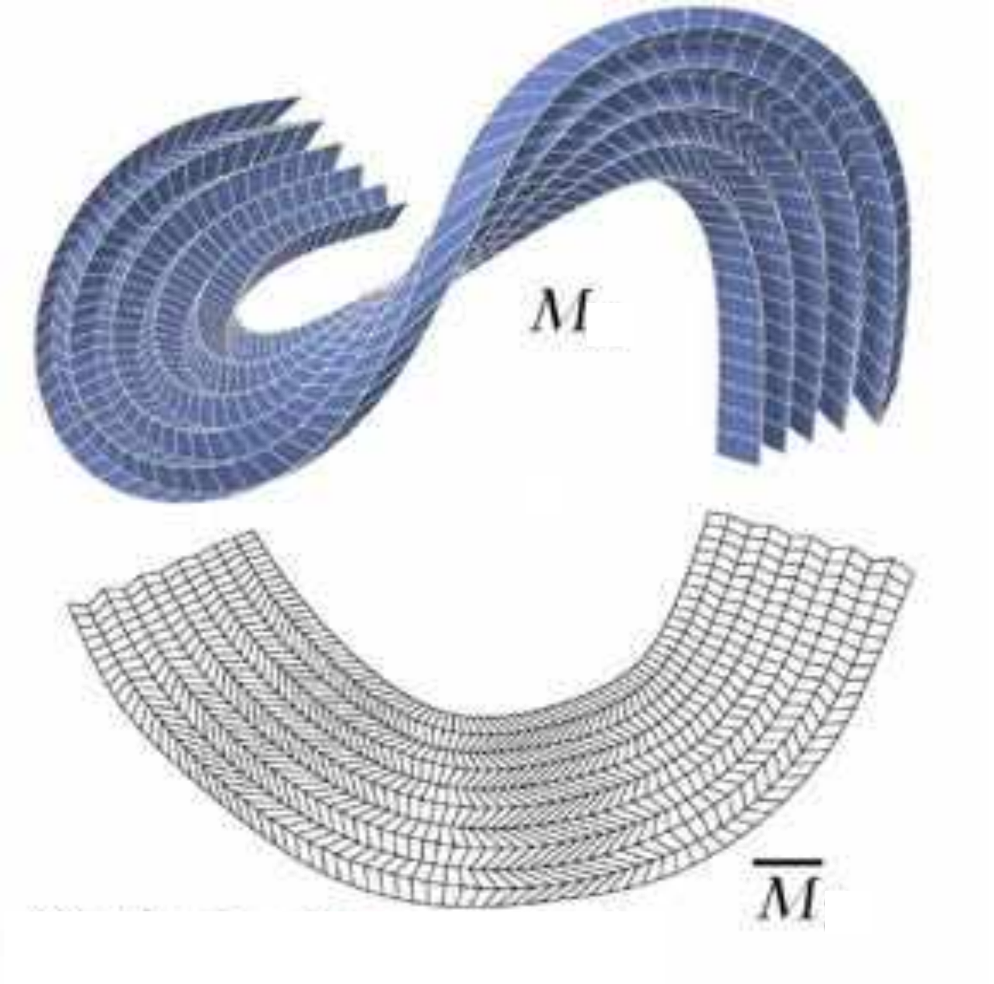


Discrete models of pleated structures

“Non-smooth” PQ mesh

- Discrete pleated structure: modeled with a PQ mesh that is isometric to a planar quad mesh.
- Developability

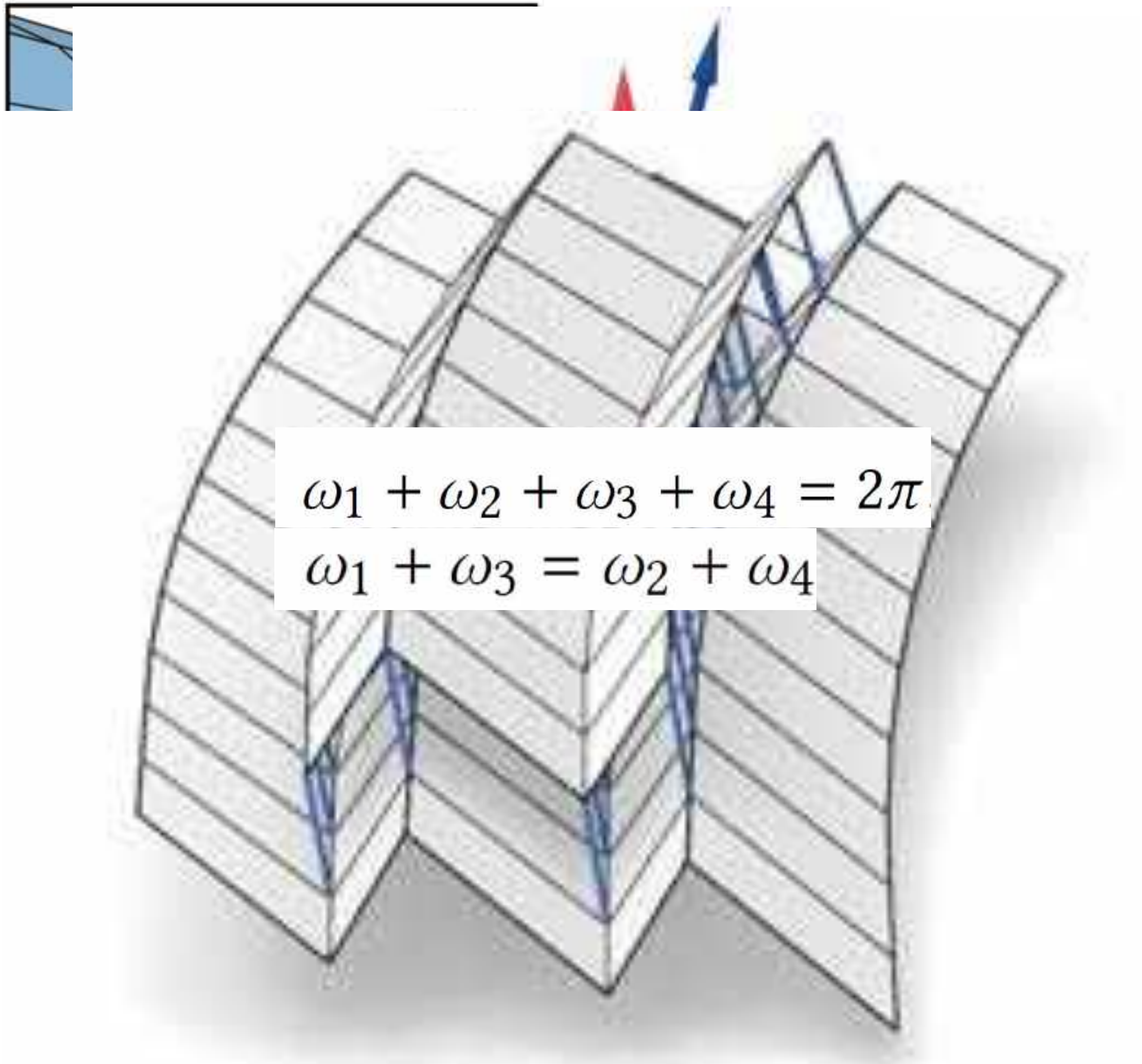
$$\omega_1 + \omega_2 + \omega_3 + \omega_4 = 2\pi$$



Conical meshes as discrete PPLS

Principal pleated structures

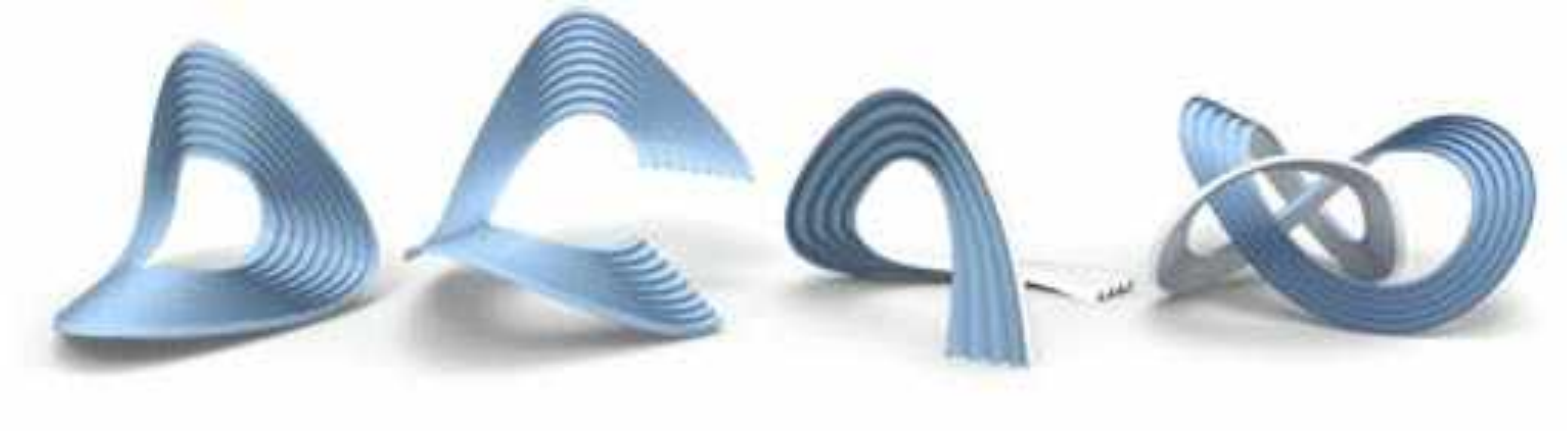
- Discrete models are special conical meshes
- Constant fold angle along each crease curve
- Offsets have the same properties

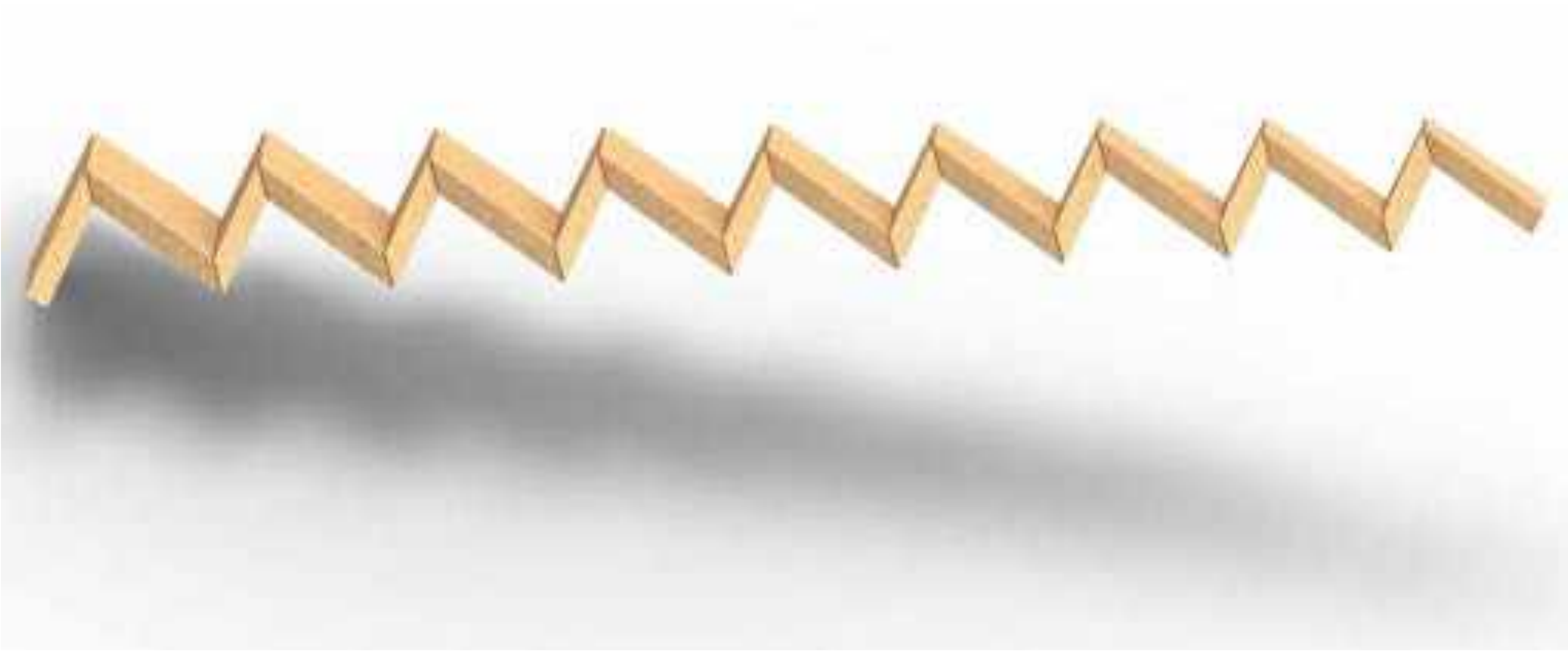


$$\omega_1 + \omega_2 + \omega_3 + \omega_4 = 2\pi$$

$$\omega_1 + \omega_3 = \omega_2 + \omega_4$$

Examples of PPLS



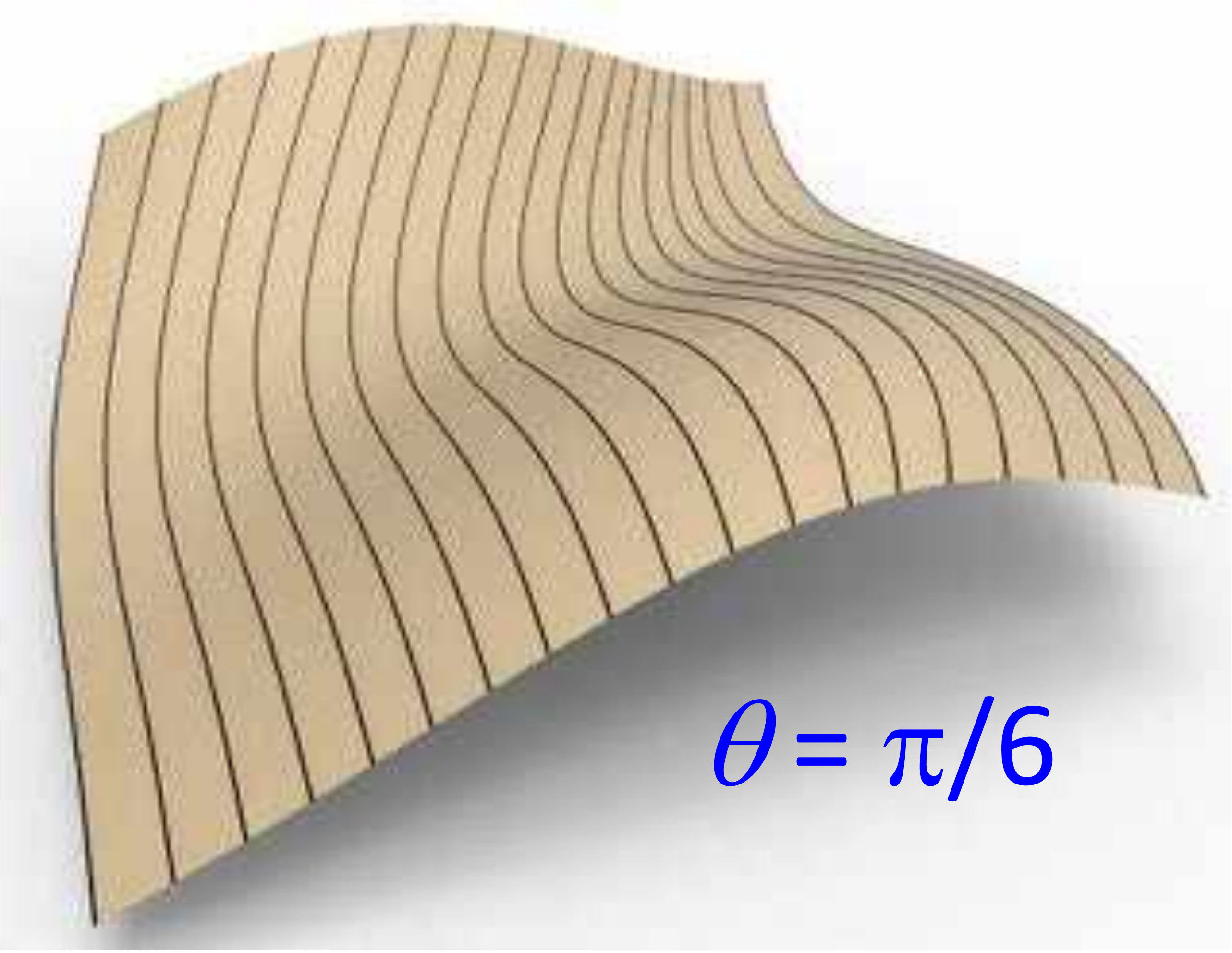


Flexible mechanism

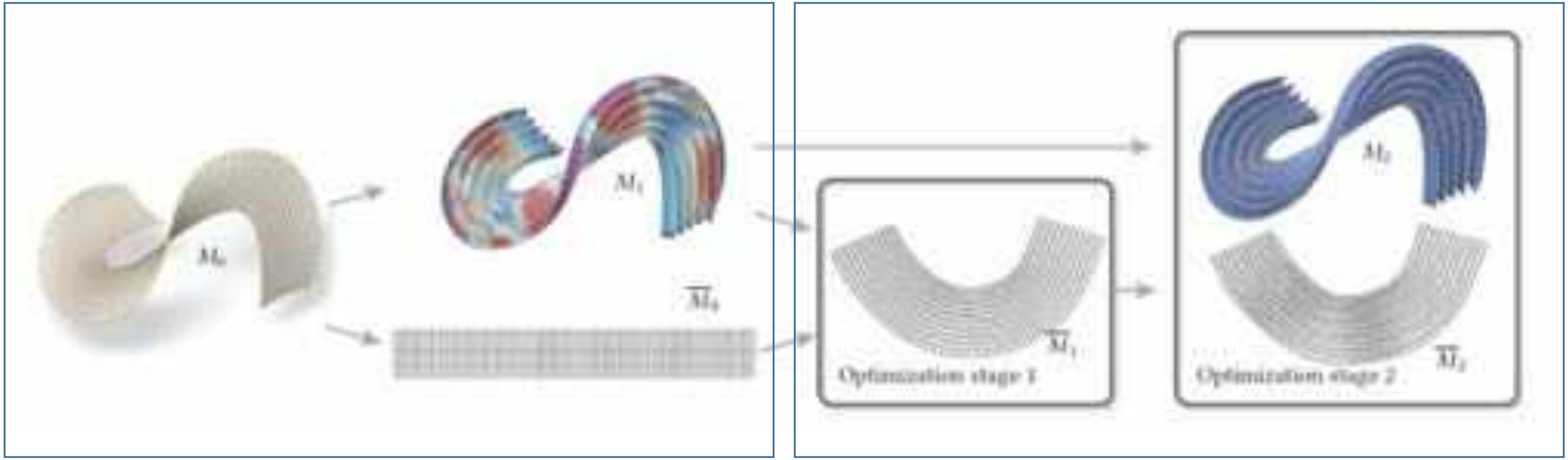
Design and reconstruction with pleated structures

Pseudo-geodesics

- Pseudo-geodesic: surface curve whose osculating planes form a constant angle θ with the surface
- Asymptotic curves (θ=0) and geodesics ($\theta = \pi/2$) are special pseudogeodesics

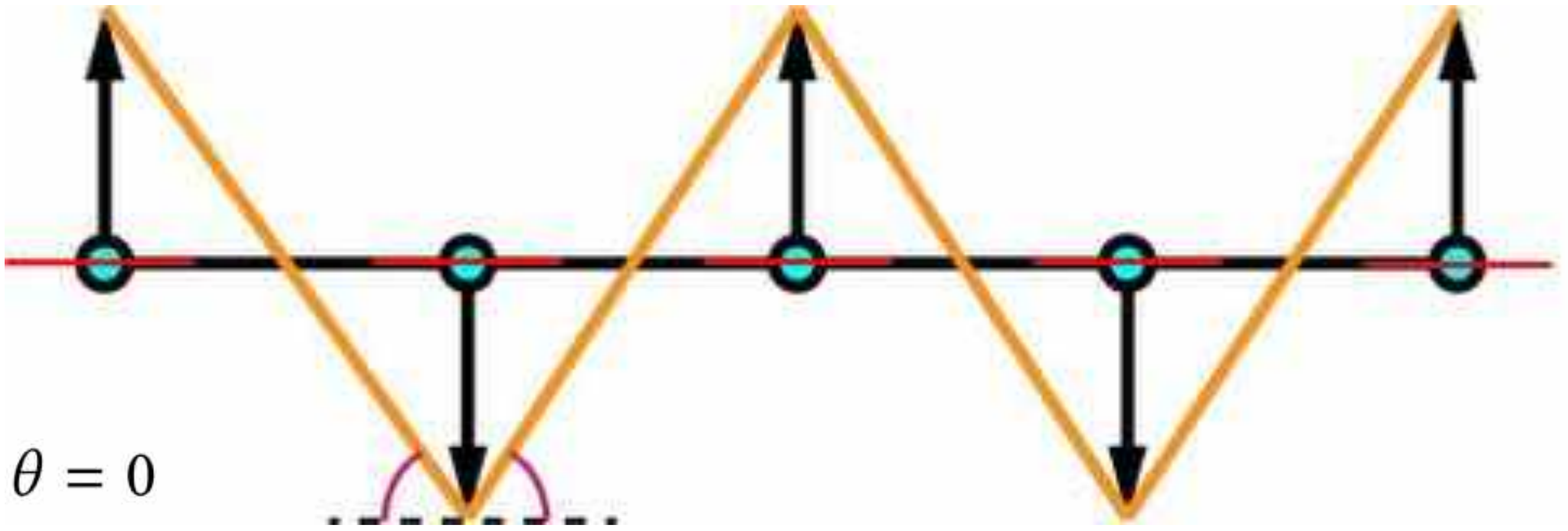


Computation pipeline



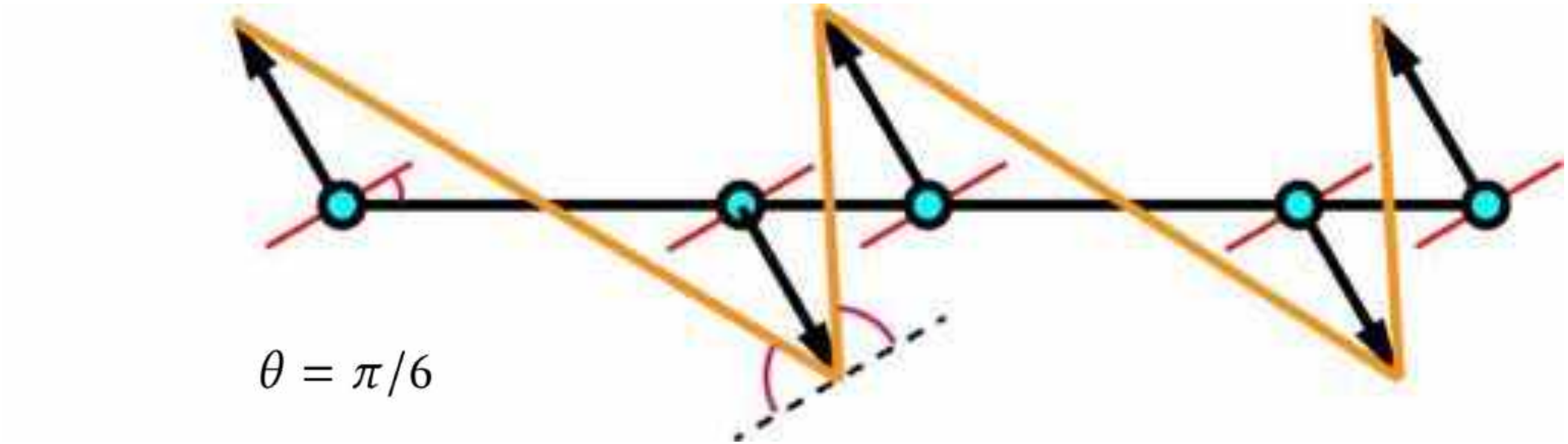
initialization optimization

Initialization



Schematic illustration of a pleated structure

Initialization



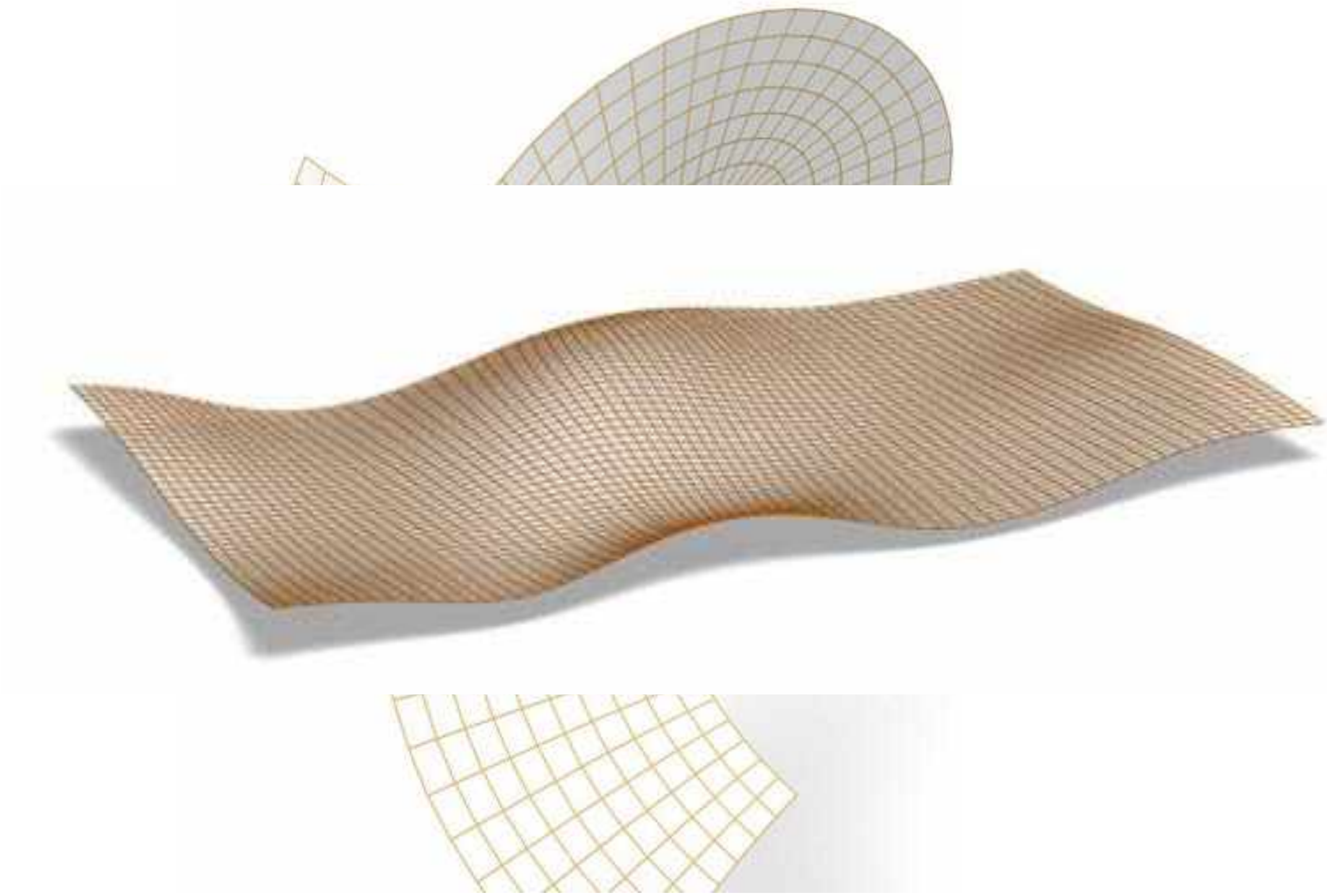
Schematic illustration of a pleated structure

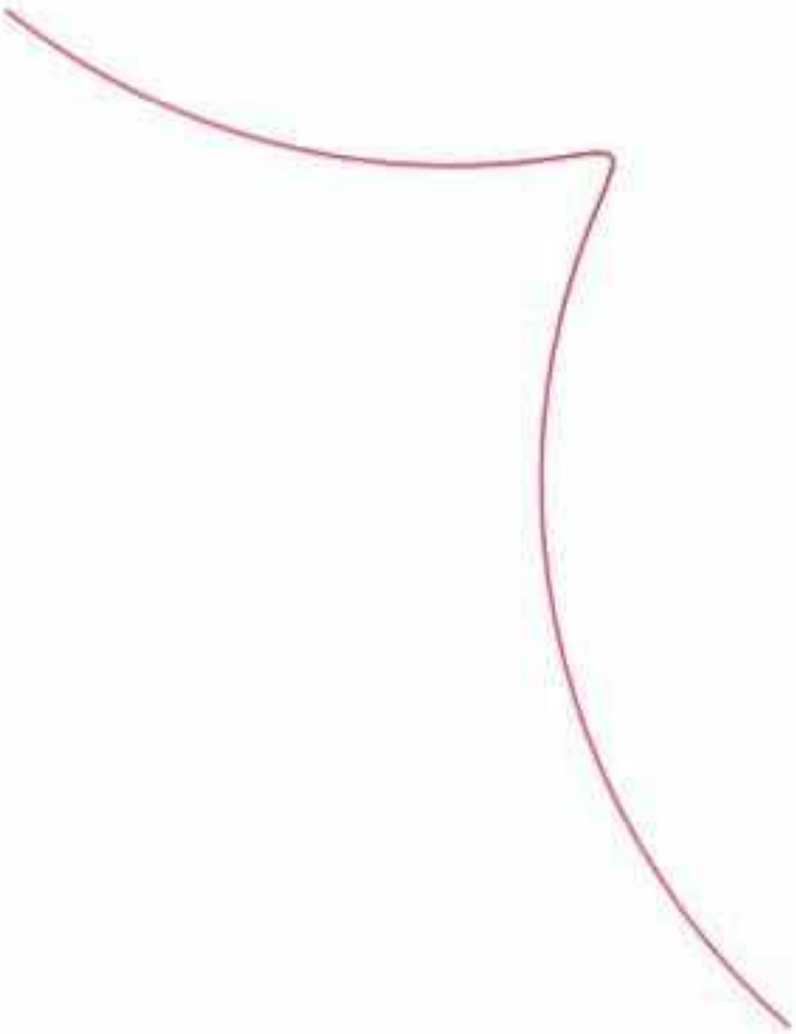
Initialization

- Generate a surface with equidistant pseudo-geodesics: evolution of a chosen curve in direction of

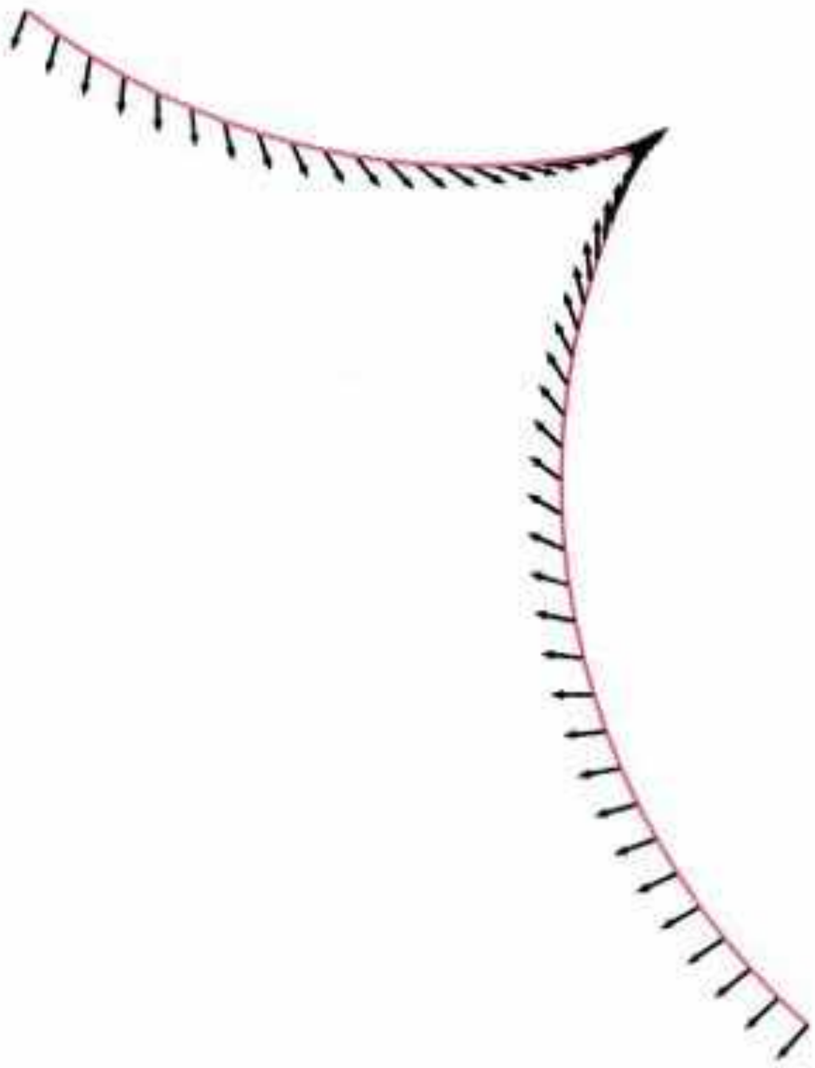
$$\mathbf{e_2} cos\theta + \mathbf{e_3} sin\theta$$

- Compute a family of nearly equidistant pseudo-geodesics on the given reference surface

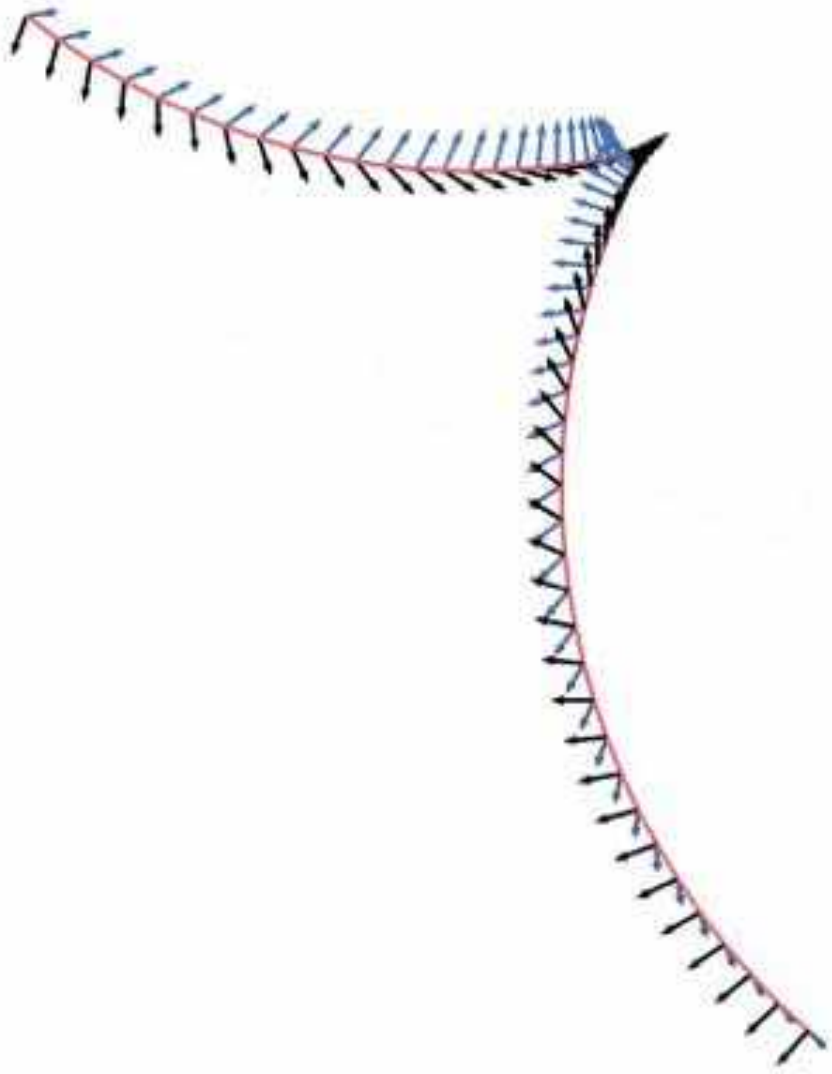




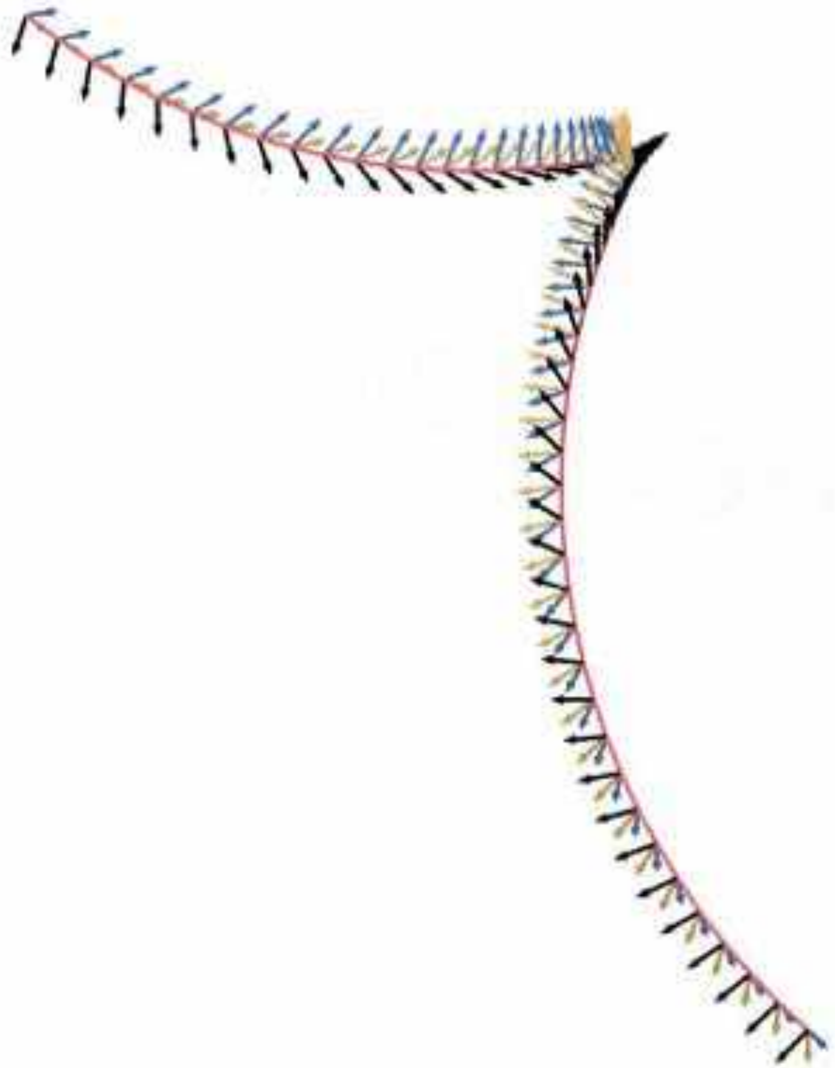
Given curve



e_2: normal direction(black)

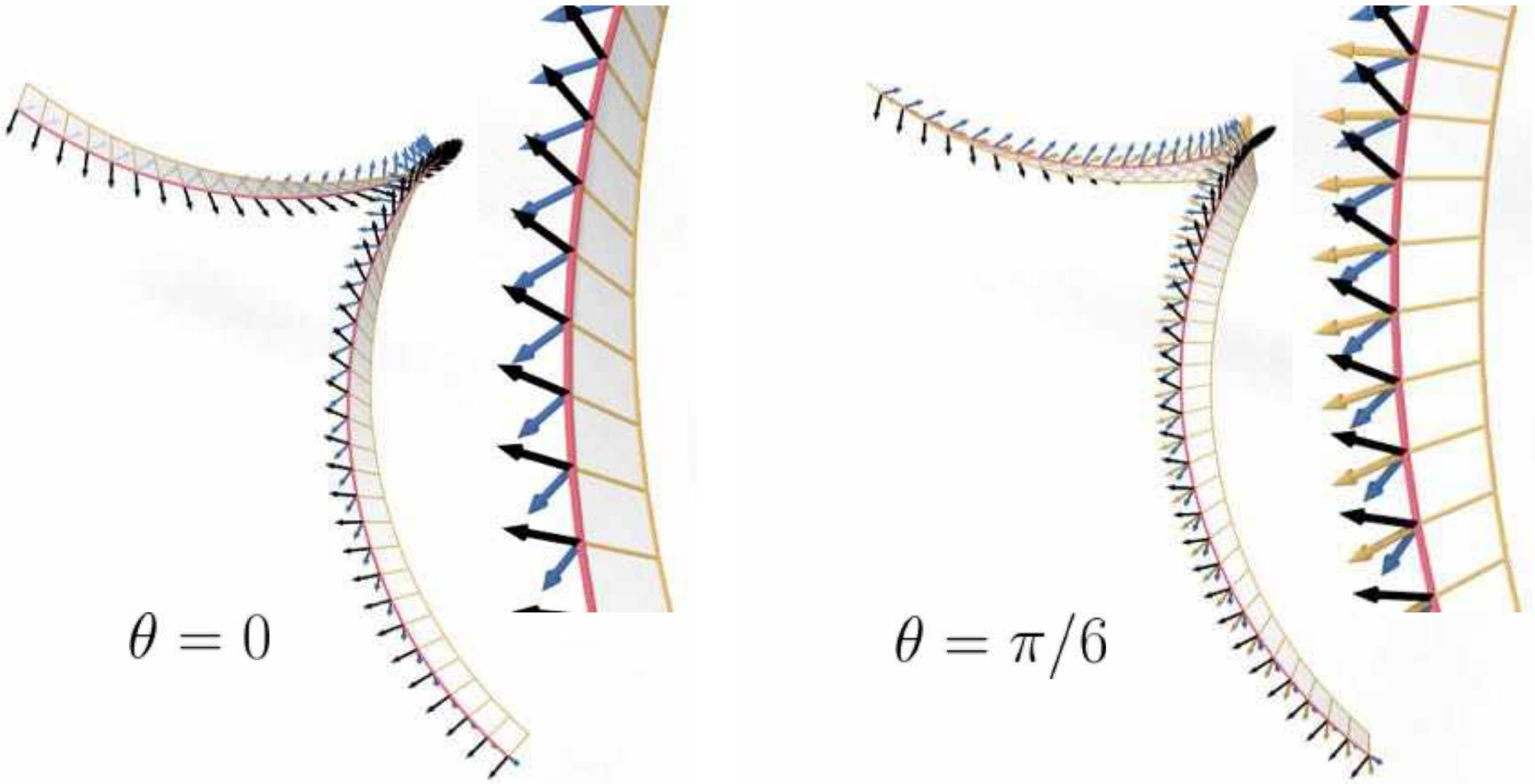


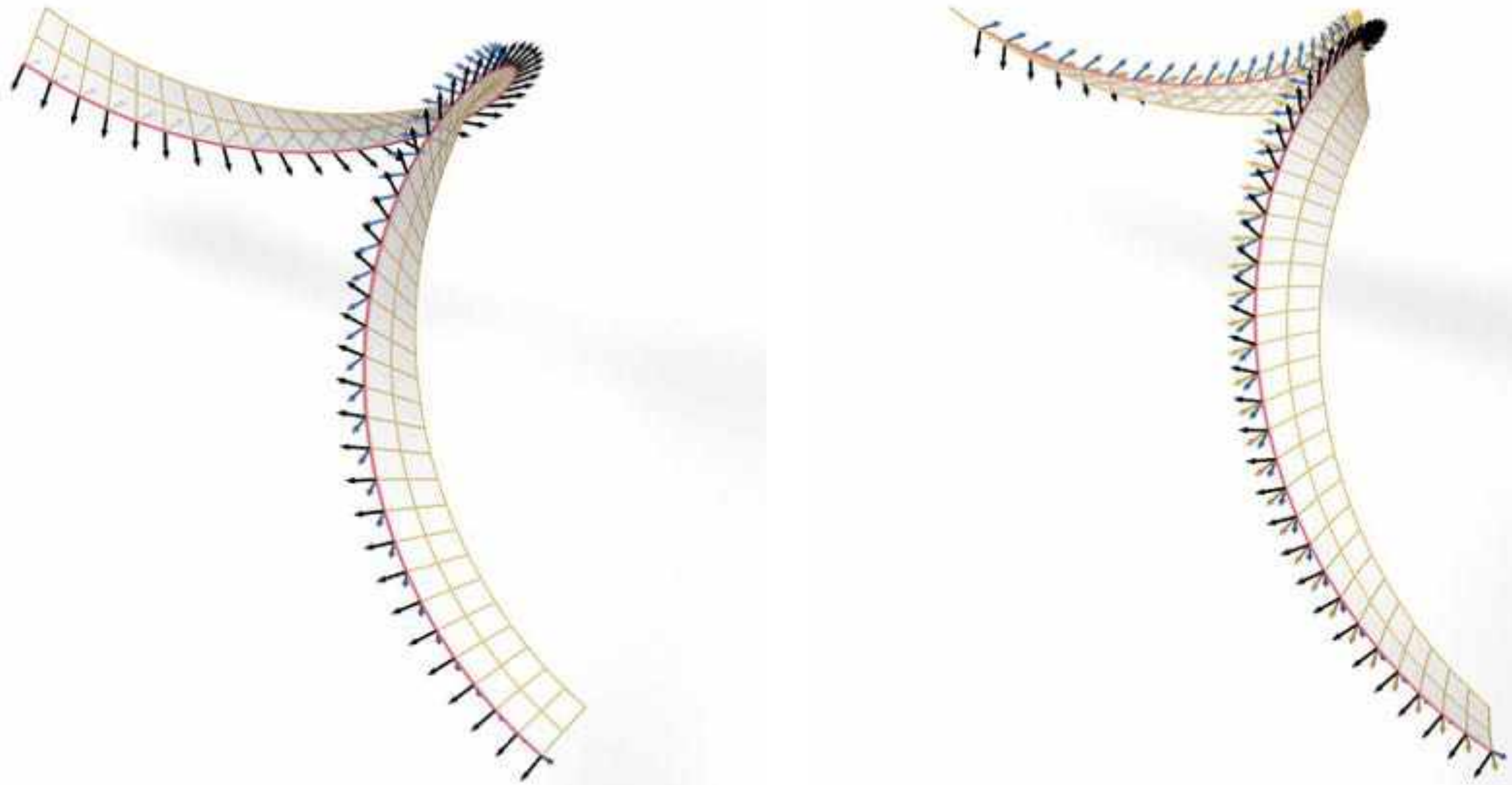
e3: bi-normal direction(blue)

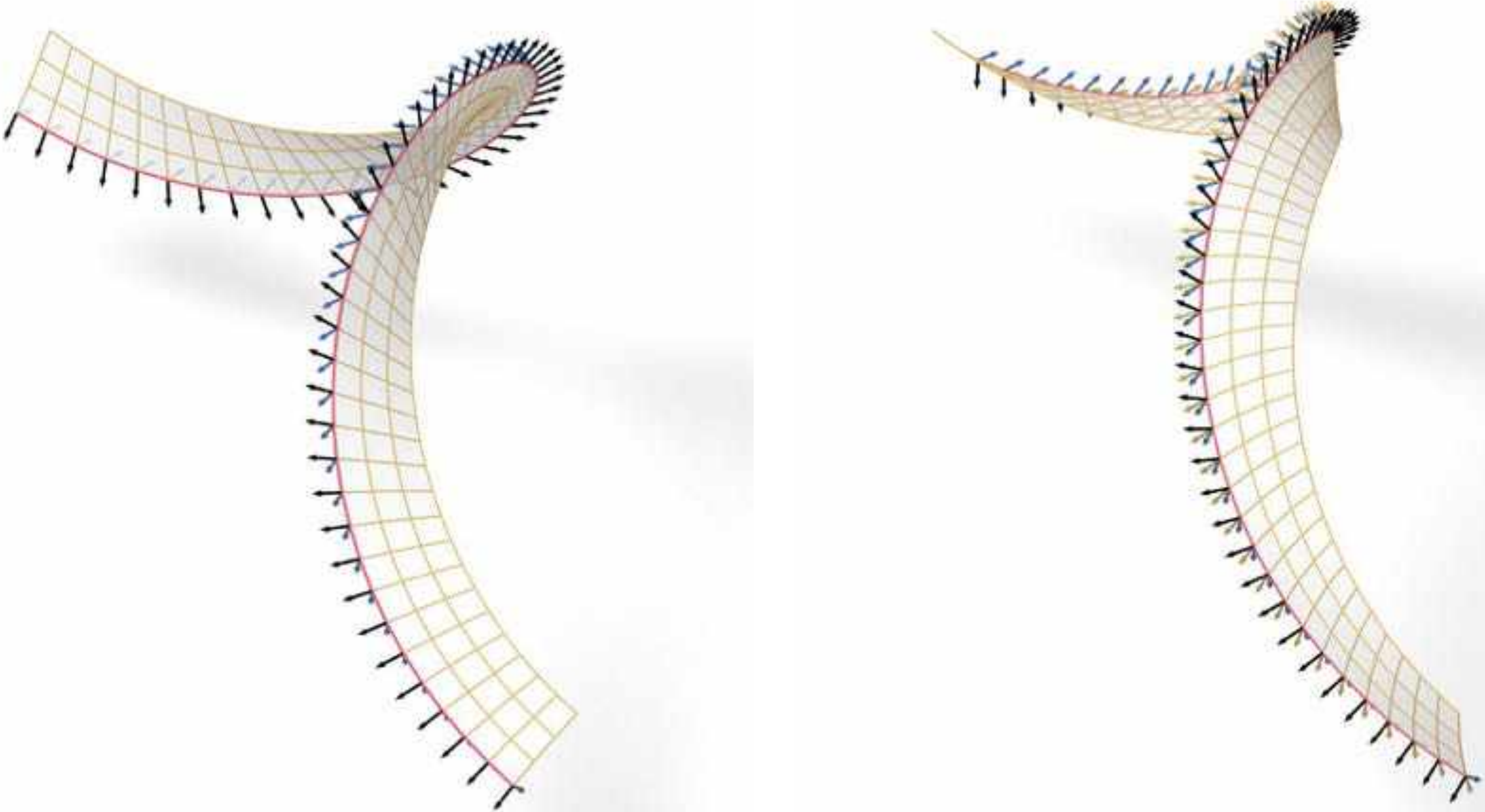


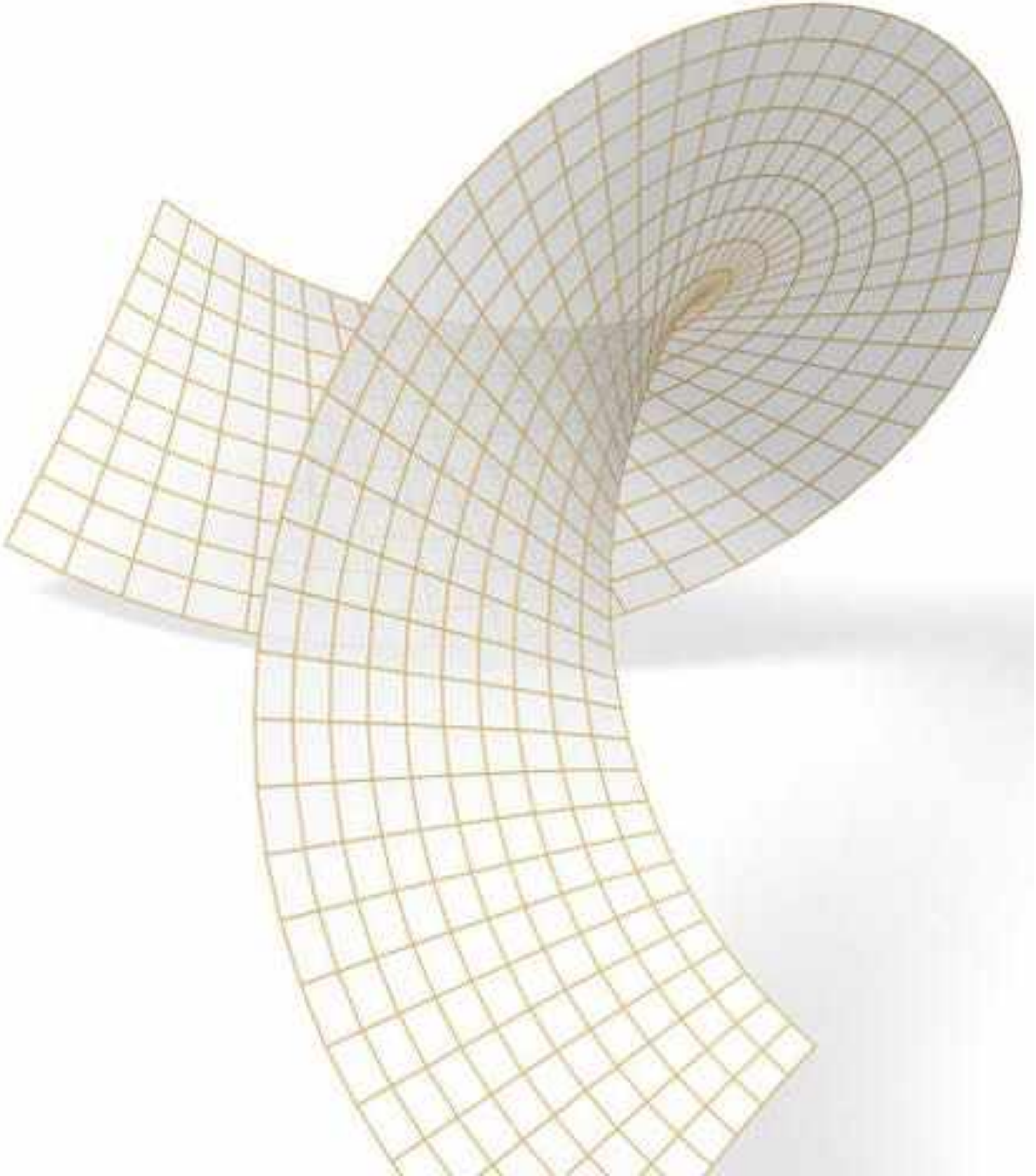
Evolution direction (yellow)

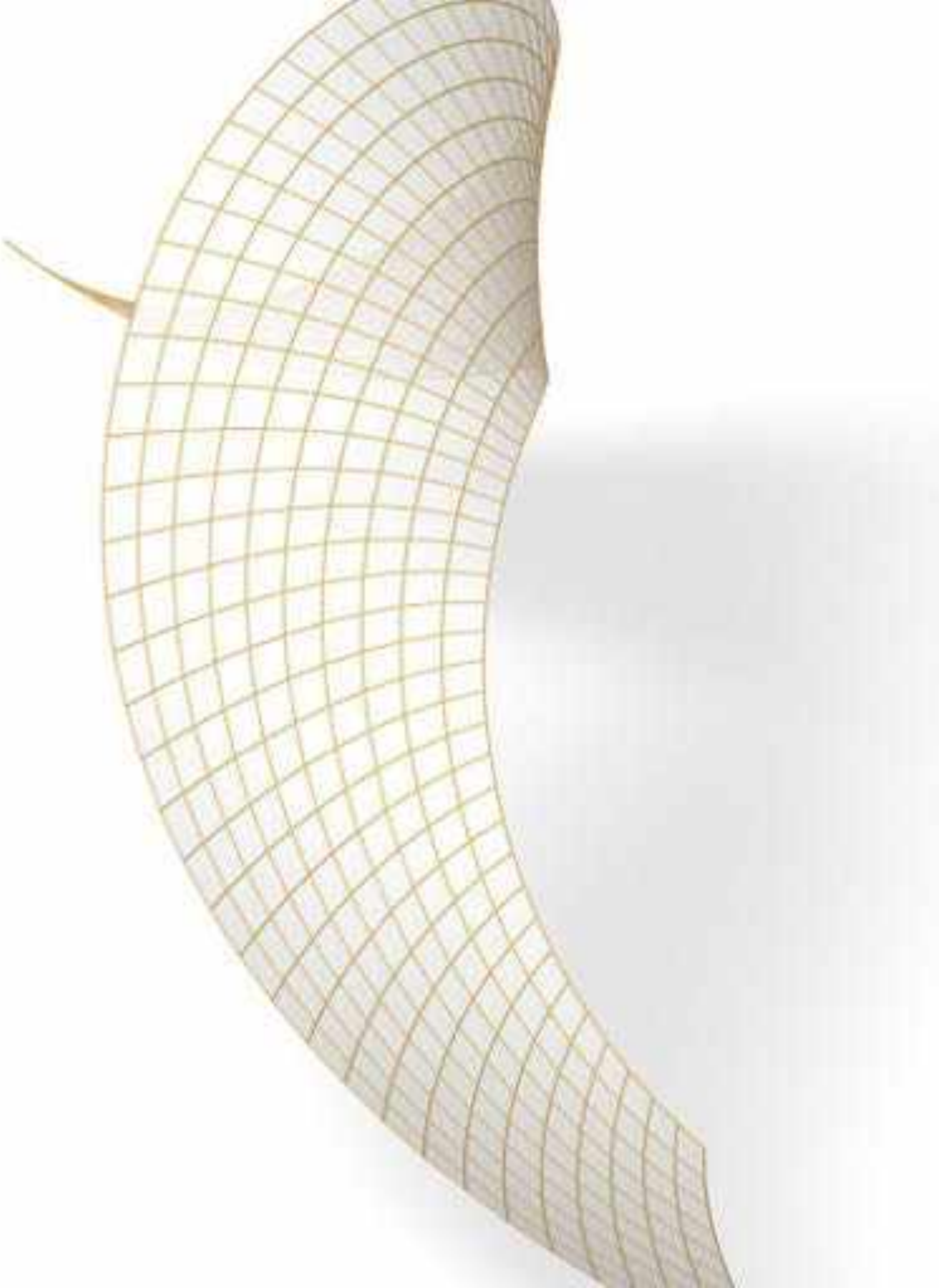
$$\mathbf{e_2} cos\theta + \mathbf{e_3} sin\theta$$

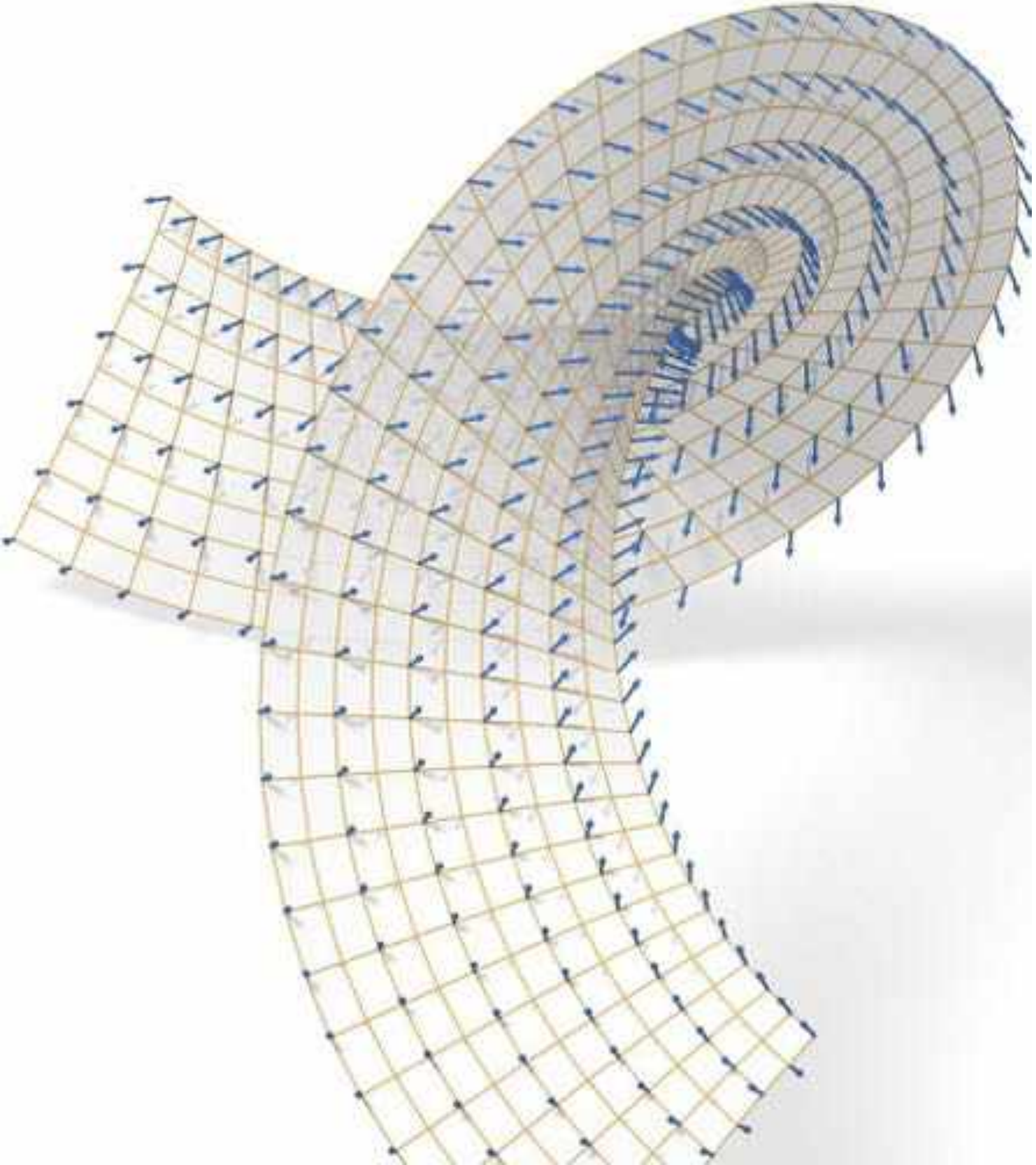

$\theta = 0$
$\theta = \pi/6$

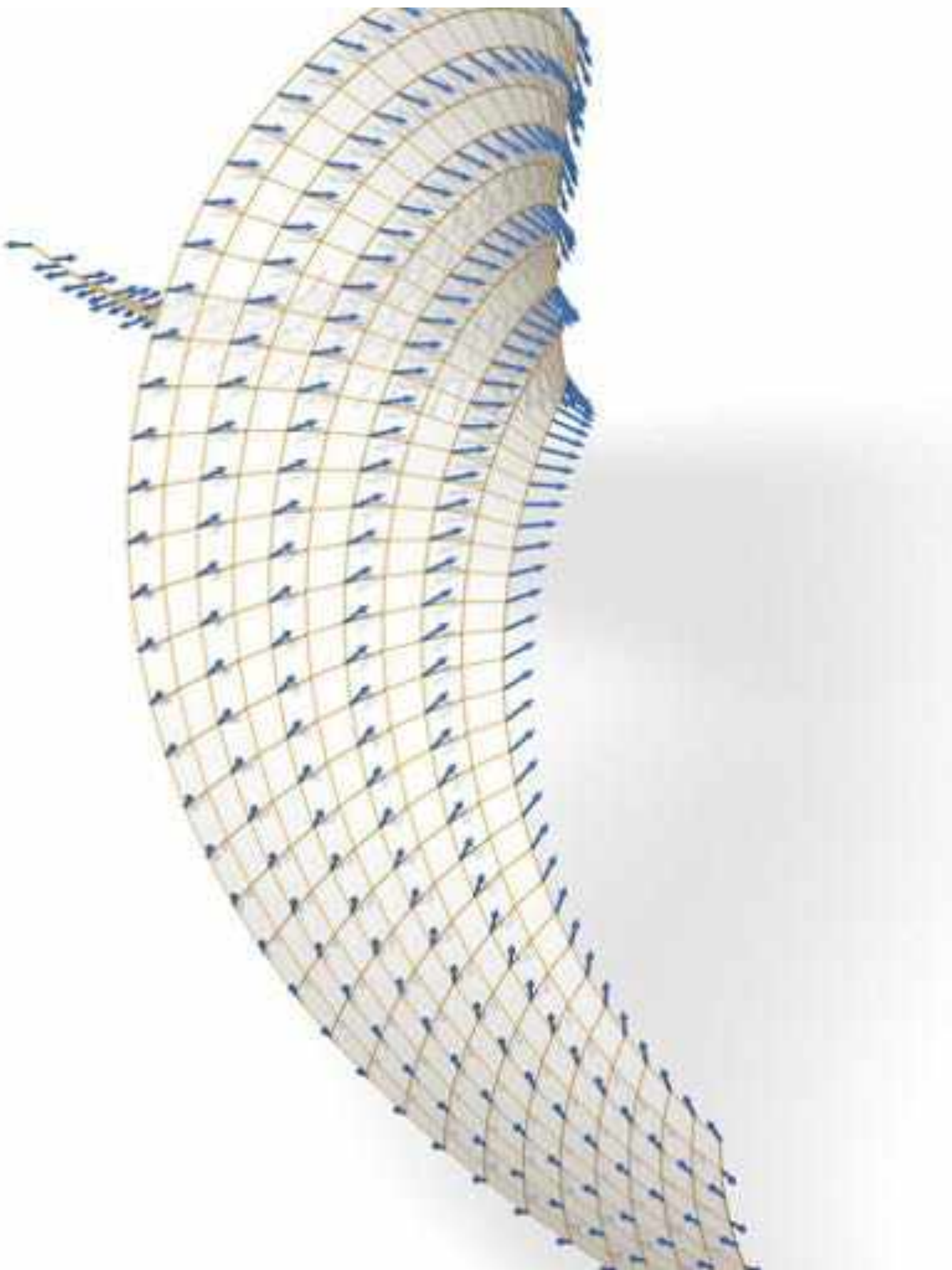


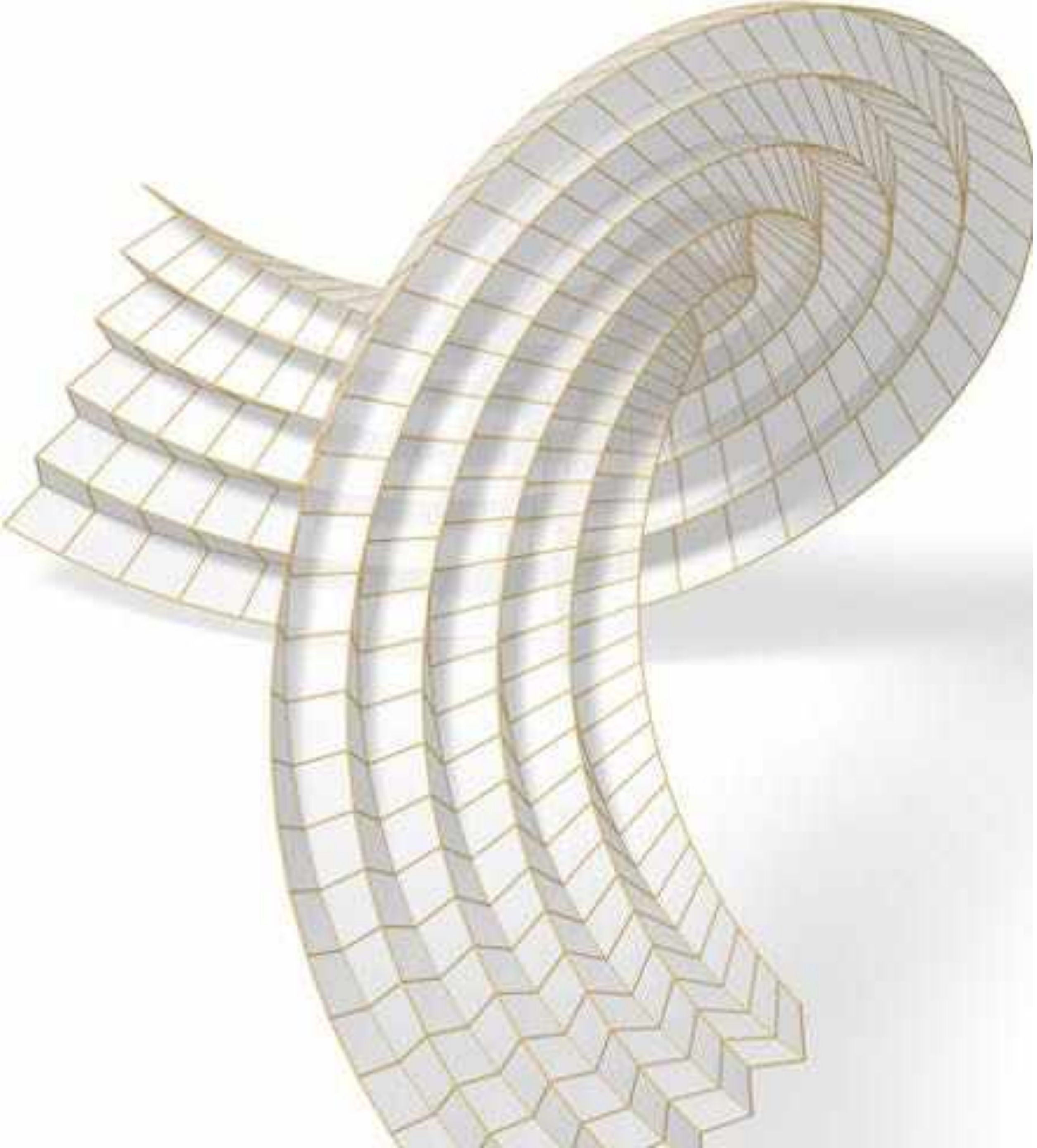


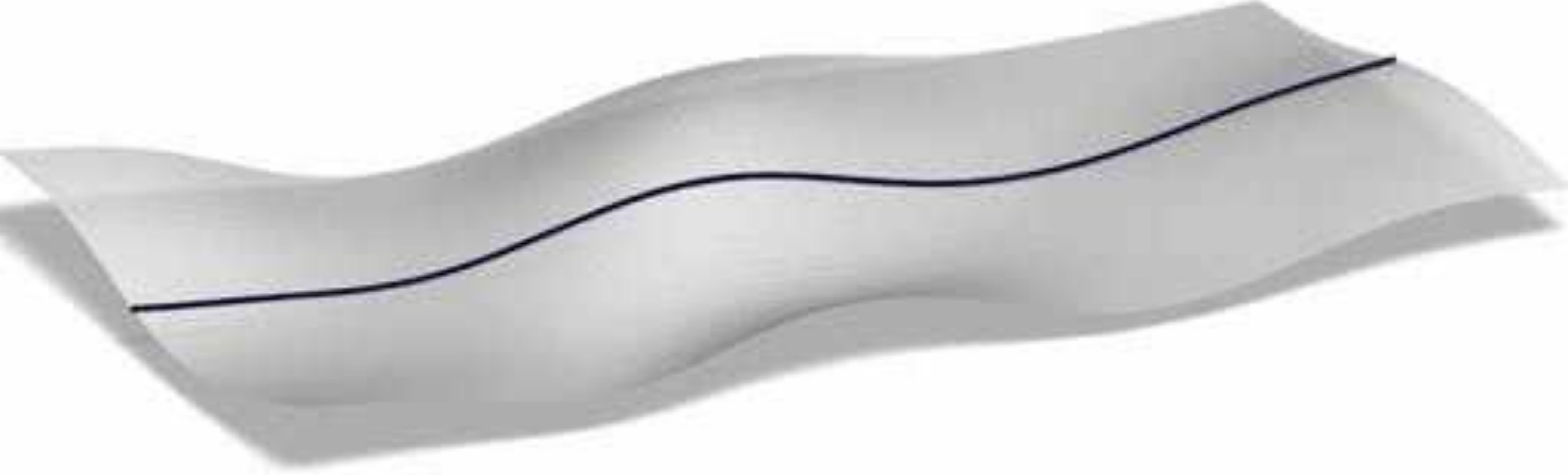


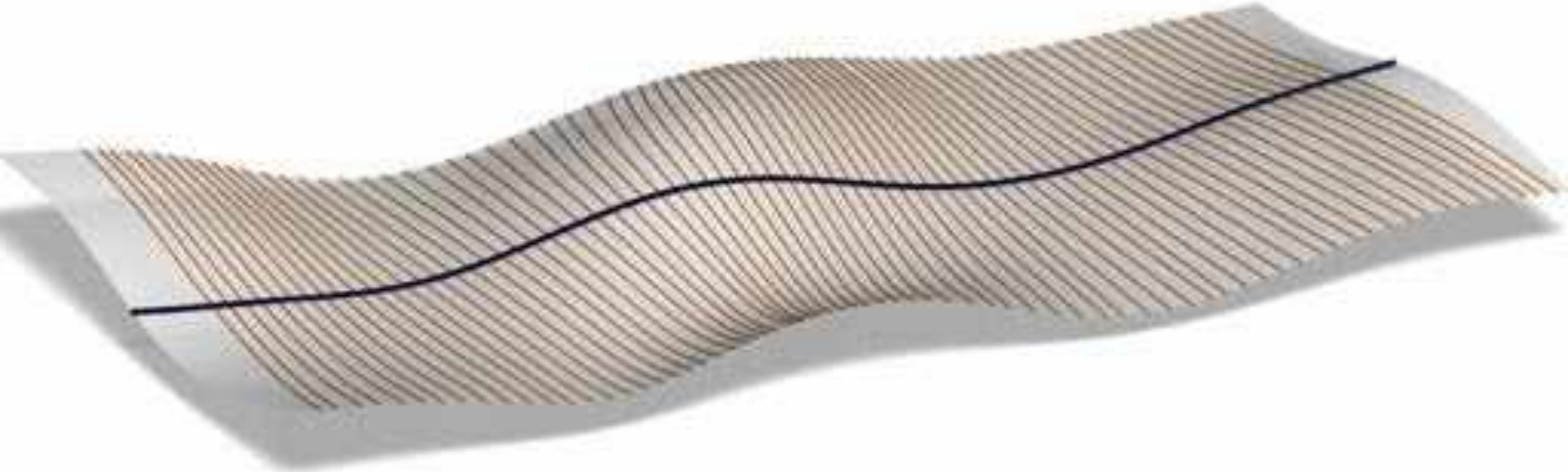








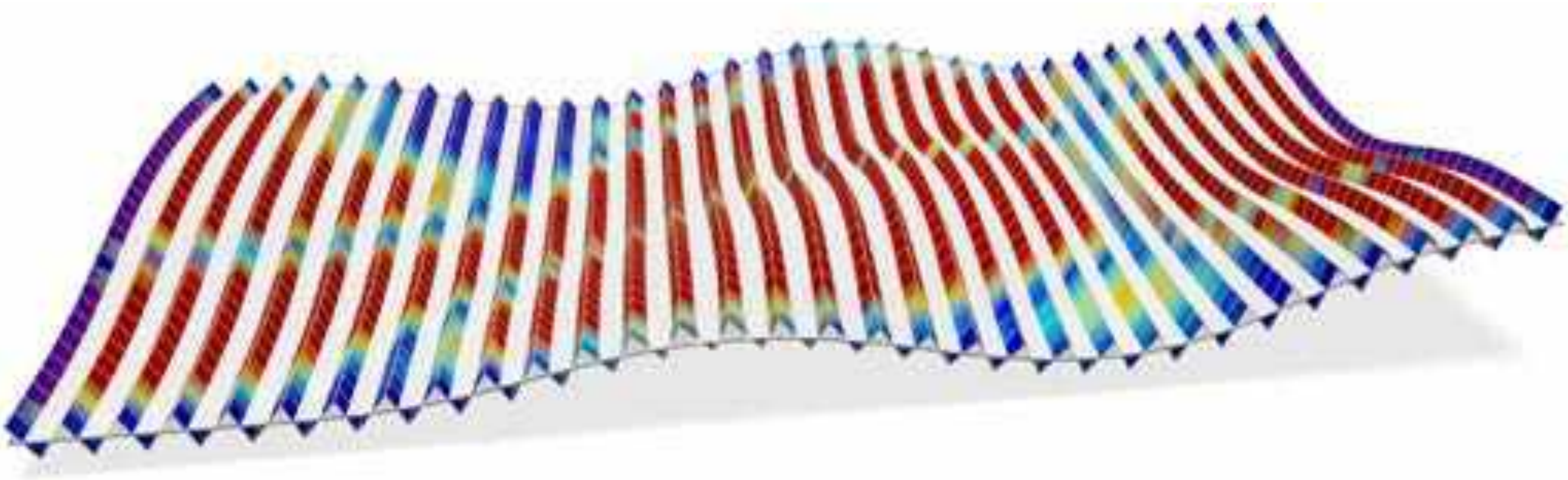





Bad initialization

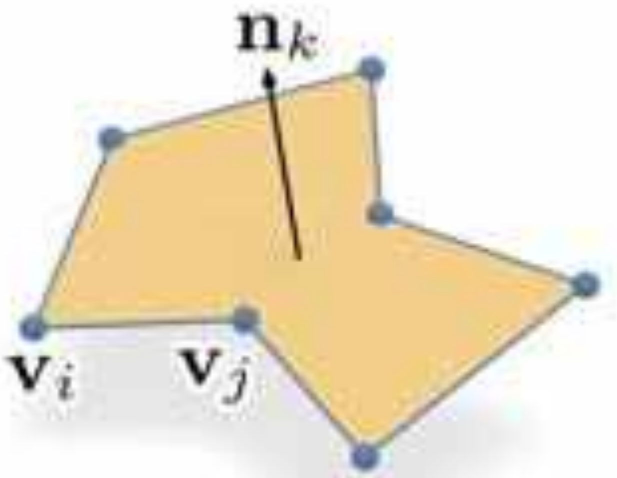
Bad initialization

Bad initialization



Optimization

- Planarity



$$E_{plan} = \sum_{f \in F} \sum_{\mathbf{v}_i \mathbf{v}_j \subset f} \langle \mathbf{v}_i - \mathbf{v}_j, \mathbf{n}_f \rangle^2 + \sum_{f \in F} \left(\|\mathbf{n}_f\|^2 - 1 \right)^2,$$

Optimization

- Developability

$$E_{isom} = \sum_{\substack{\text{edges and diagonals} \\ \mathbf{v}_i\mathbf{v}_j \text{ of faces}}} \left(\|\mathbf{v}_i - \mathbf{v}_j\|^2 - \|\bar{\mathbf{v}}_i - \bar{\mathbf{v}}_j\|^2 \right)^2 .$$

Optimization

- Closeness to polylines

$$E_{close} = \sum_{\mathbf{v}_i \in V} \langle \mathbf{v}_i - \mathbf{v}_i^*, \mathbf{n}_i^* \rangle^2$$

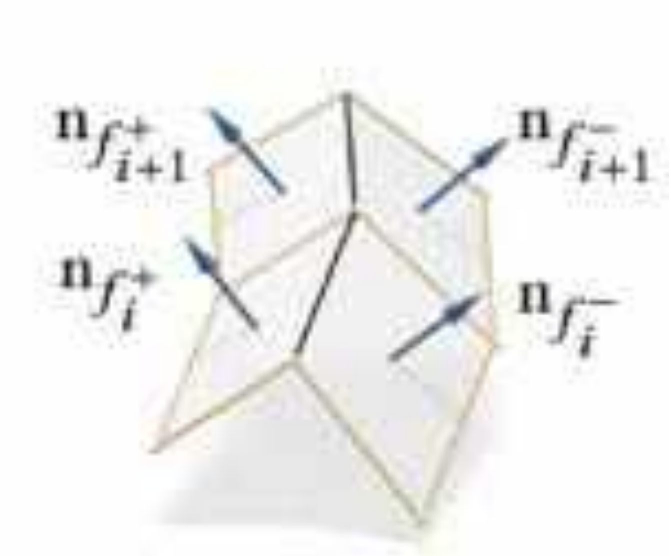
Optimization

- Fairness

$$E_{fair} = \sum_{\substack{\text{successive} \\ \text{vertices } \mathbf{v}_{i-1}\mathbf{v}_i\mathbf{v}_{i+1}}} \|\mathbf{v}_{i+1} + \mathbf{v}_{i-1} - 2\mathbf{v}_i\|^2,$$

Optimization

- Principal property



$$E_{principal} = \sum_{(f_i^+, f_i^-, f_{i+1}^+, f_{i+1}^-)} \left(\langle \mathbf{n}_{f_i^+}, \mathbf{n}_{f_i^-} \rangle - \langle \mathbf{n}_{f_{i+1}^+}, \mathbf{n}_{f_{i+1}^-} \rangle \right)^2$$

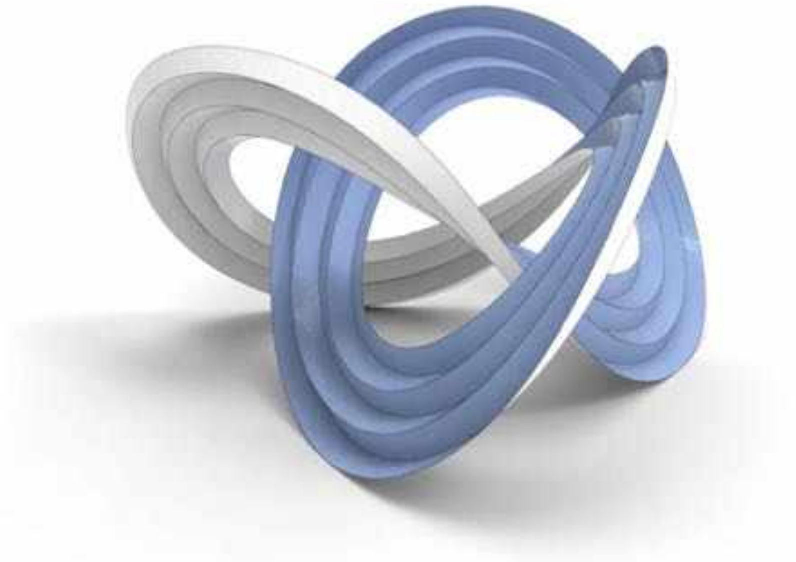
Optimization

- Objective funtion

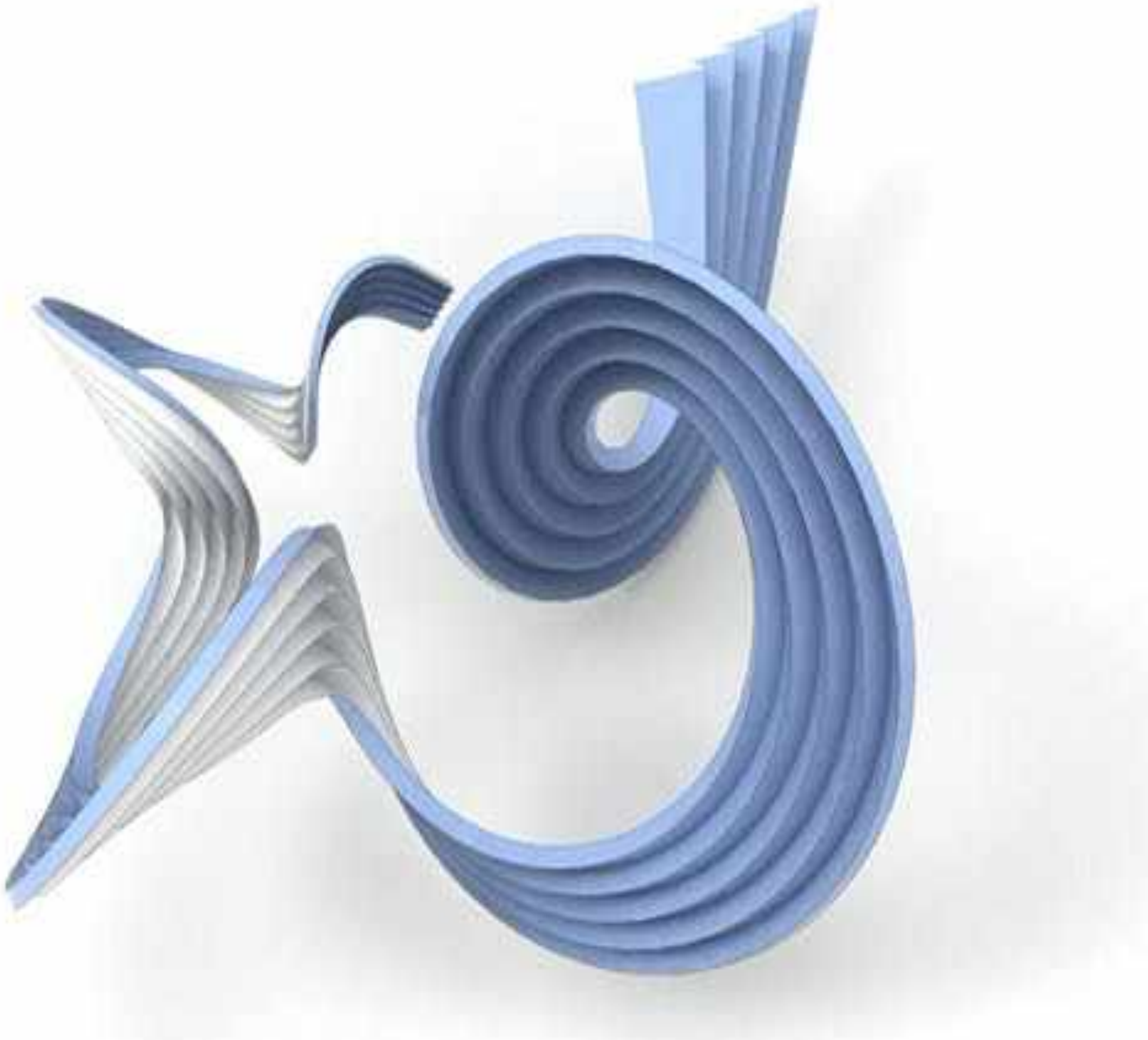
$$E = \lambda_1 E_{plan} + \lambda_2 E_{isom} + \lambda_3 E_{close} + \lambda_4 E_{fair} + \lambda_5 E_{principal}.$$

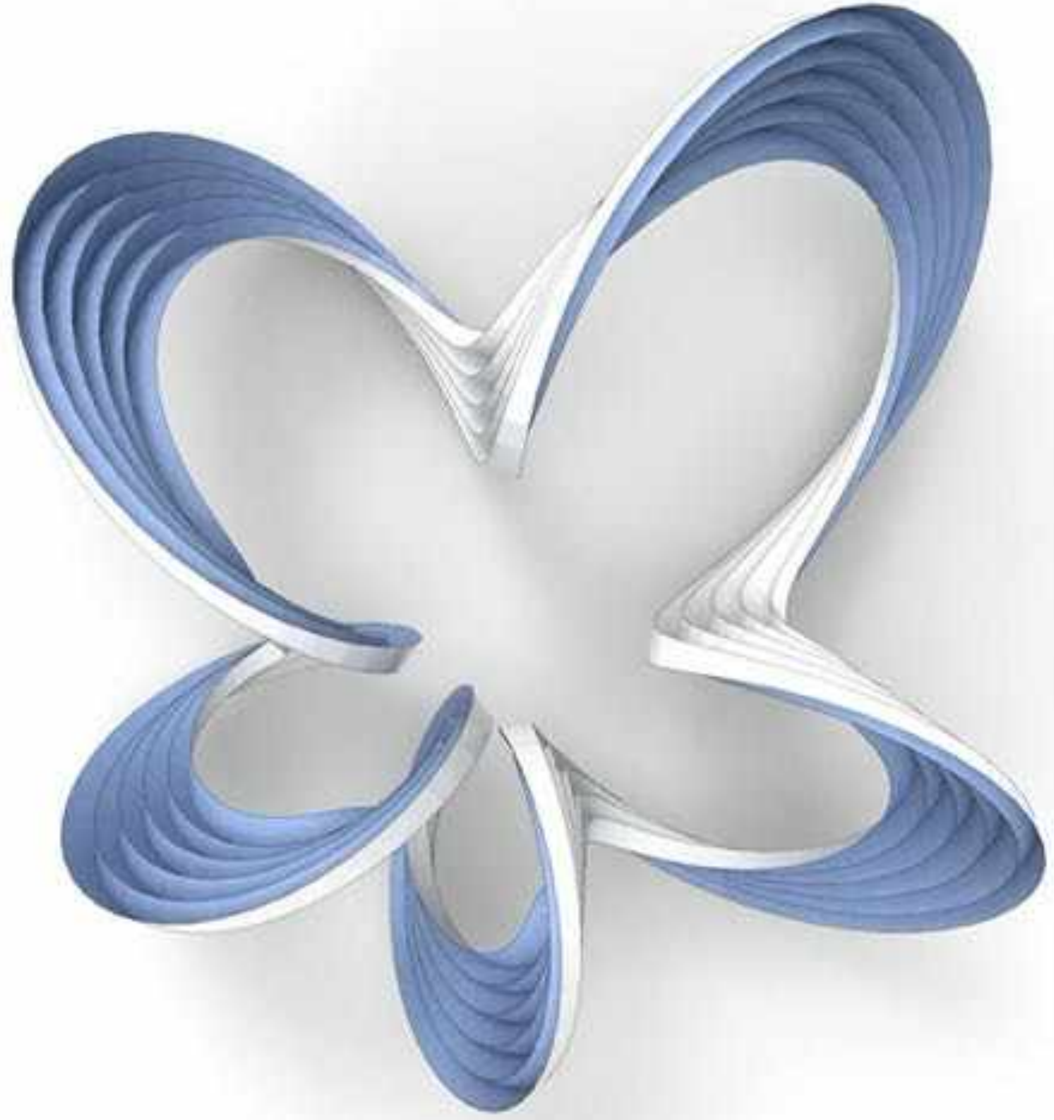
Results

Results

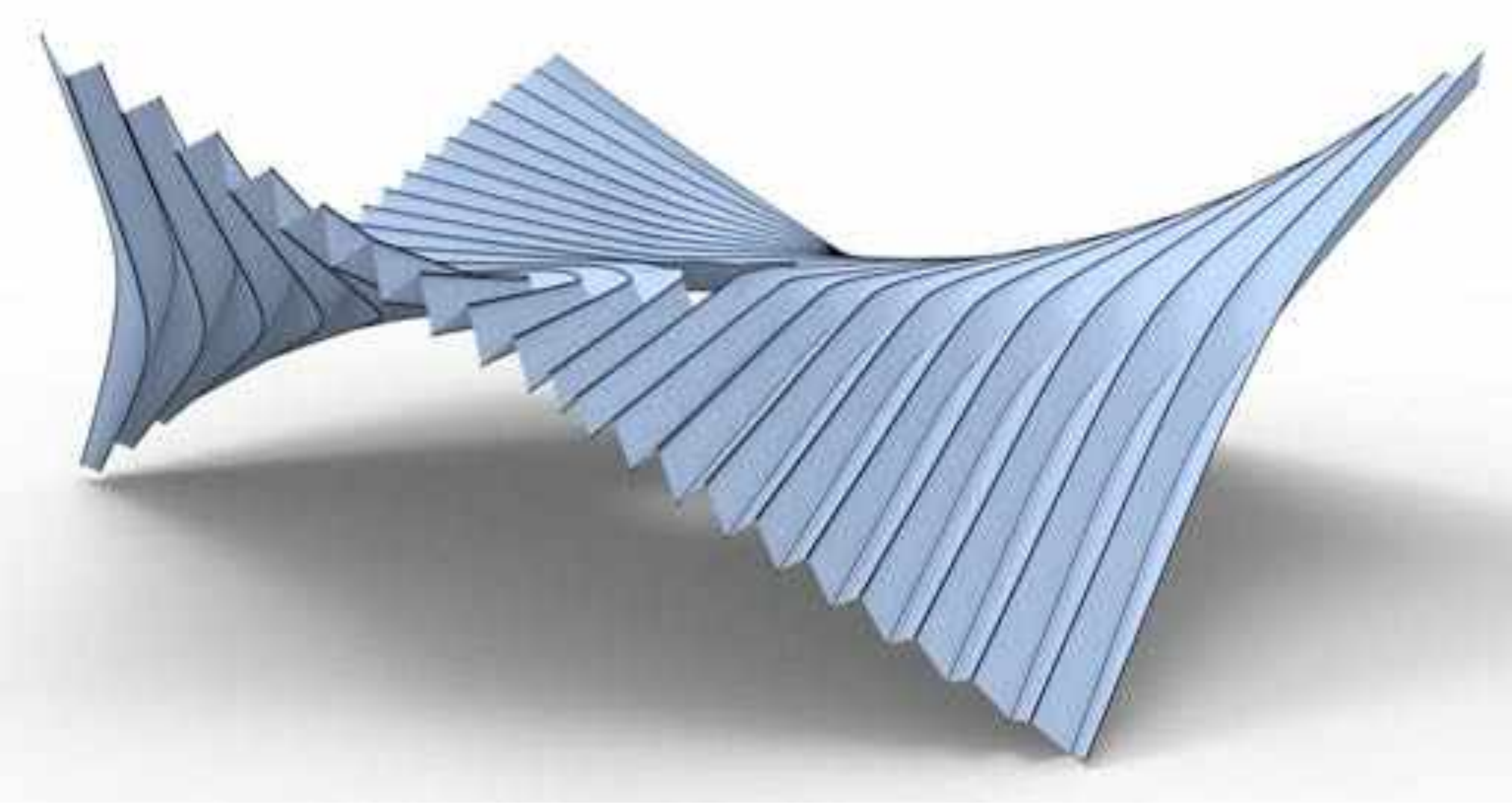


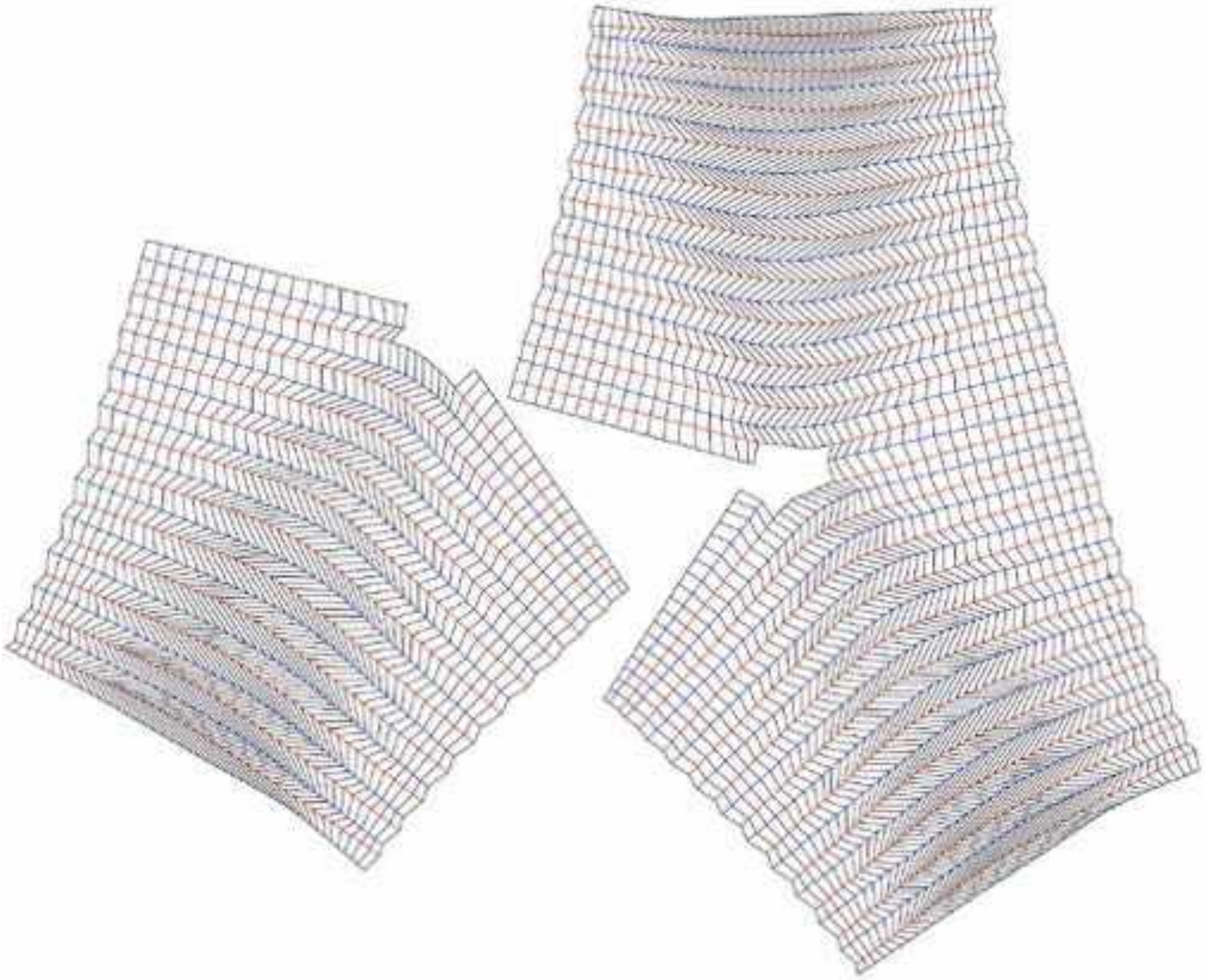
Non-uniform evolution

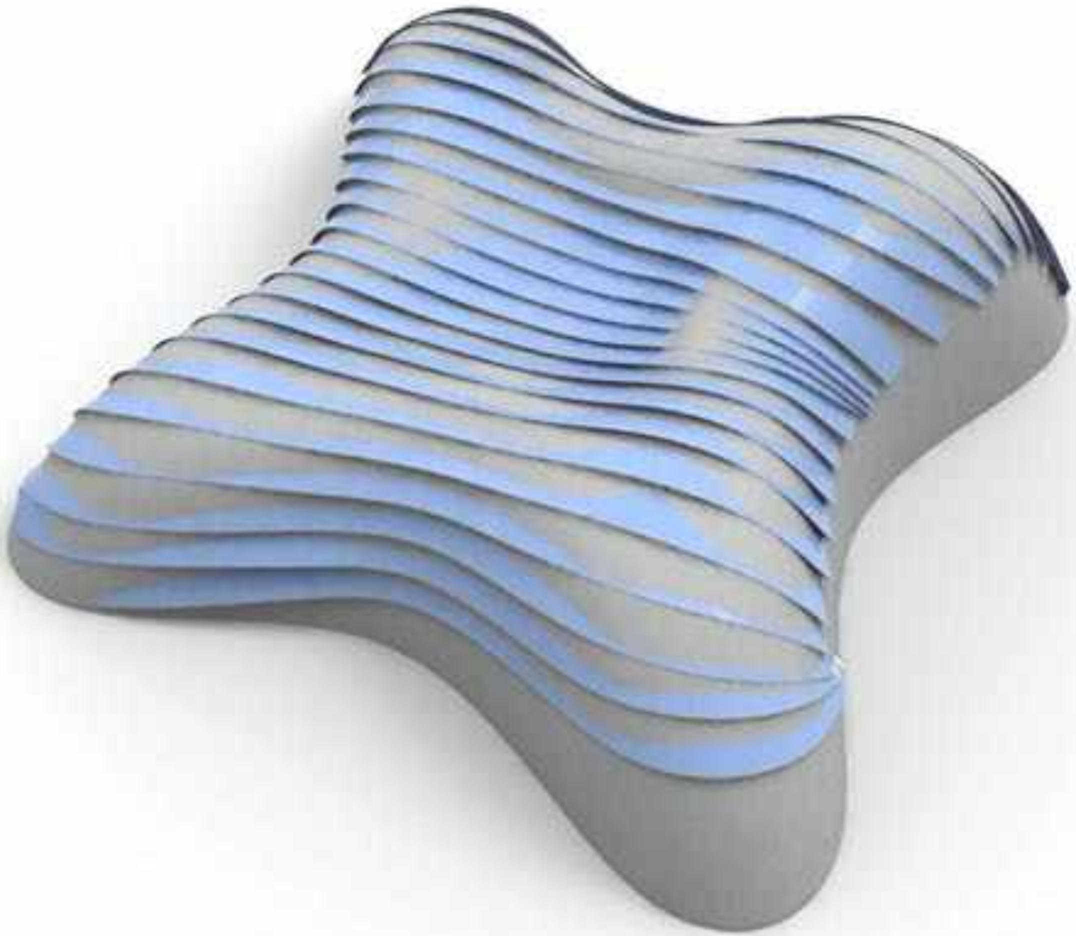


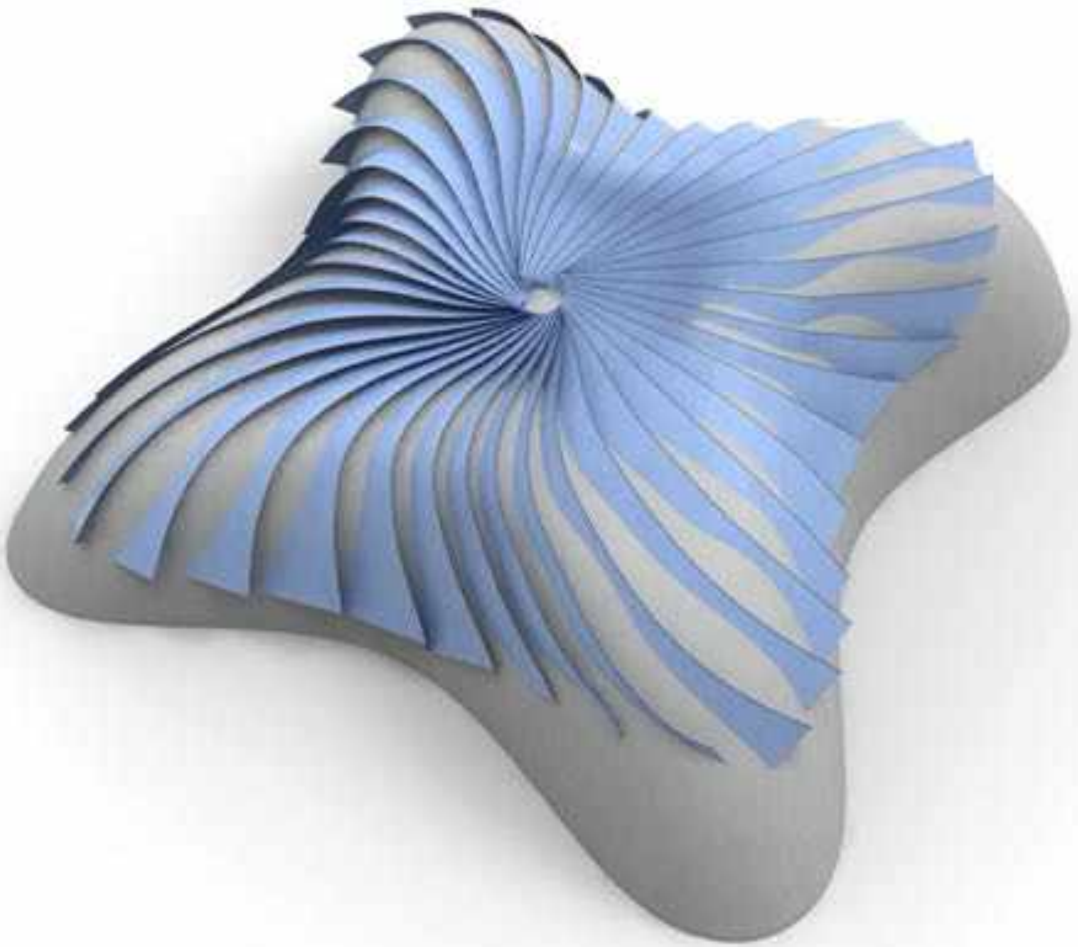


Approximation of a minimal surface









Future work

- More ways to design patterns of pseudo-geodesics for initialization
- Reconstruction with curved folded surfaces that are not pleated structures
- More connections to flat-foldable structures

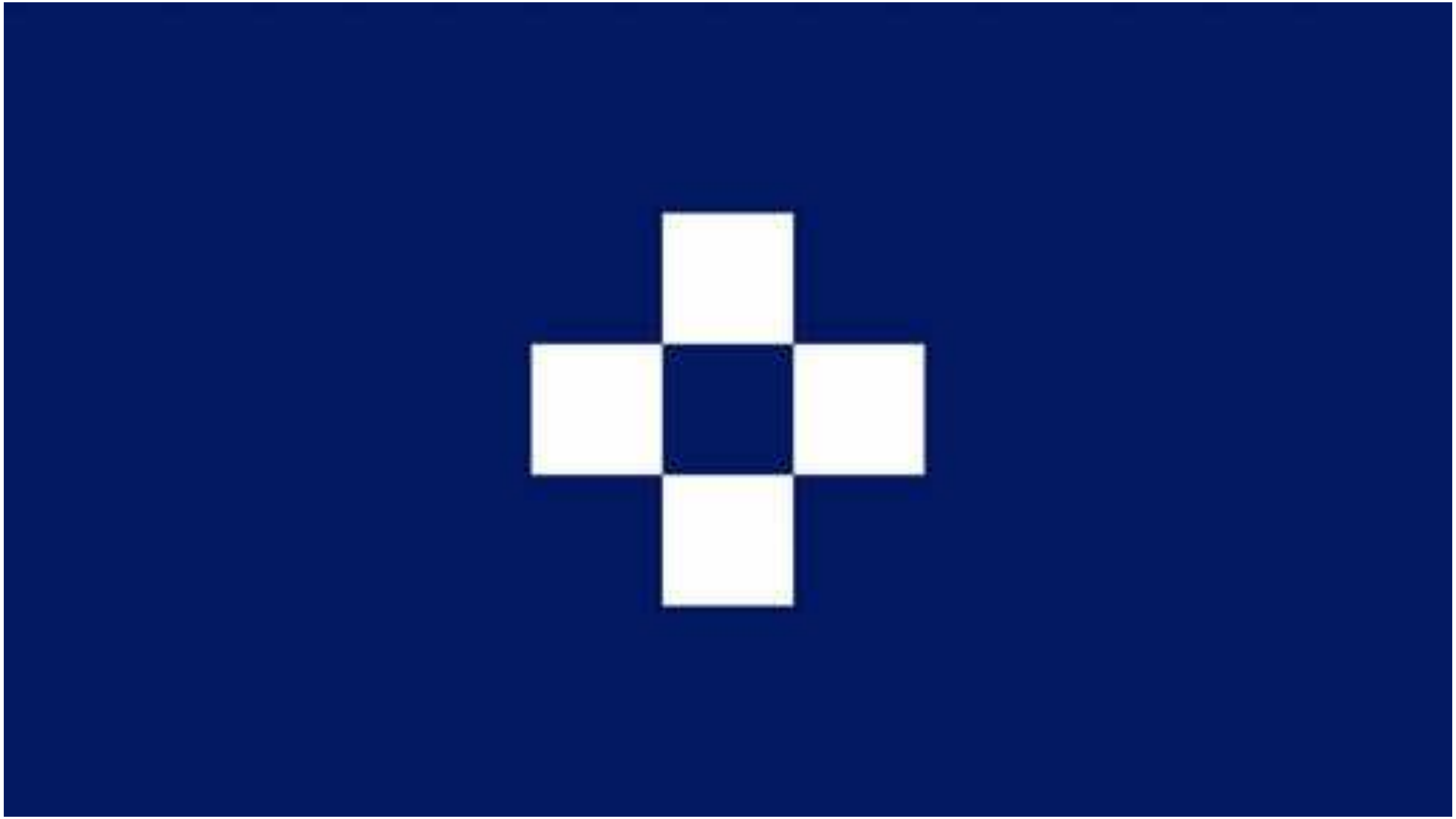
Checkerboard Patterns with Black Rectangles

(SIGGRAPH Asia 2019)
with Chi-Han Peng, Peter Wonka,and Helmut Pottmann

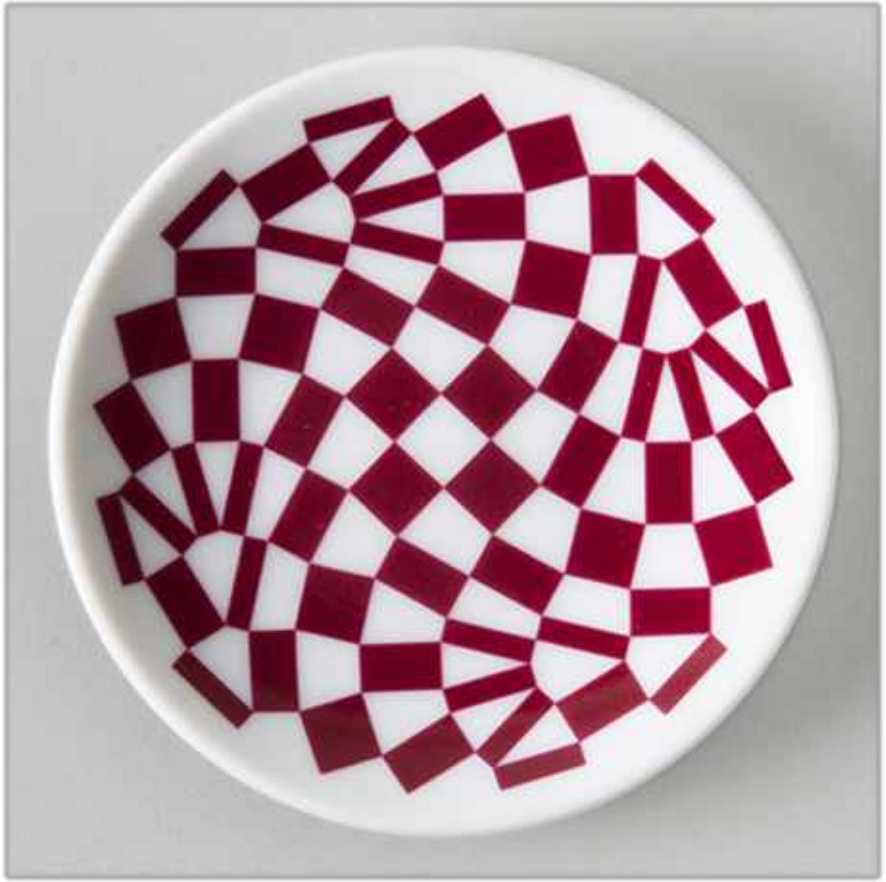
Checkerboard patterns with black rectangles

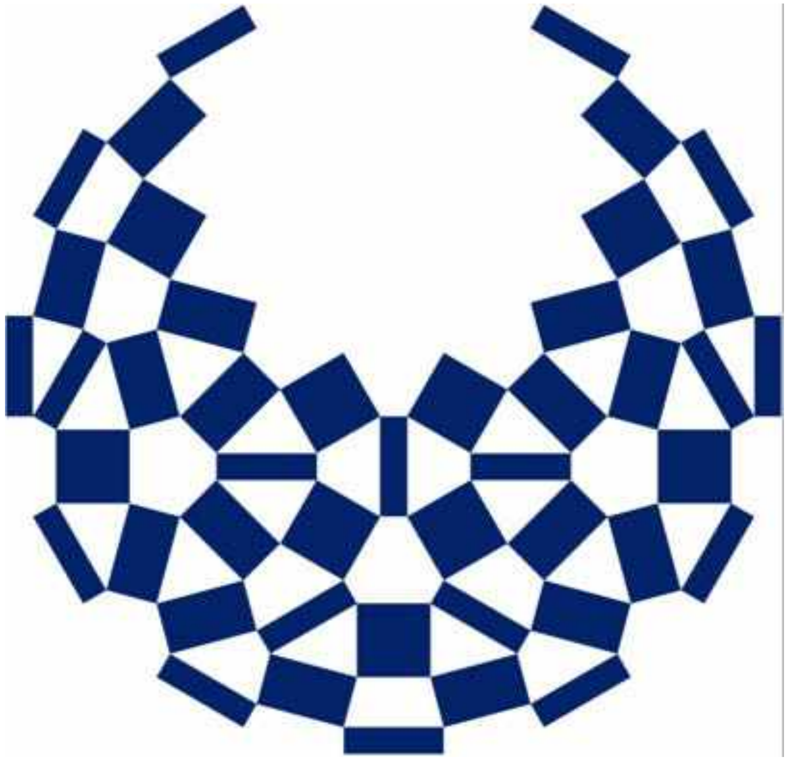
Inspiration – Tokyo 2020 Emblems

by Japanese artist Asao Tokolo

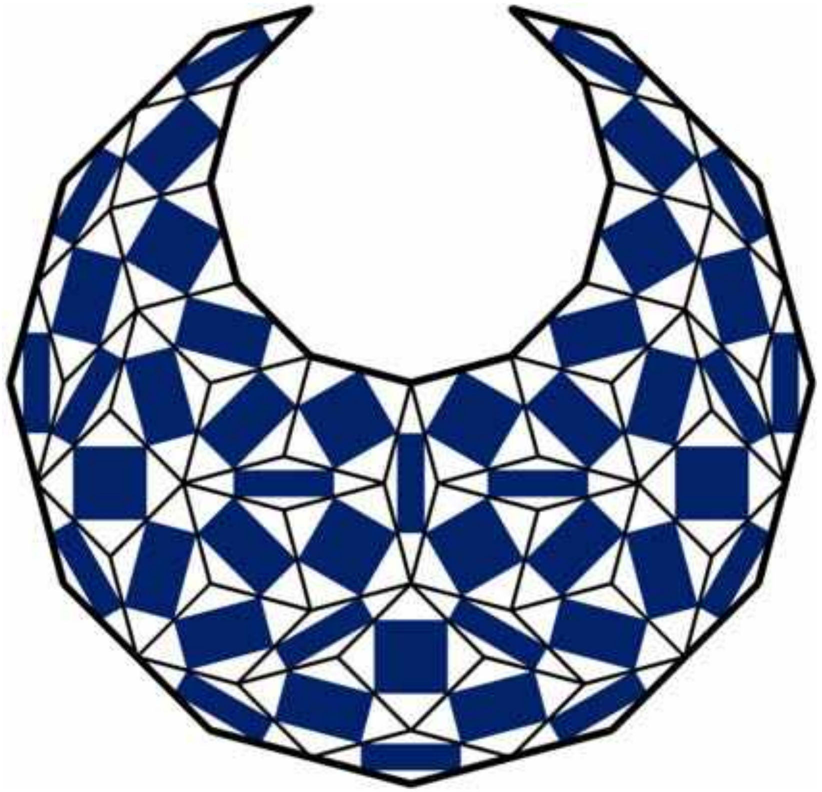


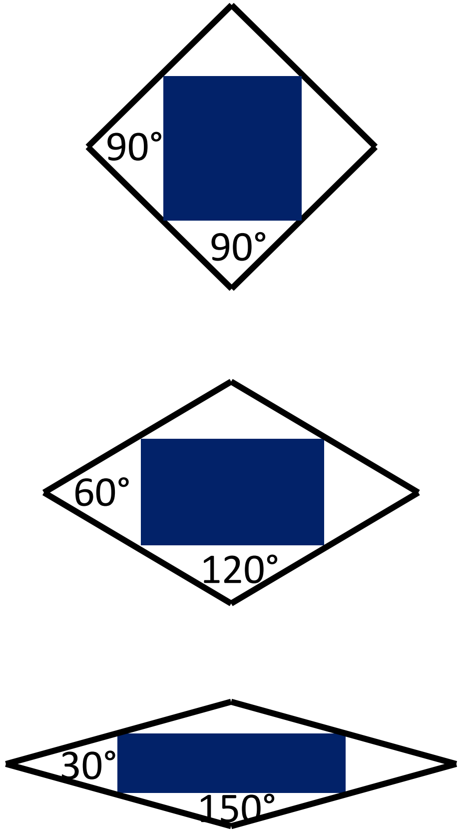
Tokyo 2020 NIPPON FESTIVAL concept video (Short version)
https://www.youtube.com/watch?v=_YVEq_GUxG0

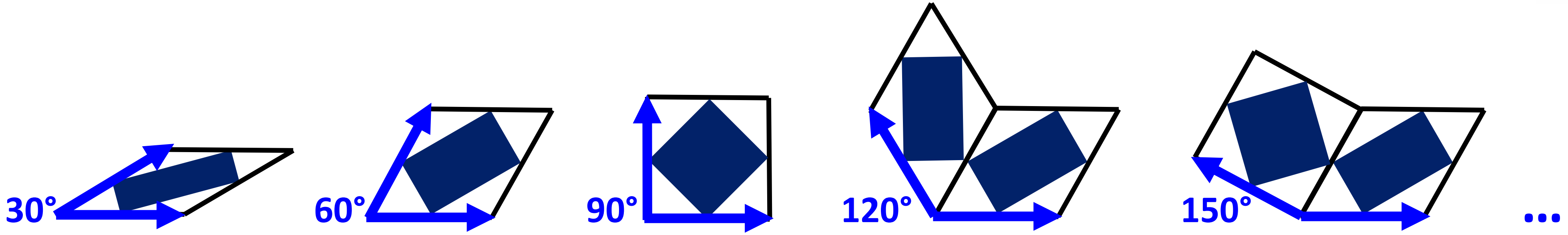




?




90°
90°
60°
120°
30°
150°


30°
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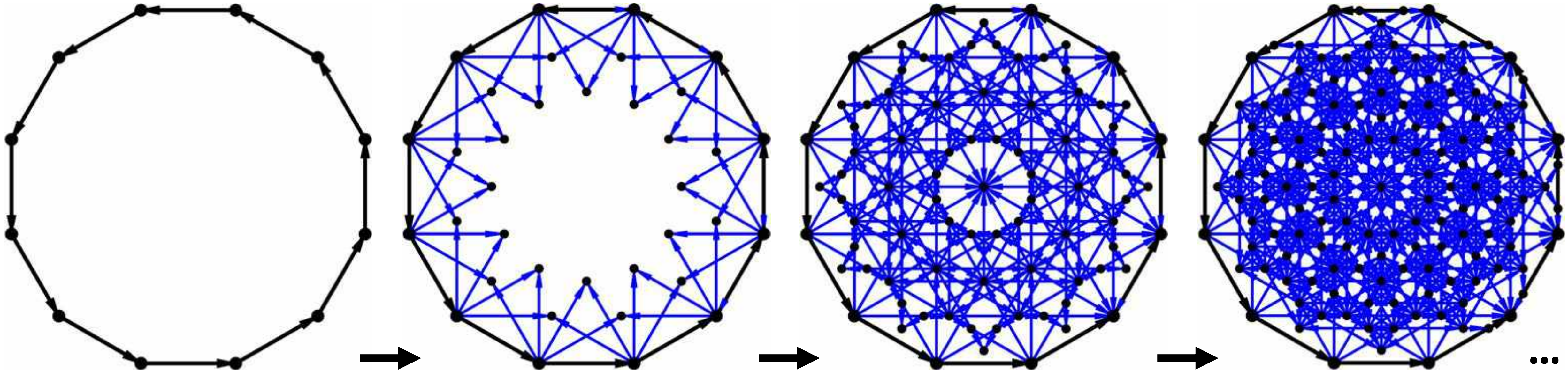
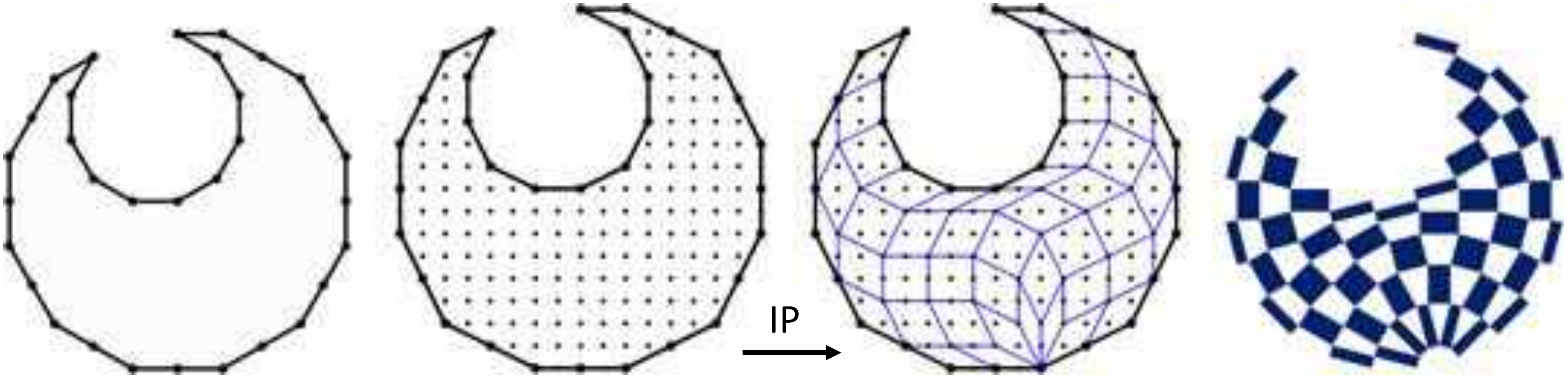

...

Pipeline

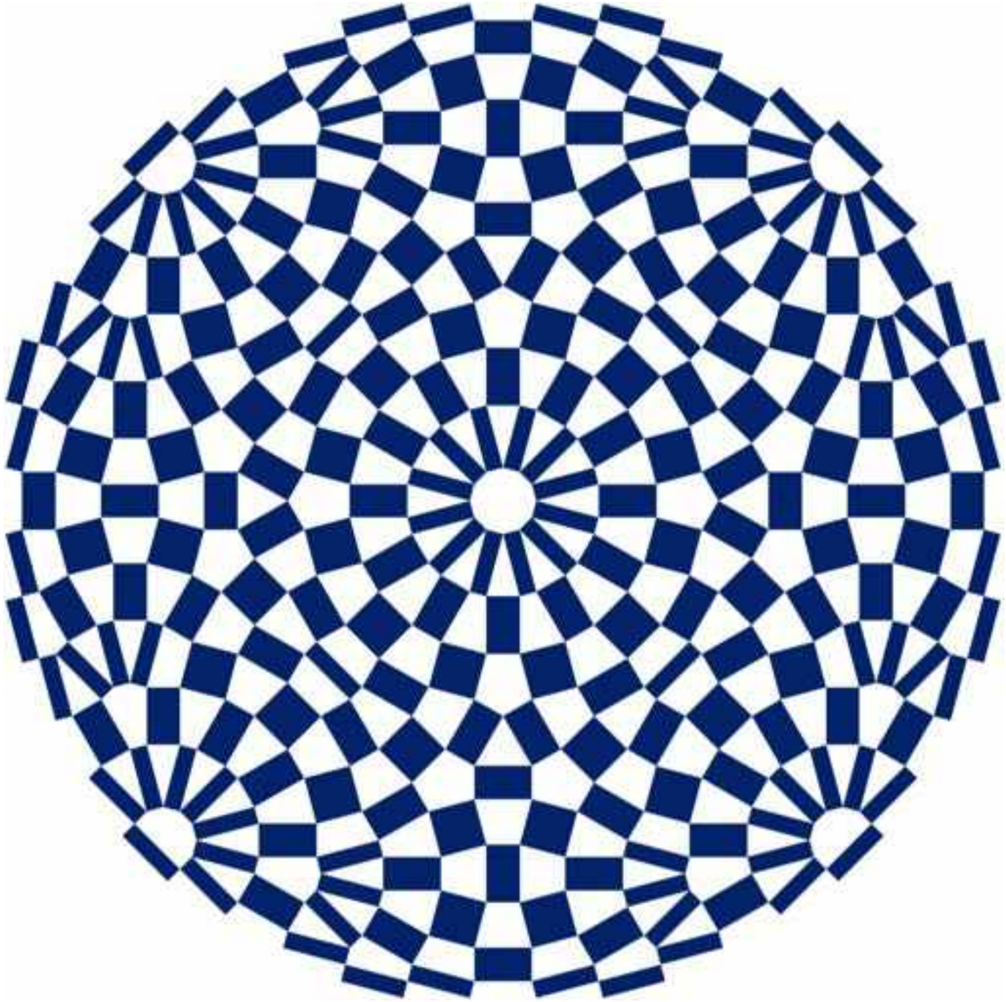


Input boundary in the Euclidean space

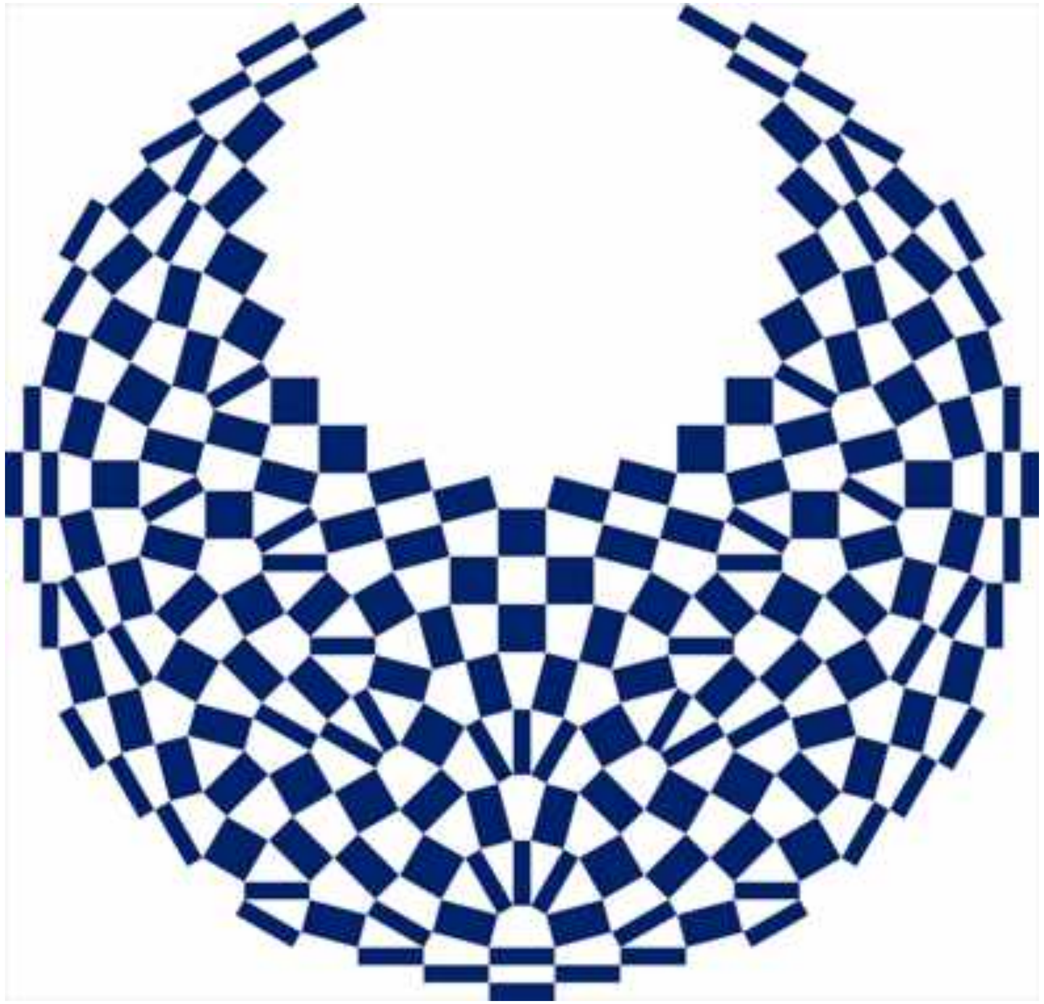
Projection space

One tiling solution found in the projection space

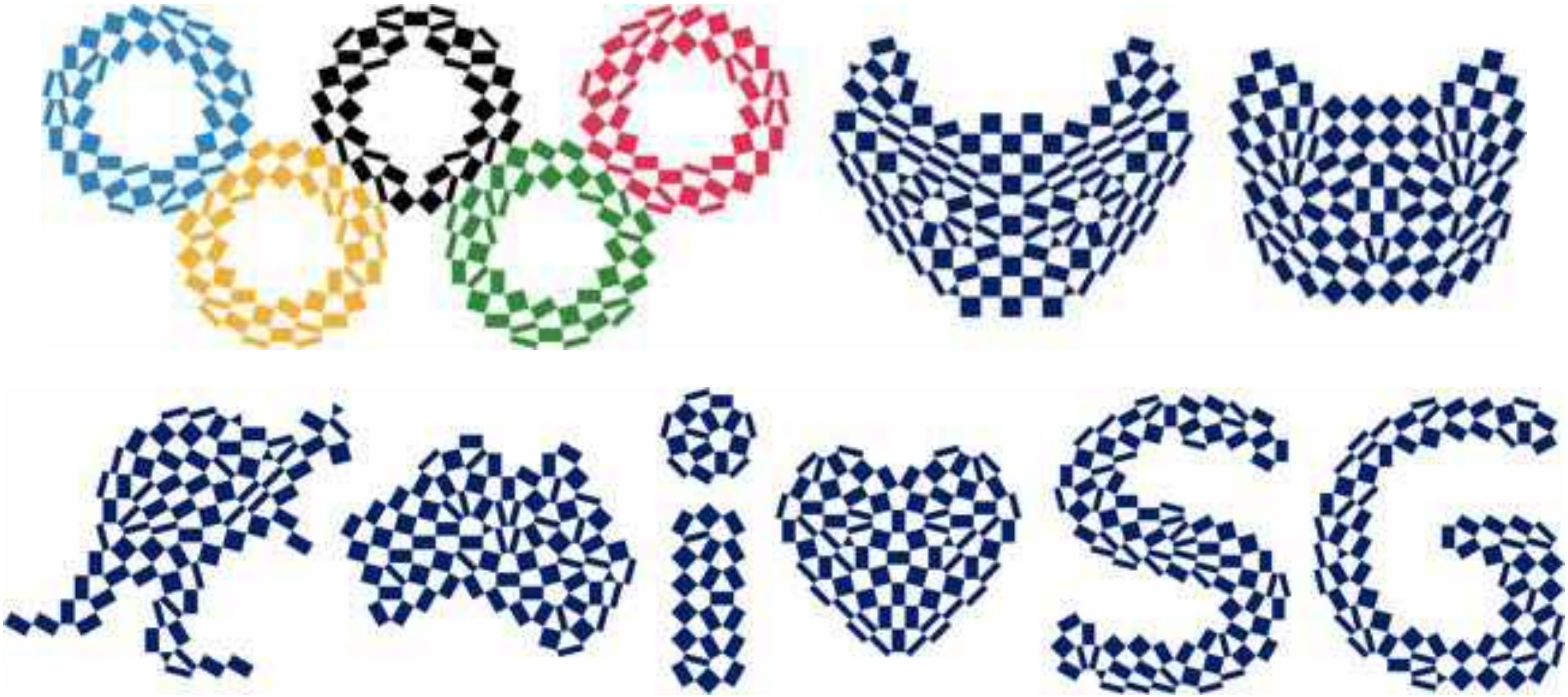
Projected back to the Euclidean space



23.75 sec

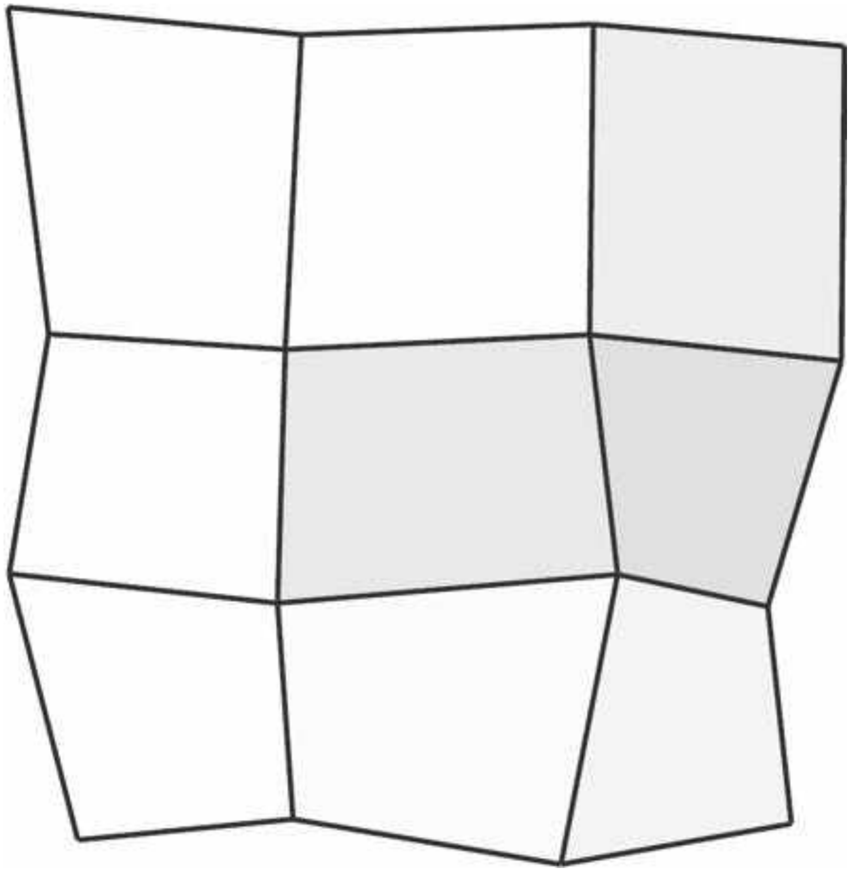


49.61 sec

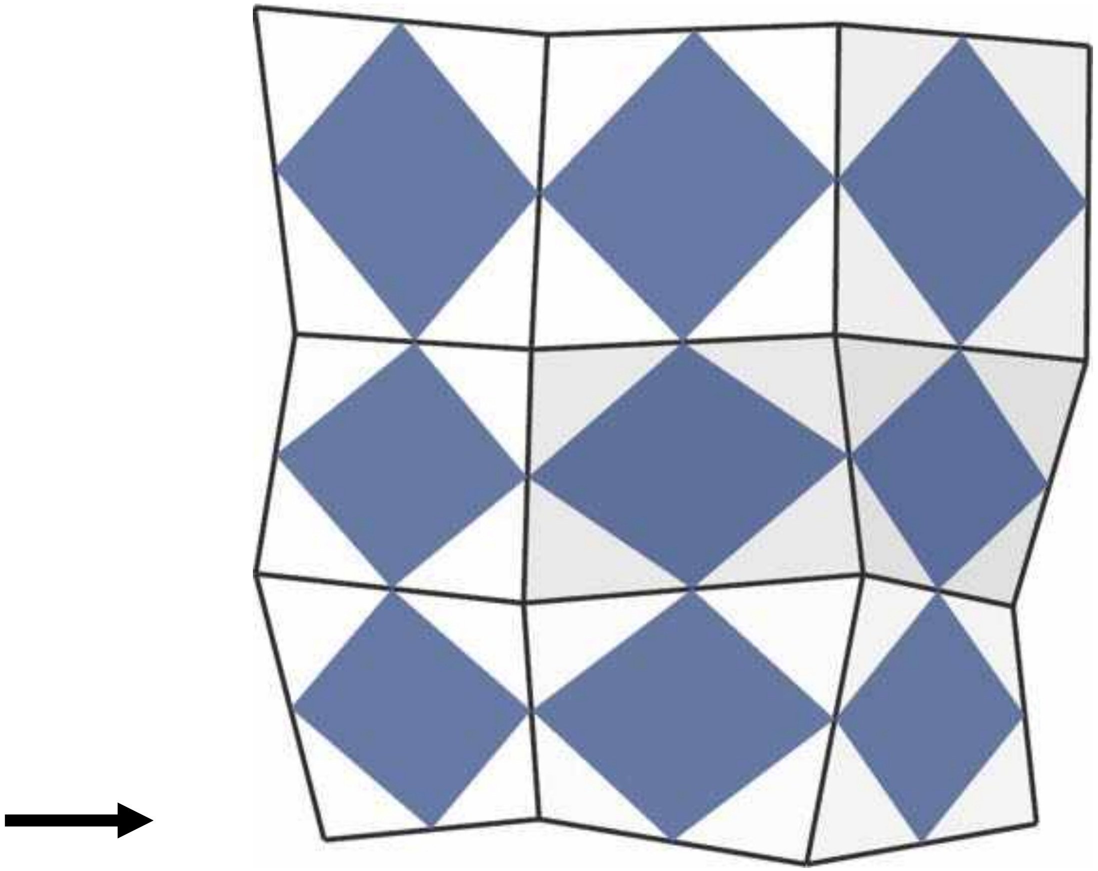


Generalization

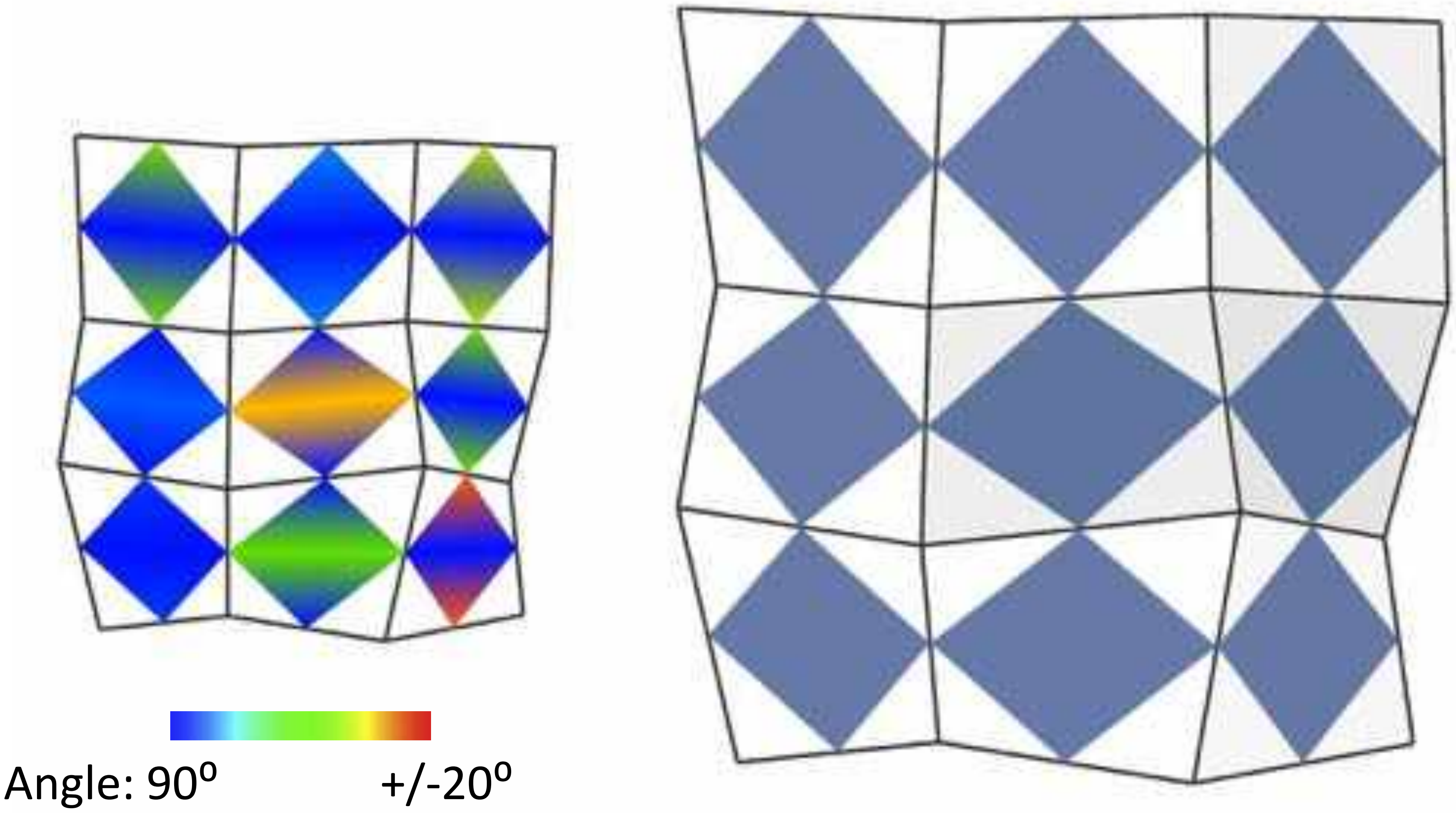
“Control mesh”

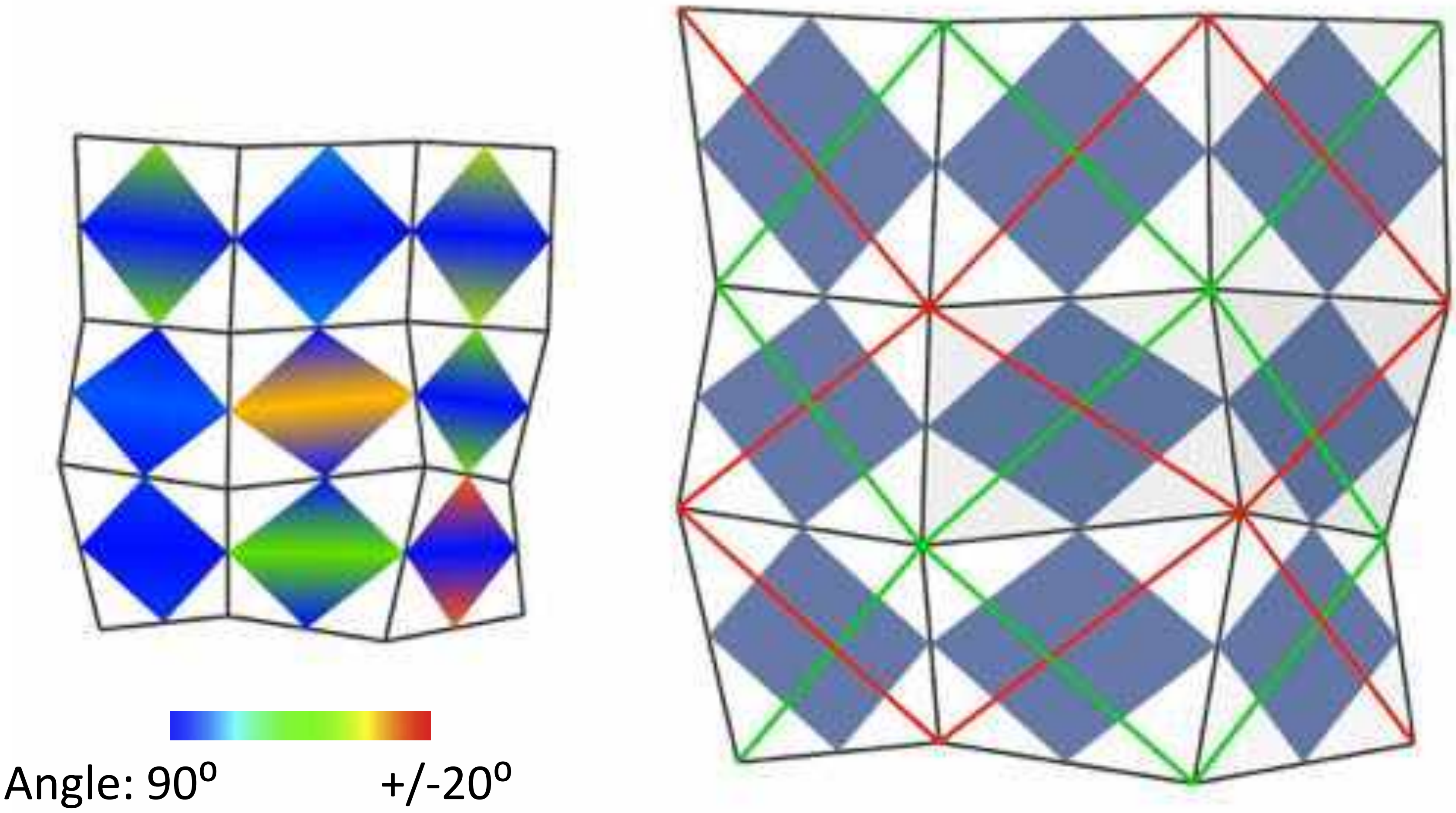


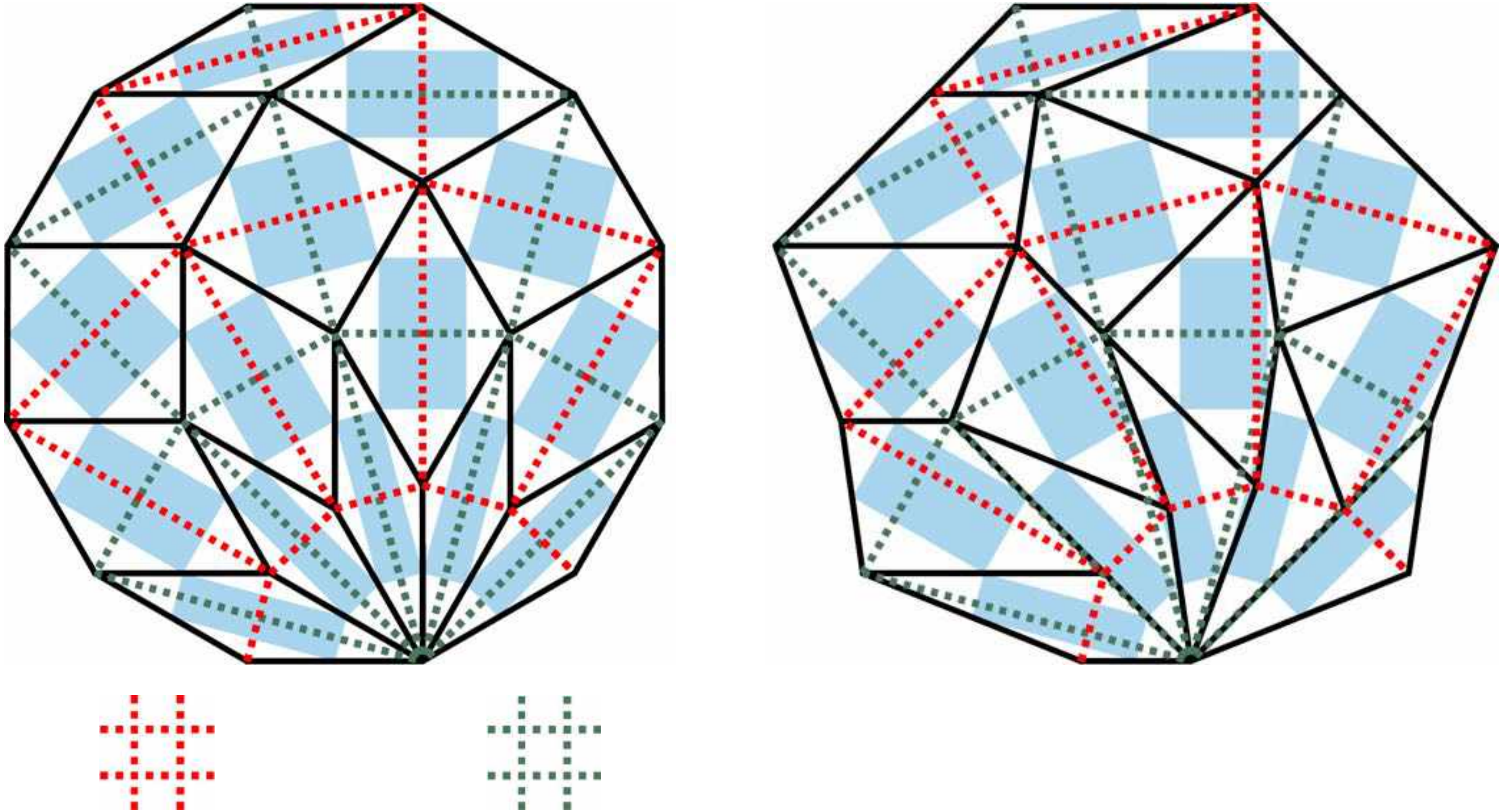
Any quad mesh



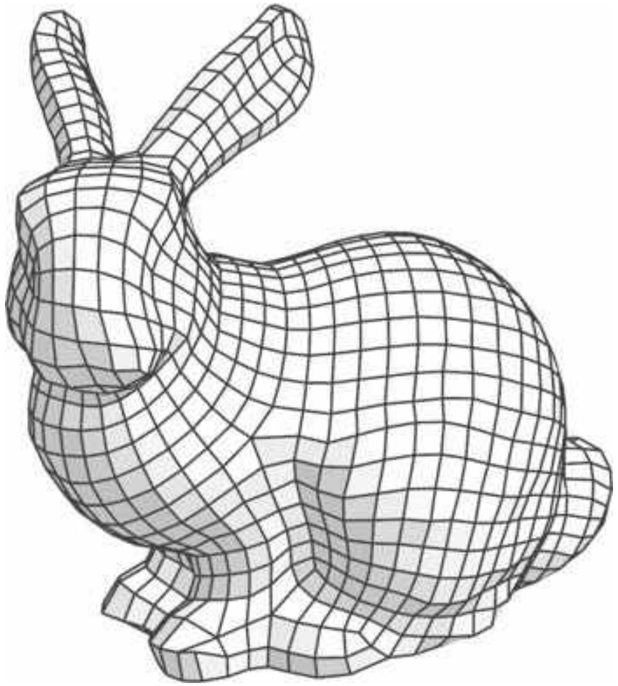
Black parallelograms


Angle: 90º
+/-20º

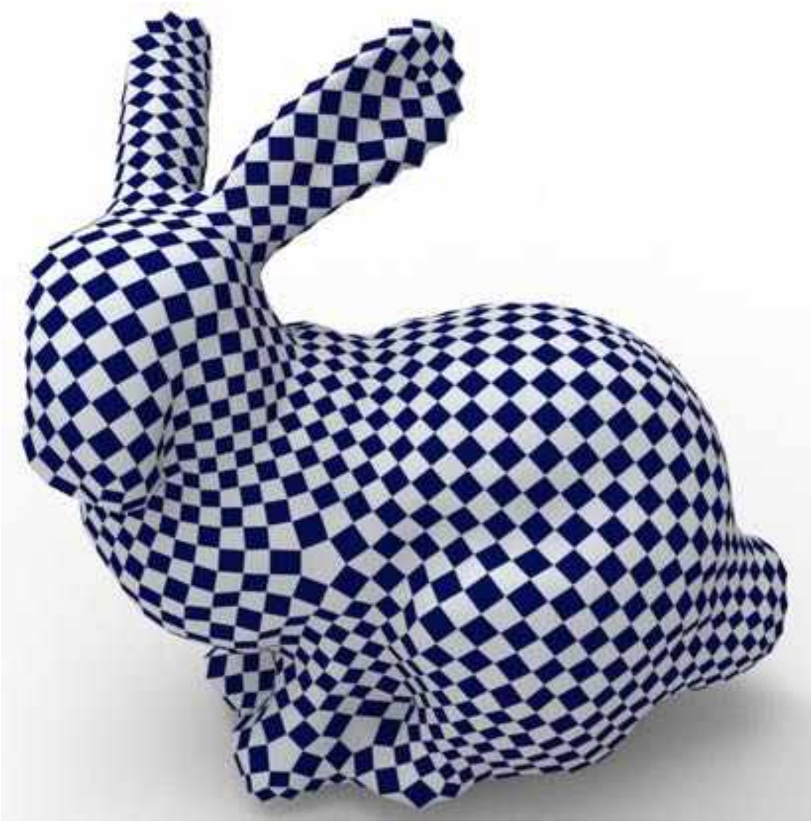

Angle: 90°
+/-20°

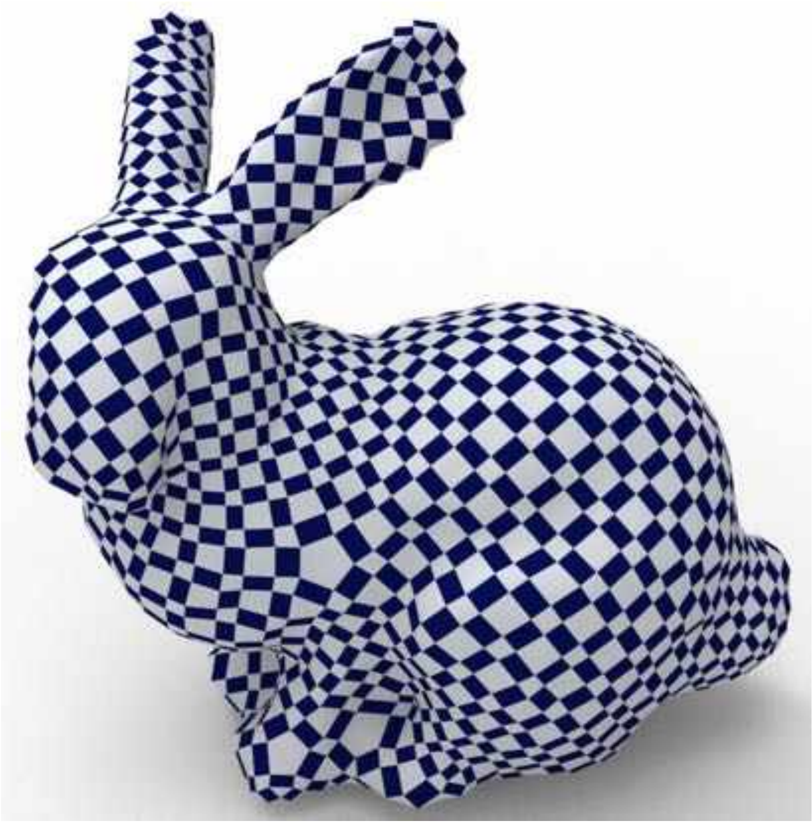


1st diagonal mesh 2nd diagonal mesh



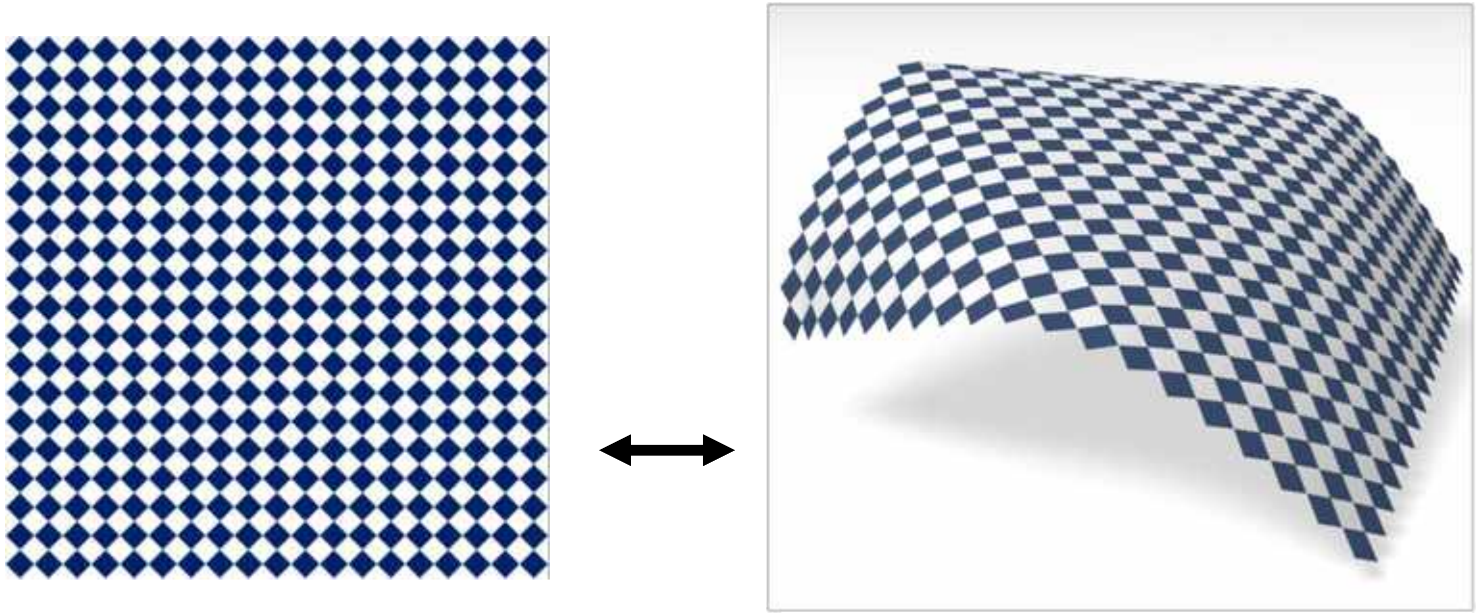
Control mesh





Developable surfaces

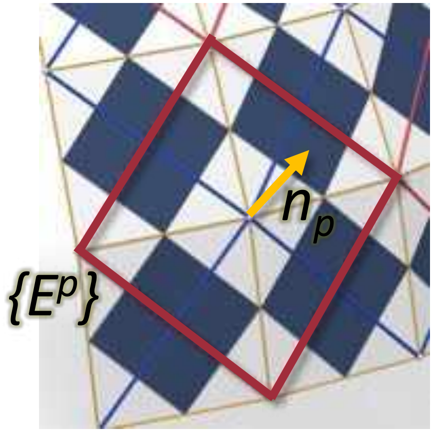
- Mapping while keeping the rectangles congruent works only if the two surface are isometric.



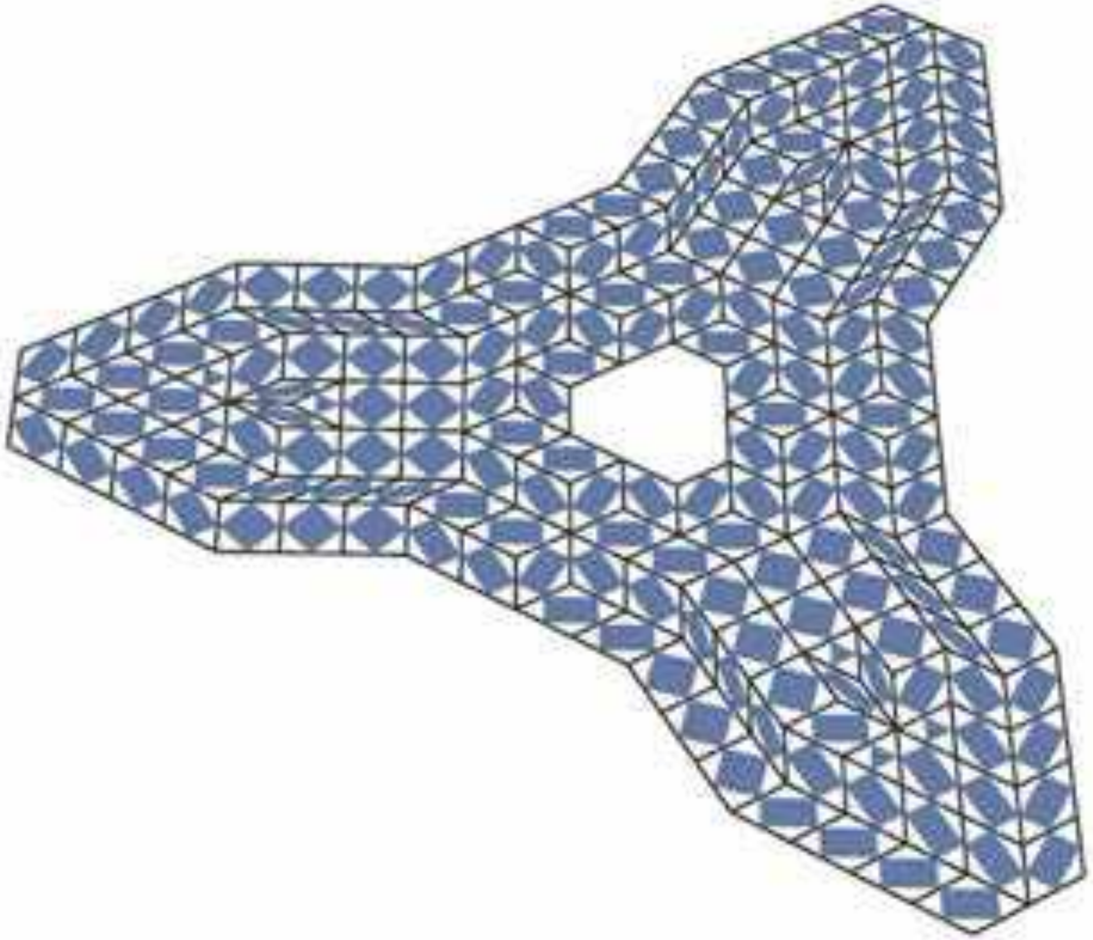
Geometric optimization

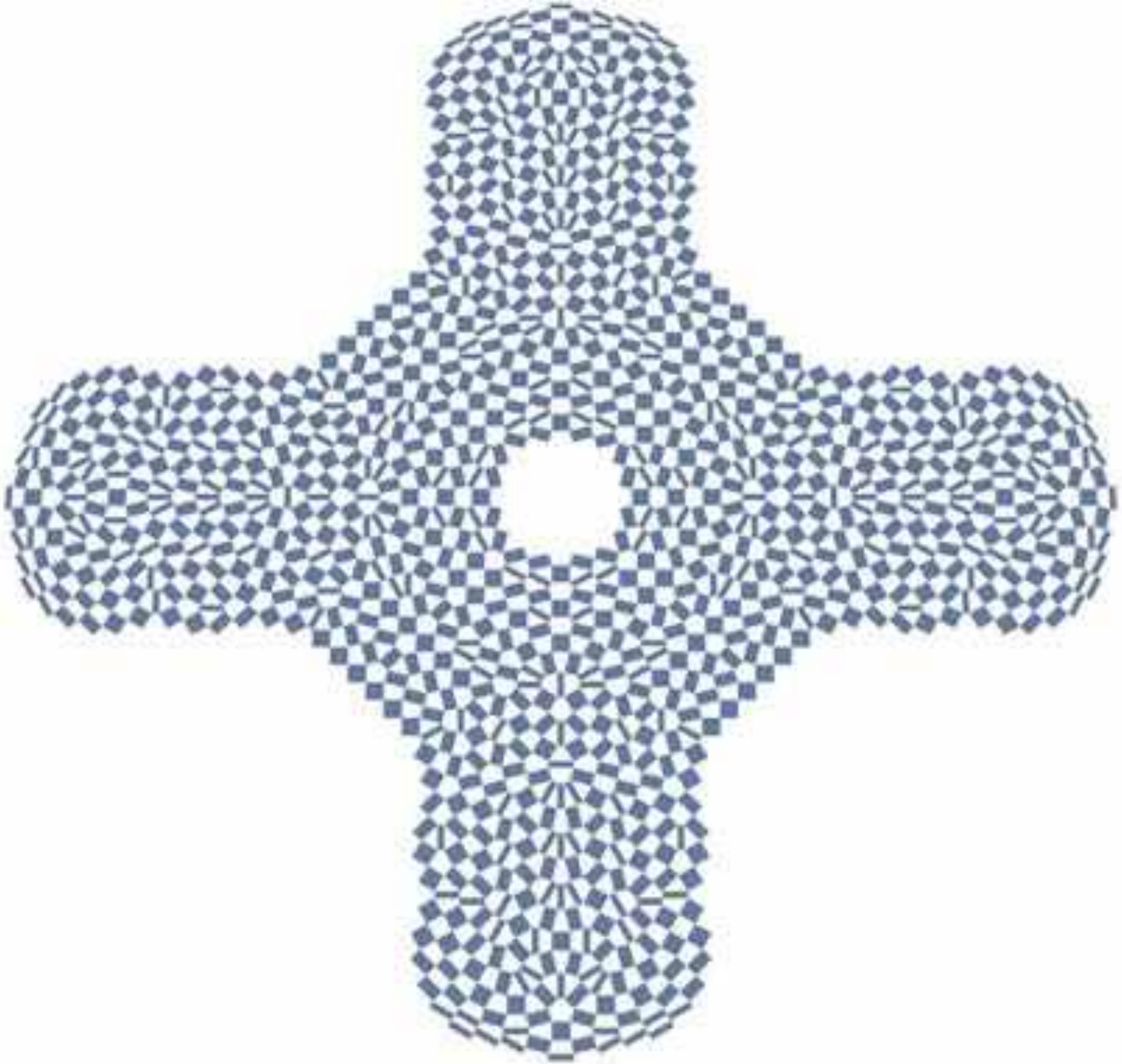
Minimize	$E_{diag\_orth} + \lambda_r E_{diag\_ratio} + \lambda_p E_{pla\_white}$
Where	$E_{diag\_orth} = \sum_{k \in F}((v_{k1} - v_{k3}) \cdot (v_{k2} - v_{k4}))^2$
	$E_{diag\_ratio} = \sum_{k \in F}((v_{k1} - v_{k3})^2 - r_k^2 (v_{k1} - v_{k3})^2)^2$
	$E_{pla\_white} = \sum_{p \in V} \sum_{(i,j) \in E^p}(n_p \cdot (v_i - v_j))^2$

v_{k1}, v_{k2}, v_{k3}, v_{k4} are vertices of quad face F_k in the control mesh

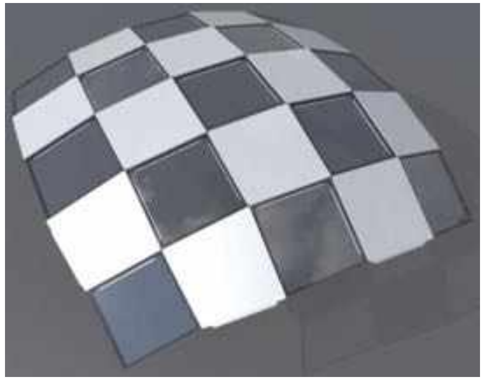


n_p is normal at v_p and E^p are diagonals surrounding v_p





Additional constraint: planar white faces



Checkerboard pattern with black squares and planar white faces

Quad-Mesh Based Isometric Mappings and Developable Surfaces

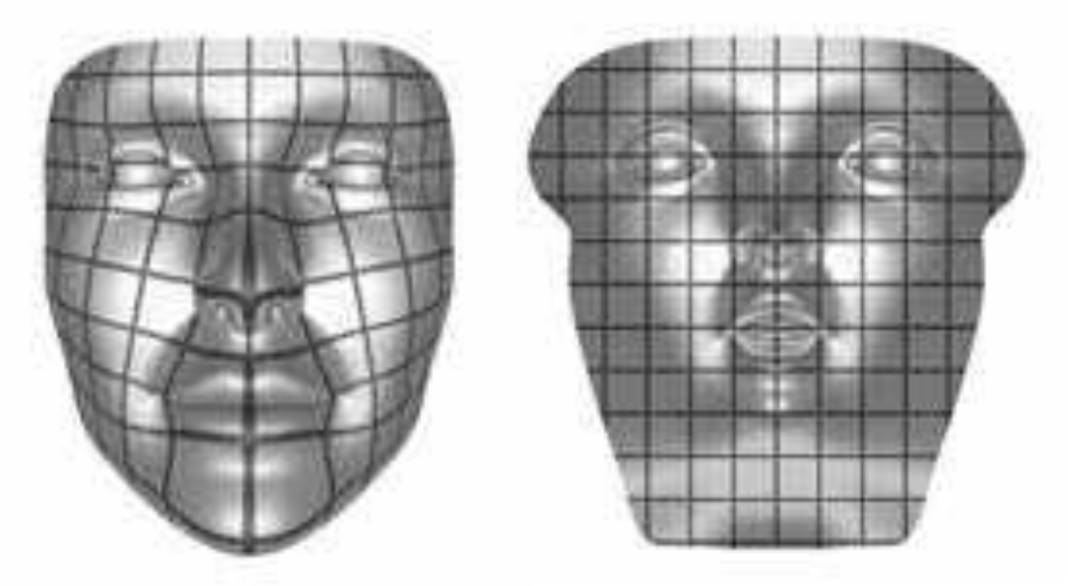
(SIGGRAPH 2020)

with Cheng Wang, Florian Rist, Johannes Wallner, and Helmut Pottmann

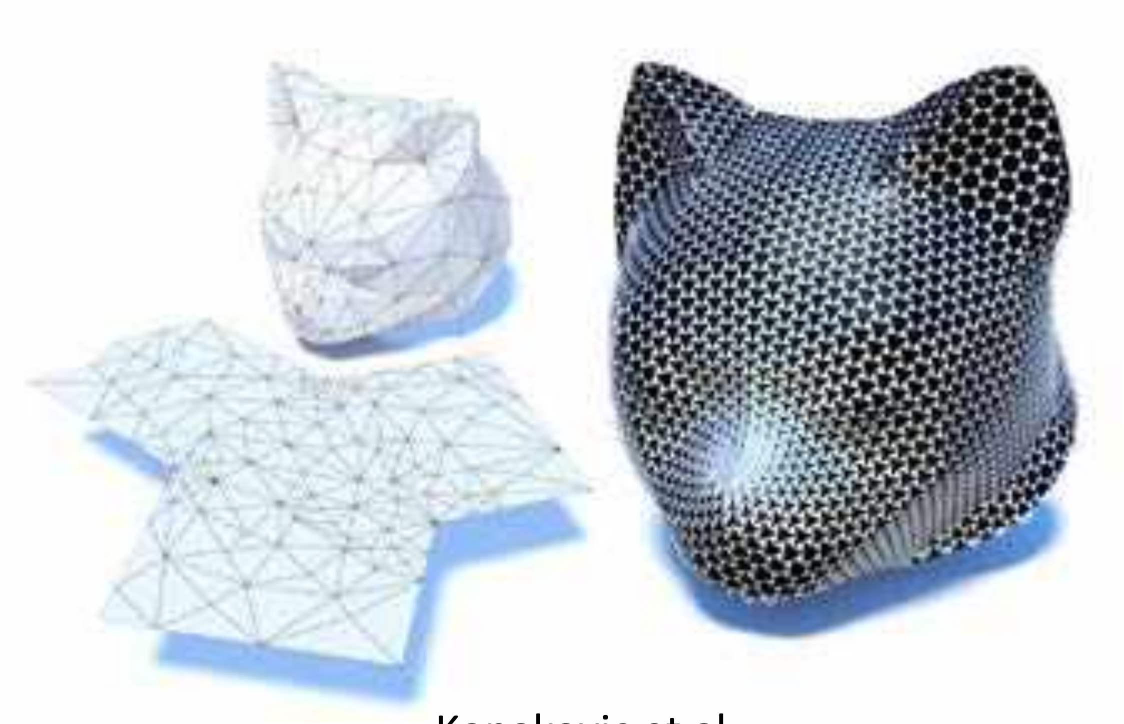
motivation

- Important topics such as mesh parametrization, texture mapping, character animation, fabrication, … are based on **special surface-to-surface maps**

  - Conformal map (angle preserving)



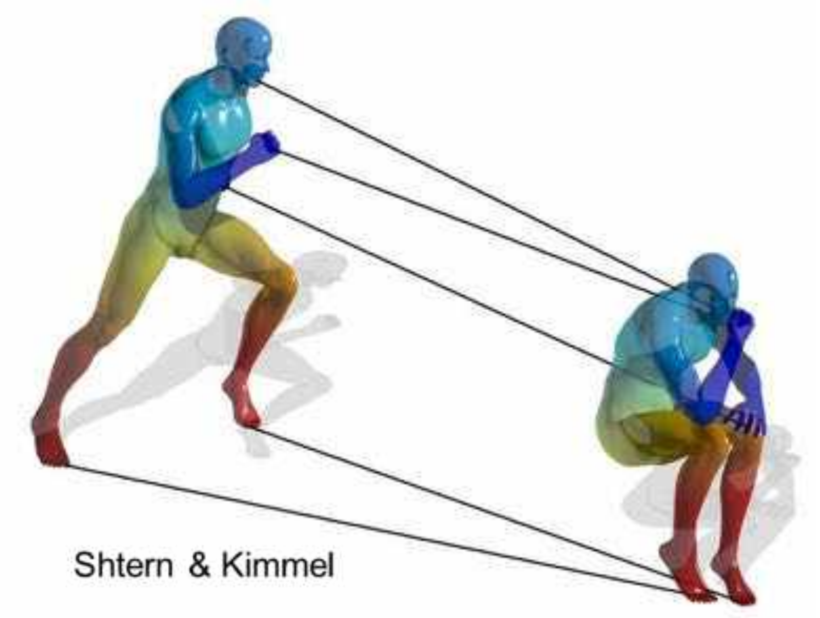
K. Crane

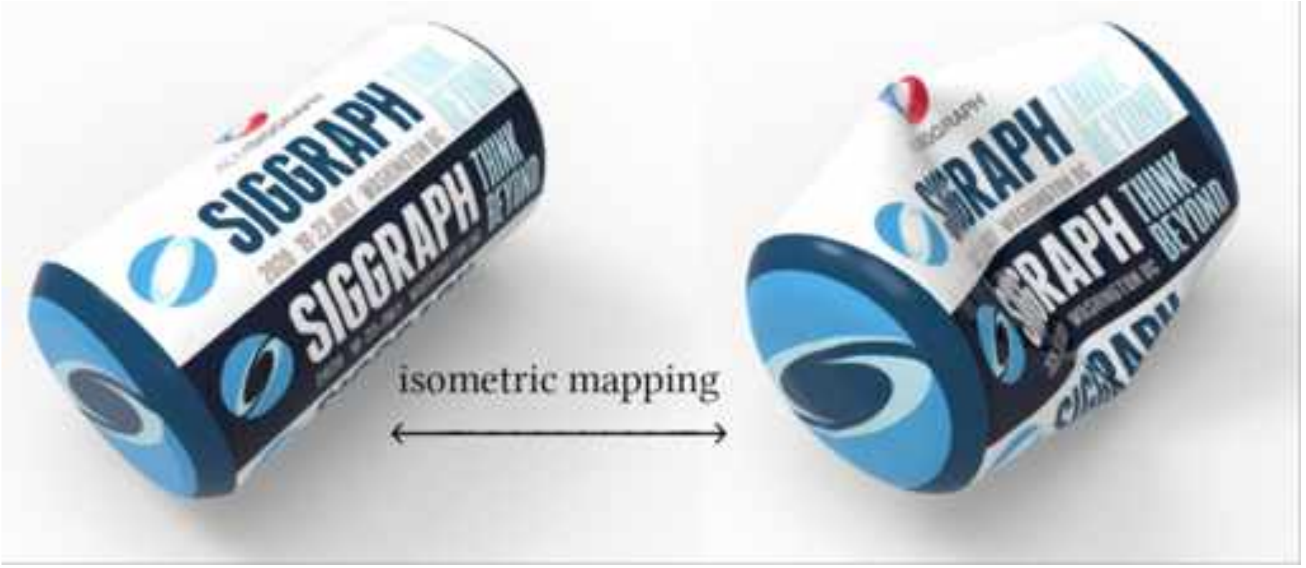


Konakovic et al.

motivation

- isometric maps (length and angle preserving = pure bending, no stretching)

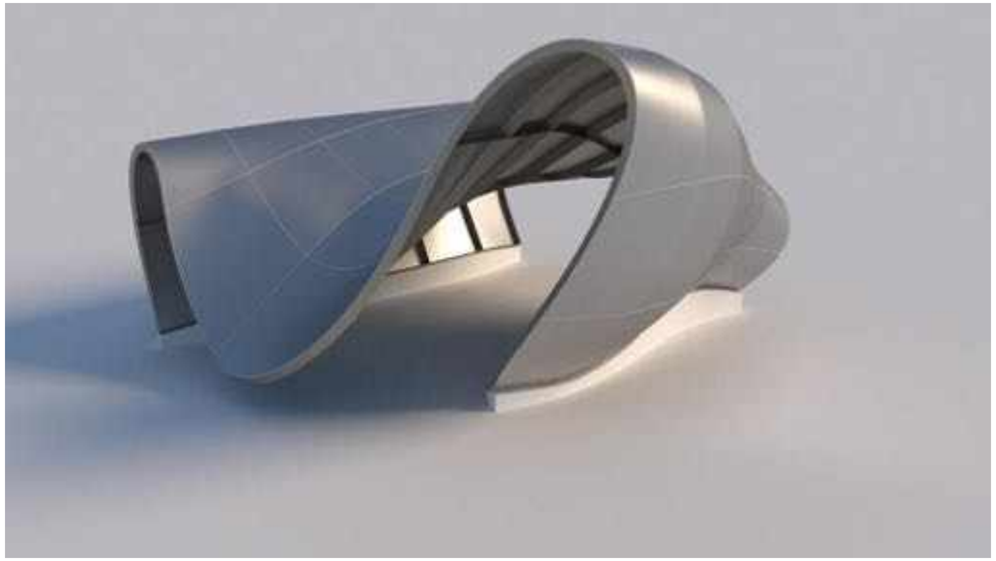


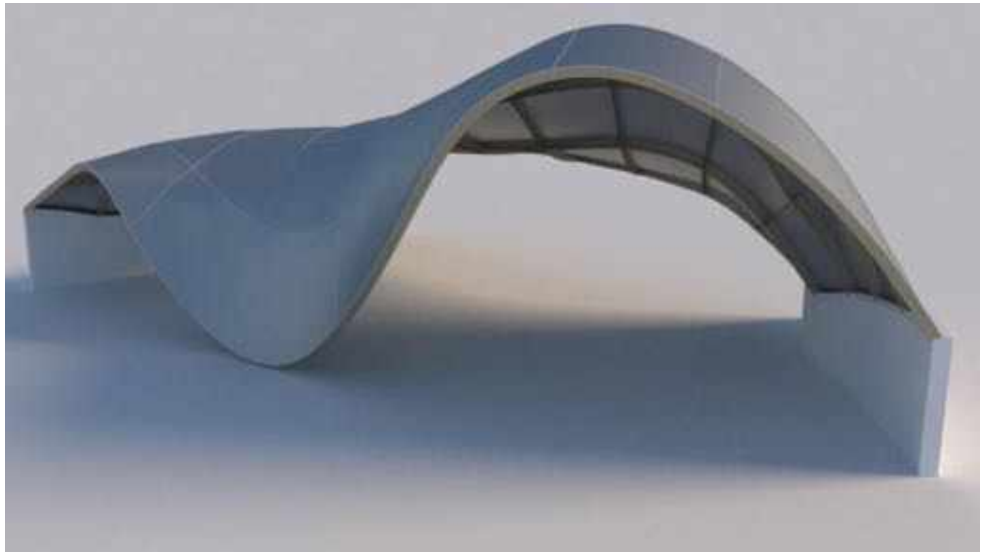


- as isometric as possible maps [Sorkine & Alexa, 2007],....

Quad meshes

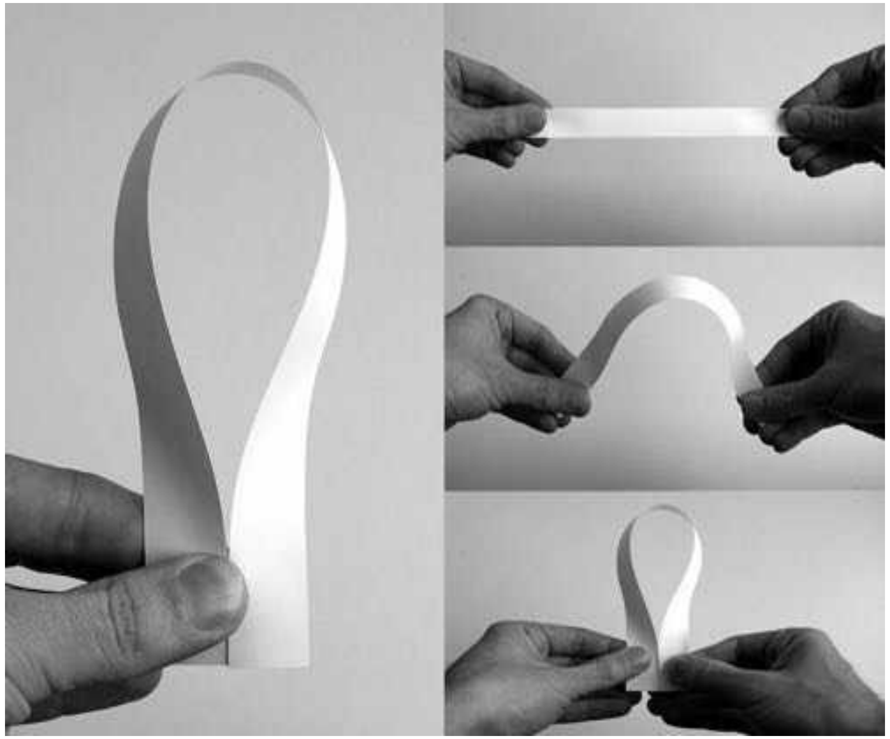
- Most research employs triangle meshes
- We present a **simple approach based on quad meshes**
- Focus on isometric maps and developable surfaces





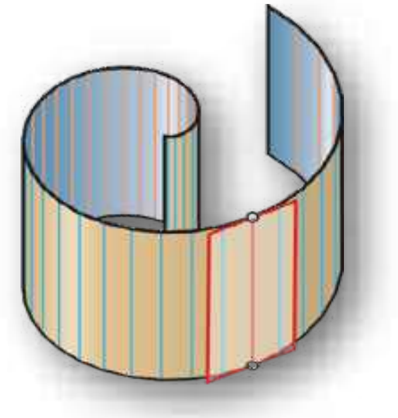
Developable surfaces

working with originally flat materials
which bend, but do not stretch

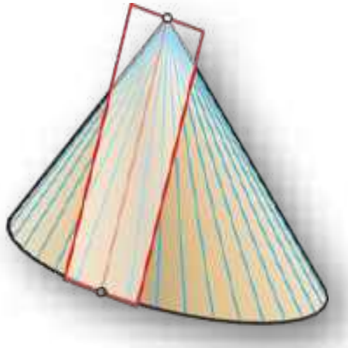


developable surfaces: Piecewise ruled

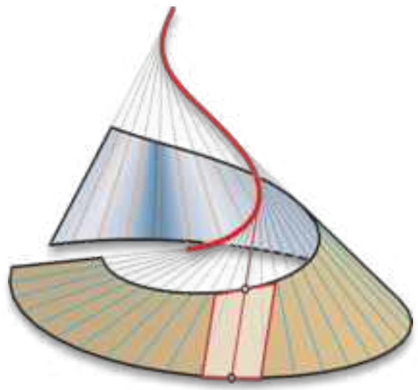
Developable surfaces are composed of planes and special ruled surfaces:



cylinder



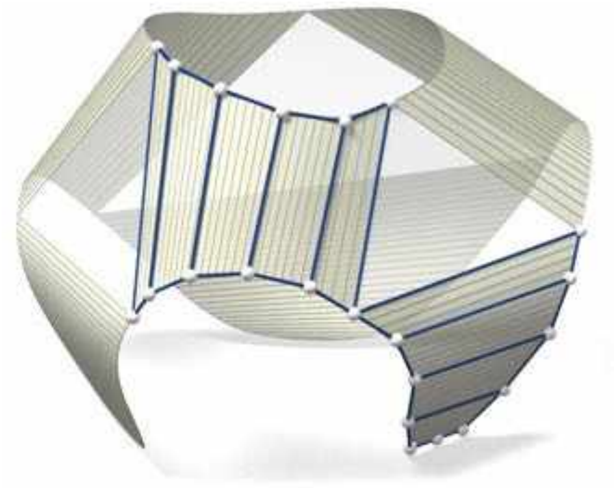
cone

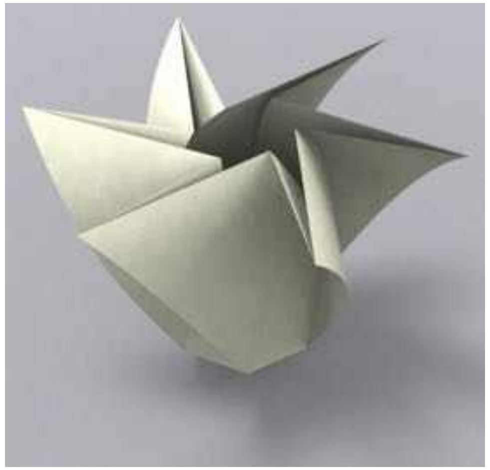


tangent surface
of a space curve

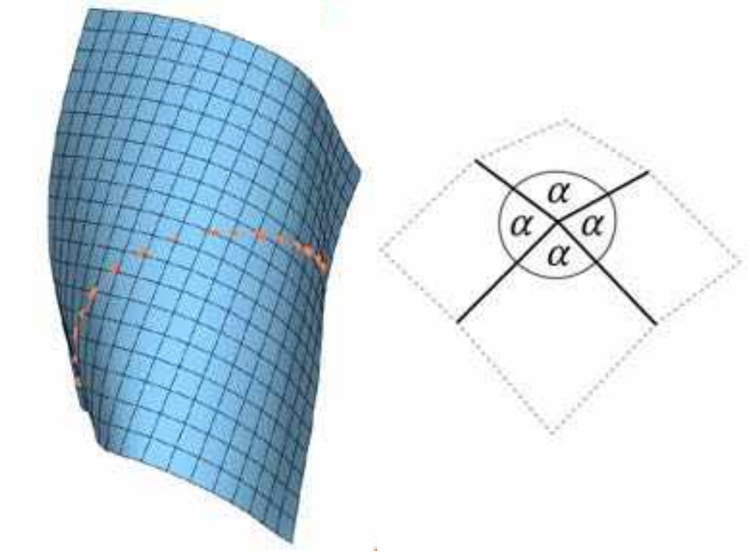
Most discrete models are based on the rulings, but the ruling pattern changes under isometric deformation. Our discrete model avoid rulings!

Recent work on modeling with developable surfaces



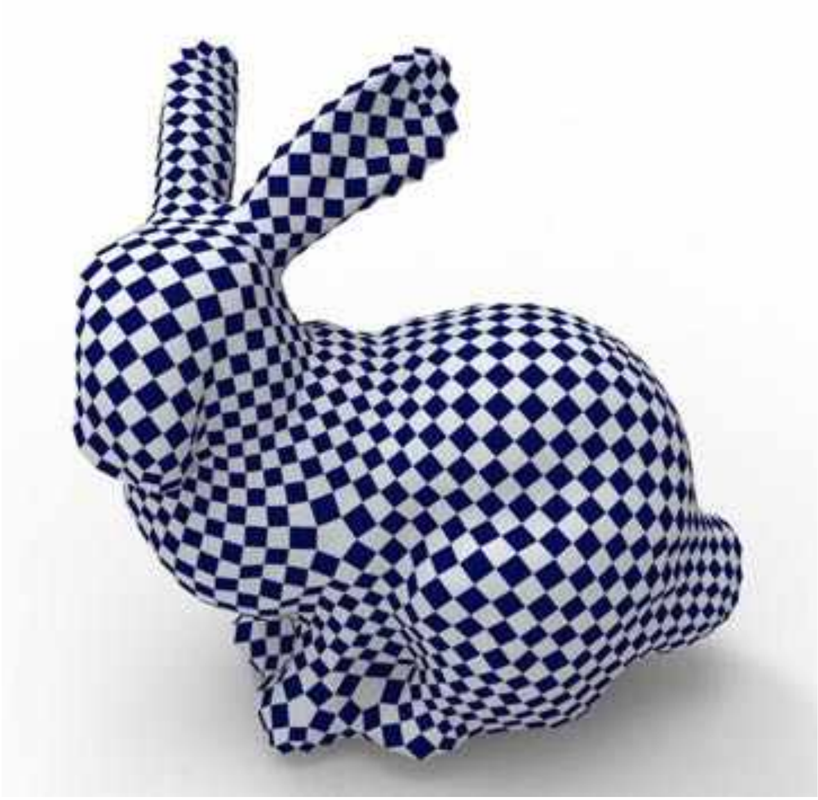


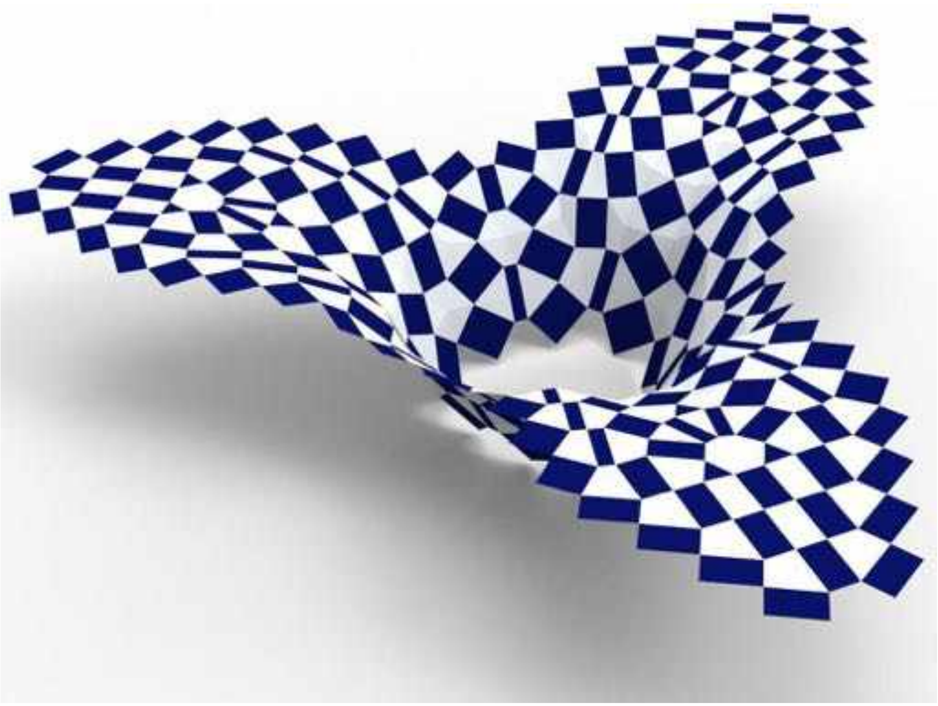
- Ruling based approach for B-spline surfaces (Tang et al. 2016)
- Developability of triangle meshes (Stein et al 2018)
- Orthogonal geodesic nets, (Rabinovich et al. 2018,2019)



Checkerboard patterns from black rectangles

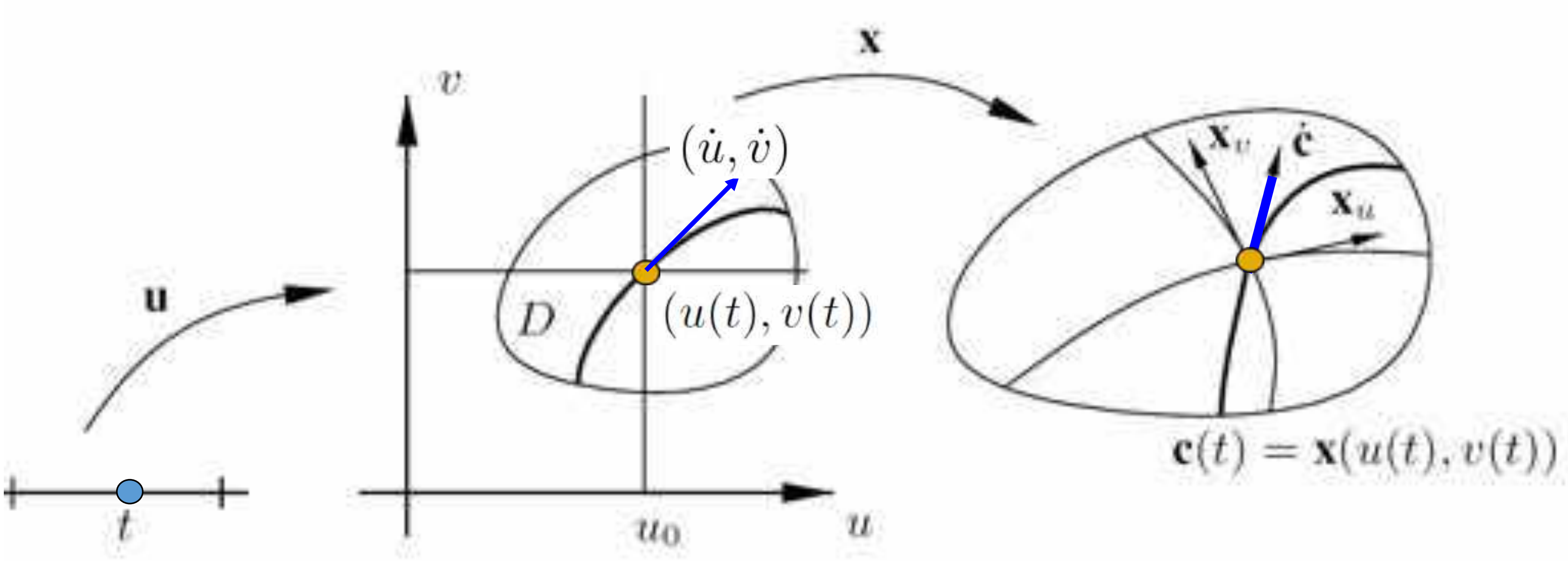
- Our approach is inspired by and generalizes work on checkerboard patterns from black rectangles [Peng et al. 2019}





Computing surface – to – surface maps via quad meshes

Curves on surfaces

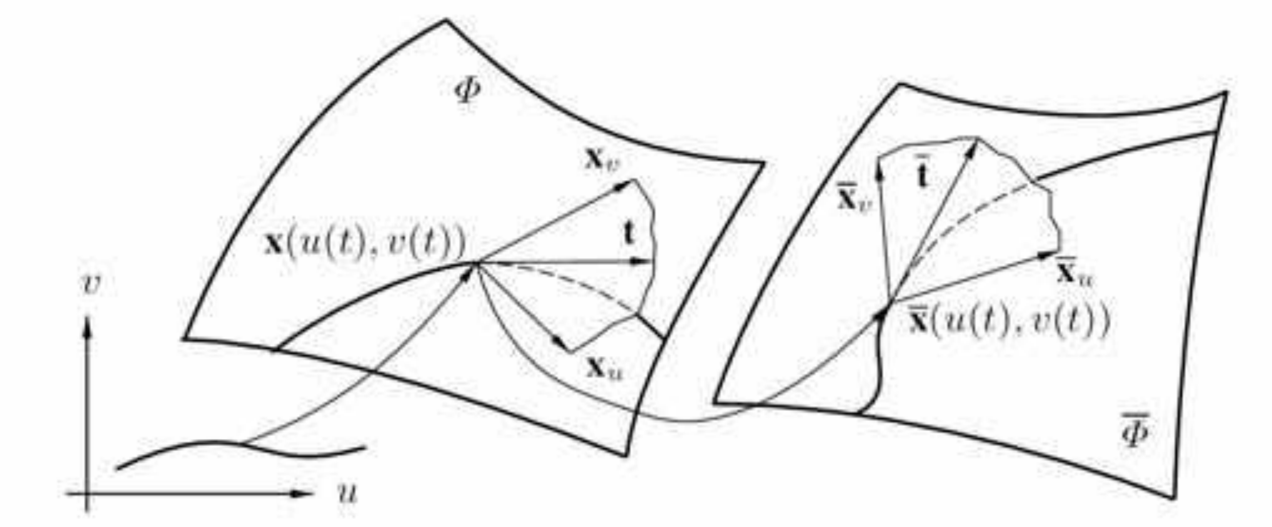


$$\dot{\mathbf{c}}(t) = \frac{d\mathbf{c}}{dt} = \frac{\partial \mathbf{x}}{\partial u}\frac{du}{dt} + \frac{\partial \mathbf{x}}{\partial v}\frac{dv}{dt} = \dot{u}\mathbf{x}_u + \dot{v}\mathbf{x}_v$$

map between two surfaces

- map via equal parameter values $\mathbf{x}(u,v) \mapsto \bar{\mathbf{x}}(u,v)$
- derivative map is **linear**

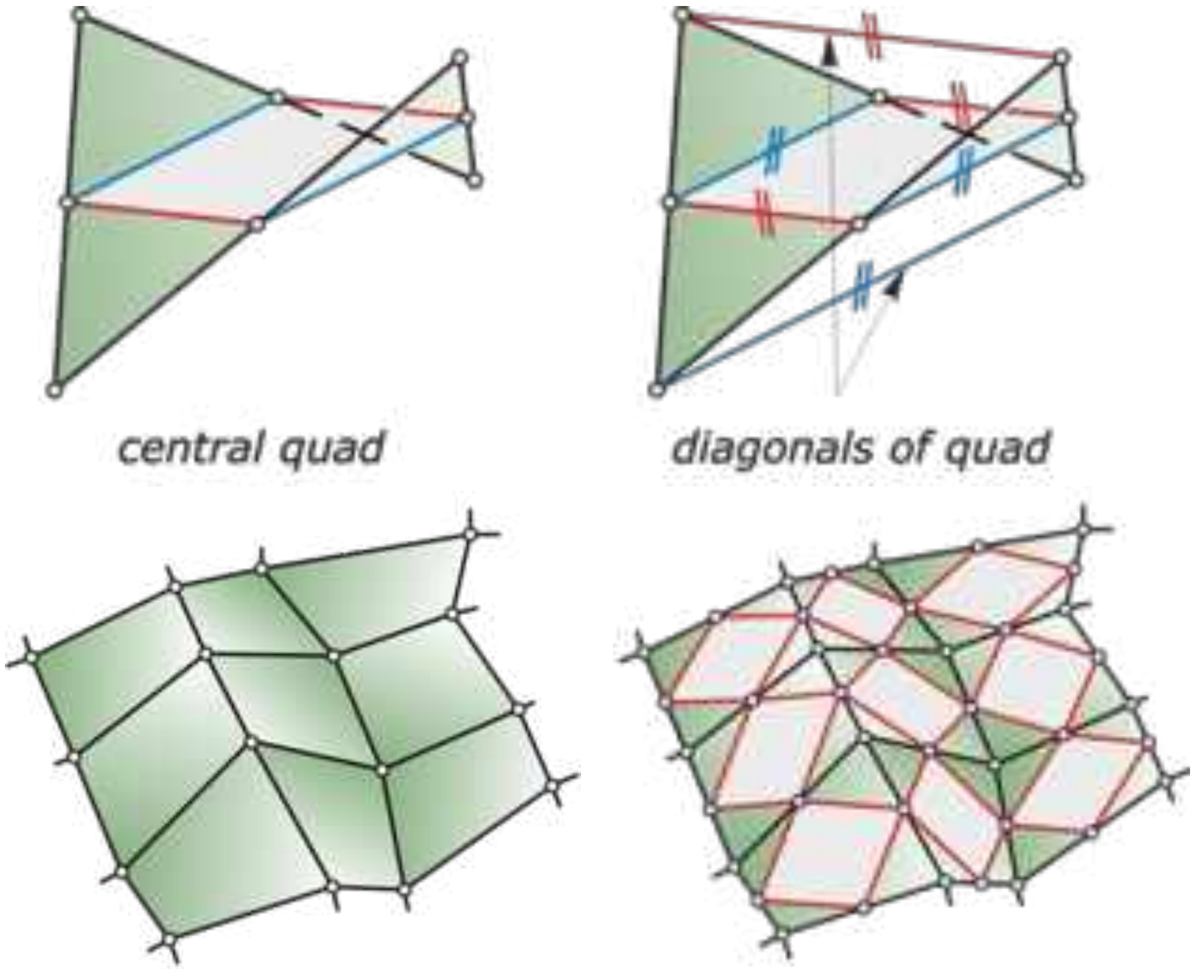
$$\mathbf{t} = \dot{\mathbf{c}} = \dot{u}\mathbf{x}_u + \dot{v}\mathbf{x}_v \longrightarrow \bar{\mathbf{t}} = \dot{\bar{\mathbf{c}}} = \dot{u}\bar{\mathbf{x}}_u + \dot{v}\bar{\mathbf{x}}_v$$



- **isometric map: derivative map = rigid body motion**
- conformal map: derivative map = similarity (rigid body motion + uniform scaling)

Mid-edge subdivision of a quad mesh

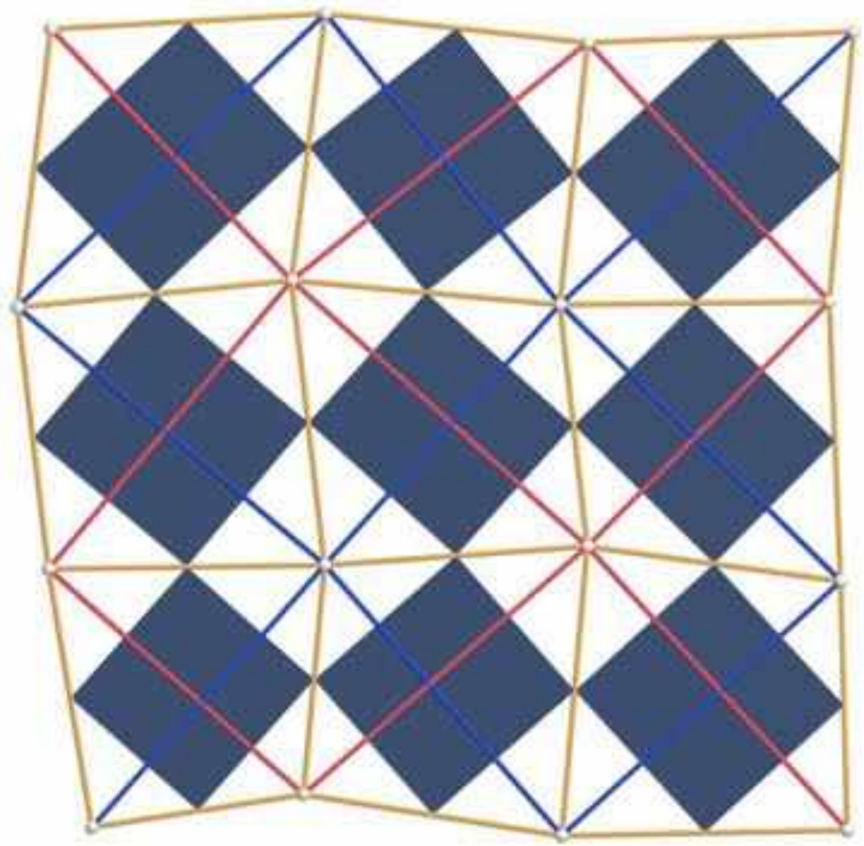
- Connecting edge midpoints of a quad Q yields a parallelogram (central quad): its edges are parallel to the diagonals of Q and have half their length
- Application of mid-edge subdivision to a quad mesh generates a checkerboard pattern (CBP) of parallelograms



meshes which play a role

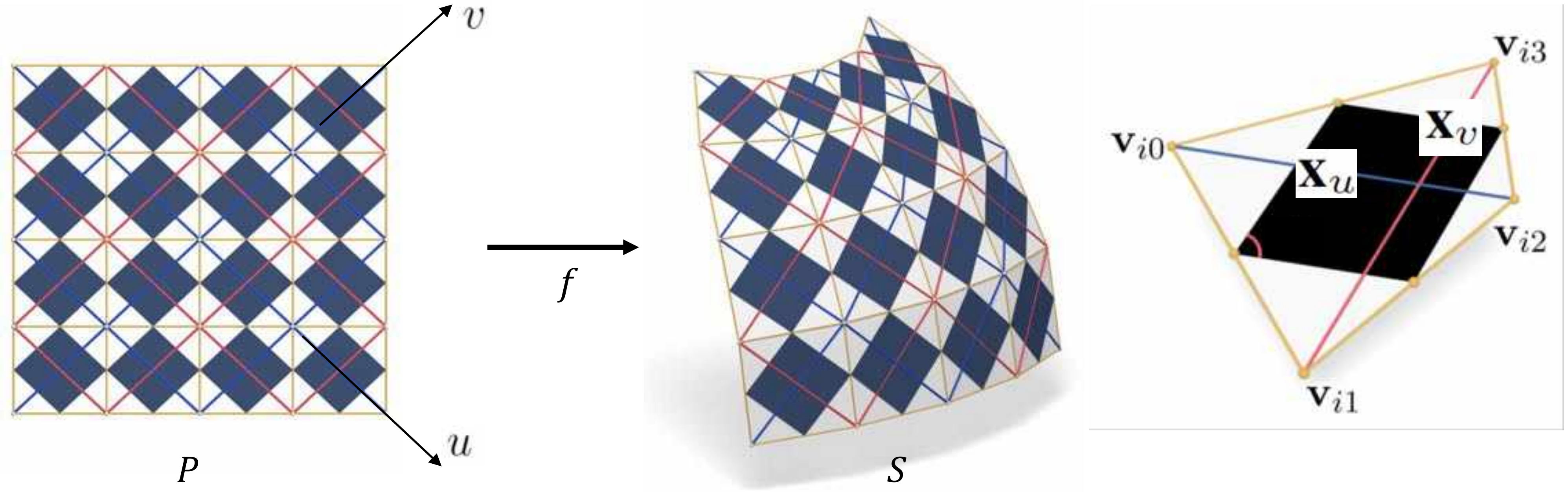
Several meshes play a role:

- The original quad mesh (called control mesh), yellow
- The result of mid-edge subdivision = checkerboard pattern of parallelograms (CBP)
- The two diagonal meshes (blue, red) of the control mesh



Regular grid as parameter domain

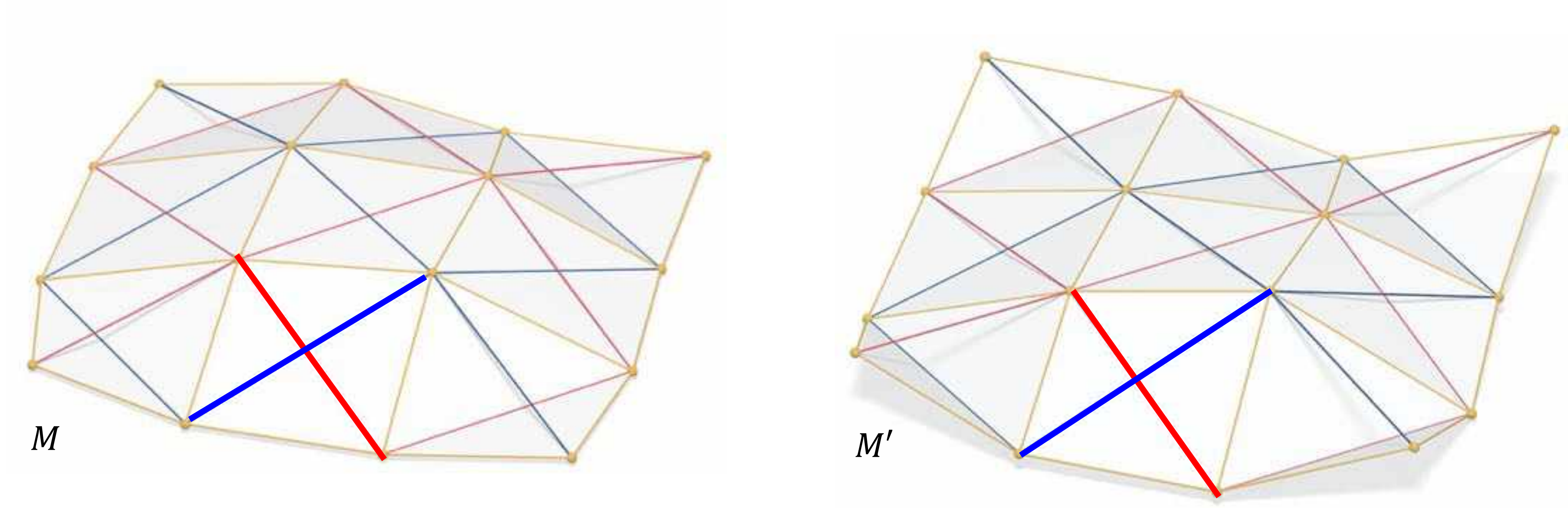
- view a regular grid as parameter domain of the control mesh C and the CBP
- obtain a discrete map f from the parameter plane P to a surface S
- The parallelograms in the CBP correspond to squares in P and are related to them by affine maps: *discrete derivative maps from parameter domain to the surface*



Quad mesh deformation via CBP

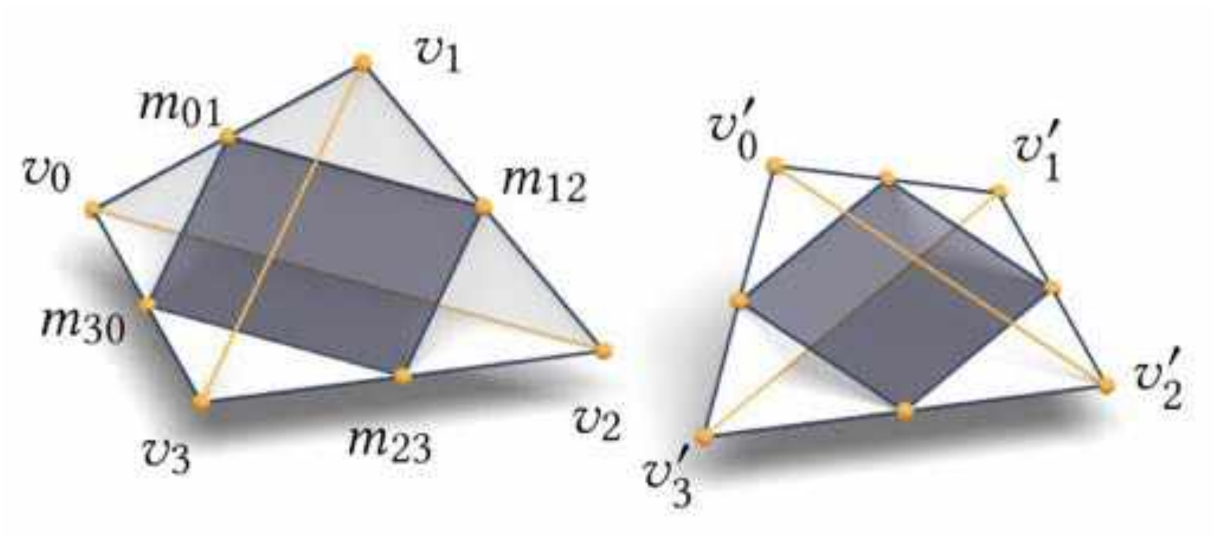
Input: quad mesh M

Goal: deform M under certain constraints to a mesh M', in particular by a conformal map or an isometric map



Discrete conformal maps via CBP

conformal map: corresponding parallelograms in the CBP are related by a similarity, i.e., diagonals in corresponding quads of M and M' possess the same angle and length ratio



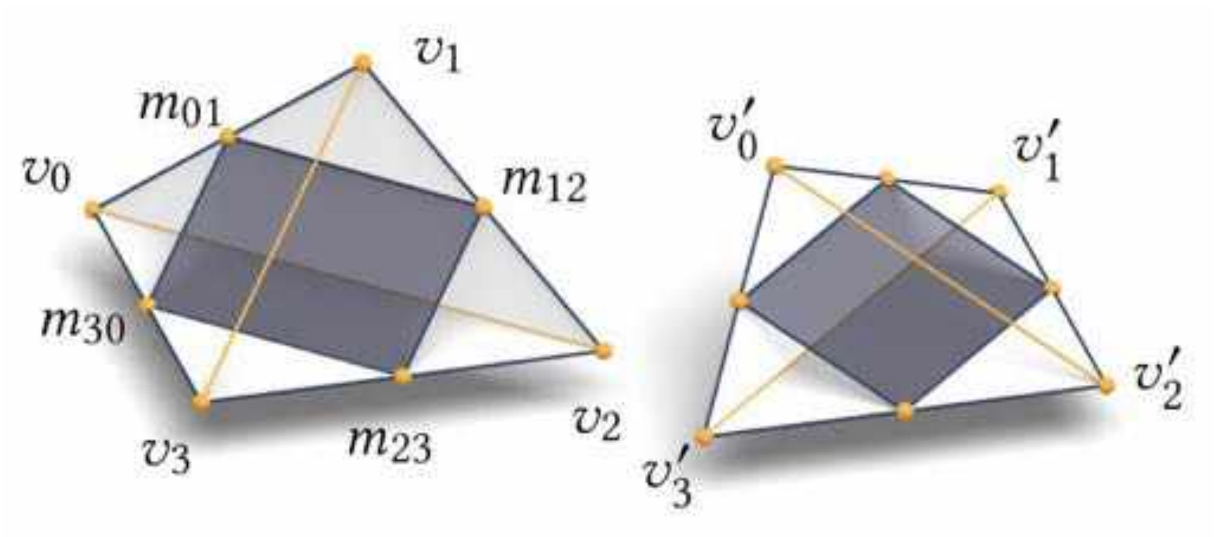
$$c_{conf,0}(f) = \lambda_f \|v_0 - v_2\|^2 - \|v_0' - v_2'\|^2 = 0,$$

$$c_{conf,1}(f) = \lambda_f \|v_1 - v_3\|^2 - \|v_1' - v_3'\|^2 = 0,$$

$$c_{conf,2}(f) = \lambda_f \langle v_0 - v_2, v_1 - v_3\rangle - \langle v_0' - v_2', v_1' - v_3'\rangle = 0$$

Discrete isometric maps via CBP

isometric map: corresponding parallelograms in the CBP are congruent, i.e., diagonals in corresponding quads of M and M' possess the same angle and lengths



$$c_{iso,0}(f) = \|v_0 - v_2\|^2 - \|v'_0 - v'_2\|^2 = 0,$$

$$c_{iso,1}(f) = \|v_1 - v_3\|^2 - \|v'_1 - v'_3\|^2 = 0,$$

$$c_{iso,2}(f) = \langle v_0 - v_2, v_1 - v_3\rangle - \langle v'_0 - v'_2, v'_1 - v'_3\rangle = 0.$$

Optimization algorithm

- The isometry constraints are expressed by $E_{iso} \rightarrow \min$

$$E_{iso} = \sum_{f \in F} \sum_{j=0}^{2} c_{iso,j}(f)^2$$

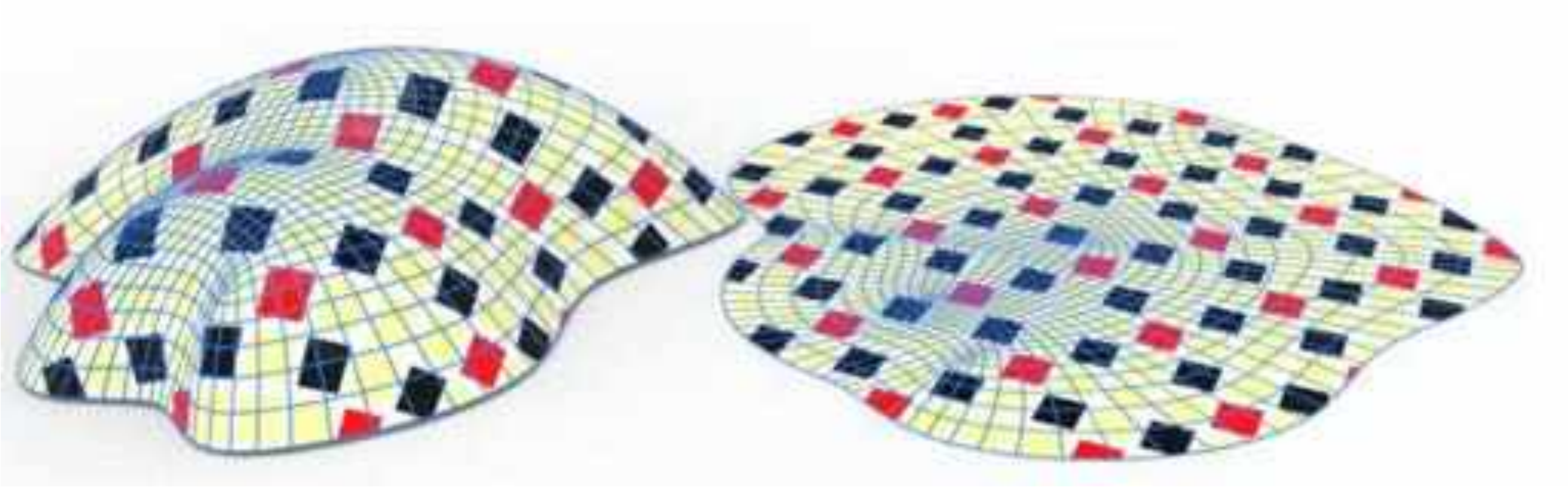
- Constraints for a conformal mapping which is as isometric as possible. $w_{conf} E_{conf} + w_{\lambda} E_{\lambda} \rightarrow \min$

$$E_{conf} = \sum_{f \in F} \sum_{j=0}^{2} c_{conf,j}(f)^2, \quad E_{\lambda} = \sum_{f \in F} (\lambda_f - 1)^2.$$

- optimized by a Levenberg-Marquardt method.

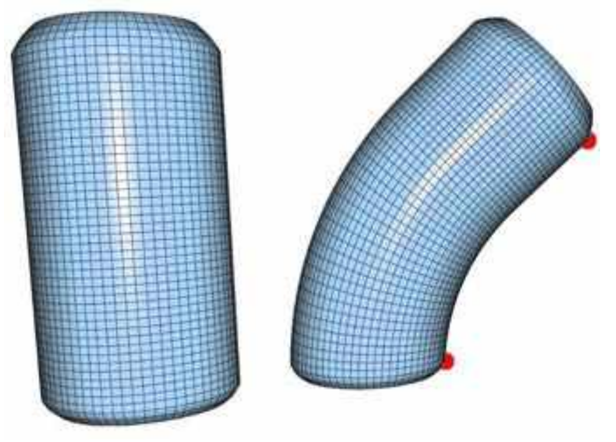
Surface parameterization for graphics

- Conformal mapping to the plane which is as isometric as possible

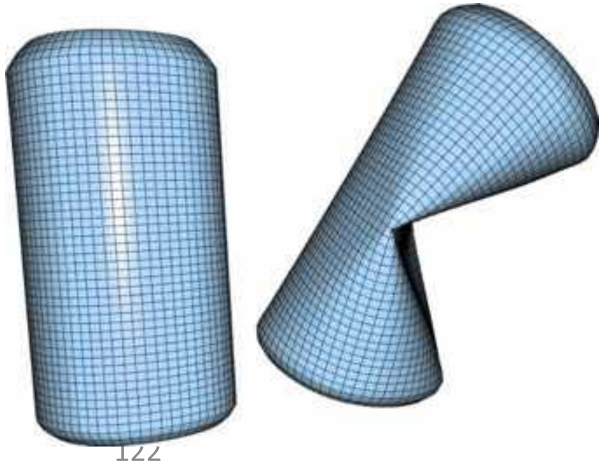


Editing of isometric deformation

deformation



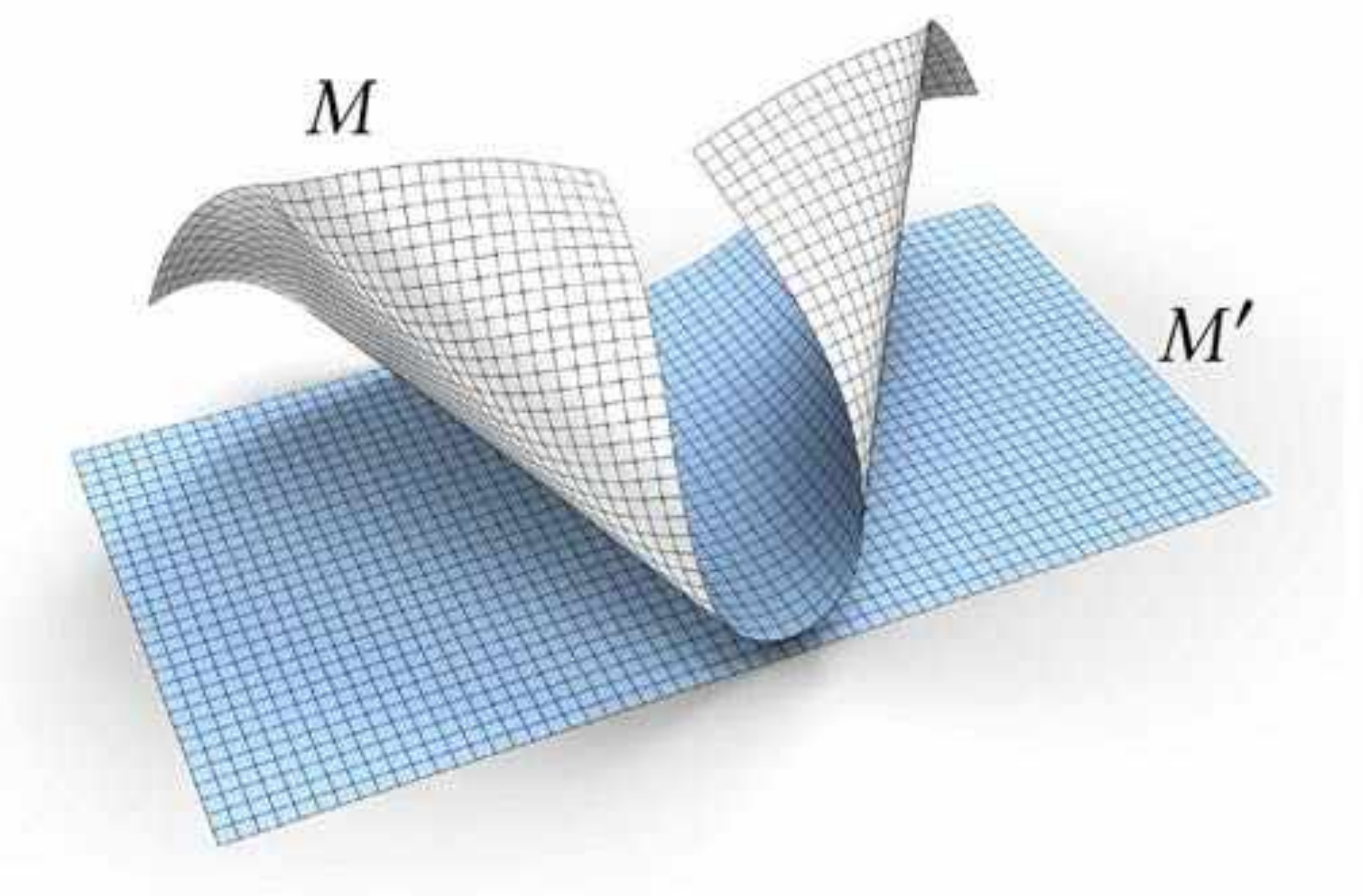
isometric deformation



Modeling developable surfaces

Discrete developable surfaces

- discrete developable surface =
- quad mesh M which is isometric to a planar mesh M'

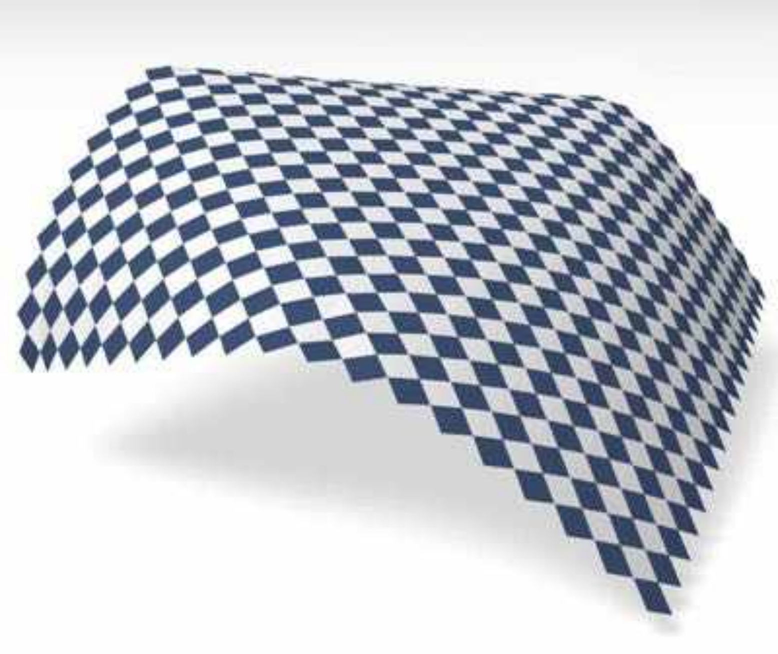


Discrete developable surface

Special case:

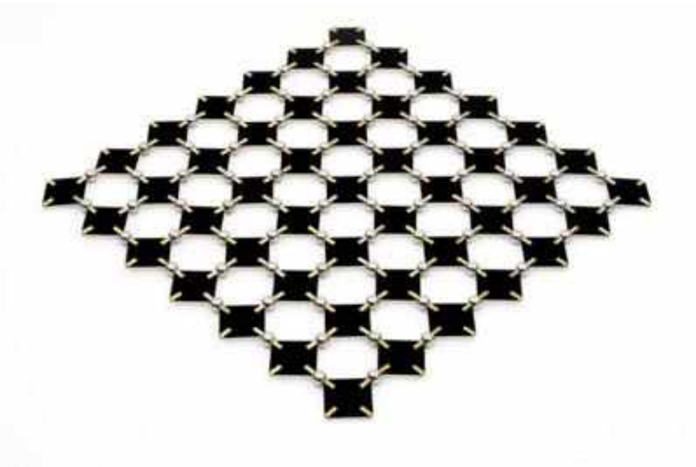
- CBP from congruent black squares (quads in *M* have orthogonal diagonals, all of the same length)
- closely related to Rabinovich et al.

optimized without fairness



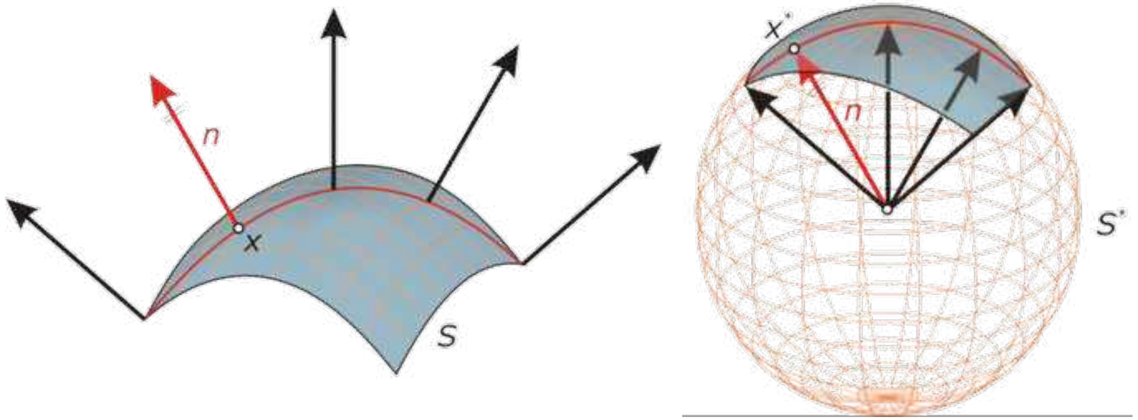
with fairness

Verification by a physical model

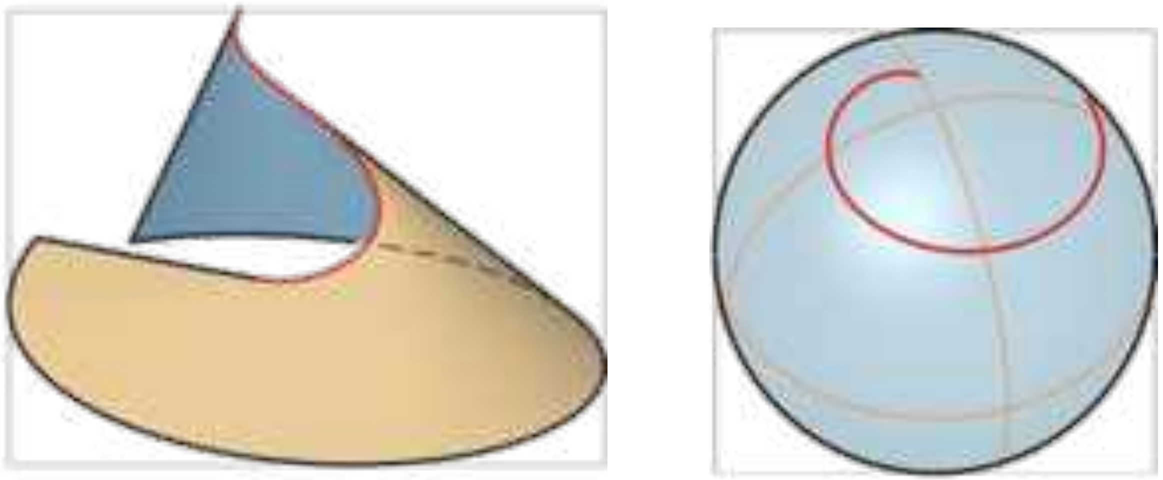


Gaussian image of a smooth developable surface

- Gauss map from a surface to the unit sphere with help of unit surface normals

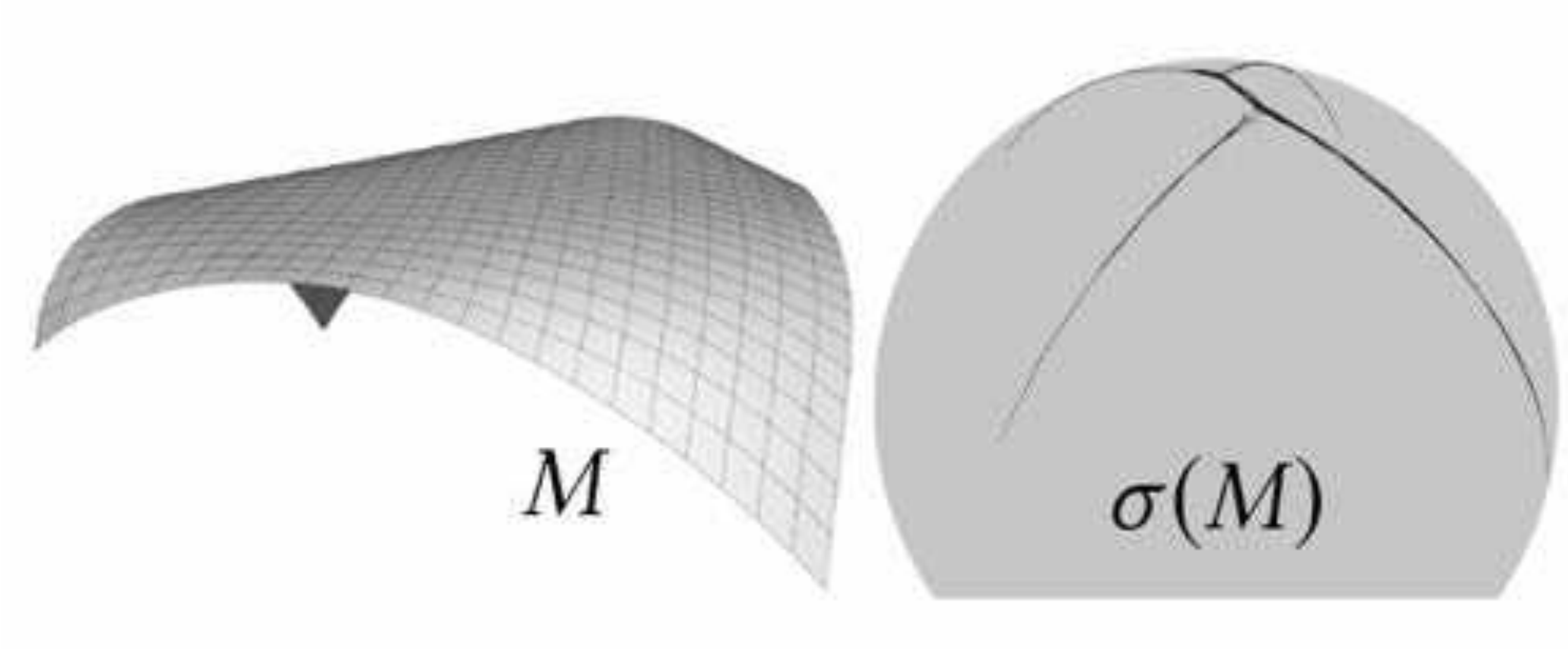


- Tangent plane and unit normal are constant along a ruling of a developable surface. Gaussian image is a curve



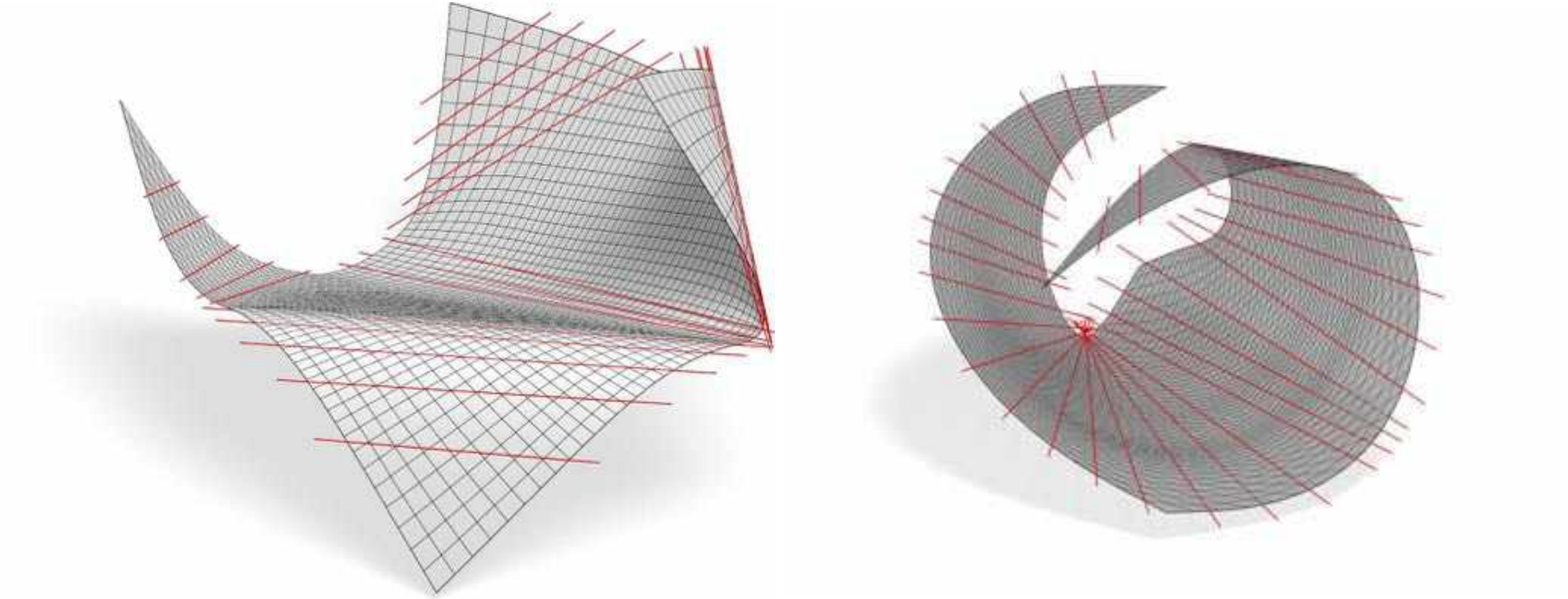
Gauss image of our discrete model

- Quality control: Gauss image (formed by normals of parallelograms) is curve-like



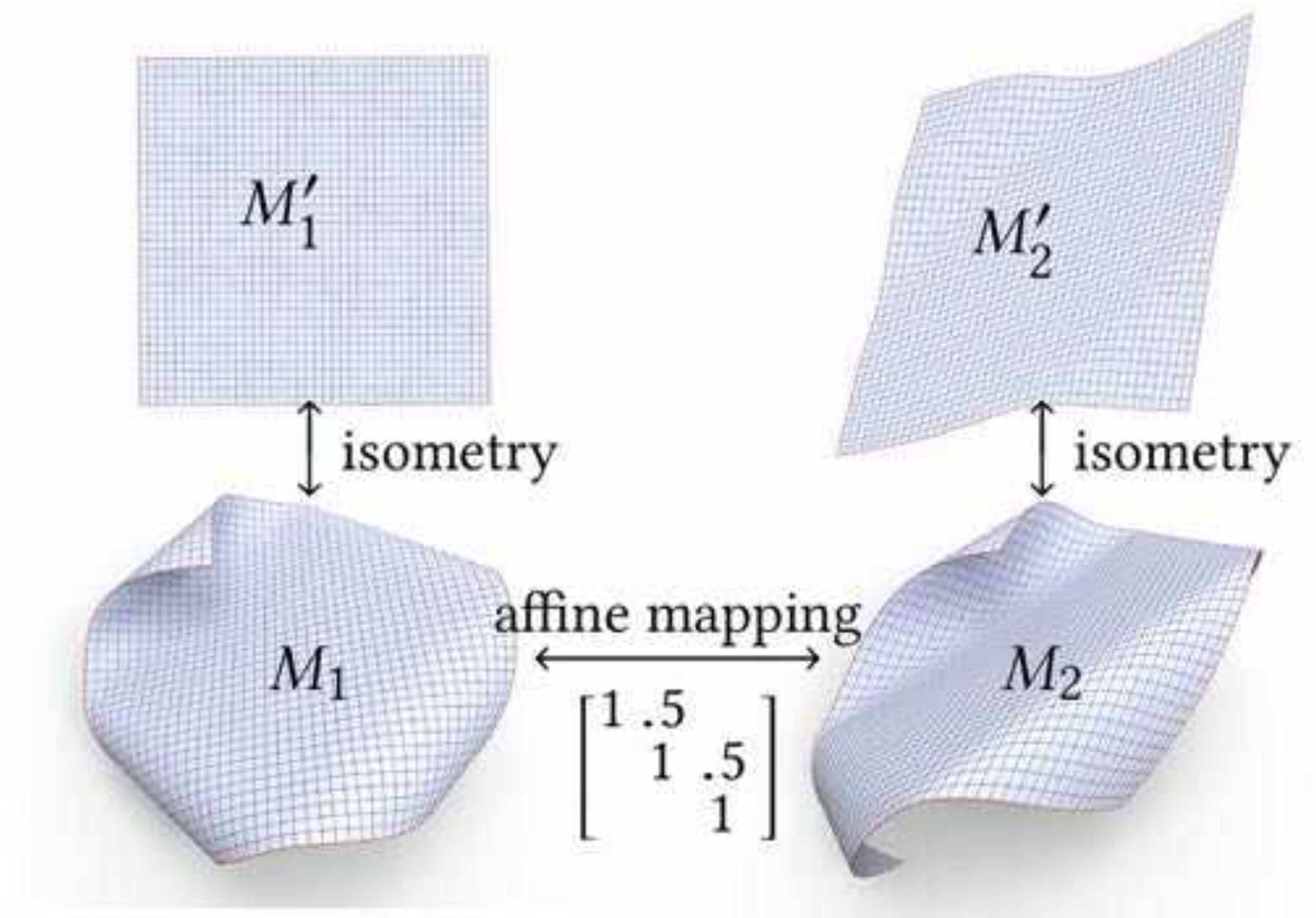
rulings

- The discrete model is not based on rulings, but there are estimated rulings which fit well

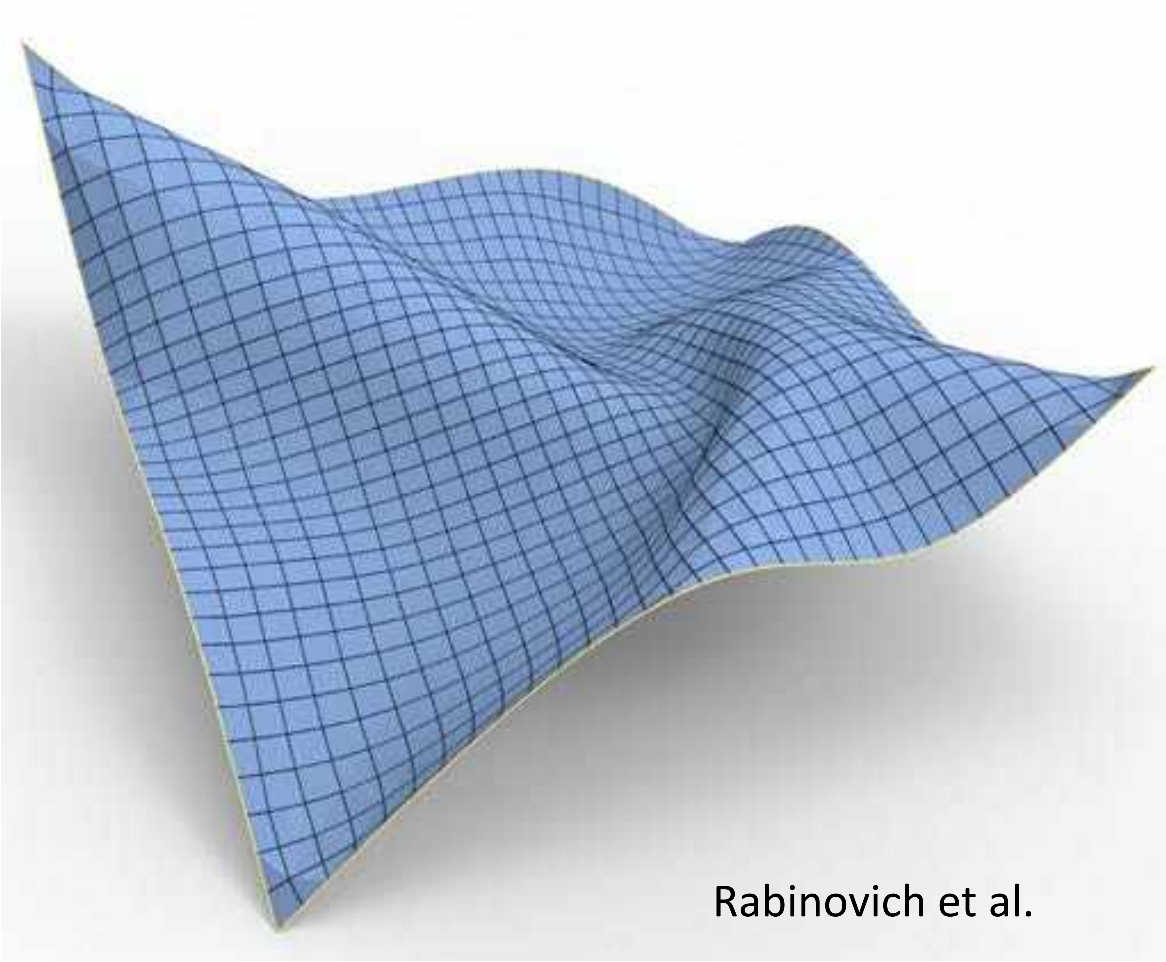


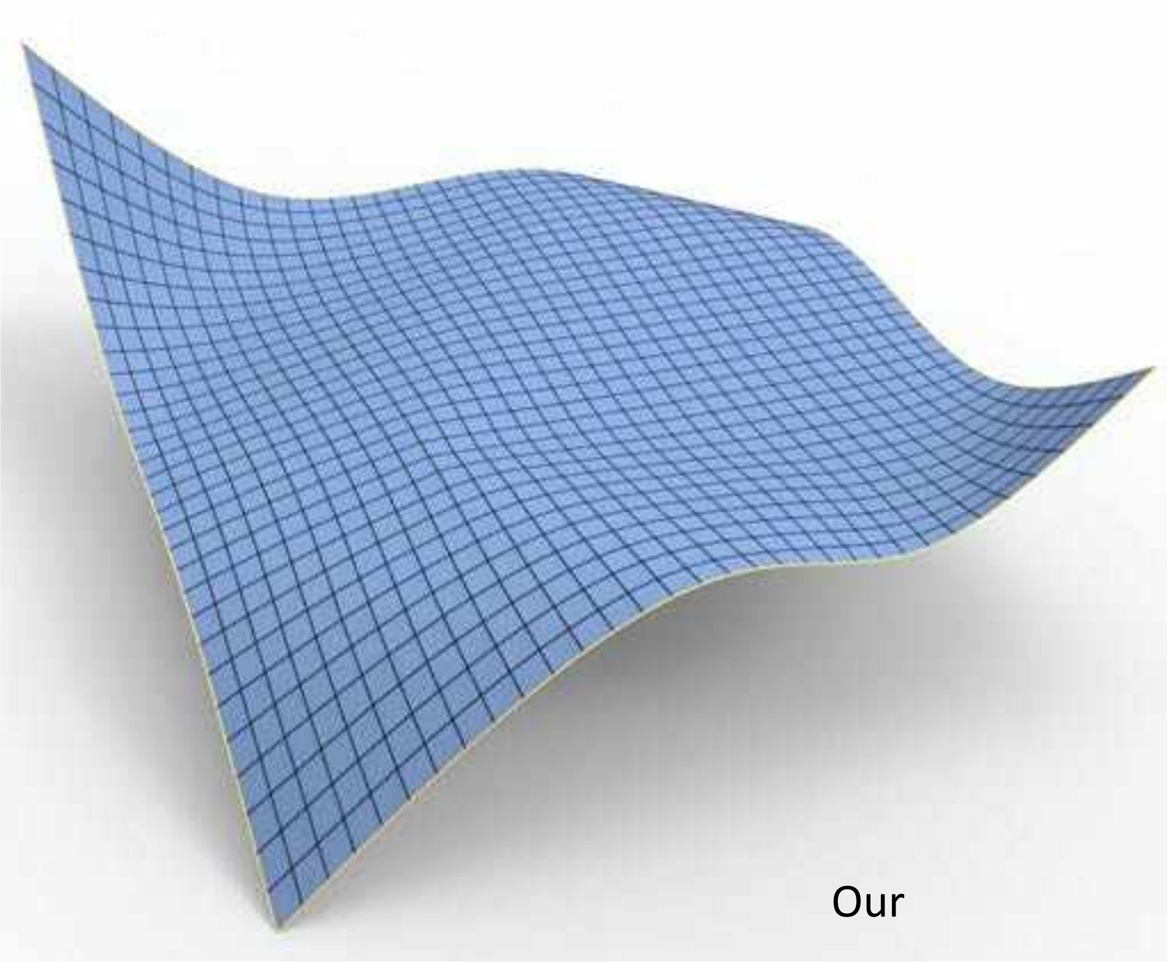
The planar mesh needs not be a regular square grid

- An affine map keeps the developability, but may change the planar unfolding dramatically



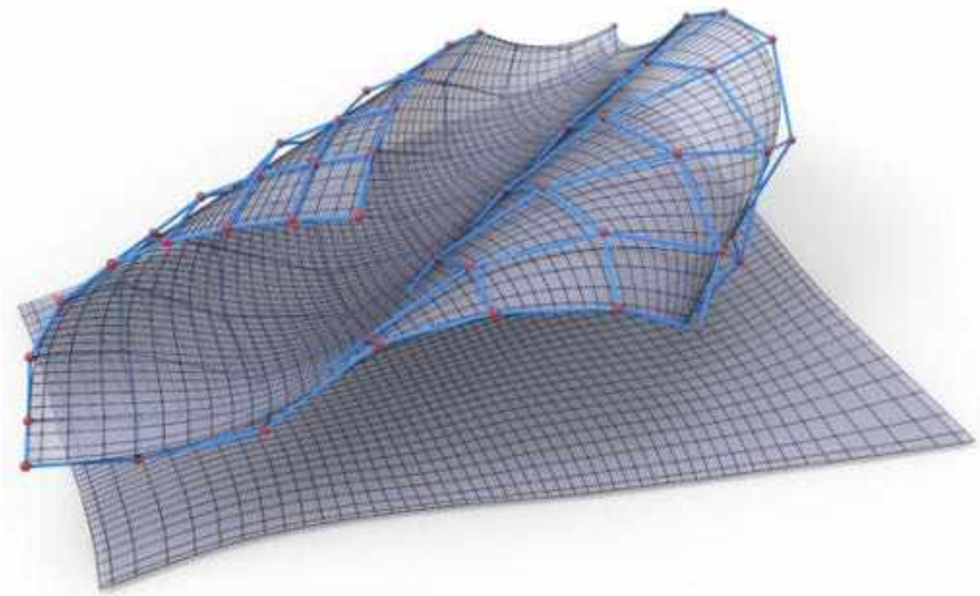
Comparison

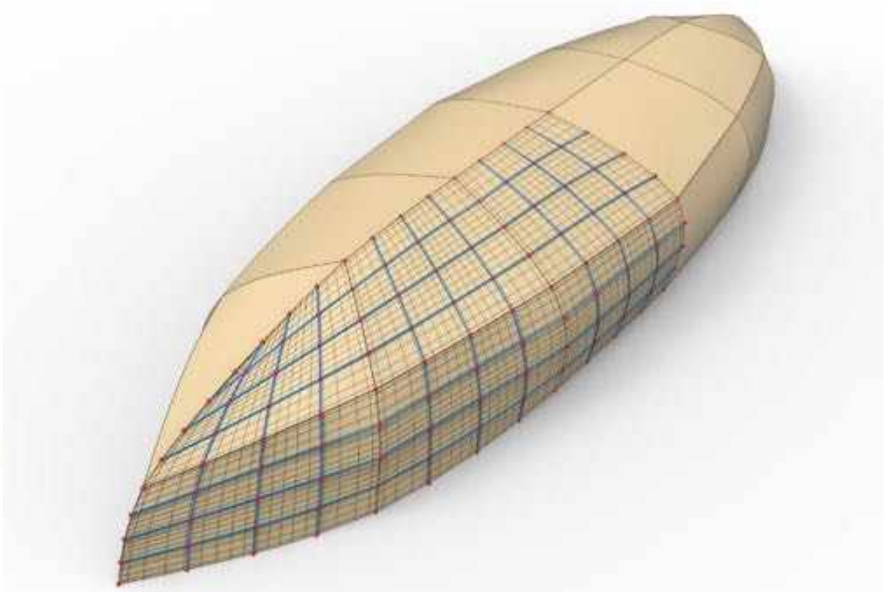




Developable B-spline surfaces for CAD/CAM

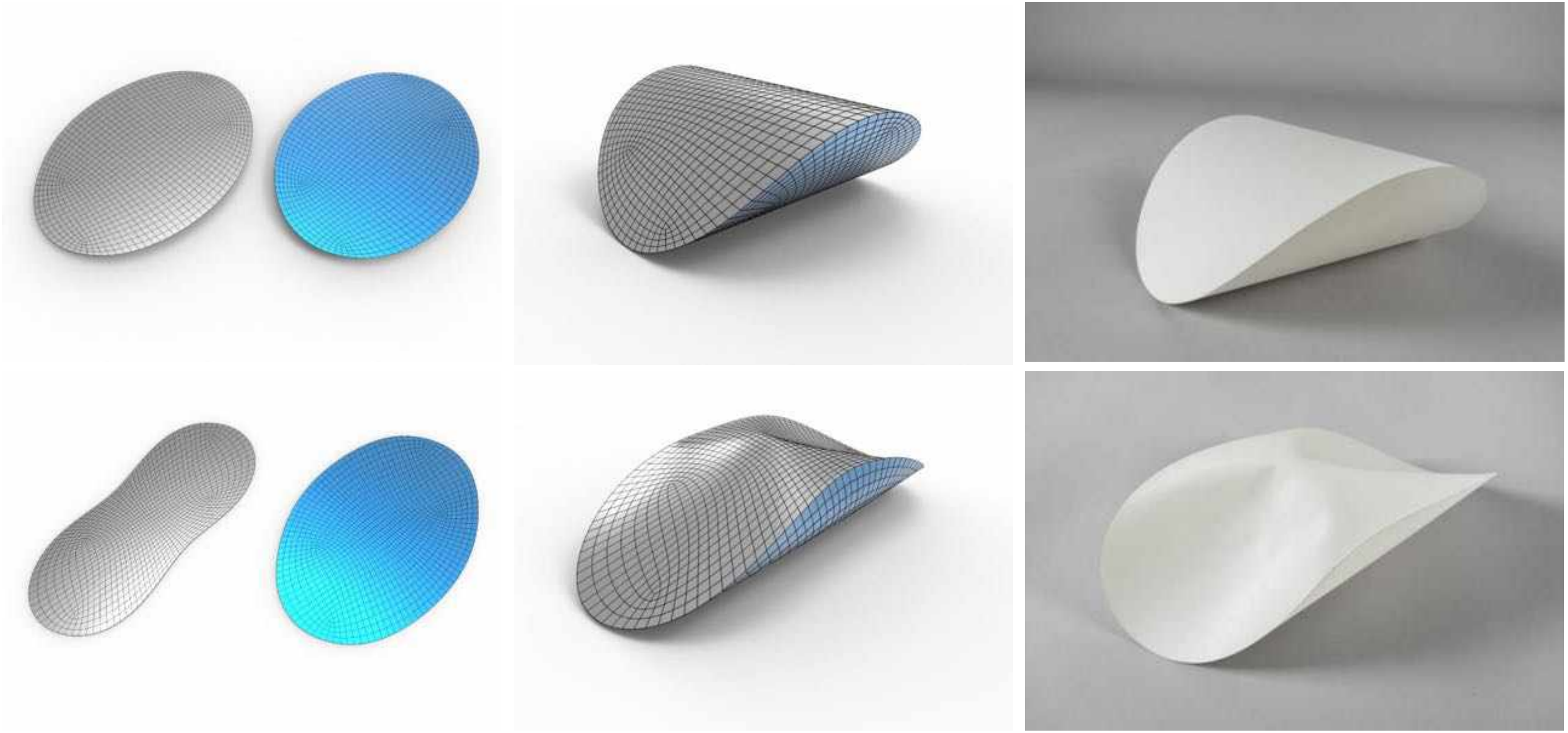
- Key idea: ensure isometry of a subdivided version of the control net to a planar mesh.
- Not possible with the discrete model of Rabinovich et al.
- Fills a gap in current NURBS-based CAD/CAM software which is weak in modeling developable surfaces





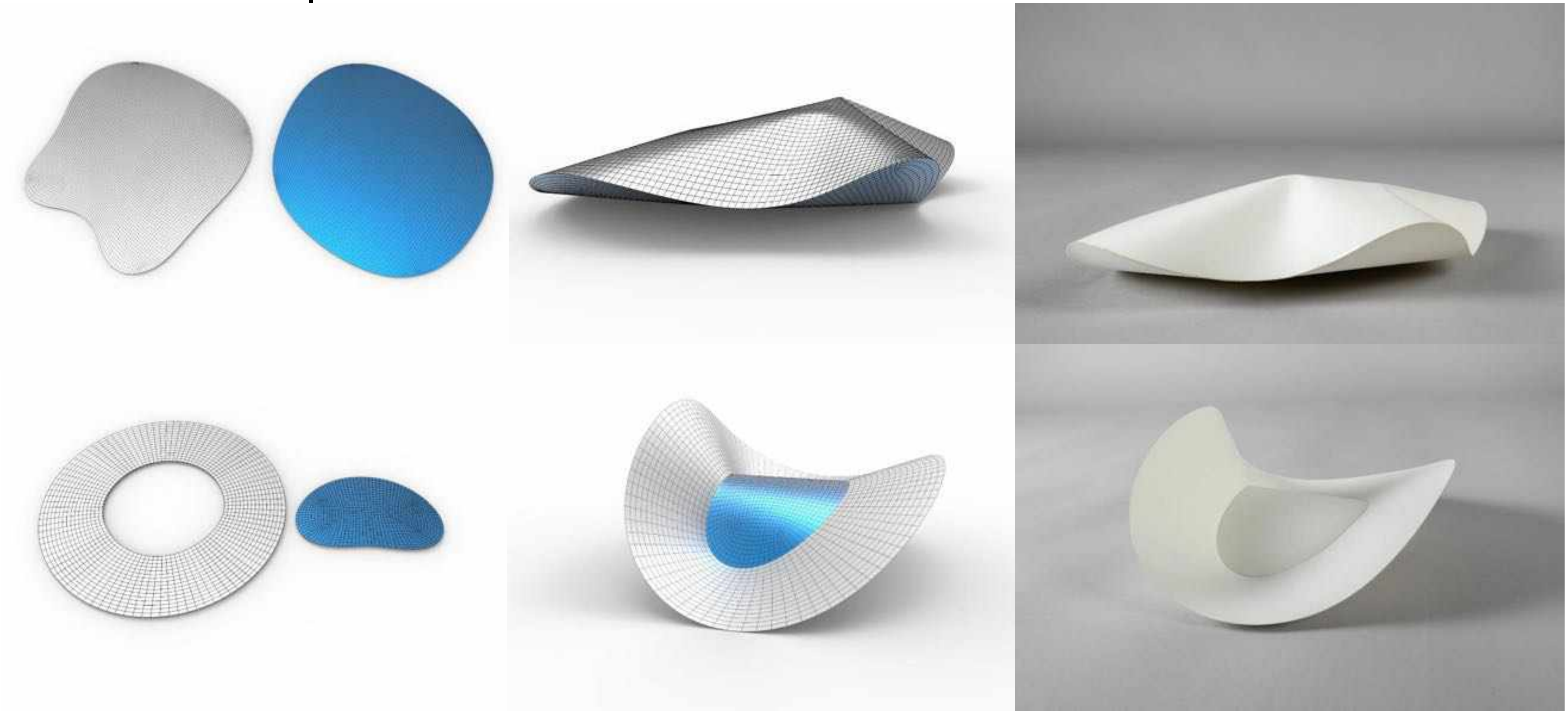
D-forms

- Gluing two planar sheets with same boundary curve length

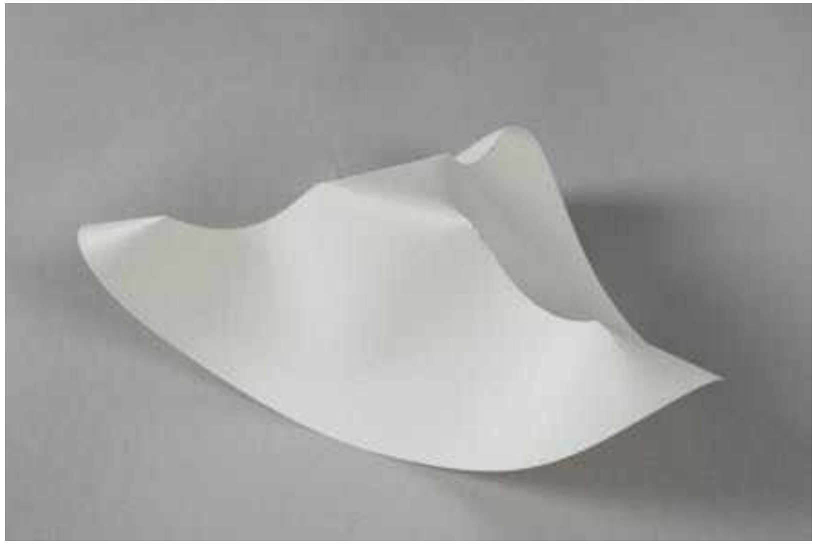


D-forms

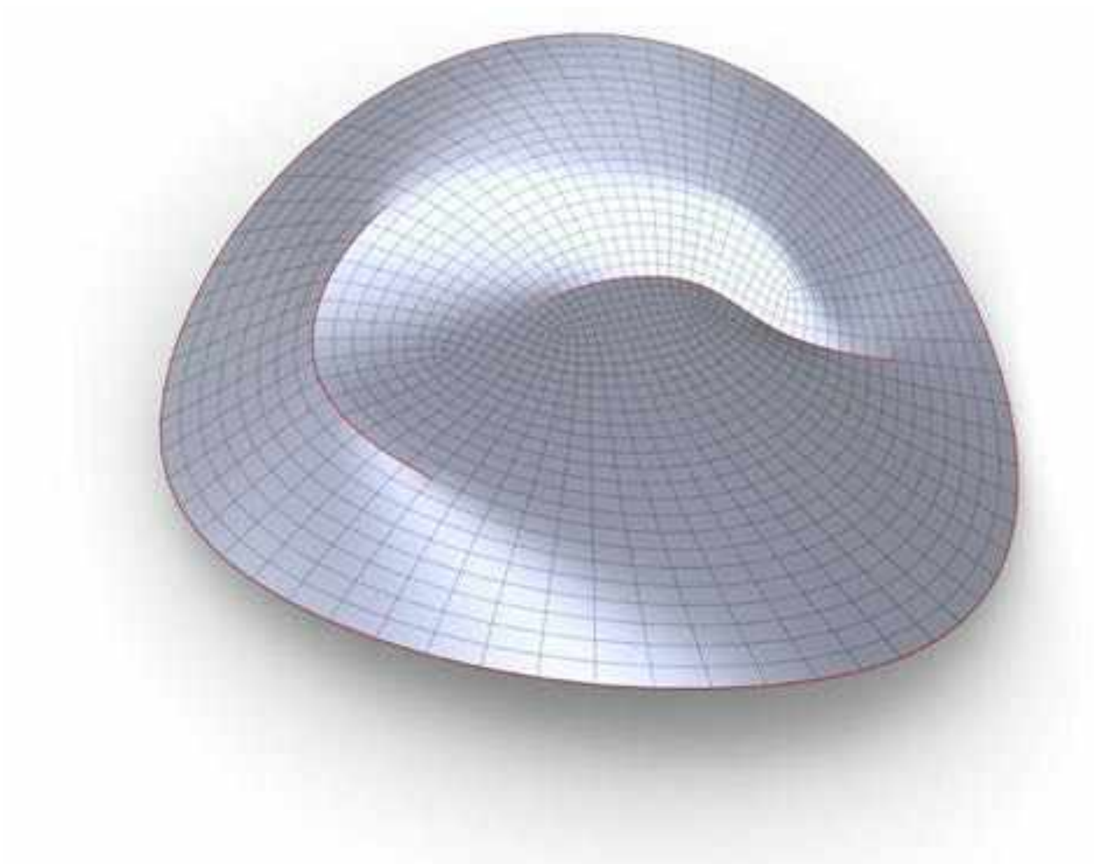
- more examples



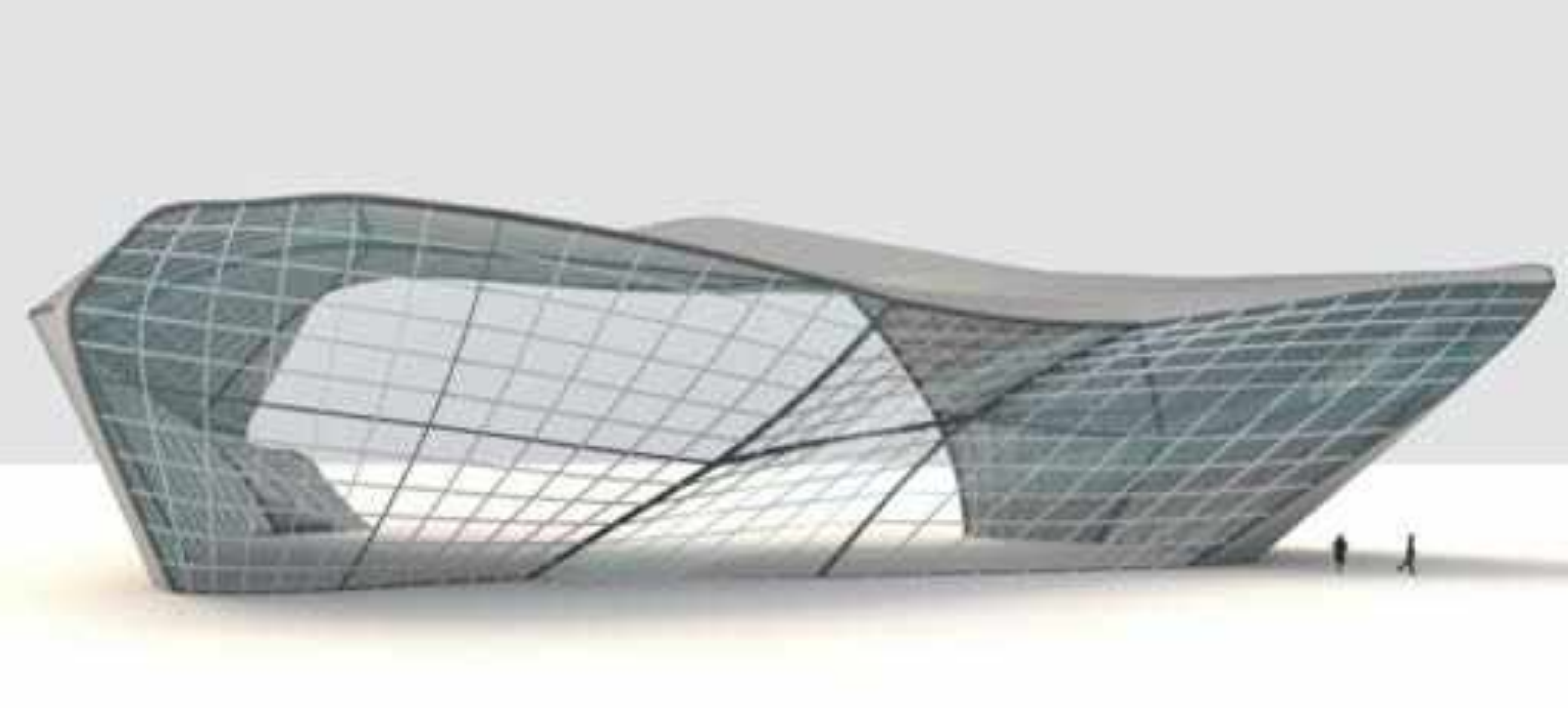
Cutting and Gluing



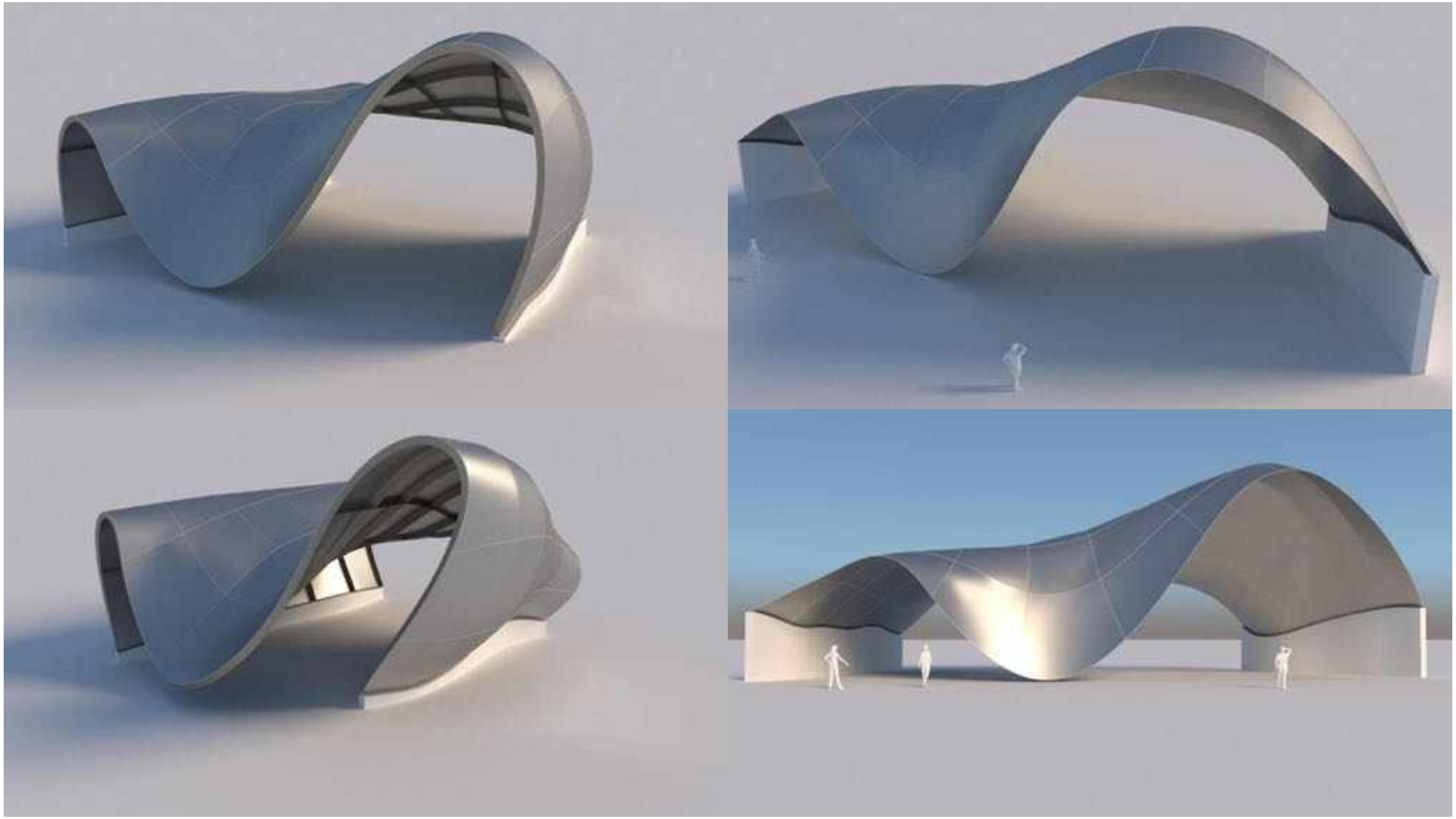
Curved folds



Paneling freeform designs in Architecture



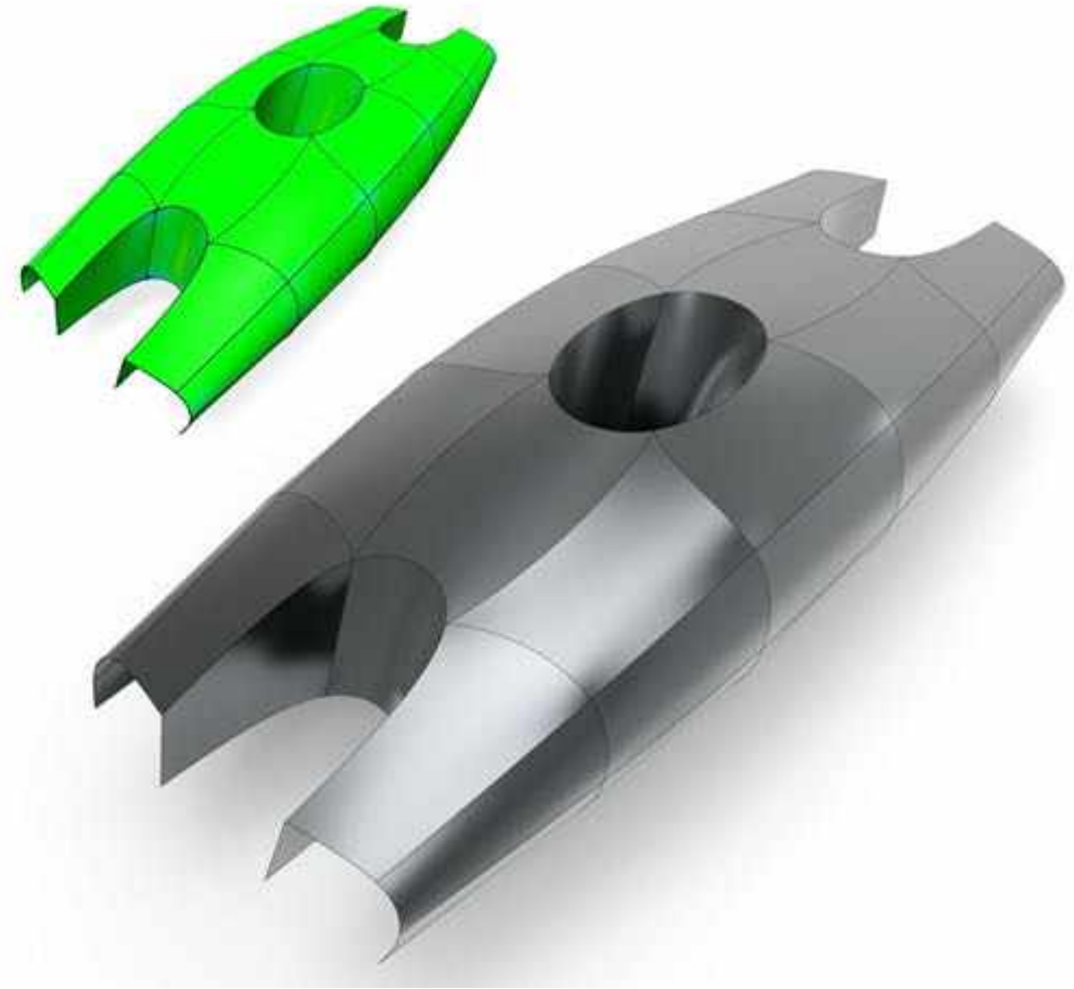
Isometric deformation of a surface formed by developable panels



Conclusion and future research

- Mappings between surfaces are easily discretized with quad meshes
- Here only first order properties; for curvatures see the paper.
- New simple and flexible discrete model of developable surfaces

- Future research directions include
- best approximation with piecewise developable surfaces – automatic segmentation
- inclusion of material properties
- more theory within discrete differential geometry



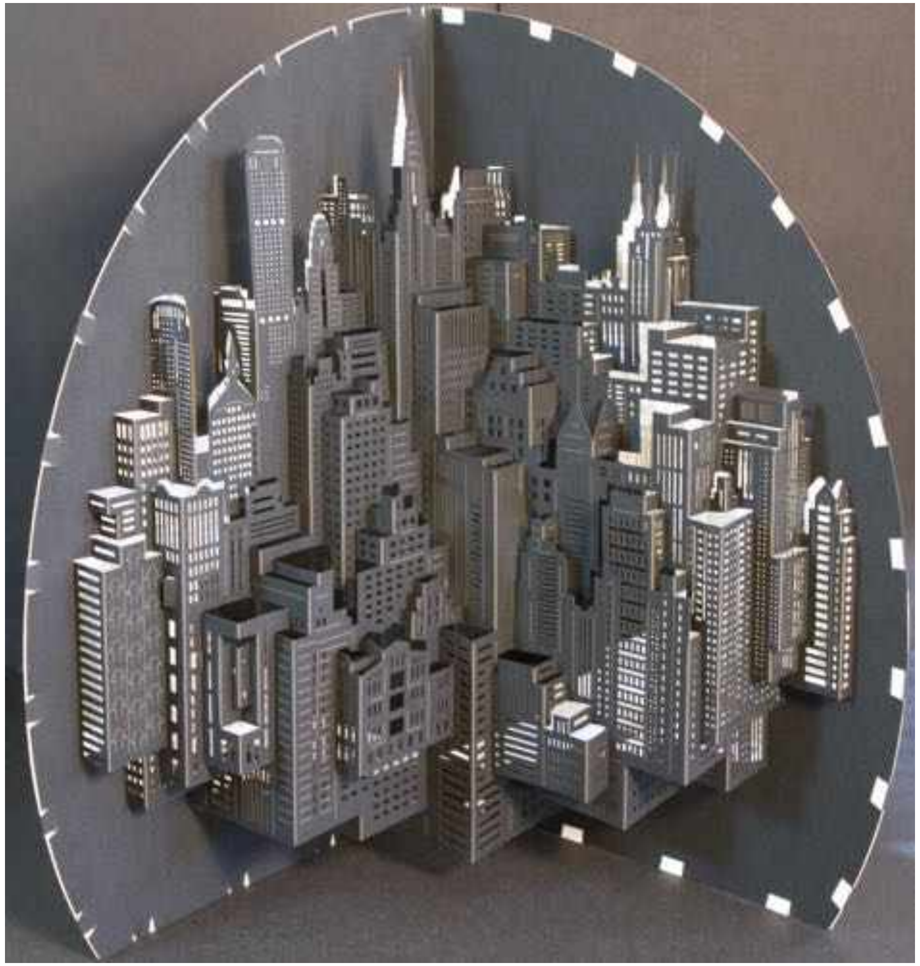
Freeform Quad-based Kirigami

(SIGGRAPH Asia 2020)
with Florian Rist, Helmut Pottmann , and Johannes Wallner

Kirigami

- A variation of Origami
- Cutting and folding
- Example: Pop-up structures

Pop-up design



Designed by Ingrid Siliakus

Popup: Automatic Paper Architectures from 3D Models

Xian-Ying Li[1] Chao-Hui Shen[1] Shi-Sheng Huang[1] Tao Ju[2] Shi-Min Hu[1]
[1]Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing
[2]Department of Computer Science and Engineering, Washington University in St. Louis

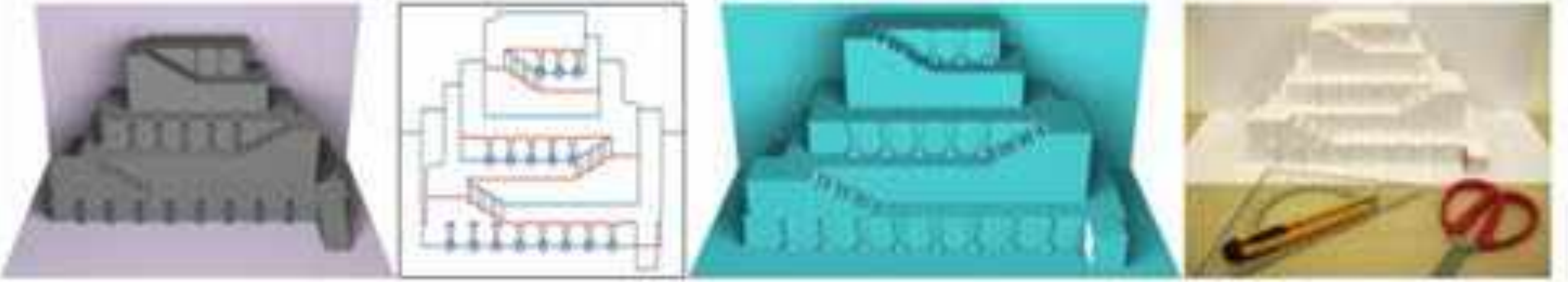
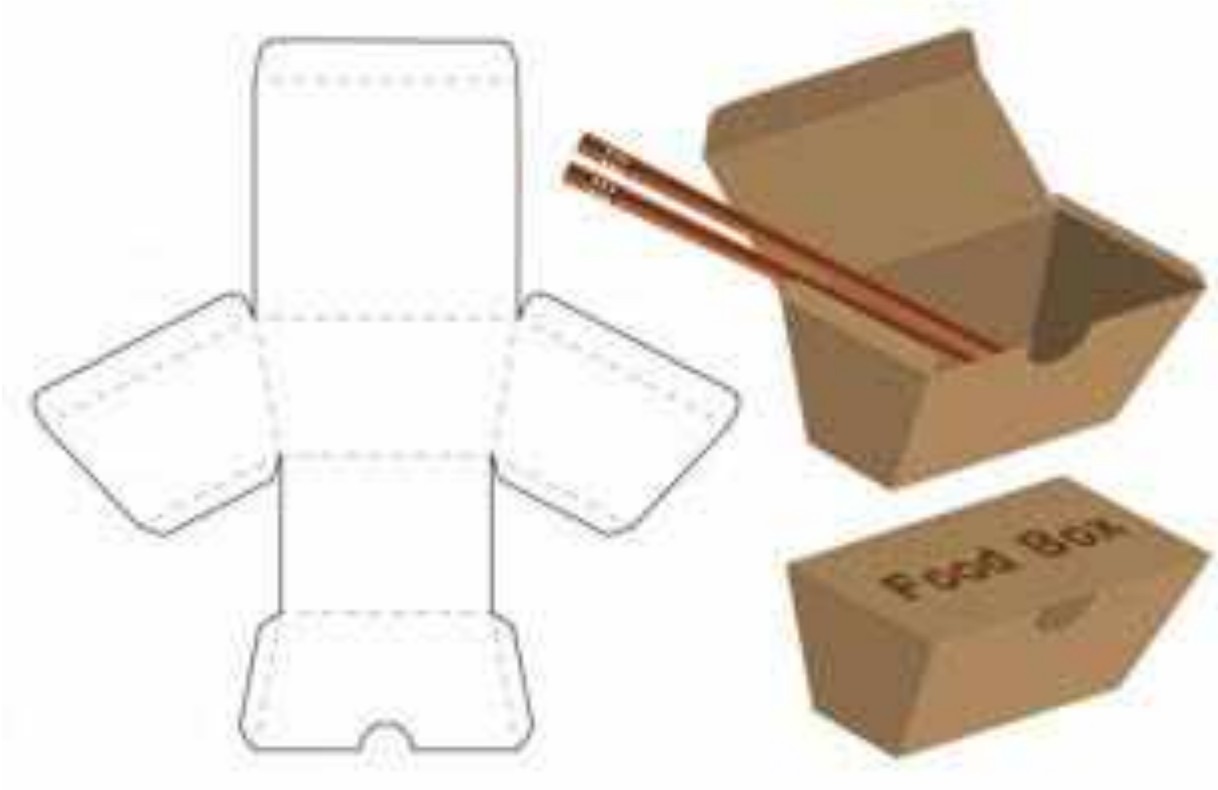


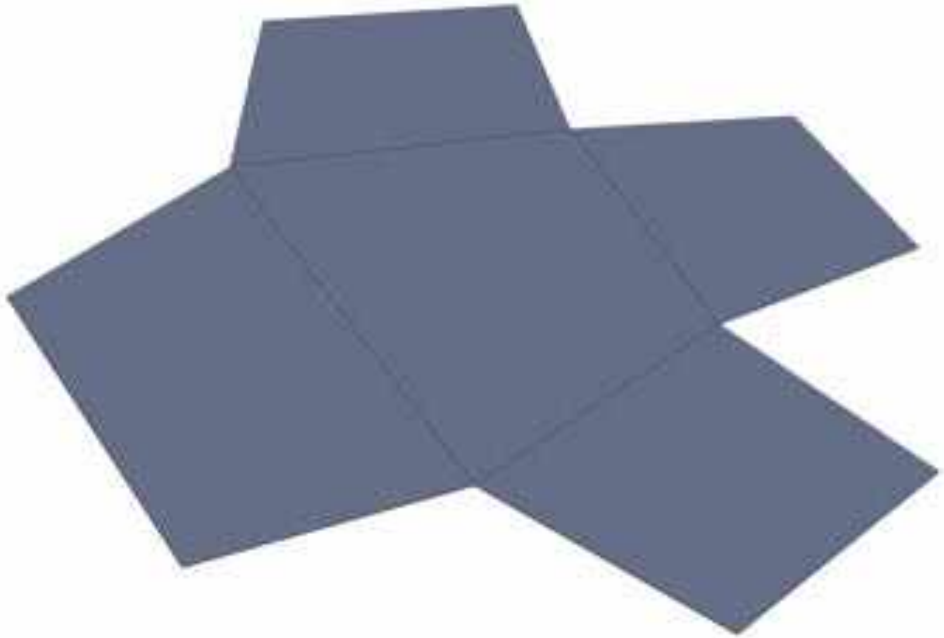
Figure 1: Given a 3D architectural model with user-specified backdrop and ground (left), our algorithm automatically creates a paper architecture approximating the model (mid-right, with the planar layout in mid-left), which can be physically engineered and popped-up (right).

SIGGRAPH 2010

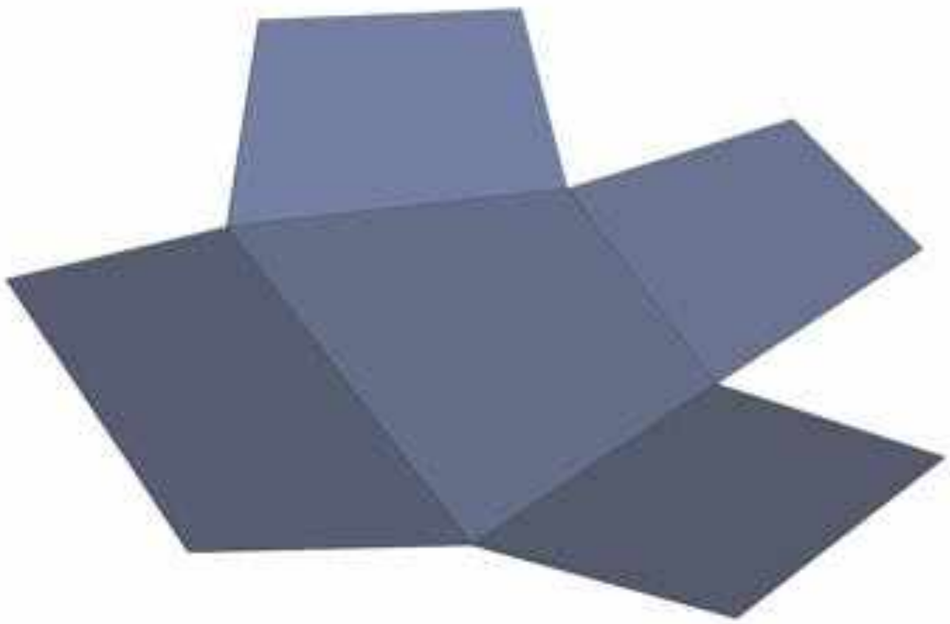
Foldable boxes



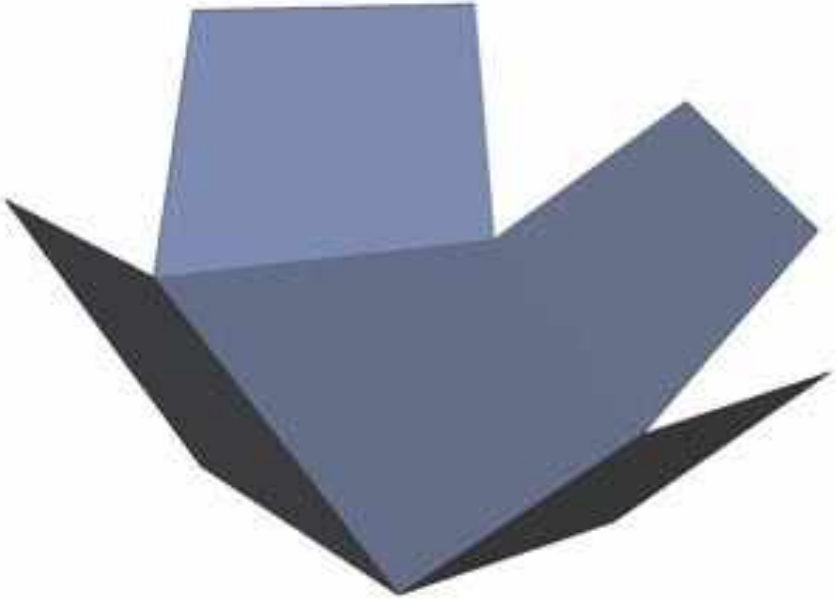
General foldable boxes



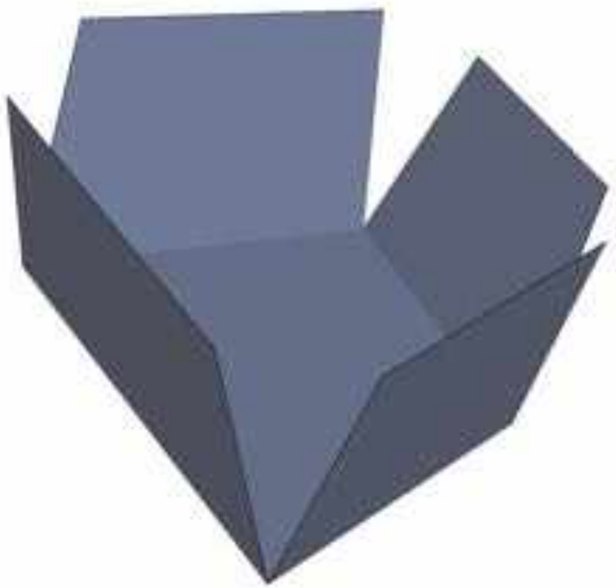
General foldable boxes



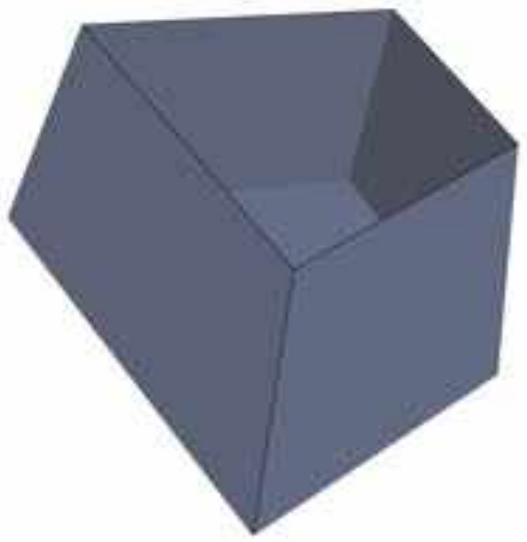
General foldable boxes



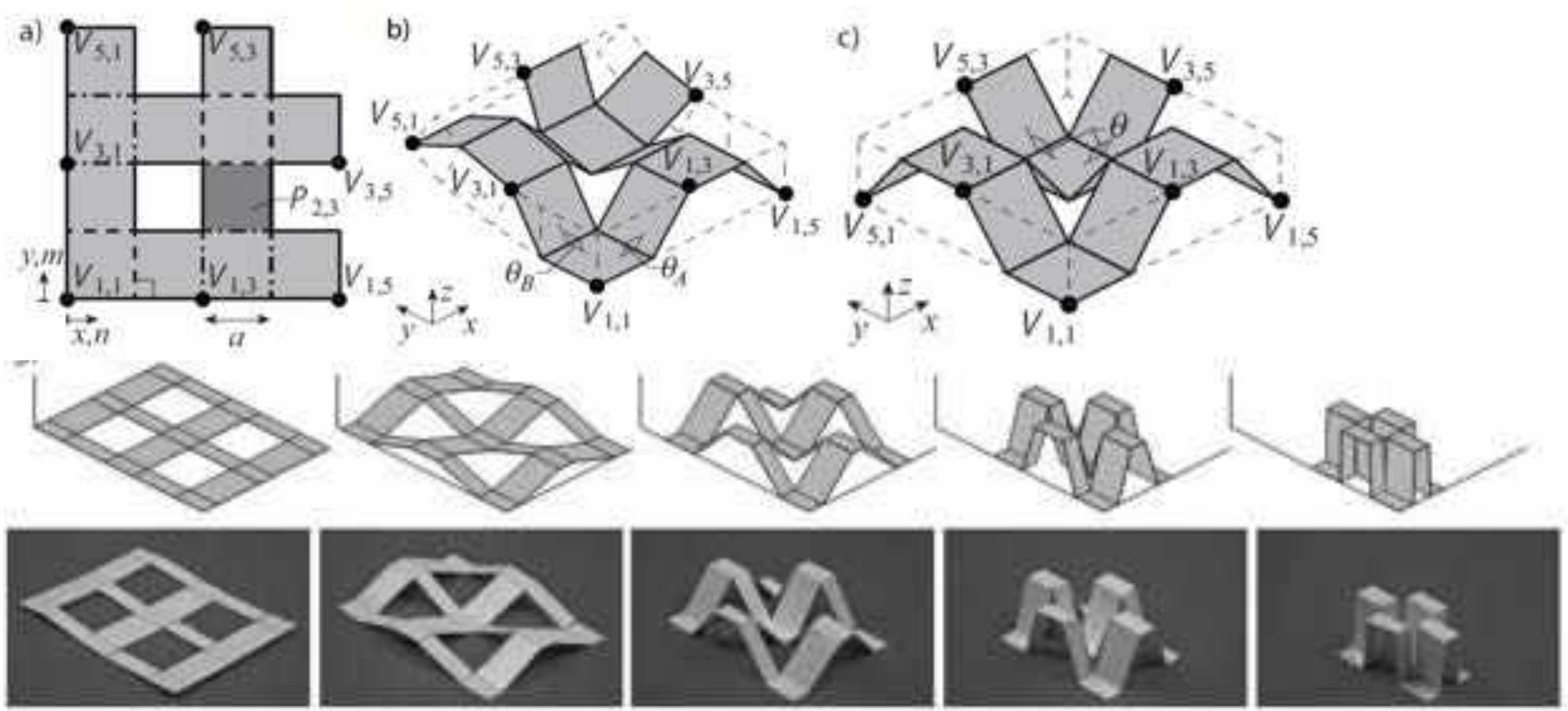
General foldable boxes



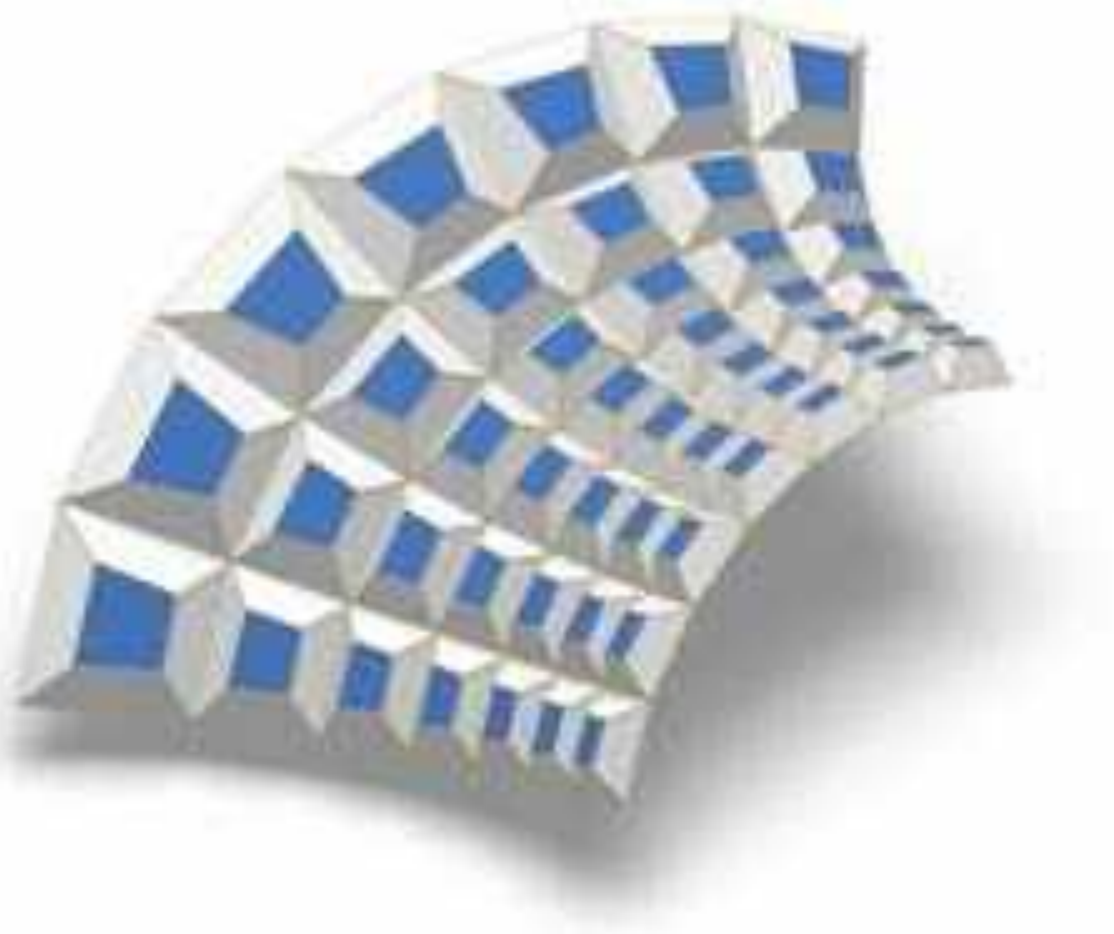
General foldable boxes



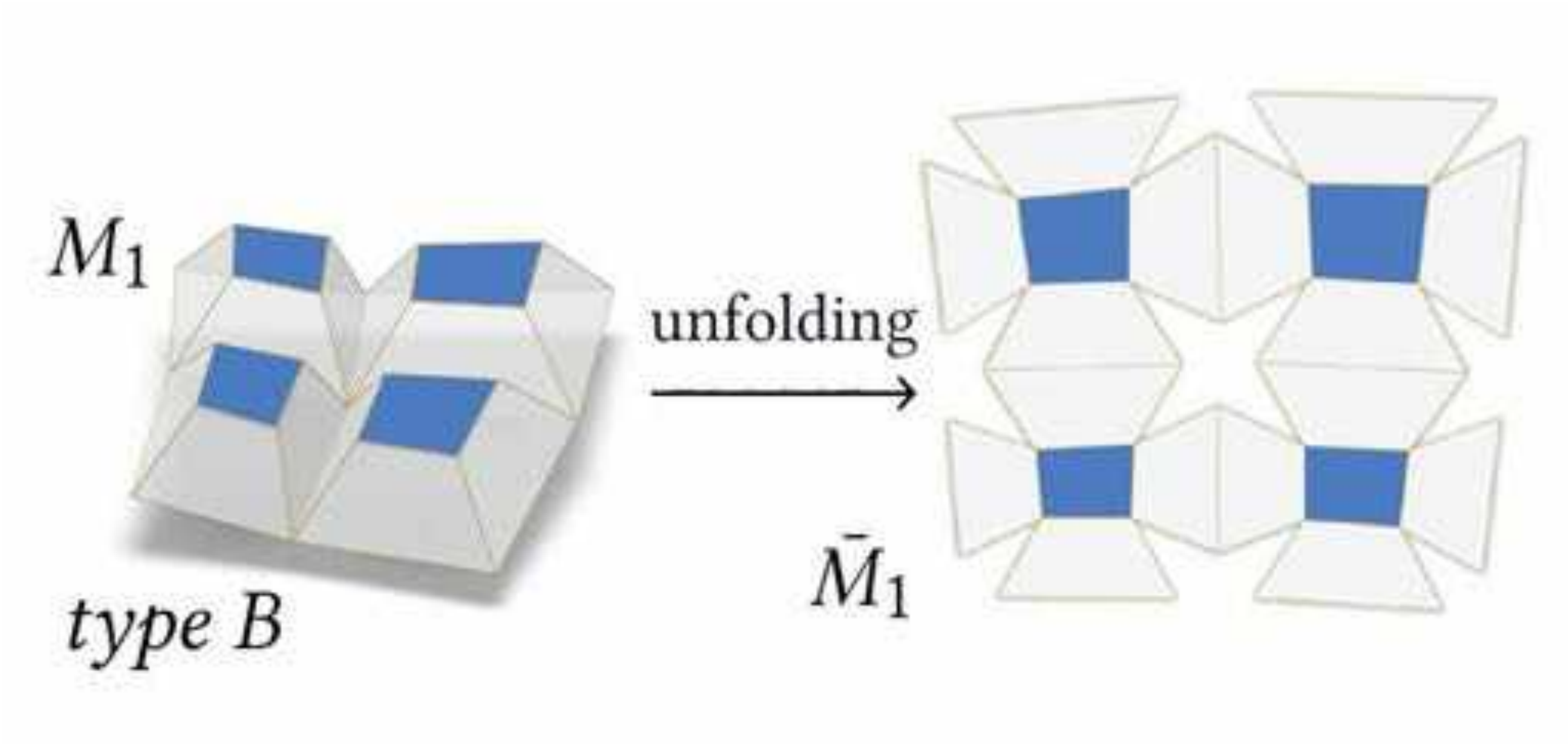
Kirigami connected by regular foldable boxes



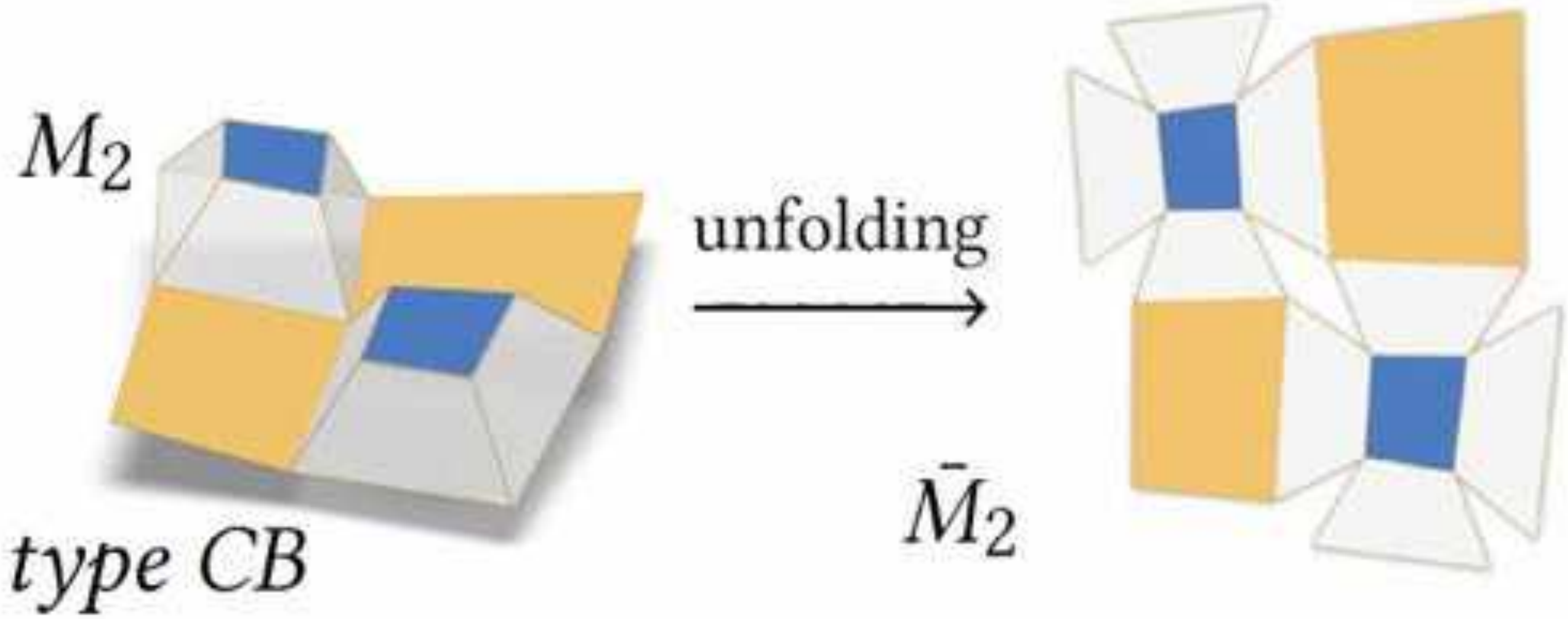
Xie, Ruikang, Chen, Yan and Gattas, Joseph M. (2015) Parametrisation and application of cube and eggbox-type folded geometries. *International Journal of Space Structures*, *30* 2: 99-110. doi:10.1260/0266-3511.30.2.99



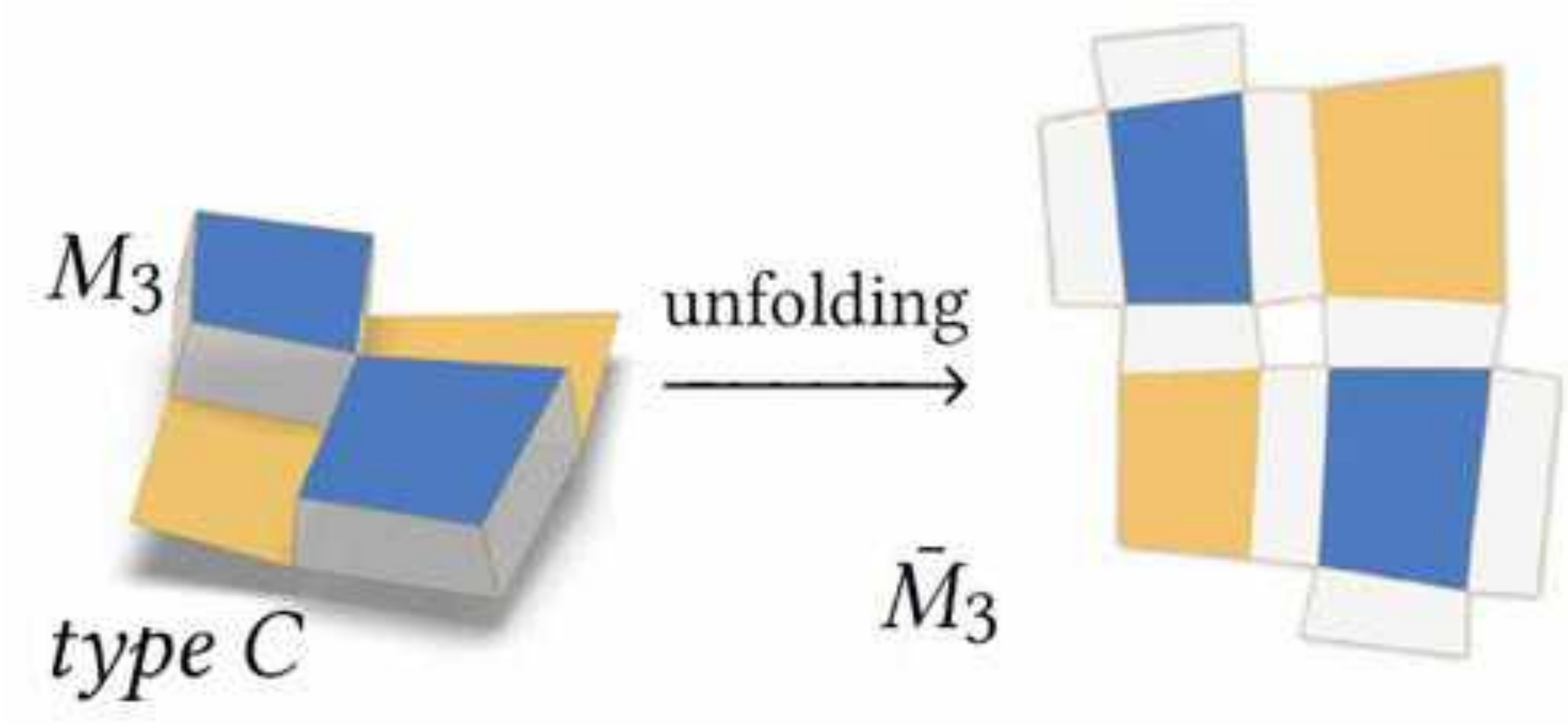
Types of kirigami structures

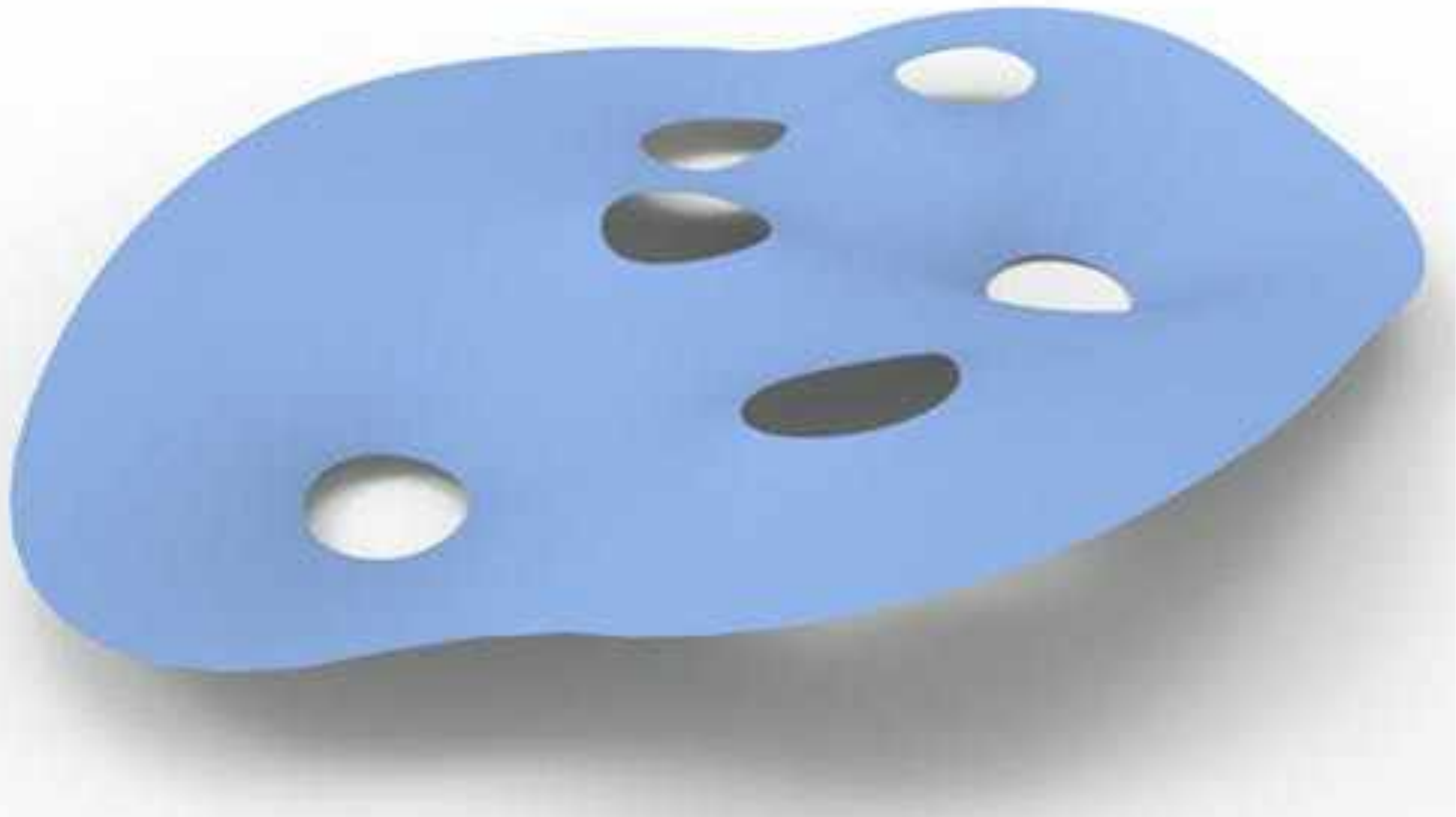


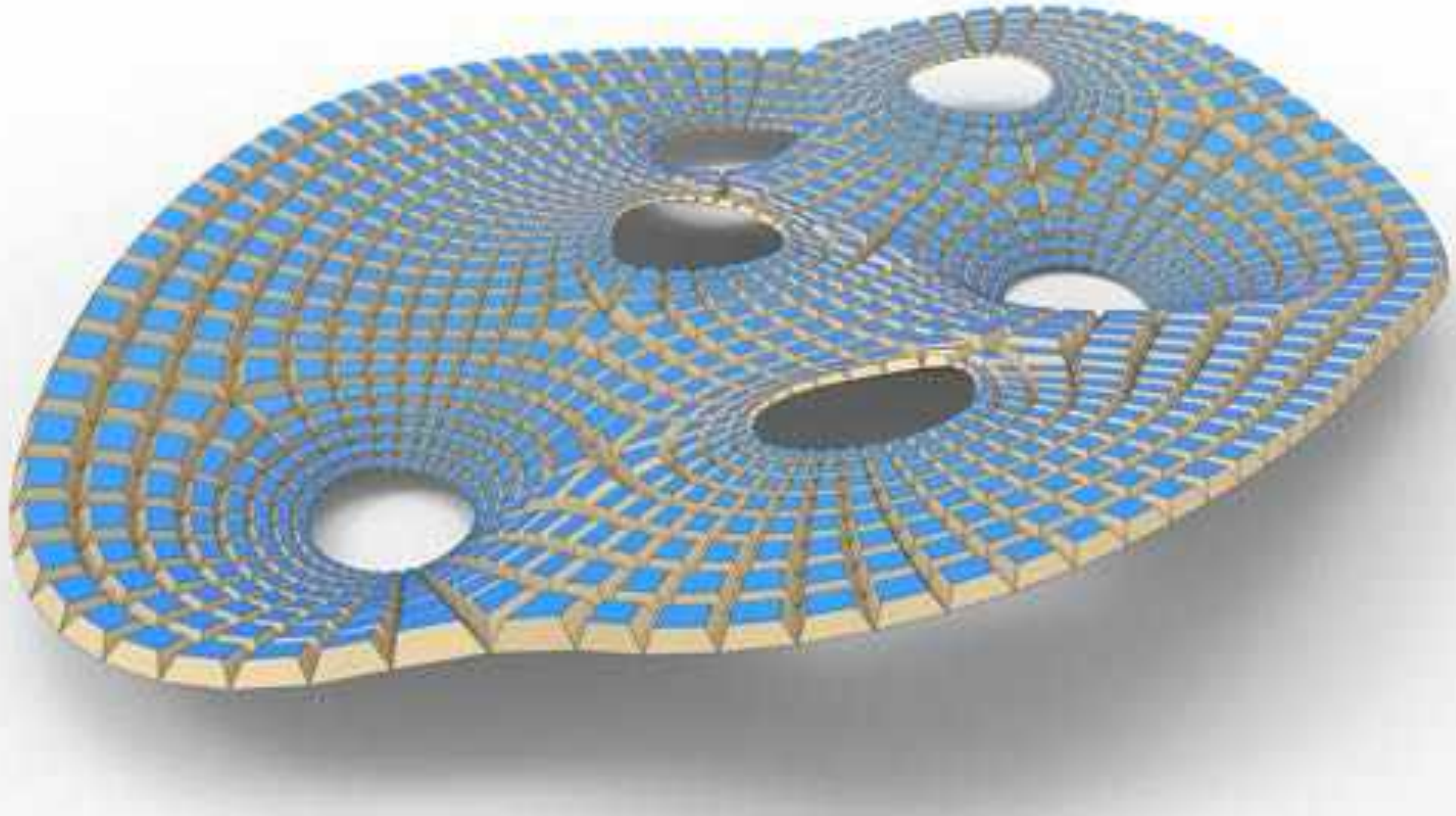
Types of kirigami structures

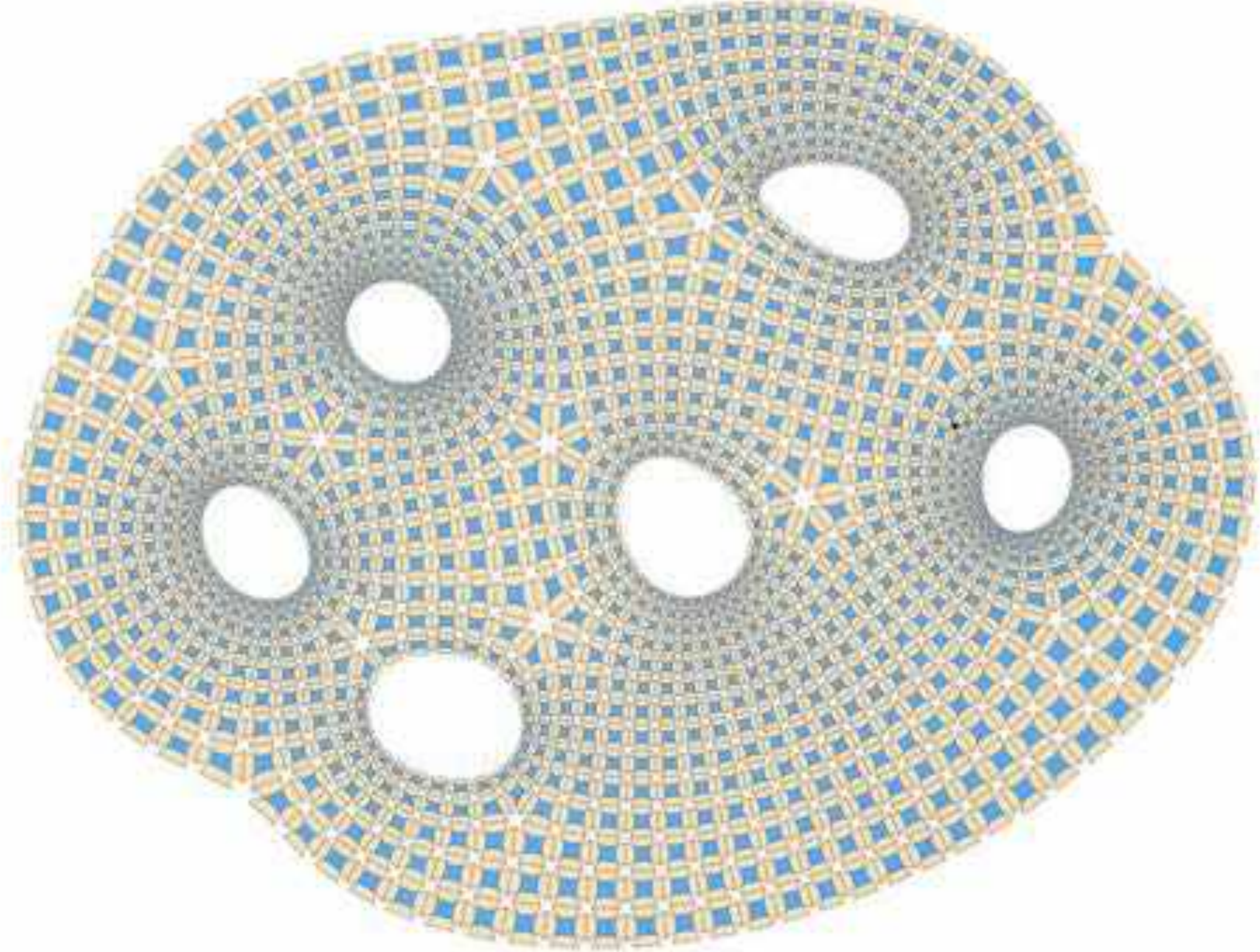


Types of kirigami structures

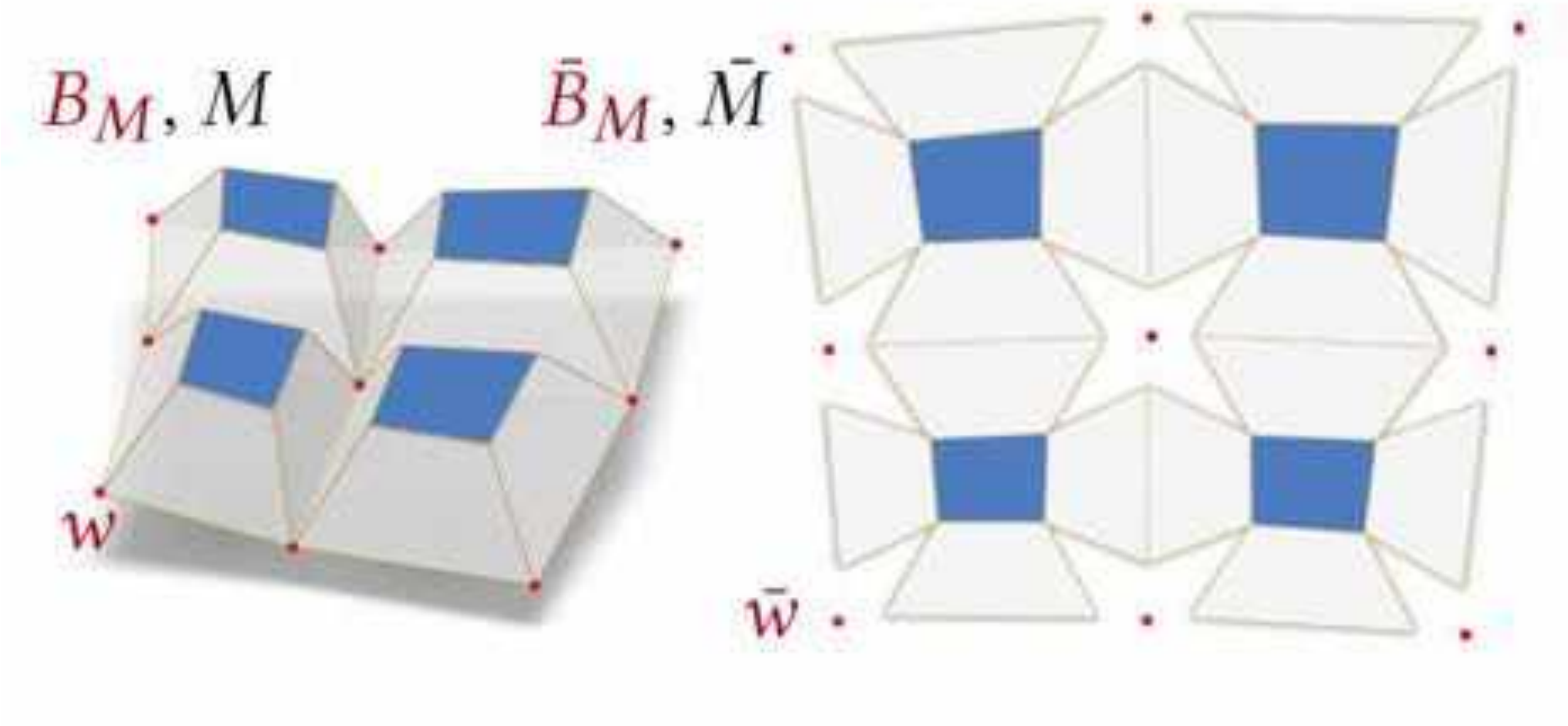








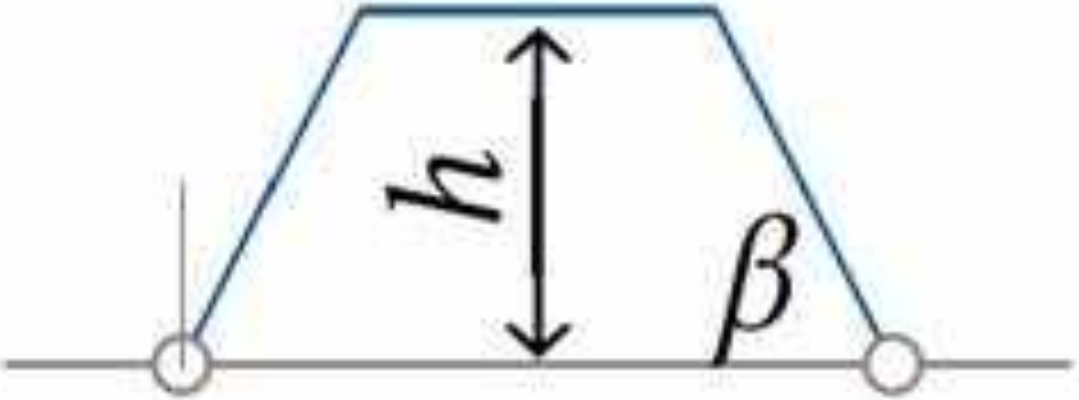
Discrete expanding mapping

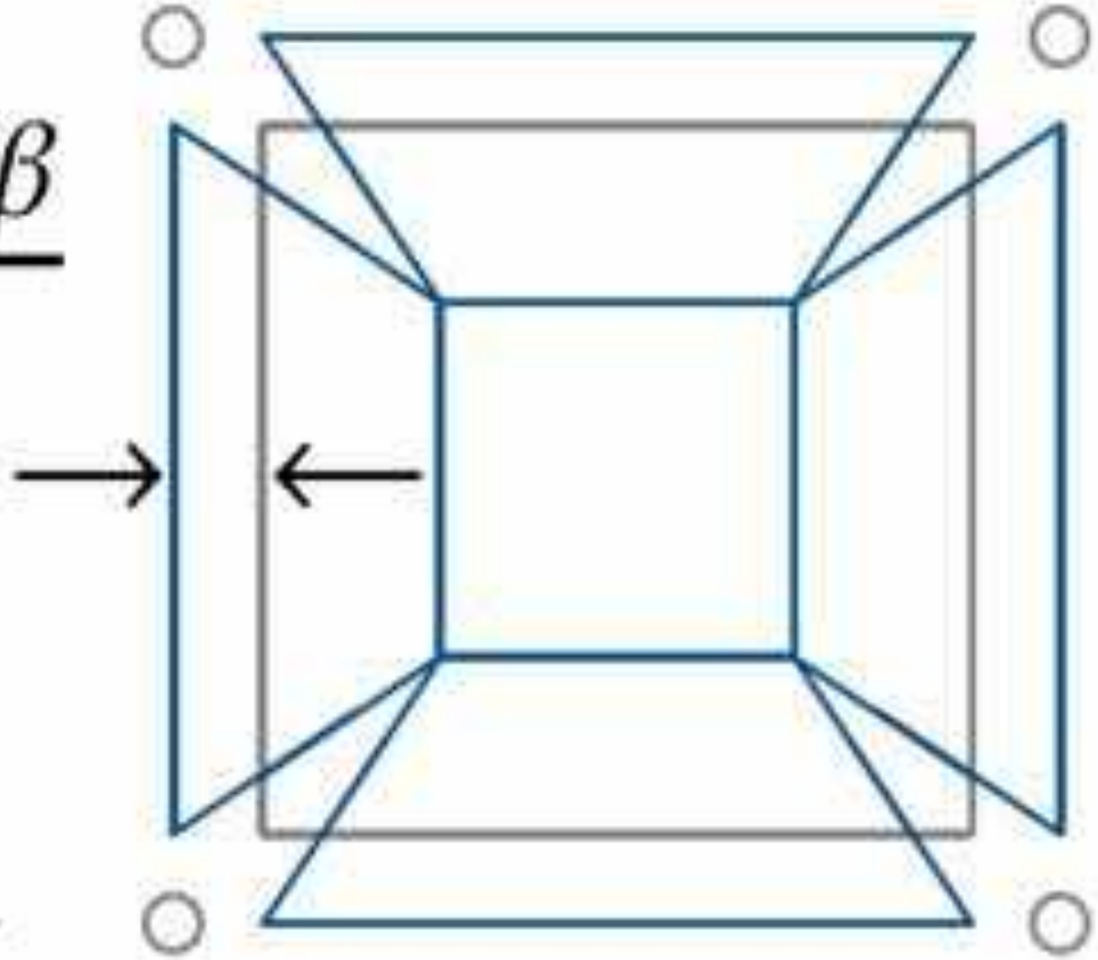


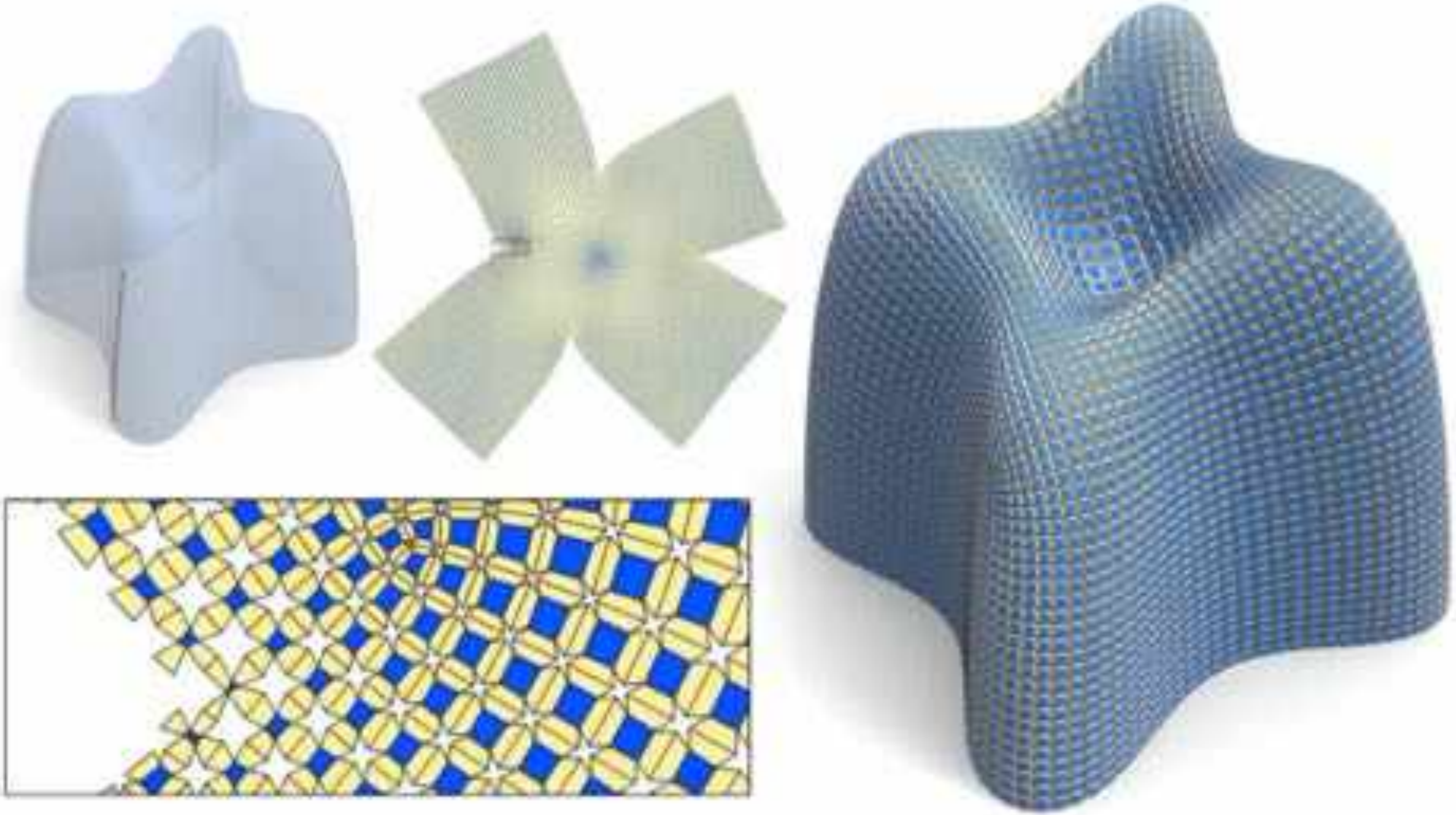
principal distortions ≥ 1.

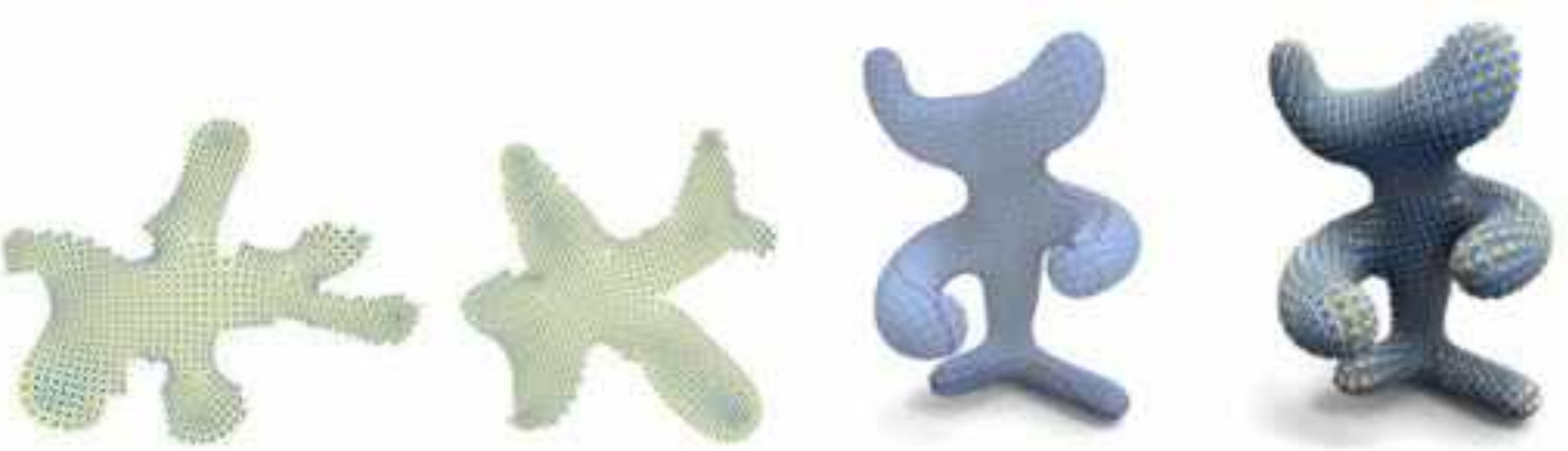
Discrete expanding mapping

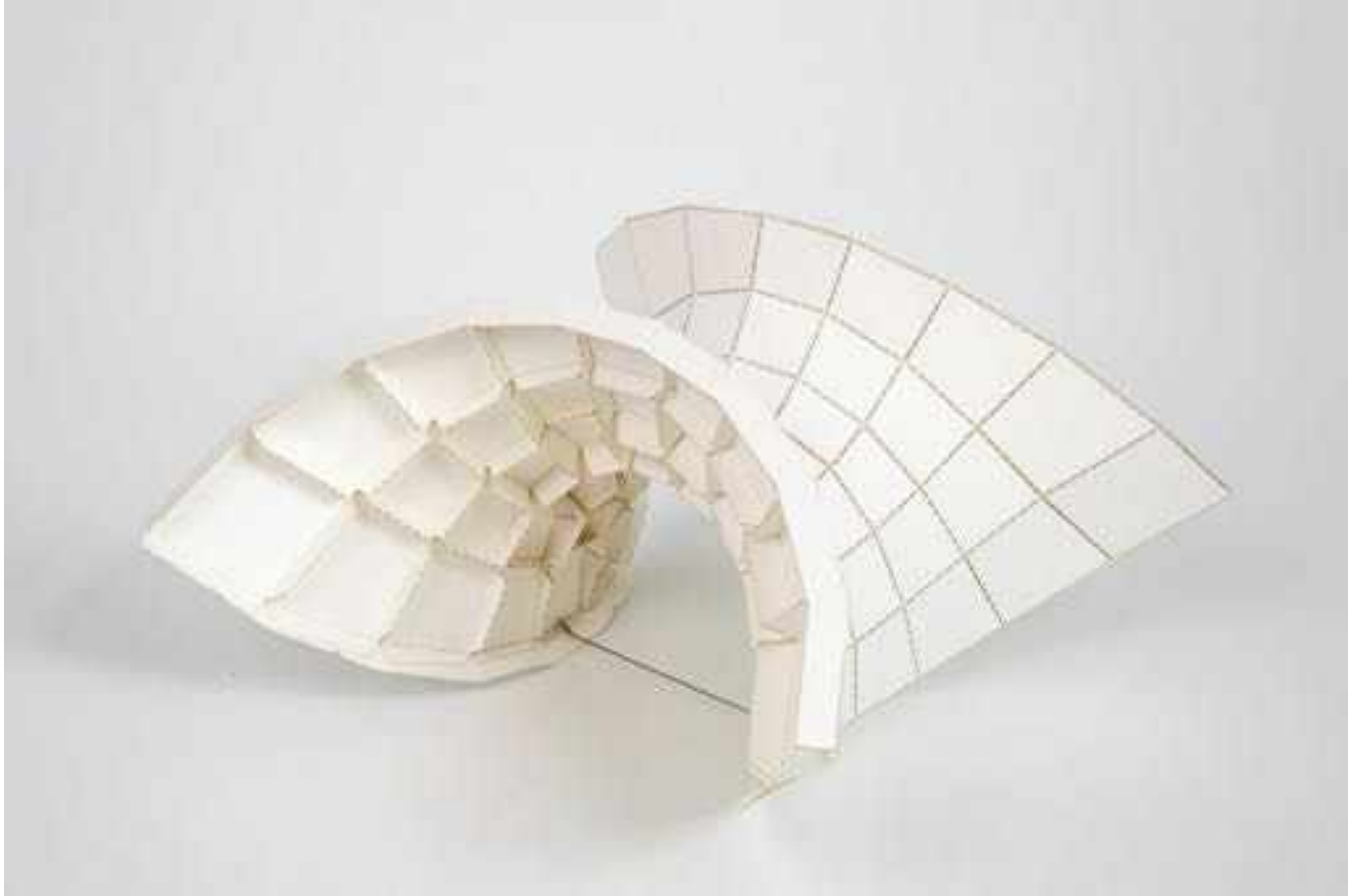
$$\eta^2 = h\frac{1-\cos\beta}{\sin\beta}$$



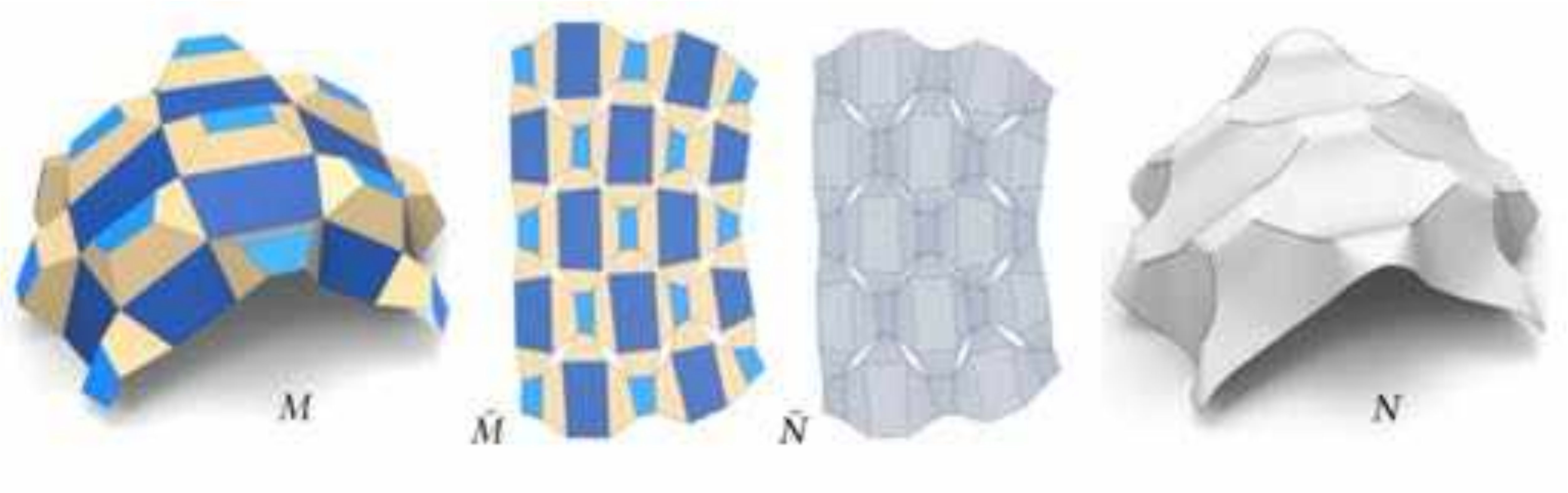


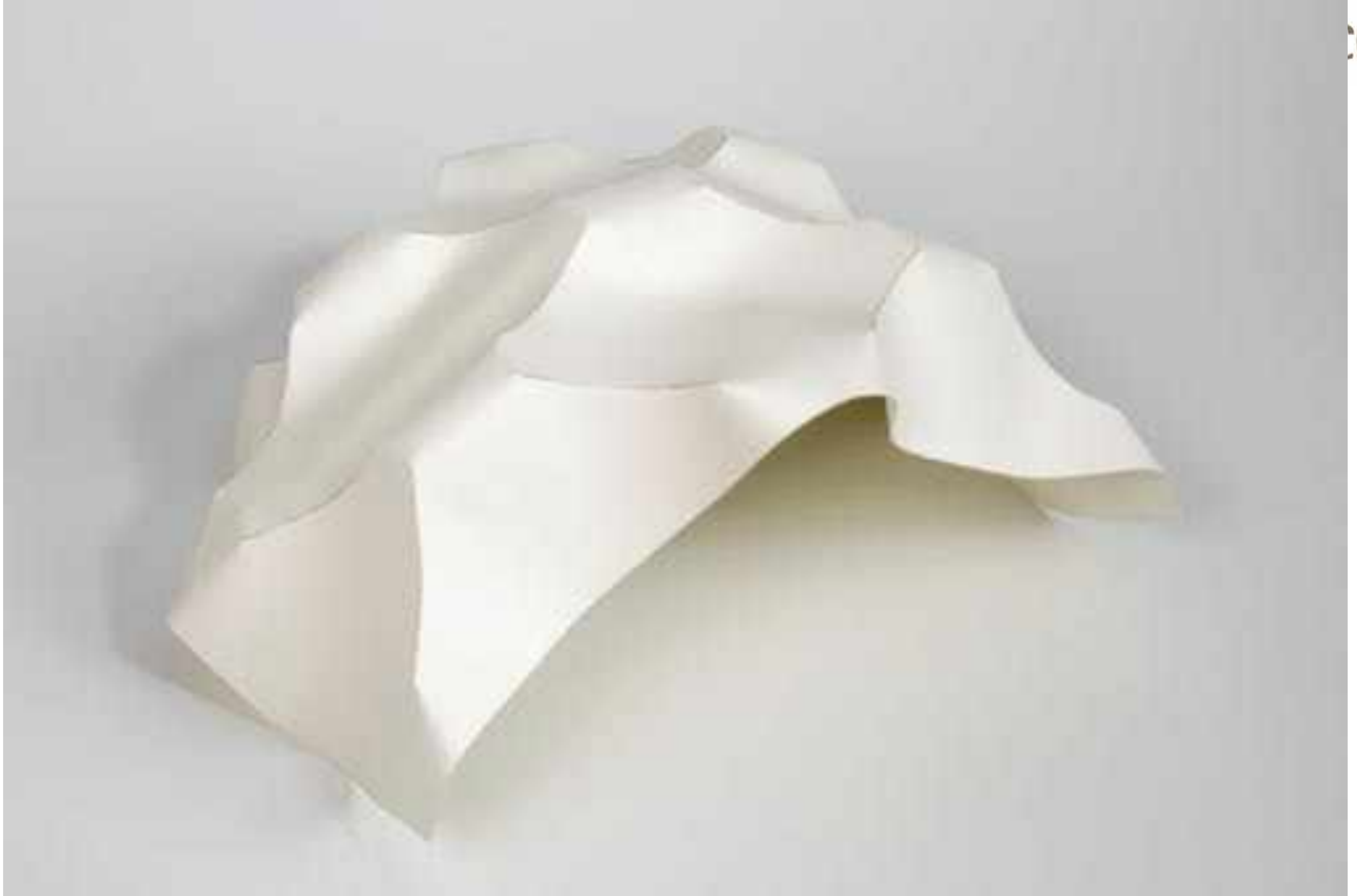






Curved Kirigami





VISUAL COMPUTING CENTER

Thank you!