



# 3D Descriptor Design and Learning for Robust Non-rigid Shape Matching

---

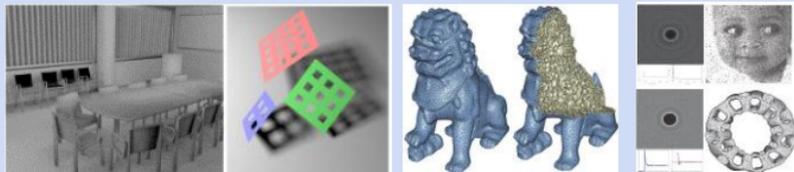


**Jianwei Guo 郭建伟**

NLPR, Institute of Automation,  
Chinese Academy of Sciences

Sept.17, 2020  
GAMES Webinar

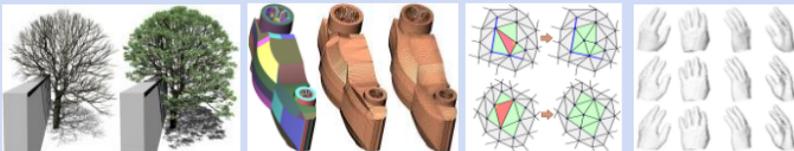
# Team--3D Visual Computing



[ACM TOG (SigAsia) 2016]

[CAGD 2016]

[TVCG 2017]



[CGF 2018]

[CAD 2019]

[TVCG 2018]

[CGF 2019]



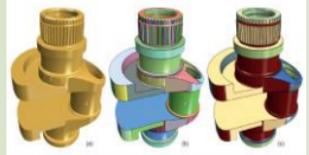
[TVCG 2020]

[TVCG 2020]

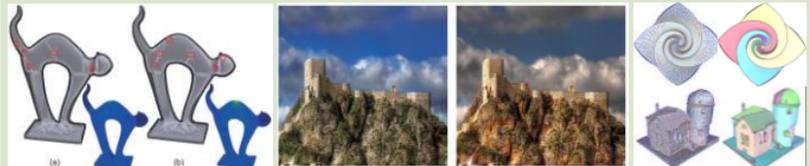
[ACM TOG 2020]



[ACM TOG (SIGGRAPH Asia) 2012]



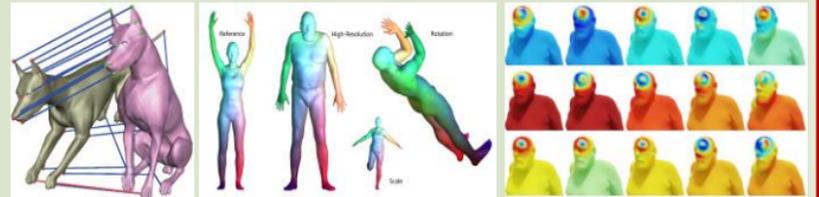
[GMP 2006, CAD 2012, JCAD 2018]



[CAGD 2018]

[IEEE TIFS 2018]

[CAD 2020]



[ECCV 2018]

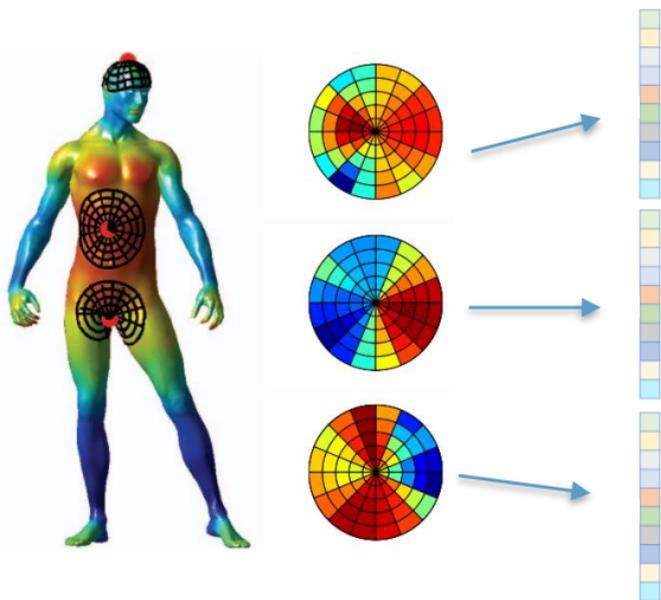
[CVPR 2019]

[ACM TOG (SIGGRAPH) 2020]

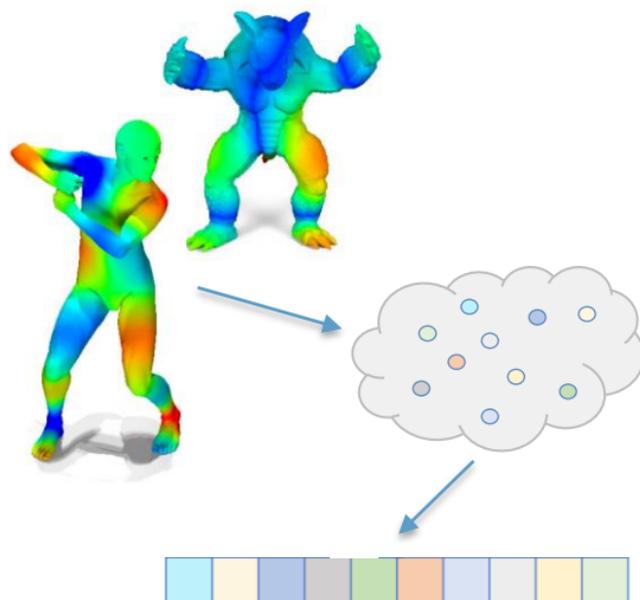
Modeling & Mesh Optimization

Shape Analysis

# 3D Shape Descriptor

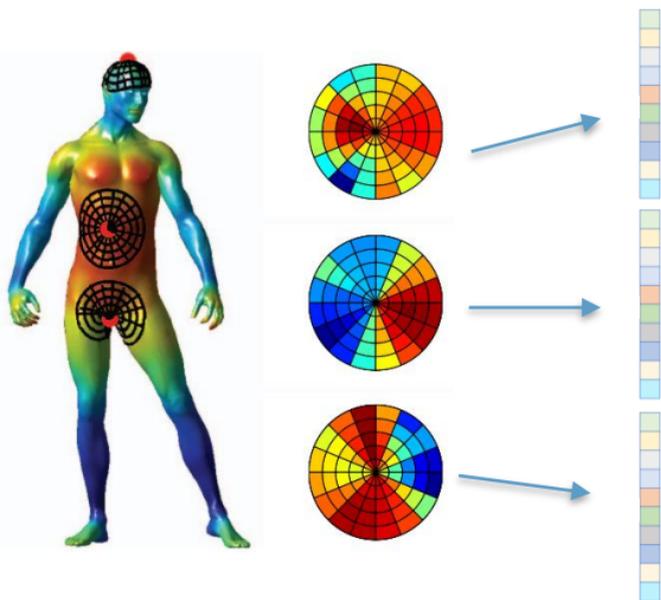


Local descriptor



Global descriptor

# 3D Shape Descriptor



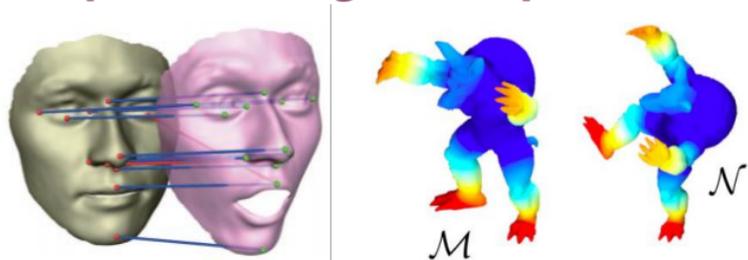
Local descriptor

Descriptor design goals:

- **Discriminative:** able to determine if a pair of vertices is similar or different
- **Robust:** work with different discretizations of a surface

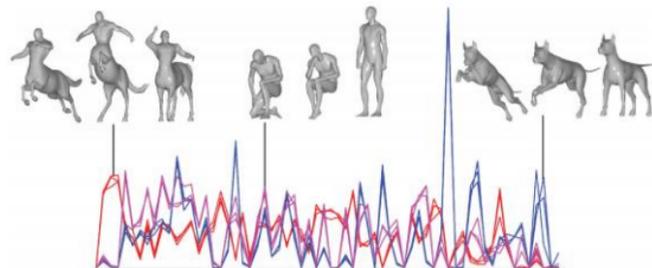
# Applications

## Shape matching/correspondence



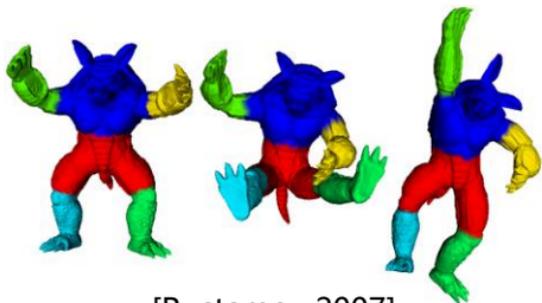
[Ovsjanikov et al. 2017; Wang et al. 2018]

## Shape retrieval



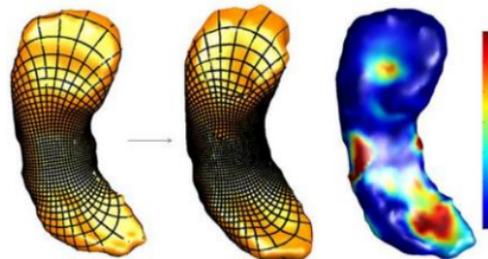
[Ovsjanikov et al. 2009]

## Shape segmentation



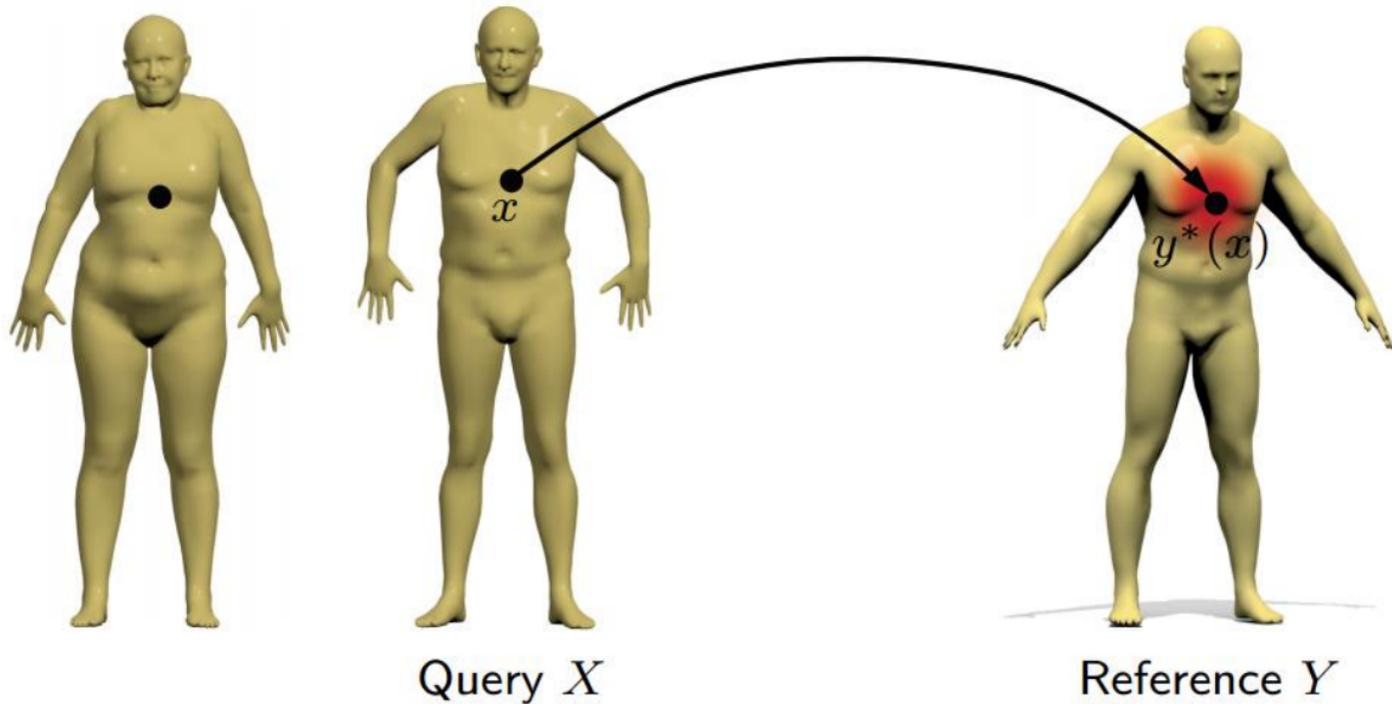
[Rustamov 2007]

## Surface registration

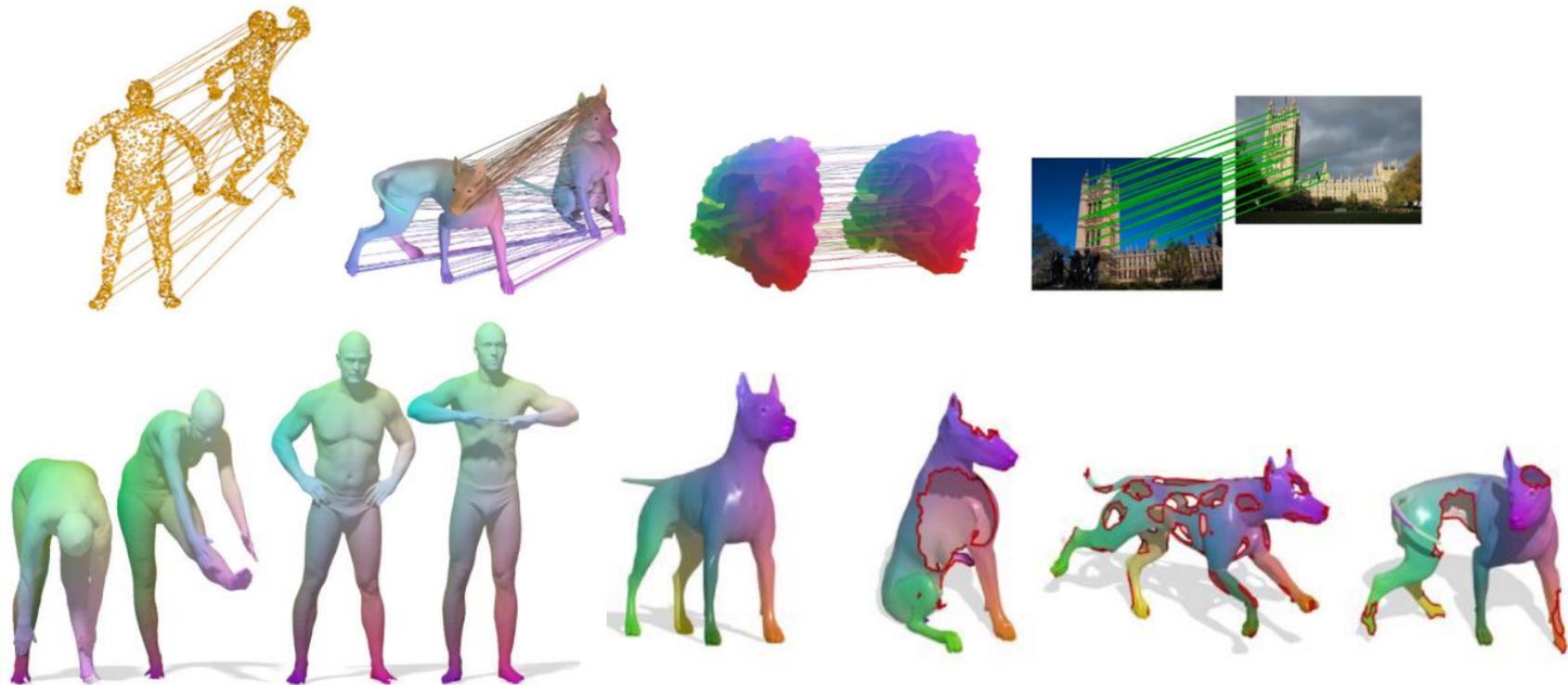


[Lui et al. 2010]

# Shape matching/correspondence



# Shape matching/correspondence



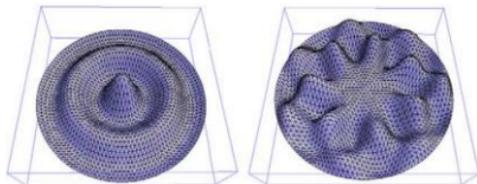
Van Kaick et al.: *A survey on shape correspondence*. CGF, 2011.

Ovsjanikov et al.: *Computing and processing correspondences with functional maps*. SIGGRAPH ASIA 2016 Courses.

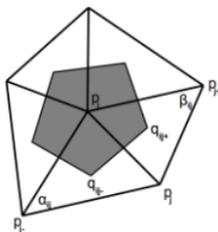


# Related Work

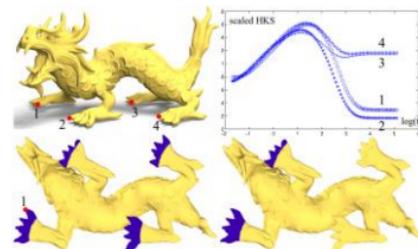
## ❖ Spectral domain approaches



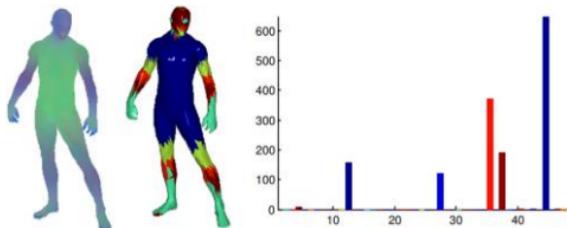
Shape-DNA [1999Reuter et al. 2006]



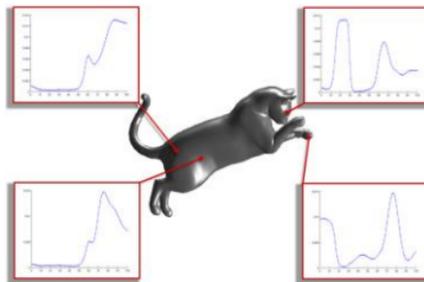
GPS [Rustamov 2007]



HKS [Sun et al. 2010]



Scale-invariant HKS [Bronstein and Kokkinos 2010]



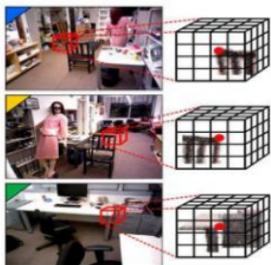
WKS [Aubry et al. 2011]



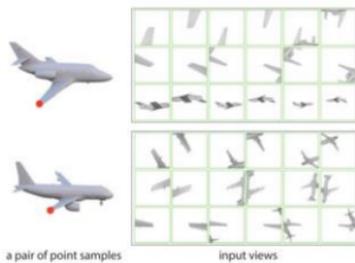
DTEP [Melziet al. 2018]

# Related Work

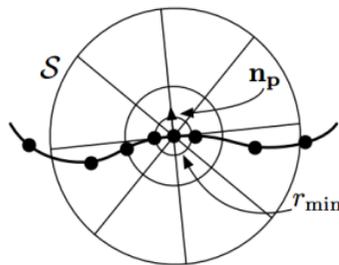
## ❖ Deep learning approaches



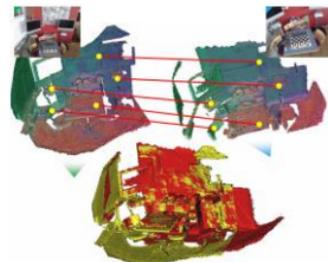
[Zeng et al. 2017]



[Huang et al. 2018]

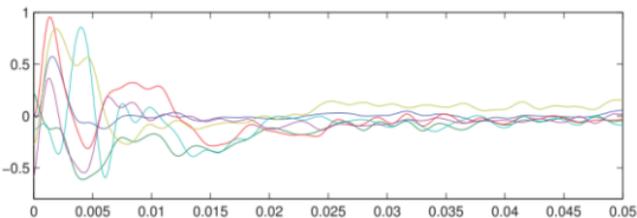


CGF [Khoury et al. 2017]

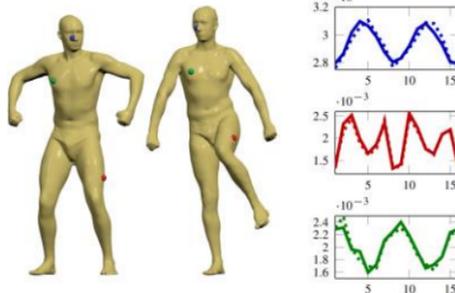


PPFNet [Deng et al. 2018]

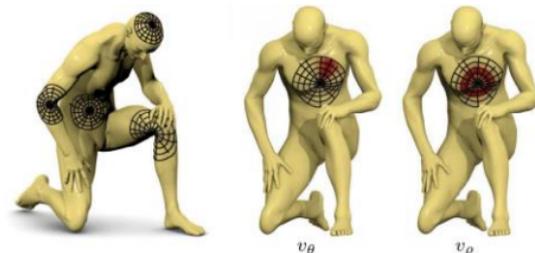
- Mesh-aware



OSD [Litman and Bronstein 2014]



[Boscaini et al. 2015, 2016]



[Masci et al. 2018]

# Related Work

## ❖ Non-learned descriptors

- + **pros** : WKS and HKS are **robust**
- - **cons** : **not as discriminative** as supervised descriptors

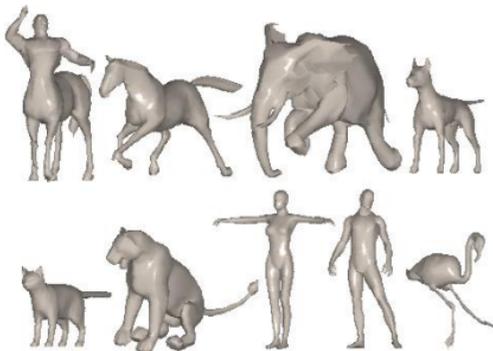


## ❖ Supervised descriptors

- + **pros**: GNNs compute **discriminative** descriptors
- - **cons** : **less robust** to different surface discretizations



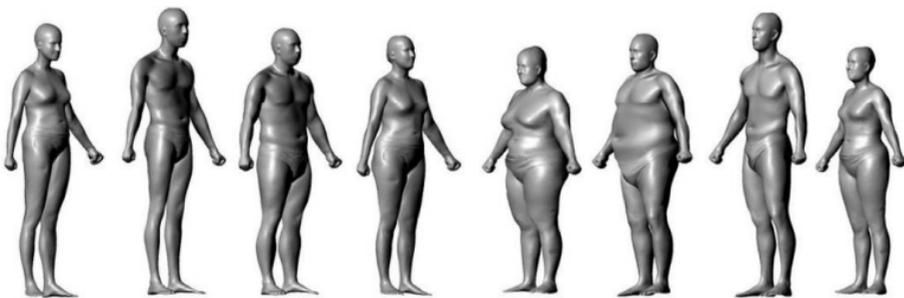
# Datasets



TOSCA [Bronstein et al. 2008]



SCAPE [Anguelov et al. 2005]



SPRING [Yang et al. 2014]

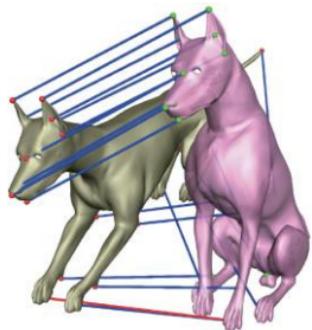


FAUST [Bogo et al. 2014]

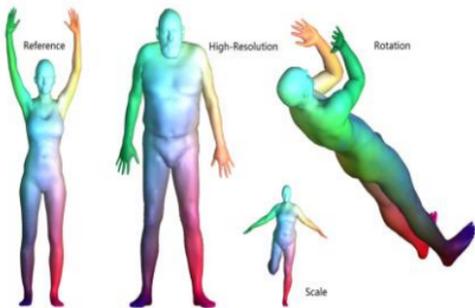
# Our Work

## ❖ Contributions

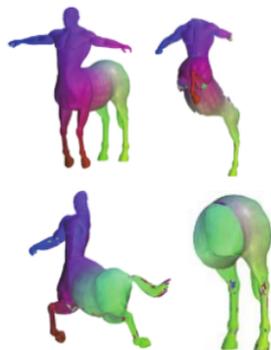
- Two new non-learned features, namely, *Local Point Signature* (LPS) and *Wavelet Energy Decomposition Signature* (WEDS). They exhibit high resilience to changes in mesh resolution, triangulation, scale, and rotation.
- Two supervised frameworks to transform the non-learned features to more discriminative descriptors



[ECCV 2018]



[CVPR 2019]



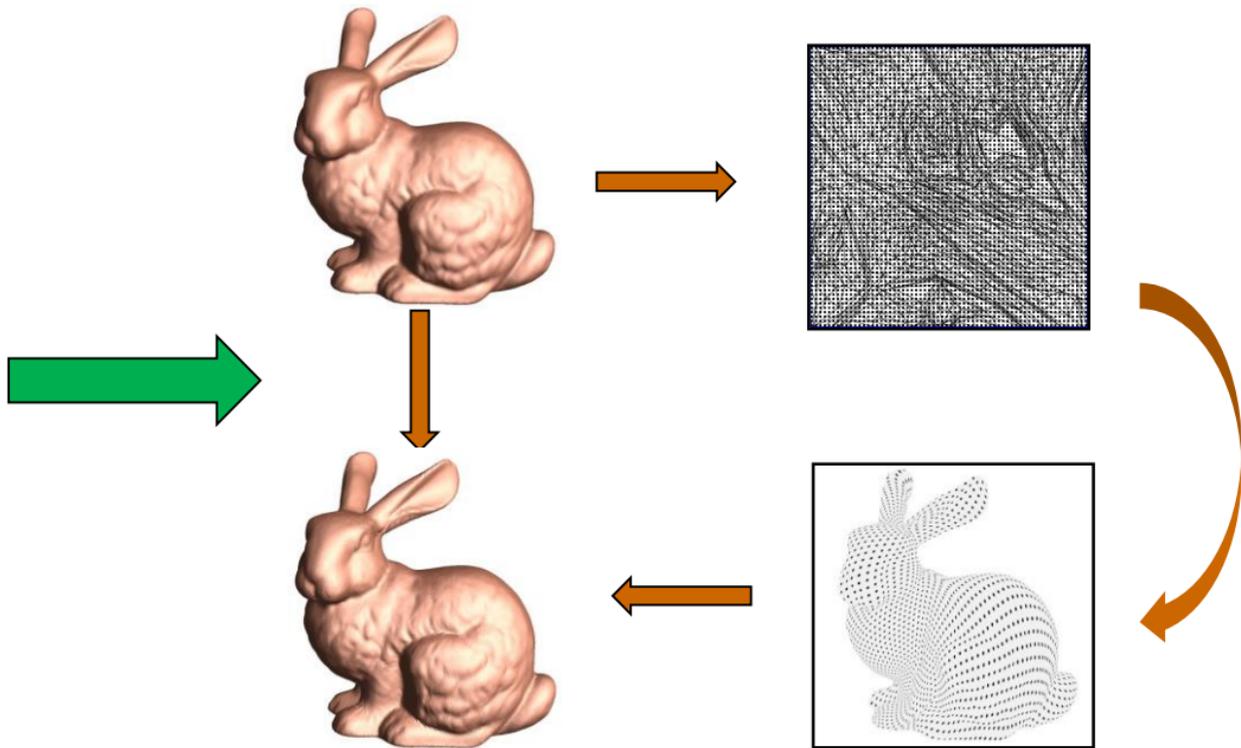
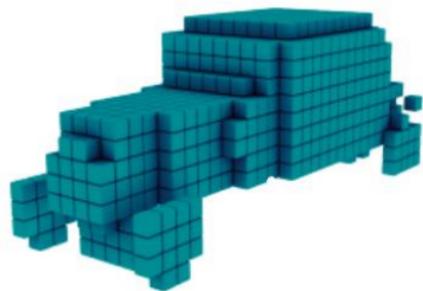
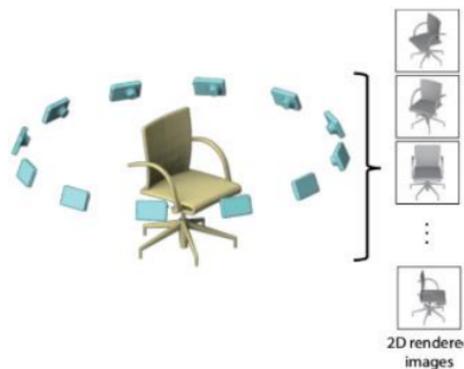
[CVMJ 2020]



[ACM TOG (SIGGRAPH) 2020]

# 1-Descriptor Learning using Geometry Images

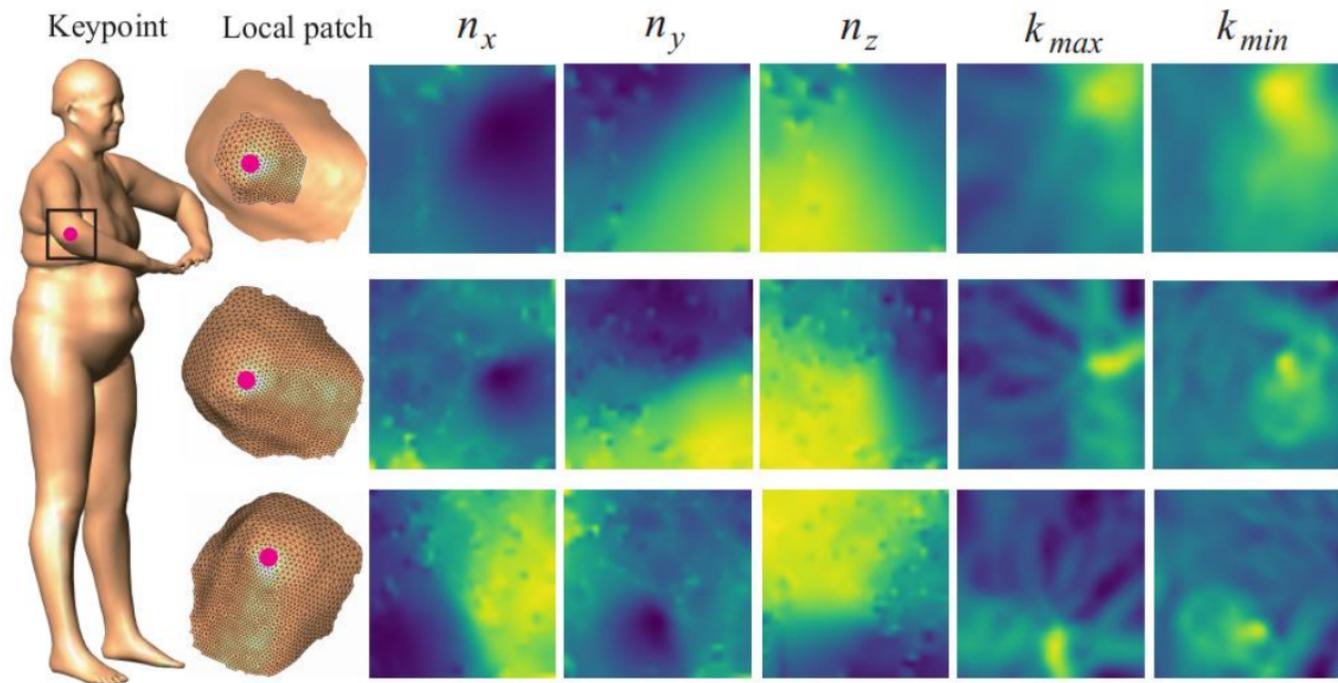
## ❖ Motivation



Geometry images [Gu et al. 2002]

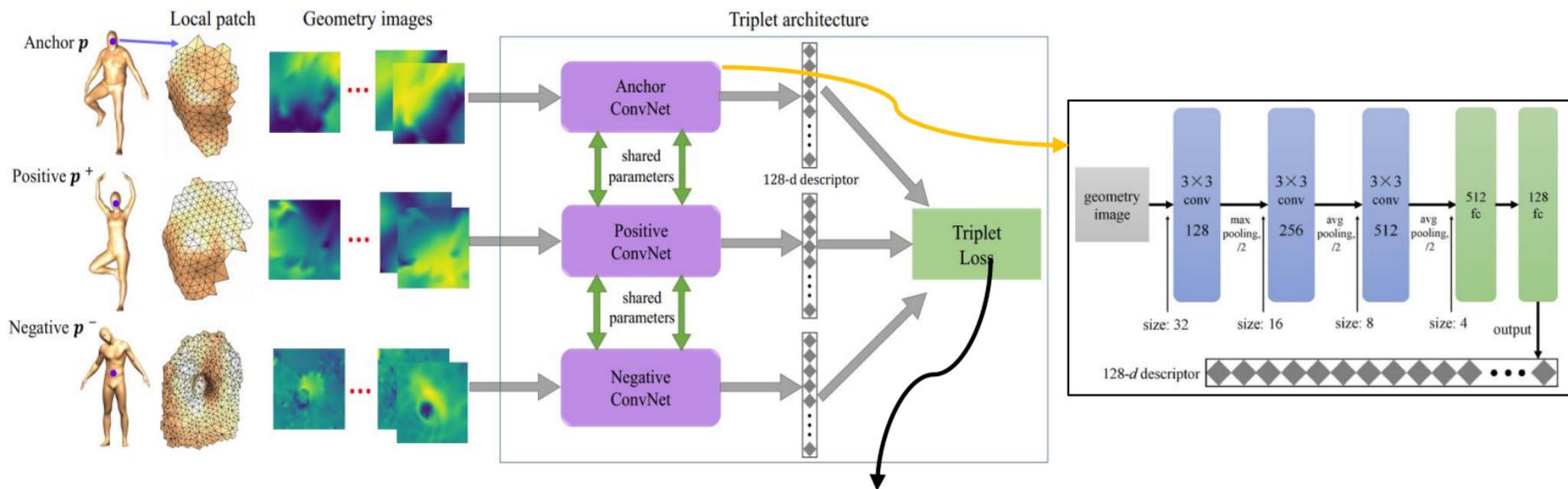
# 1-Descriptor Learning using Geometry Images

## ❖ Motivation



# 1-Descriptor Learning using Geometry Images

## Triplet Neural Network



$$L_{Min-CV} = \lambda \frac{\sigma(D_{pos})}{\mu(D_{pos})} + \sum_{i=1}^N [D_{pos}^i - D_{neg}^i + \alpha]_+$$

# 1-Descriptor Learning using Geometry Images

## ❖ Triplet Loss

- Classic triplet loss:

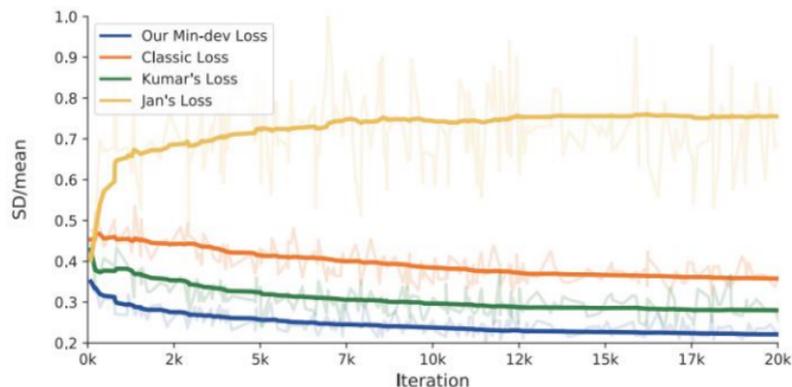
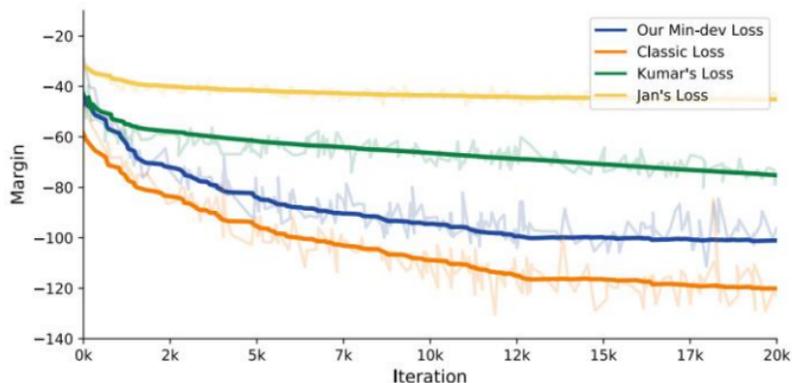
$$L = \sum_{i=1}^N [D_{\text{pos}}^i - D_{\text{neg}}^i + \alpha]_+$$

$$D_{\text{pos}}^i = D(f(\mathbf{p}_i), f(\mathbf{p}_i^+))$$

$$D_{\text{neg}}^i = D(f(\mathbf{p}_i), f(\mathbf{p}_i^-))$$

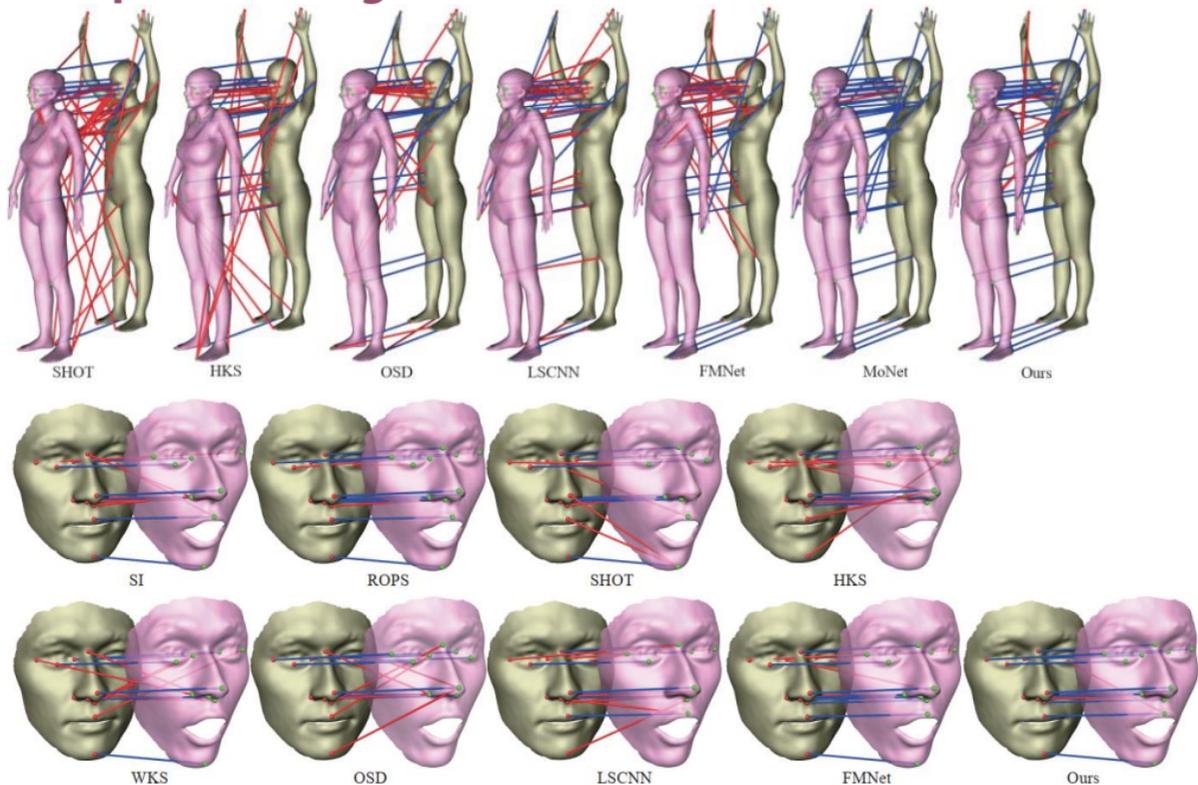
- Min-Coefficient of Variation (Min-CV) loss:

$$L_{\text{Min-CV}} = \lambda \frac{\sigma(D_{\text{pos}})}{\mu(D_{\text{pos}})} + \sum_{i=1}^N [D_{\text{pos}}^i - D_{\text{neg}}^i + \alpha]_+$$



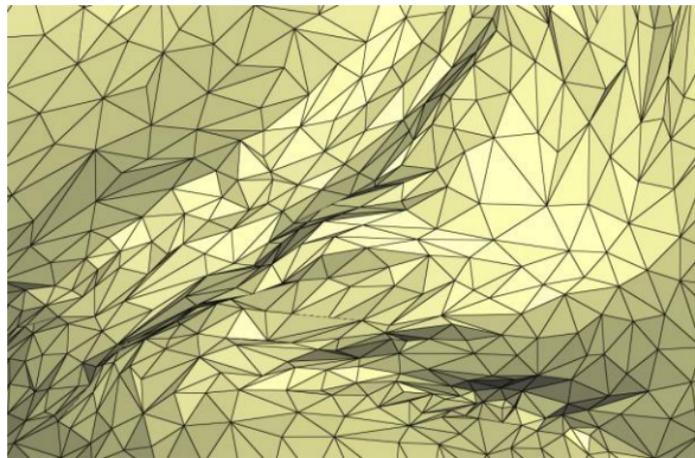
# 1-Descriptor Learning using Geometry Images

## ❖ Results: shape matching

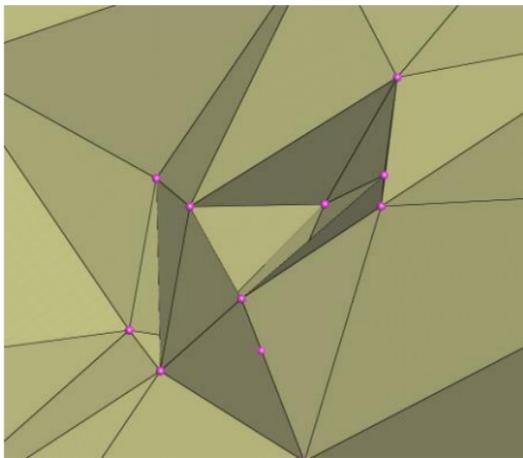


# 1-Descriptor Learning using Geometry Images

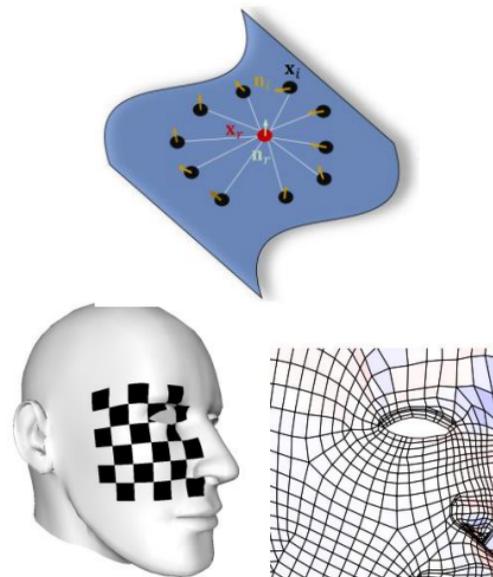
## ❖ Discrete Geodesic Polar Coordinates (DGPC)



ill-shaped triangles



degenerate triangles

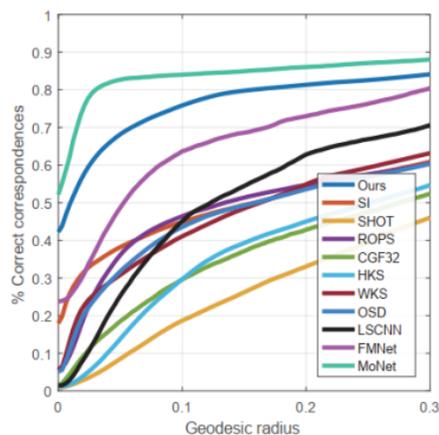
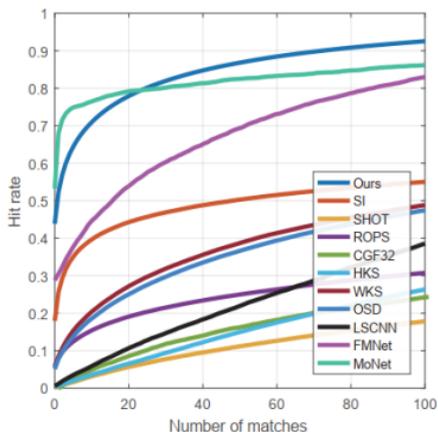


DGPC [Melvæ  
and Reimers 2012]

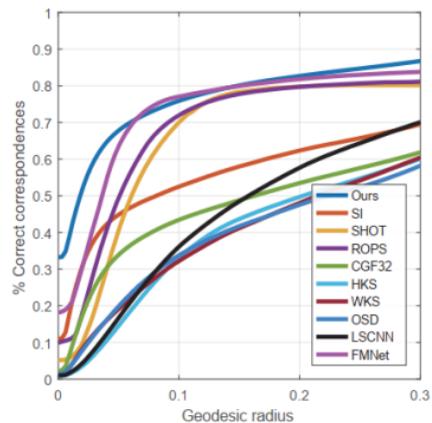
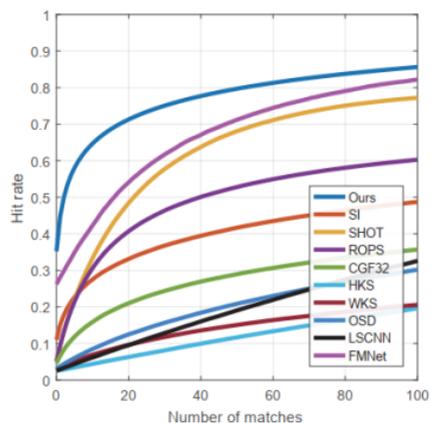
# 1-Descriptor Learning using Geometry Images

## ❖ Results: shape correspondence

- Cumulative match characteristic (CMC)
- Cumulative geodesic error (CGE)



FAUST



SPRING

# 1-Descriptor Learning using Geometry Images

## ❖ Results: shape correspondence



Source



Ground truth



SI



SHOT



ROPS



CGF-32



HKS



WKS



OSD



LSCNN



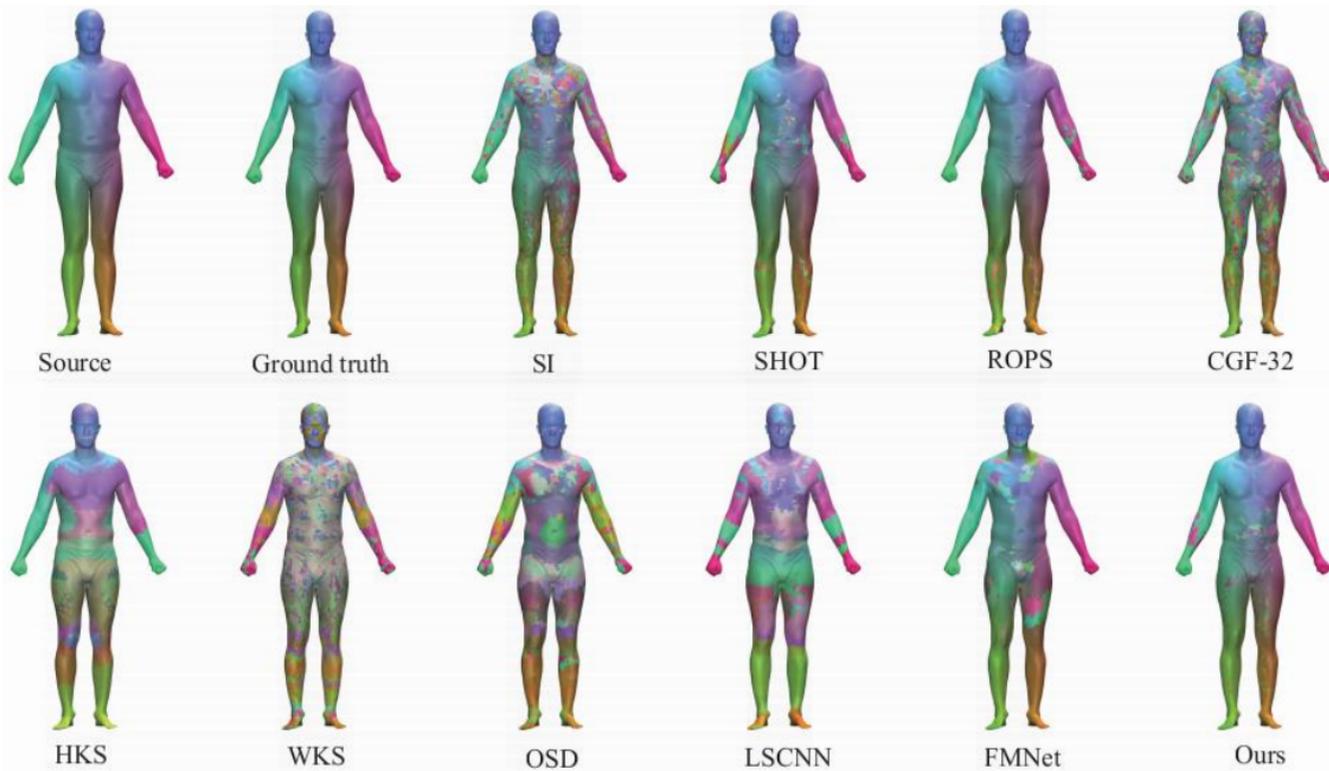
FMNet



Ours

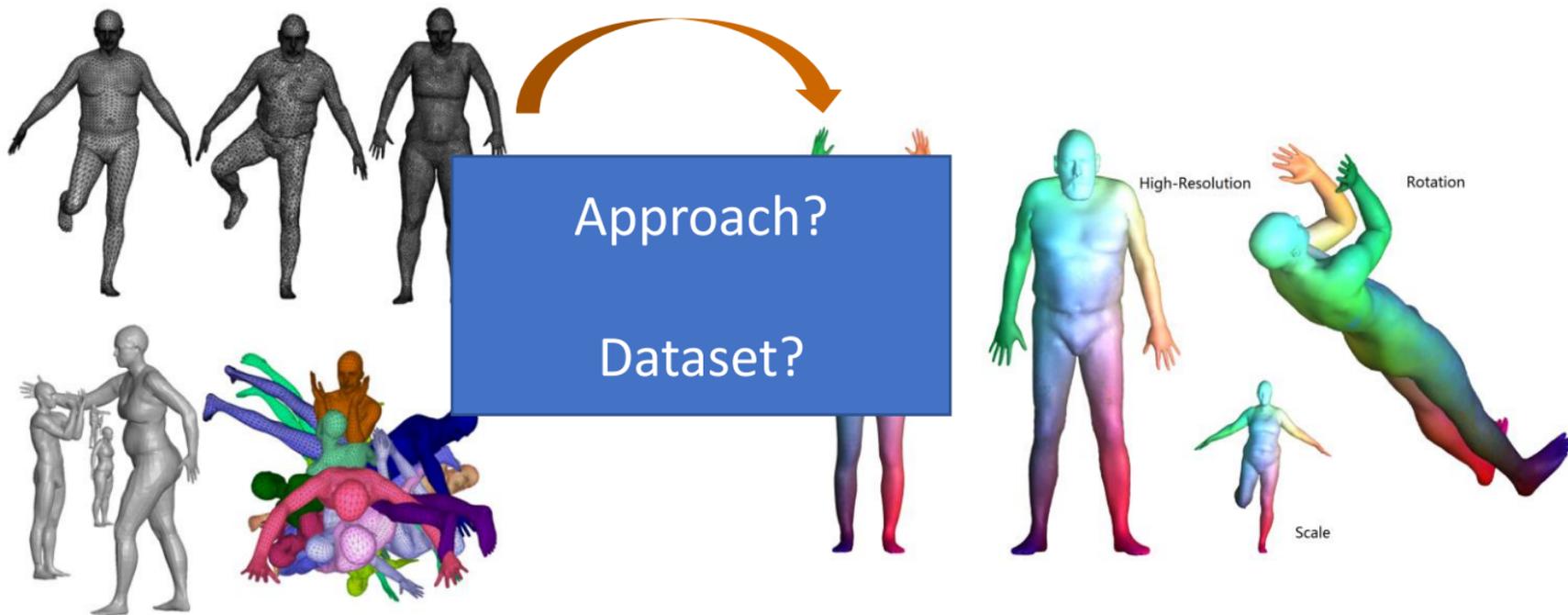
# 1-Descriptor Learning using Geometry Images

## ❖ Results: shape correspondence



# 2-Robust Local Spectral Descriptor

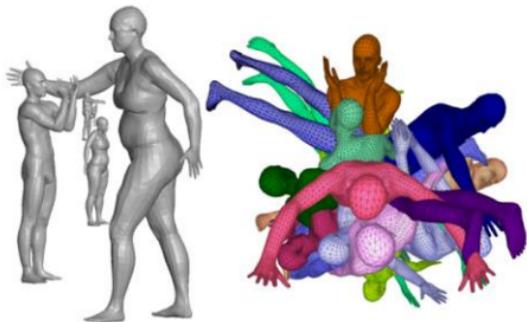
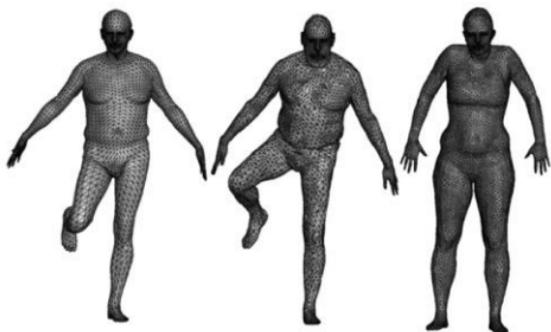
## ❖ Motivation



Resolutions, triangulations, scales, rotations

# 2-Robust Local Spectral Descriptor

## ❖ Motivation



Dirichlet energy

$$f : S \rightarrow R$$

$$E(f) = \int_S |\nabla f(v)|^2 dv = \int_S f(v) \Delta f(v) dv$$

# 2-Robust Local Spectral Descriptor

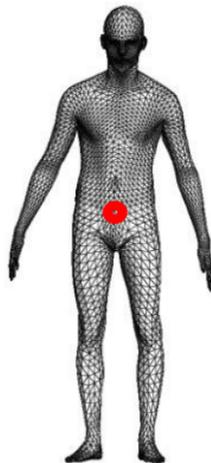
## ❖ Local Point Signature

$$E(f) = \int_S |\nabla f(v)|^2 dv = \int_S f(v) \Delta f(v) dv \quad \longrightarrow \quad \Delta f = -\text{div}(\nabla f)$$

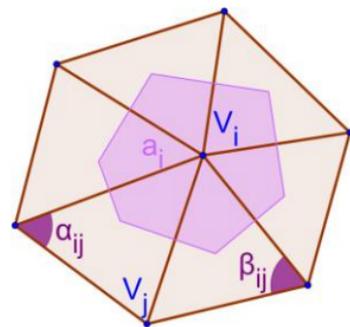
Laplace-Beltrami operator

$$\mathbf{L}_{ij} = \begin{cases} -\frac{\cot \alpha_{ij} + \cot \beta_{ij}}{2a_i} & \text{if } i, j \text{ are adjacent} \\ \sum_k \frac{\cot \alpha_{ik} + \cot \beta_{ik}}{2a_i} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{L}\Phi_i = \lambda_i \Phi_i, i = 0, 1, \dots, k-1 \quad \{\lambda_i | i = 0, 1, \dots, k-1\}$$



Mesh  $M$   
 $\tilde{f}: V \rightarrow \mathbb{R}$



$$\Delta \tilde{f}(v_i) = \frac{1}{a_i} \sum_{j \in N(v_i)} \frac{\cot \alpha_{ij} + \cot \beta_{ij}}{2} (\tilde{f}(v_i) - \tilde{f}(v_j))$$

# 2-Robust Local Spectral Descriptor

## ❖ Local Point Signature

$$\mathbf{T}\Phi_i = \lambda_i \mathbf{A}\Phi_i, i = 0, 1, \dots, k-1$$

$$\langle \Phi_i, \Phi_j \rangle_{\mathbf{A}} = \Phi_i^T \mathbf{A}\Phi_j$$

$$\tilde{E}(\tilde{f}) = \tilde{f}^T \mathbf{A}\tilde{f} = \sum_{j=0}^{N-1} \sigma_j^2 \lambda_j$$

$$\sigma_j = \langle \tilde{f}, \Phi_j \rangle_{\mathbf{A}} = \tilde{f}^T \mathbf{A}\Phi_j$$

Spectral coefficients

$$\tilde{E}(\tilde{F}) = \sum_{i=1}^d \sum_{j=0}^{N-1} \sigma_{ij}^2 \lambda_j = \sum_{j=0}^{N-1} \lambda_j \sum_{i=1}^d \sigma_{ij}^2$$

$$\tilde{F} = (\tilde{f}_1, \tilde{f}_2, \dots, \tilde{f}_d) : V \rightarrow R^d$$

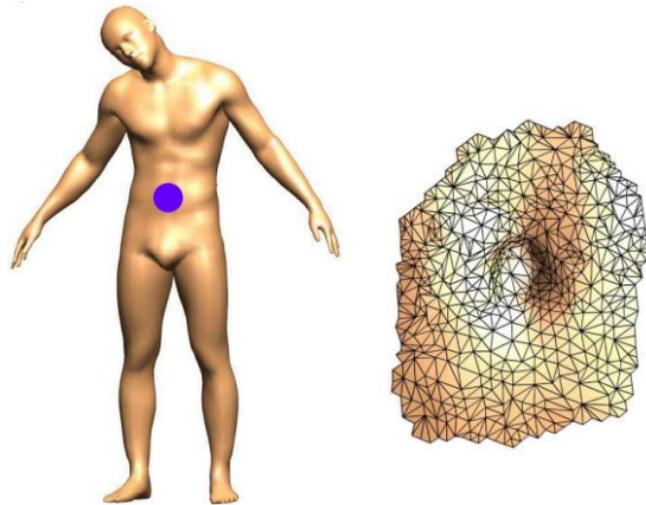
$$sf = \left( \lambda_1 \sum_{i=1}^d \sigma_{i1}^2, \lambda_2 \sum_{i=1}^d \sigma_{i2}^2, \dots, \lambda_{N-1} \sum_{i=1}^d \sigma_{iN-1}^2 \right)$$

# 2-Robust Local Spectral Descriptor

## ❖ Local Point Signature

- Build a local patch mesh around a vertex
- Compute Laplacian eigenvectors and eigenvalues
- Compute spectral coefficients

$$\sigma_j = \langle \tilde{f}, \Phi_j \rangle_{\mathbf{A}} = \tilde{f}^T \mathbf{A} \Phi_j \quad X = (x_1, x_2, x_3) : V \rightarrow R^3$$

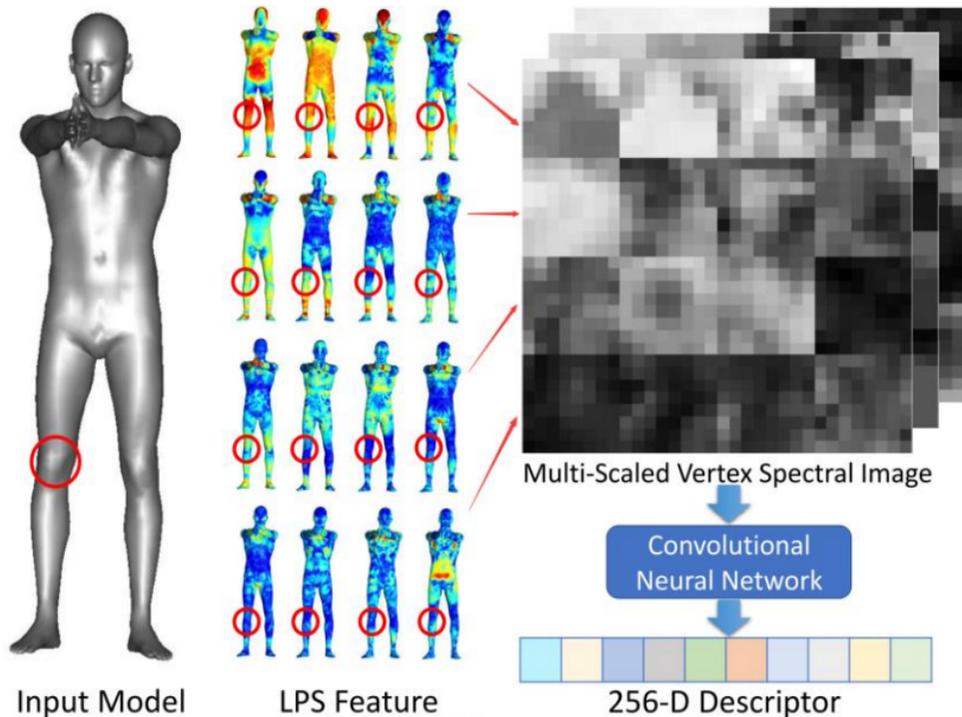


- LPS is derived from Dirichlet energy and expressed as follows

$$LPS = \left( \lambda_1 \sqrt{\sum_{i=1}^3 \sigma_{i1}^2}, \lambda_2 \sqrt{\sum_{i=1}^3 \sigma_{i2}^2}, \dots, \lambda_{16} \sqrt{\sum_{i=1}^3 \sigma_{i16}^2} \right)$$

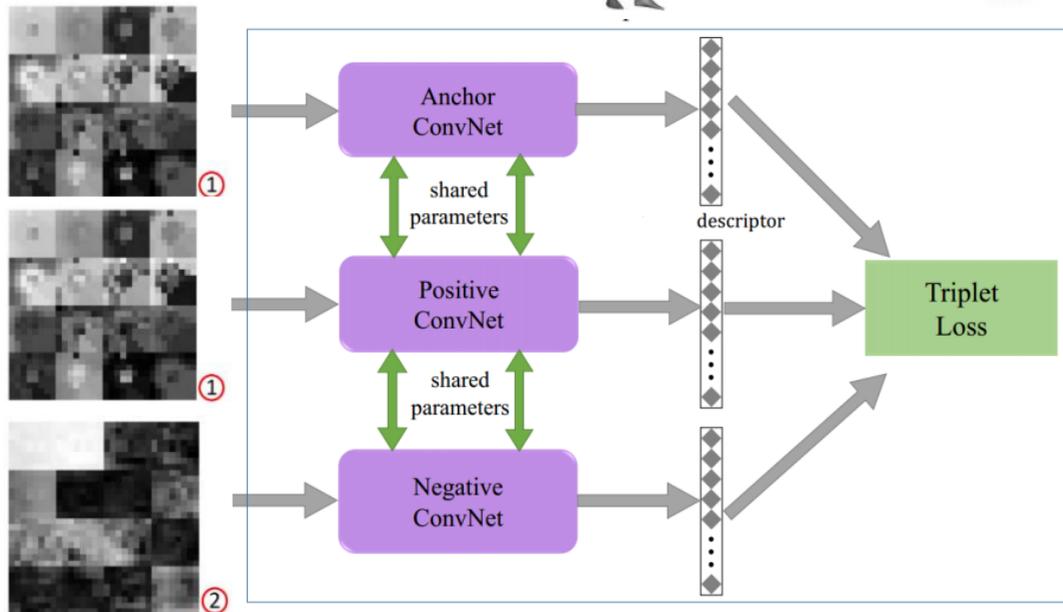
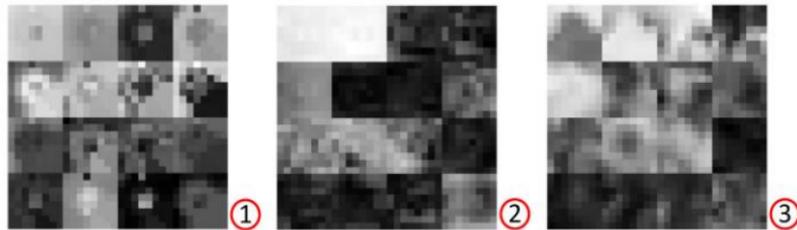
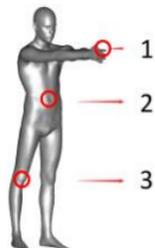
# 2-Robust Local Spectral Descriptor

## ❖ Descriptor learning



# 2-Robust Local Spectral Descriptor

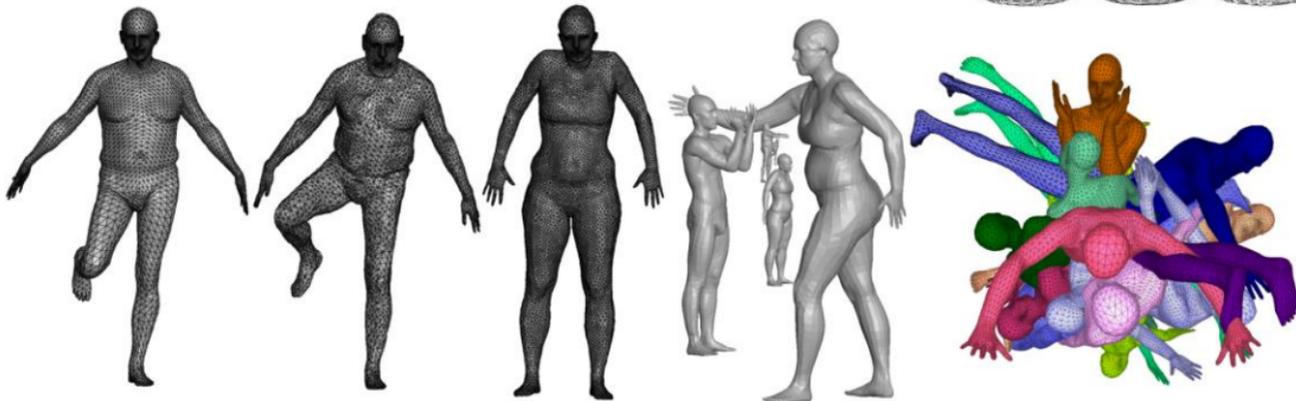
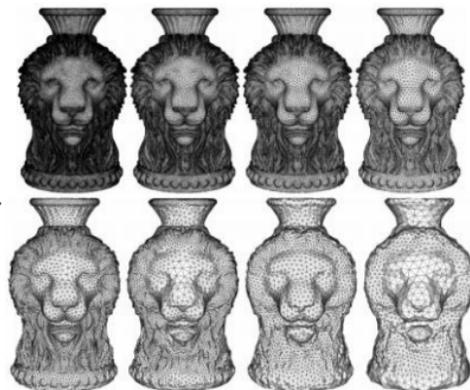
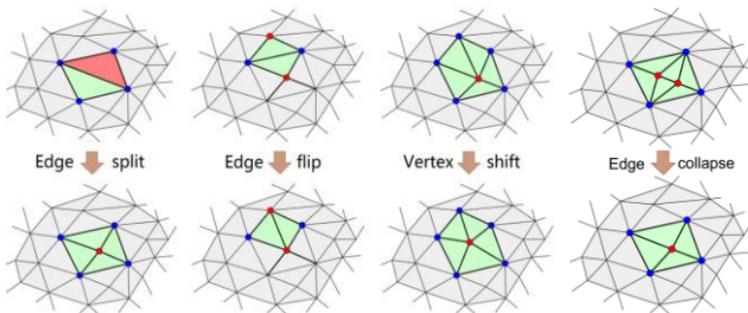
## ❖ Descriptor learning



# 2-Robust Local Spectral Descriptor

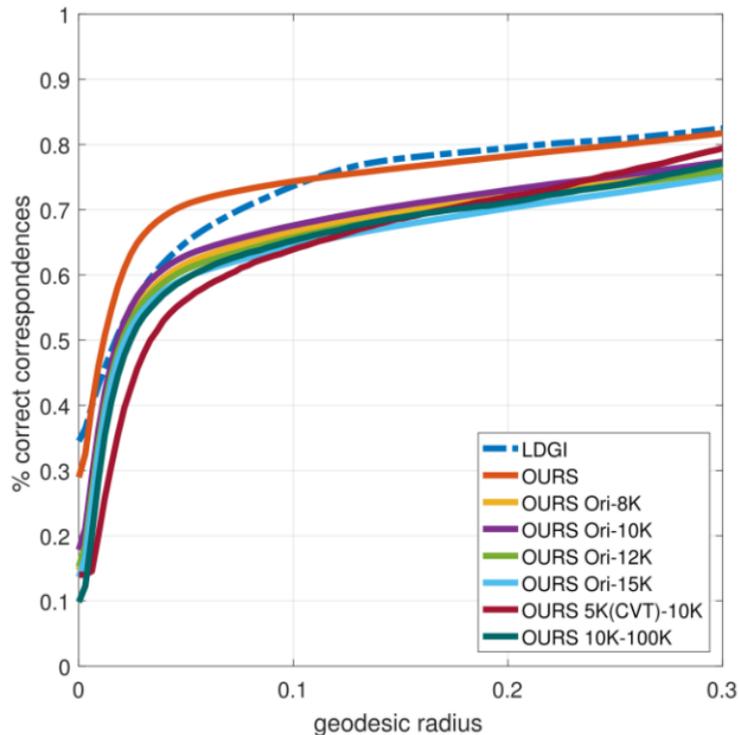
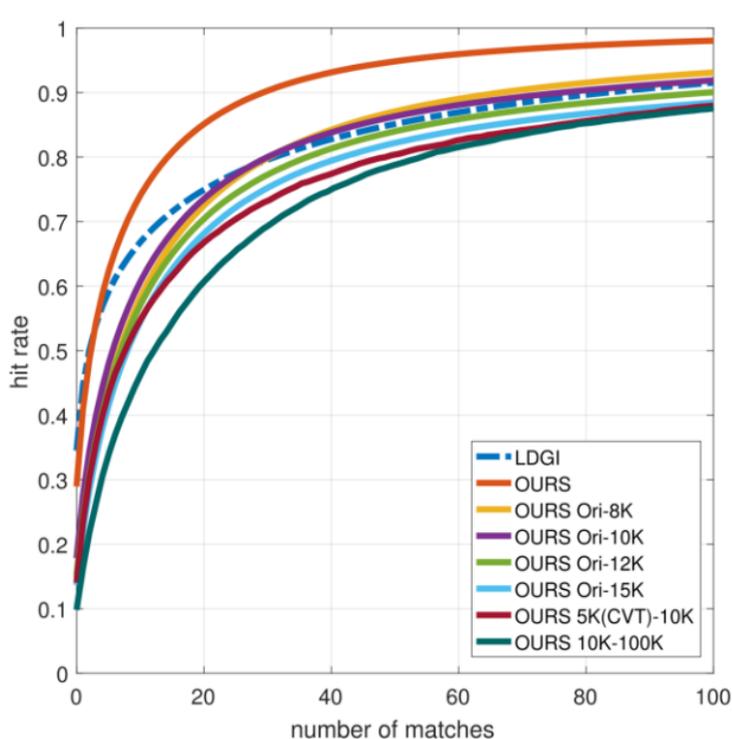
## Dataset

Discrete optimization method (Wang et al. 2019)



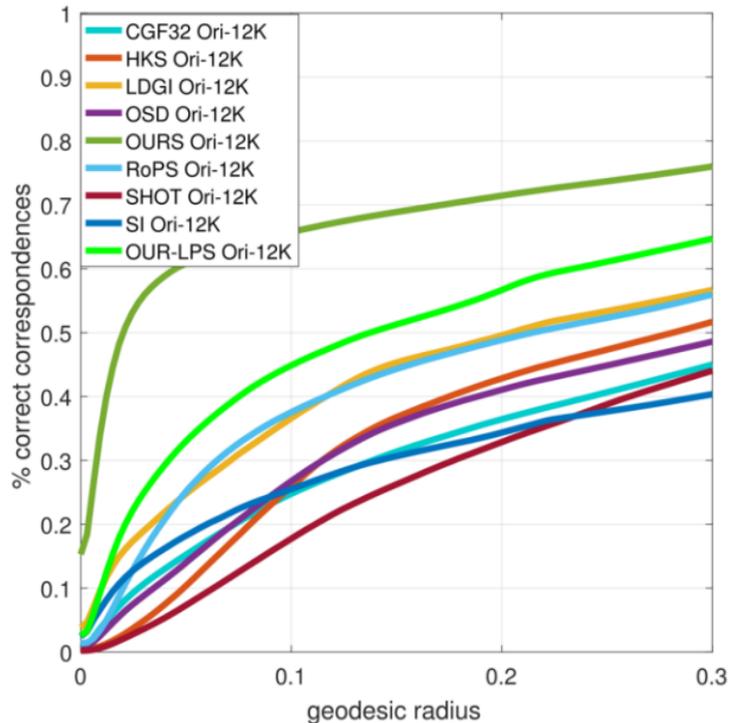
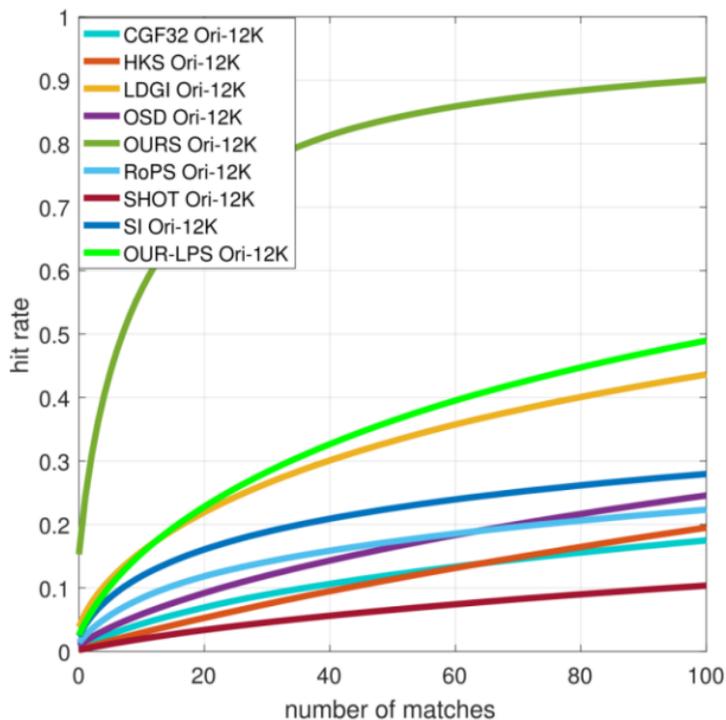
# 2-Robust Local Spectral Descriptor

## ❖ Results: Robust to resolution and triangulation



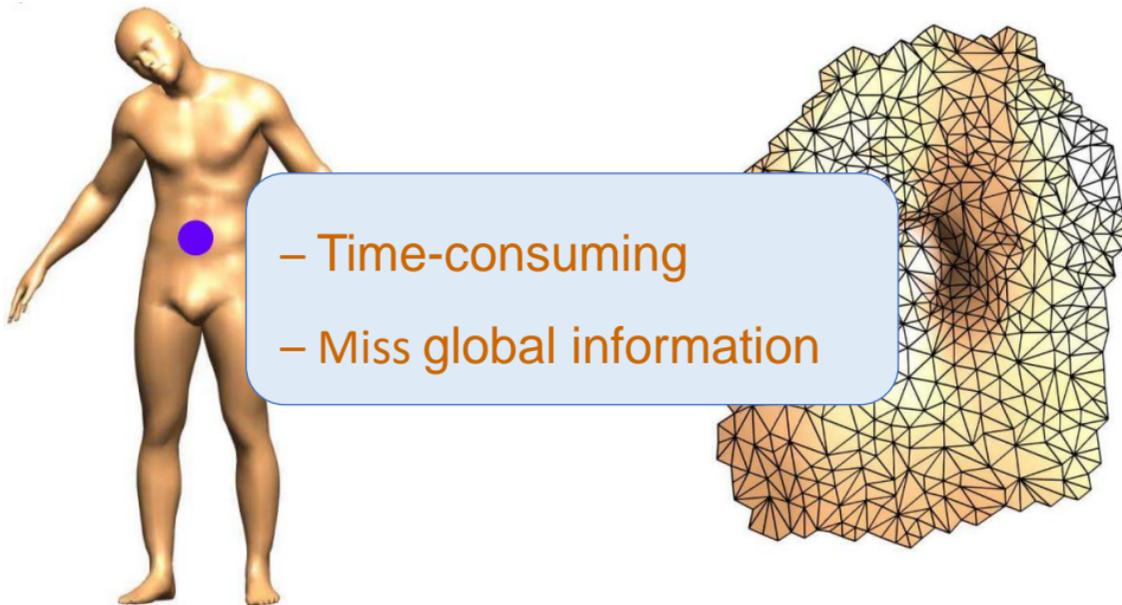
# 2-Robust Local Spectral Descriptor

## ❖ Results: Comparison



# 3-Descriptor Learning using Multiscale GCNs

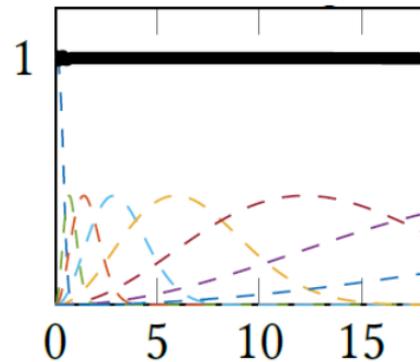
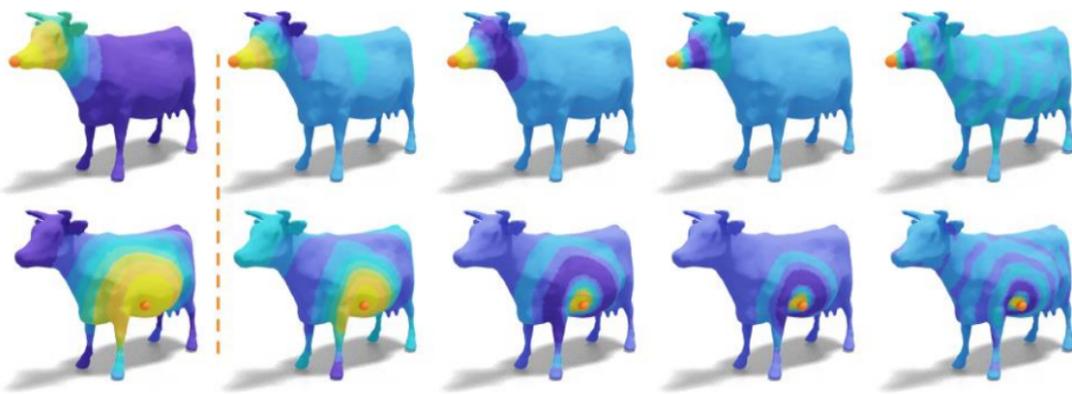
## ❖ Motivation



cut a geodesic disk

# 3-Descriptor Learning using Multiscale GCNs

## ❖ Graph wavelets



$$\xi_v = \sum_{j=0}^{N-1} a(v) h(\lambda_j) \phi_j(v) \phi_j$$

Scaling functions

$$\psi_{t,v} = \sum_{j=0}^{N-1} a(v) g(t\lambda_j) \phi_j(v) \phi_j$$

Wavelet functions

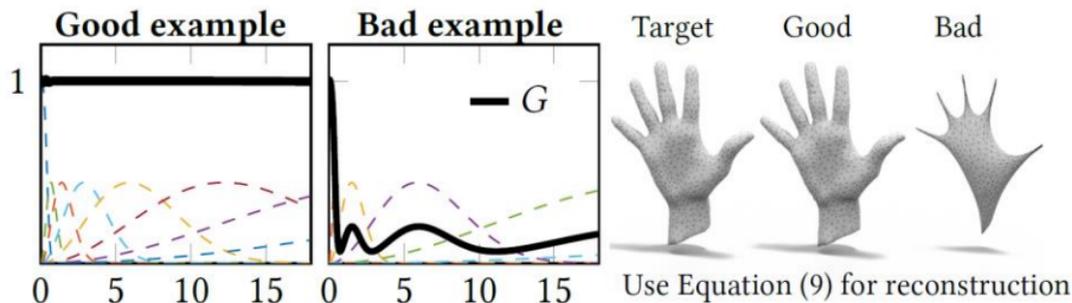
Wavelet filter functions

- Multiscale property of wavelet functions

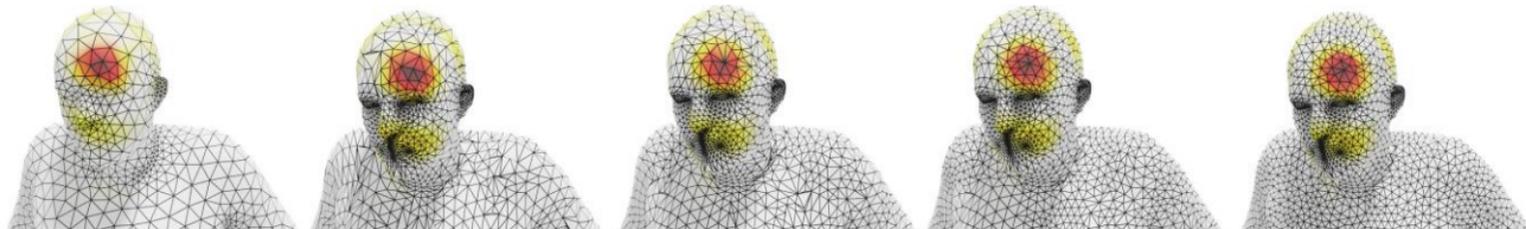
# 3-Descriptor Learning using Multiscale GCNs

## ❖ Graph wavelets

- Reconstruction property of multiscale wavelet functions



- Robustness to the change of resolution and triangulation



# 3-Descriptor Learning using Multiscale GCNs

## ❖ Wavelet Energy Decomposition Signature (WEDS)

Reconstruct discrete Dirichlet energy

Restructure into a sum per vertex

Collect the local energy using multiscale wavelets

$$E(\mathbf{f}) = \mathbf{f}^T \mathbf{L} \mathbf{f} = \sum_{j=0}^{N-1} \lambda_j \left( \sum_{m=0}^K \sum_v \gamma_j(t_m, v) \right)^2$$

$$\begin{aligned} E(\mathbf{X}) &= \sum_{i=1}^d \sum_{j=0}^{N-1} \lambda_j \sum_{m=0}^K \sum_v \gamma_{ij}(t_m, v) \omega_{ij} \\ &= \sum_{m=0}^K \sum_v \left[ \sum_{j=0}^{N-1} \lambda_j \sum_{i=1}^d \gamma_{ij}(t_m, v) \omega_{ij} \right] \end{aligned}$$

$$WEDS_{t_s}(v) = \left\{ \sum_x \psi_{t_s, v}^*(x) \varepsilon_{t_m}(x) \right\}, m \in [0, K]$$

# 3-Descriptor Learning using Multiscale GCNs

## ❖ Wavelet Energy Decomposition Signature (WEDS)

Pose 1  
 $n = 7K$



Pose 2  
 $n = 15K$



# 3-Descriptor Learning using Multiscale GCNs

## ❖ Multiscale Graph Convolution Network

- Convolutions on graphs are defined as:

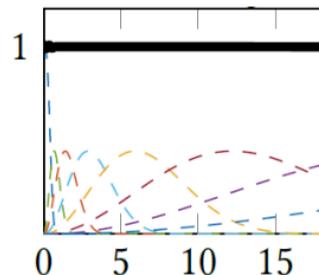
$$\mathbf{x}^{\text{out}} = \mathbf{y} *_{\mathbf{w}} \mathbf{x}^{\text{in}} = \Phi \left( \left( \Phi^T \mathbf{y} \right) \odot \left( \Phi^T \mathbf{x}^{\text{in}} \right) \right) = \Phi \mathbf{w}_{\theta} \Phi^T \mathbf{x}^{\text{in}}$$

- ChebyNet approximates  $\mathbf{w}_{\theta}$  using  $m$ -order polynomials:

$$\mathbf{w}_{\theta} = \sum_{m=0}^{K-1} \theta_m \text{diag} \left( \{ \lambda_j \}_{j=0}^{k-1} \right)^m = \sum_{m=0}^{K-1} \theta_m \Lambda^m$$

- MGCN approximates  $\mathbf{w}_{\theta}$  using multiscale wavelet filter basis:

$$\mathbf{w}_{\theta} = \sum_{m=0}^K \theta_m \text{diag} \left( \{ g_{t_m}(\lambda_j) \}_{j=0}^k \right) = \sum_{m=0}^K \theta_m g_{t_m}(\Lambda)$$

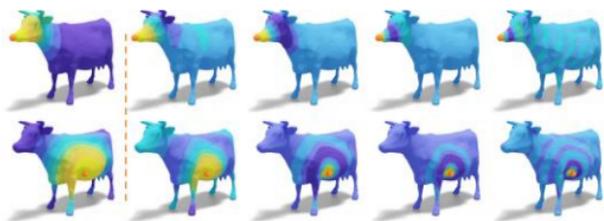


# 3-Descriptor Learning using Multiscale GCNs

## ❖ Multiscale Graph Convolution Network

- The multiscale convolution can be simplified as follows:

$$\mathbf{x}^{\text{out}} = \mathbf{y} *_{\mathbf{w}} \mathbf{x}^{\text{in}} \approx \sum_{t_s \in S_{t_s}} \theta_{t_s} \overline{\Psi}_{t_s}^T \mathbf{x}^{\text{in}}$$



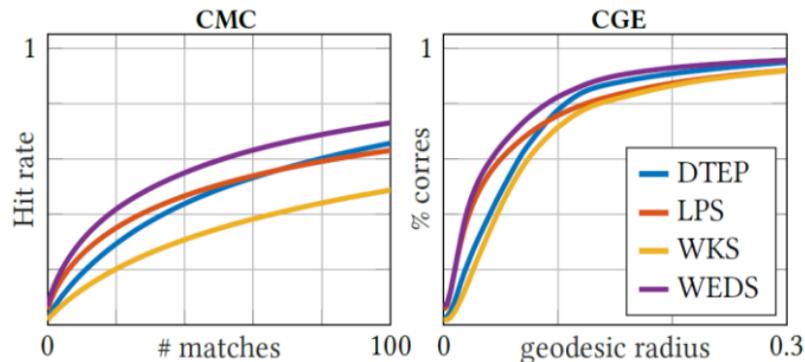
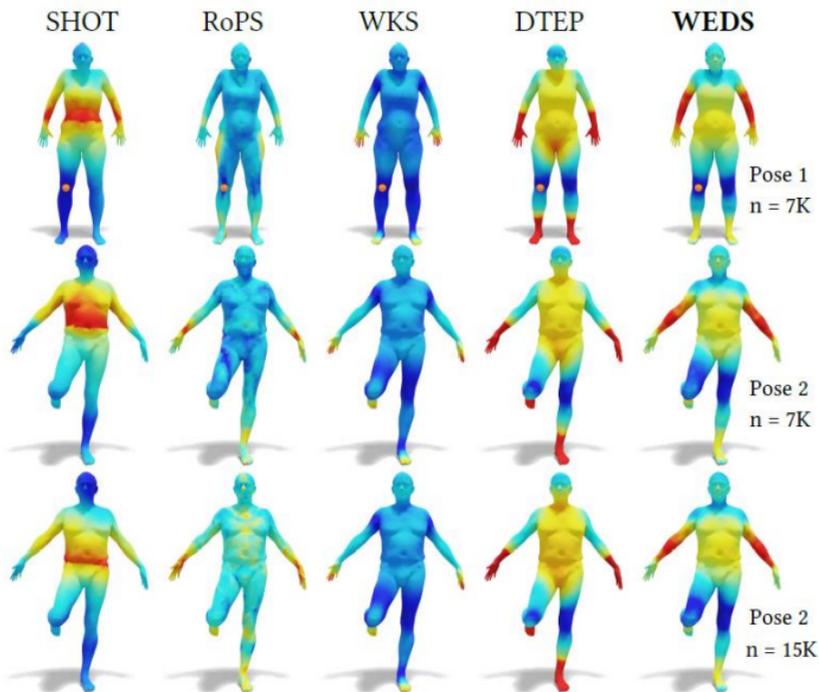
Convolve local and global information



Robustness to discretization

# 3-Descriptor Learning using Multiscale GCNs

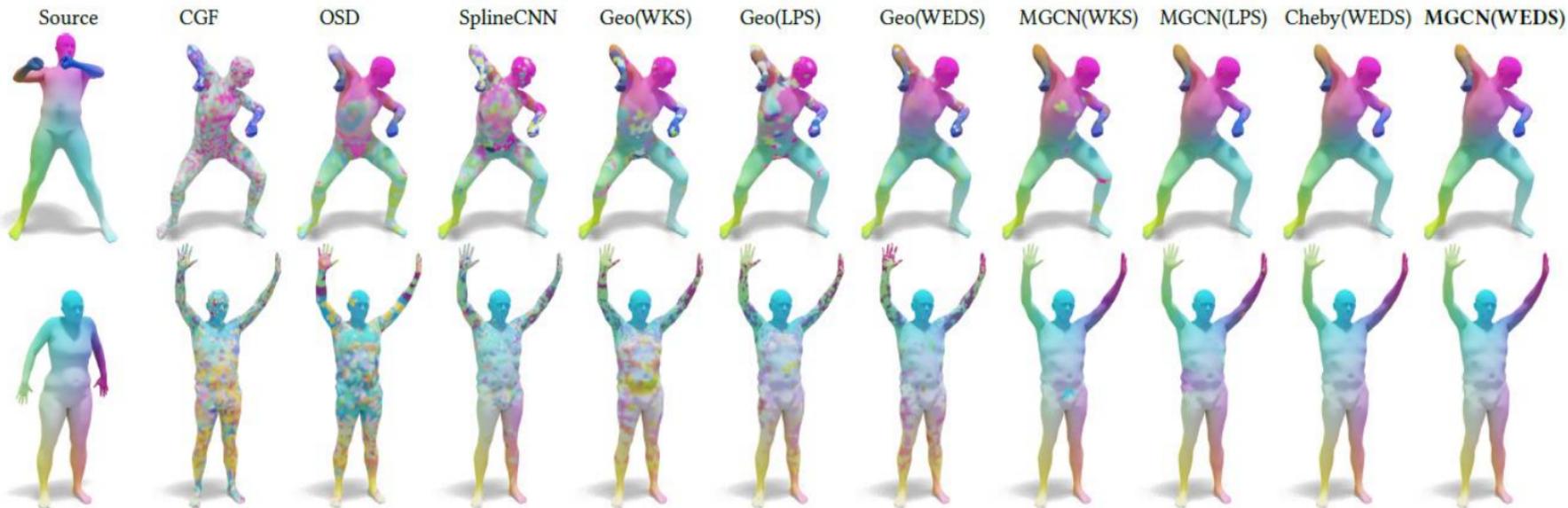
## ❖ Results: performance of WEDS



Descriptors	Dataset	
	FAUST(6890)	SCAPE(12.5K)
SI	352 / 153	380 / 251
SHOT	381 / 262	271 / 196
RoPS	346 / 252	267 / 187
HKS	511 / 396	507 / 409
WKS	335 / 118	260 / 72
LPS	325 / 97	227 / 71
DTEP	312 / 89	267 / 76
WEDS	<b>287 / 69</b>	<b>225 / 66</b>

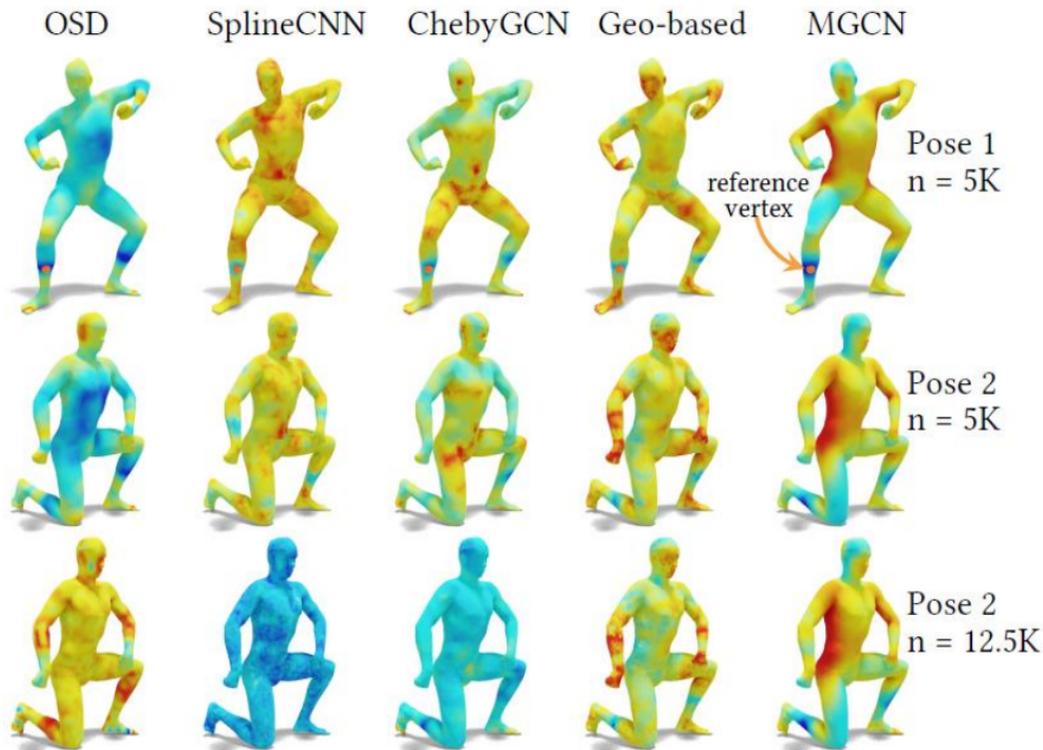
# 3-Descriptor Learning using Multiscale GCNs

## ❖ Results: shape correspondence



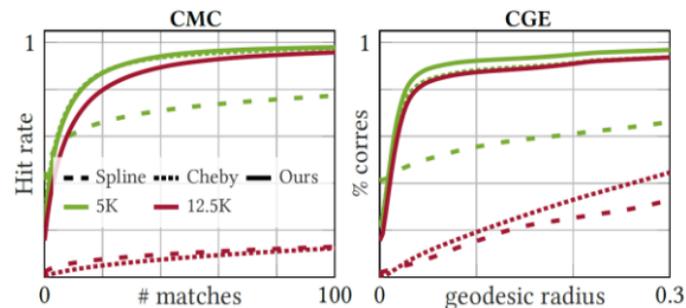
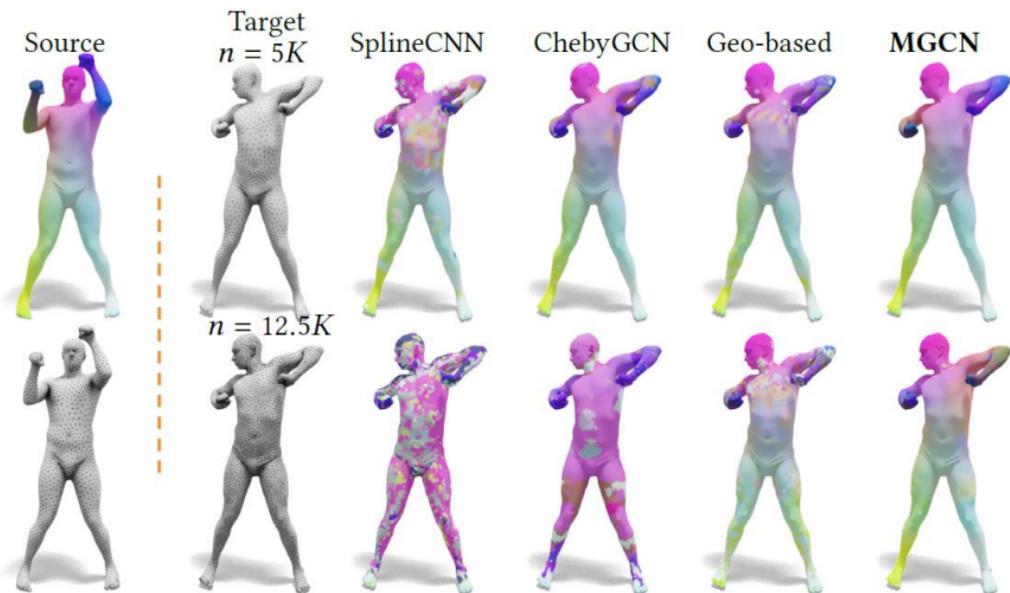
# 3-Descriptor Learning using Multiscale GCNs

## ❖ Results: robust to resolution and triangulation



# 3-Descriptor Learning using Multiscale GCNs

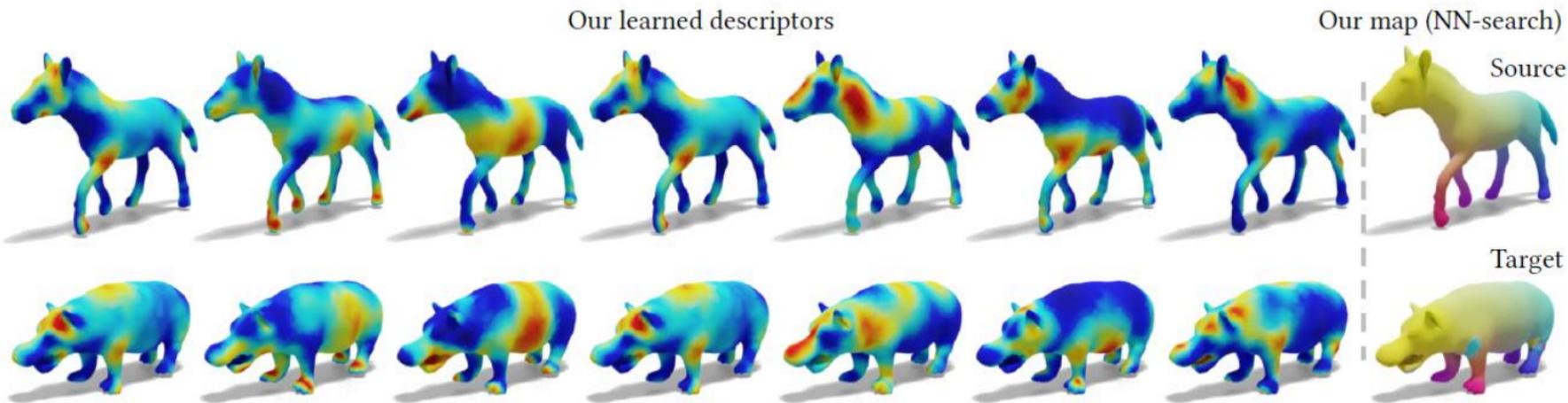
## ❖ Results: robust to resolution and triangulation



Descriptors	Network	#Resolution	
		SCAPE 5K	SCAPE 5K-12.5K(Test)
Histogram	CGF	374 / 264	428 / 323
-	OSD	259 / 94	835 / 742
-	SplineCNN	297 / 180	503 / 353
WEDS	ChebyGCN	68 / 29	458 / 332
WKS	Geo-based	185 / 72	192 / 80
LPS	Geo-based	175 / 68	182 / 74
WEDS	Geo-based	163 / 65	179 / 71
WKS	MGCN	67 / 46	98 / 64
LPS	MGCN	54 / 26	82 / 44
WEDS	MGCN	48 / 17	73 / 39

# 3-Descriptor Learning using Multiscale GCNs

## ❖ Results: non-isometric shapes



Shape Pairs	BIM	SplineCNN + NN	MGCN + NN
<b>Average (8×7 pairs)</b>	59	225	<b>45</b>
Cow, Wolf	37	219	<b>18</b>
Tiger, Dog	39	227	<b>38</b>
Fox, MaleLion	104	221	<b>51</b>
Horse, Hippo	<b>76</b>	244	80

# Conclusions

## ❖ Contributions

- Two non-learned features: *Local Point Signature* (LPS) and *Wavelet Energy Decomposition Signature* (WEDS).
- Two supervised frameworks to transform the non-learned features to more discriminative descriptors

## ❖ Future work

- Extend to point clouds and triangle soups
- Industrial applications

# Acknowledgements

## Co-authors:

Dong-Ming Yan, CASIA

Yiqun Wang, CASIA

Hanyu Wang, University of Maryland-College Park

Jing Ren, Peter Wonka, KAUST



Tencent 腾讯

CCF-腾讯犀牛鸟科研基金

## Code and Data:

LDGI: <https://github.com/jianweiguo/local3Ddescriptorlearning>

LPS: <https://github.com/yiqun-wang/LPS>

MGCN: <https://github.com/yiqun-wang/MGCN>

# *Thank you!*