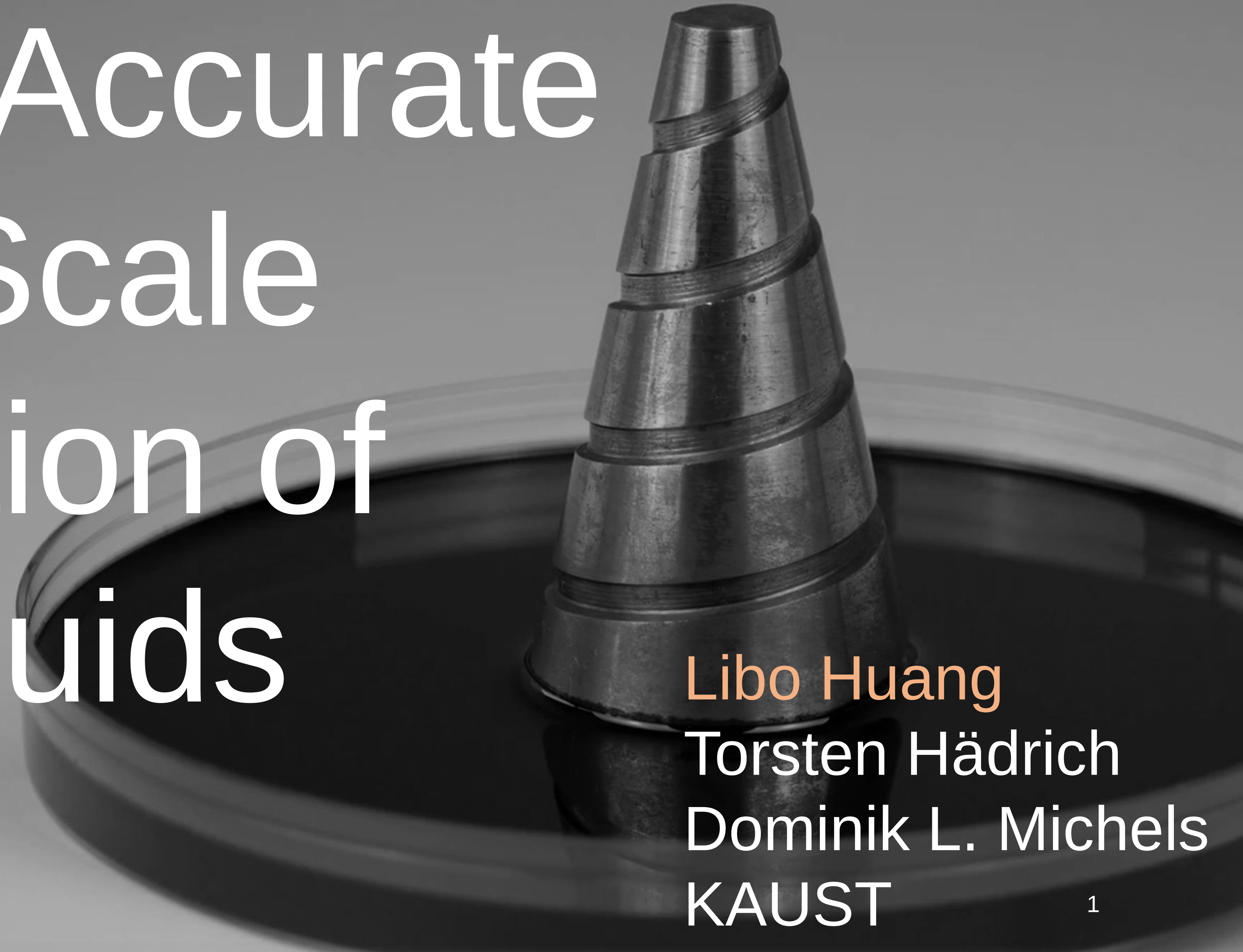
On the Accurate Large-Scale Simulation of Ferrofluids

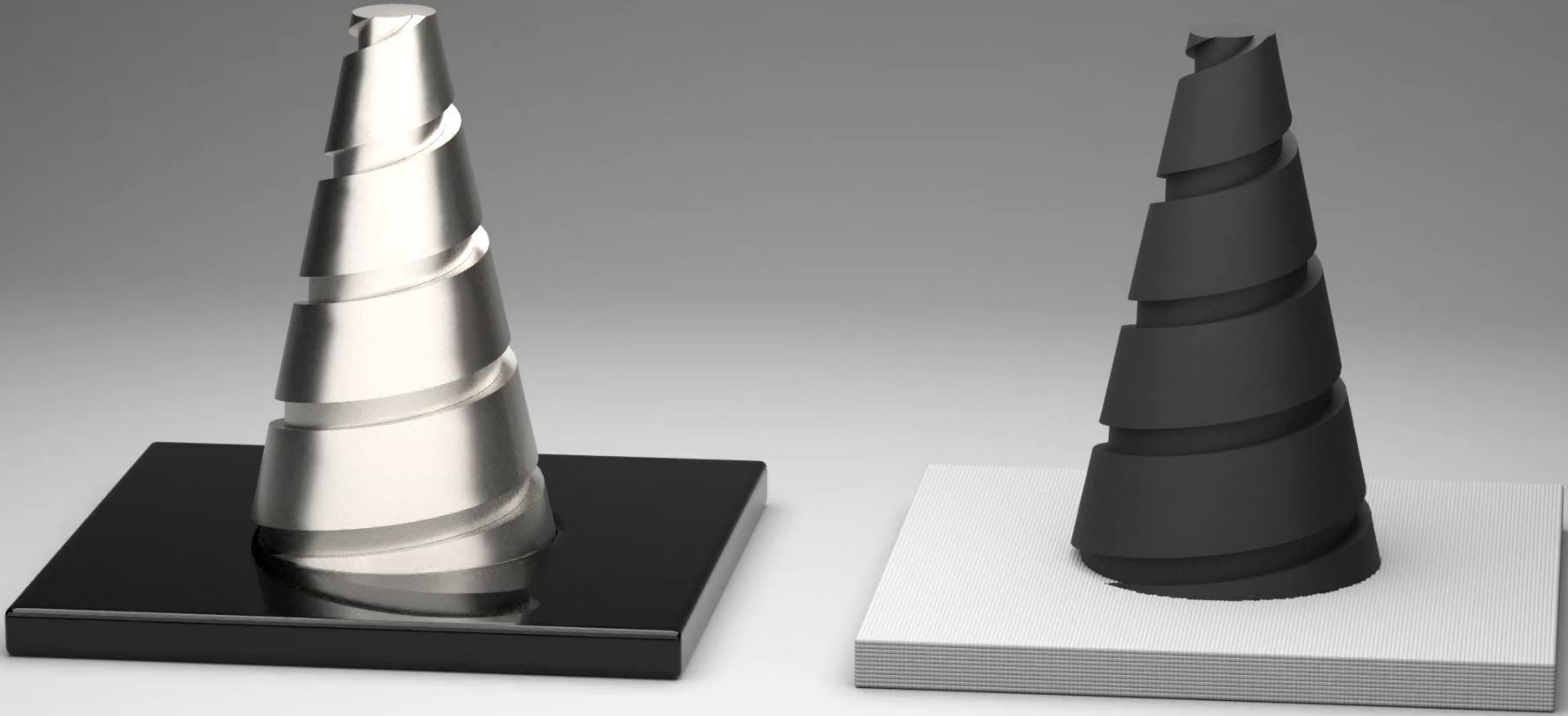
Fe
26 Iron



Libo Huang
Torsten Hädrich
Dominik L. Michels
KAUST

Real footage

Simulation



Meshed View

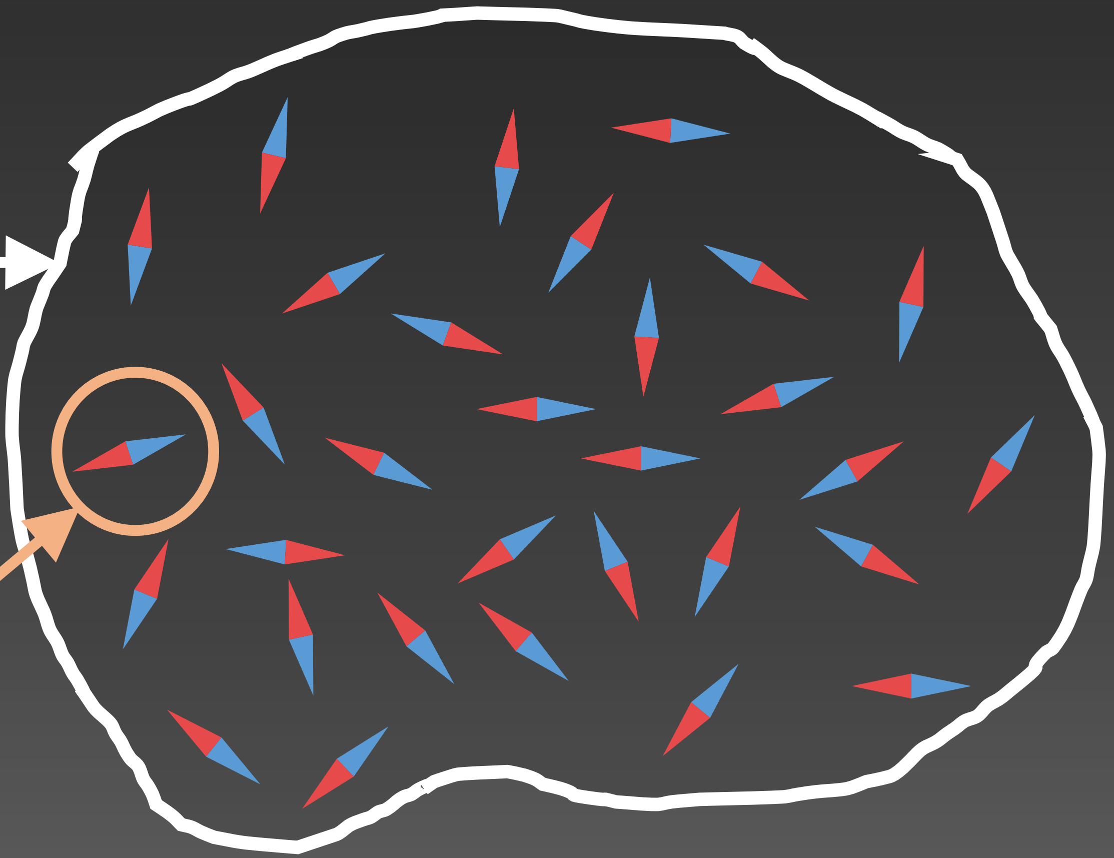
Particle View

Outline

- Why it has spikes?
- Related work
- Our method (physically based)
- Results & Discussion

No external magnetic field

Fe
26 Iron
rrofluid

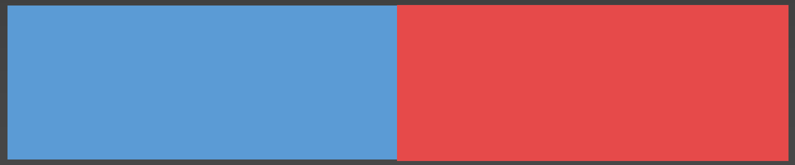


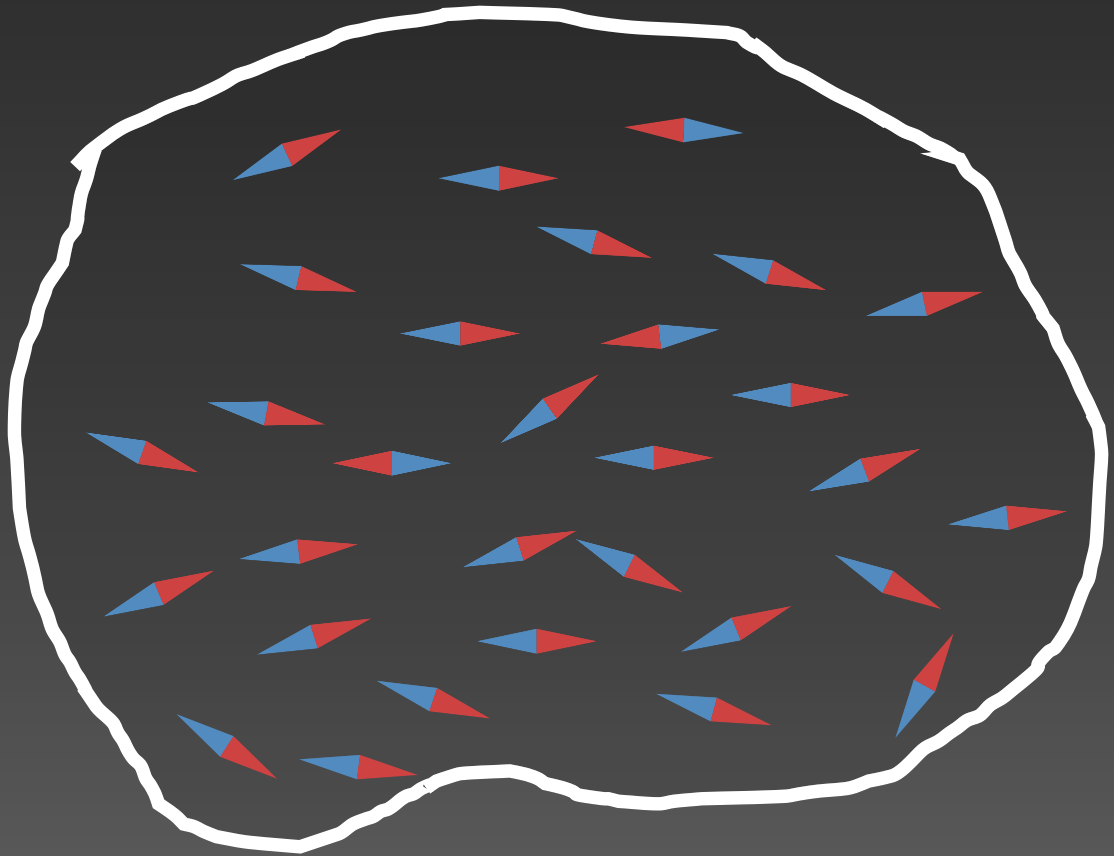
nanoparticles

Fe_3O_4

random direction

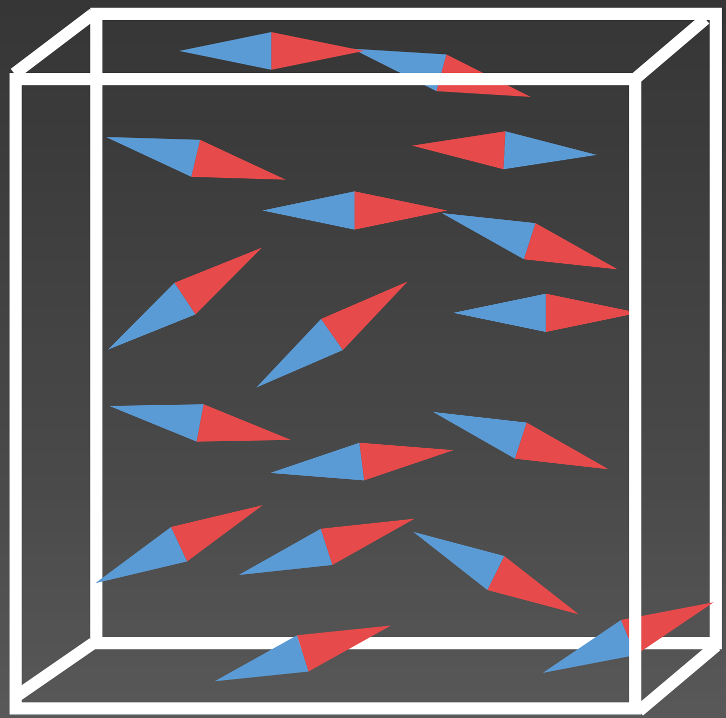
With external magnetic field



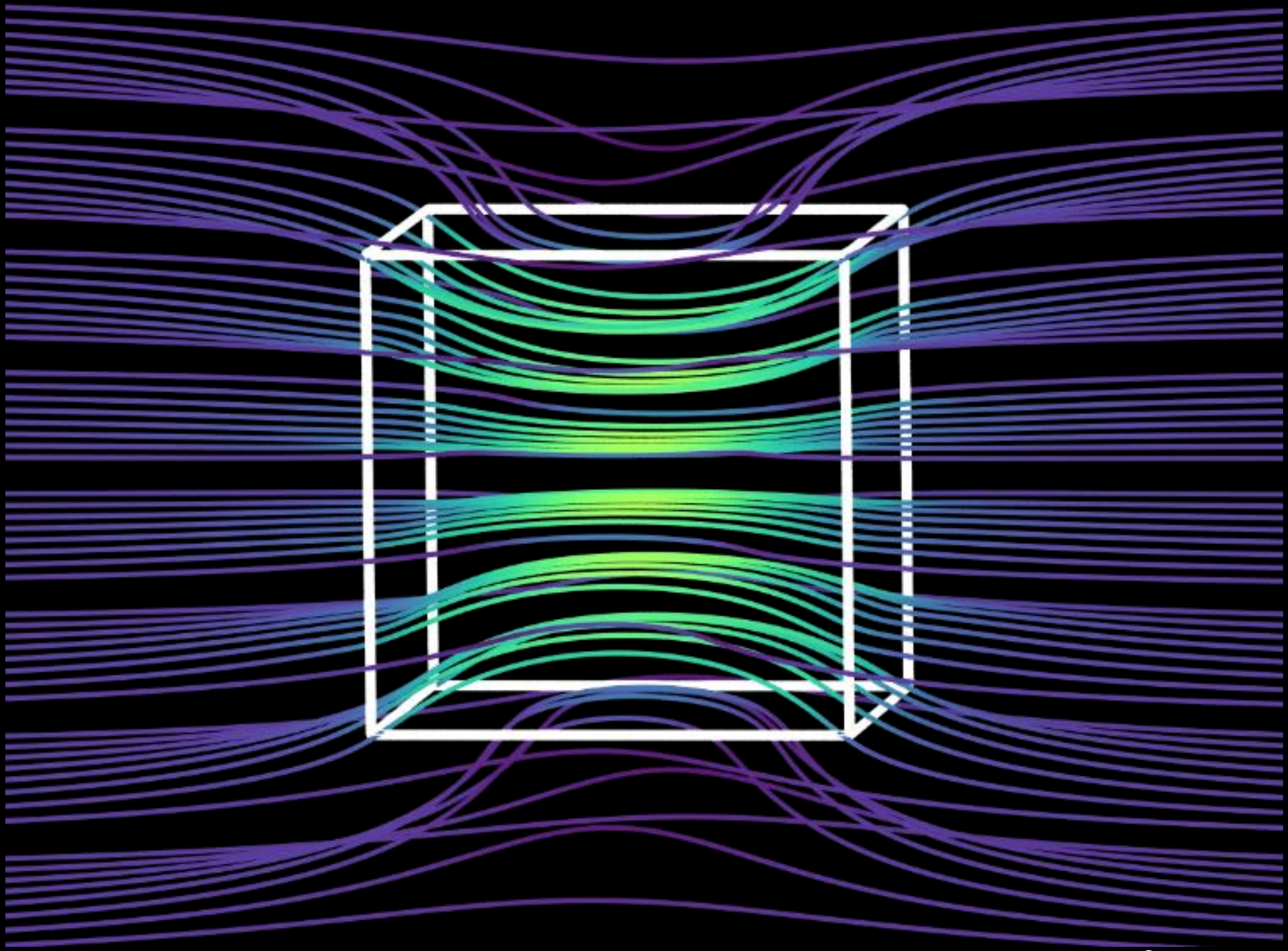


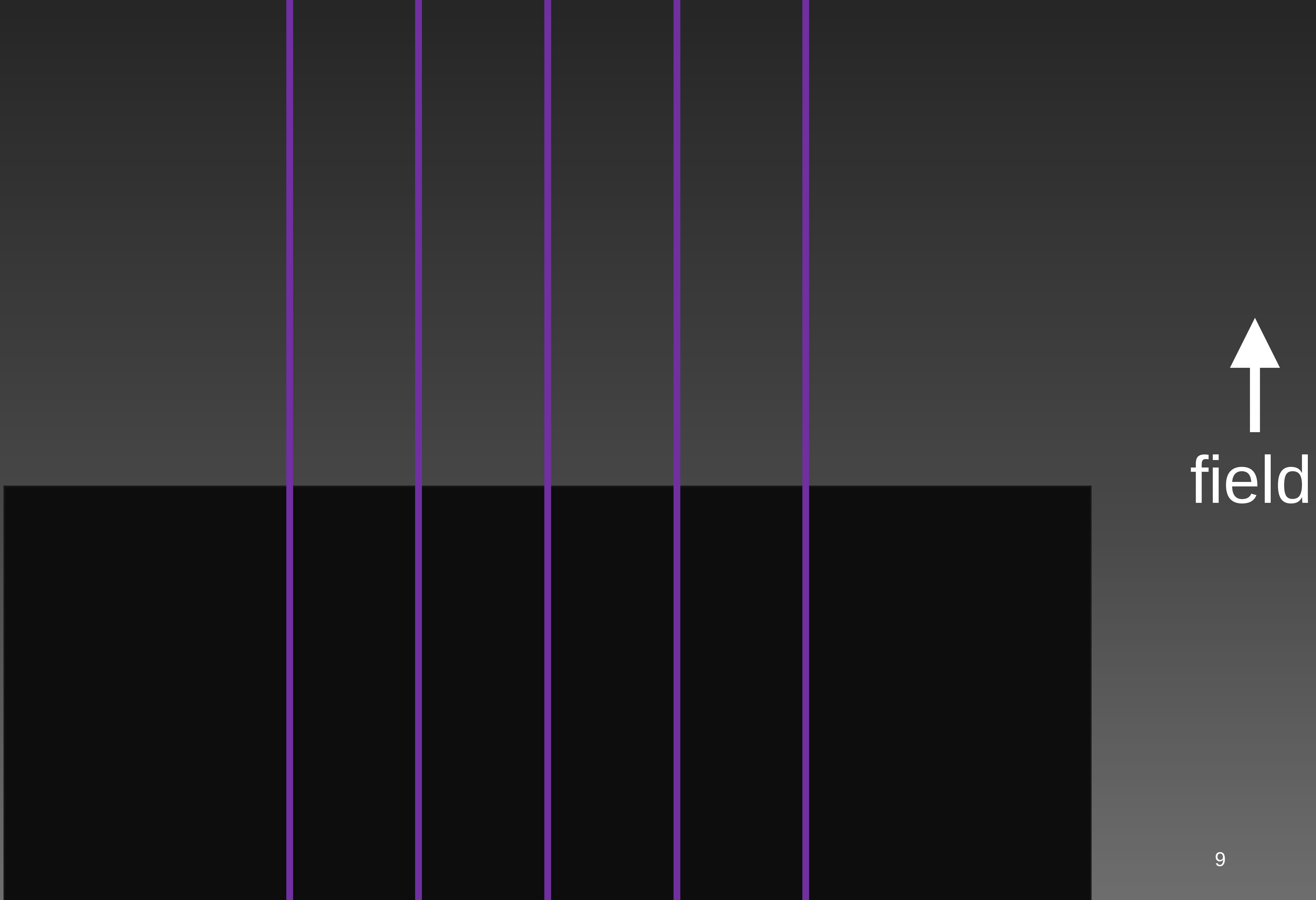
dominant direction

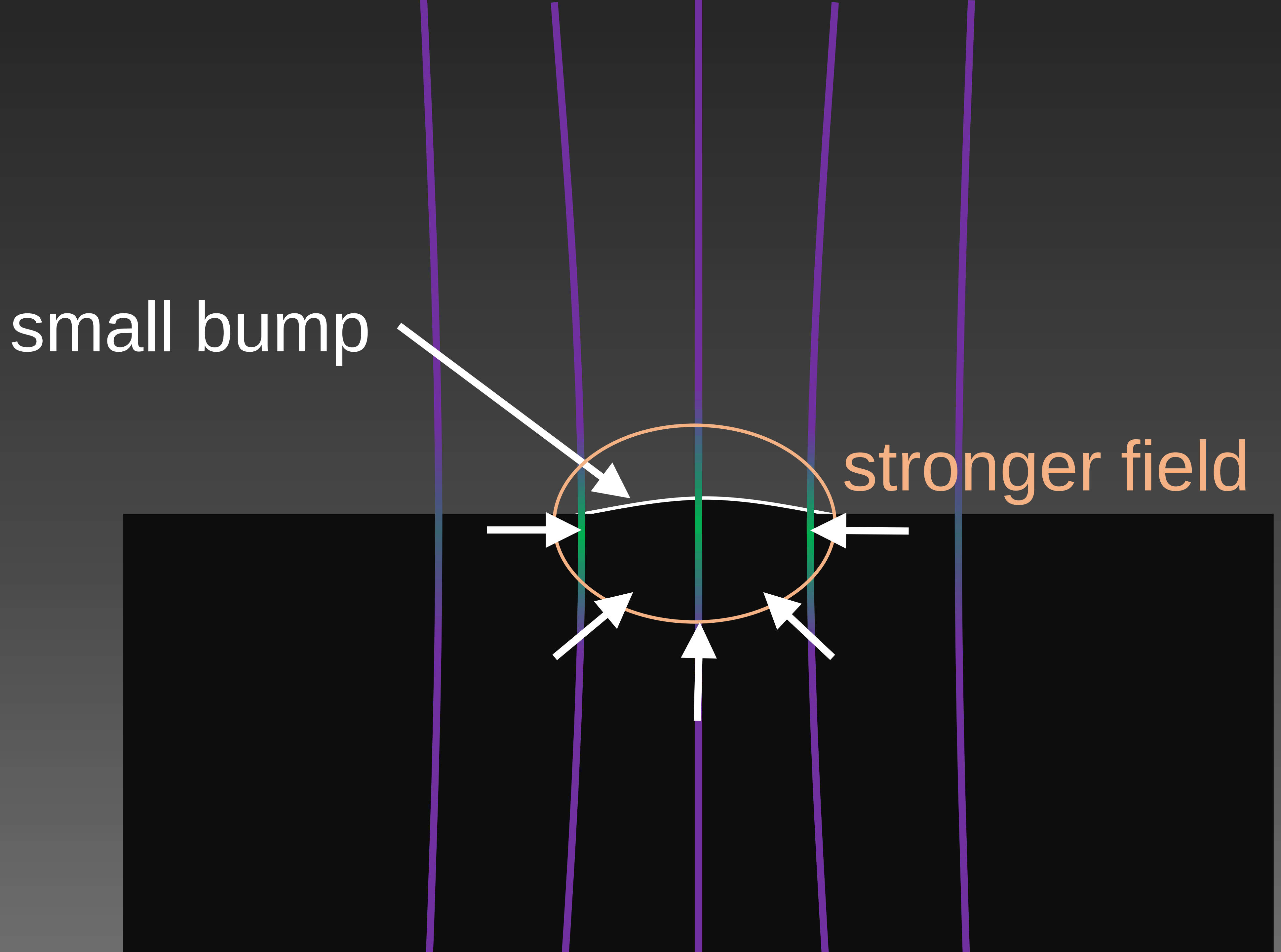
With external magnetic field



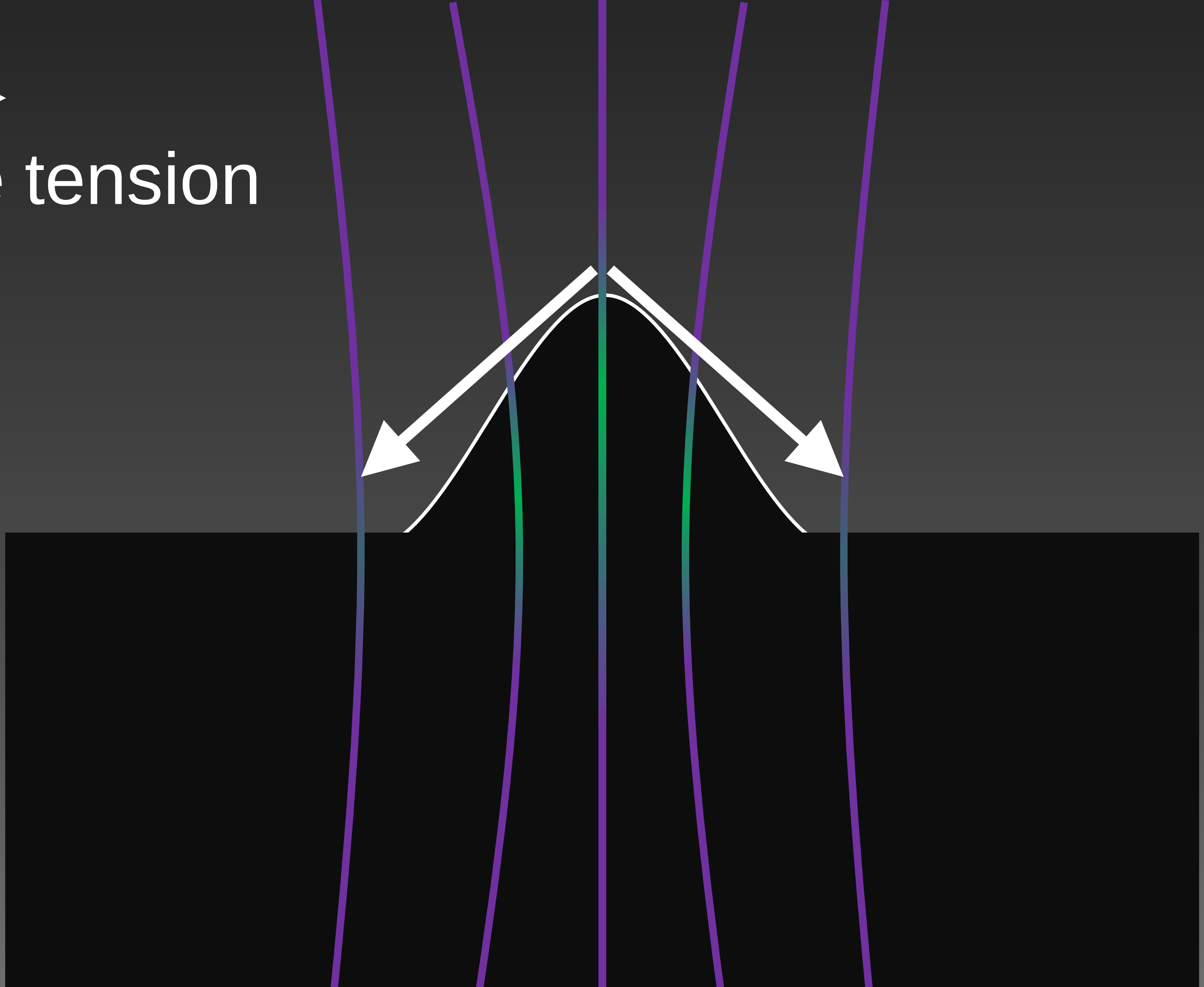
constant
magnetic field



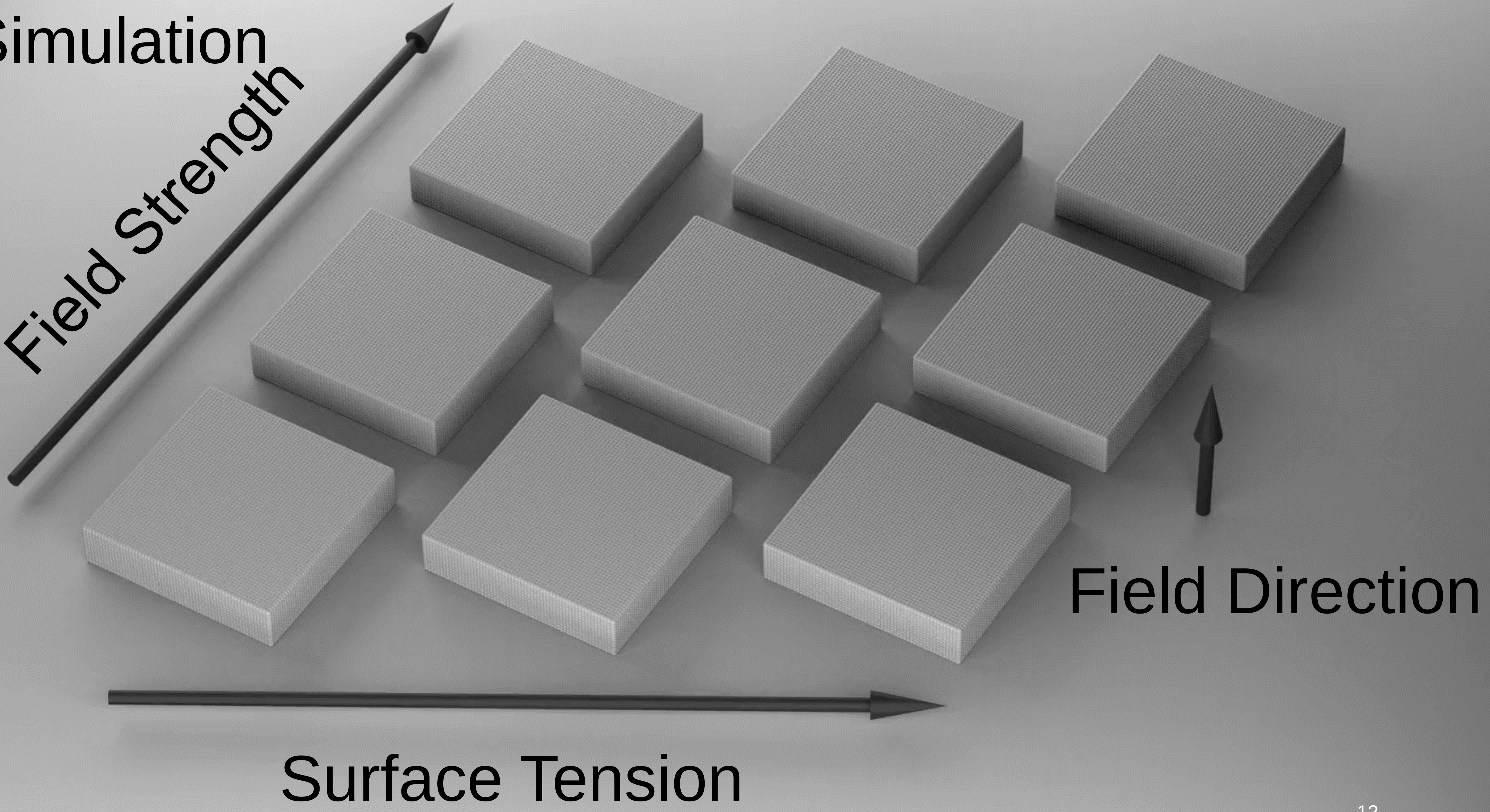

field


small bump
stronger field

surface tension



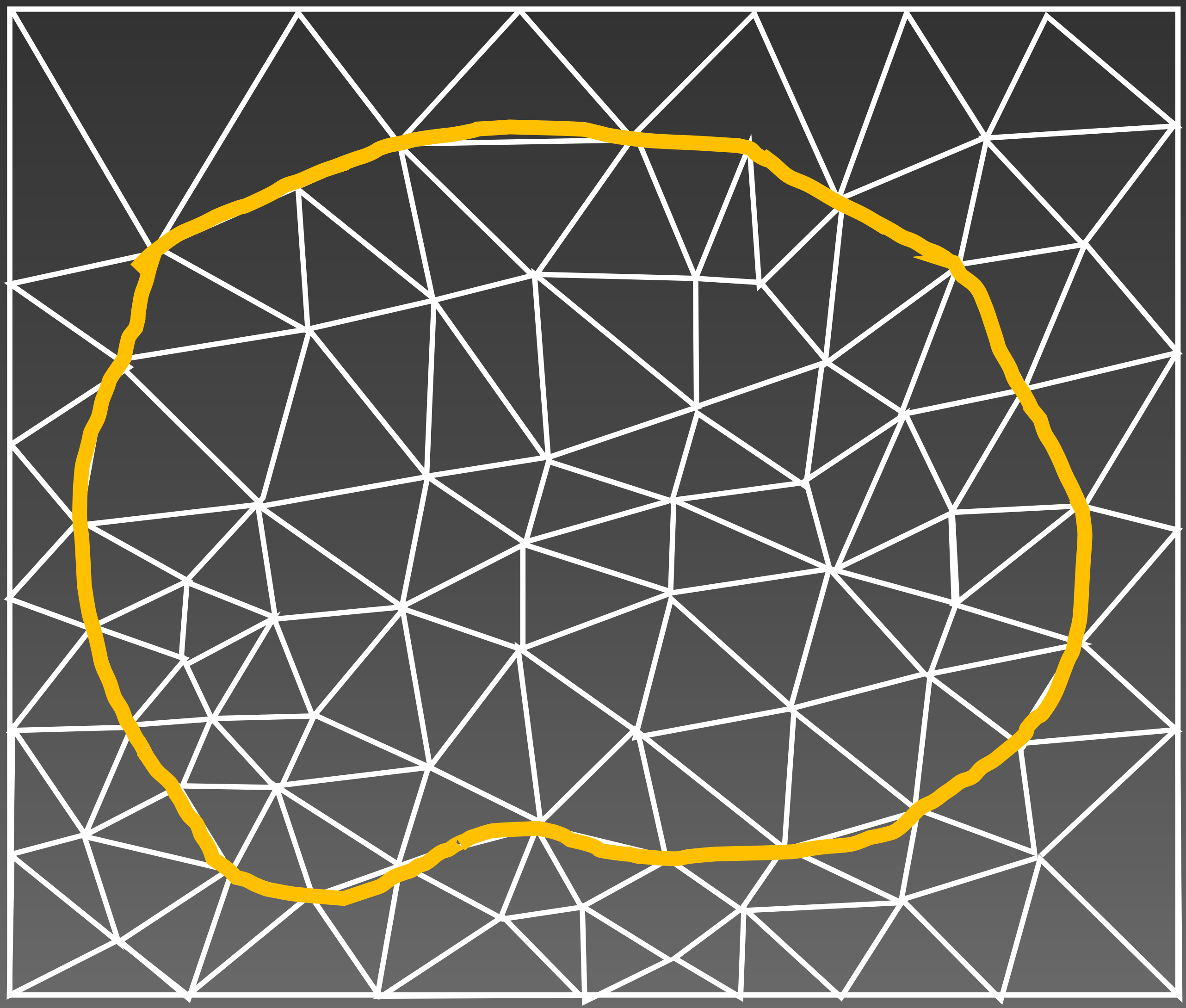
Simulation



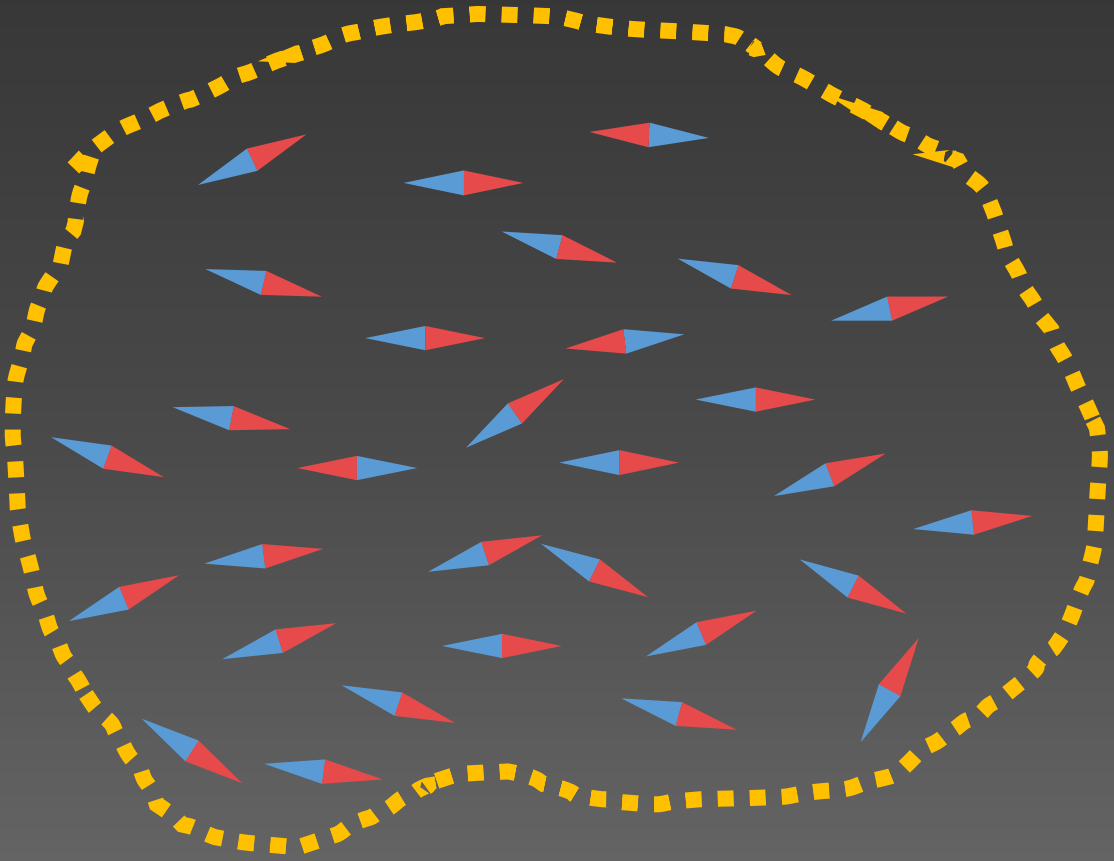
$$F = F_{\mathrm{surface}} + F_{\mathrm{fluid}}$$

$$\boxed{+ F_{\mathrm{magnet}}}$$

Finite Element Method



Particle



Challenges

FEM

- Remeshing the fluid and air

Particle

- Approximating continuous ferrofluid
- Accurate and stable magnetic forces

Our solution

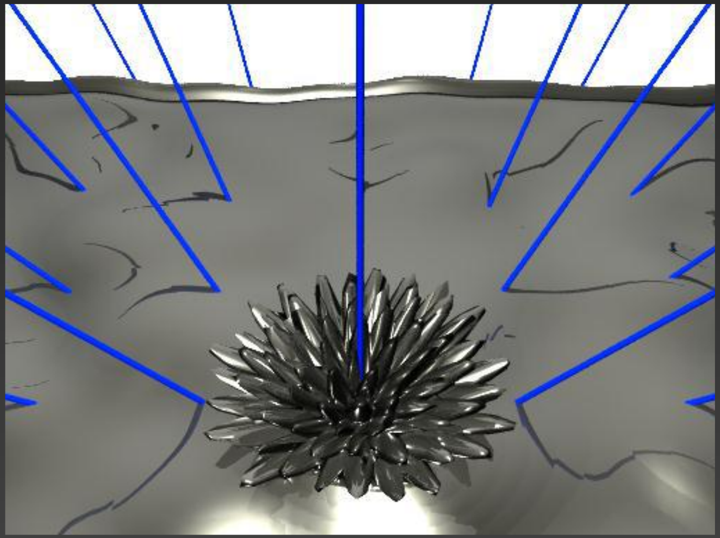
Only particles, no re-meshing

1. Smooth magnets, continuous fluid
2. Forces of smooth magnets, accurate, stable
3. Fast multipole method, $O(N^2) \rightarrow O(N)$

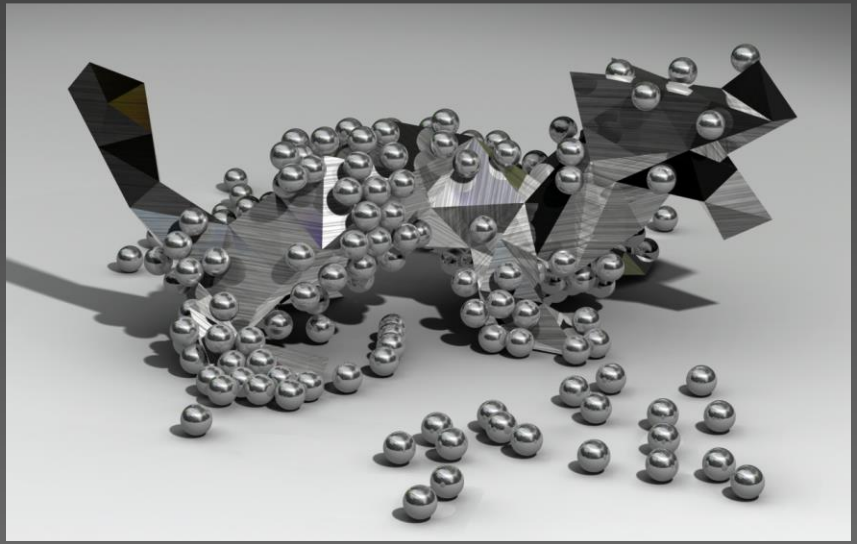
Related Work

From visual computing

- Ferrofluid: [Ishikawa et al. 2012, 2013]
- Rigid magnet: [Thomaszewski et al. 2008]
- Rigid magnet: [Kim et al. 2018]



Post processing



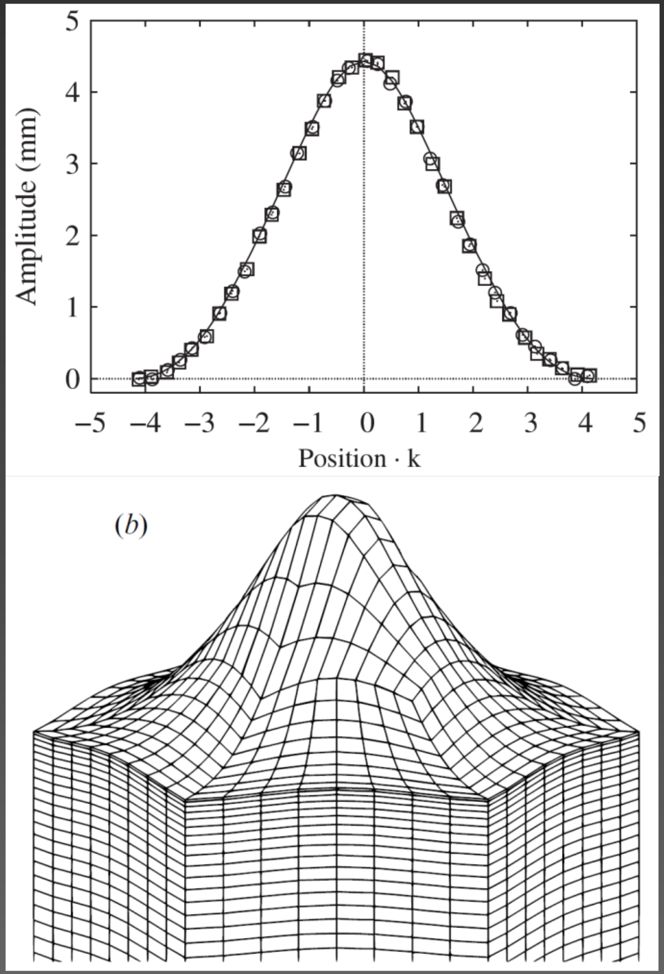
Rigid magnet

Rigid magnet

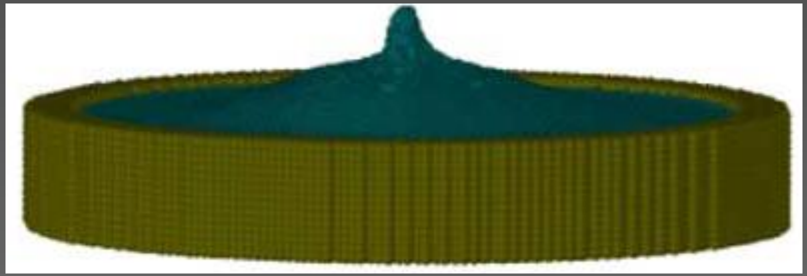
Related Work

From math & physics

- [Nochetto et al. 2016]
- [Yoshikawa et al. 2010]
- [Lavrova et al. 2006, Gollwitzer 2006]



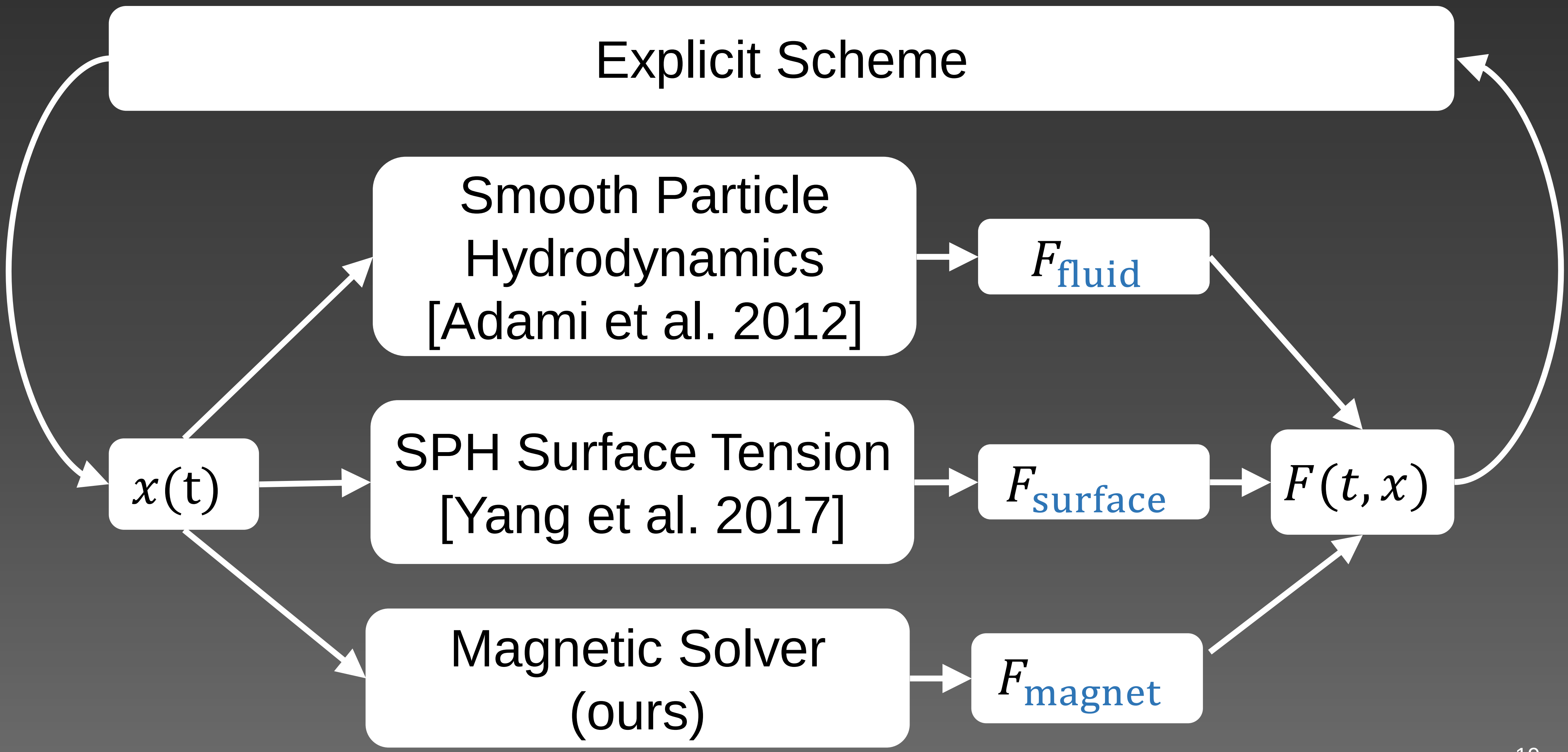
2D dynamic



One spike

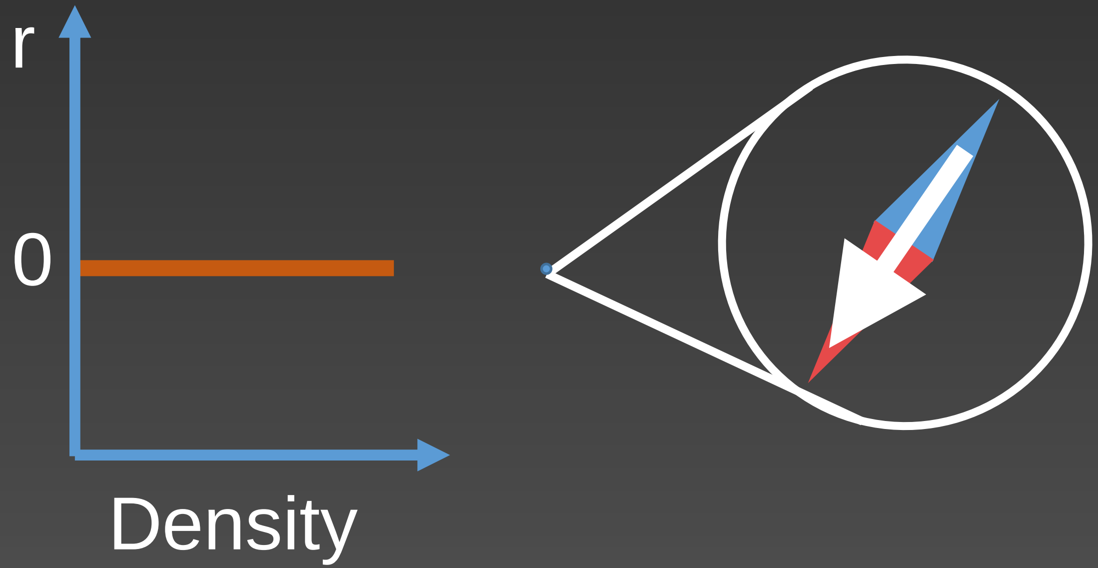
Static

The simulator



Smooth Magnet

Point

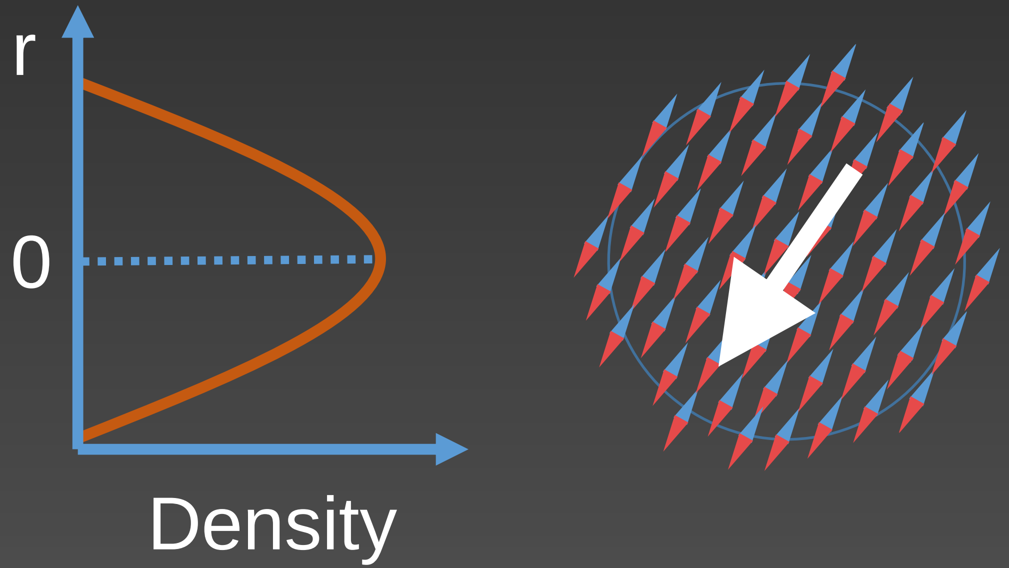


Infinite small point

Near center $B(r) \propto \frac{1}{r^3}$

$B(0)$ undefined

Smooth

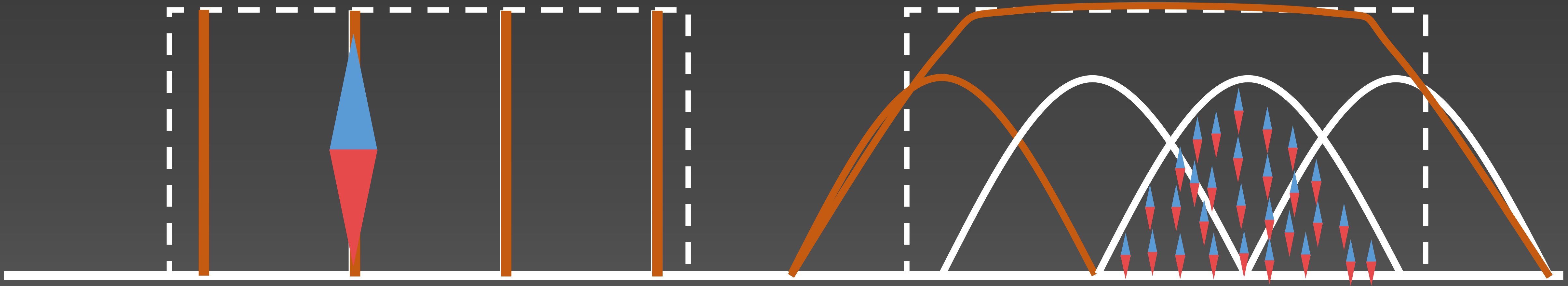


Finite size cloud

Near center $B(r) \propto \mathrm{Density}(r)$

$B(0)$ well-defined

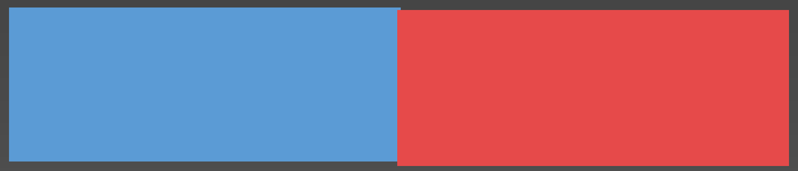
Point   Smooth



Discontinuous   Continuous

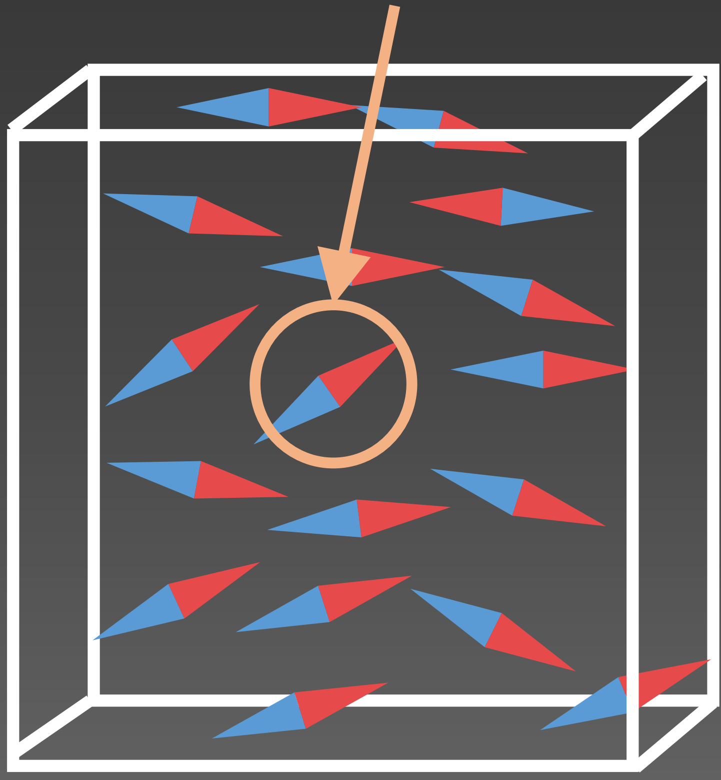
Solve Magnetization

Input



magnetic field

Output: directions

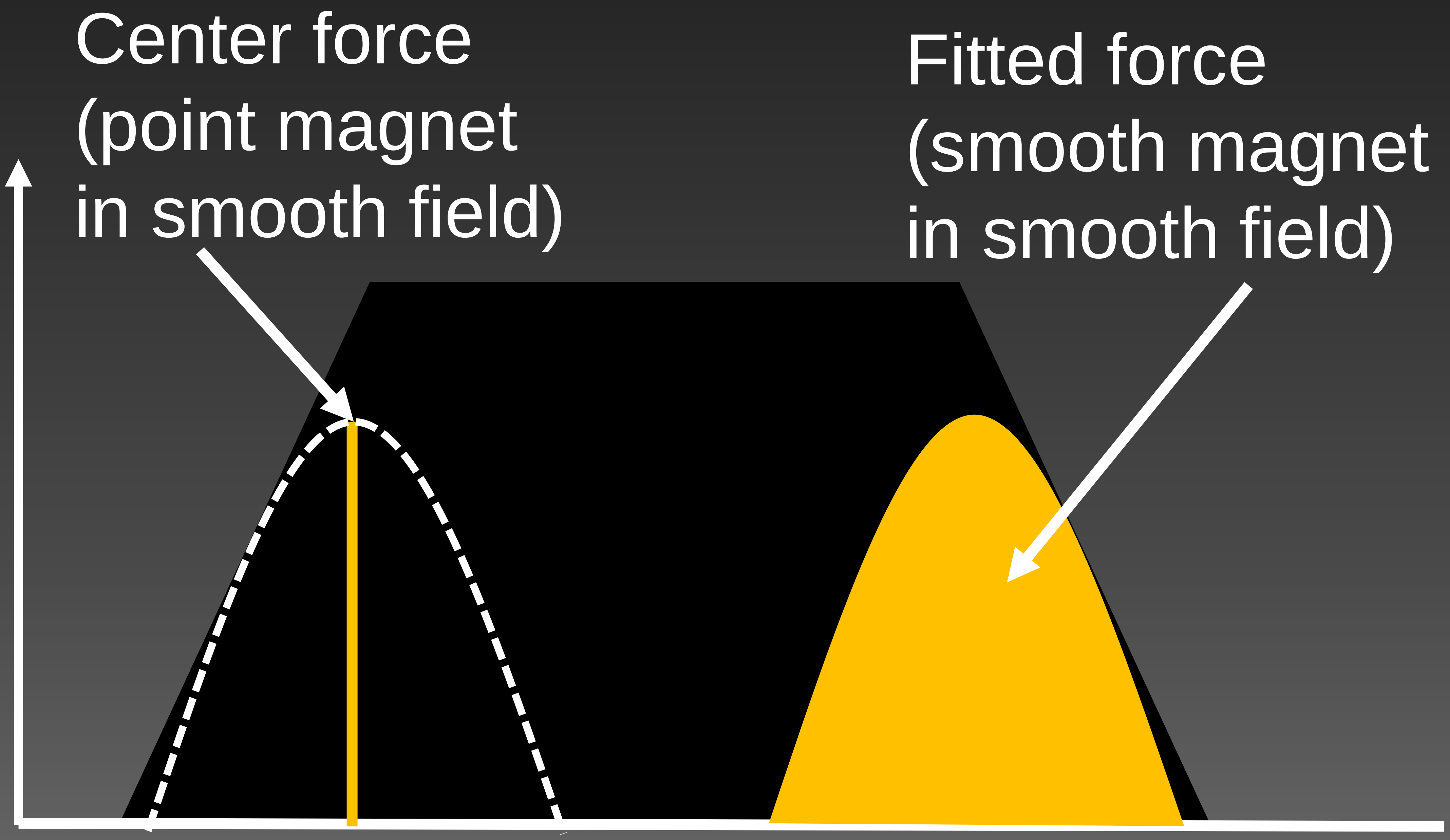


Solve Magnetization

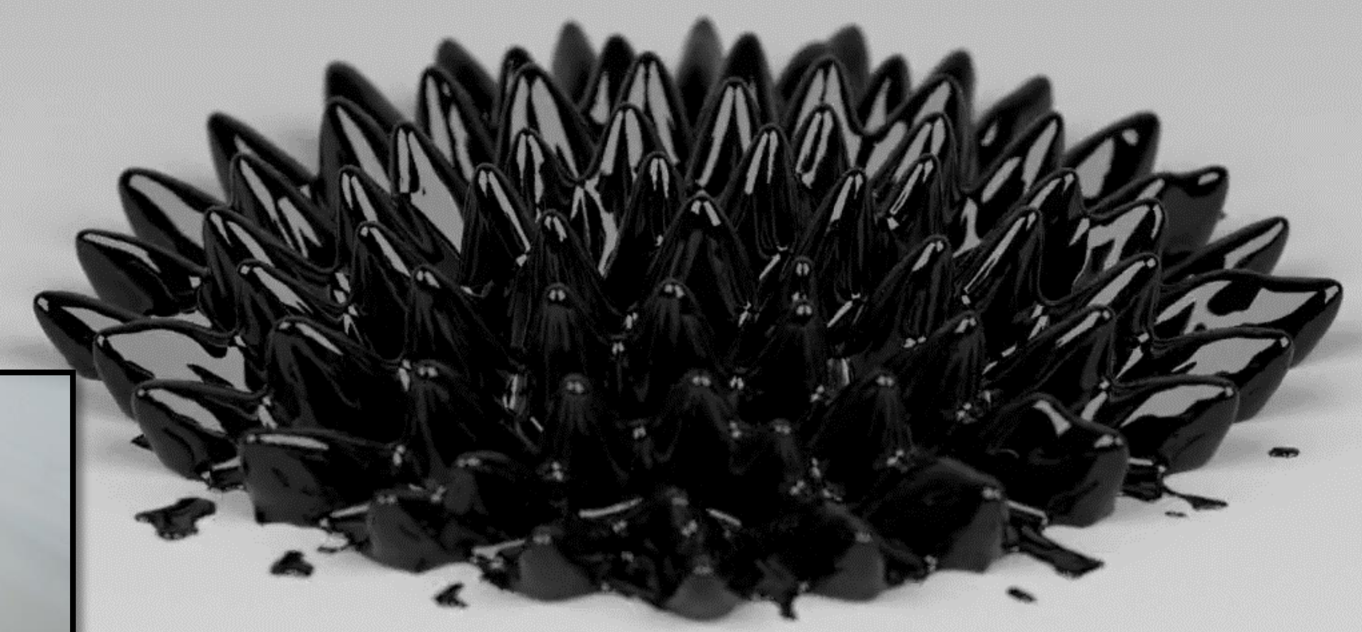
- Note: each smooth magnet affects others
- An optimization problem:
  - Best dominant directions satisfy physics laws.
  - Least square conjugate gradient
  - Fast multipole, $O(N^2) \to O(N)$

Force Principles

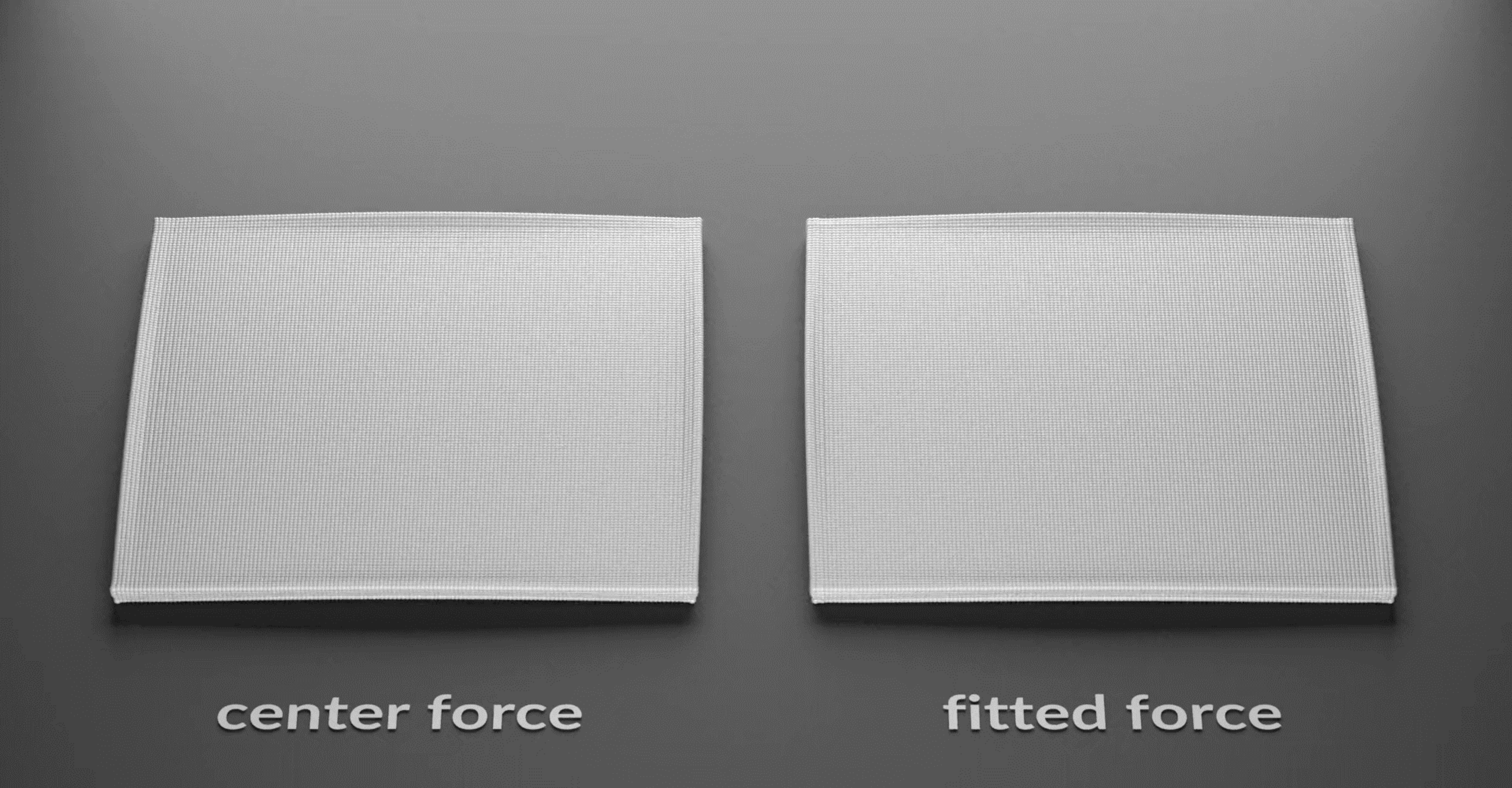
1. $\forall$ nanoparticle $\rightarrow$ magnetic field
2. $\forall$ nanoparticle $\leftarrow$ magnetic forces


Center force
(point magnet
in smooth field)
Fitted force
(smooth magnet
in smooth field)

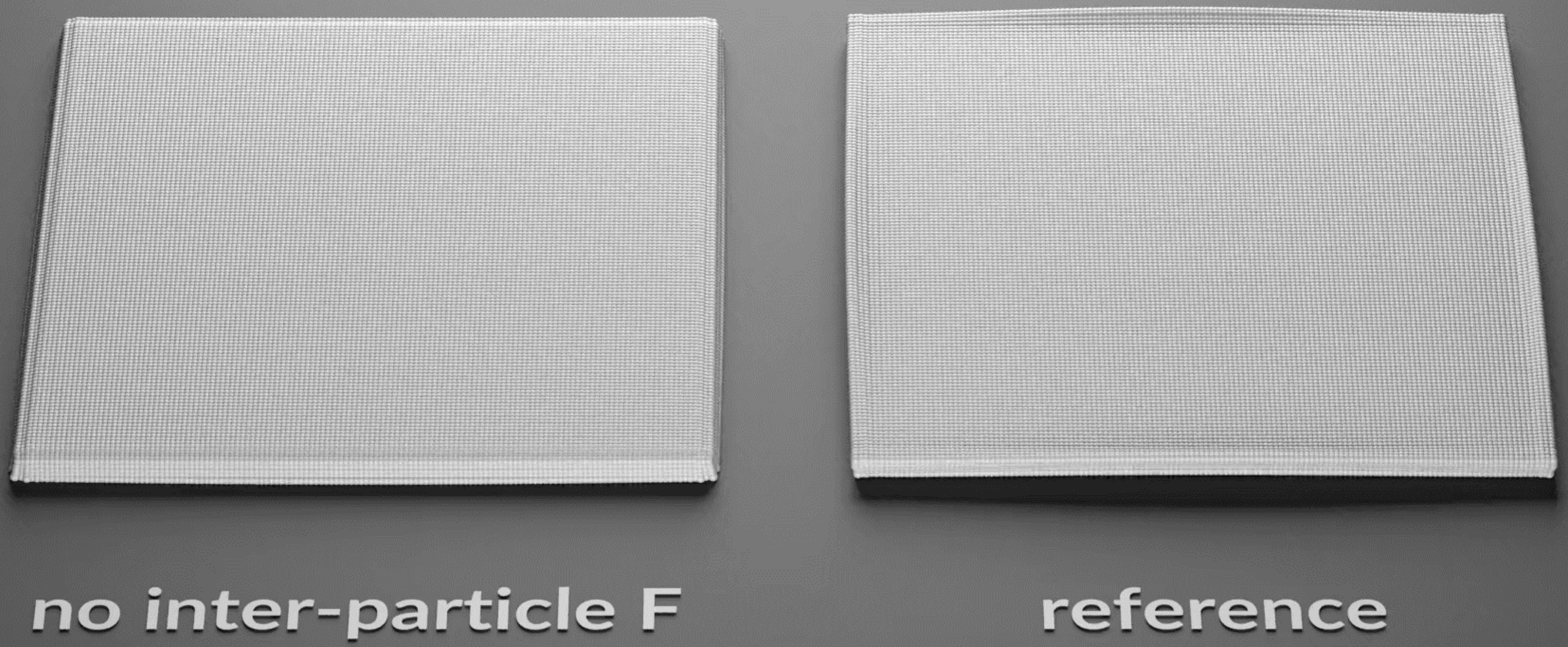
Simulation




Real footage

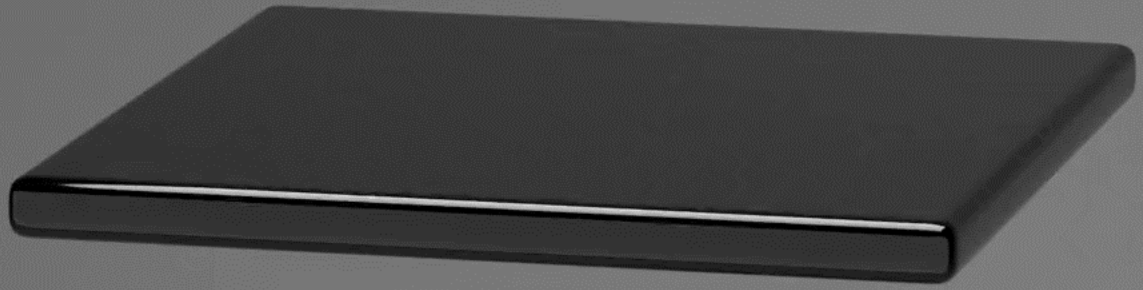

center force
fitted force

Simulation



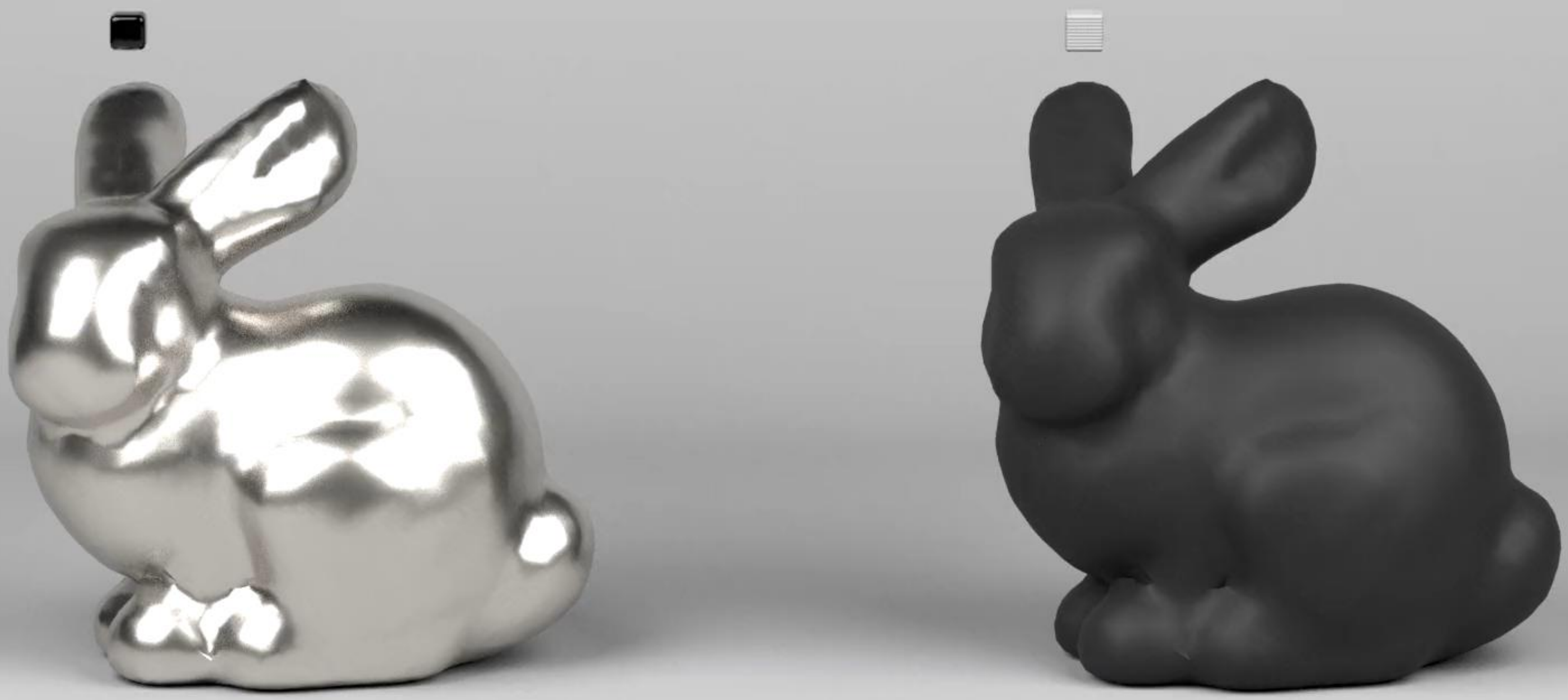
Real footage

Simulation



Simulation

Simulation



Conclusion

3D dynamic ferrofluid simulator using smooth magnet.

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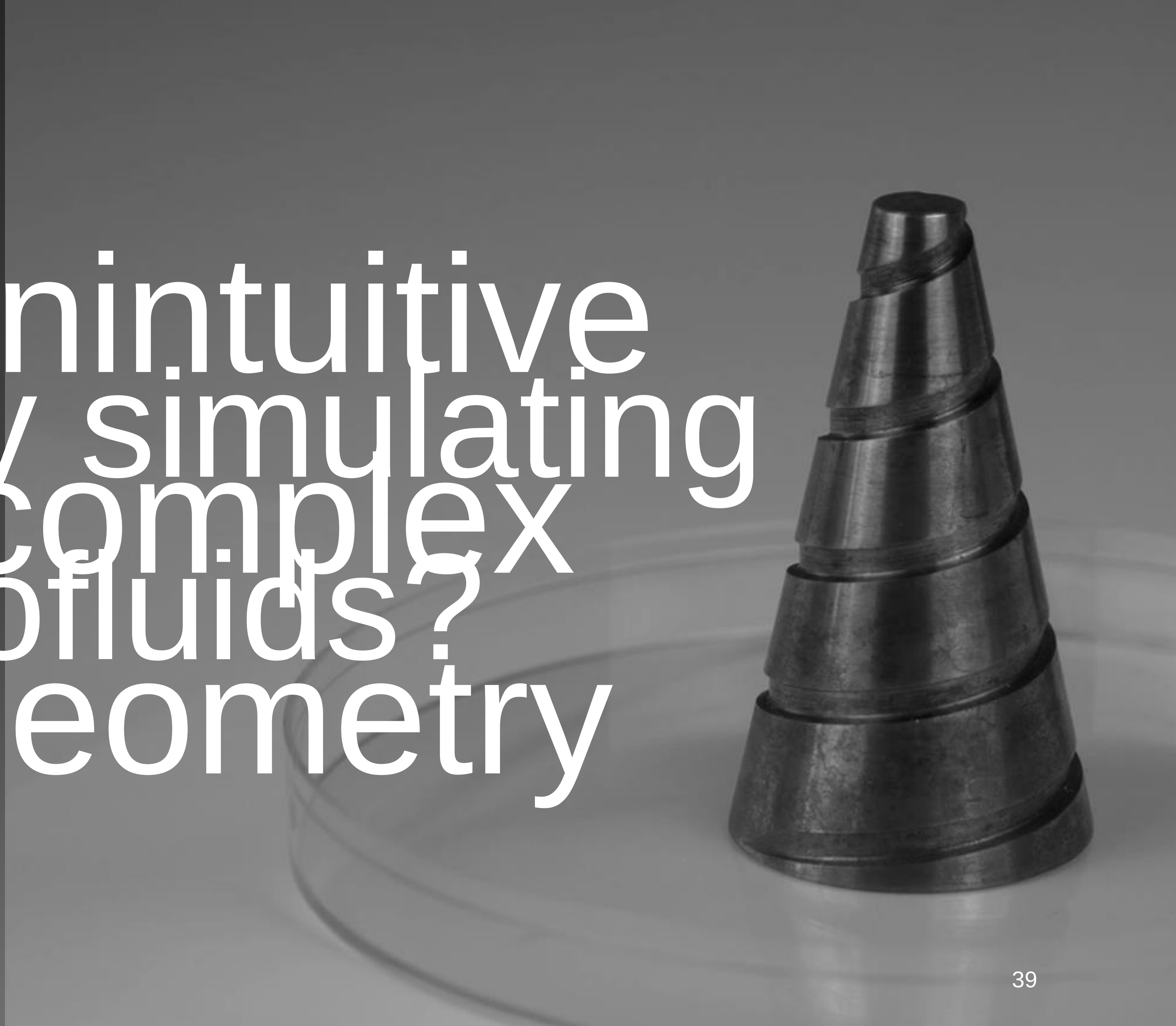
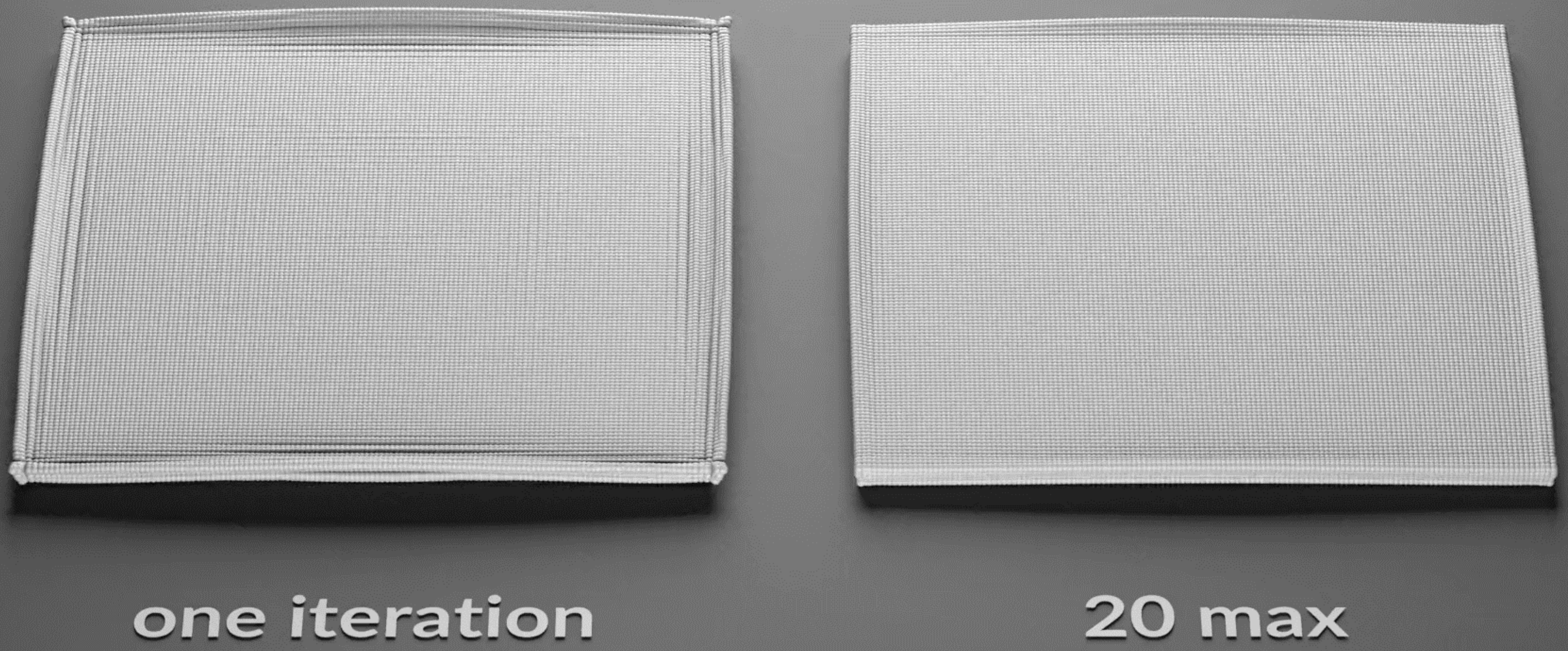
Questions?

Why simulating ferrofluids?

Unintuitive complex geometry

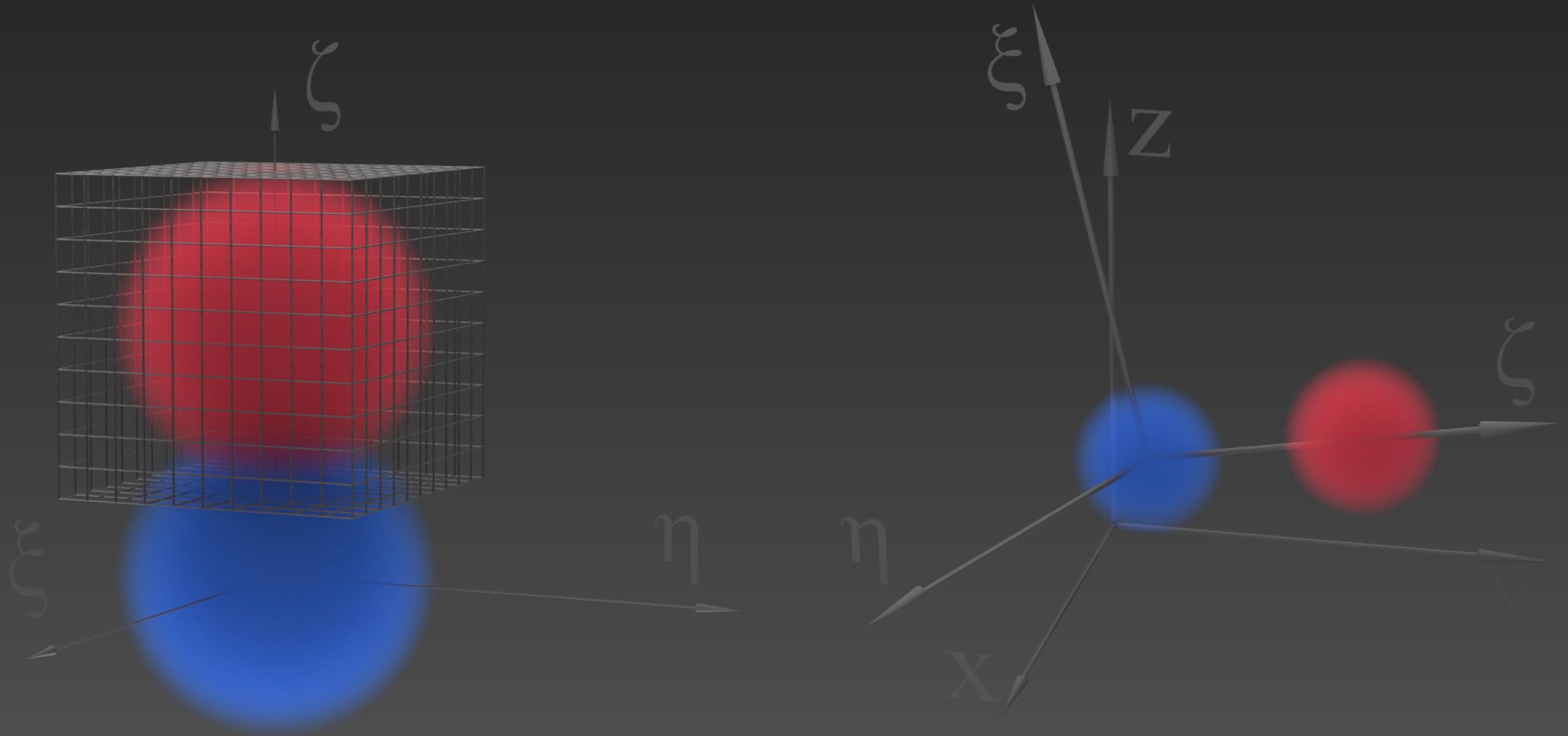


Simulation



Target

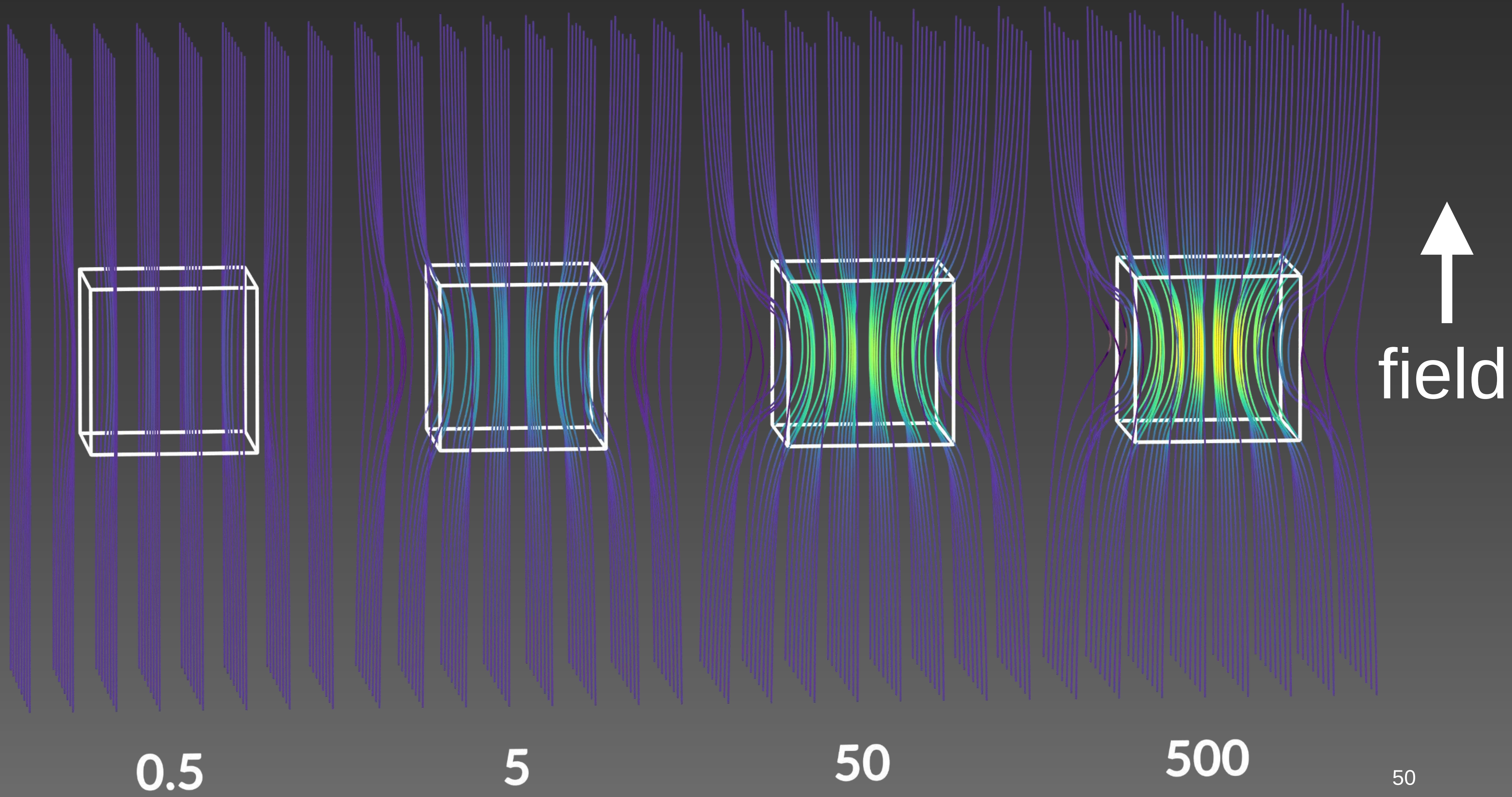
Source



In local coordinates $F^{i}_{s \to t} = \Lambda^{ijk}(r) m^{j}_{s} m^{k}_{t}$

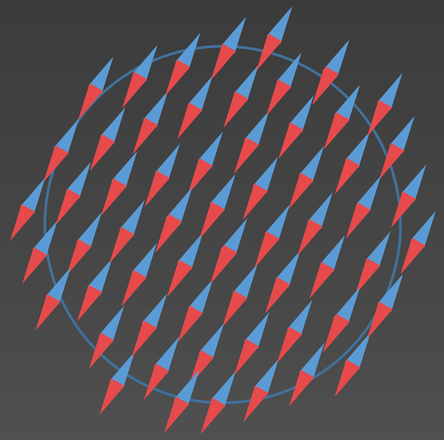
A third-order tensor (to be measured) gives forces

Susceptibility ∝ Nanoparticle Density



How to describe ferrofluid?

Particle i



$$\text{Magnitude} \times \text{Direction} = m_i \in \mathbb{R}^3$$

1. Particles Generate Magnetic Fields

$$b_i^{\text{fluid}} = \sum_{j=1}^{N} b_{j\to i}^{\text{fluid}} = \sum_{j=1}^{N} G_{ij} m_j$$

$$G_{ij} \in \mathbb{R}^{3\times 3}$$

2. Magnetic Fields Influence Particles

$$m_i = c(b_i^{\text{fluid}} + b_i^{\text{external}})$$

$$c \in \mathbb{R}, \text{constant}$$

A correct particle state m generates a field b^{fluid}, which combined with external field b^{external} lead to the same state m.

$$b_i^{\text{fluid}} = \sum_{j=1}^{N} G_{ij} m_j$$

$$m_i = c\left(b_i^{\text{fluid}} + b_i^{\text{external}}\right)$$

$$\min_m \left\| m - c\left(Gm + b^{\text{external}}\right) \right\|_2$$

Center force:

All nanoparticles moved to particle center to calculate force (bounded but inaccurate).

Fitted force:

All nanoparticles contribute to the force.
Pre-calculated, stored as fitted polynomial (accurate surface force).

Fast Multipole Method