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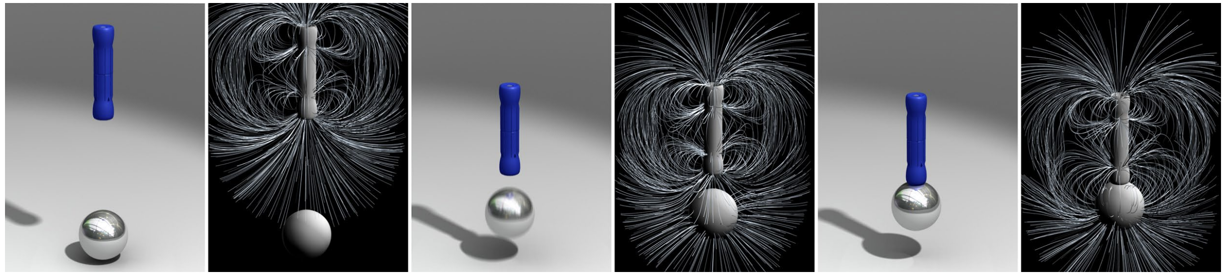
A LEVEL-SET METHOD FOR MAGNETIC SUBSTANCE SIMULATION

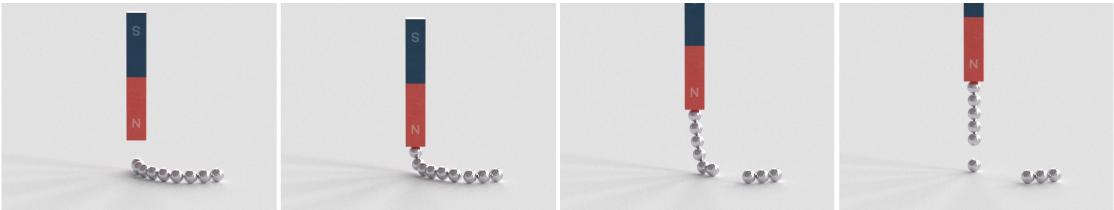
Xingyu Ni, Peking University
Bo Zhu, Dartmouth College
Bin Wang, Beijing Film Academy
Baoquan Chen, Peking University

MAGNETIC SIMULATION

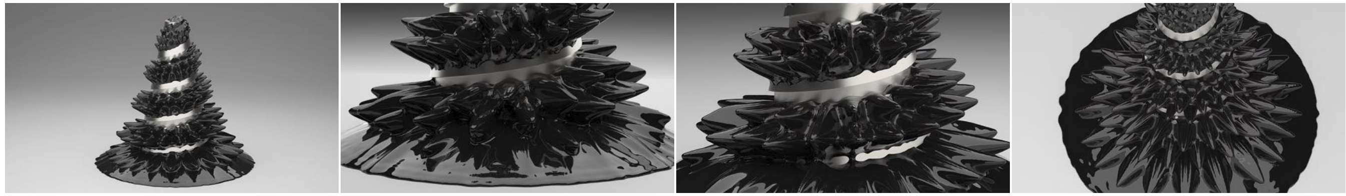
- Magnetic interaction
  - Phenomena in life
  - Applications in industry
- Magnetic simulation in graphics
  - Magnetic rigid bodies
    - [Thomaszewski et al. 2008]
    - [Kim et al. 2018]
  - Magnetic liquids (ferrofluids)
    - [Huang et al. 2019]
- Observation
  - Need for a unified framework
  - All based on Lagrangian views



[SIGGRAPH 2008] Magnets in Motion



[SIGGRAPH 2018] Magnetization Dynamics for Magnetic Object Interactions



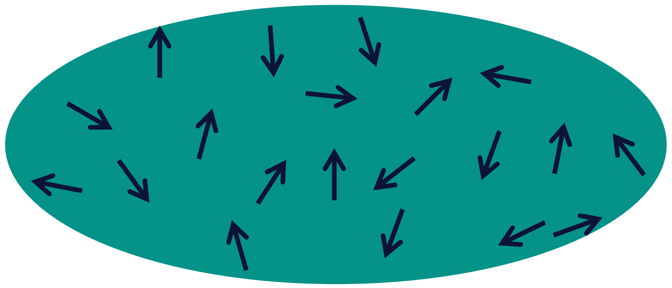
[SIGGRAPH 2019] On the Accurate Large-scale Simulation of Ferrofluids

CONTRIBUTION

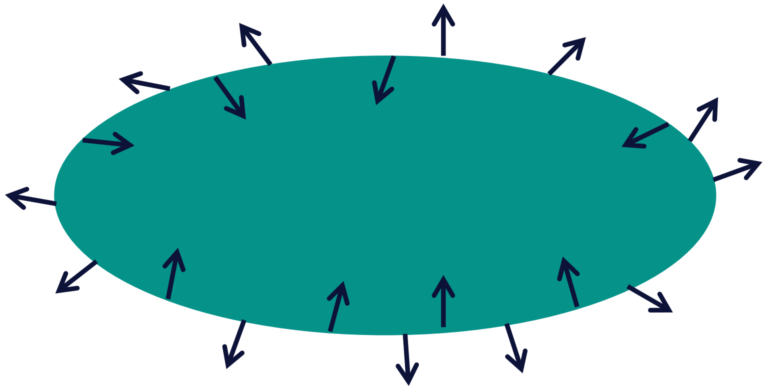
- Treat magnetic interaction as interfacial force
  - Solving the magnetic-mechanical coupling problem efficiently
- Based on an Eulerian view
  - Taking advantage of the level-set method
- Model a broad range of magnetic phenomena in a unified way
  - Treating classical simulators simply as black boxes
  - Supporting fluids, solids, and their couplings
- Include an efficient, precise numerical scheme
  - Introducing magnetic force by solving a Poisson equation with jump conditions
  - Especially easy to be incorporated into a standard Euler fluid solver

HOW TO PORTRAY MAGNETIC FORCE

- The first step to simulate the motion of a physical system
  - To establish the dynamics of this system
  - Equivalent to determining what the form of force is in Newtonian mechanics
- Magnetic force:
  - Yet to be definitely answered



Volumetric (throughout the substance)



Interfacial (only on the surface)

HOW TO PORTRAY MAGNETIC FORCE

- The Maxwell stress tensor $\boldsymbol{T}_{\mathrm{m}}$
  - $\boldsymbol{T}_{\mathrm{m}} = \frac{1}{\mu_0}\left(\boldsymbol{B} \otimes \boldsymbol{B} - \frac{1}{2}B^2\boldsymbol{I}\right)$ in vacuum, with the electric terms omitted
    - μ_0 is the constant *vacuum permeability*
    - $\boldsymbol{B}$ is the *magnetic induction intensity*
    - $\boldsymbol{I}$ is the second-order unit tensor
  - Physicists compute magnetic force by taking $\boldsymbol{f}_{\mathrm{m}} = \nabla \cdot T_{\mathrm{m}}$
- Abraham–Minkowski controversy
  - What the form of $\boldsymbol{T}_{\mathrm{m}}$ is in matter
  - Einstein–Laub form vs. Minkowski form

Form	The Maxwell stress tensor	Magnetic force
Einstein–Laub	$\boldsymbol{T}_{\mathrm{m}}^{\mathrm{E}} = \boldsymbol{B} \otimes \boldsymbol{H} - \frac{\mu_0}{2}H^2\boldsymbol{I}$	$\boldsymbol{f}_{\mathrm{m}}^{\mathrm{E}} = \mu_0\boldsymbol{M} \cdot \nabla\boldsymbol{H}$
Minkowski	$\boldsymbol{T}_{\mathrm{m}}^{\mathrm{M}} = \boldsymbol{B} \otimes \boldsymbol{H} - \frac{1}{2}(\boldsymbol{B} \cdot \boldsymbol{H})\boldsymbol{I}$	$\boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}} = \boldsymbol{B} \cdot \nabla\boldsymbol{H} - \frac{1}{2}\nabla(\boldsymbol{B} \cdot \boldsymbol{H})$

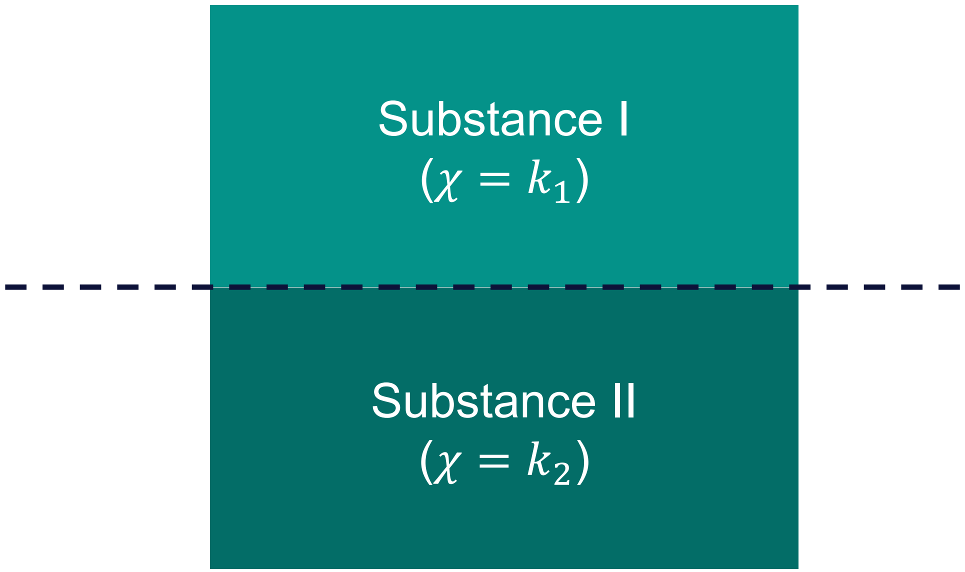
* $\boldsymbol{H}$ is the *magnetic field intensity* and $\boldsymbol{M}$ is the *magnetization intensity*, satisfying $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

$\boldsymbol{f}_{\mathrm{m}}^{\mathrm{E}}$ is called the *Kelvin force*,
while $\boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}}$ is called the *Helmholtz force*

Totally Different!

HOW TO PORTRAY MAGNETIC FORCE

- Assume that the magnetic substances in the system are all linear, isotropic: $\boldsymbol{M} = \chi \boldsymbol{H}$
  - χ is called the *magnetic susceptibility*, constant within each substance
  - Kelvin force vs. Helmholtz force
    - $\boldsymbol{f}_{\mathrm{m}}^{\mathrm{E}} = \frac{\mu_0}{2} \chi \nabla (H^2)$
      - Compatible with the Lagrangian view
        - Let $\chi_i = k_1$ if Particle i belongs to Substance I and $\chi_i = k_2$ if Particle i belongs to Substance II
        - The magnetic force exerted on Particle i is $\frac{\mu_0}{2} \chi_i \nabla (H^2)$
        - These are what related works adopted
    - $\boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}} = -\frac{\mu_0}{2} H^2 \nabla \chi$
      - Only on the interface!



* k_1 and k_2 are both constants

HOW TO PORTRAY MAGNETIC FORCE

Question: since the two forms are totally different, can they both be right?

- Take fluid simulation as an example
  - Need solving the pressure to enforce incompressibility
    - $\nabla \cdot [\rho u + (\boldsymbol{f} - \nabla p)\mathrm{d}t] = 0$
      - ρu is the advected momentum density
    - BC: $p = 0$ outside the fluid
  - Provided that there is an external force $\boldsymbol{f}'$
    - $\boldsymbol{f}' = \nabla \Phi$, satisfying $\Phi = 0$ outside the fluid
  - Theorem: $\boldsymbol{f}'$ does not influence the motion
    - $p^* = p + \Phi$ is a new solution of the pressure
      - $\nabla \cdot [\rho u + (\boldsymbol{f} + \boldsymbol{f}' - \nabla p^*)\mathrm{d}t] = 0$
      - $p^* = 0$, outside the fluid
    - The total force leaves the same
      - $\boldsymbol{f} - \nabla p = \boldsymbol{f} + \boldsymbol{f}' - \nabla p^*$

$$\boldsymbol{f}' = \boldsymbol{f}_{\mathrm{m}}^{\mathrm{E}} - \boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}} = \nabla\left(\frac{\chi \mu_0 H^2}{2}\right)$$

- $\boldsymbol{f}_{\mathrm{m}}^{\mathrm{E}}$ and $\boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}}$ are indistinguishable in effect

- The essence of the A–M controversy
  - How to divide physical quantities between ferromagnetic system and mechanical system

Ferromagnetic force	Undecided $\boldsymbol{f}'$	Mechanical force

PHYSICAL MODELING

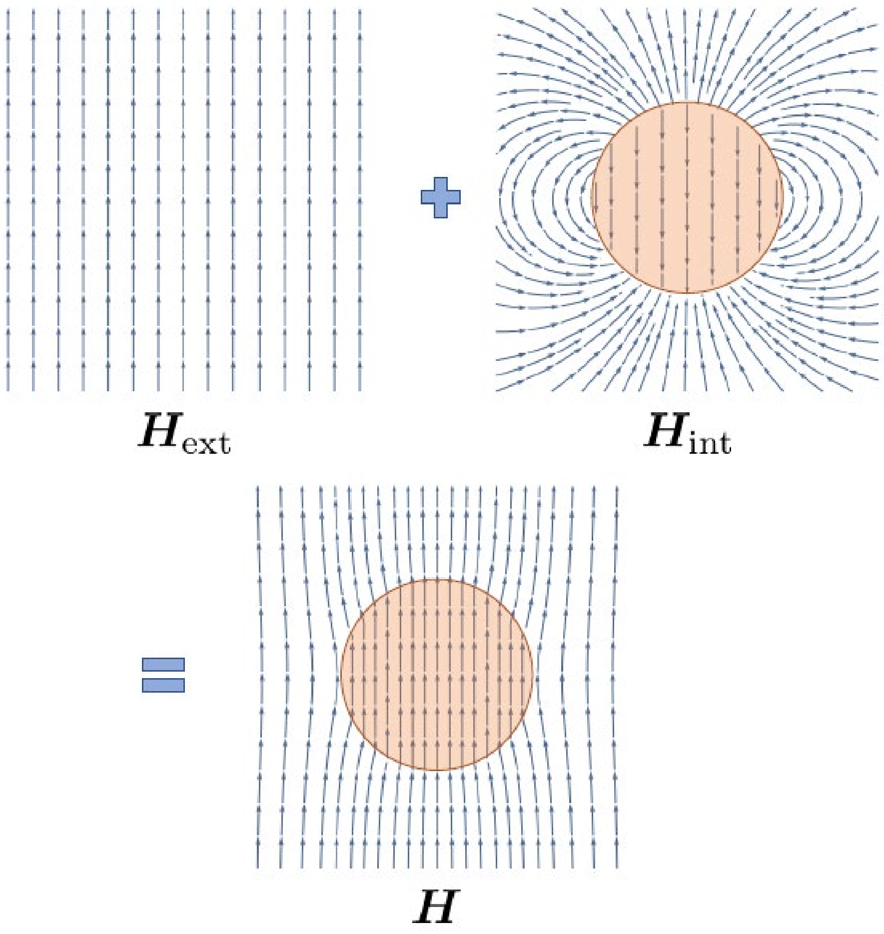
- Eulerian view & Helmholtz force
  - Capture the surface accurately by the level-set
  - Reduce $\boldsymbol{f}_{\mathrm{m}}$ from 3D to 2D
- Four-step interaction
  - Magnetization
    - $\boldsymbol{H}_{\mathrm{ext}}$ magnetizes the immersed magnetic substance
      - According to its current shape and position
  - Induction
    - $\boldsymbol{H}_{\mathrm{int}}$ is induced
    - $\boldsymbol{H} = \boldsymbol{H}_{\mathrm{ext}} + \boldsymbol{H}_{\mathrm{int}}$
  - Exertion
    - Apply the magnetic force $\boldsymbol{f}_{\mathrm{m}} = \boldsymbol{f}_{\mathrm{m}}^{\mathrm{M}}$
  - Reshaping
    - The state of the physical system is updated

* Simultaneously and repeatedly

Time Splitting

PHYSICAL MODELING

- Magnetization & induction
  - Solve for $\boldsymbol{H}_{\text{int}}$ based on $\boldsymbol{H}_{\text{ext}}$ and χ
  - Governed by Maxwell's equations
    - $\nabla \cdot \boldsymbol{B} = 0$
    - $\nabla \times \boldsymbol{H} = \boldsymbol{j}_{\text{f}} + \frac{\partial \boldsymbol{D}}{\partial t}$
  - Equivalent to a Poisson's equation
    - $\nabla \cdot (1 + \chi)\nabla\psi = \nabla \cdot \chi \boldsymbol{H}_{\text{ext}}$
    - $\boldsymbol{H}_{\text{int}} = -\nabla\psi$
  - (Discretized into a linear system)



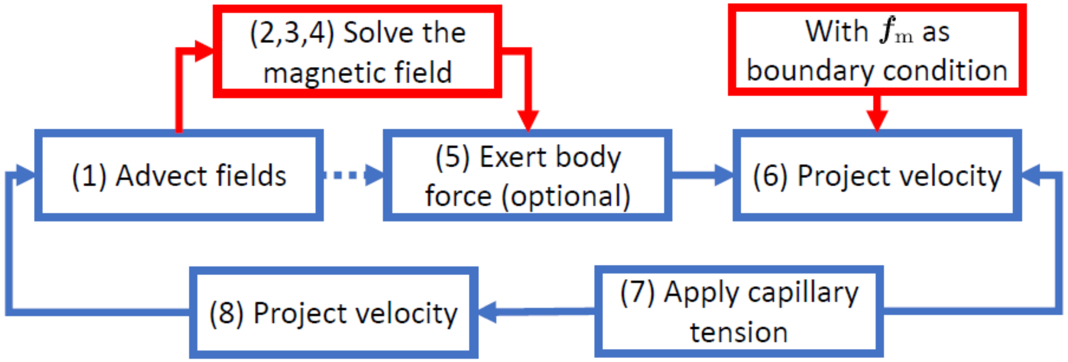
PHYSICAL MODELING

- Exertion
  - The precise formula of Helmholtz force
    - $\boldsymbol{f}_{\mathrm{m}} = \frac{\mu_0}{2} k \left[\boldsymbol{H}^2 + \frac{k^2}{4k+4} (\boldsymbol{H} \cdot \widehat{\boldsymbol{n}})^2 \right] \delta_\Sigma(\boldsymbol{r}) \widehat{\boldsymbol{n}}$
      - δ_Σ is the generalized *Dirac delta function*
      - $\widehat{\boldsymbol{n}}$ is the unit normal vector on the interface
    - Derived by taking the weak form of δ_Σ
    - Refer to the paper for details
  - Always perpendicular to the interface
- Reshaping
  - For fluids (with surface tension)
    - $\rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) = -\nabla p + \rho \boldsymbol{g} + \boldsymbol{f}_{\mathrm{m}} - \sigma \kappa \widehat{\boldsymbol{n}}$
    - $\nabla \cdot \boldsymbol{u} = 0$
  - For elastic bodies
    - $\rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} + \boldsymbol{f}_{\mathrm{m}}$
  - For rigid bodies
    - $m_\rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = m_\rho \boldsymbol{g} + \oiiint_\Omega \boldsymbol{f}_{\mathrm{m}} \mathrm{d}V$
    - $\boldsymbol{I}_\rho \frac{\mathrm{D}\boldsymbol{\omega}}{\mathrm{D}t} + \boldsymbol{\omega} \times \left(\boldsymbol{I}_\rho \cdot \boldsymbol{\omega} \right) = \oiiint_\Omega (d \times \boldsymbol{f}_{\mathrm{m}}) \mathrm{d}V$

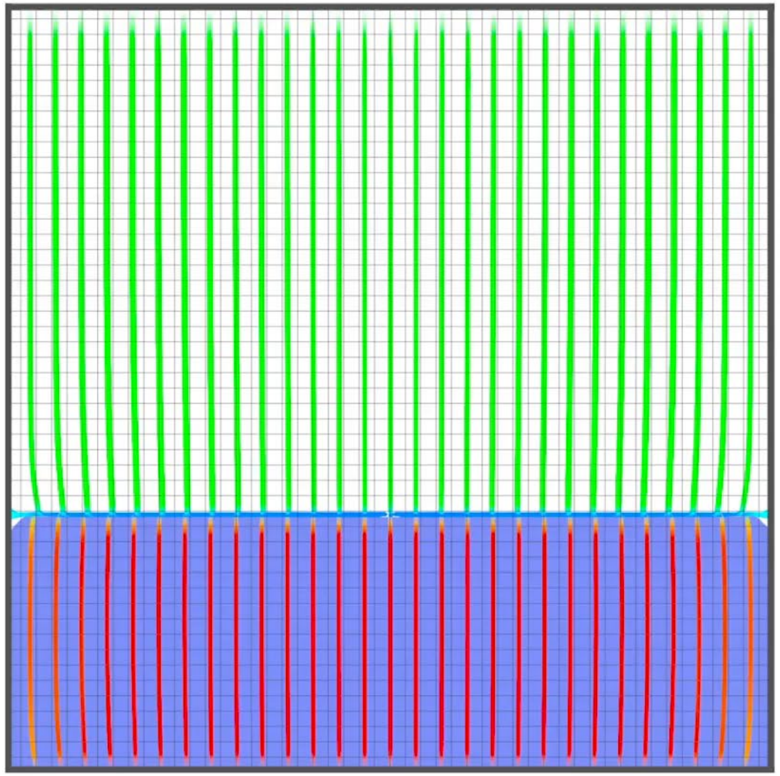
NUMERICAL SCHEME

- Solve the magnetic field (a linear system)
  - Conjugate gradient method
    - Multi-grid preconditioned
- Exert the magnetic force
  - For fluids
    - Treat the interfacial Helmholtz force as pressure jump
      - Young–Laplace Equation
      - $\boldsymbol{f}_{\mathrm{m}}\,\mathrm{d}V = \Delta p\,\mathrm{d}\boldsymbol{s}$
    - Involved as BC in projection step
  - For solids
    - Apply on surface triangles

* The pipeline sketch of ferrofluid simulation with new components painted red



SIMULATION RESULTS



Rosensweig instability (2D)

CONCLUSION

- Summary
  - The first versatile level-set method
  - A novel computational approach
    - With interfacial Helmholtz force introduced
  - An efficient numerical scheme
- Limitations
  - Explicit magnetic force
    - Potential numerical instability
  - Weak coupling
- Future work
  - Boundary element method
  - Apply Helmholtz force implictly

Thank you for listening!