

# COMPLEMENTARY DYNAMICS

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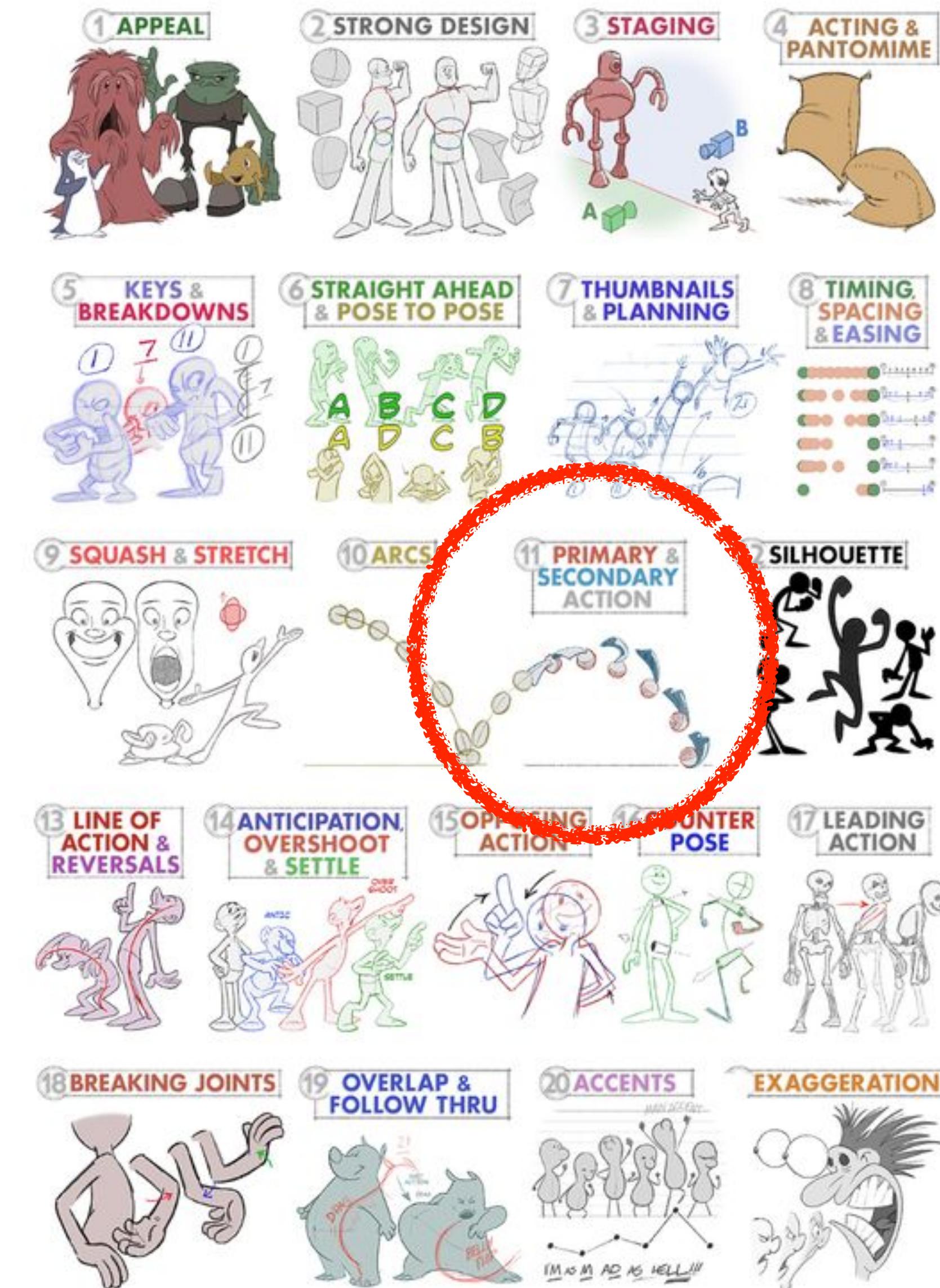
dgp

Dynamic Graphics Project  
University of Toronto  
[www.dgp.toronto.edu](http://www.dgp.toronto.edu)



# The 21 Foundations of Animation

BY DERMOT O'CONNOR  
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**With Secondary Motion**

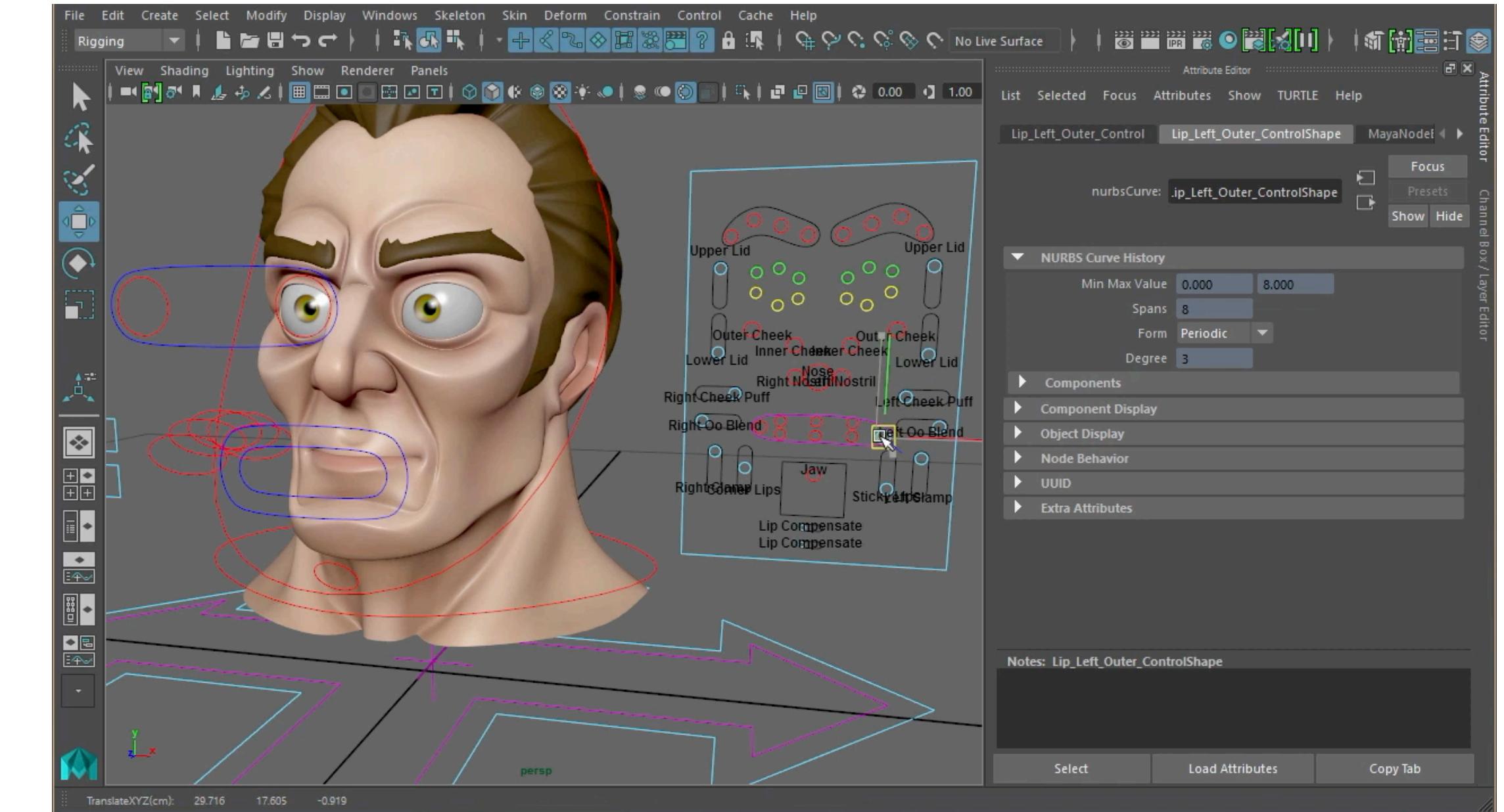


**Primary Motion Only**



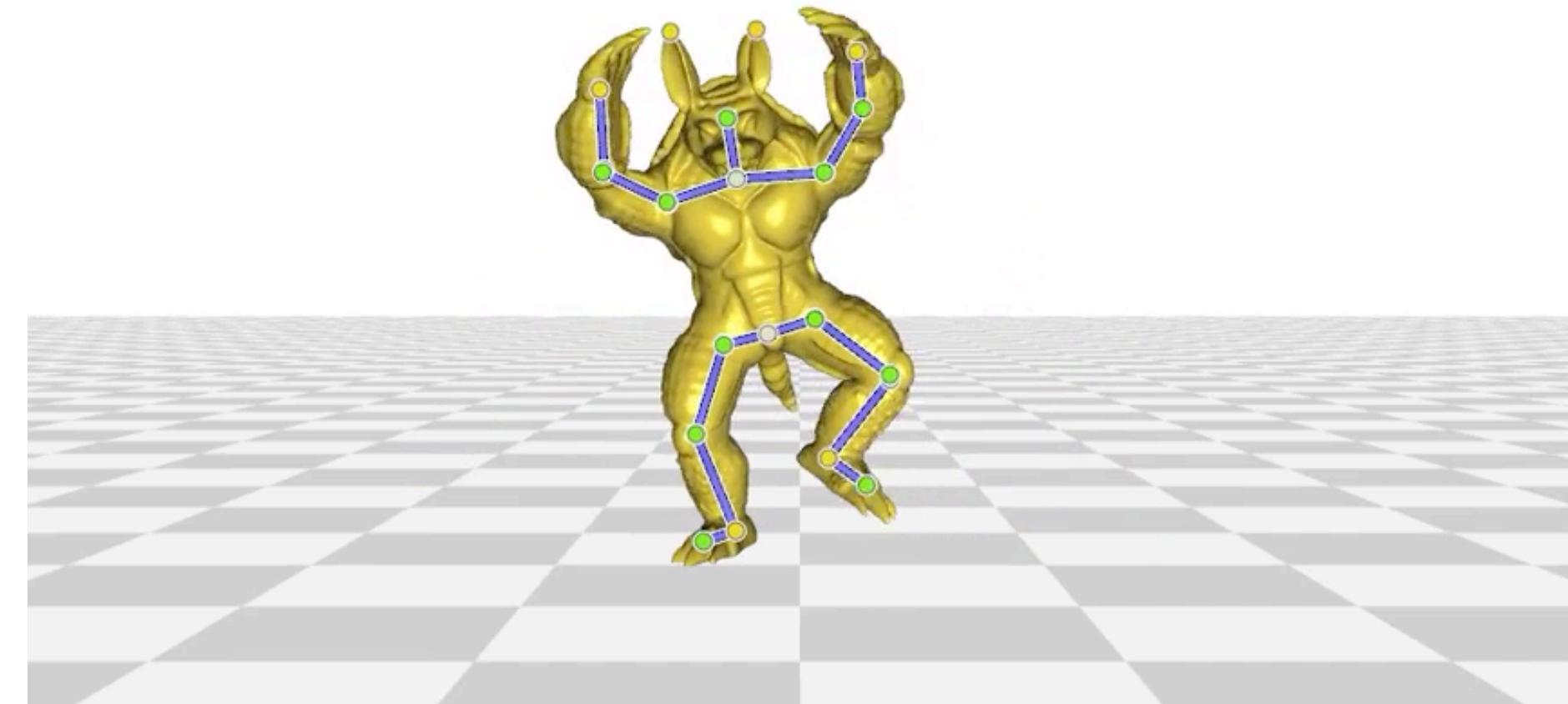
# Fully Animated

- Geometric
- Controllable
- Whatever intended
- ...



[https://www.youtube.com/watch?v=pliiuA\\_8pM8&t=957s](https://www.youtube.com/watch?v=pliiuA_8pM8&t=957s)

- Requires creativity
- Labor consuming
- Hard to do complex motion
- ...

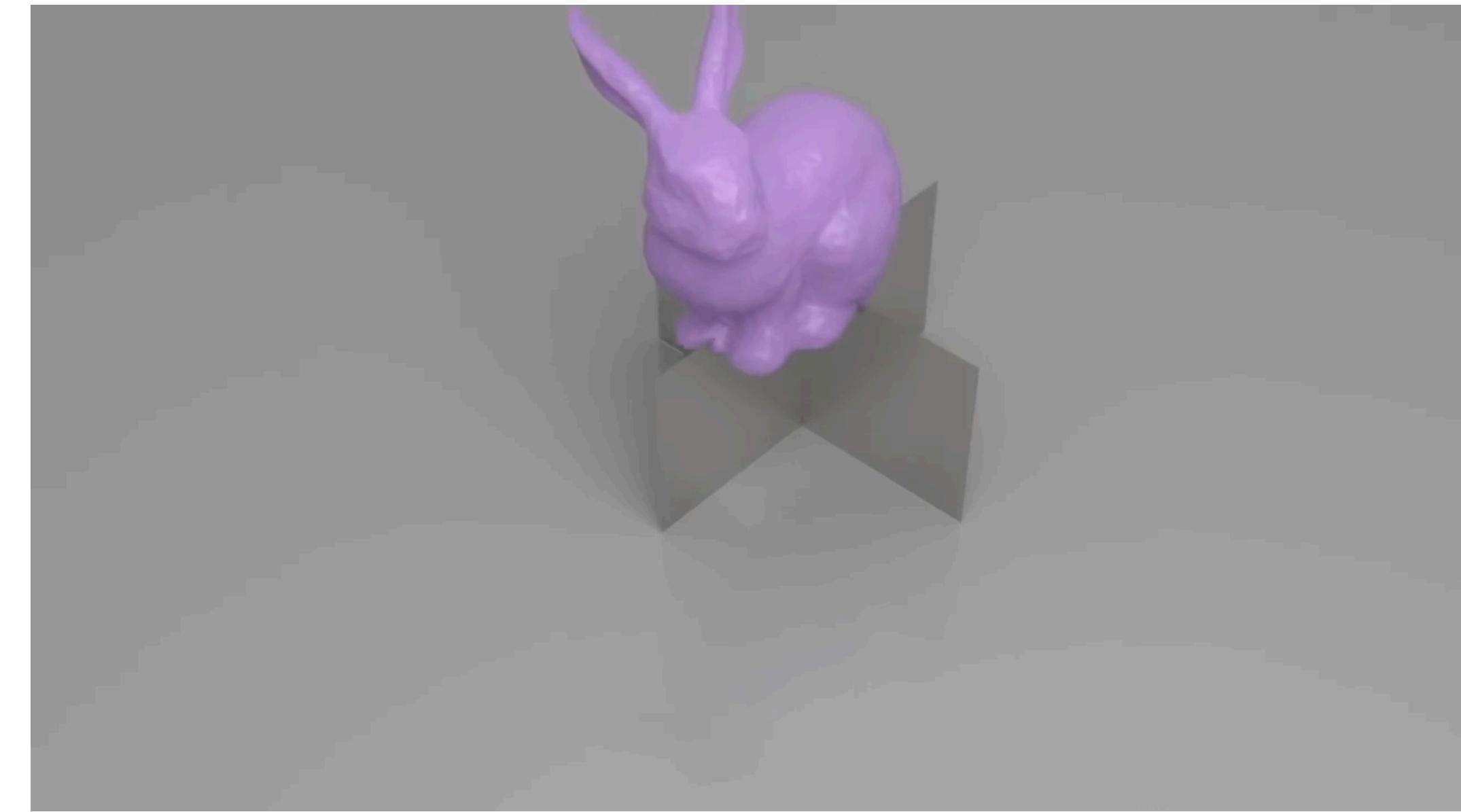


[Jacobson et al. 2012]

# Fully Simulated



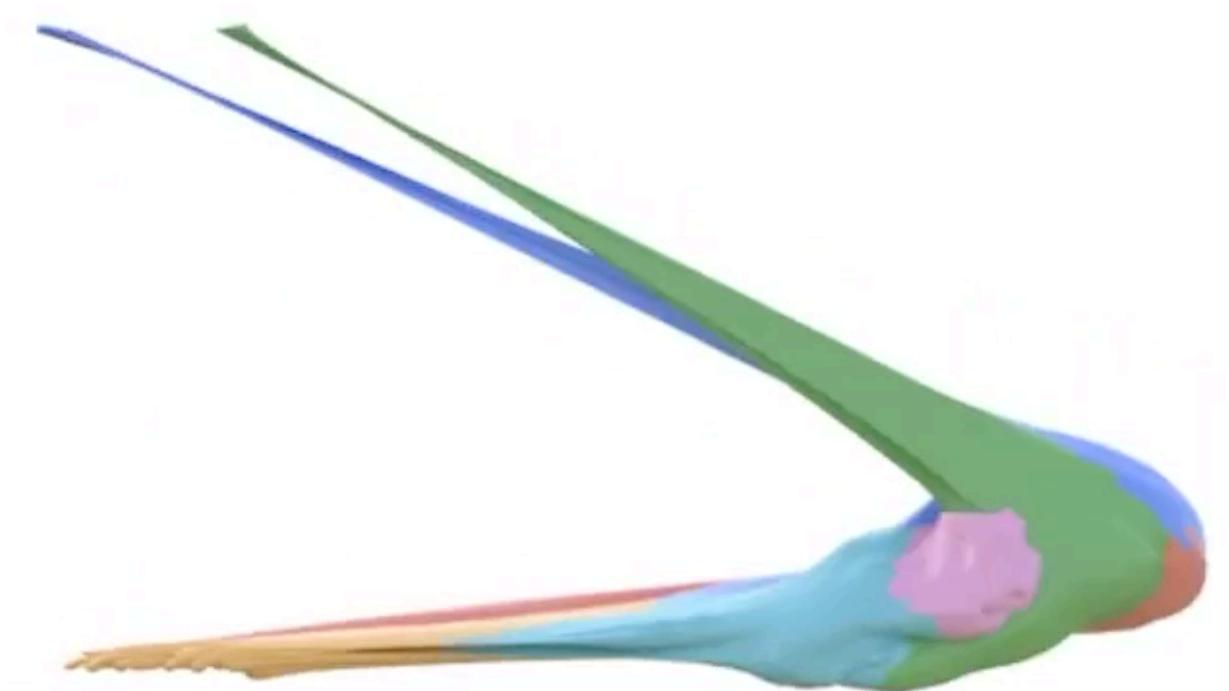
- Physically accurate
- Automatic
- Happy accidents
- ...



[Hu et al. 2018]

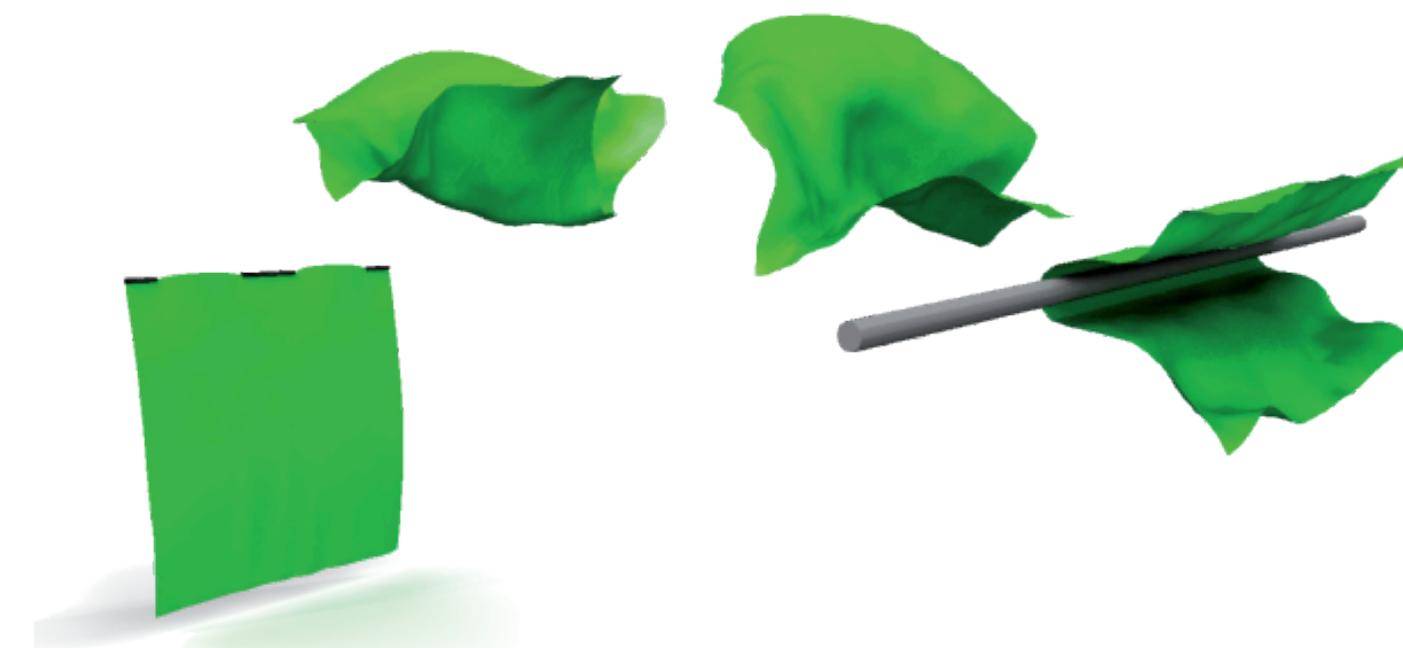


- No direct control
- No artistic intention
- Iterations can take long
- ...

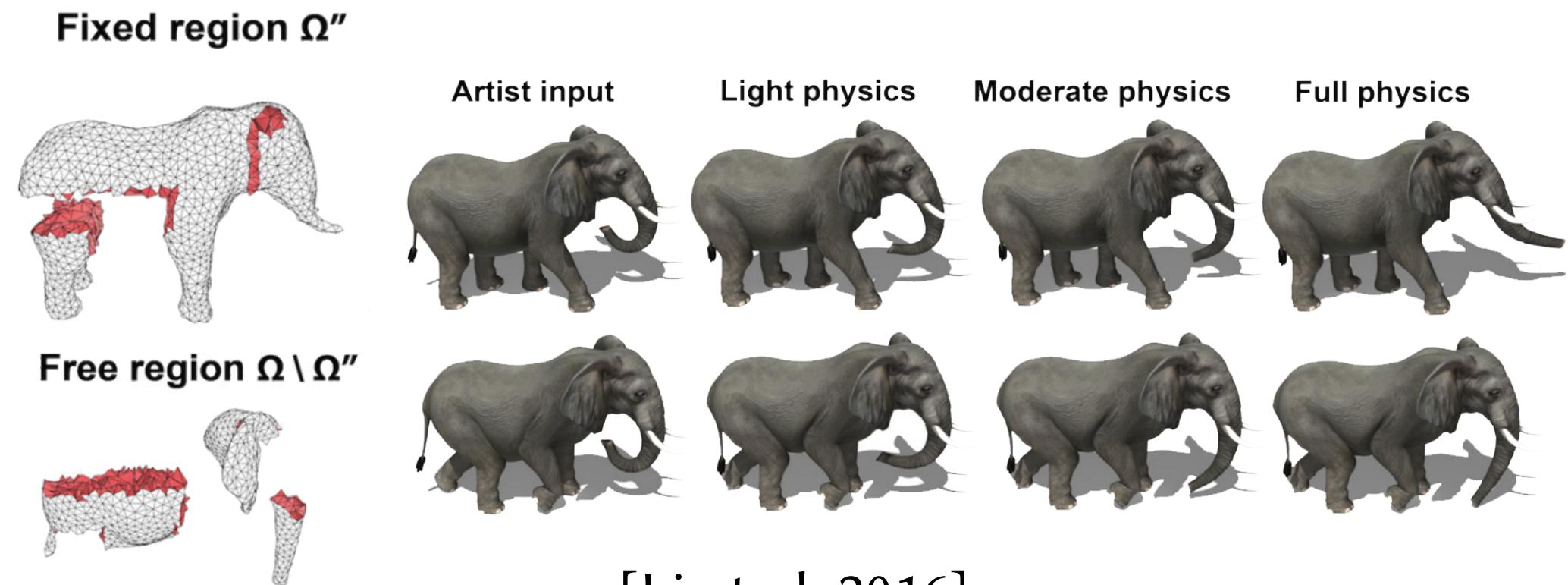


[Li et al. 2019]

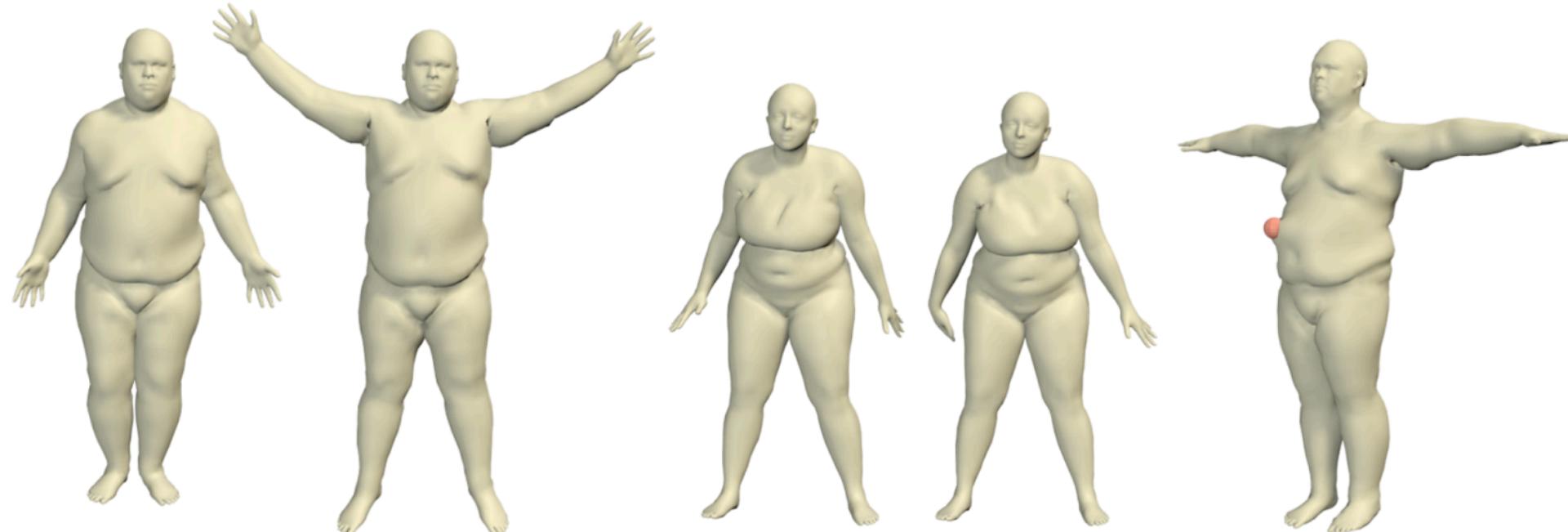
# Simulation under artistic control?



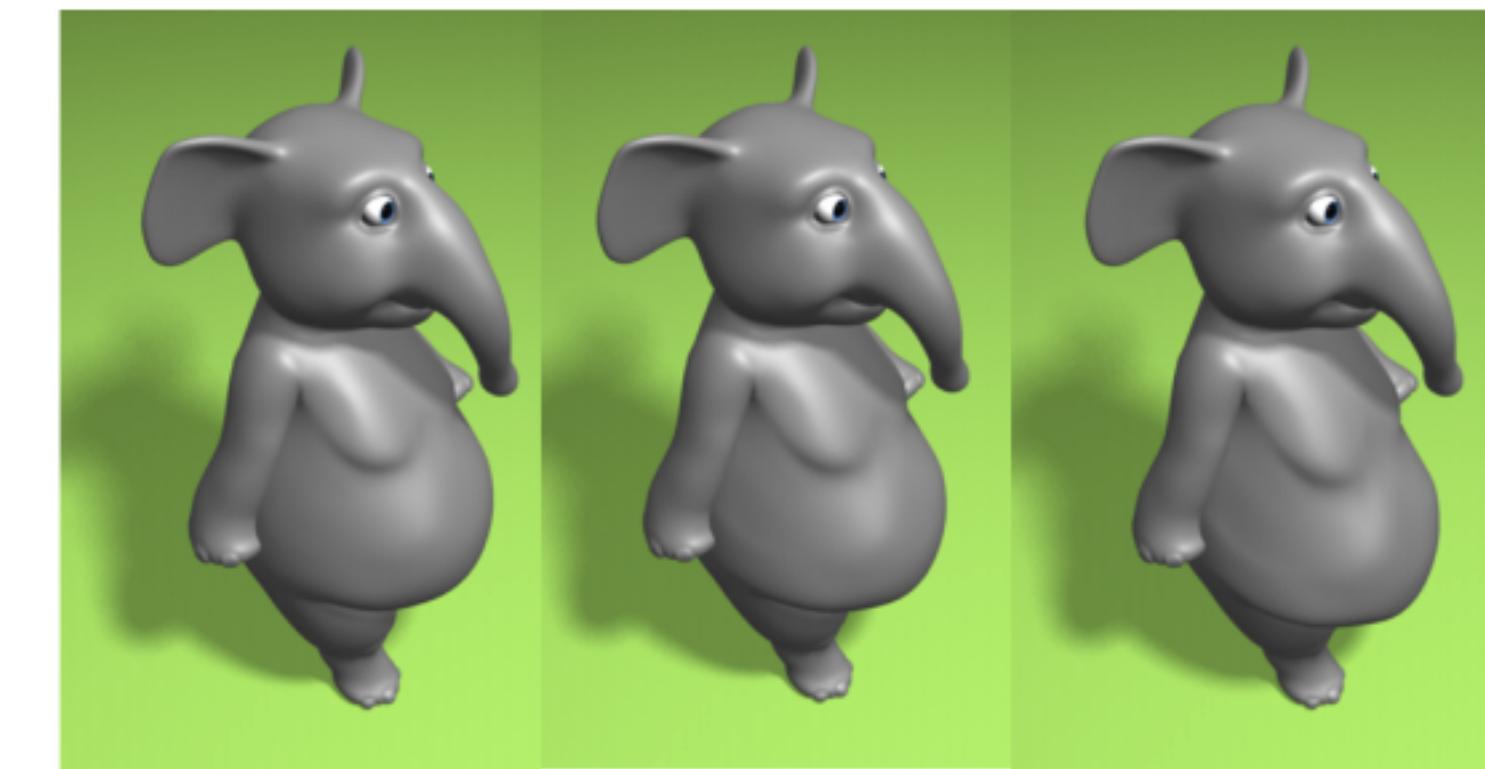
[Hildebrandt et al. 2012]



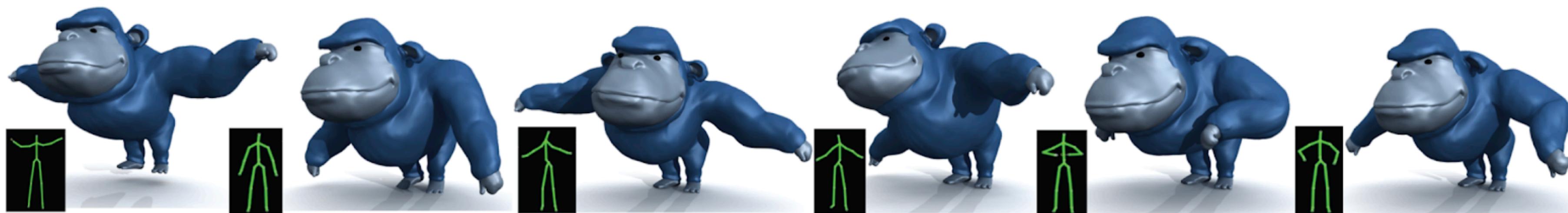
[Li et al. 2016]



[Kim et al. 2017]



[Hahn et al. 2012]



[Xu et al. 2016]

...

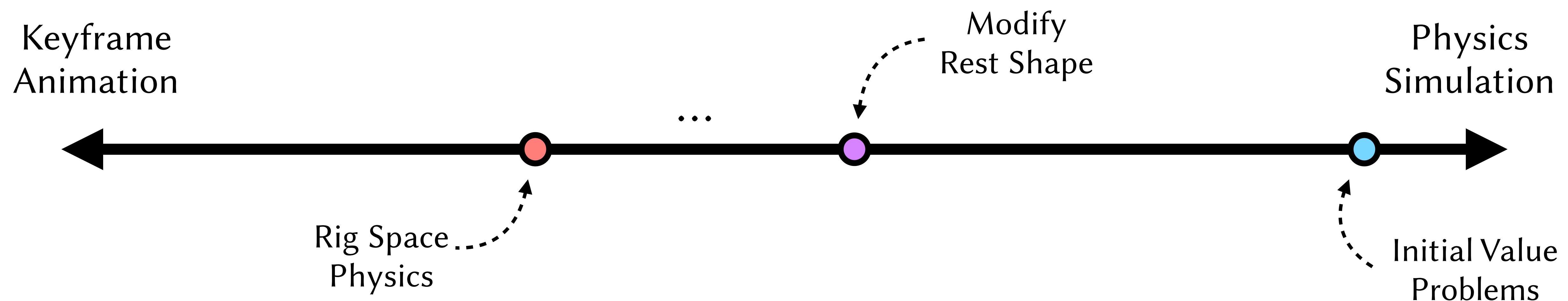
# The *False* Linear Spectrum

Keyframe  
Animation

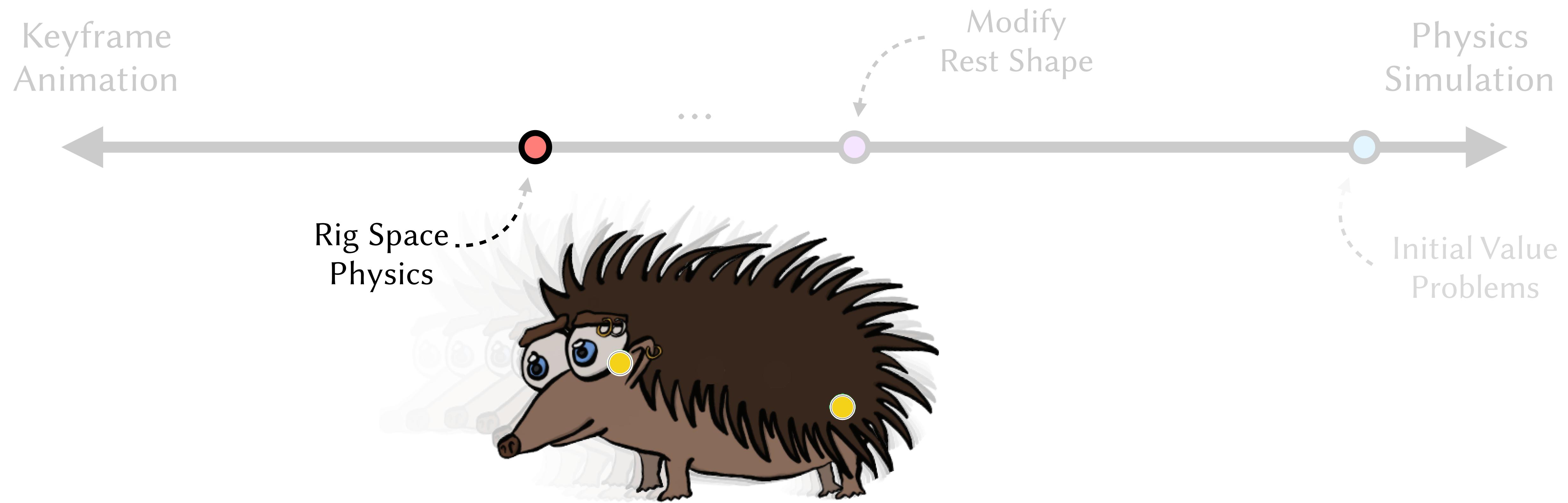


Physics  
Simulation

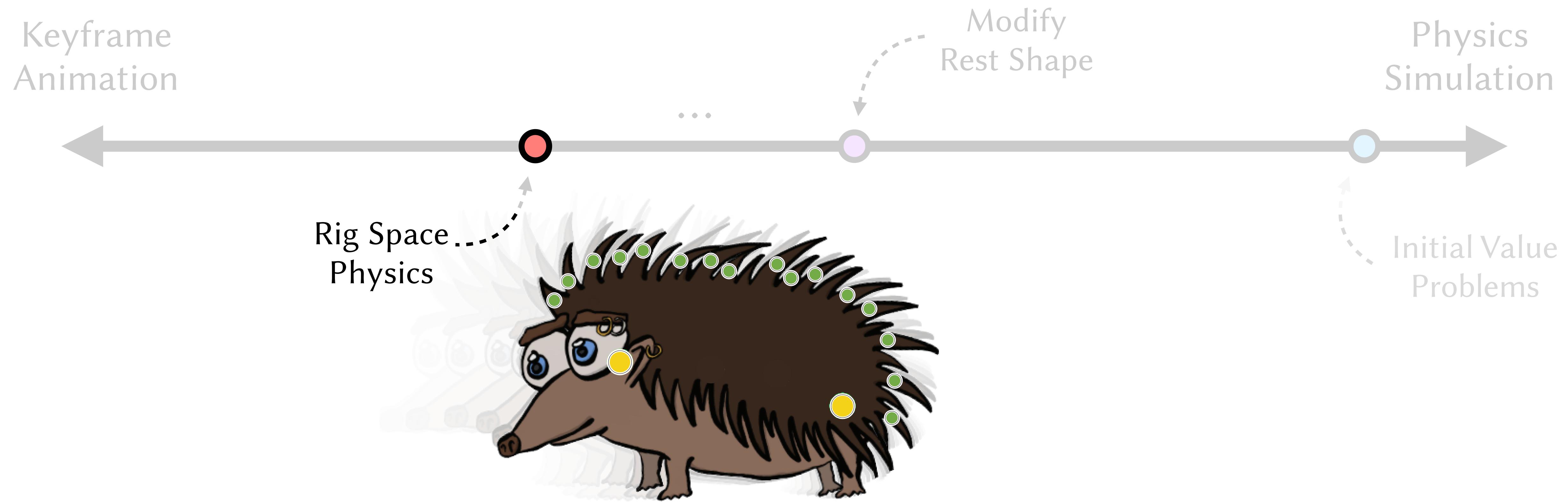
# The *False* Linear Spectrum



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# The *False* Linear Spectrum



# The *False* Linear Spectrum

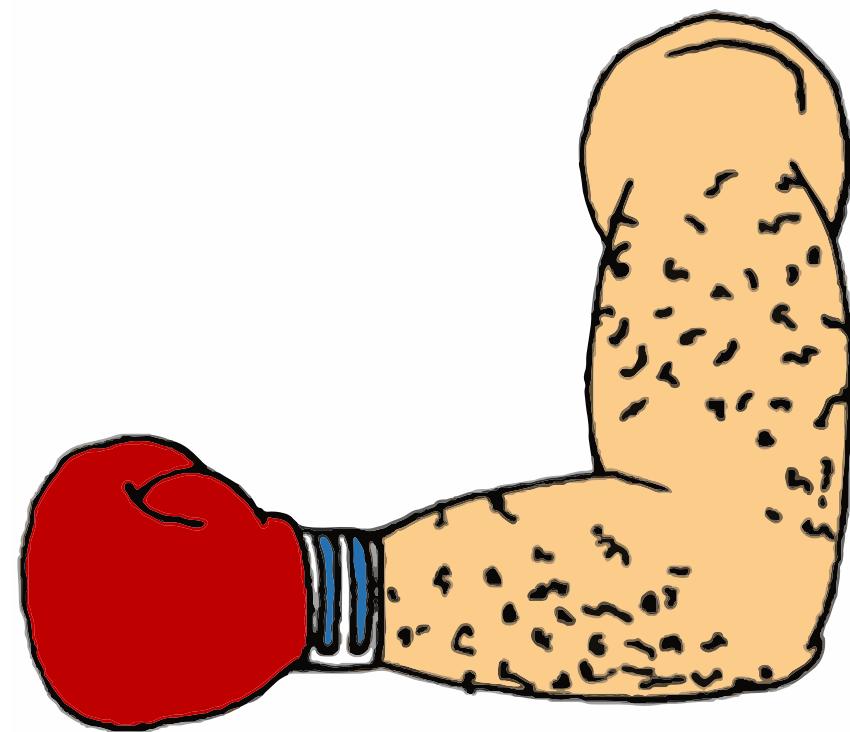
Keyframe Animation



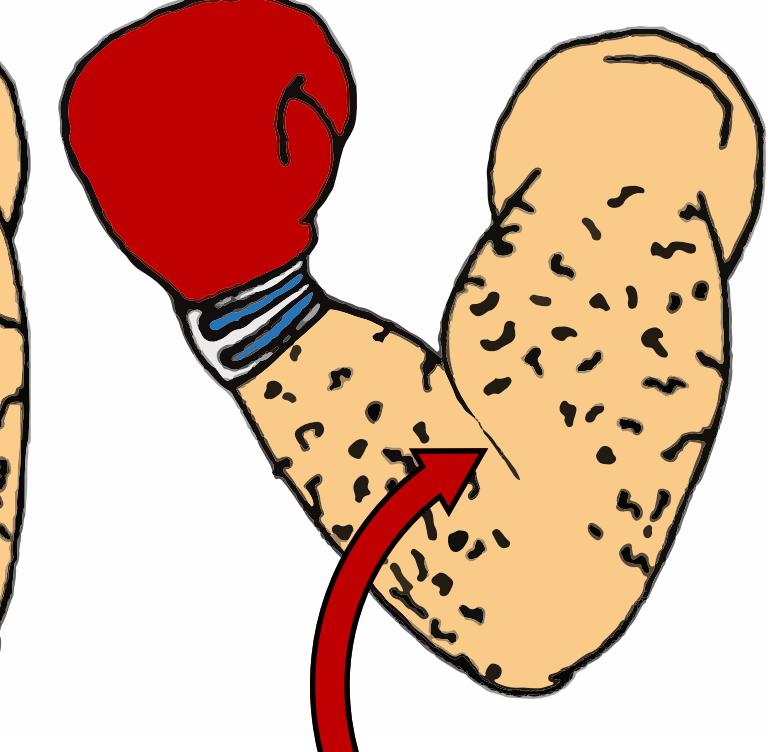
Rig Space Physics



rest pose



rig pose



*inverted elements*

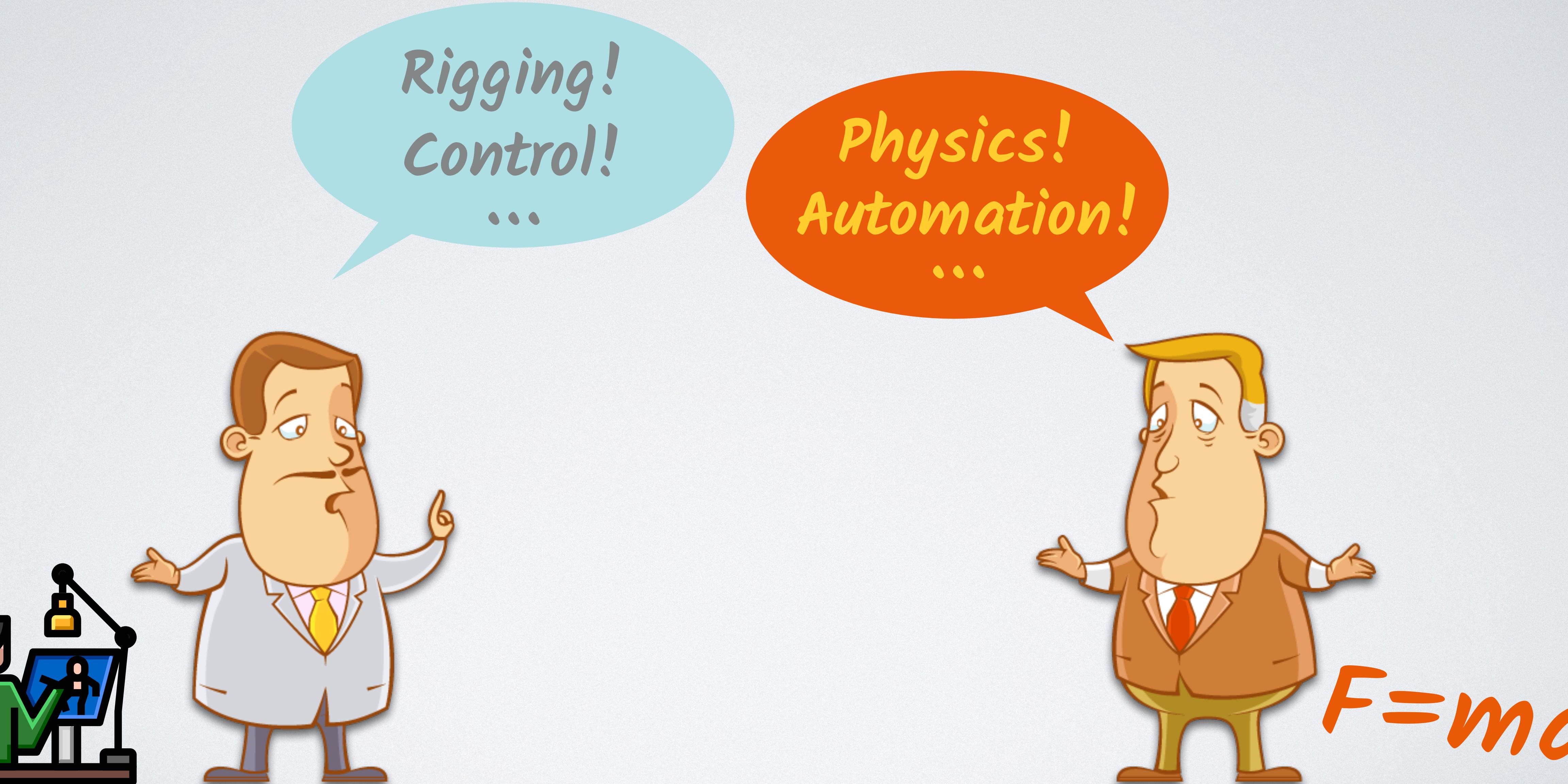
Modify Rest Shape

Physics Simulation

Initial Value Problems



Rigging!  
Control!  
...



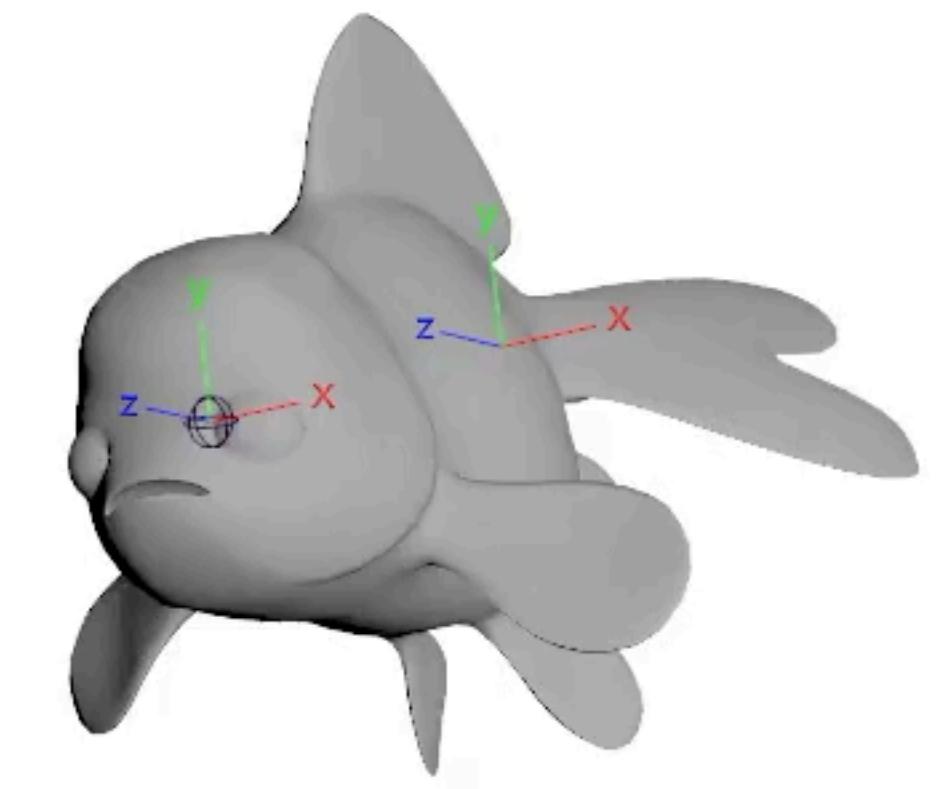
Physics!  
Automation!  
...



$$F=ma$$

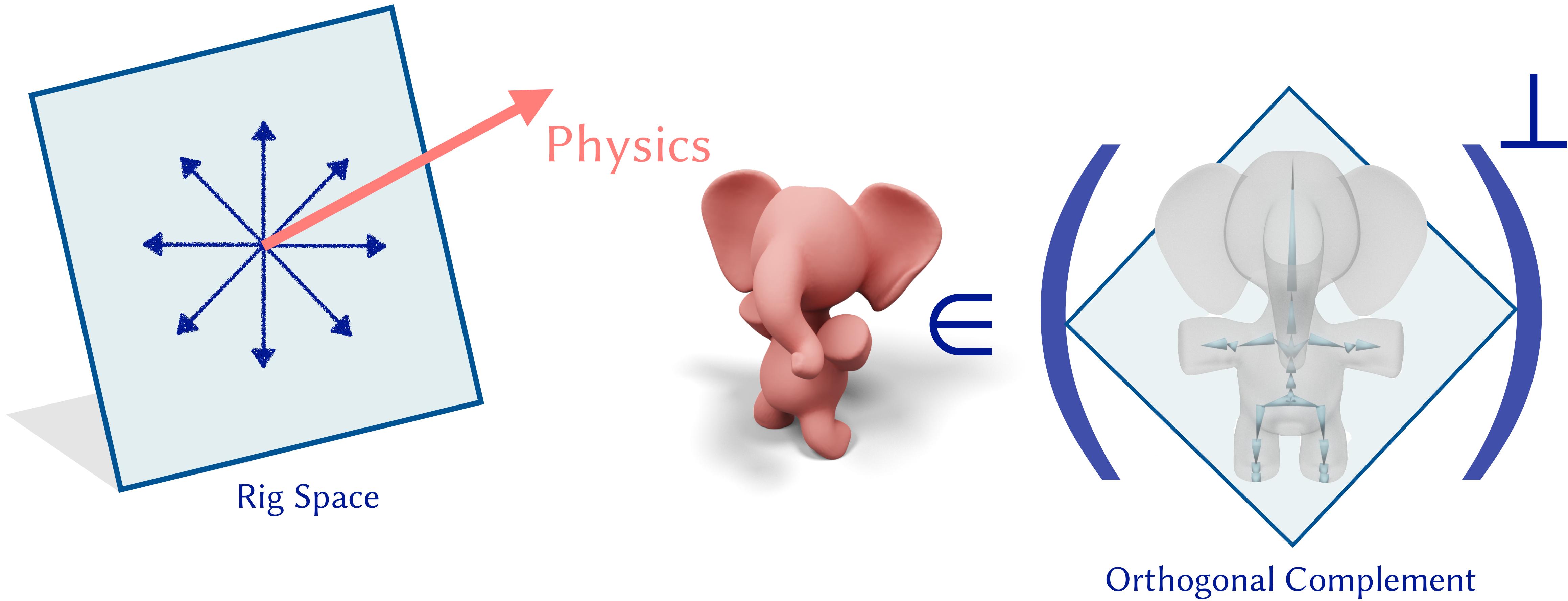
“Creative primary effects and physical secondary effects are not **contradictory**, but rather they are *complementary*”

# Primary Motion + Our Complementary Secondary Motion

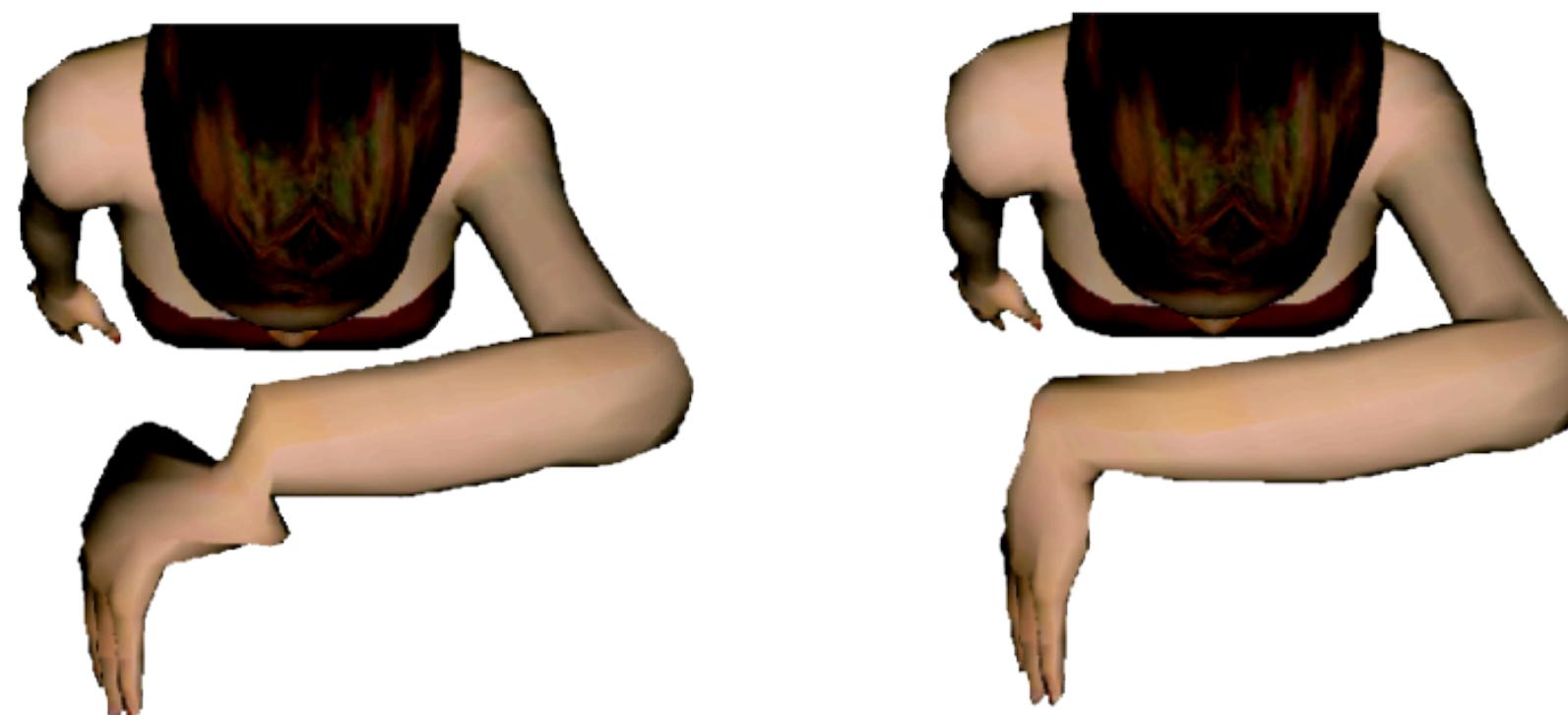


# OUR METHOD

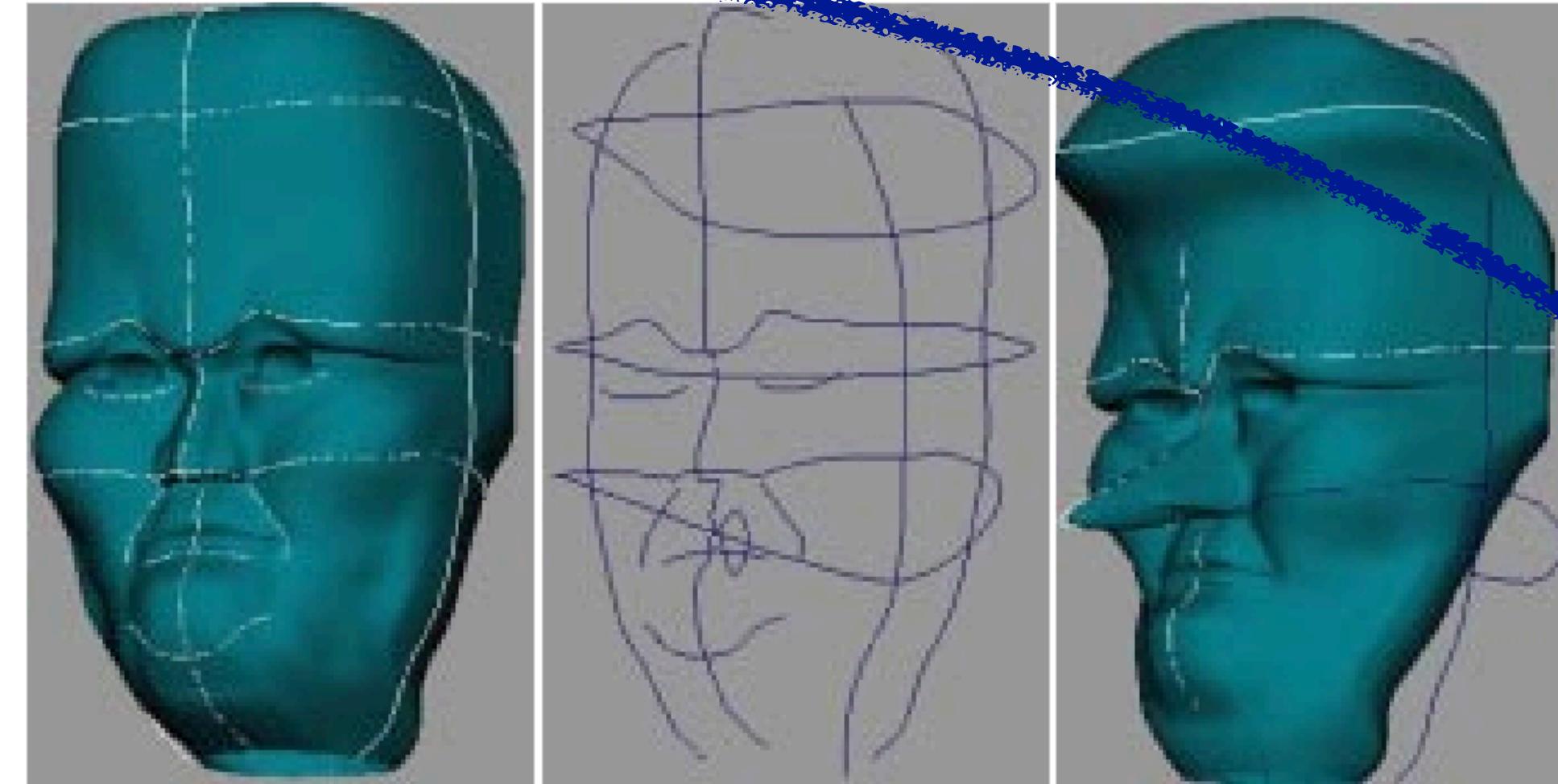
# Main Idea



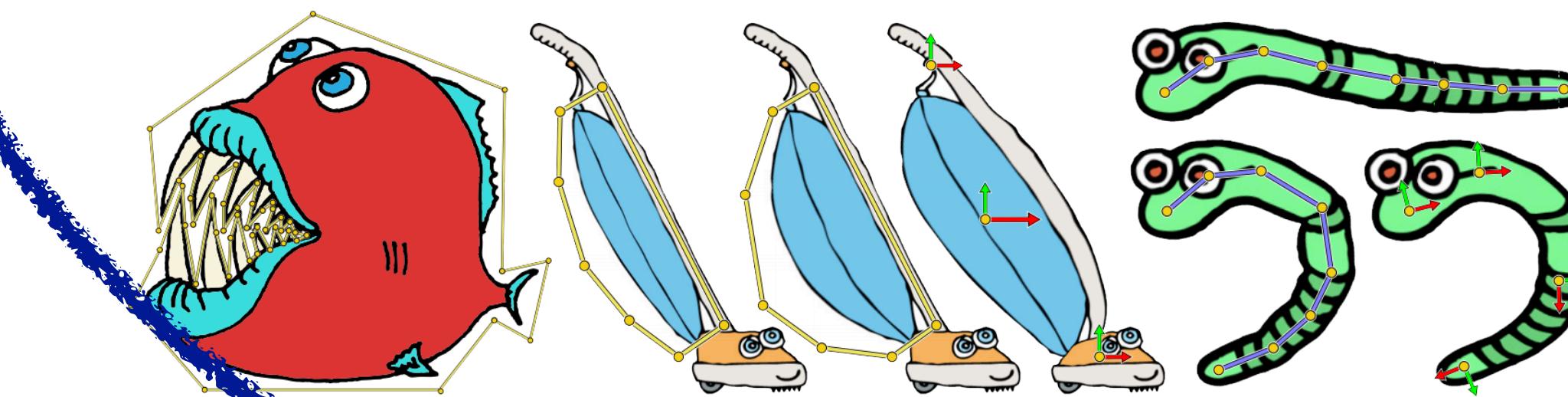
# Input Rig Function



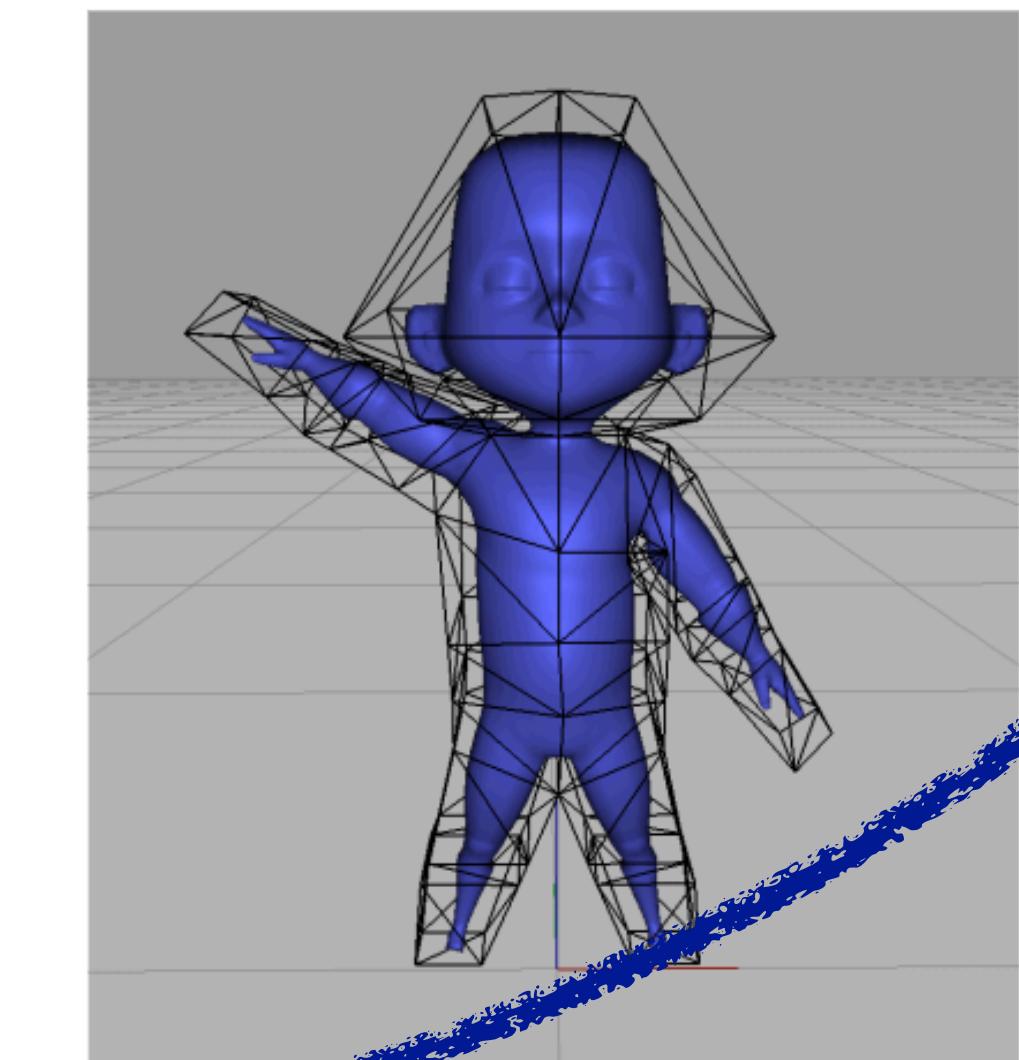
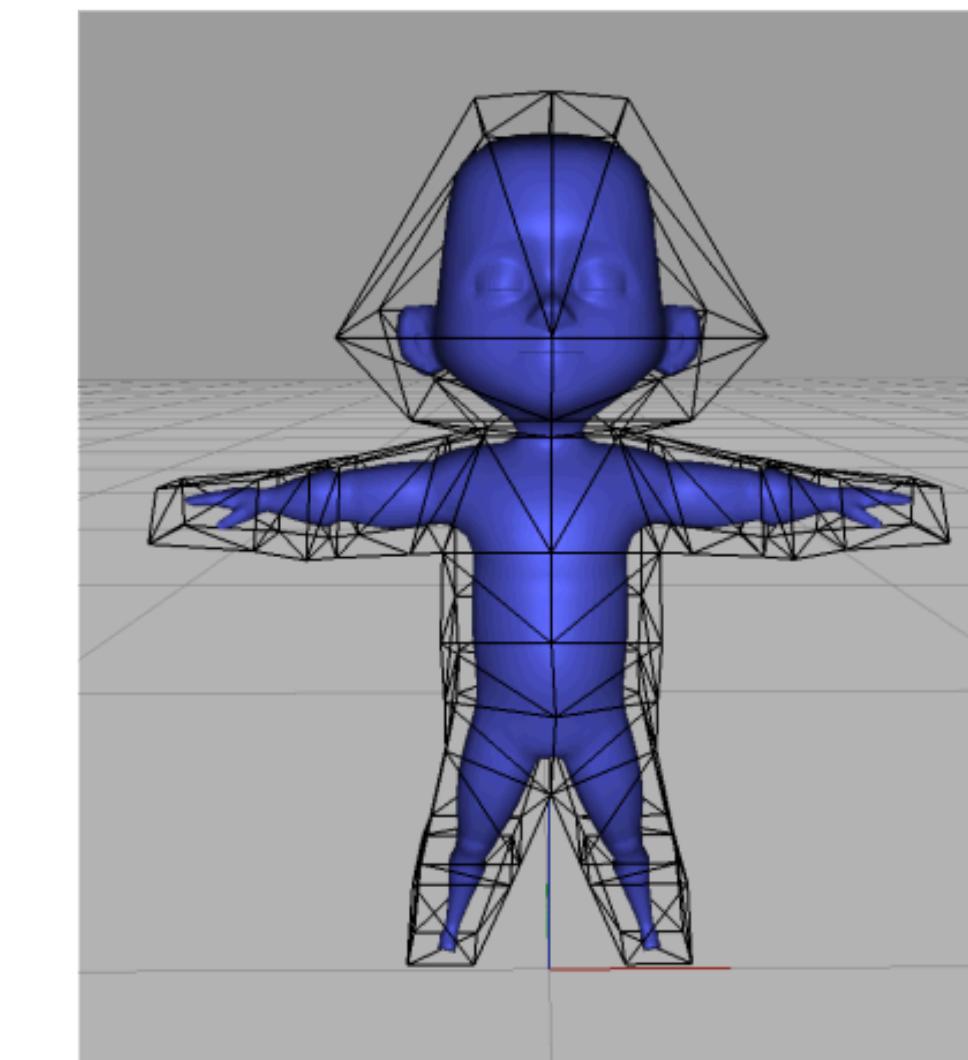
[Kavan et al. 2007]



[Singh and Fiume 1998]

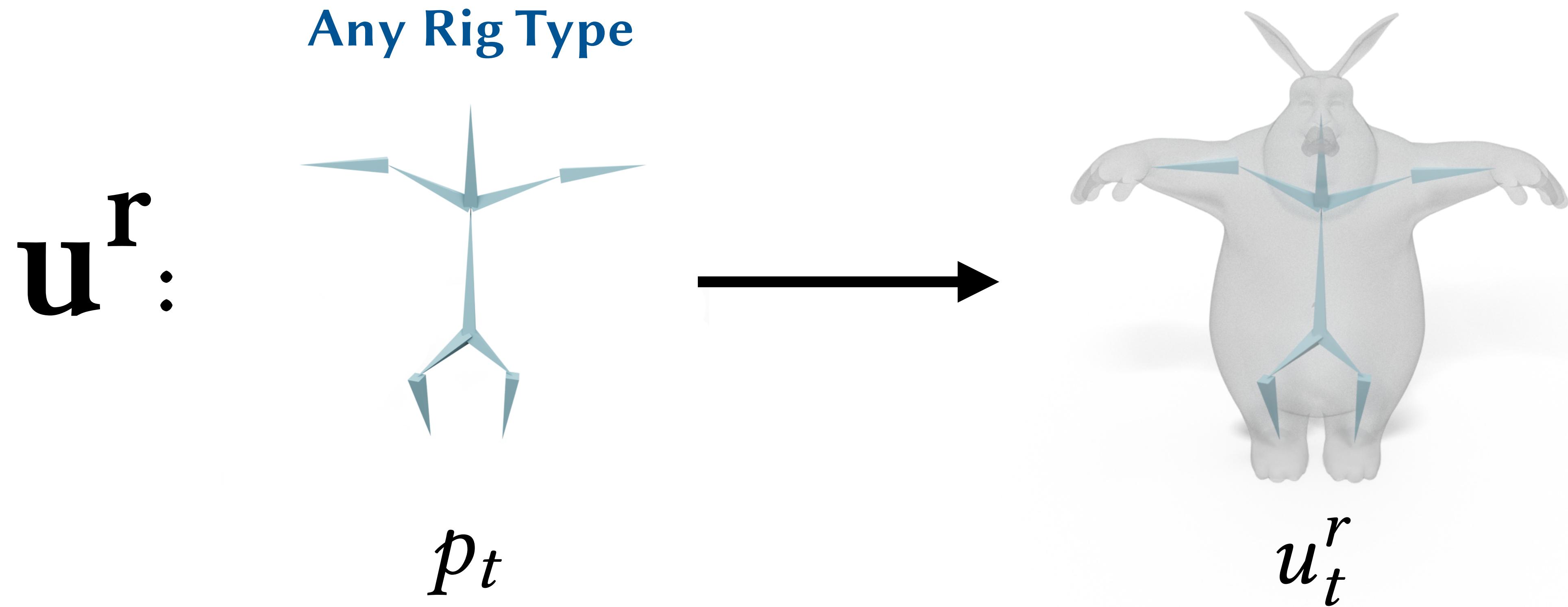


[Jacobson et al. 2011]

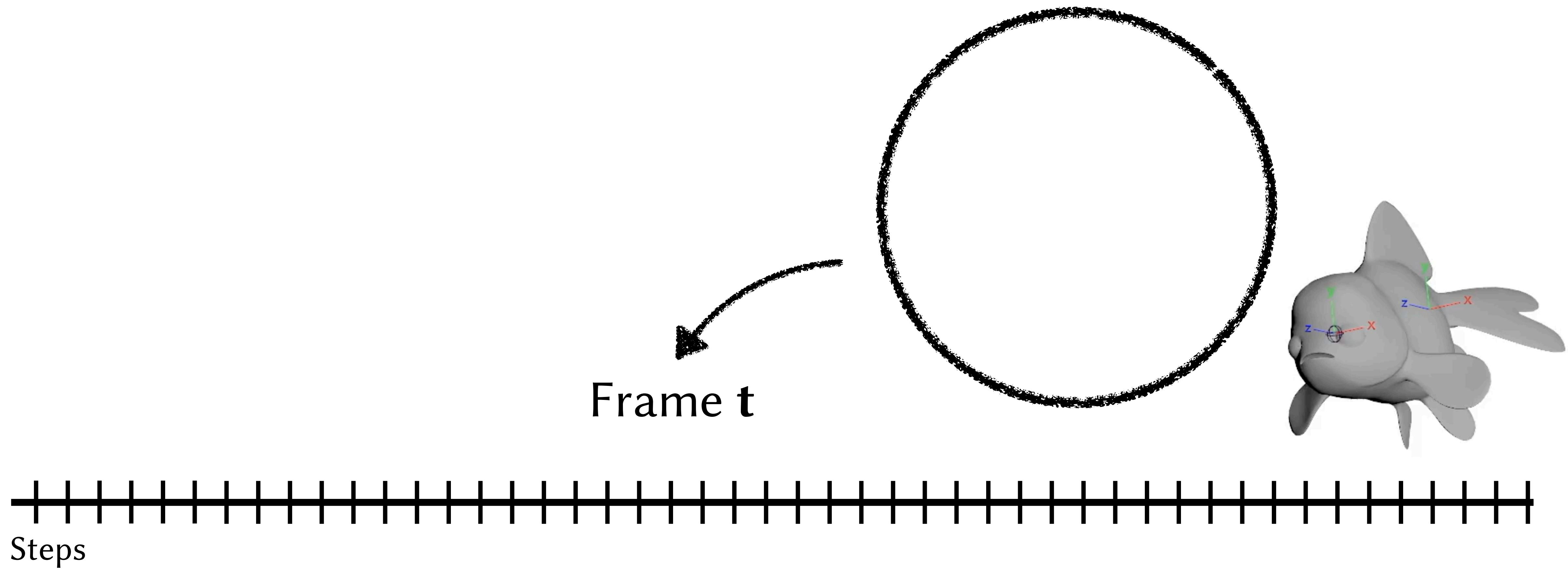


[DeRose and Meyer 2007]

# Input Rig Function



# Input Rig Parameters Sequence



# Our Goal

$$u_t = u_t^r + u_t^c$$

Final Displacement      Rig Displacement      Complementary Displacement

The diagram illustrates the decomposition of a goldfish's final displacement into two components. On the left, a 3D model of a goldfish with a red cap is labeled  $u_t$ . This is followed by an equals sign. To the right of the equals sign is another 3D model of the goldfish, which is gray and has a local coordinate system (x, y, z axes) attached to its head. This is labeled  $u_t^r$ . To the right of a plus sign is a large red question mark, labeled  $u_t^c$ . Below each term are descriptive labels: 'Final Displacement' under the first term, 'Rig Displacement' under the second term, and 'Complementary Displacement' under the third term.

# Two Guidelines for finding appropriate $U^c$

- $U^c$  should react to internal and external forces
- $U^c$  should not undo the rig displacement

$u_t^c$ ?

Complementary  
Displacement

# Time Integration

$$E(u_t) = \frac{1}{2} \|u_t - \tilde{u}\|_M^2 + h^2 \Psi(u_t)$$

Inertia Term

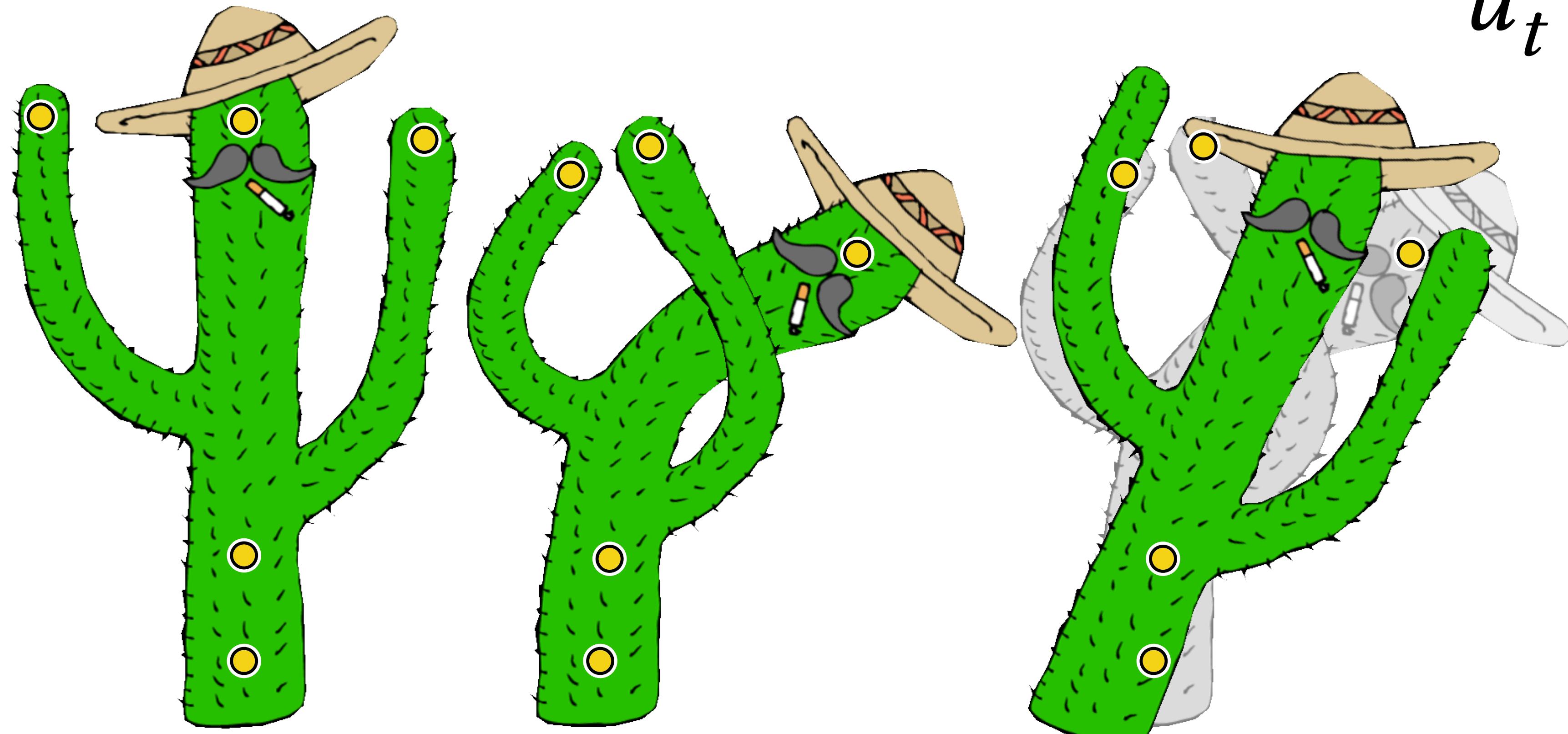
Elasticity Potential

Any Physical Model

$$u_t^c = \underset{u_t^c}{\operatorname{argmin}} E_t(u_t^r + u_t^c)$$

# Unconstrained

$$u_t^c = -u_t^r$$

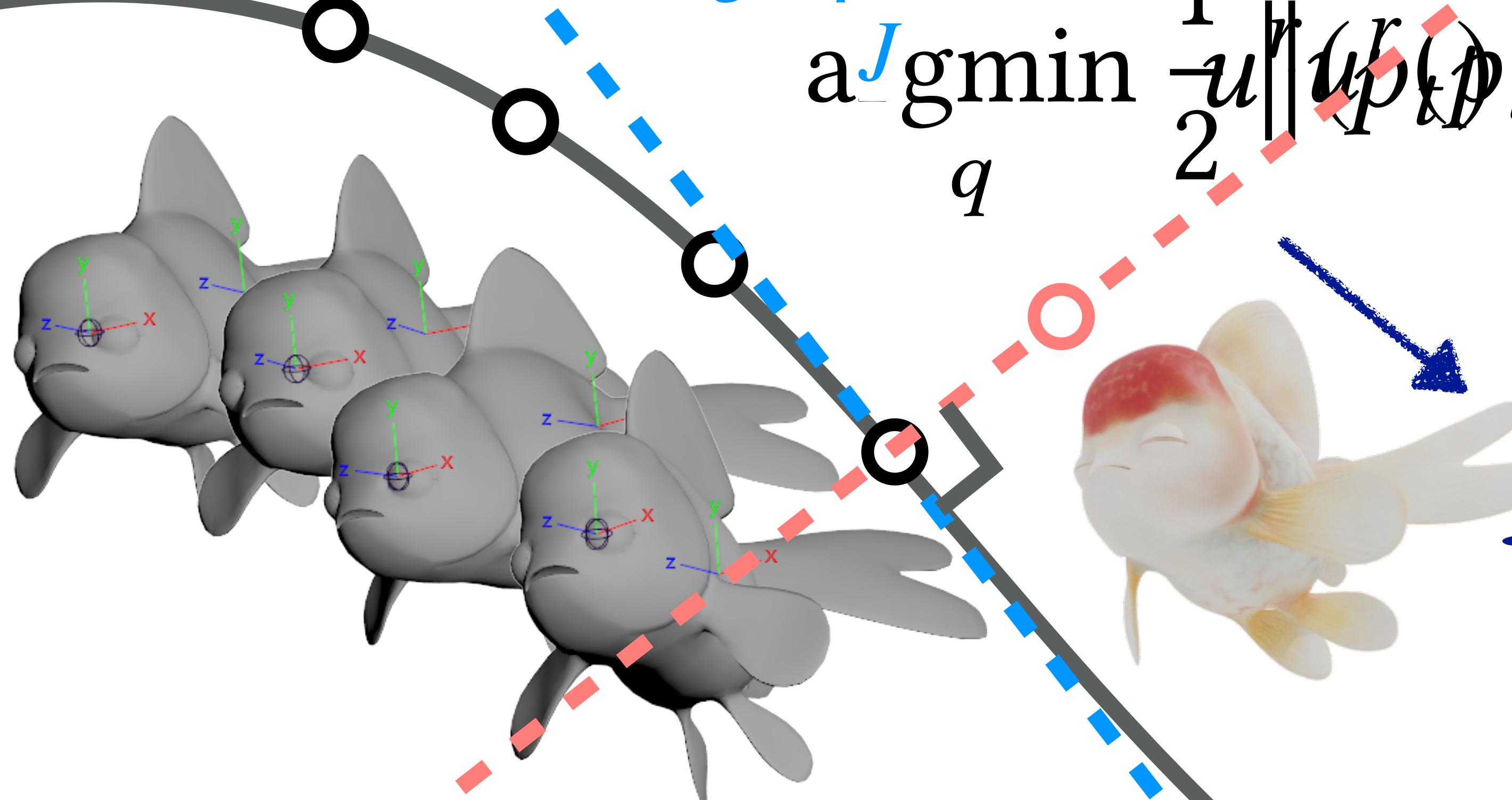


rest pose

rig pose

physics *undoes* rig pose

# Orthogonal Constraint



Jacobian of  
Rig displacement

$$\text{argmin}_q \frac{1}{2} \| u^r(p_t) + u^c_t - u^r(q) \|_M^2 = p_t$$



$$J_t^\top M u_t^c = 0$$

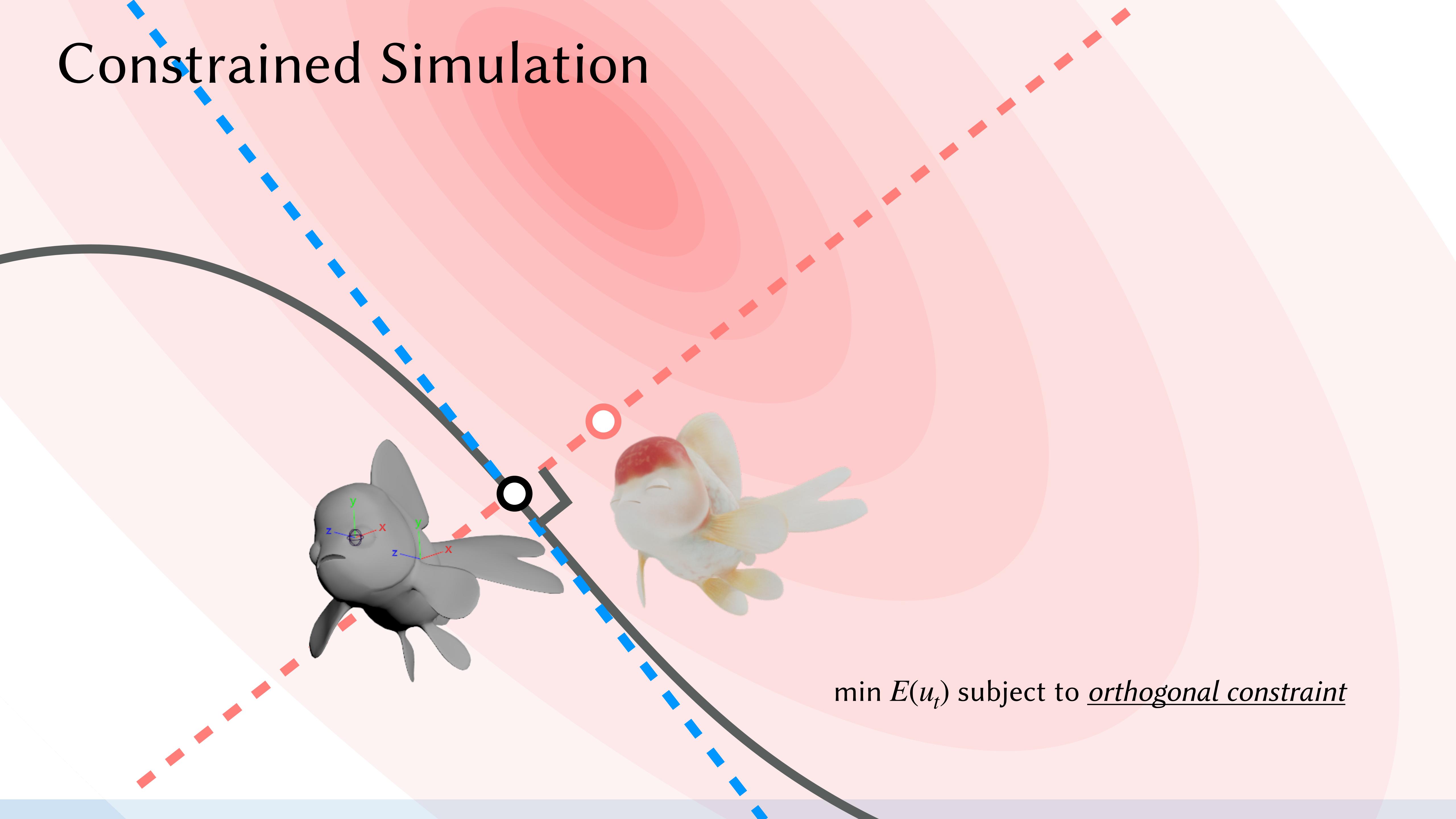
linear!

$J^\perp$

# Constrained Simulation

$\min E(u_t)$  subject to *orthogonal constraint*

# Constrained Simulation



# Compared to Typical Simulation

Given  $u_t^r$

$$u_t = \underset{u_t}{\operatorname{argmin}} E_t(u_t)$$

# Compared to Typical Simulation

Given  $u_t^r$

$$u_t^c = \underset{u_t^c}{\operatorname{argmin}} E_t(u_t^r + u_t^c)$$

# Compared to Typical Simulation

Given  $u_t^r$

Update  $J_t$

$$u_t^c = \underset{u_t^c}{\operatorname{argmin}} E_t(u_t^r + u_t^c) \text{ subject to } J_t^\top M u_t^c = 0$$

$$u_t = u_t^r + u_t^c$$

So simple!

# Nonlinear Optimization with Equality Constraint

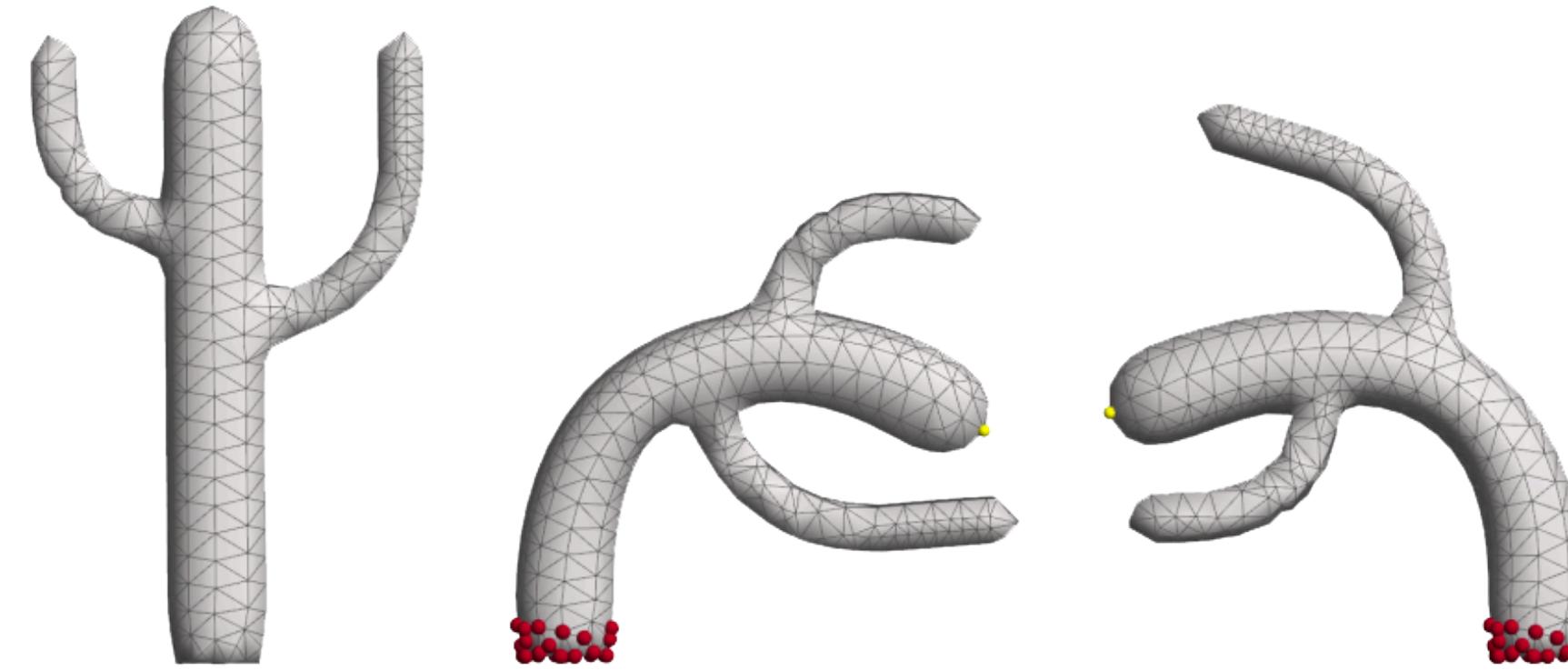
## → Newton's method

- Update the gradient and hessian  $\mathbf{g}, \mathbf{H}$
  - Build and solve the KKT system
  - Do line search  $s$
-

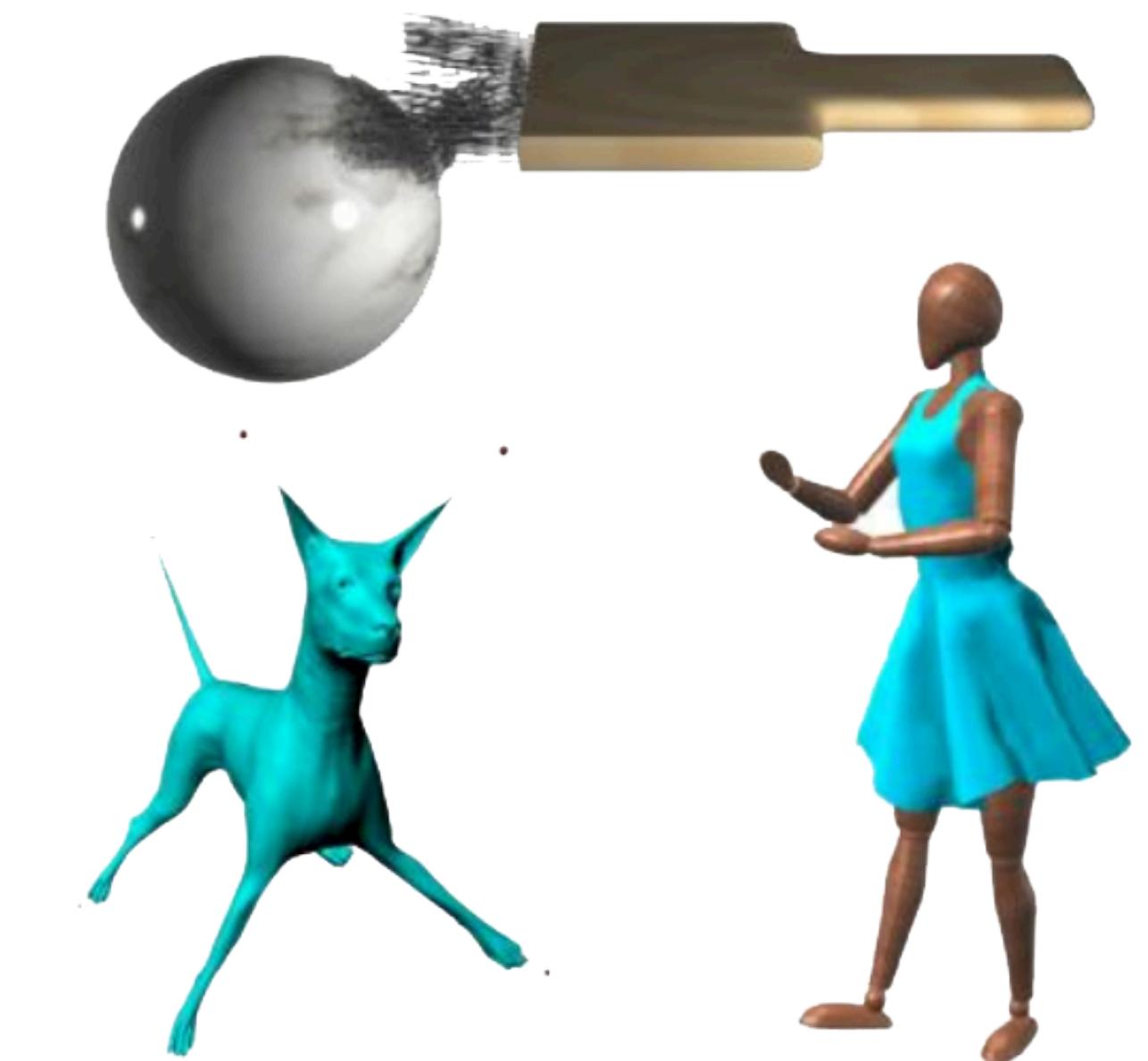
# Nonlinear Optimization with Equality Constraint

→ ADMM-type of Solvers

- Local Step
- Global Step

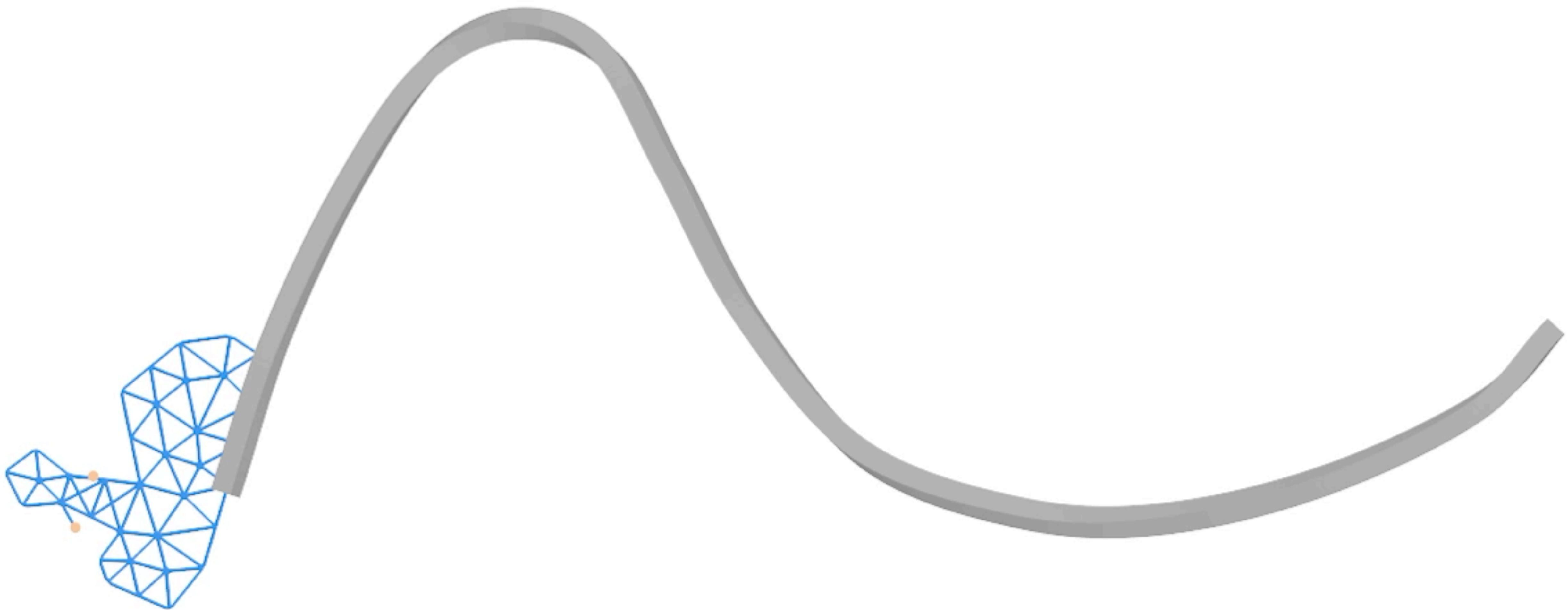


ARAP



Fast Mass Spring

# Fast Mass Spring



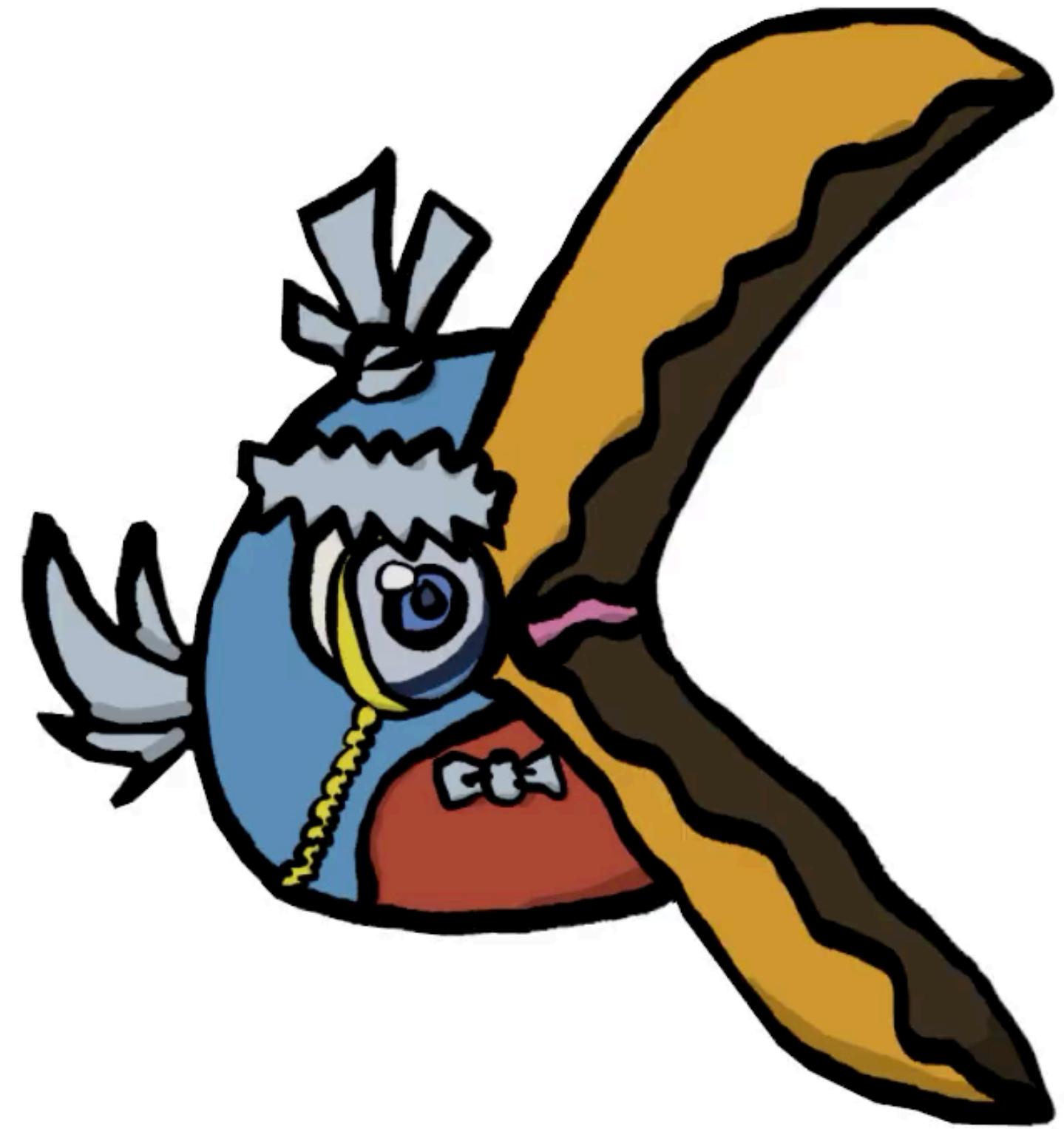
# MORE RESULTS

ANY RIG TYPE

# Bone Handle

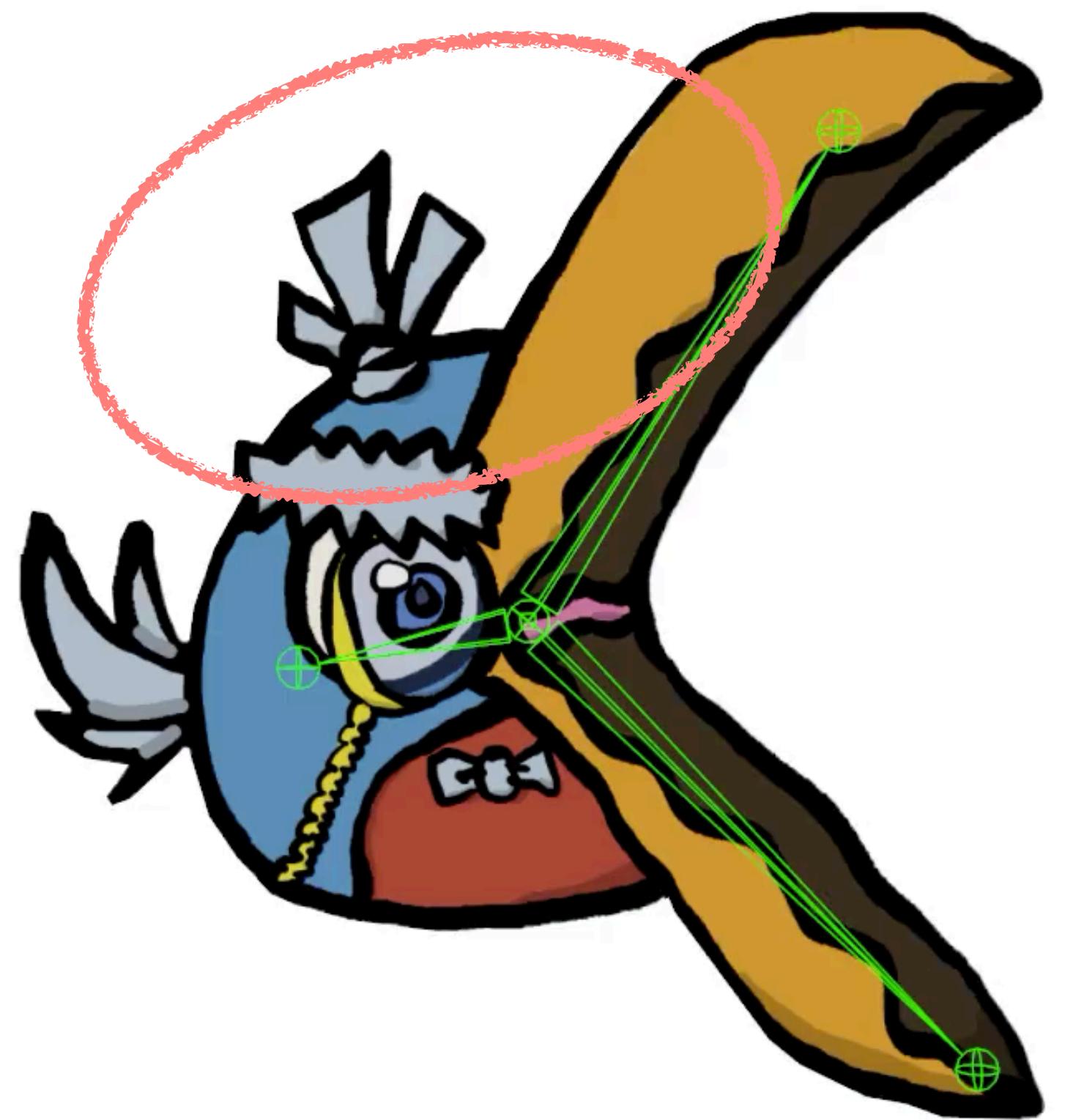


# Bone Handle

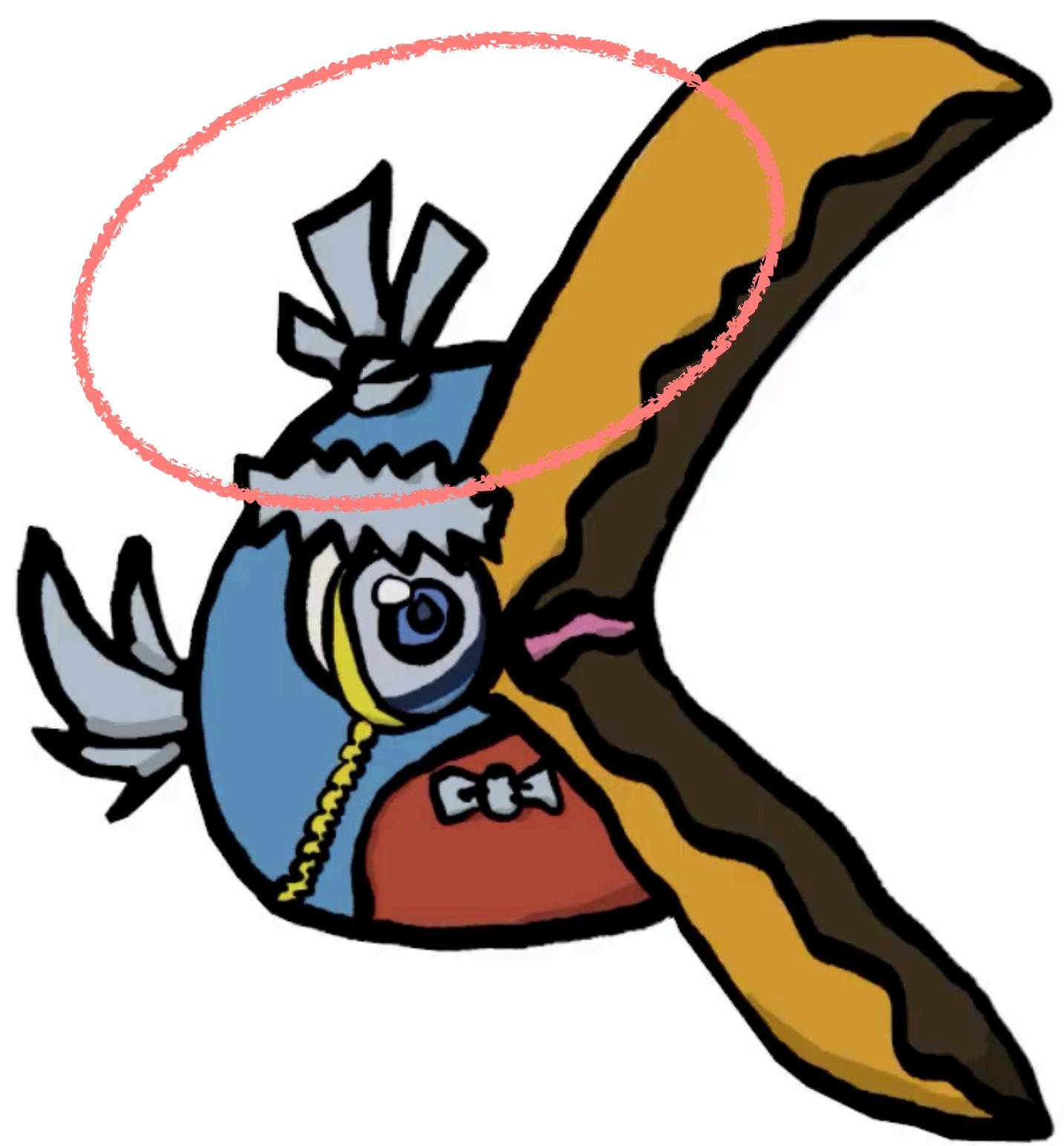


+ Our Complementary Secondary Motion

# Bone Handle

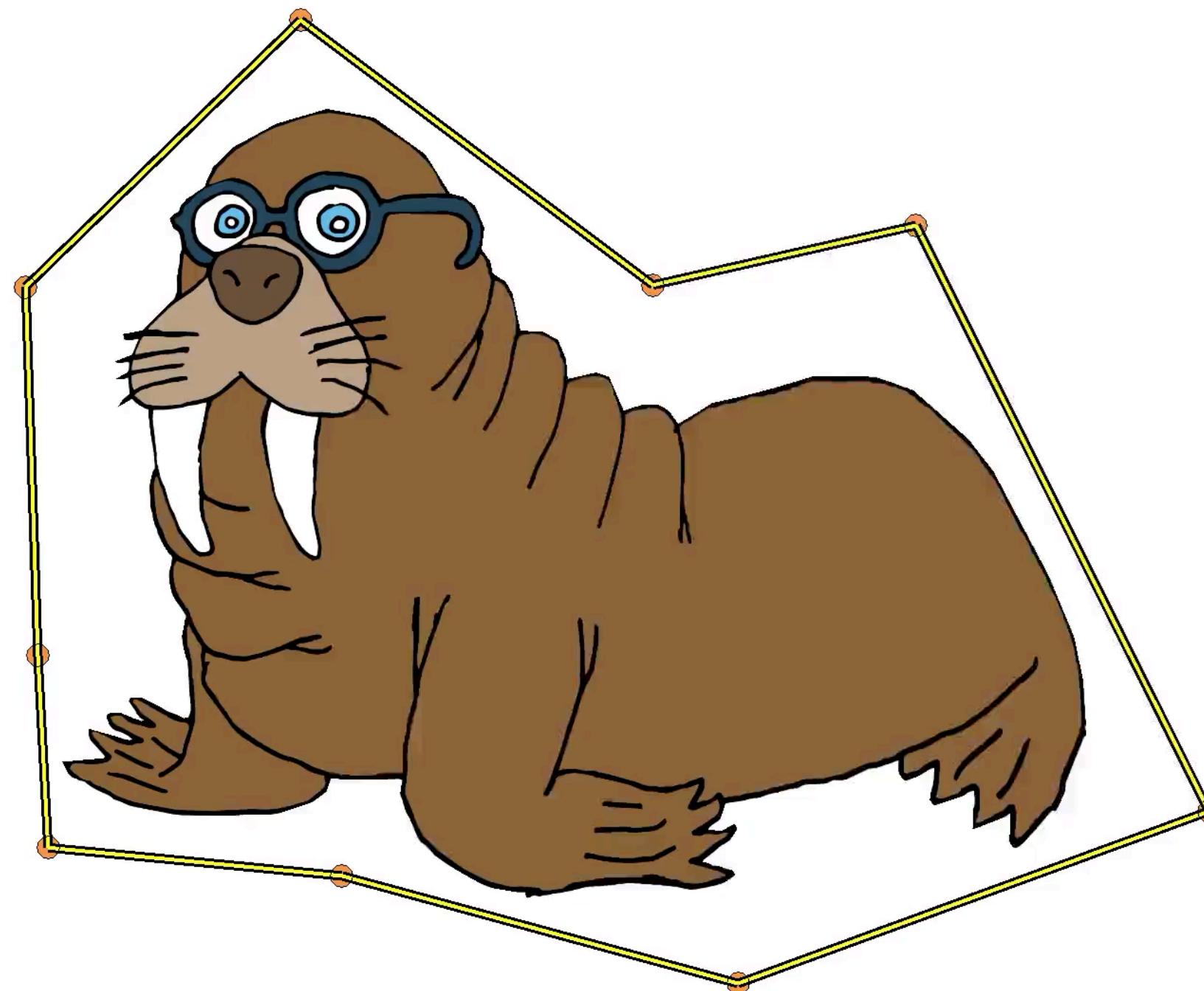


Input



Output

# Cage Handle

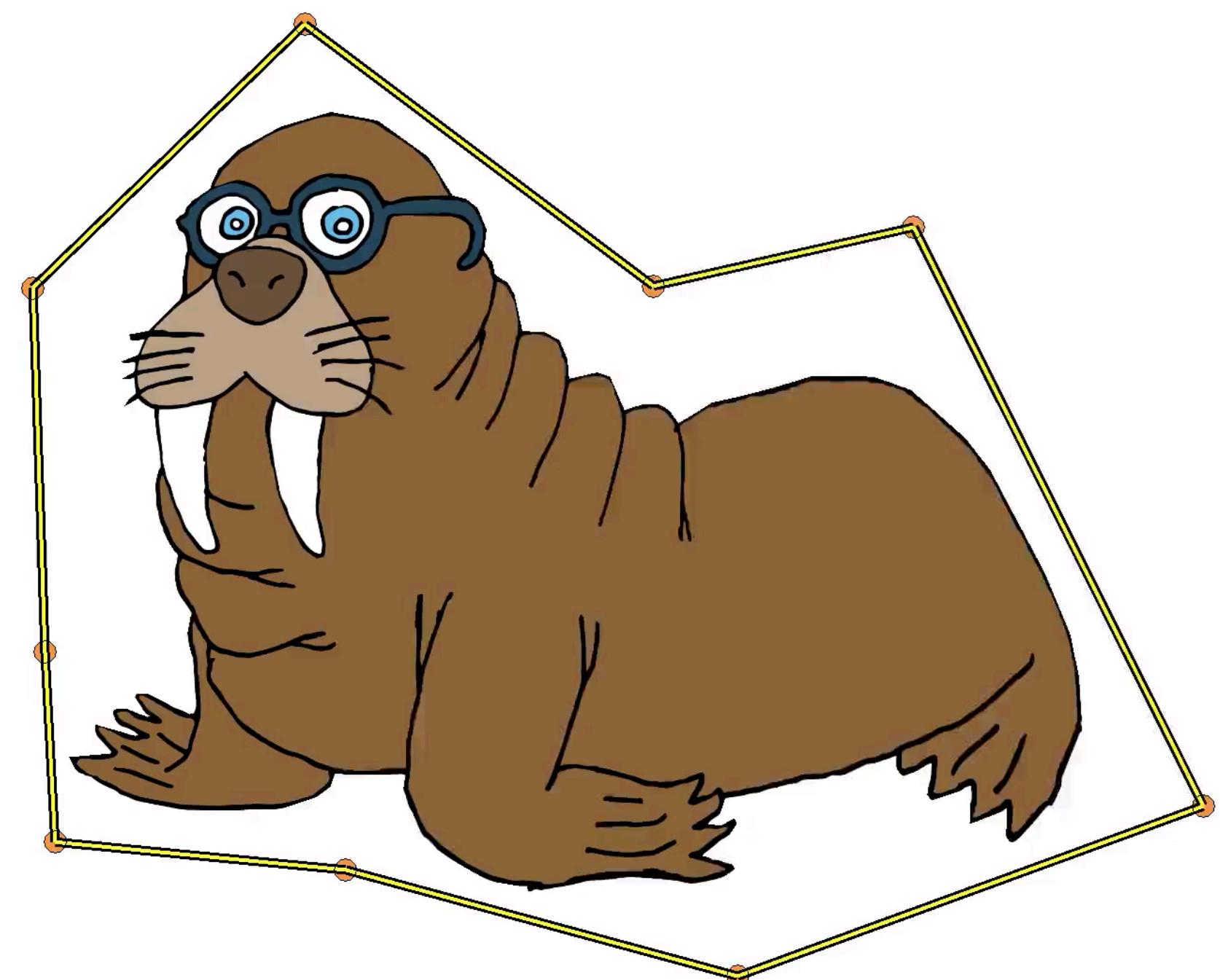


# Cage Handle



+ Our Complementary Secondary Motion

# Cage Handle

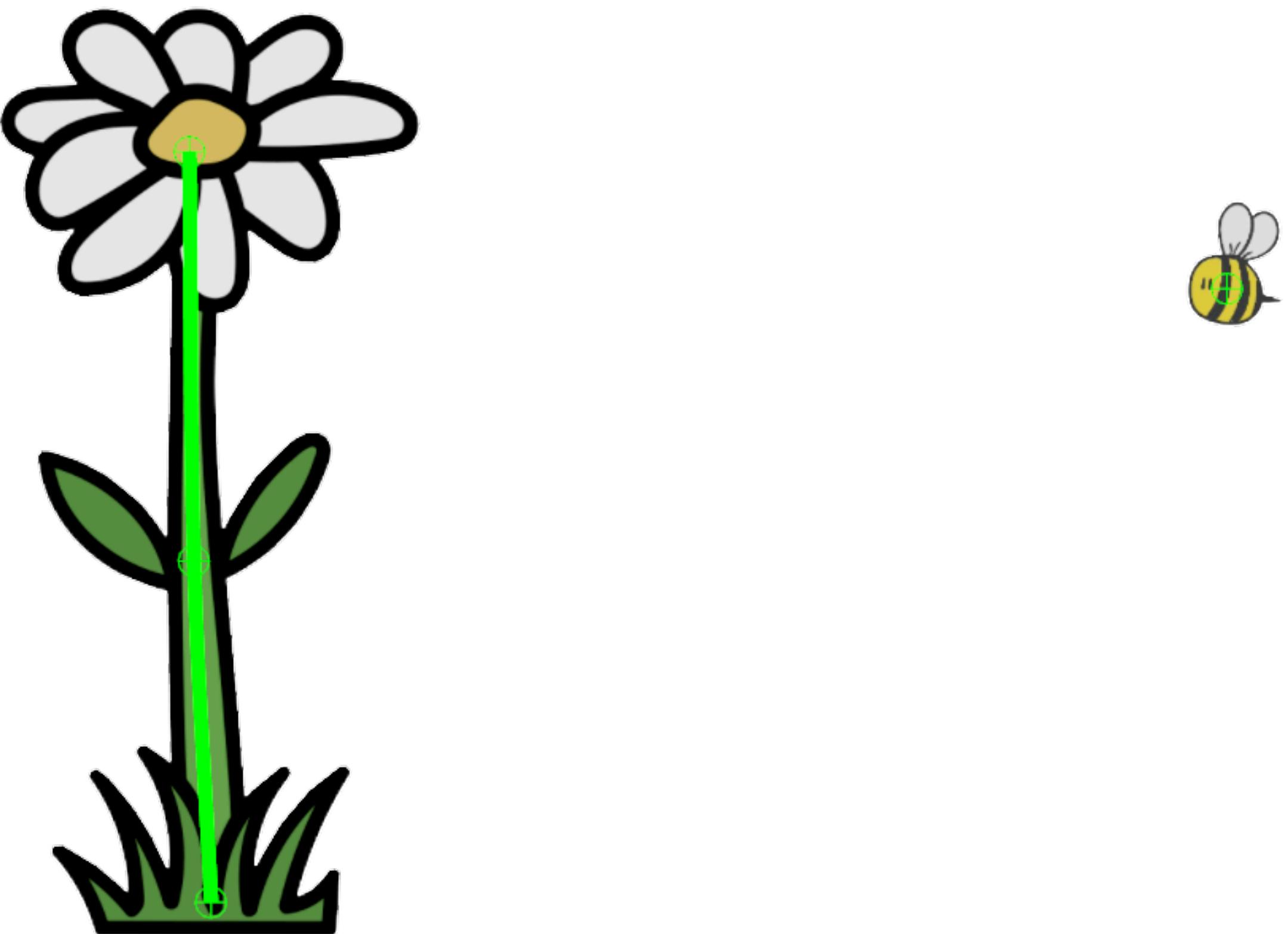


Input

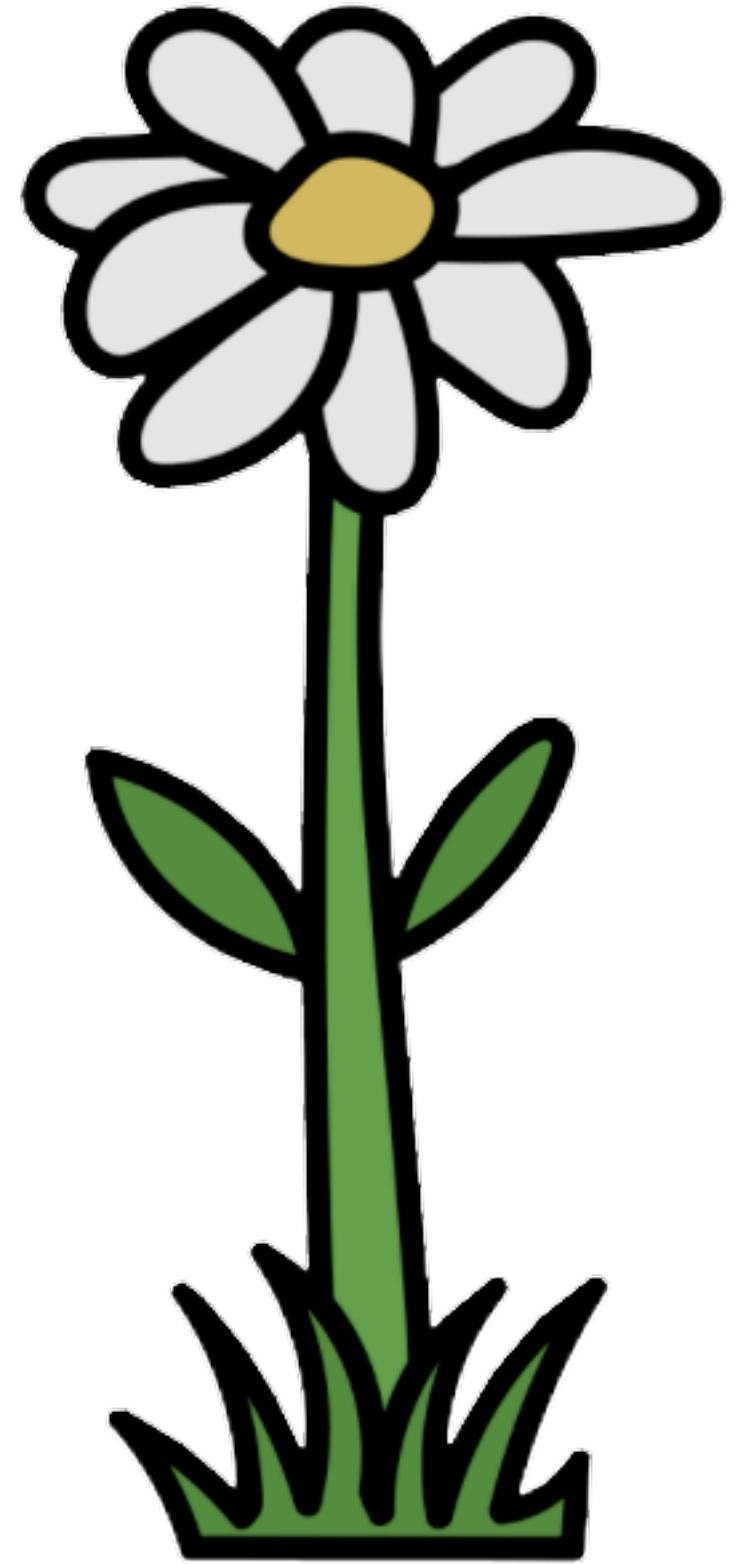


Output

# Wire Deformer

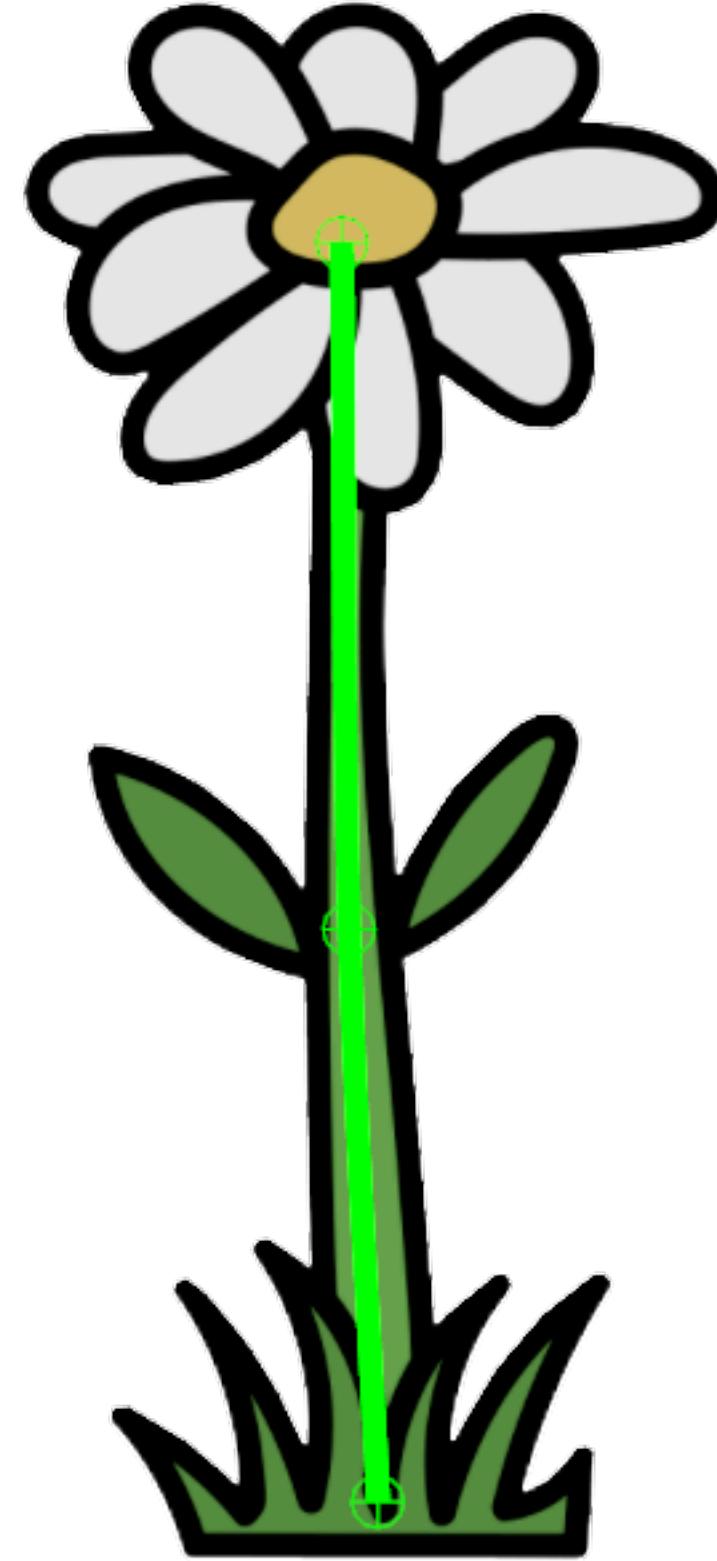


# Wire Deformer

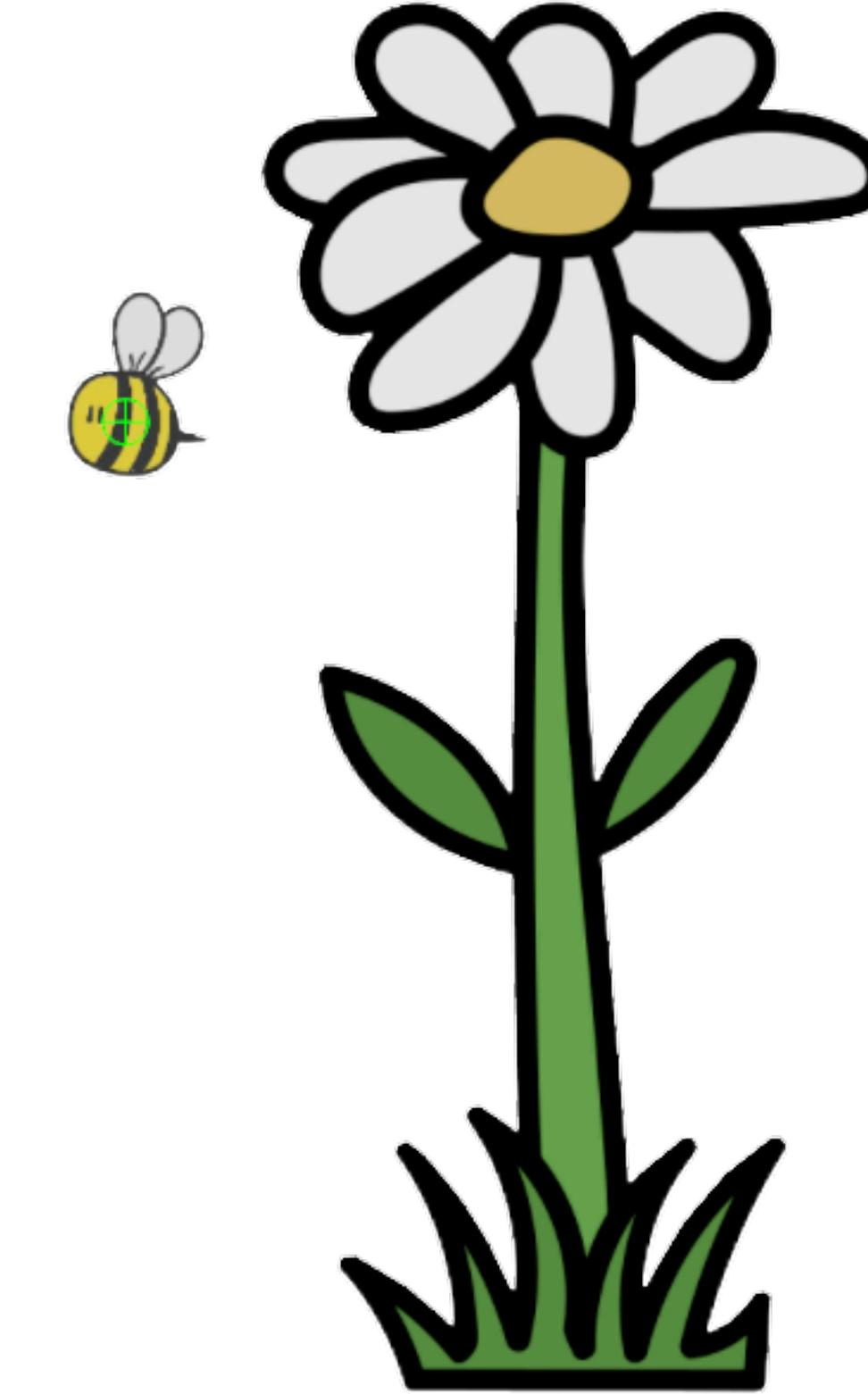


+ Our Complementary Secondary Motion

# Wire Deformer



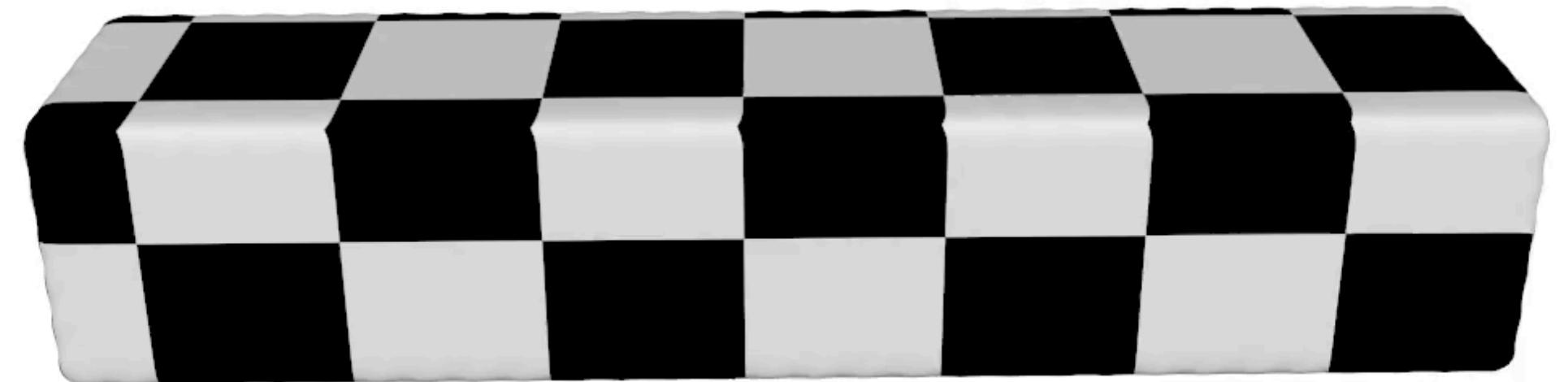
Input



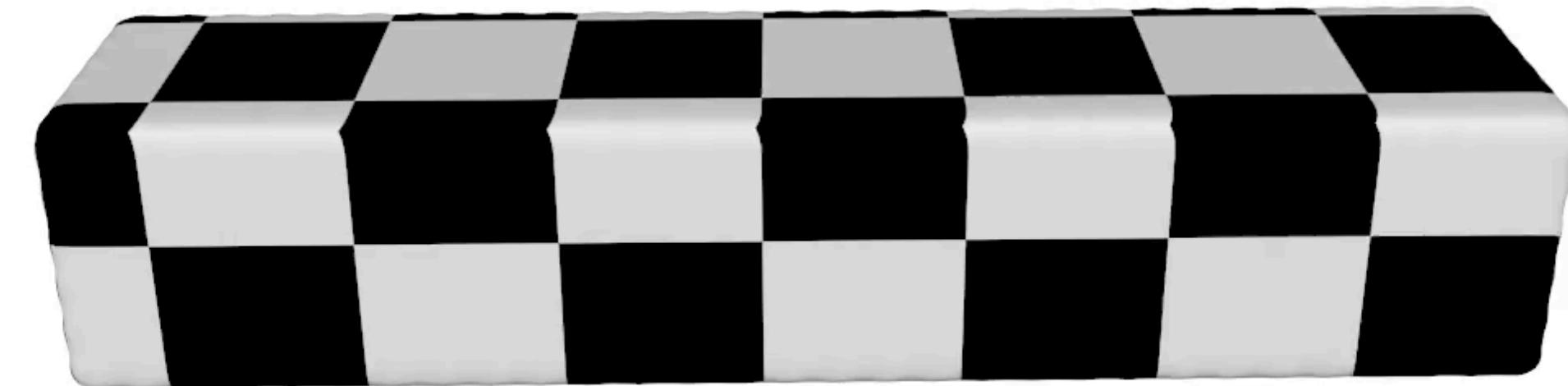
Output



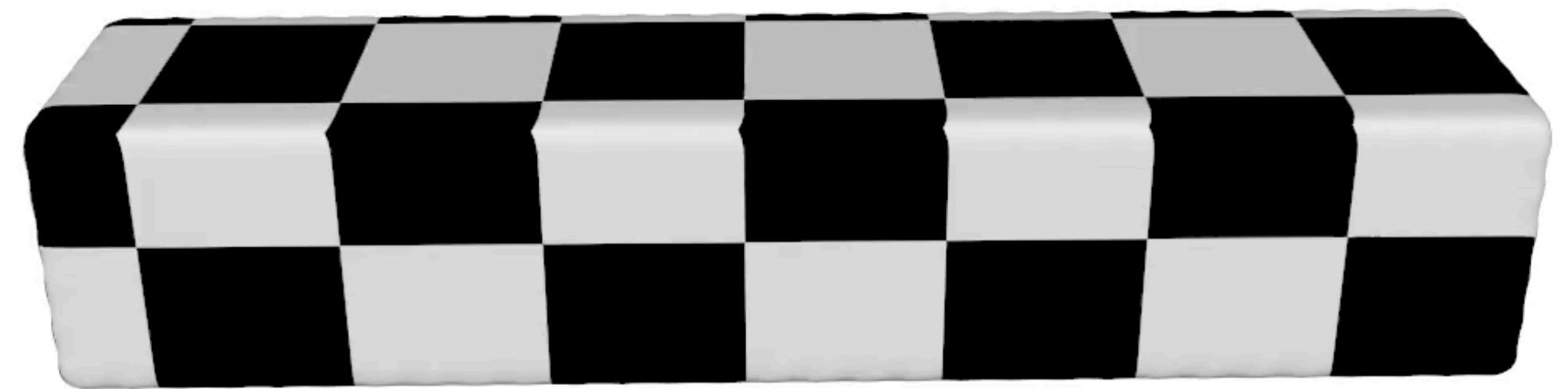
# Dual Quaternion Skinning



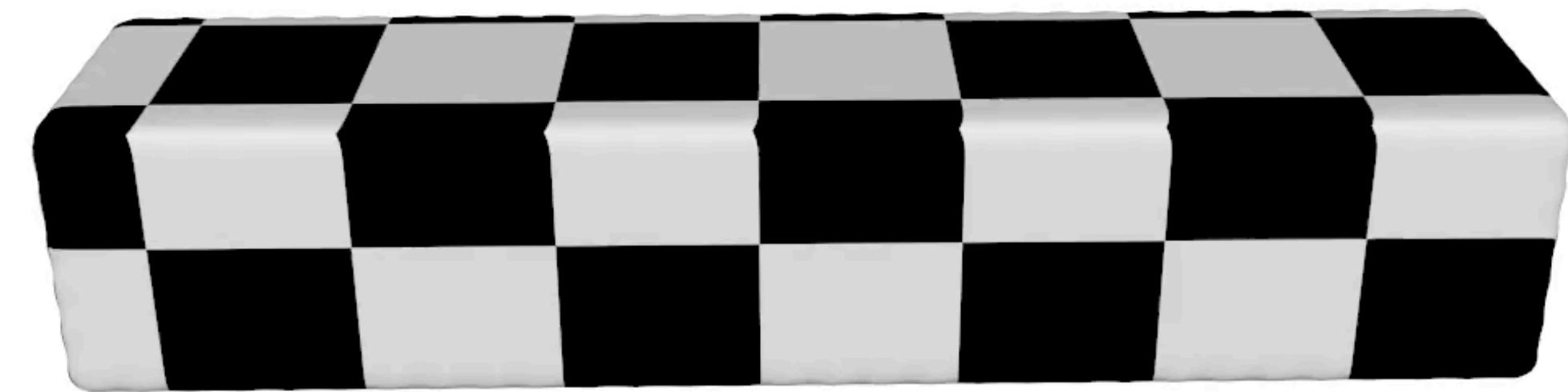
Input



Output

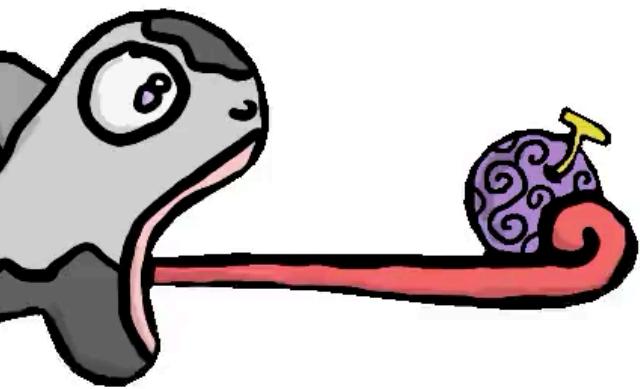


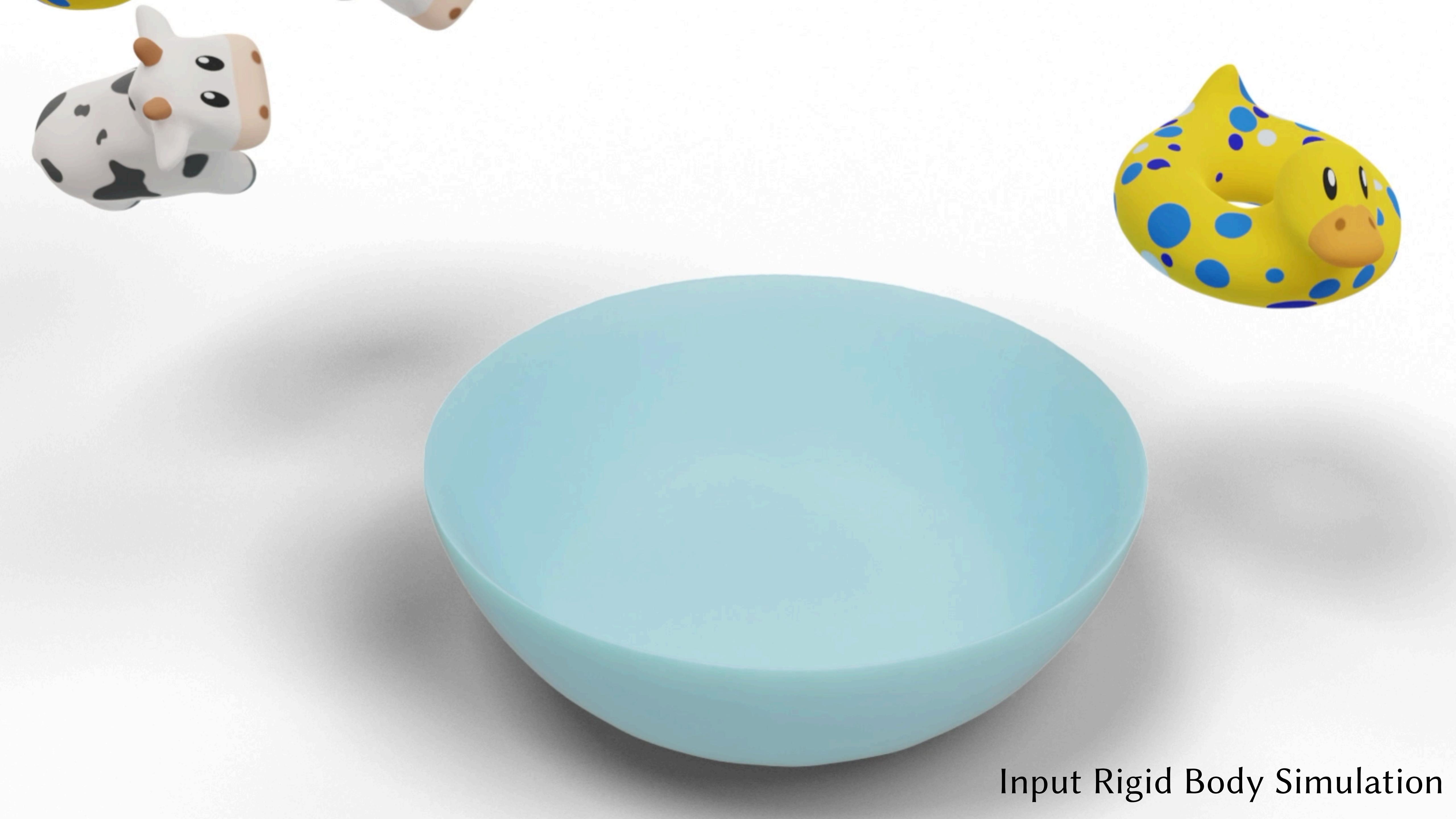
Input



Output

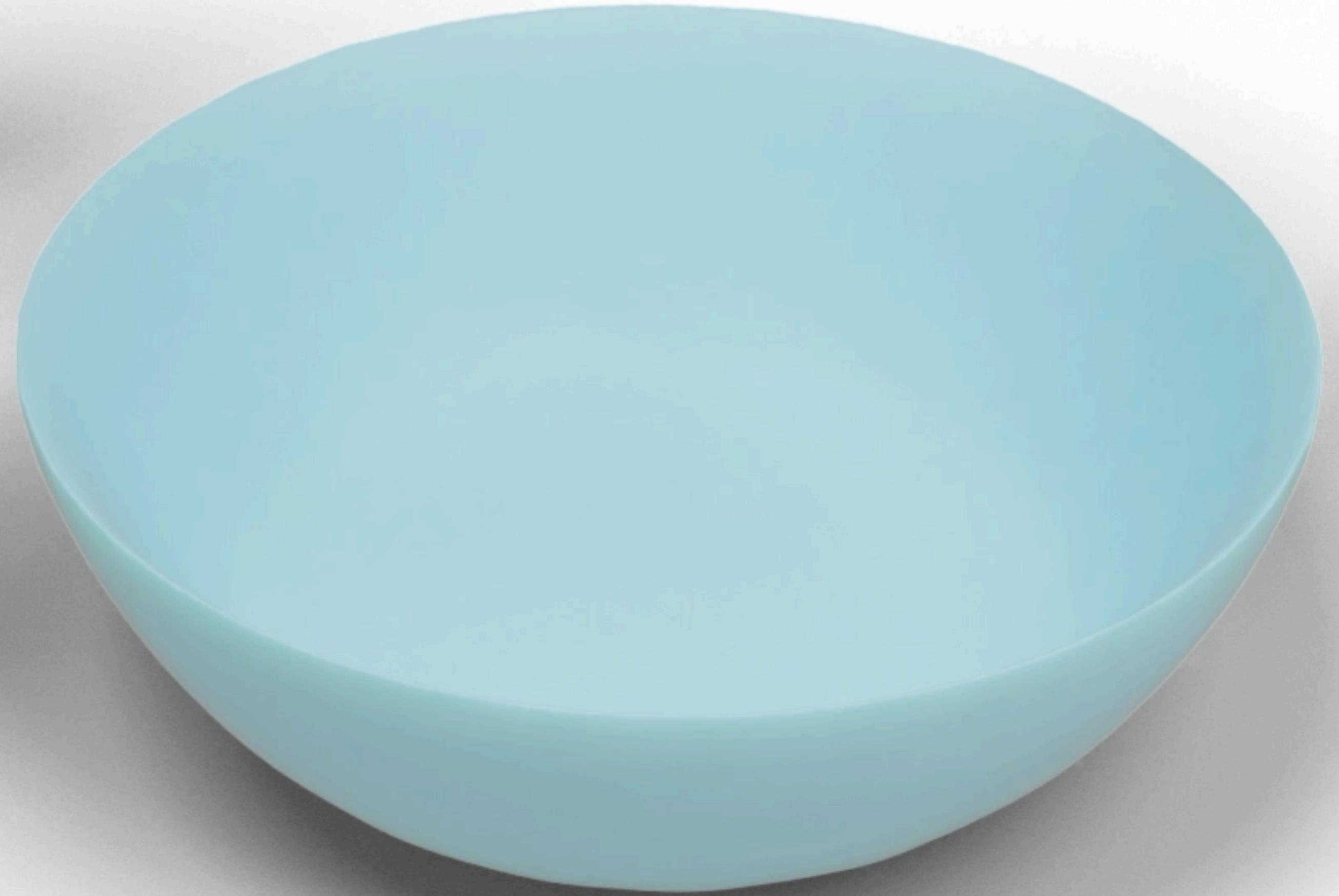
# Keyframe Animation





Input Rigid Body Simulation

Ours



2x slower



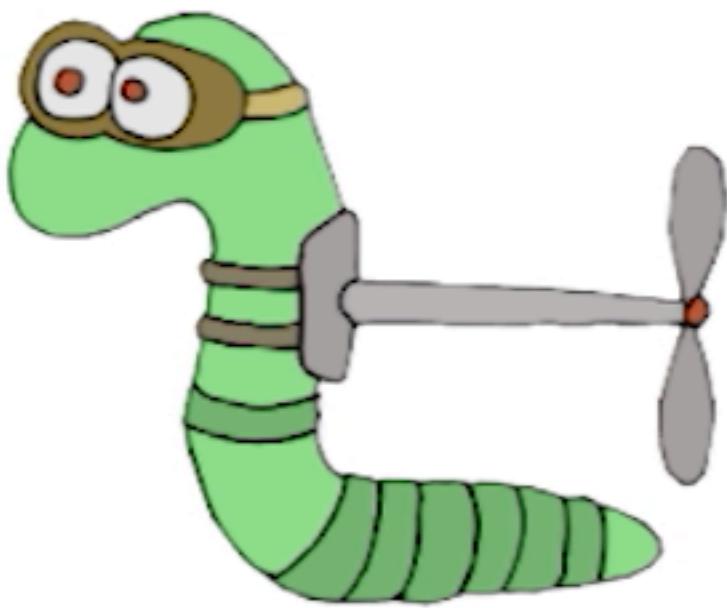
2x slower



# Material & Environment

# Heterogeneous Material

Input



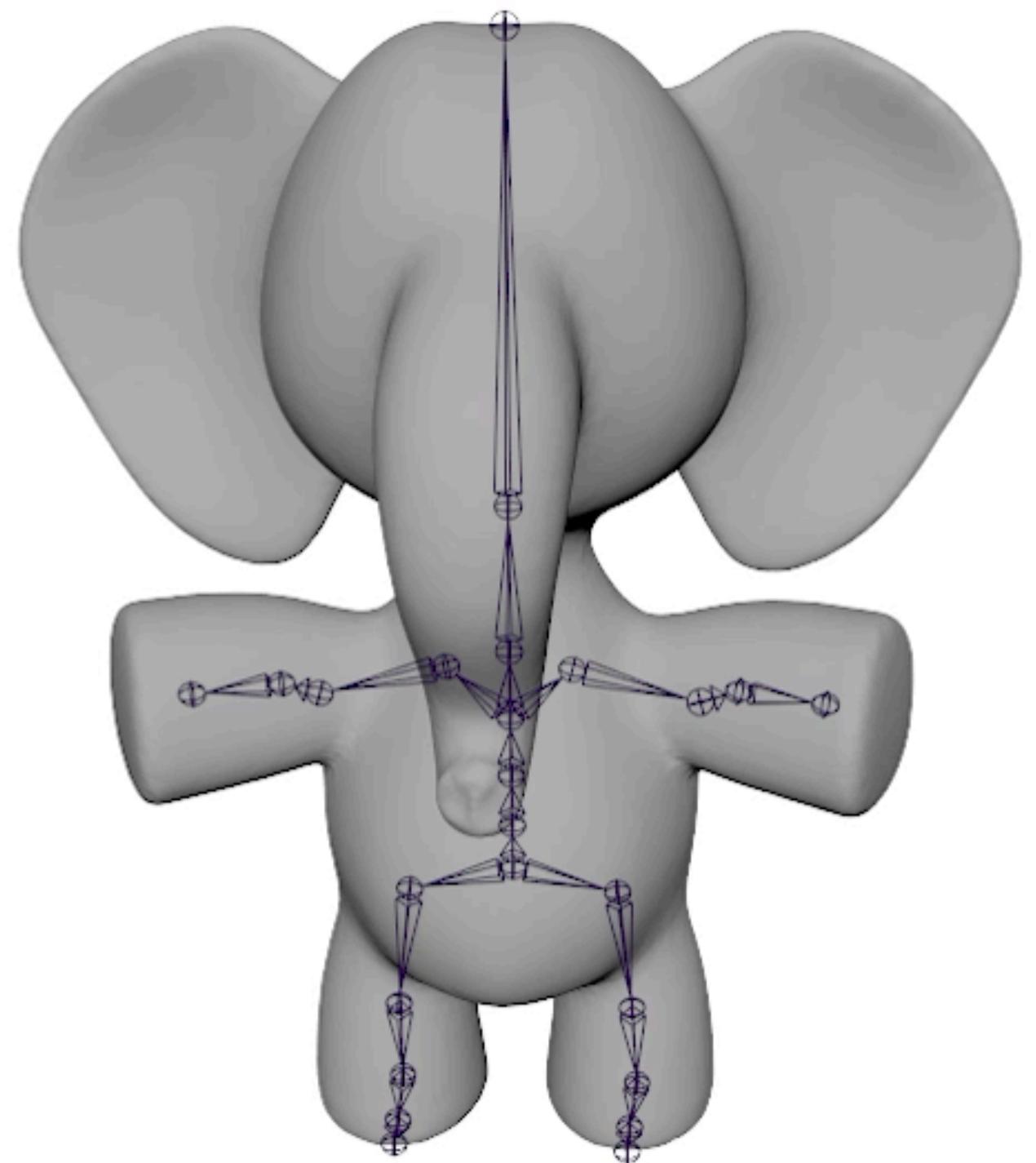
All soft material



Soft body,  
Stiff propellor



# Mocap Data

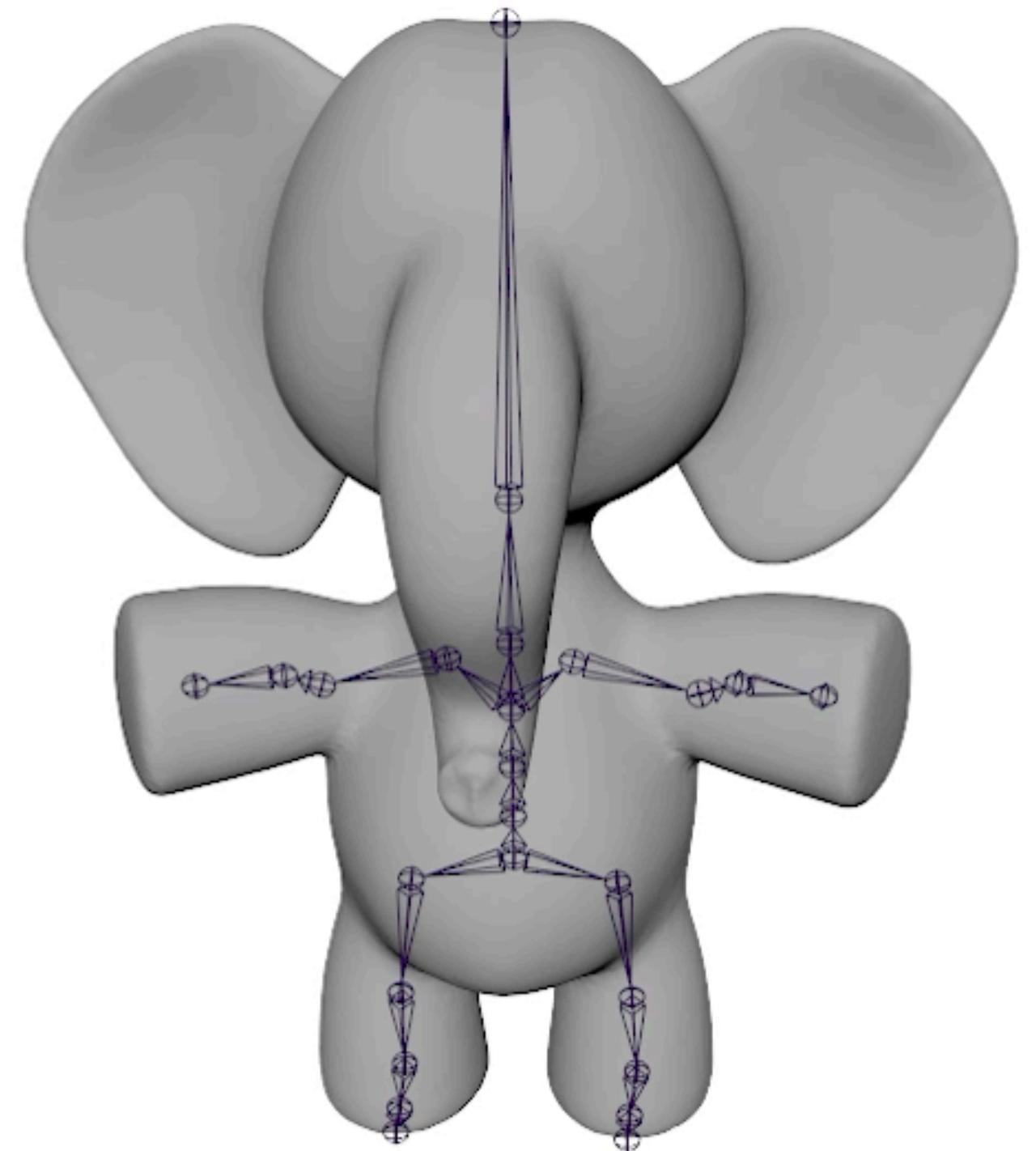


Input



Output

# Softer Material

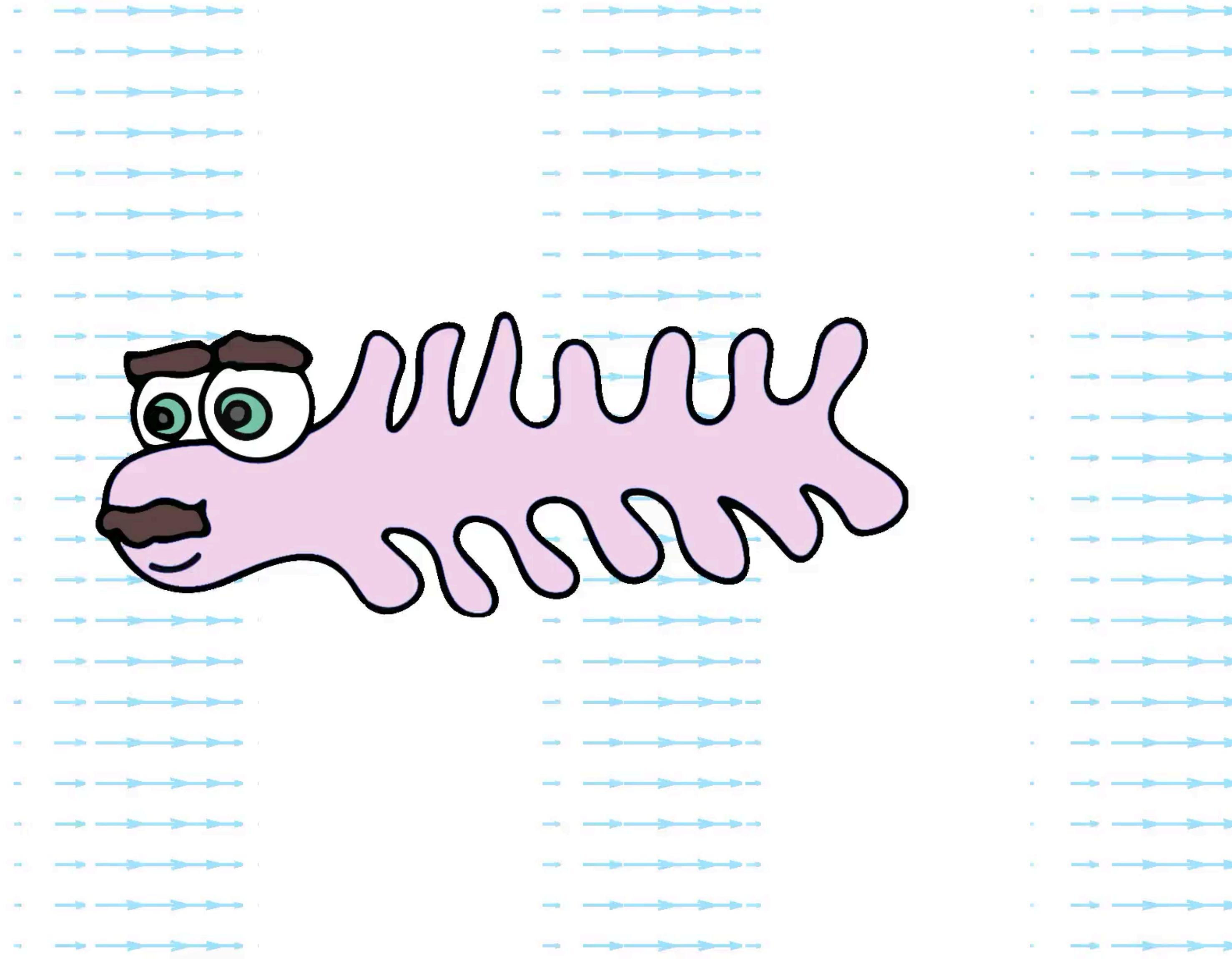


Input

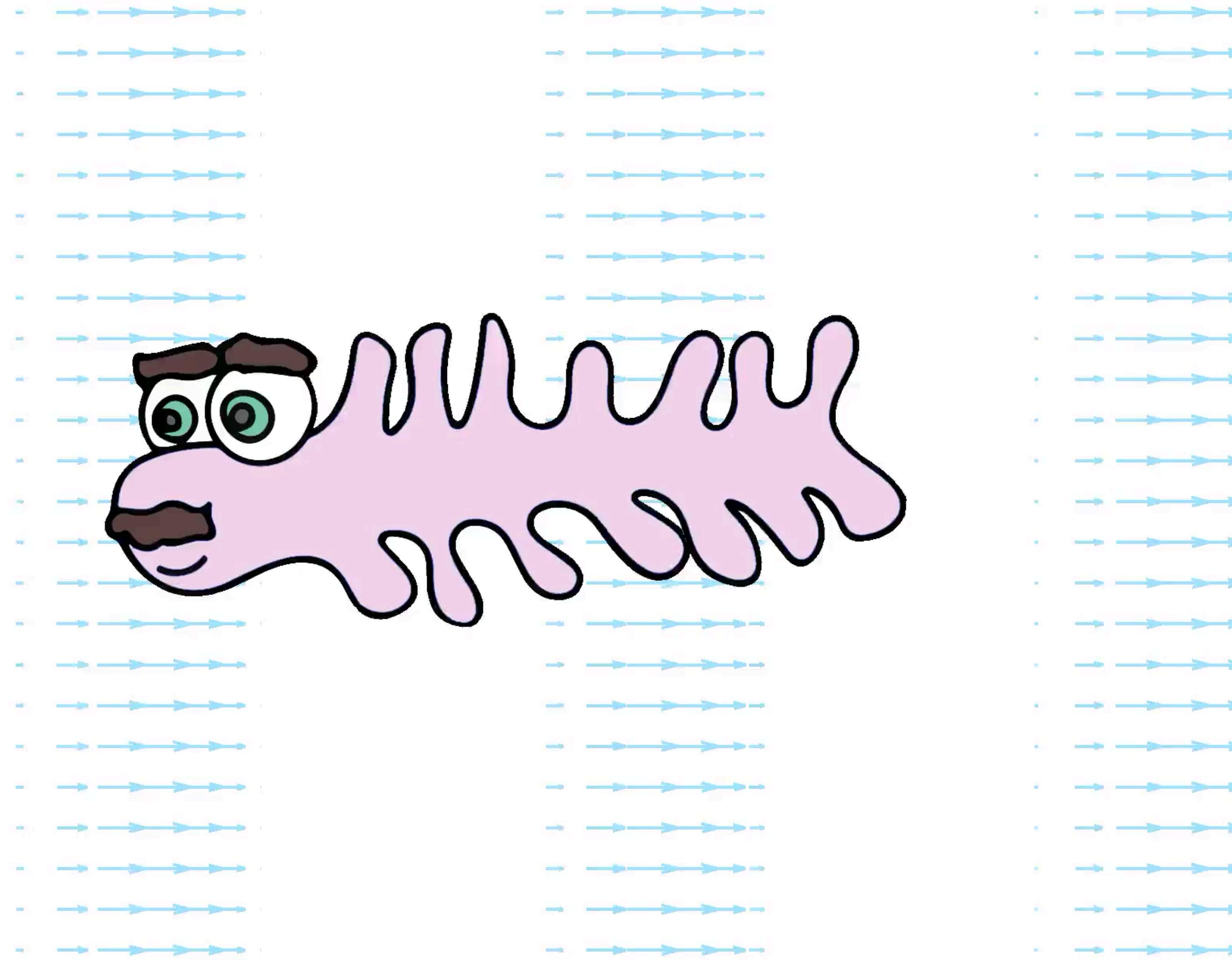


Output

# Background Force

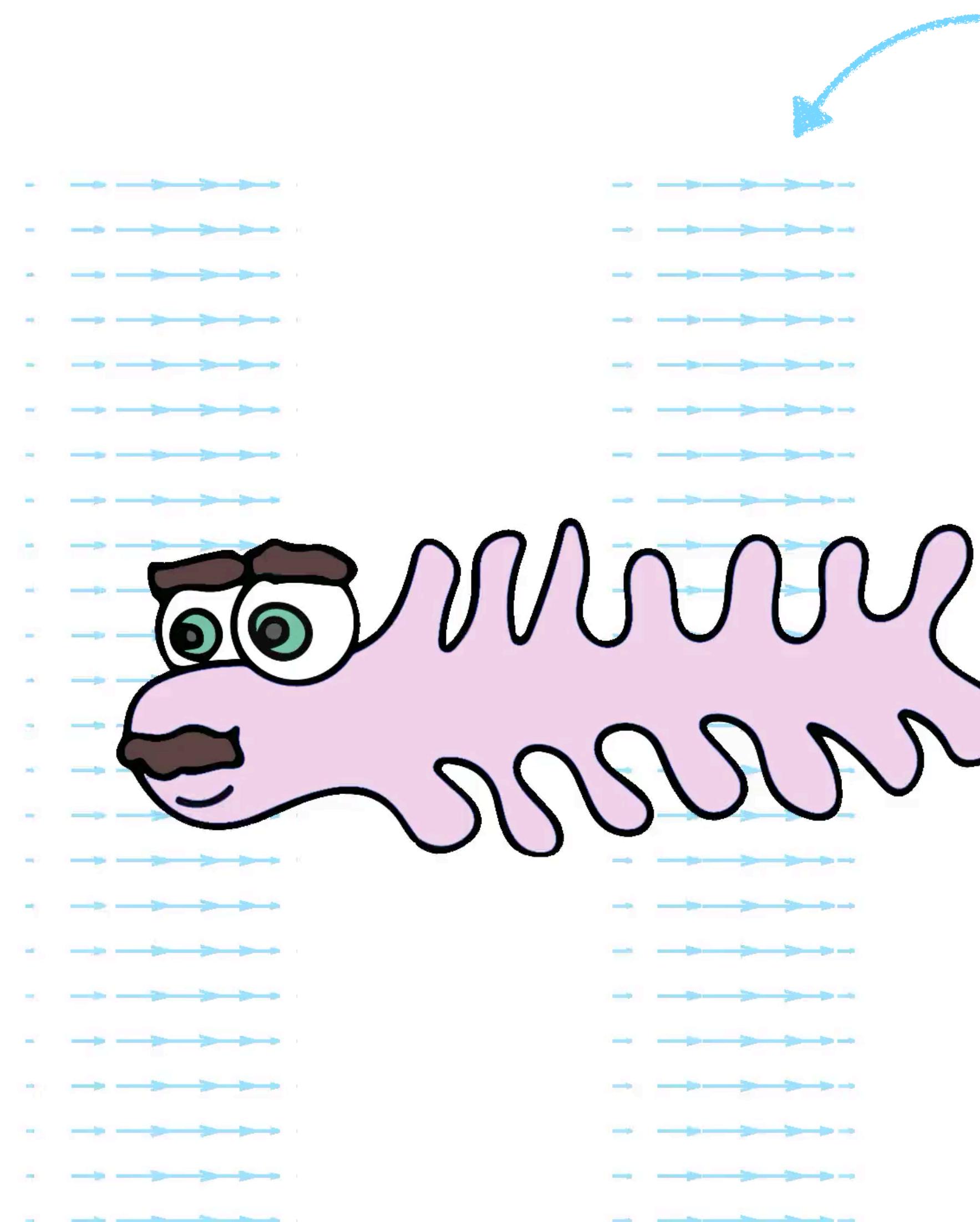


# Background Force

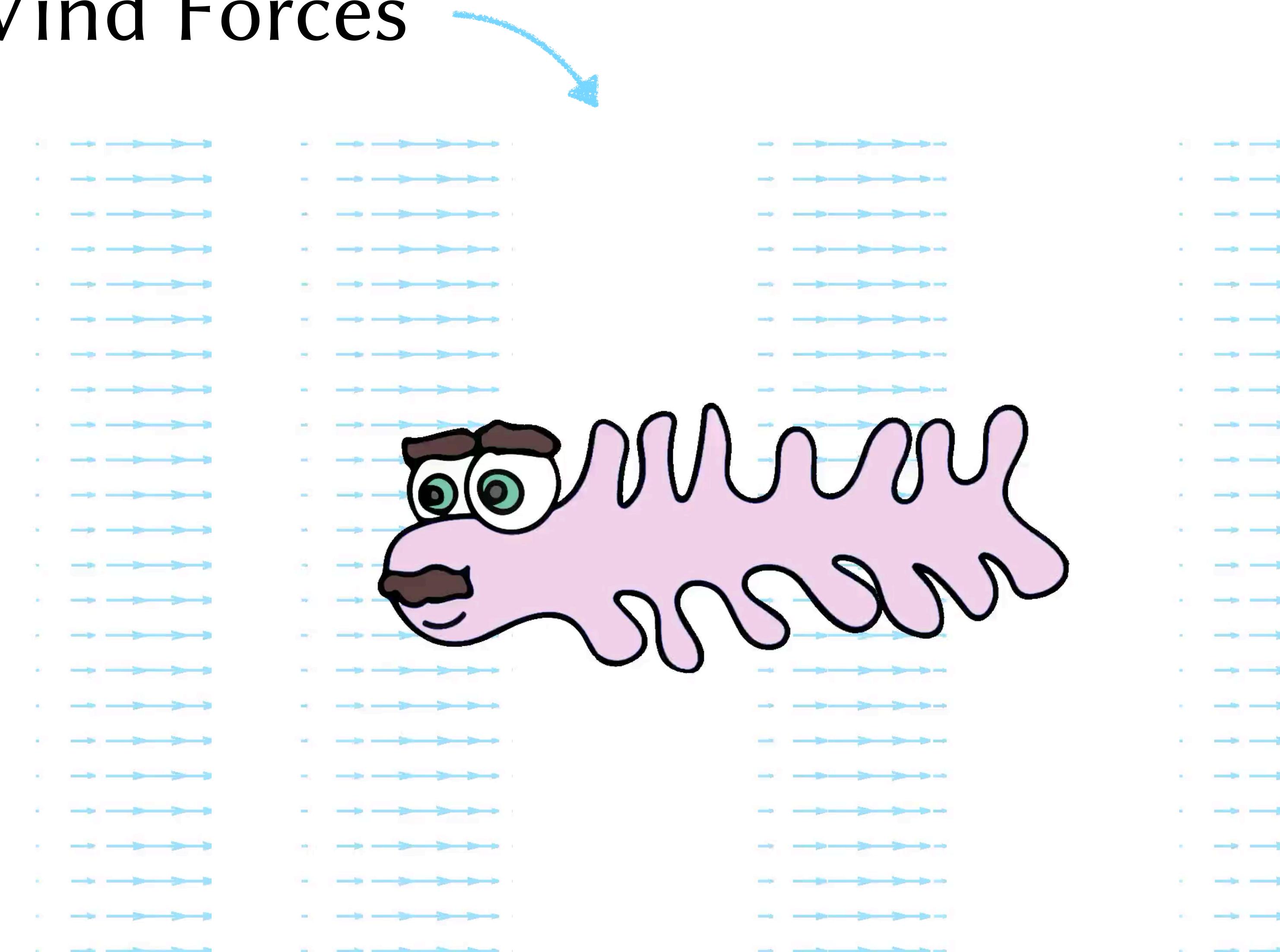


+ Our Complementary Secondary Motion

# Wind Forces

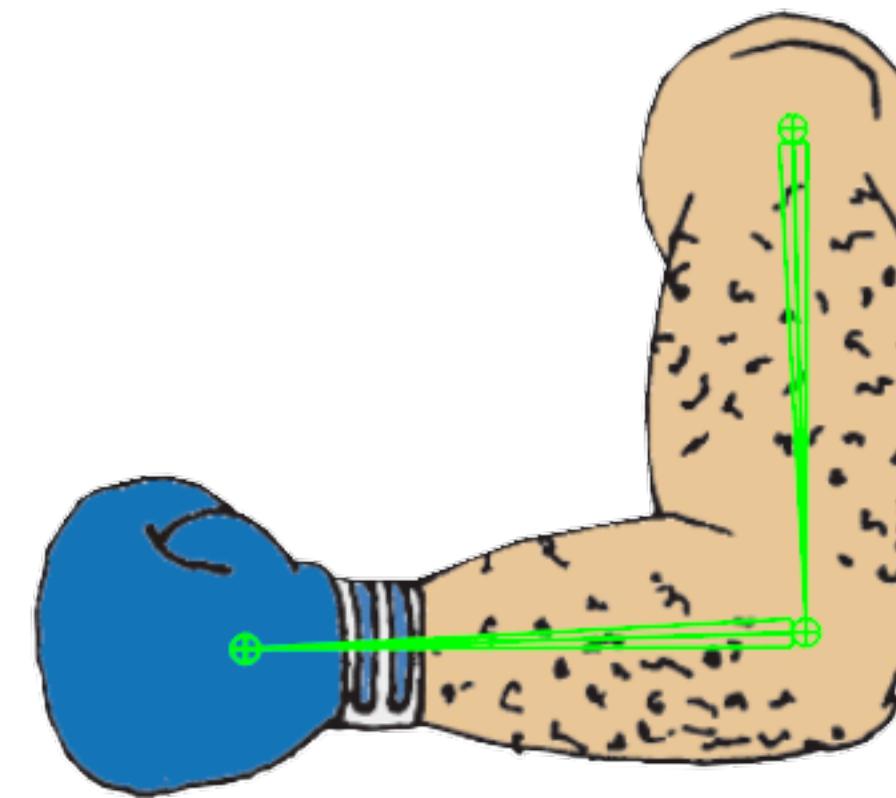
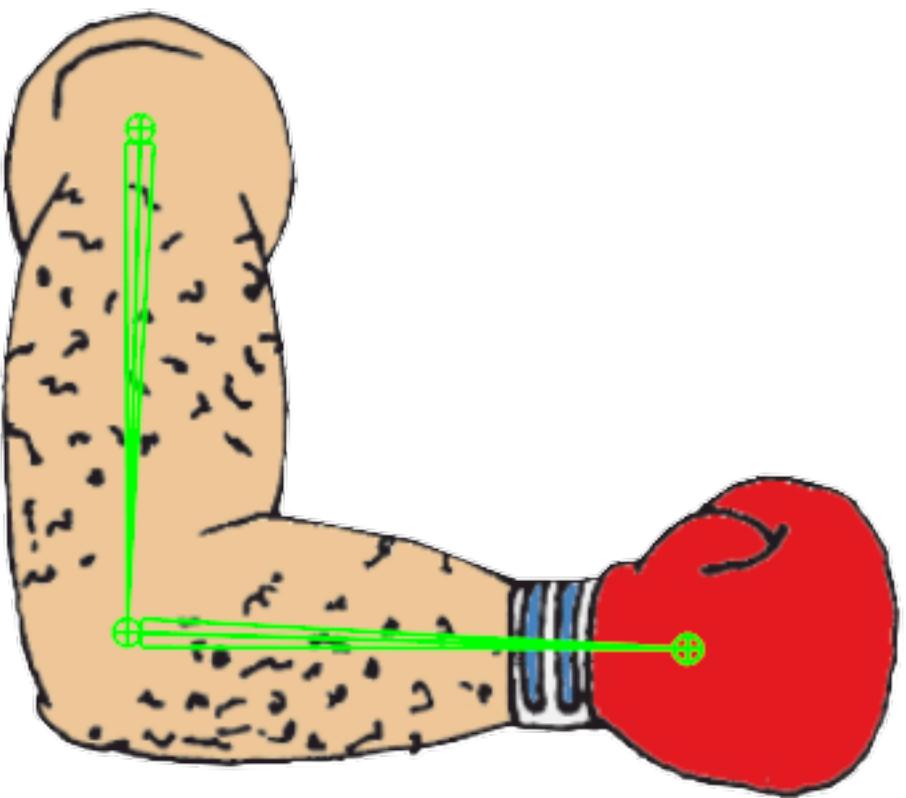


Input

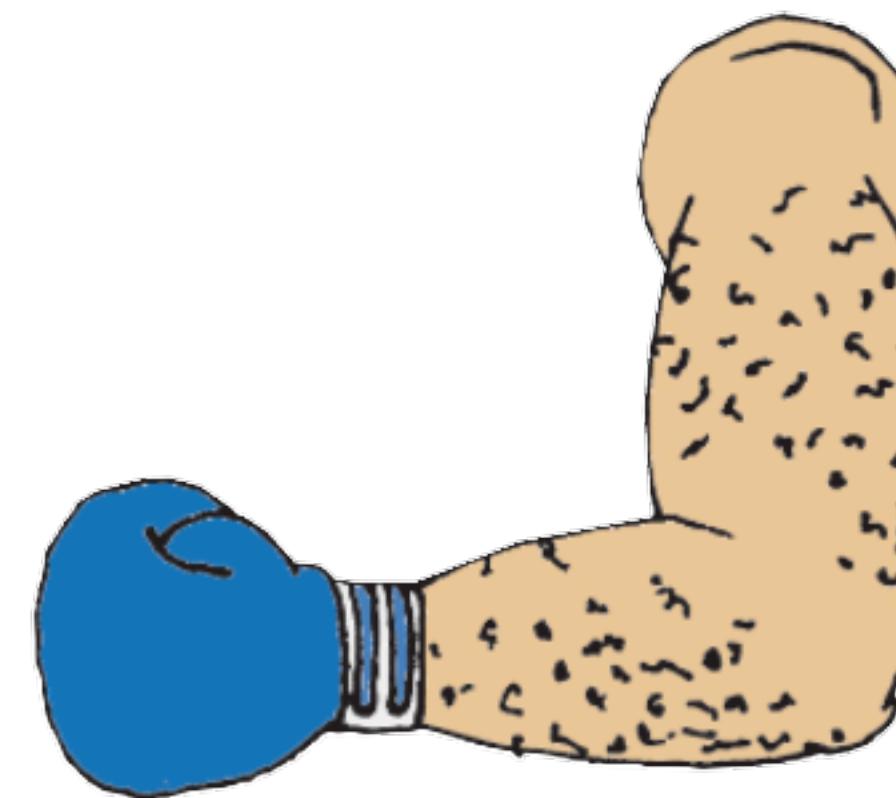
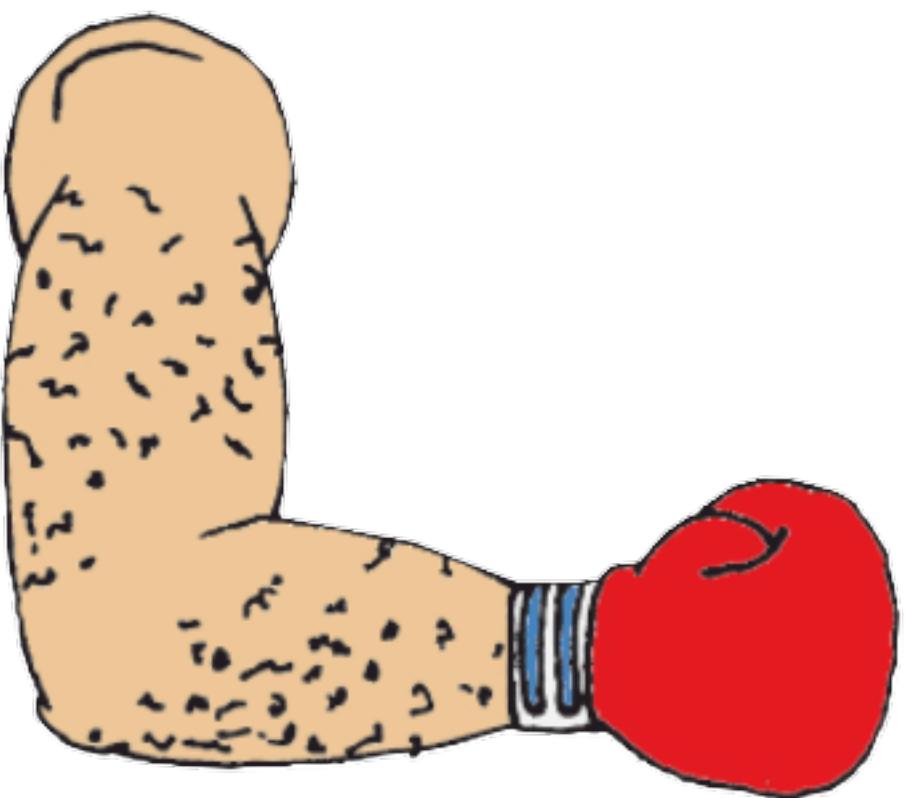


Output

# Contact forces



Input



Output

# External Forces



**Input**

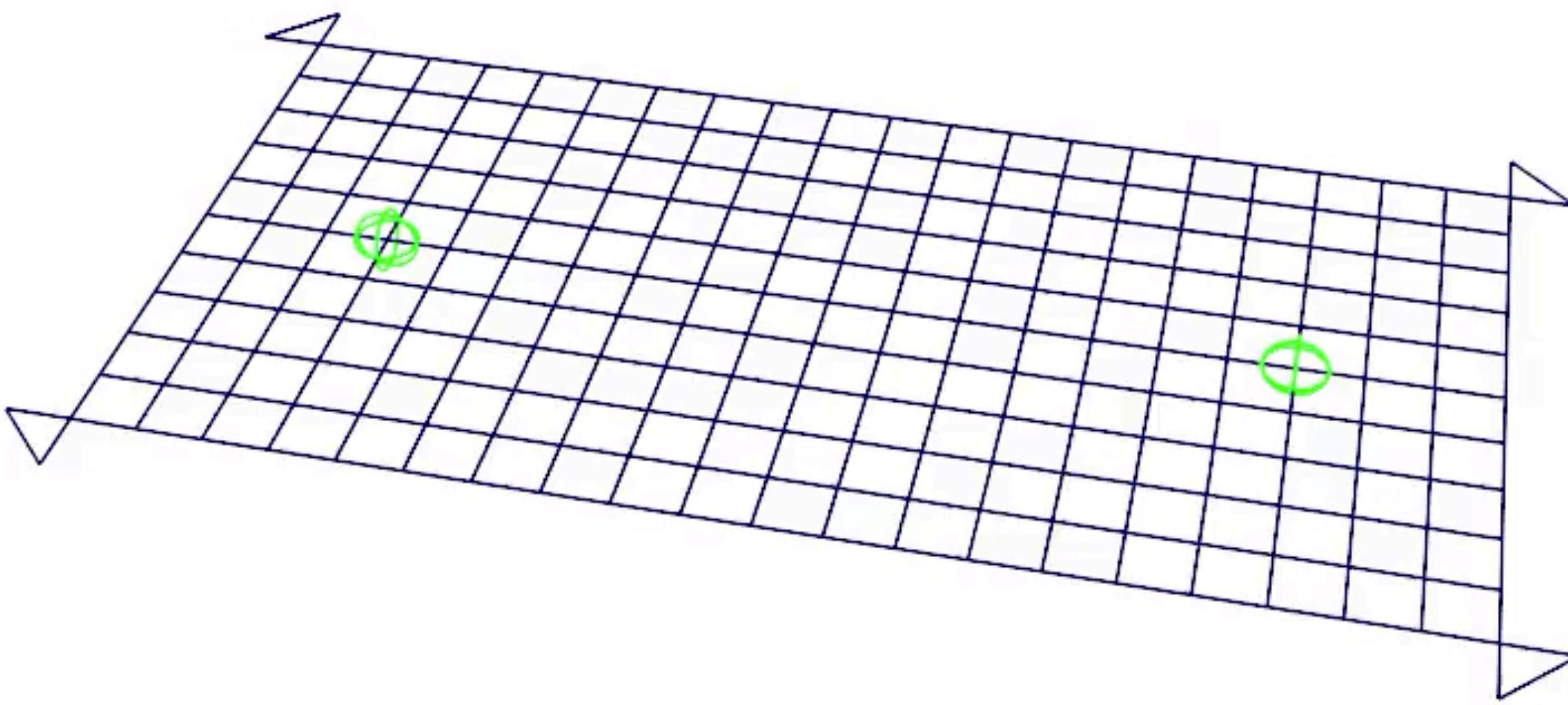


**No Collision**



**Collision**

# Cloth Simulation

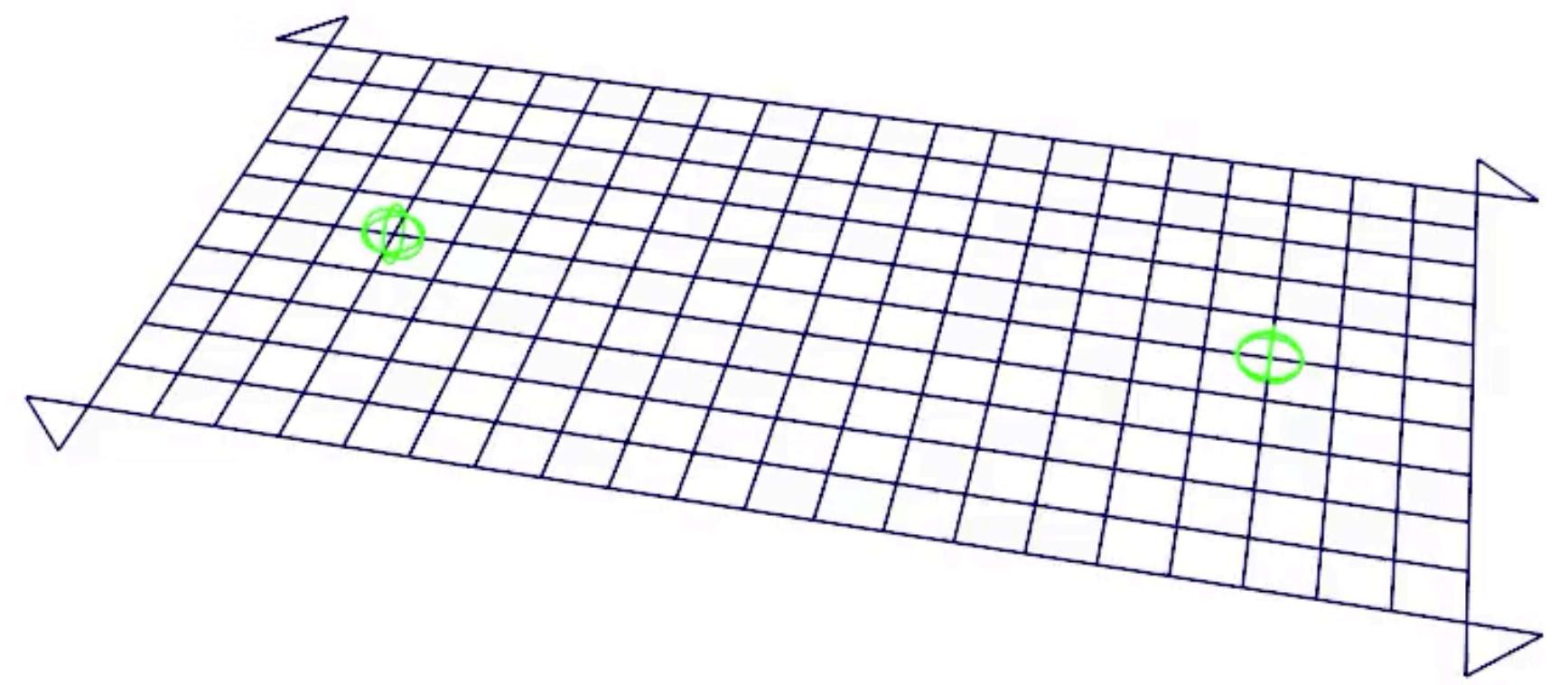


# Cloth Simulation



+ Our Complementary Secondary Motion

# Cloth Simulation



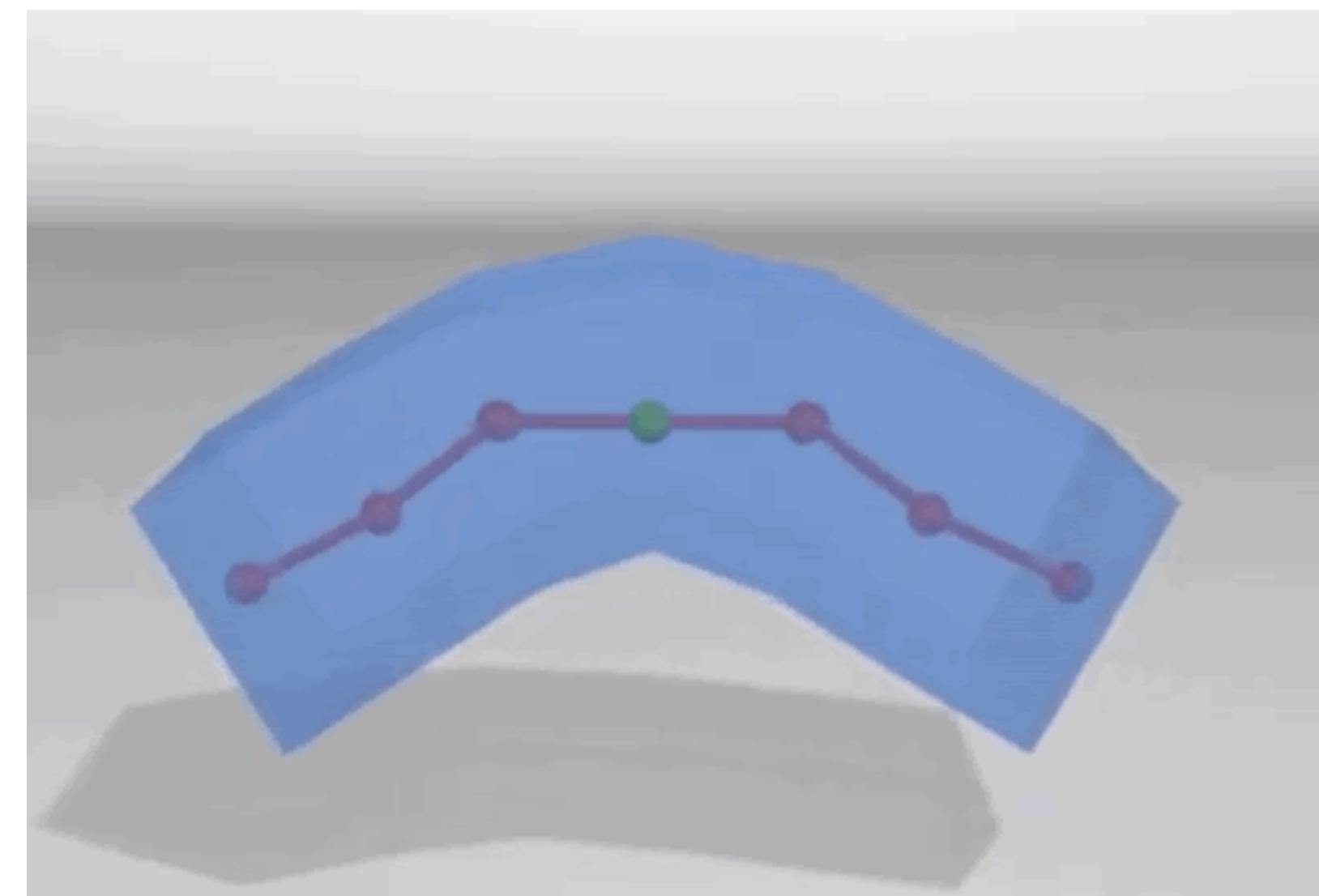
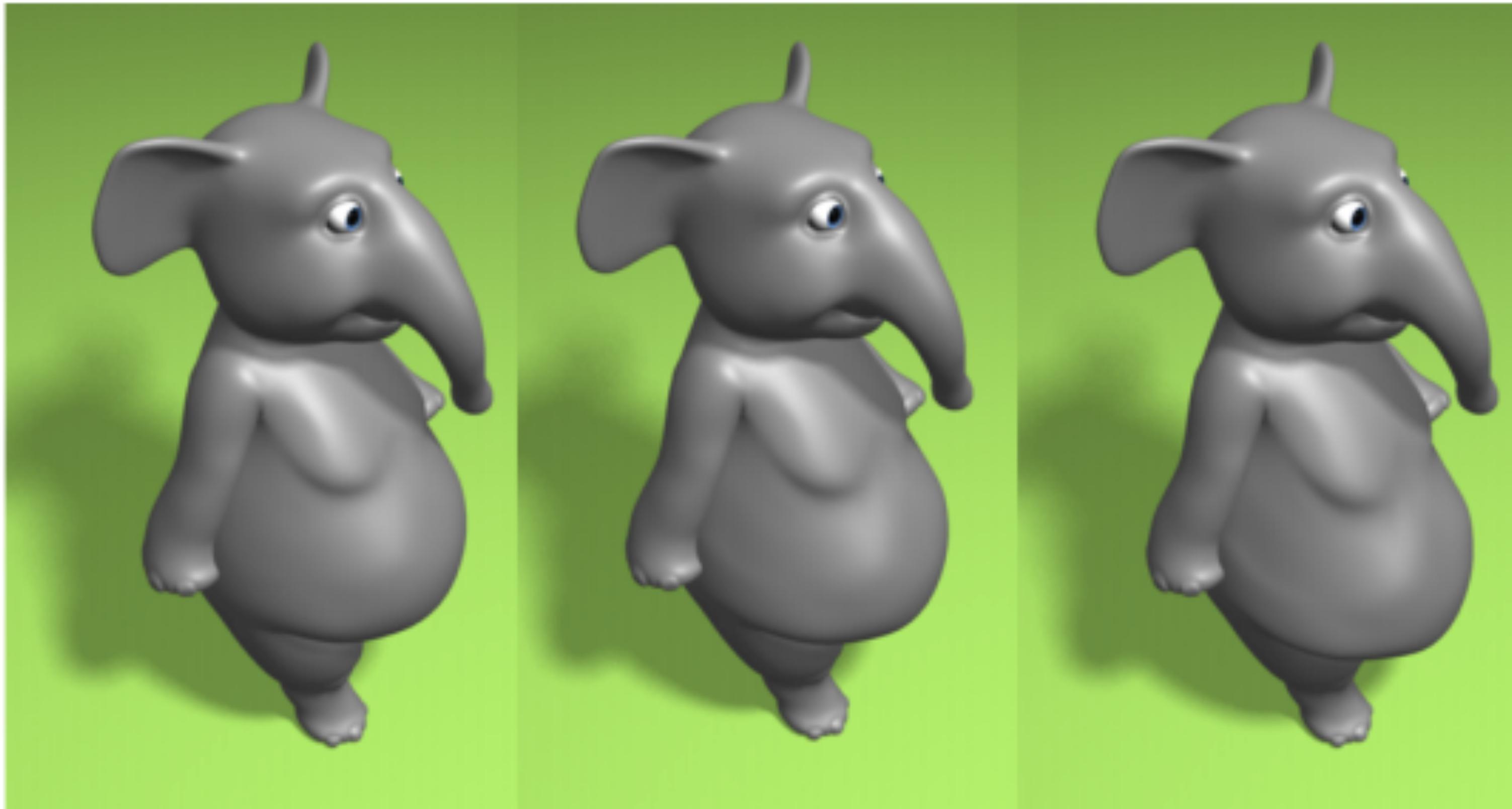
Input



Output

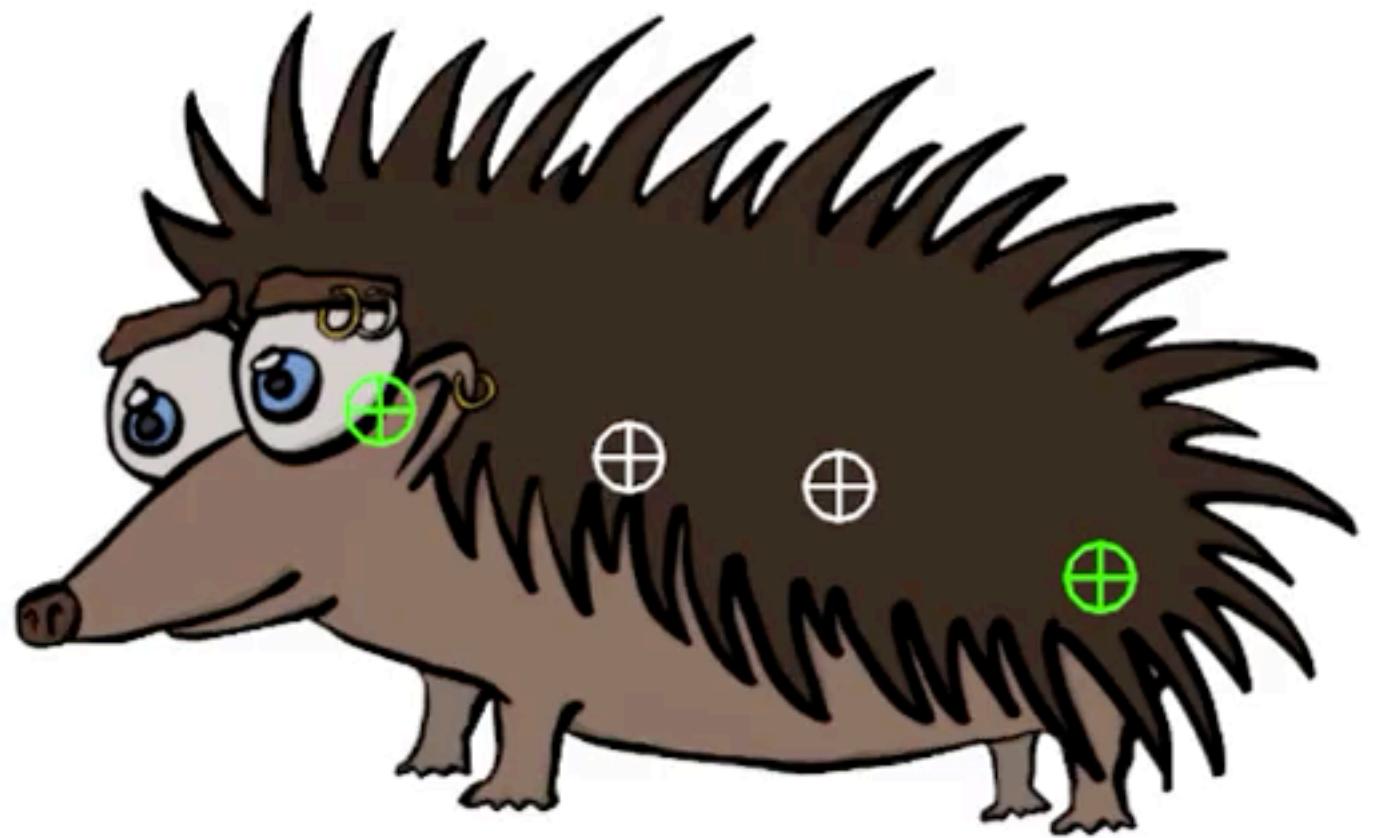
# Comparison with Previous Methods

# Comparing with Rig Space Physics

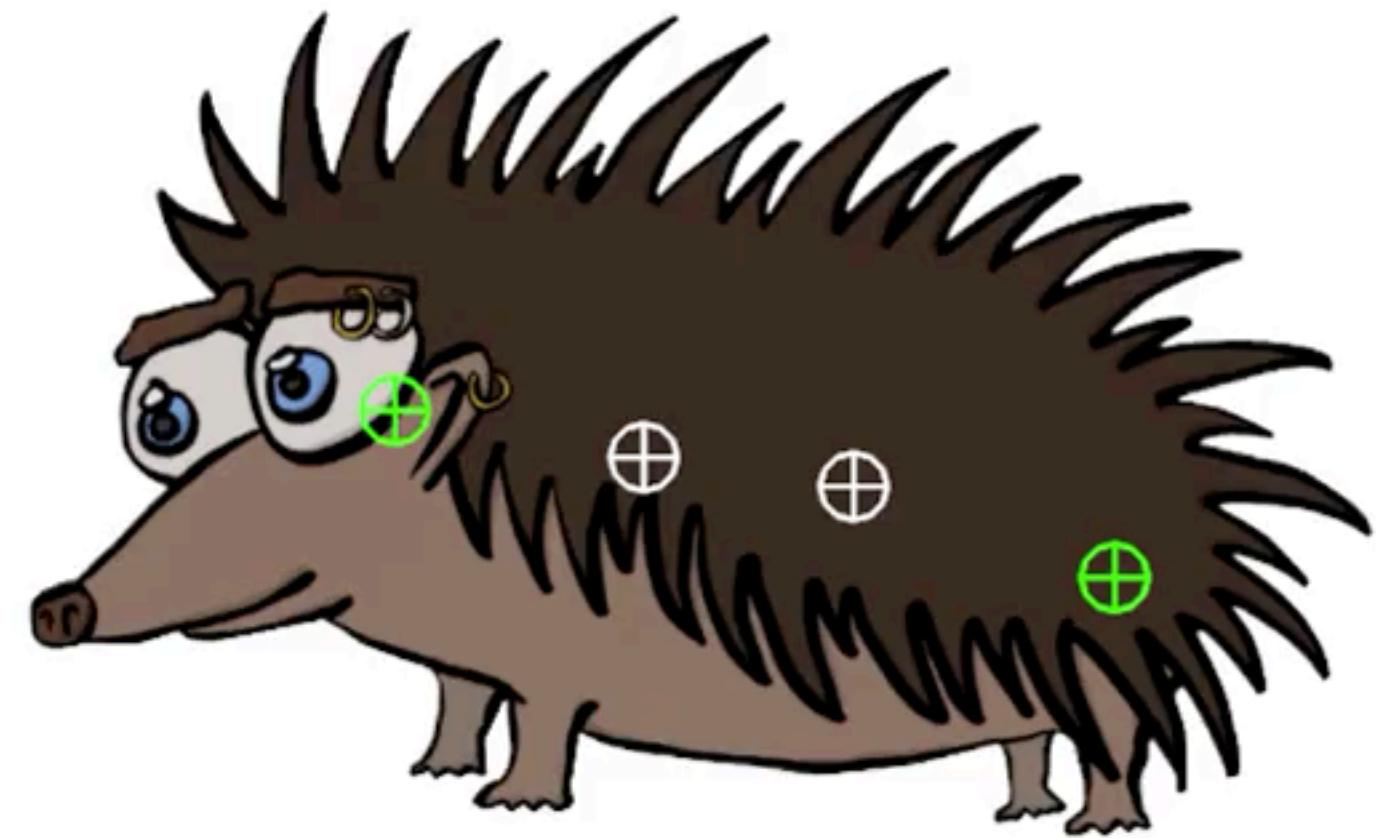


[Hahn et al. 2012]

# Comparing with Rig Space Physics

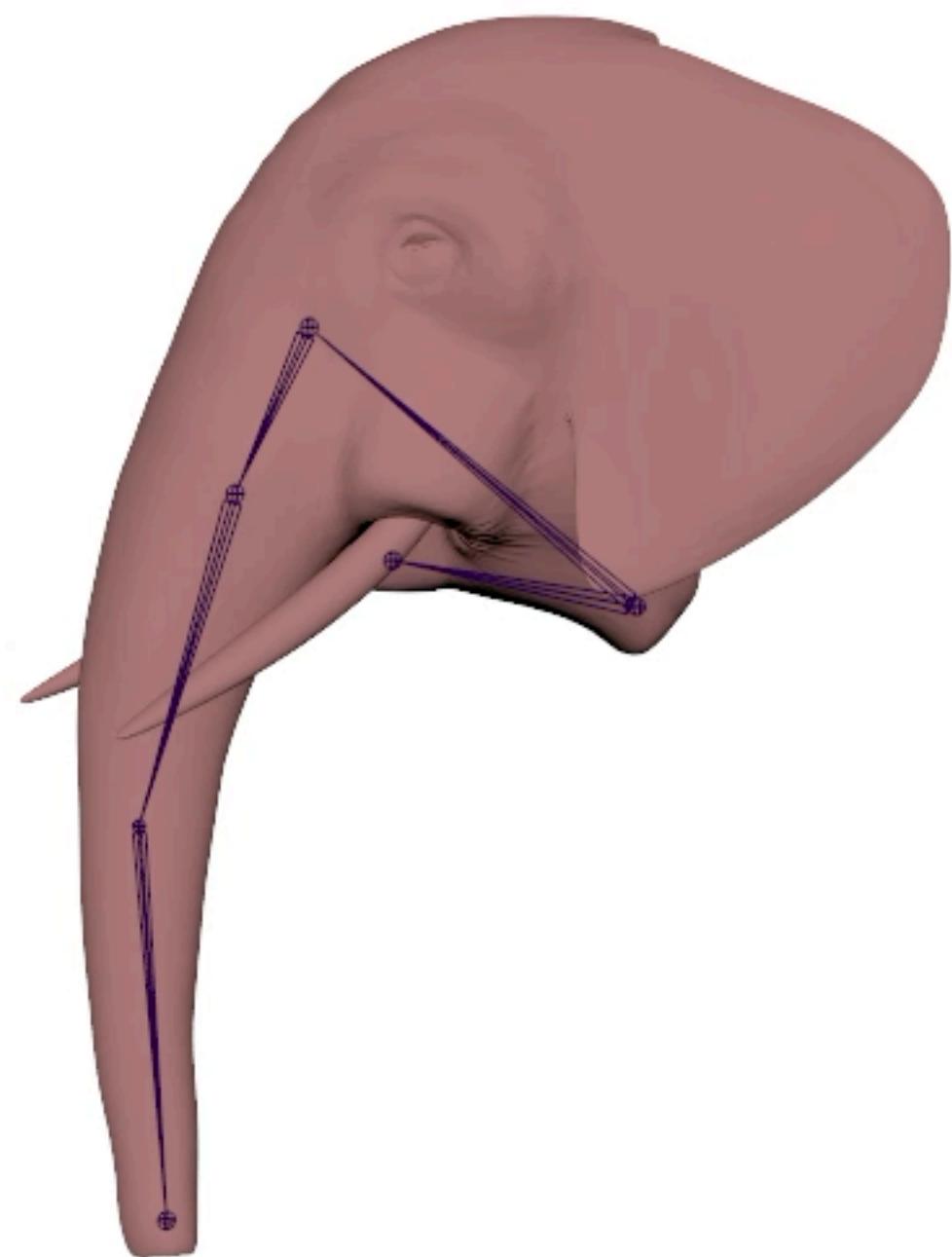


Rig Space Physics

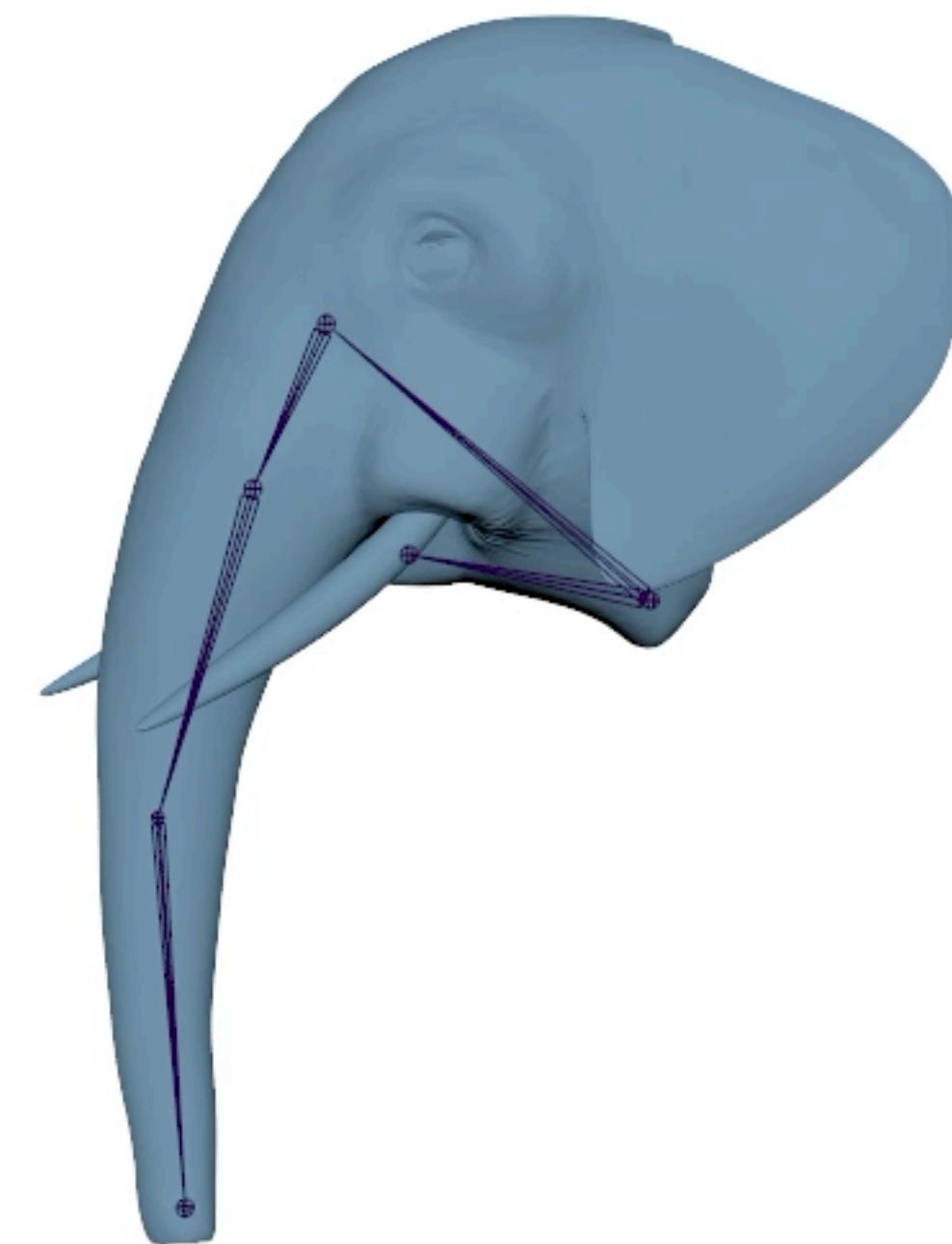


Ours

# Comparison with Positional Constraint-based Method



“Steel Bone”



Ours

# Comparison with Tracking-based Method

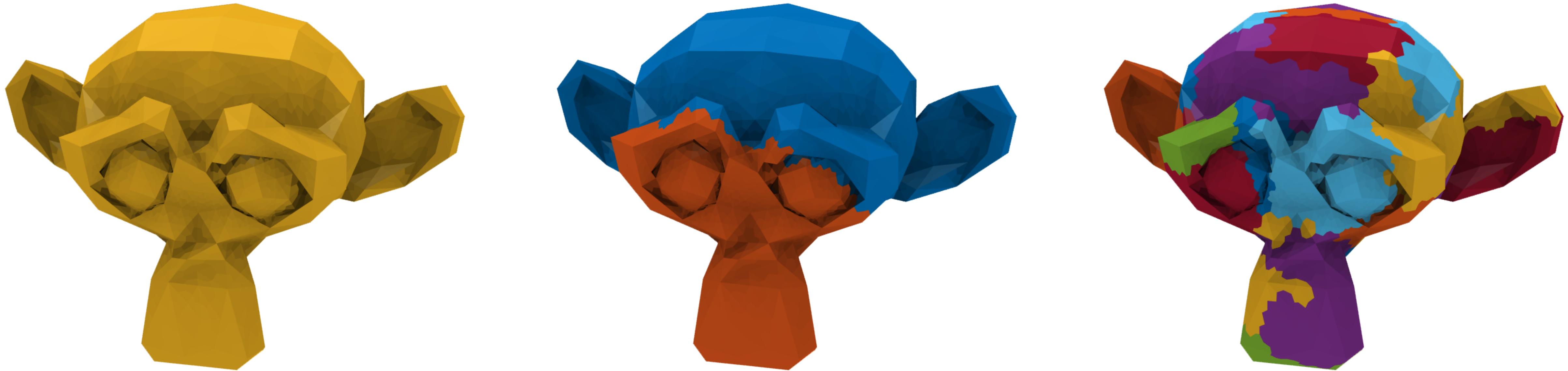


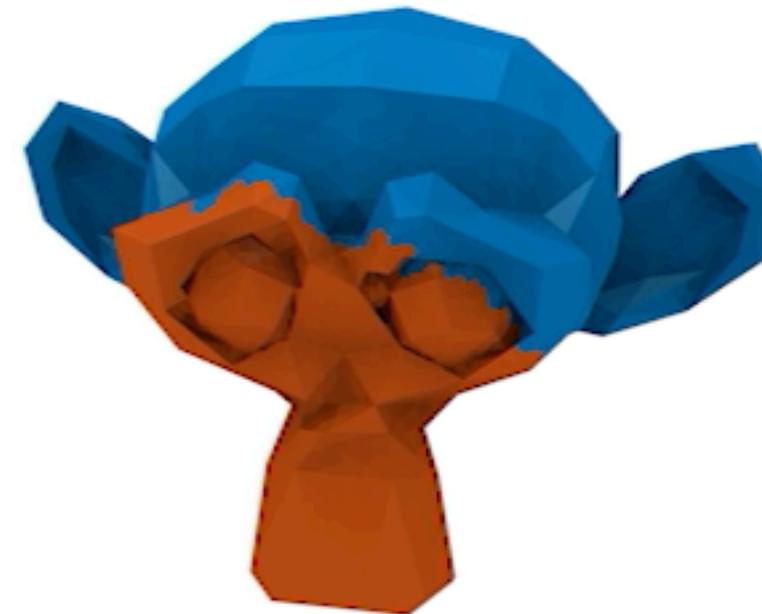
Input



Output

# Segmentation





1 Cluster

2 Clusters

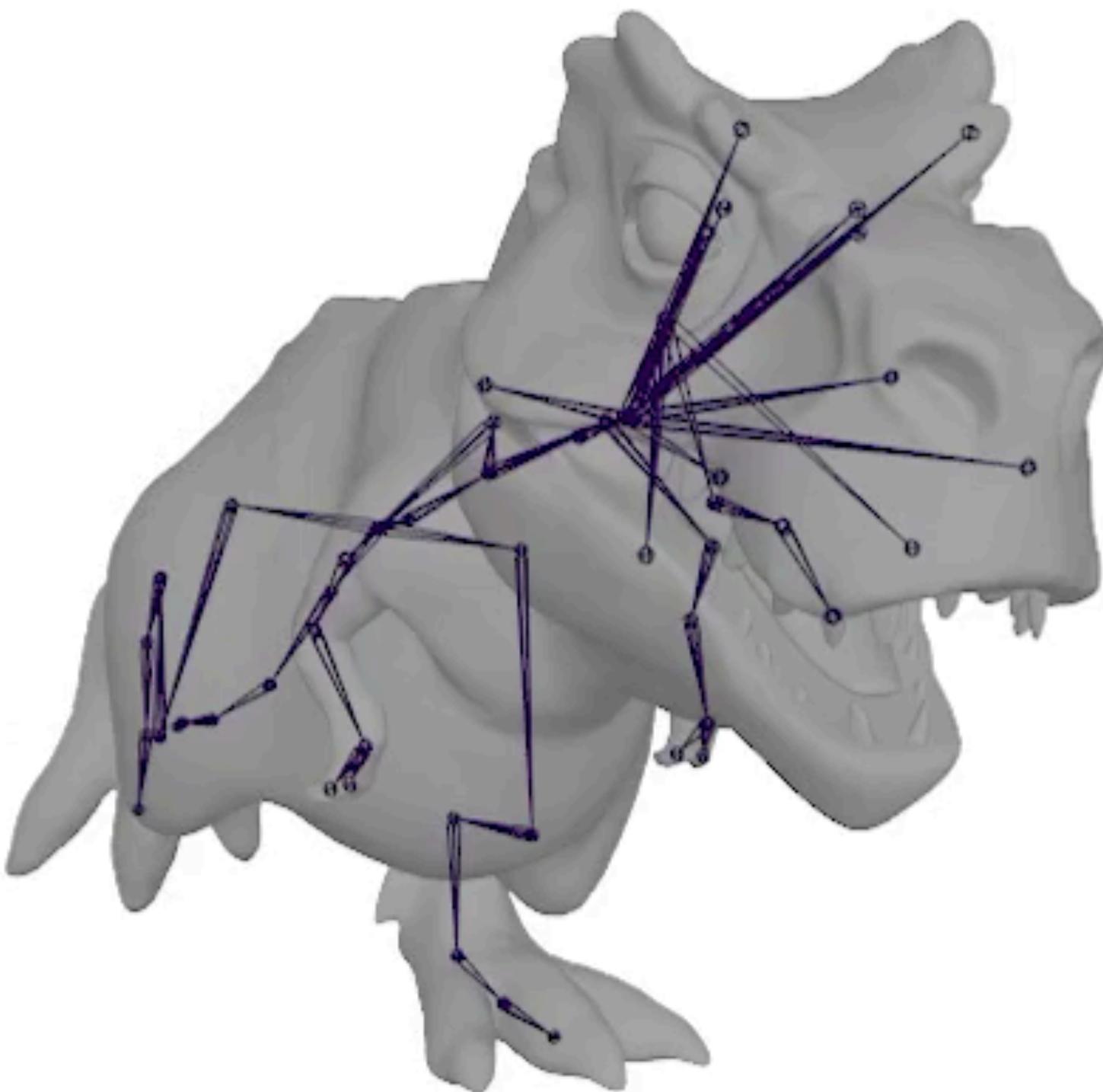
50 Clusters

Tracked Simulation [Bergou et al. 2007]

Ours



# Rig in the Wild

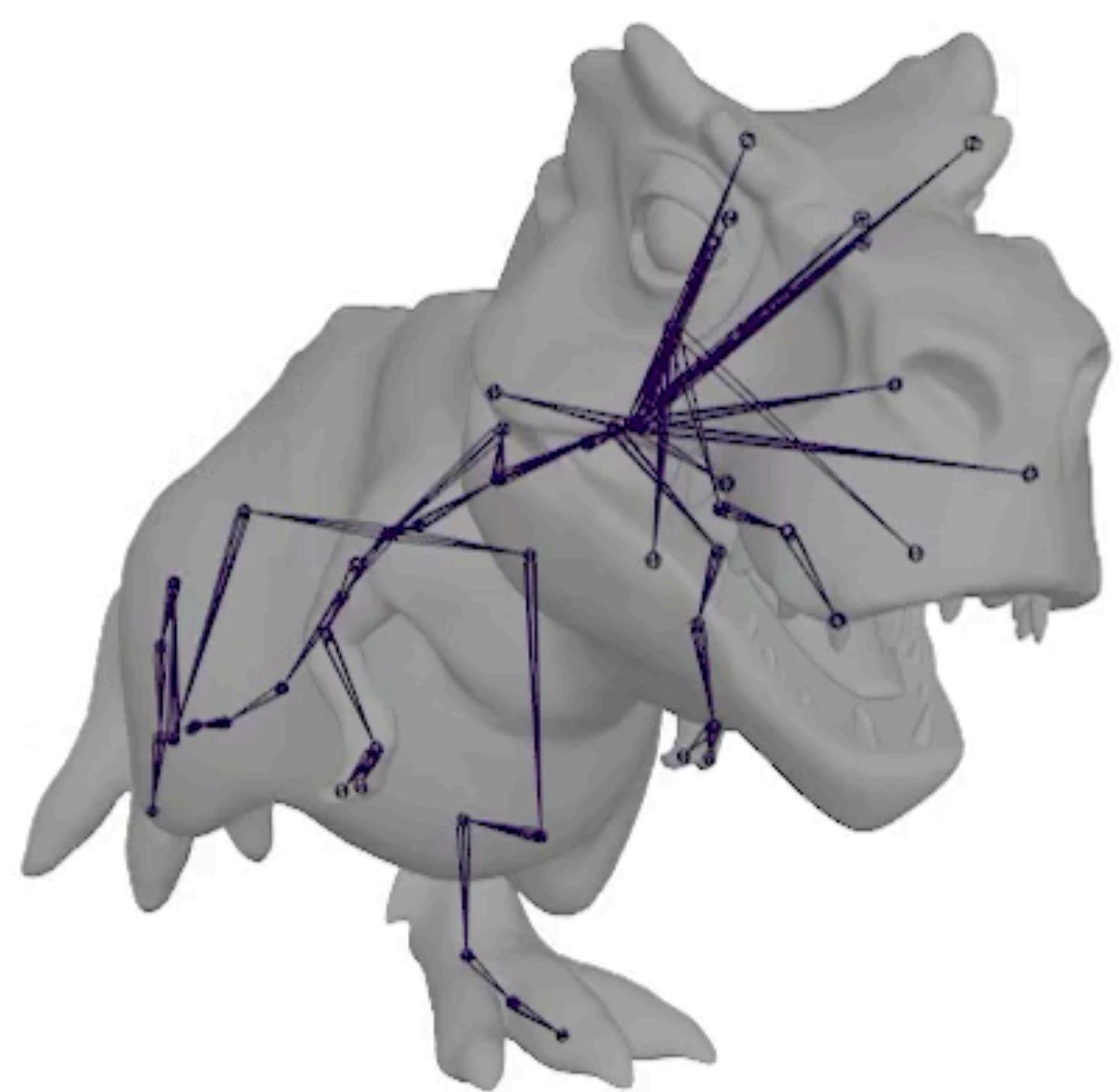


# Rig in the Wild



+ Our Complementary Secondary Motion

# Rig in the Wild



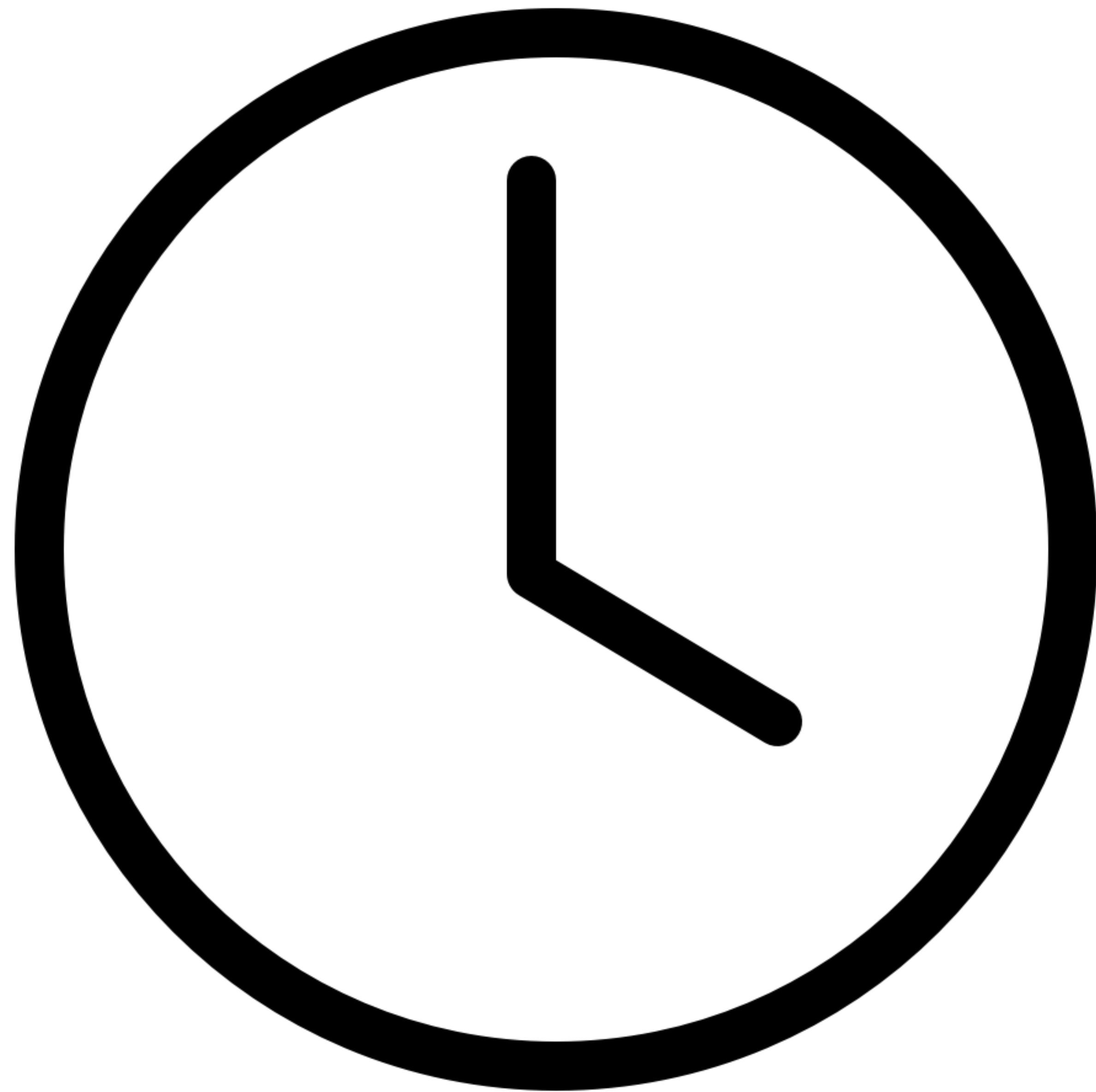
Input



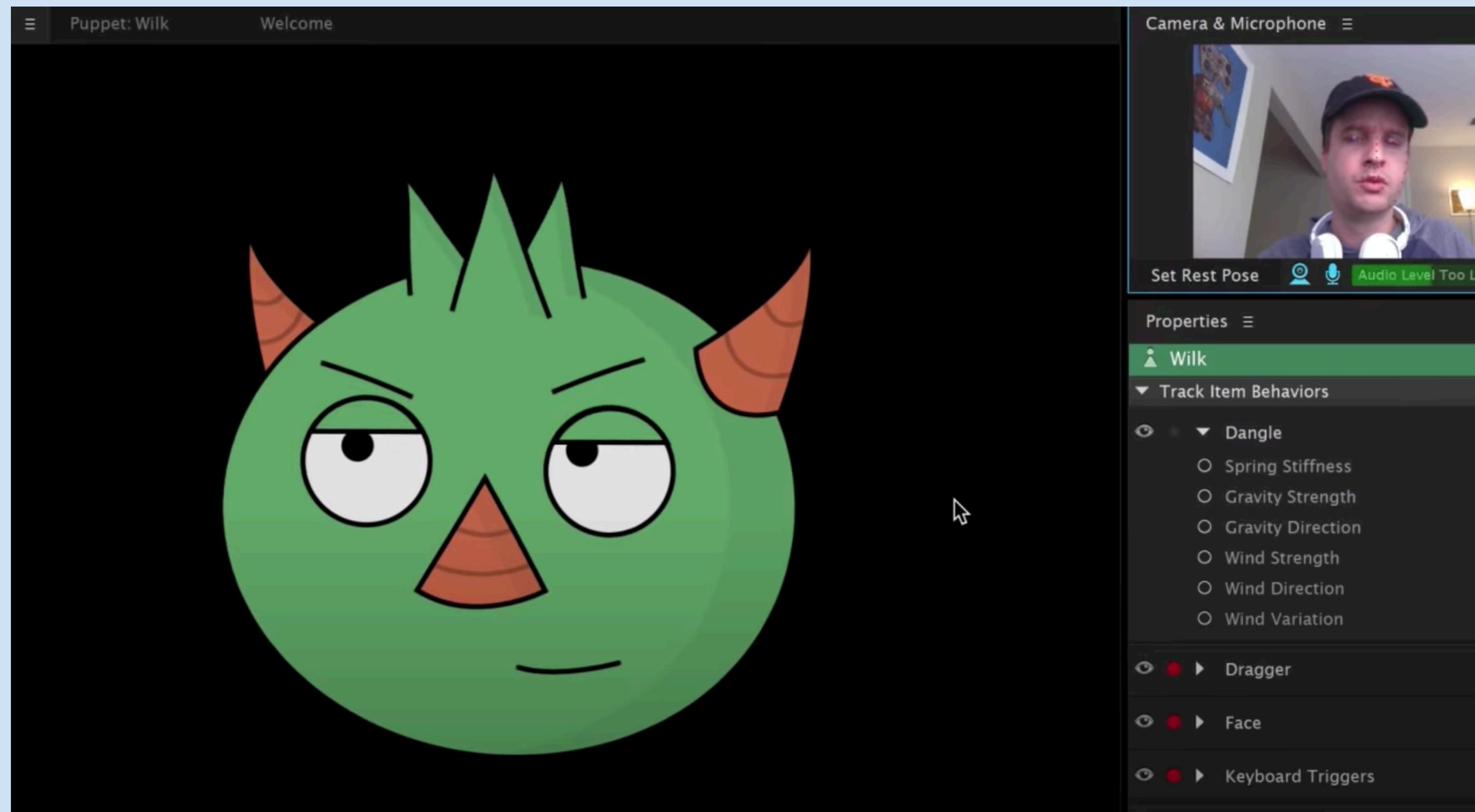
Output

# Limitation & Future Work

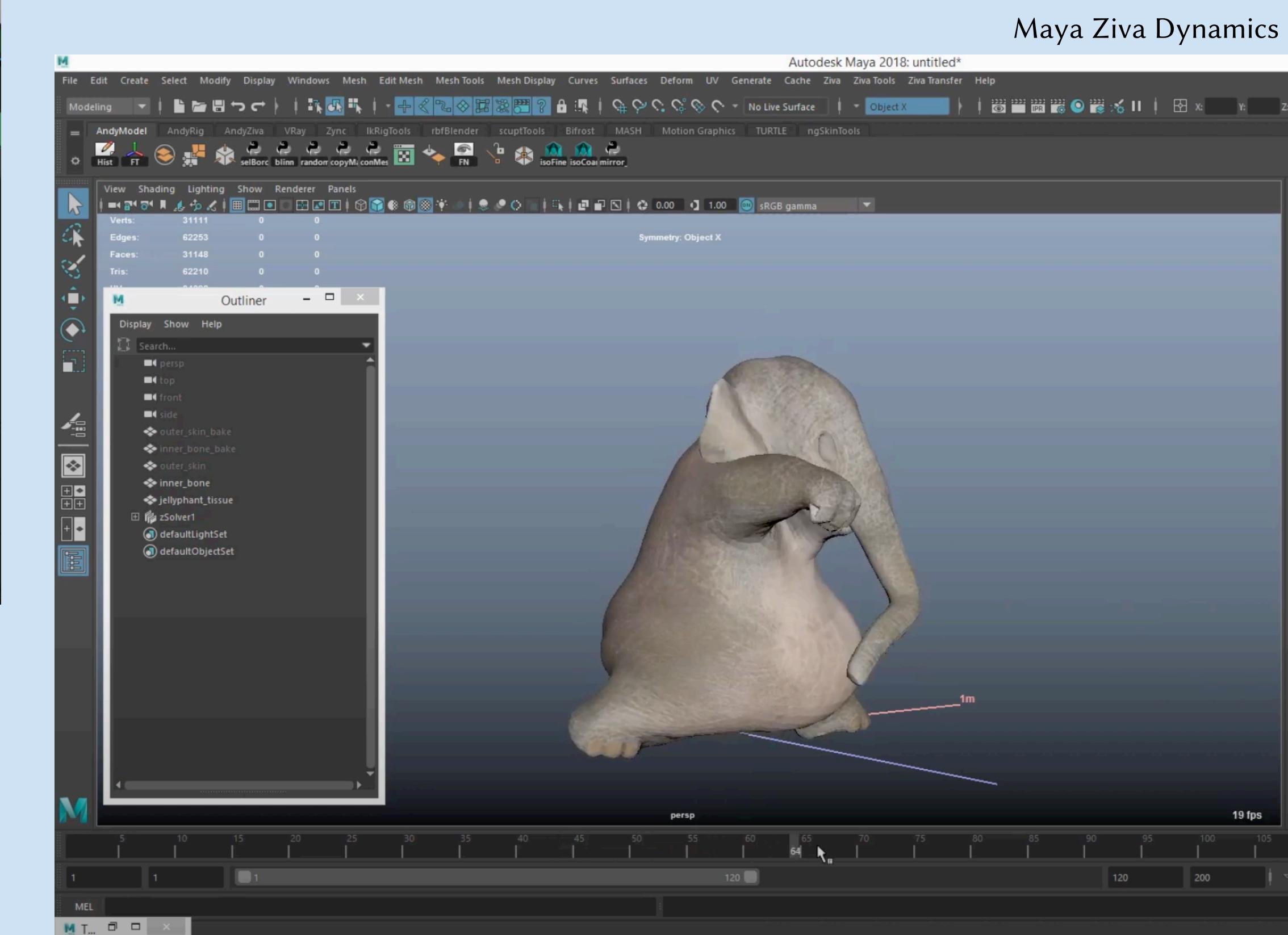
“Alas, complementary dynamics is not  
complimentary”



# Live Performance Environment



Adobe Character Animator



Maya Ziva Dynamics



“Complementary dynamics turn physics simulation into the artist’s respectful partner, rather than an unruly party crasher”

# ACKNOWLEDGEMENTS



Dynamic Graphics Project  
University of Toronto  
[www.dgp.toronto.edu](http://www.dgp.toronto.edu)



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AUTODESK.



MESH



Adobe

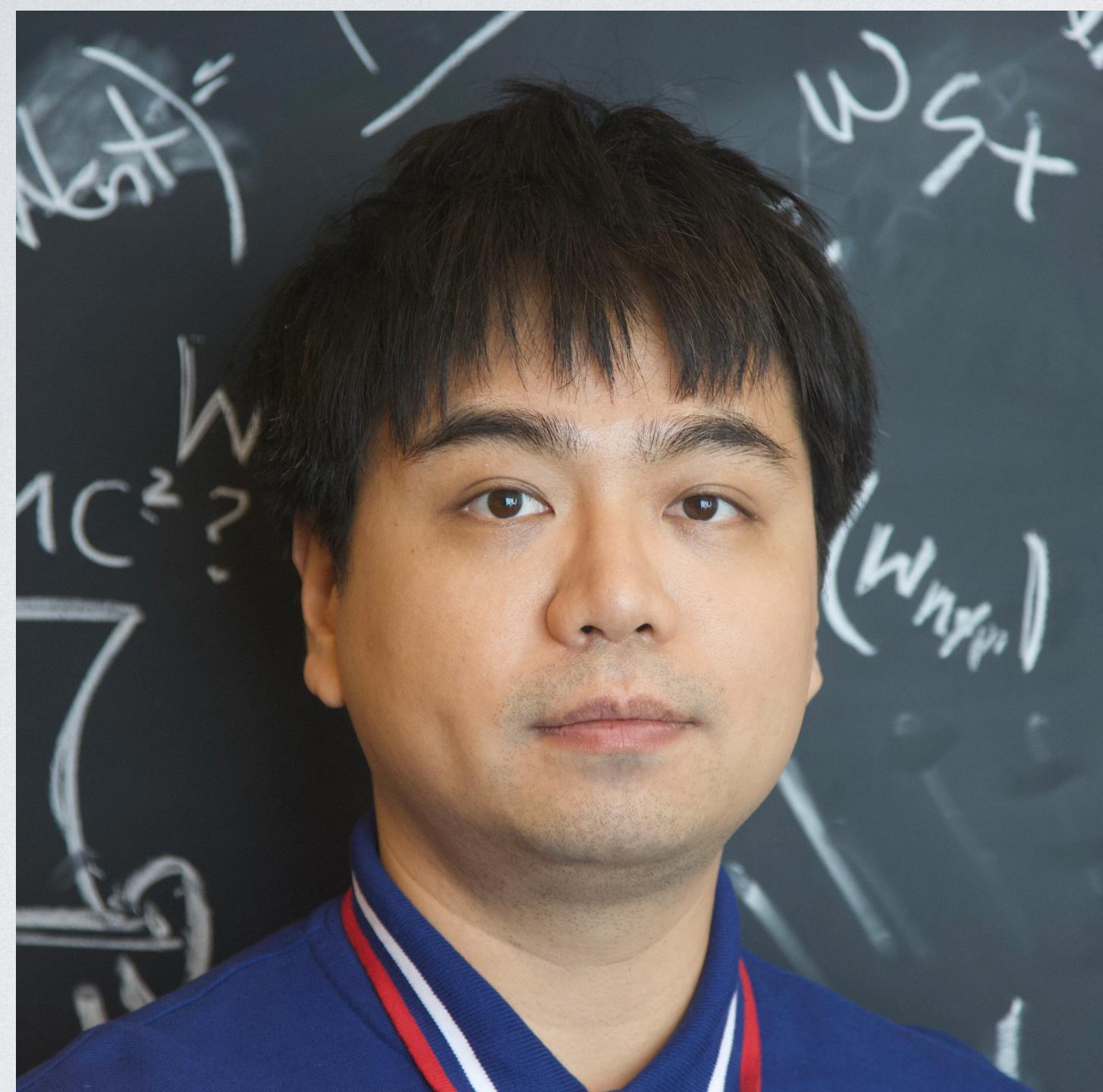


**NSERC**  
**CRSNG**



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# COMPLEMENTARY DYNAMICS

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<https://www.dgp.toronto.edu/projects/complementary-dynamics/>