Cosserat Rod with rh-Adaptive Discretization

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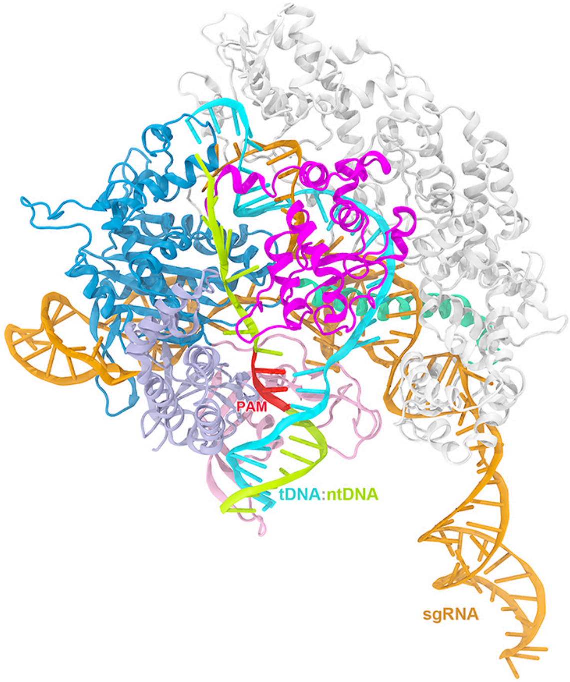
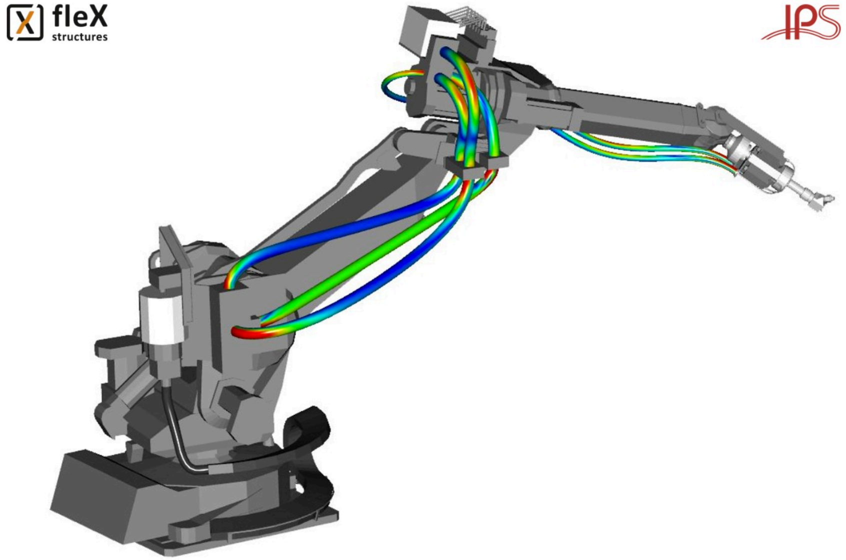
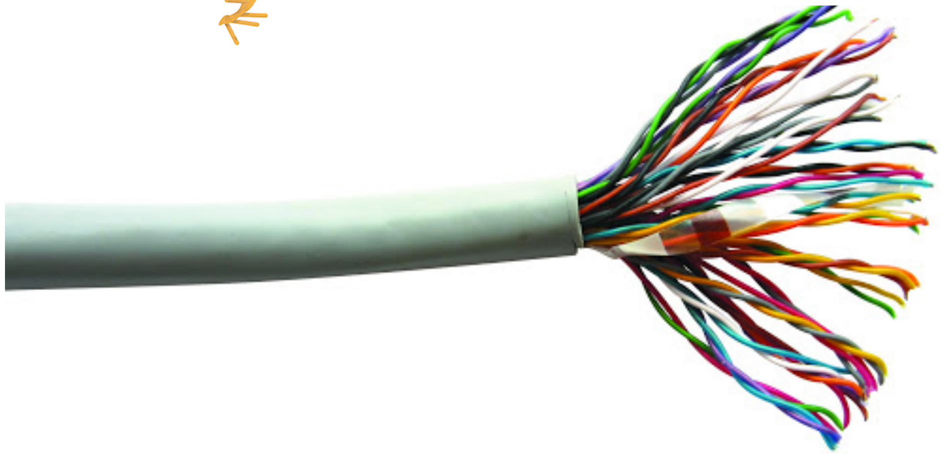
Background

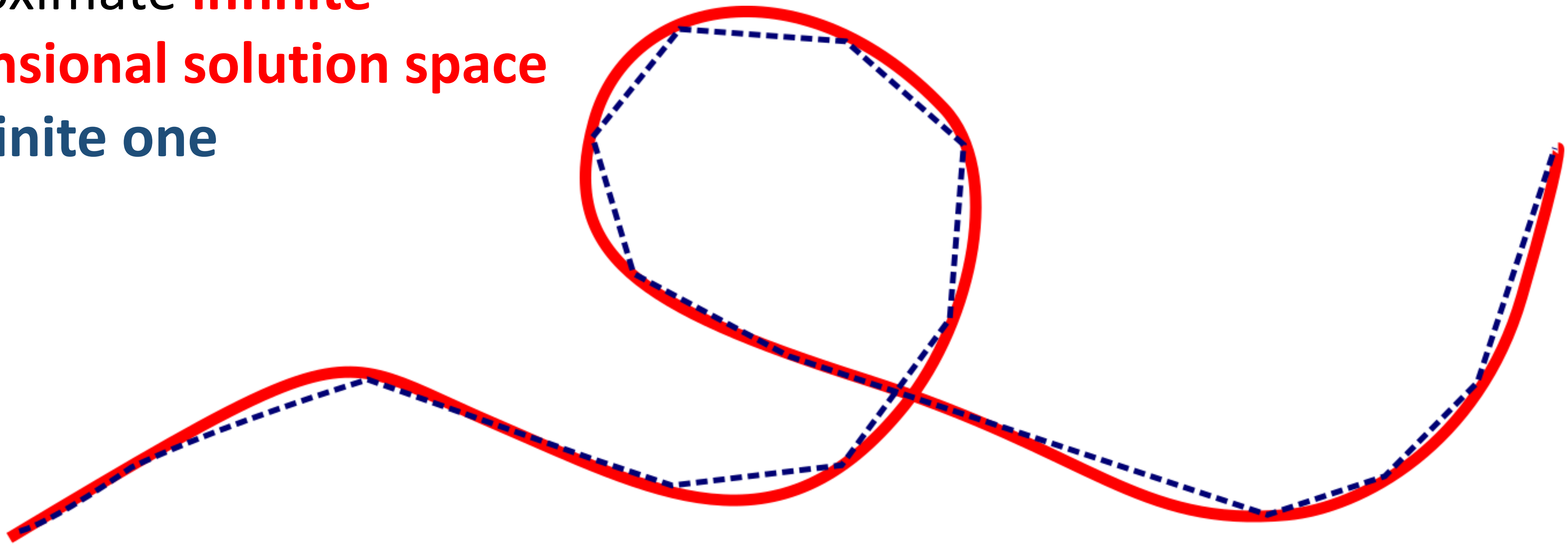






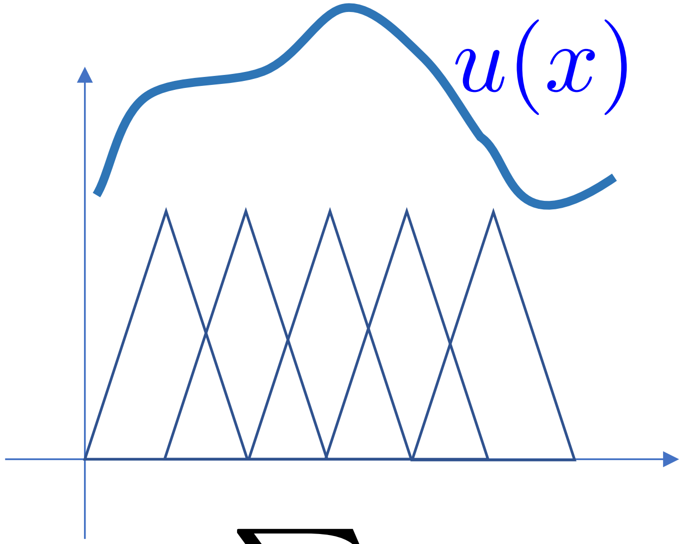
Discretization

Approximate **infinite dimensional solution space** by a **finite one**

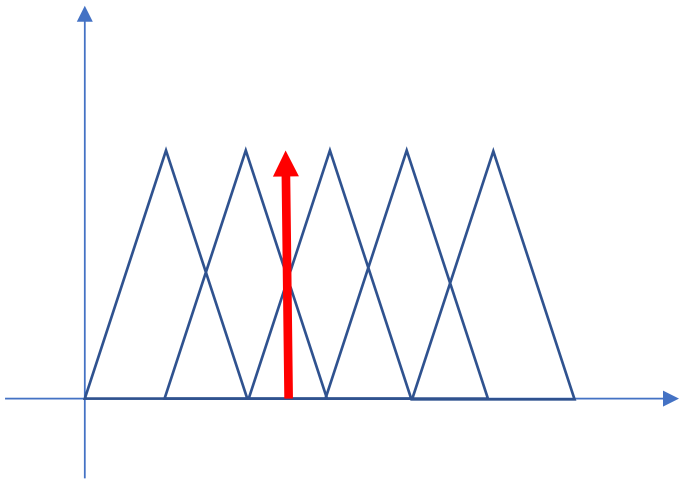


Motivation

- Problem with the finite solution space: **Scale**



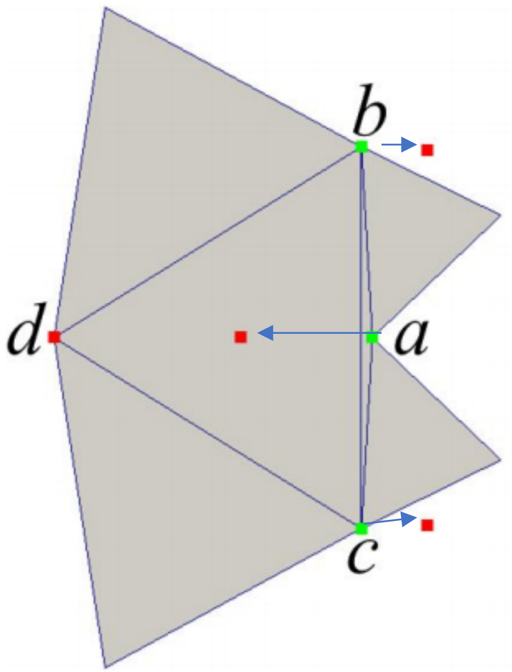
$$u(x) \approx \sum_i u_i \varphi_i(x)$$

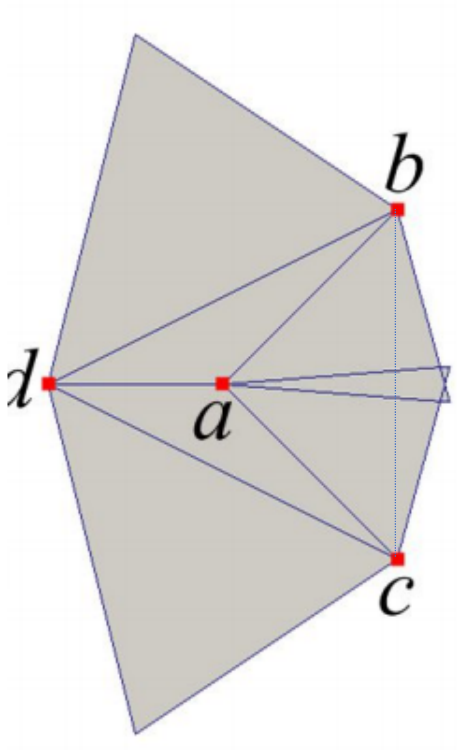


$$\delta(x - x_0)$$

Motivation

- Problem with the finite solution space: **Tessellation**

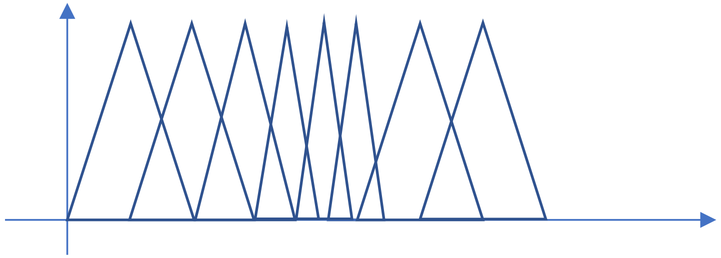




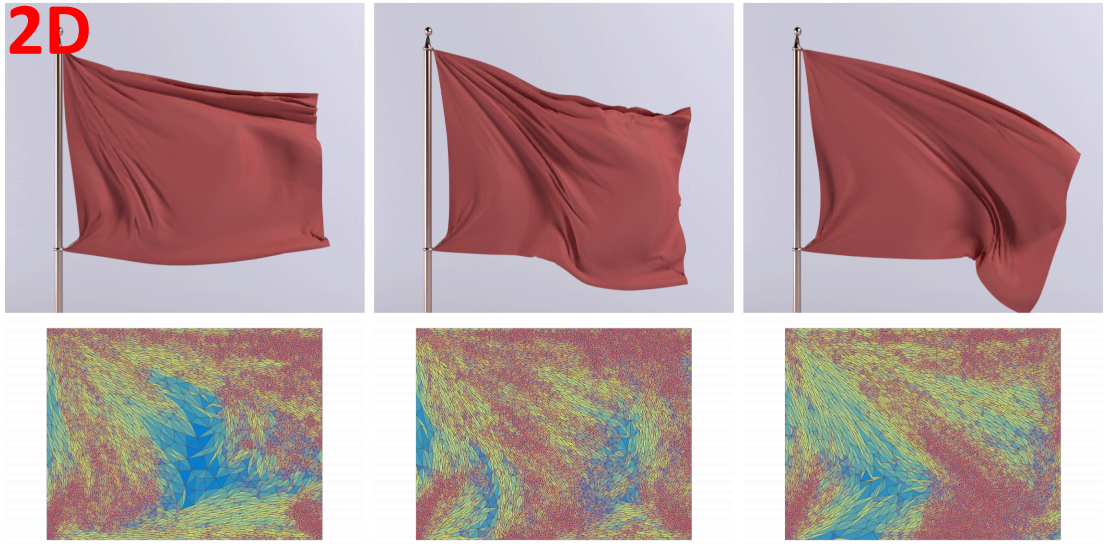
Adaptive tessellation method

- H-adaptive method
  - Dynamically adding or removing vertices

1D

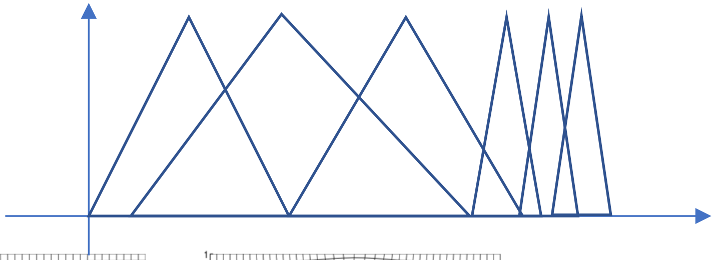


2D

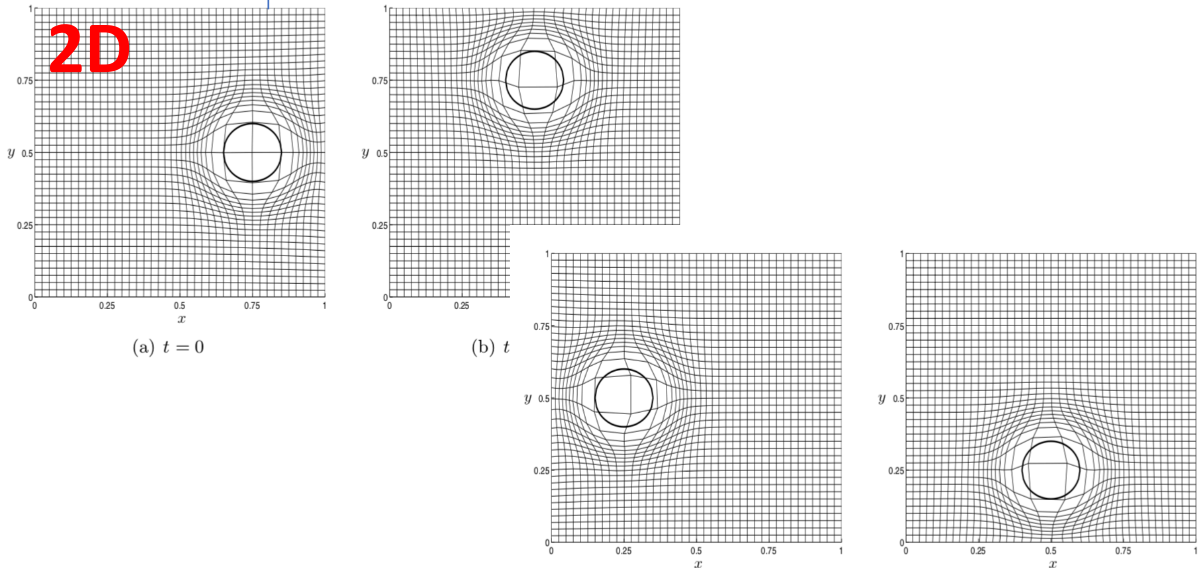


- R-adaptive method
  - Dynamically changing vertex distribution and connectivity

1D

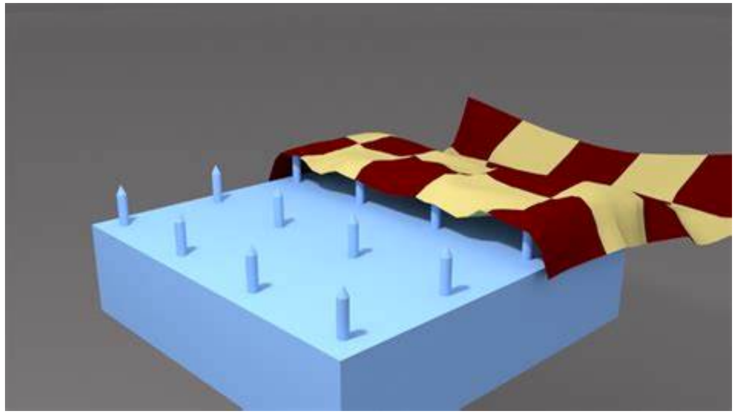


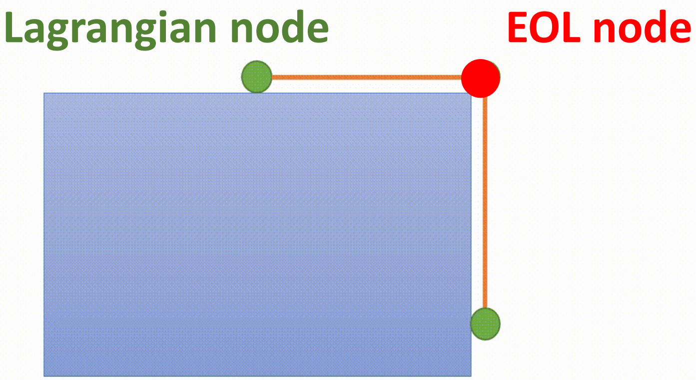
2D



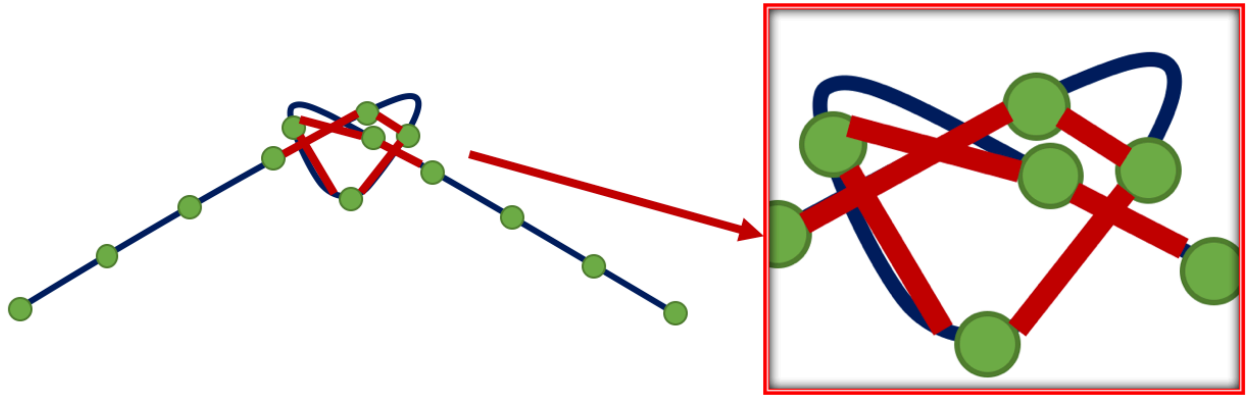
Eulerian-on-Lagrangian method

- Get applied for rods, cloth
  - Contact-oriented





Eulerian-on-Lagrangian method



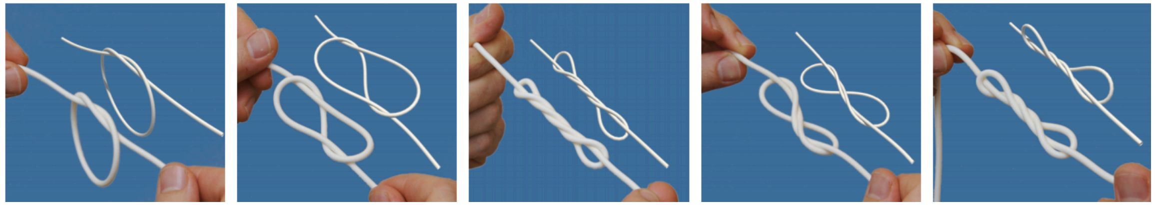
- Get applied for rods, cloth
  - Contact-oriented
  - Only EOL node can move
- Limited in adaptivity
  - More to consider: intricate geometry
- **Our extension**
  - **Unify both EOL and Lagrangian node**
  - **Drive the motion of material points in more flexible ways**

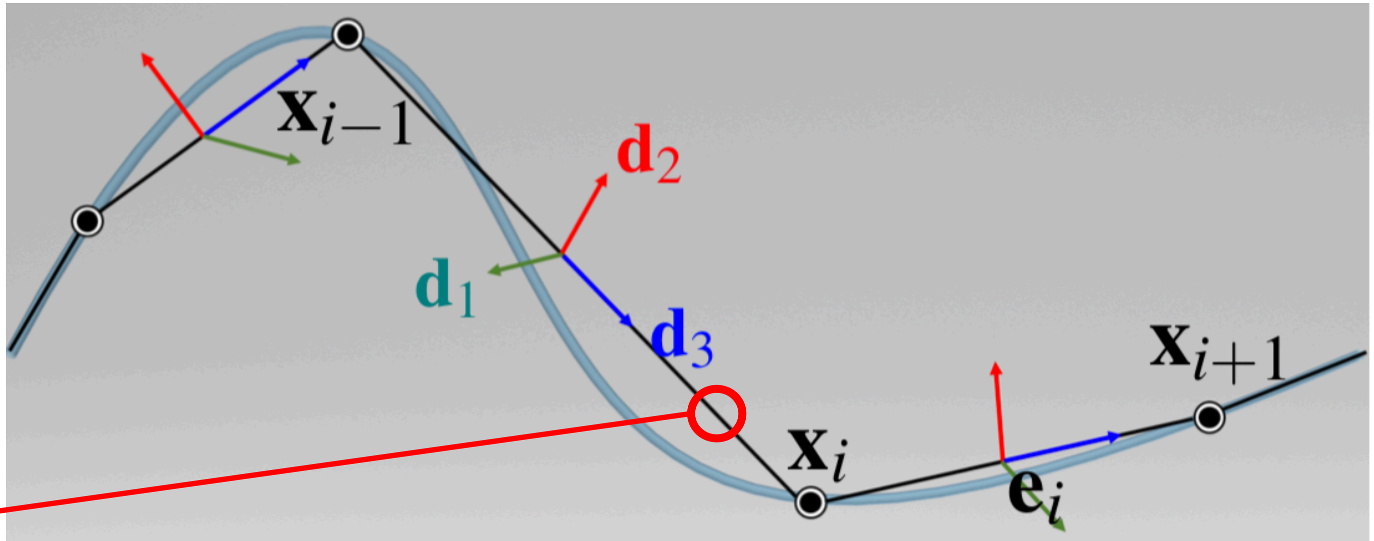
How to correctly formulate the dynamics?

How to automatically evolve those material points?

Moving mesh discretization

- Focus on Cosserat rod
  - Easy to bend and twist
  - Accuracy is largely limited by its initial discretization





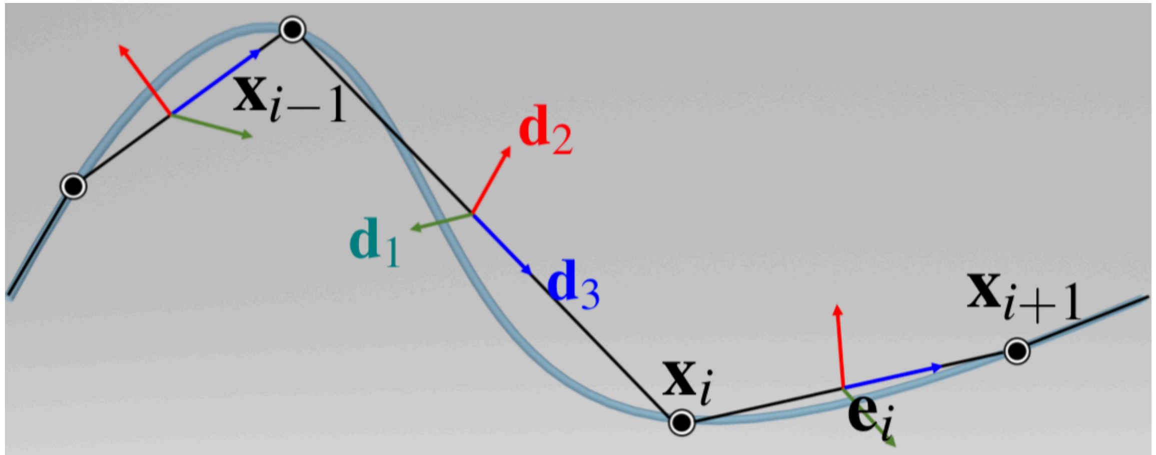
$$\mathbf{x}(u) = \frac{u - u_{i+1}}{u_i - u_{i+1}}\mathbf{x}_i + \frac{u_i - u}{u_i - u_{i+1}}\mathbf{x}_{i+1}, \quad u \in [u_i, u_{i+1}].$$

$$\mathbf{v}(\mathbf{x}, u, \dot{\mathbf{x}}, \dot{u}) = \dot{\mathbf{x}} - \nabla_u \mathbf{x} \cdot \dot{u}.$$

System Lagrangian

$$T_k(\mathbf{x}, u, \dot{\mathbf{x}}, \dot{u}) = \frac{1}{2}\int_0^L \rho\pi r^2 \|\mathbf{v}\|^2 du,$$

$$V_s(\mathbf{x}, u) = \frac{1}{2}\int_0^L k_s(\|\mathbf{F}\| - 1)^2 du,$$



$$T_r(\mathbf{q}, \dot{\mathbf{q}}, u) = \frac{1}{2}\int_0^L \|\mathbf{J}\boldsymbol{\omega}(\mathbf{q}, \dot{\mathbf{q}})\|^2 du,$$

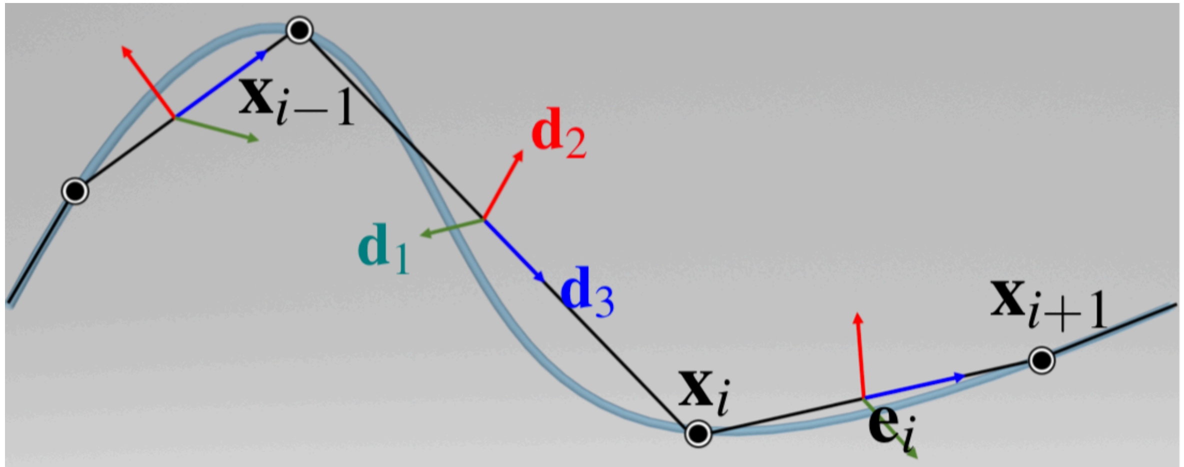
$$V_b(\mathbf{q}, u) = \frac{1}{2}\int_0^L \|\mathbf{K}(\boldsymbol{\varepsilon}(\mathbf{q}) - \bar{\boldsymbol{\varepsilon}})\|^2 du,$$

$$C_p(\mathbf{x}, u, \mathbf{q}) = \frac{\mathbf{F}}{\|\mathbf{F}\|} - \mathbf{d}_3(\mathbf{q}) = \mathbf{0},$$

$$C_q = \|\mathbf{q}\| - 1 = 0.$$

Equation of motion

Euler-Lagrange equation $$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = g$$

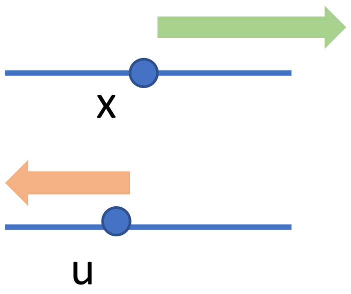


For positional variables r=[x, u]

$$\mathrm{EoM}(\mathbf{r}, \dot{\mathbf{r}}): \quad \boxed{\dot{\mathbf{M}}(\mathbf{r})\dot{\mathbf{r}}} + \mathbf{M}(\mathbf{r})\ddot{\mathbf{r}} - \boxed{\frac{\partial T_k}{\partial \mathbf{r}}} + \frac{\partial V_s}{\partial \mathbf{r}} = \mathbf{g},$$

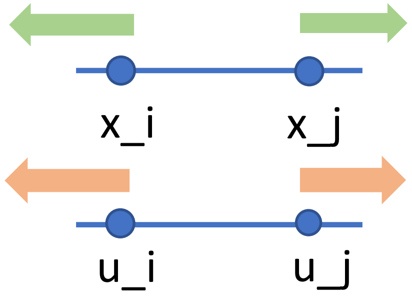
Singularities in dynamic system

$$\boxed{\mathbf{v}(\mathbf{x}, u, \dot{\mathbf{x}}, \dot{u}) = \dot{\mathbf{x}} - \nabla_u \mathbf{x} \cdot \dot{u}.}$$



$$\mathbf{M}_i(\mathbf{x}_i, \mathbf{x}_{i+1}, u_i, u_{i+1}) = \frac{\rho\pi r^2 (u_{i+1} - u_i)}{6} \begin{bmatrix} 2\mathbf{I}_{3\times3} & \mathbf{I}_{3\times3} & -2\mathbf{F} & -\mathbf{F} \\ \mathbf{I}_{3\times3} & 2\mathbf{I}_{3\times3} & -\mathbf{F} & -2\mathbf{F} \\ -2\mathbf{F}^T & -\mathbf{F}^T & 2\mathbf{F}^T\mathbf{F} & \mathbf{F}^T\mathbf{F} \\ -\mathbf{F}^T & -2\mathbf{F}^T & \mathbf{F}^T\mathbf{F} & 2\mathbf{F}^T\mathbf{F} \end{bmatrix},$$

$$\boxed{\mathbf{F}_i = \frac{\mathbf{x}_i - \mathbf{x}_{i+1}}{u_i - u_{i+1}}}$$



$$\|\mathbf{F}\| - 1 \equiv 0$$

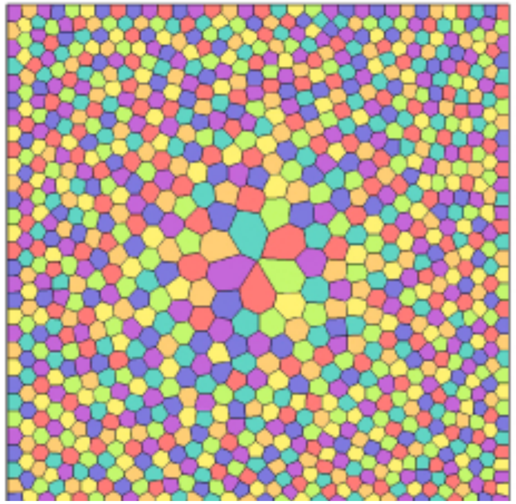
Constraint of material points

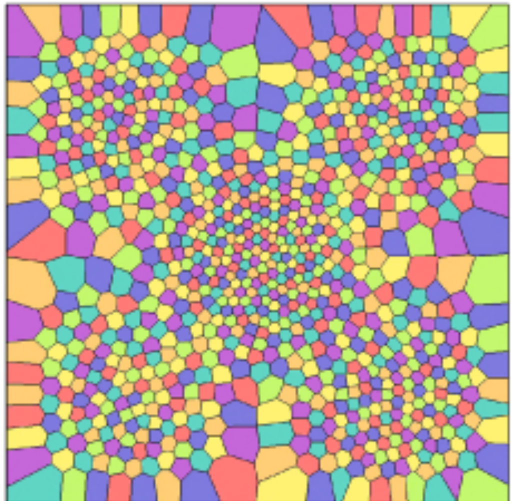
- How to constrain material points? Fixed?

region having a larger curvature should be more densely sampled.

$$R(\mathbf{u}) = \sum_i \int_{\Omega_i} \theta(u) \|u - u_i\|^2 du,$$

- Centroid Voronoi Tessellation

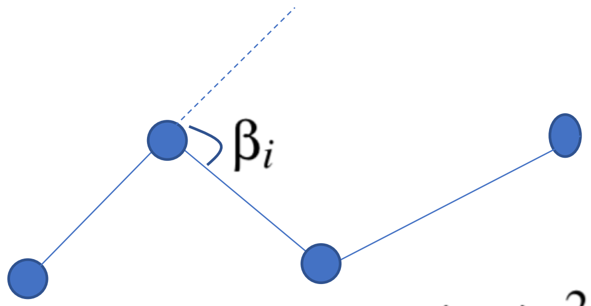




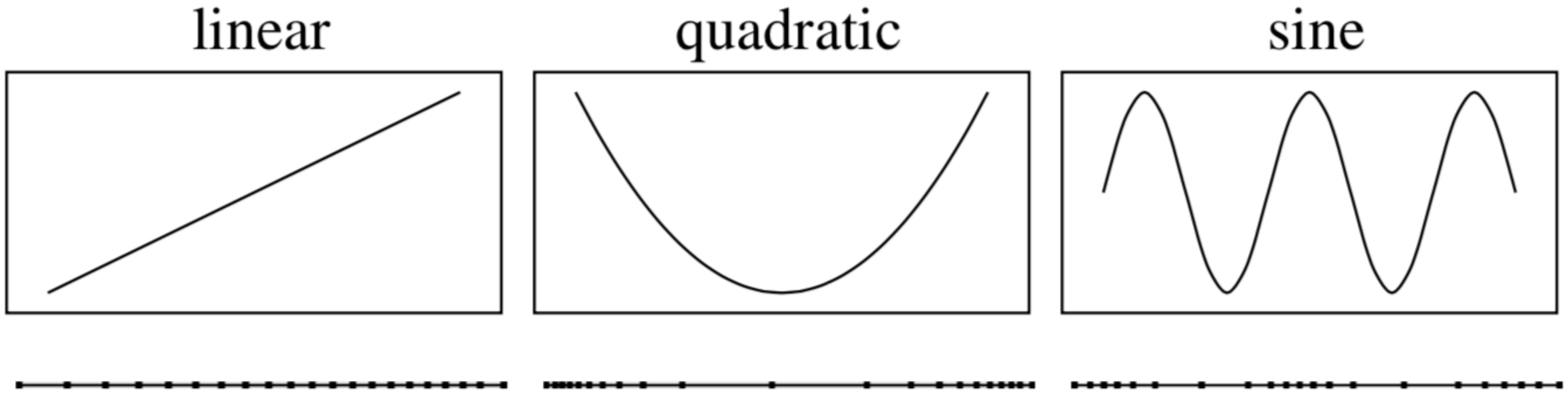
$$R(\mathbf{u}) = \sum_i \int_{\frac{u_{i-1}+u_i}{2}}^{\frac{u_i+u_{i+1}}{2}} \theta_i \|u - u_i\|^2 du$$
$$= \sum_i \frac{\theta_i}{24} \left[(u_{i+1} - u_i)^3 - (u_i - u_{i-1})^3 \right],$$

How to select the density function

- Curvature aware density function



$$\theta_i = \theta_{min} + \theta_{max}\left(\frac{\beta_i}{\pi}\right)^2,$$



Constrained optimization

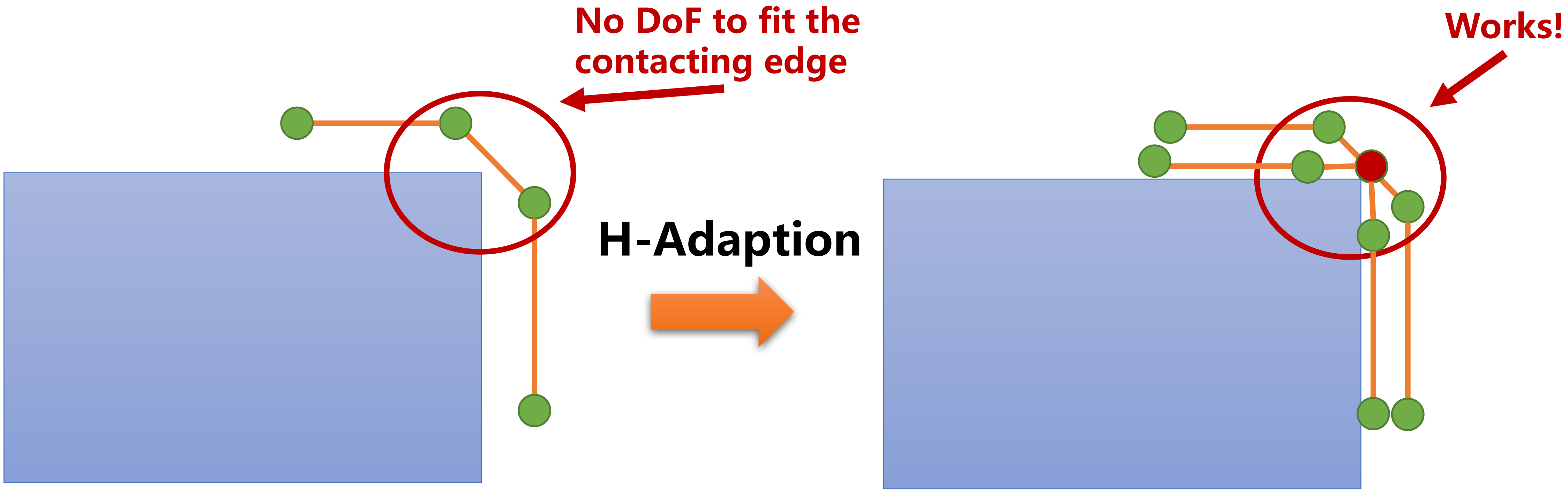
- Formulation: r=[u, x]

$$
\begin{aligned}
\underset{\mathbf{r}^{n+1}}{\arg\min} \quad & R(\mathbf{u}^{n+1};\boldsymbol{\theta}^{n}) \\
\text{s.t.} \quad & \mathrm{EoM}(\mathbf{r}^{n+1};\mathbf{r}^{n},\dot{\mathbf{r}}^{n}) = 0, \\
& \mathrm{EoM}(\mathbf{q}^{n+1};\mathbf{q}^{n},\dot{\mathbf{q}}^{n}) = 0, \\
& C(\mathbf{r}^{n+1},\mathbf{q}^{n+1}) = 0, \\
& W(\mathbf{r}^{n+1},\mathbf{q}^{n+1}) \geq 0,
\end{aligned}
$$

- Numerical solve
  - Simplify EoM by neglecting some terms
  - Turn EoM constraint into soft penalties
  - Solved by SQP

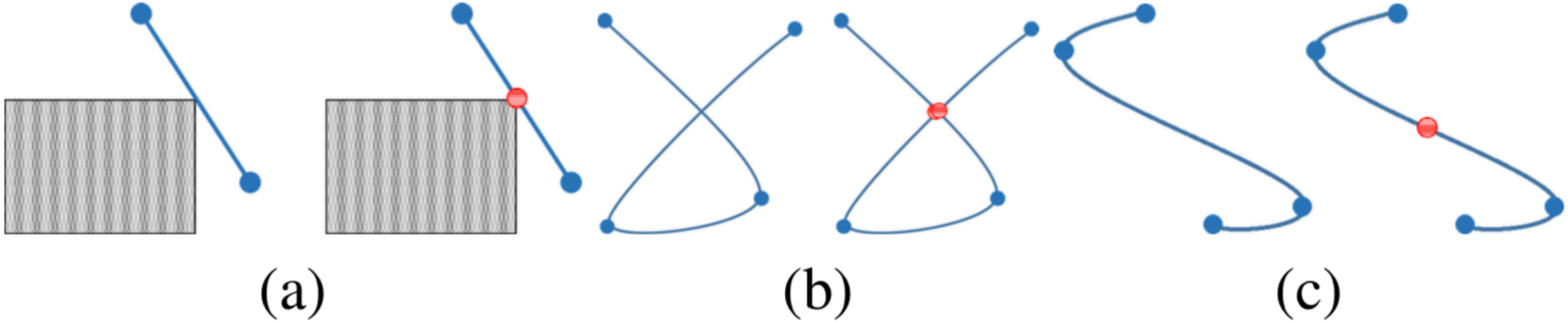
Is r-adaption enough?

- Nyquist sampling theorem
  - Sampling frequency >= 2*signal frequency

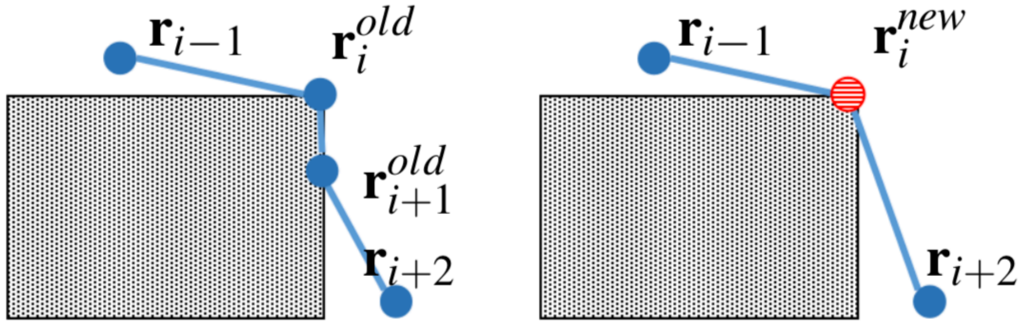


H-adaption

Node insertion



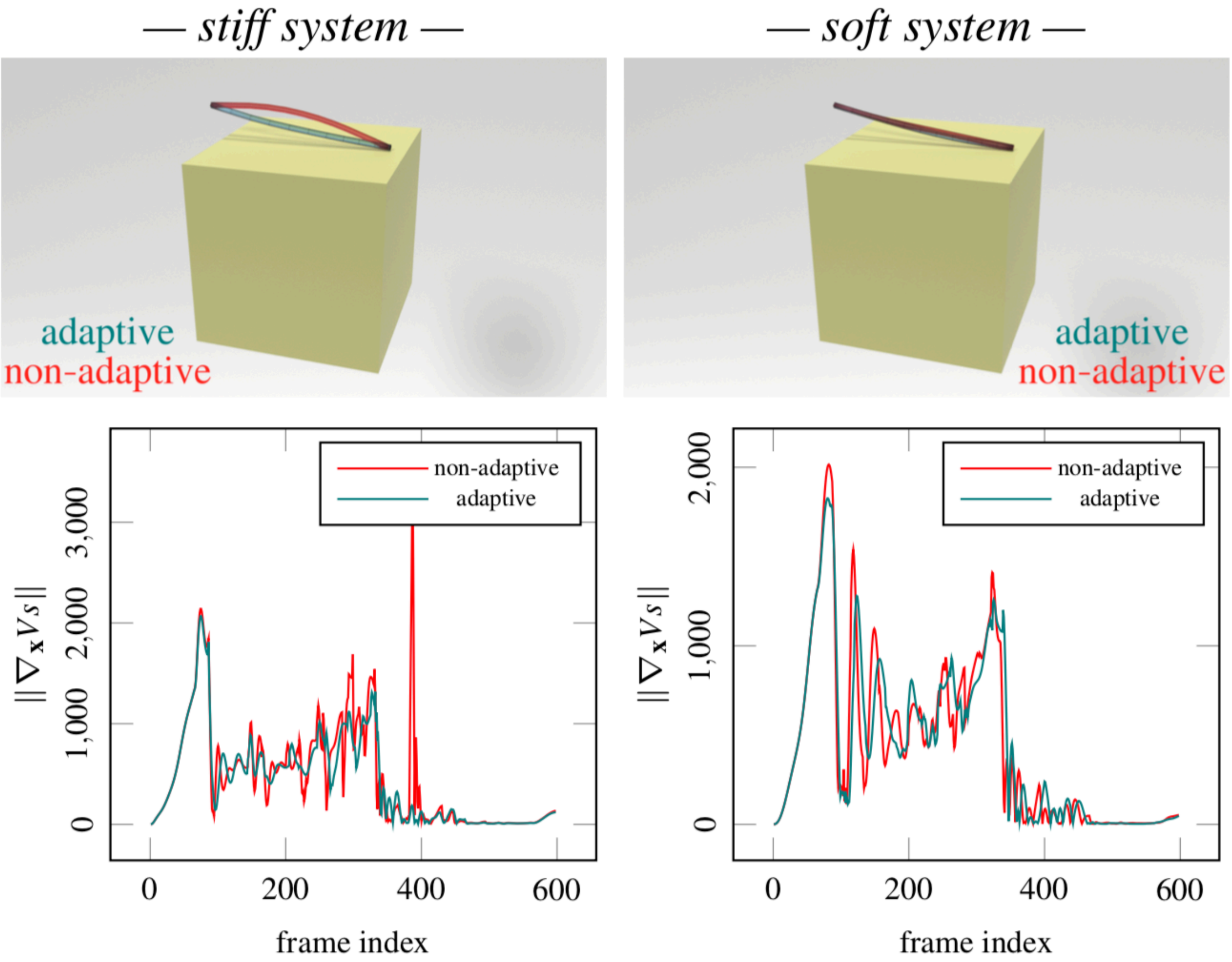
Node deletion



Merging tiny segments for

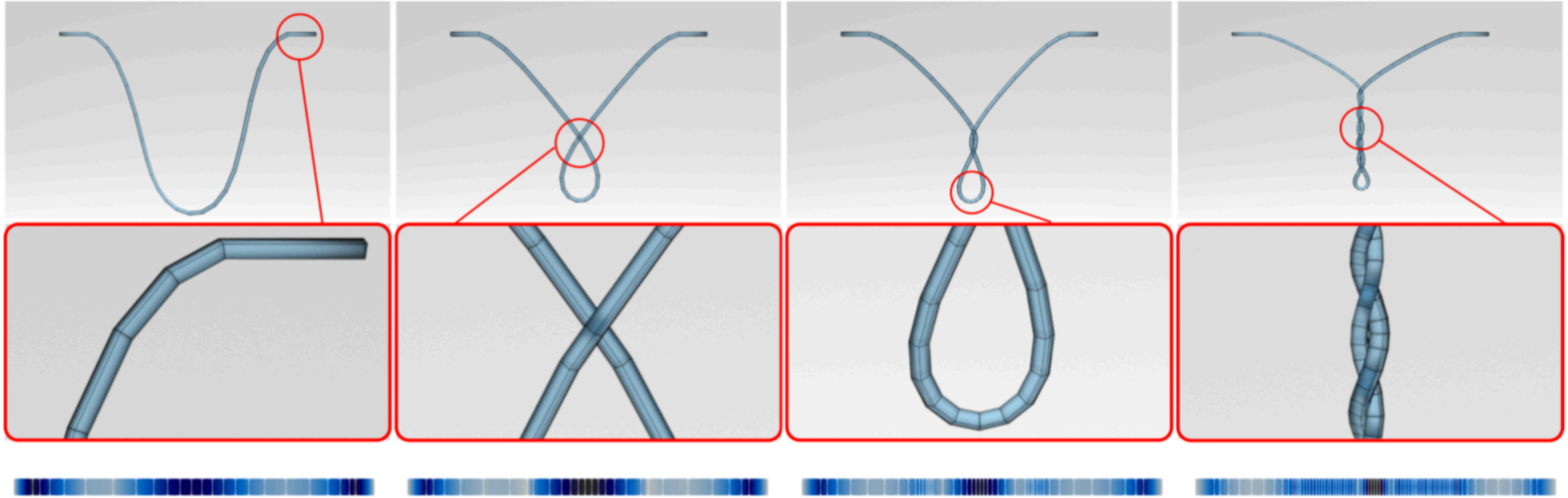
- Efficiency
- Avoiding infinitely large stiffness

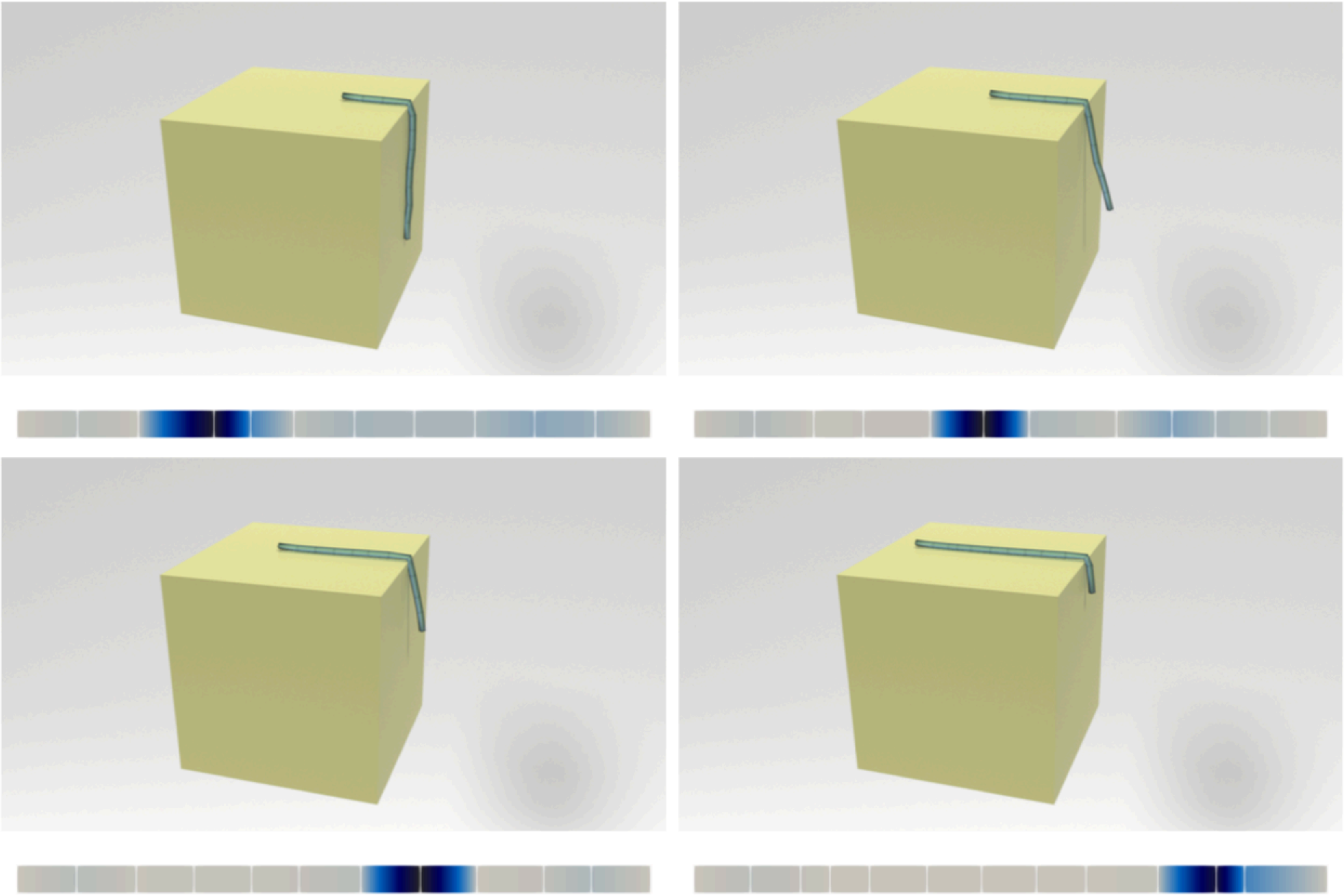
Stiffness adjustment



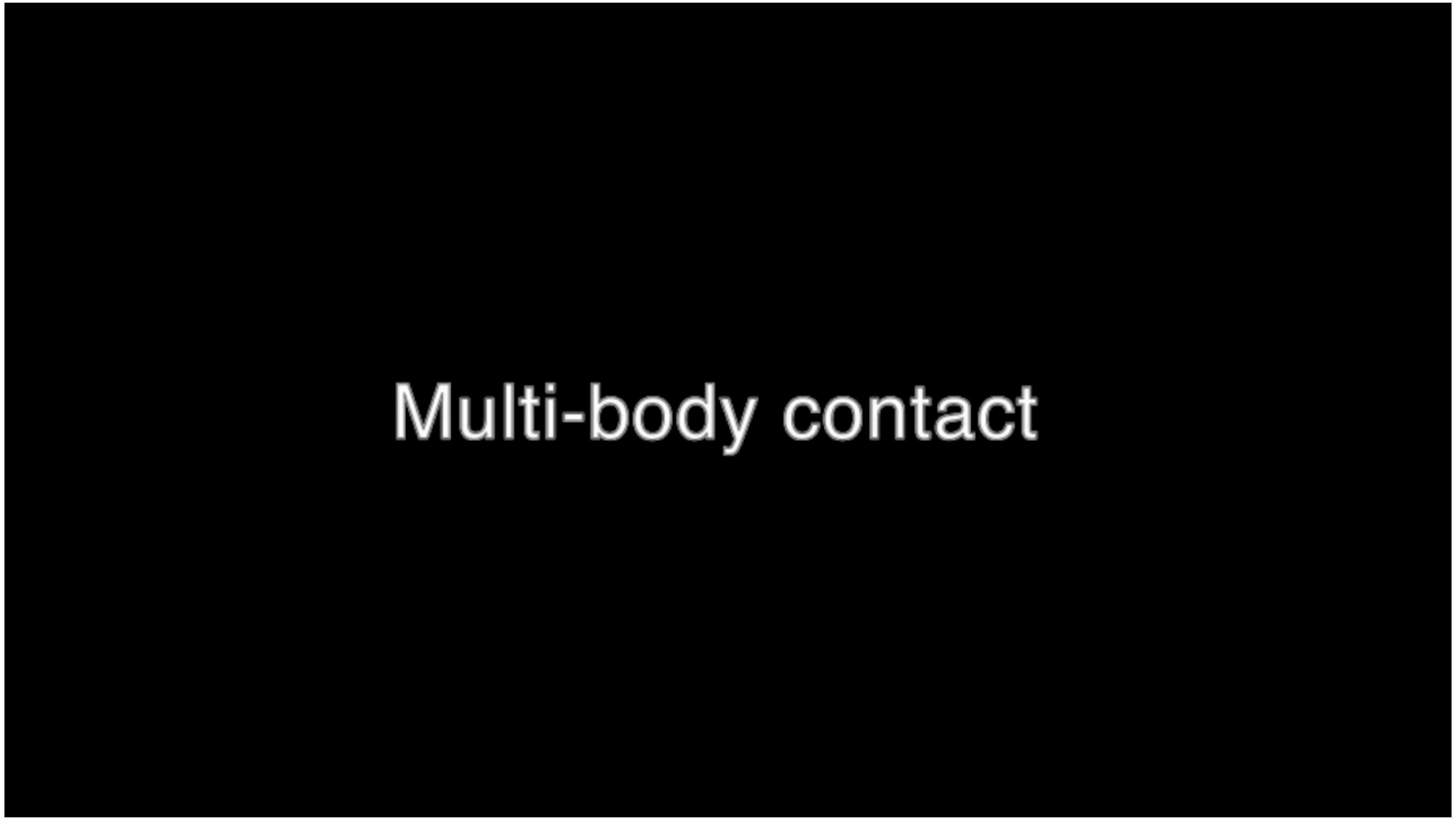
Results

Excessive twisting





Rigid-body contacts



Knotting

Future works

- Better prior or posterior knowledge to drive dynamics of material points
- Accurate handling of equation of motion
- R-adaption in a higher dimensional space
  - Energy smoothness
- Customized numerical solver
  - Statics-dynamics separation
- H-adaption guided by multiscale theory

Thanks!

Q&A