INCREMENTAL POTENTIAL CONTACT: INTERSECTION- AND INVERSION- FREE LARGE DEFORMATION DYNAMICS

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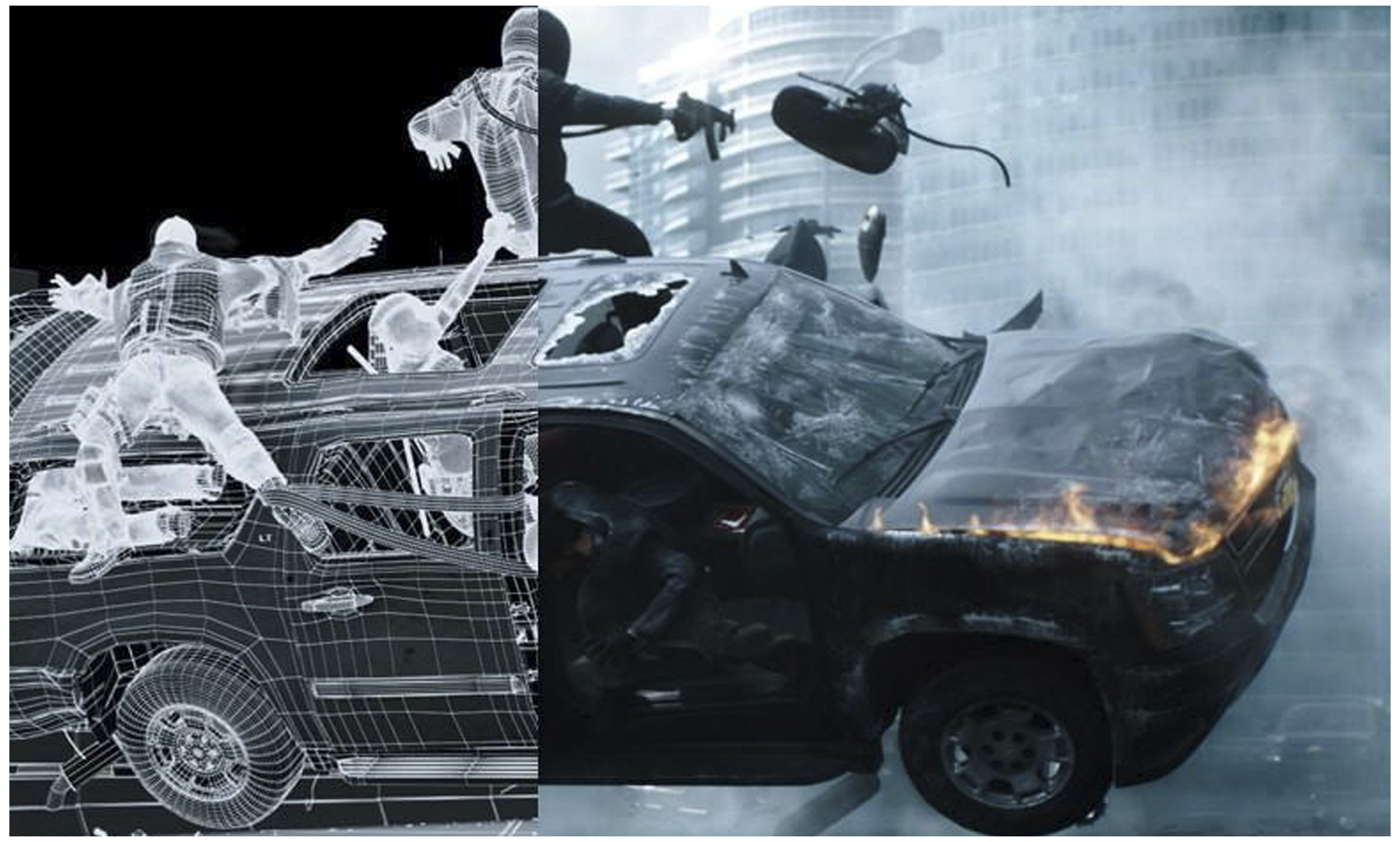
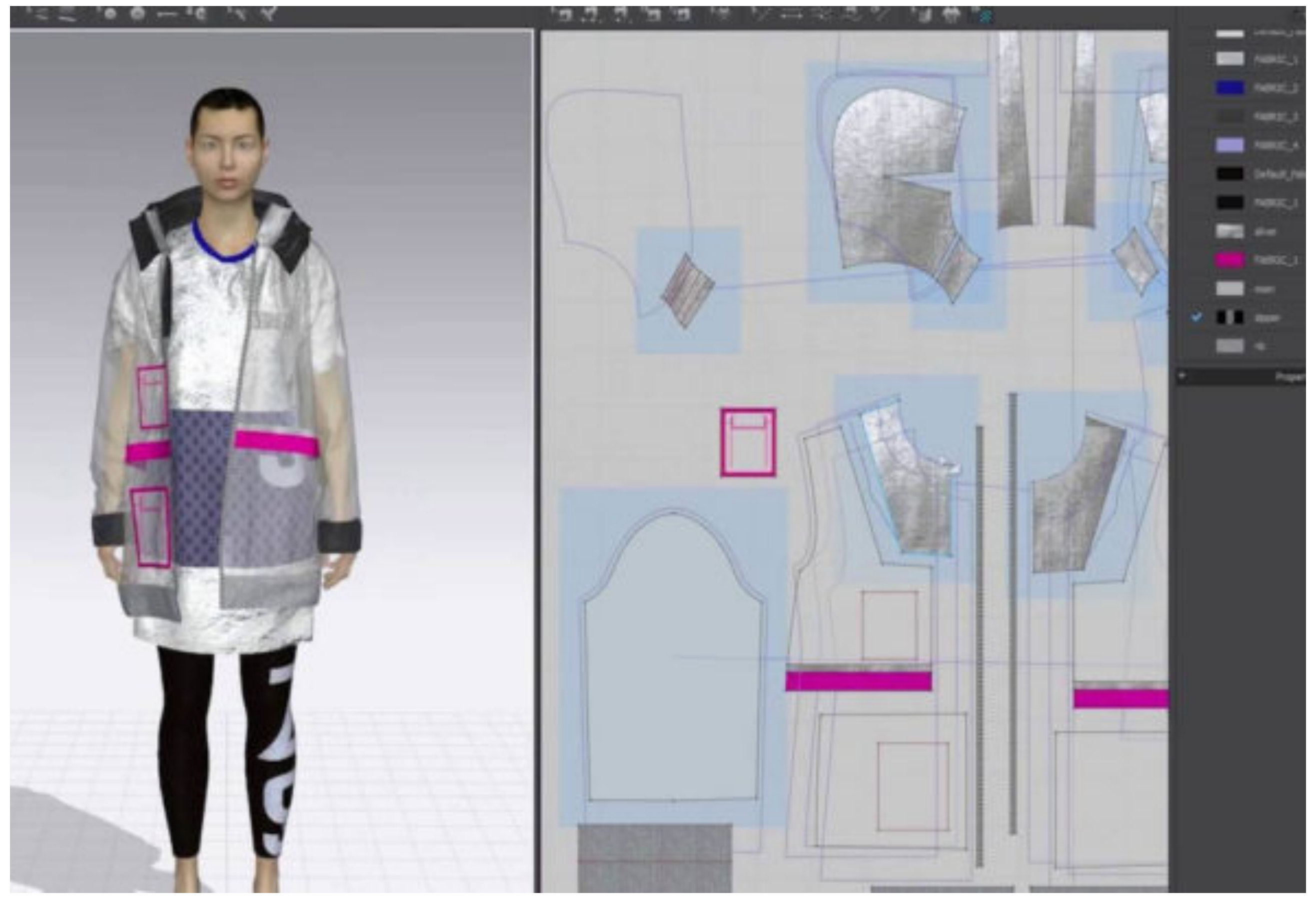
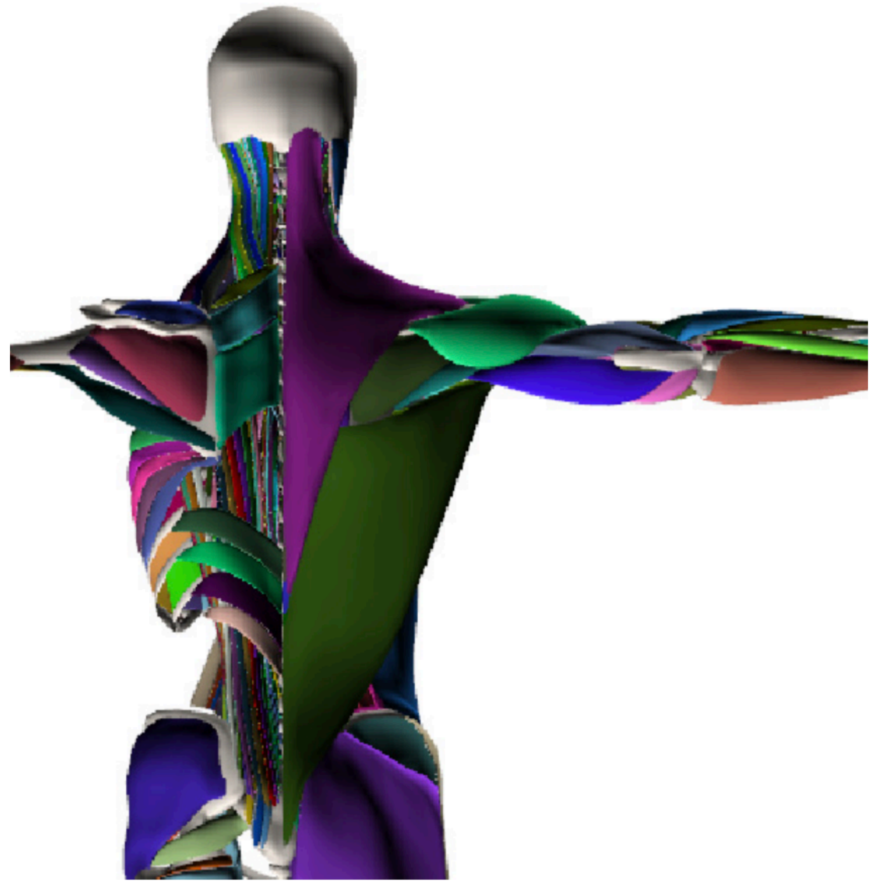
CONTACT SIMULATION APPLICATIONS



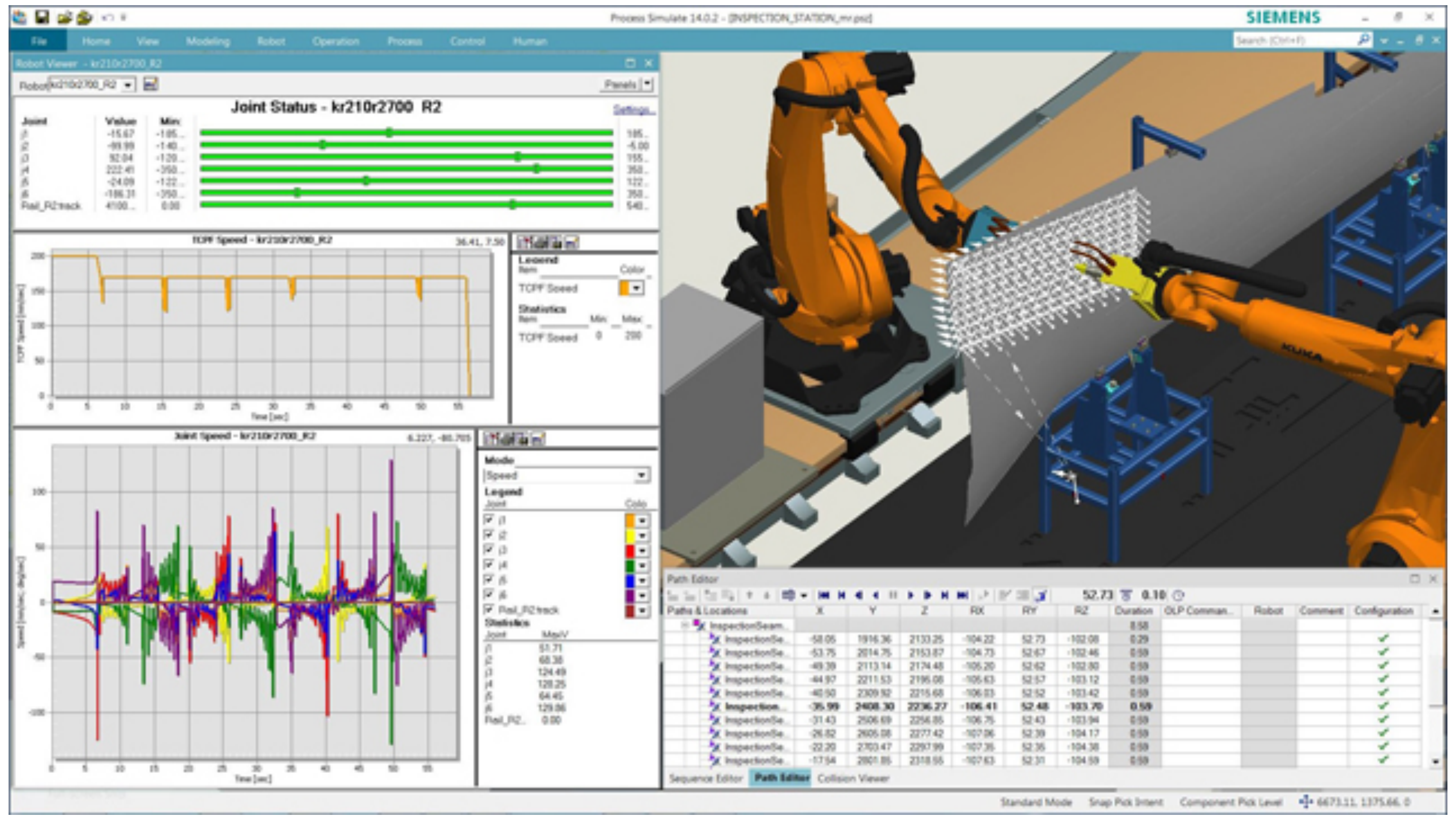
Visual effects & animations



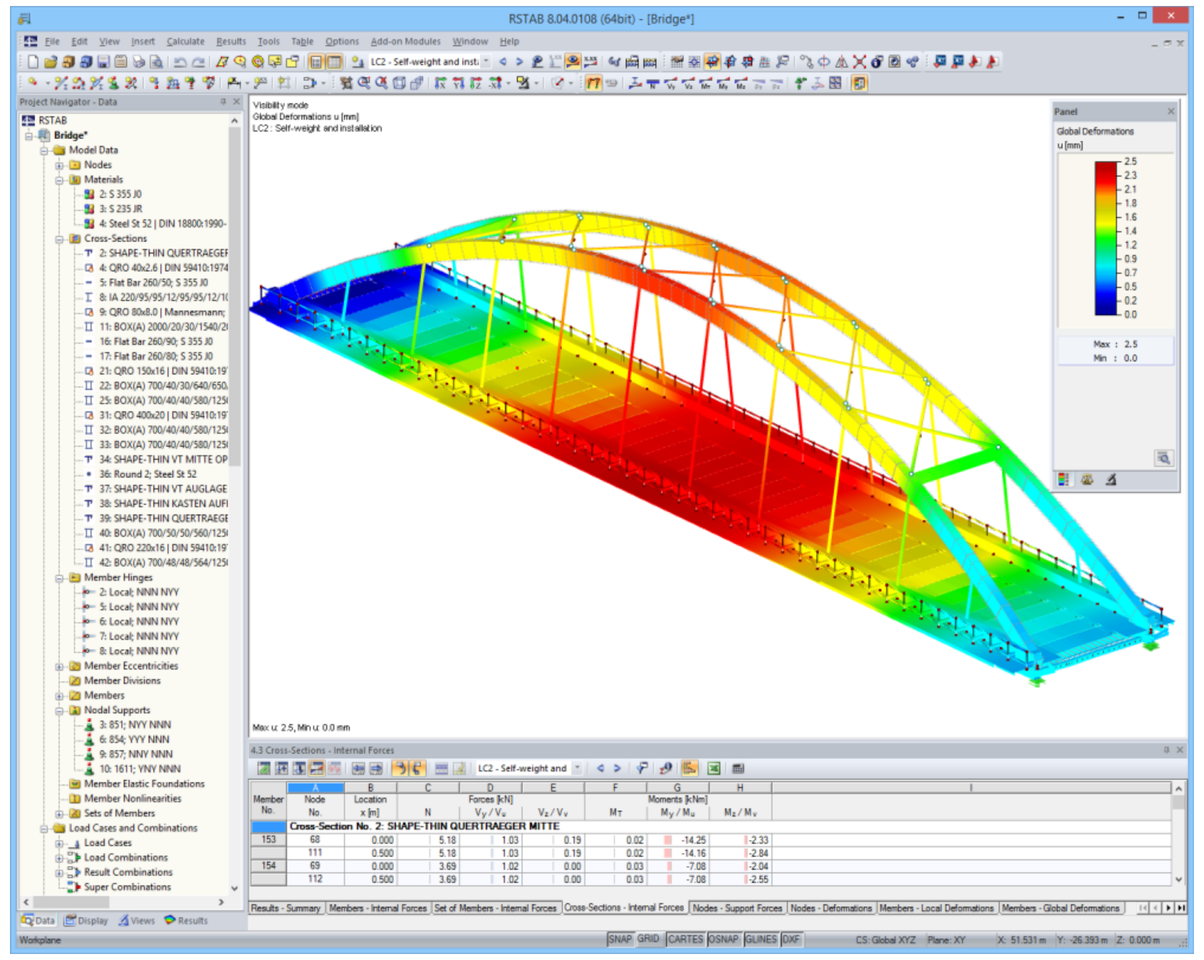
Fashion



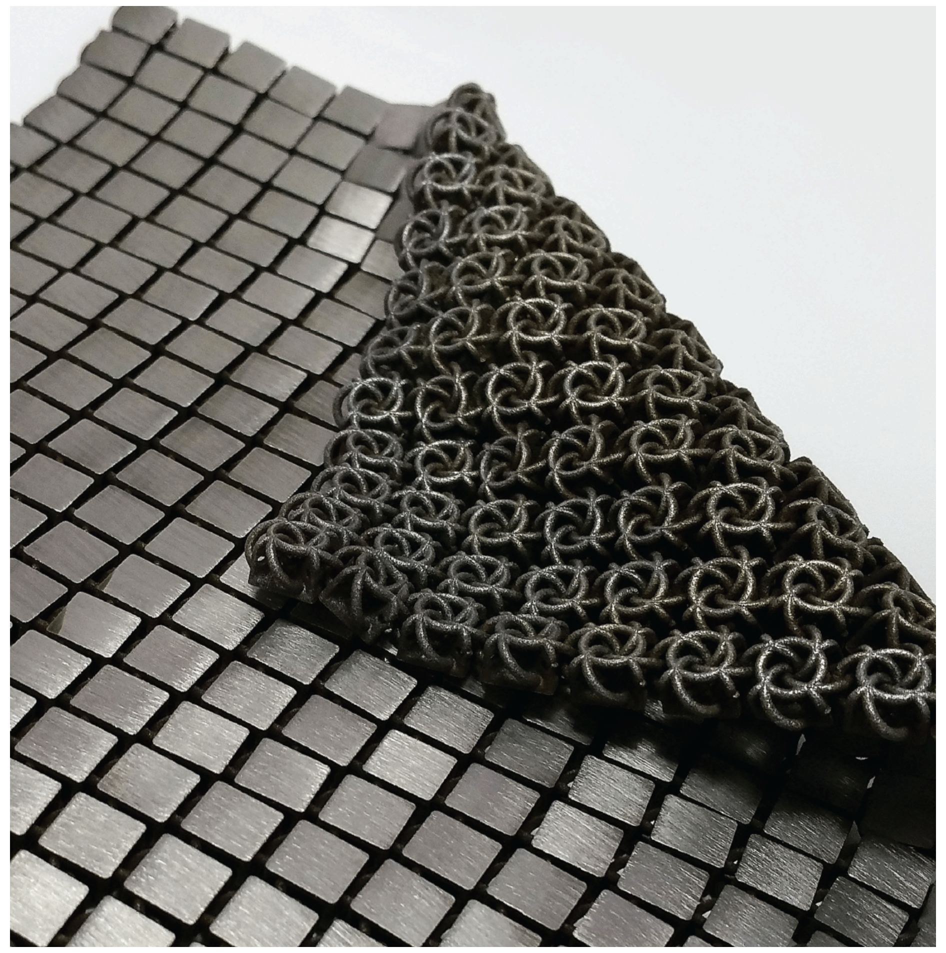
Biomechanics



Robotics



Engineering



Material science

TIME STEPPING

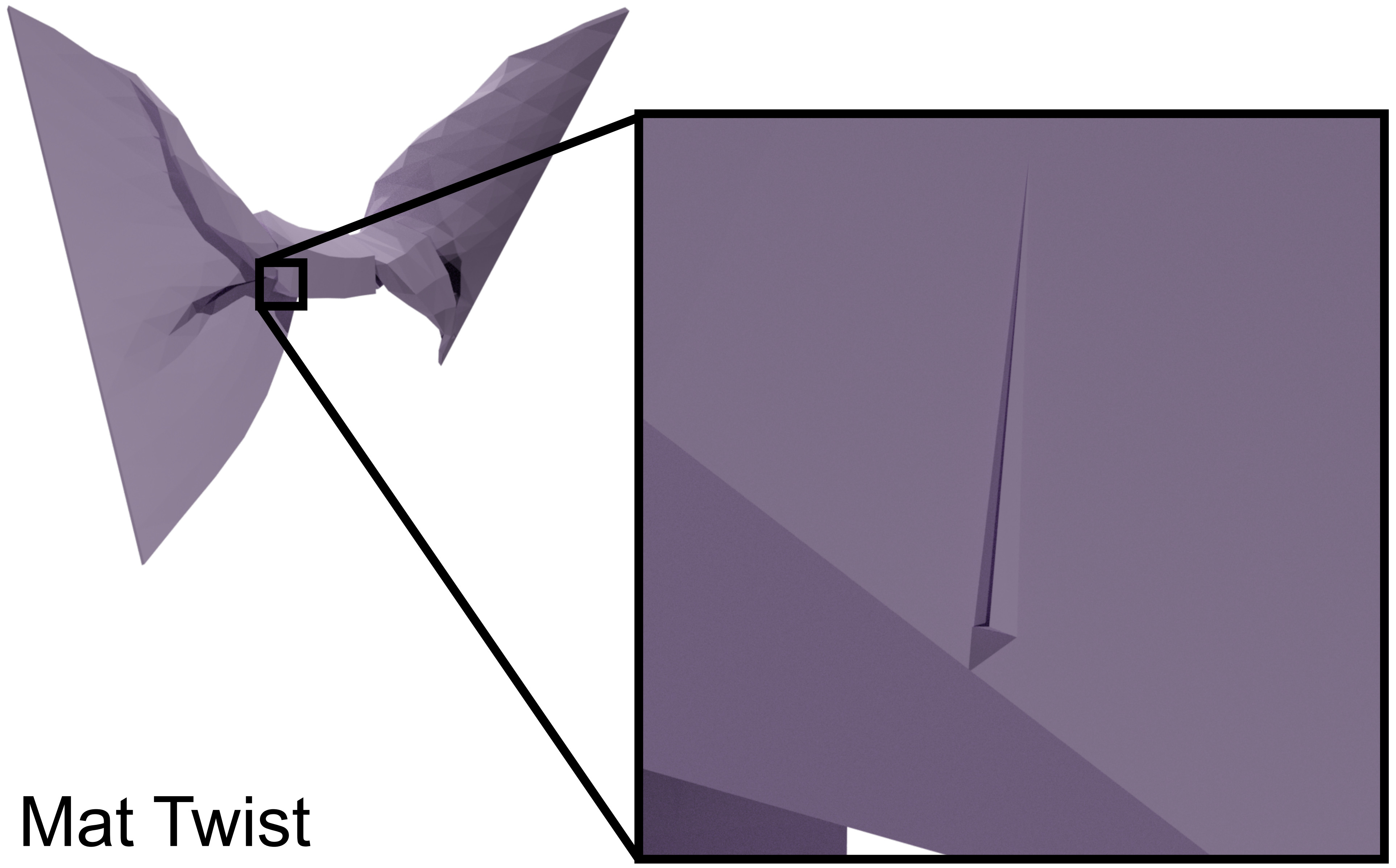
DOT [Li et al. 2019]

CHALLENGES WITH CONTACT

Existing methods

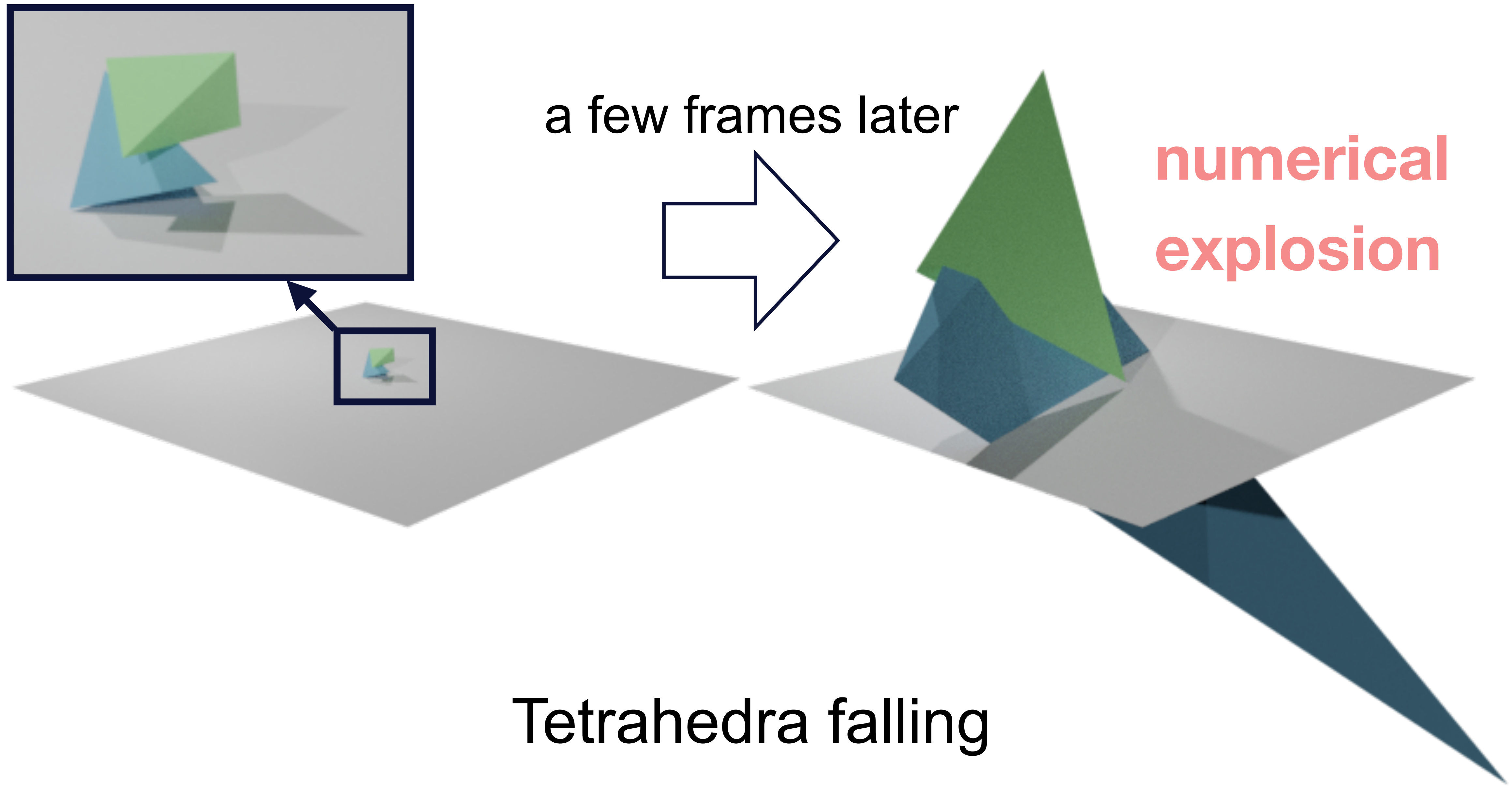
- need small time step sizes for stability and to avoid failures
- need per-example nonphysical parameter/mesh tuning

Common failures



Mat Twist

interpenetration

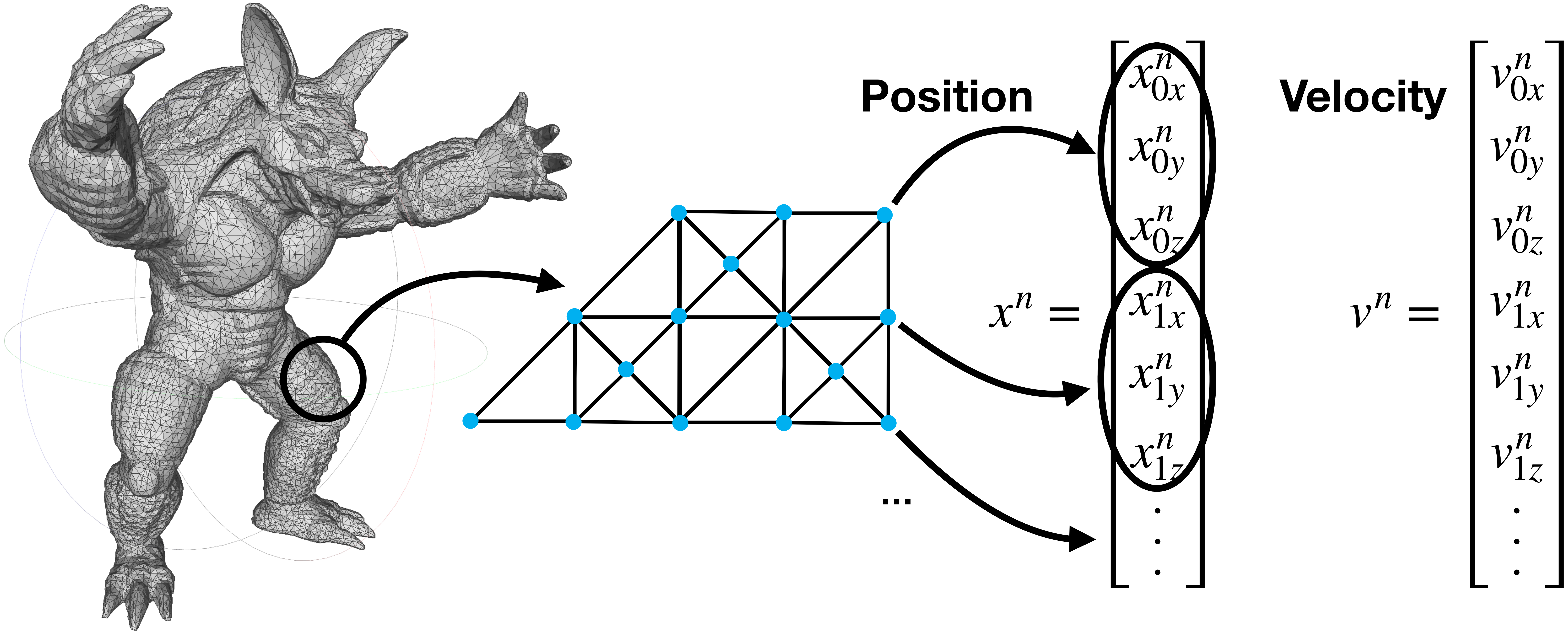


Tetrahedra falling

INCREMENTAL POTENTIAL CONTACT (IPC)

Mat Twist (100s)
45K nodes
133K tets
h: 0.04s
4x playback speed

SPATIAL DISCRETIZATION



IMPLICIT TIME STEPPING

$\{x^0, v^0\} \xrightarrow{f} \{x^1, v^1\} \xrightarrow{f} \{x^2, v^2\} \xrightarrow{f} \dots \xrightarrow{f} \{x^n, v^n\} \xrightarrow{f} \dots$

Given x^0, v^0**, for time steps** $n = 0,1,2,\dots$

$$\begin{cases} v^{n+1} = v^n + hM^{-1}f(x^{n+1}) & \textbf{Velocity update} \\ x^{n+1} = x^n + hv^{n+1} & \textbf{Position update} \end{cases}$$

$$x^{n+1} - h^2M^{-1}f(x^{n+1}) = x^n + hv^n$$

$$\Rightarrow \min_{x^{n+1}} \frac{1}{2}||x^{n+1} - \tilde{x}^n||_M^2 + h^2\Psi(x^{n+1})$$

where

$$\Psi(x) = -\int f(x)dx$$

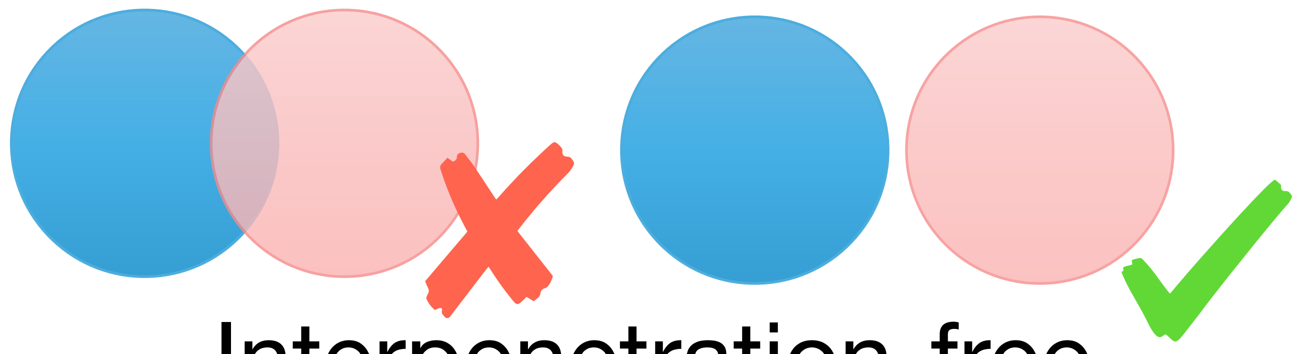
$$\tilde{x}^n = x^n + hv^n$$

$$\Psi = \sum_e V_e(\frac{\mu}{2}(\sum_i S_{ii}^2 - d) - \mu \ln(\det A_e) + \frac{\lambda}{2}(\ln(\det A_e))^2)$$

$A = USV$ is the transformation matrix of an element

OPTIMIZATION TIME INTEGRATION

For each time step t



Interpenetration-free

Predictive position — Mass matrix — Time step Size

$$x^{t+1} = \textbf{argmin}_x E(x) = \frac{1}{2}||x - \tilde{x}||^2_M + h^2\Psi(x) \qquad s.t. \quad g(x) \geq 0$$

New node positions — Incremental potential — Inertia term — Elasticity Potential

[Ortiz and Stainier 1999]

Nonlinear, nonconvex! Hard!

Without contact:
Solve with line search methods with 2nd-order information
[Liu et al. 2017, Li et al. 2019]

Challenges with contact:

1. How to **define** $g(x)$

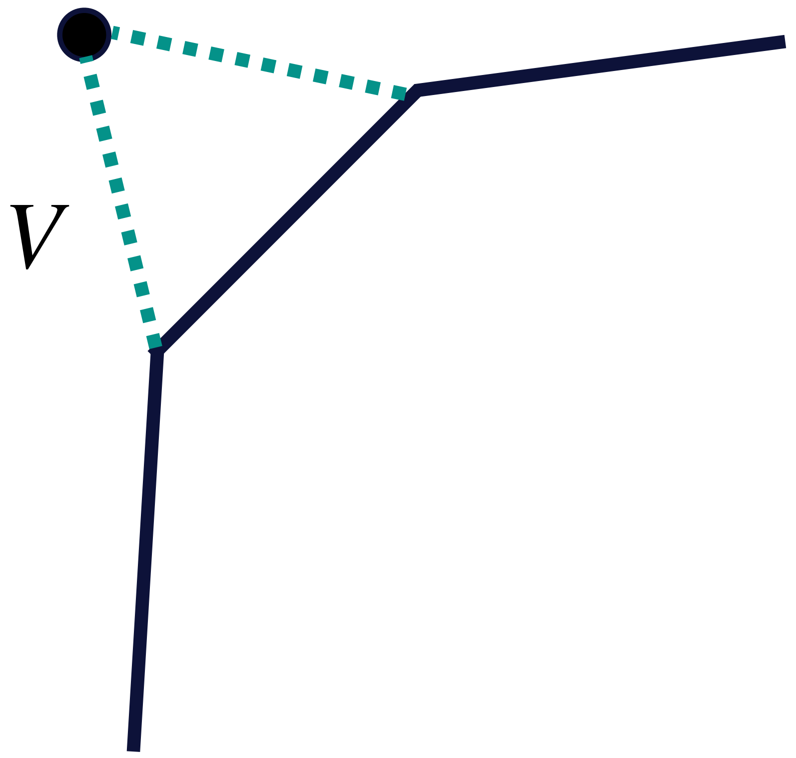
2. How to **solve** the constrained problem

RELATED WORK

Constraint definition:

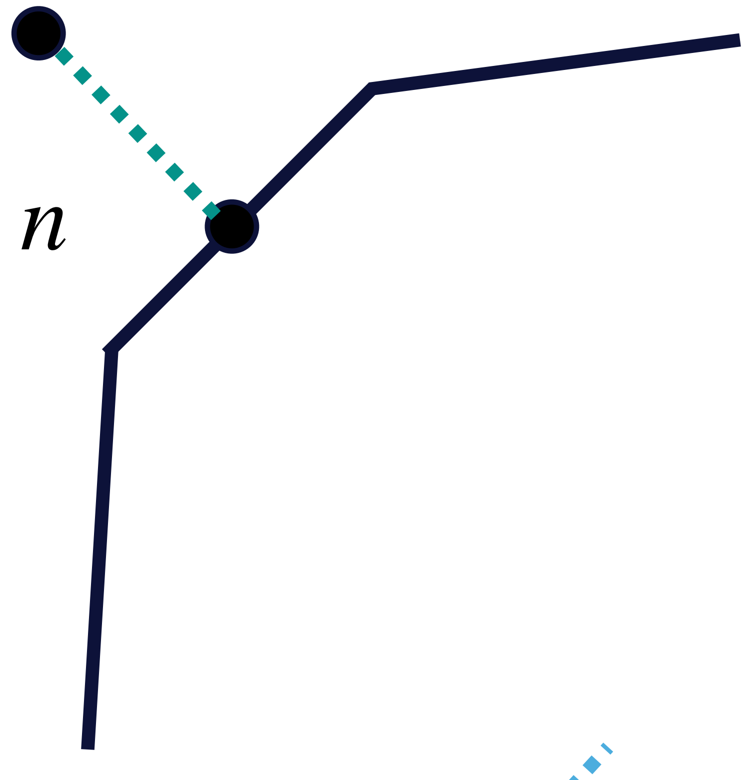
Volume constraints [Kane 1999, Müller 2015, etc]:

$g(x) = V$

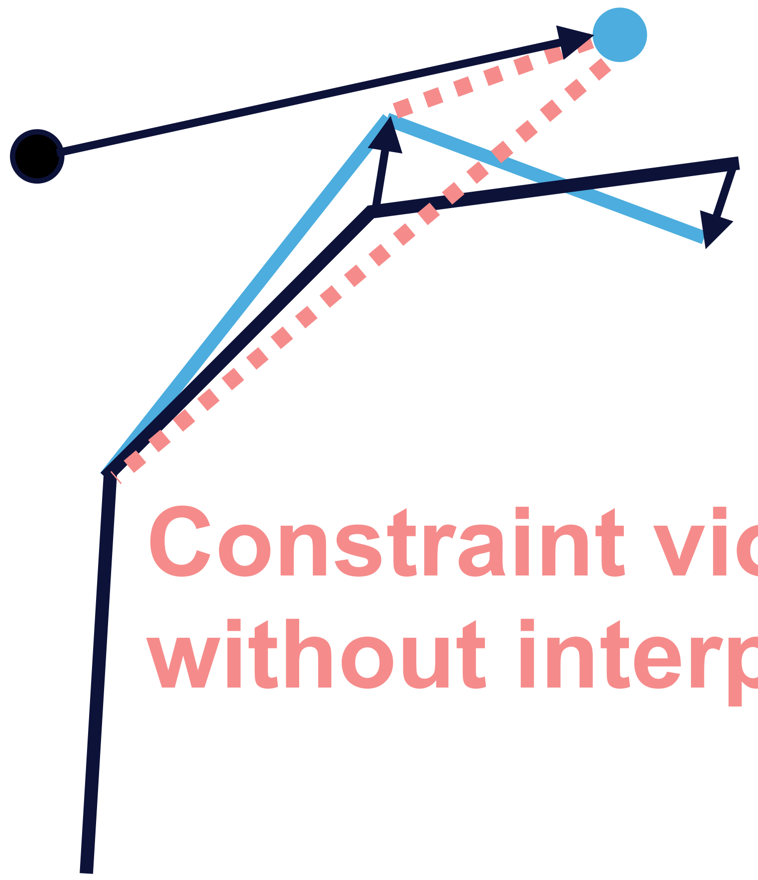


Gap constraints [Harmon 2008, Verschoor 2019, Otaduy 2009, etc]:

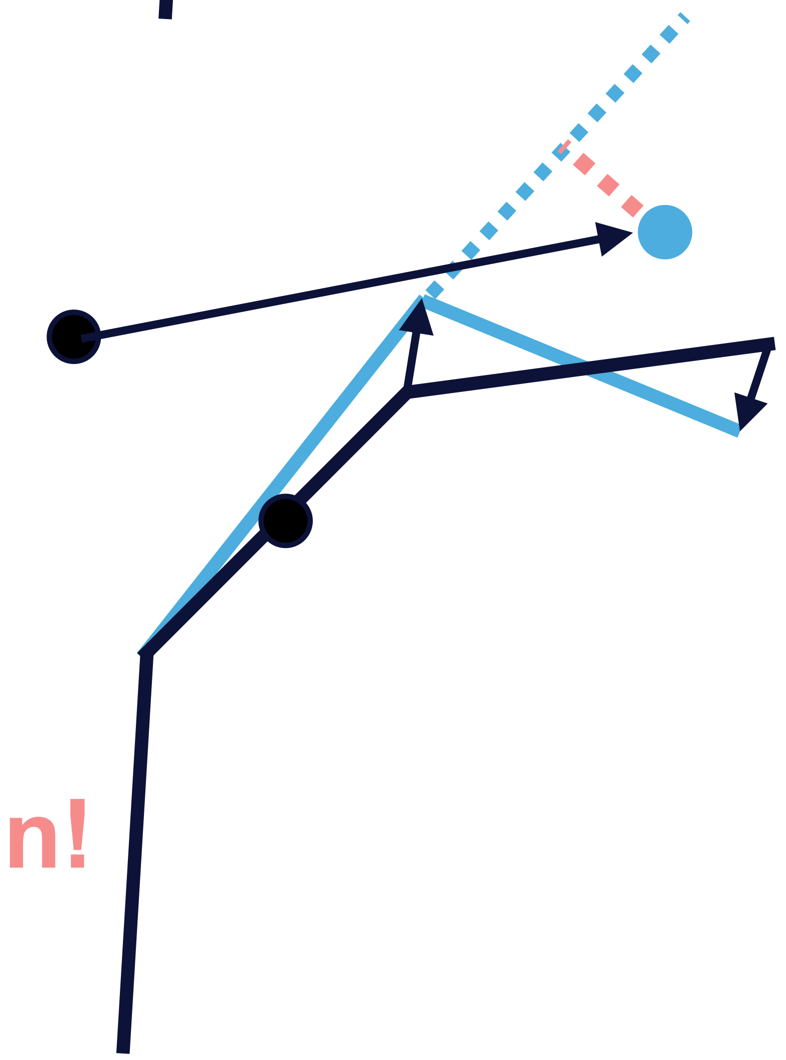
$g(x) = d \cdot n$



Large displacement:



Constraint violated without interpenetration!



SQP solve [Kane 1999, Kaufman 2008, Otaduy 2009, Verschoor 2019, etc]:

For iteration i in a time step:

$$x^{i+1} = \text{argmin}_x \frac{1}{2} x^T \nabla^2 E(x^i) x + x^T \nabla E(x^i)$$

$$s.t. \quad \forall k \in C(x^0), \quad g_k(x^i) + x^T \nabla g_k(x^i) \geq \epsilon$$

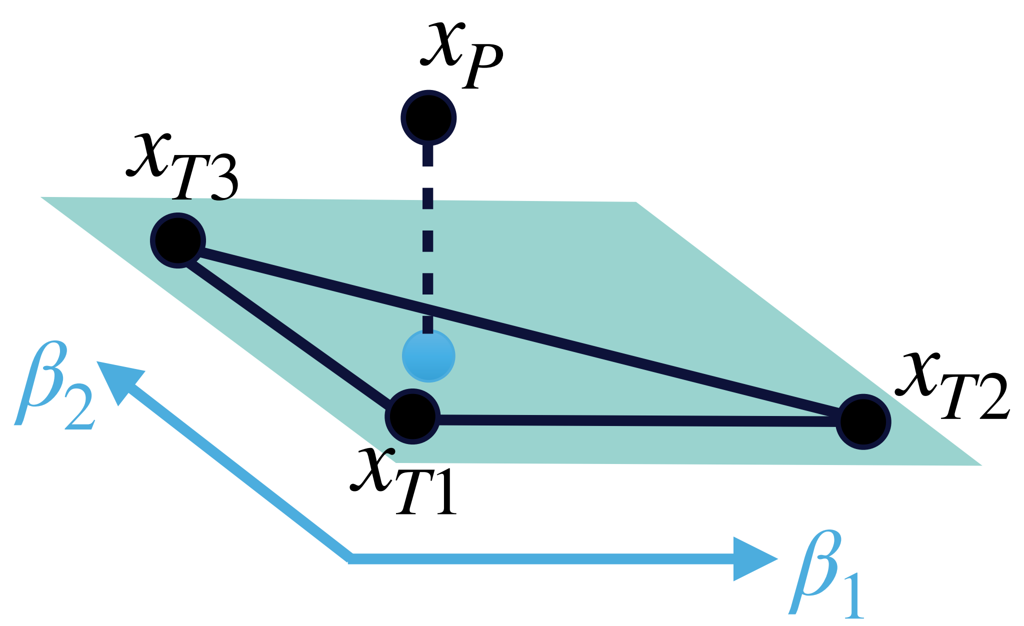
Heuristically updating C, require small h

Houdini FEM

SOFA [Faure et al. 2012]

IPC METHOD

CONSISTENT UNSIGNED DISTANCE



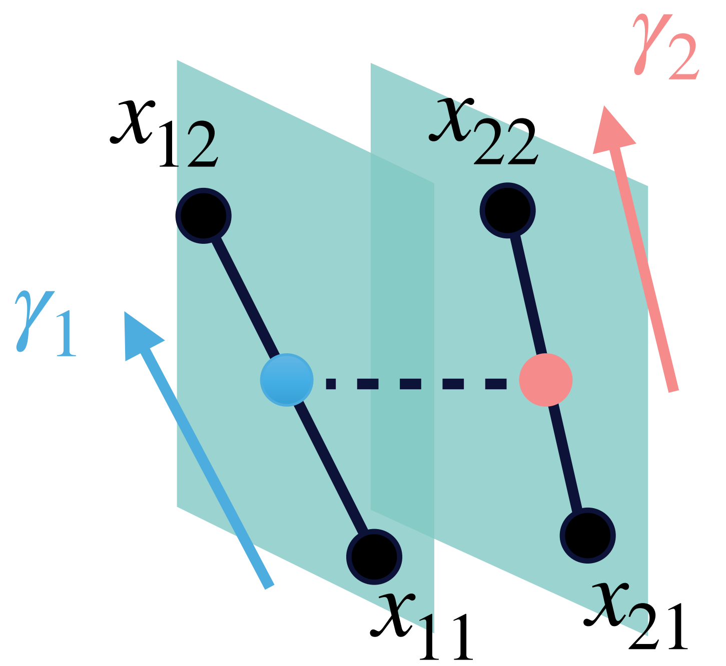
Always compute the precise distance

k-th point (x_P) - triangle ($x_{T1}x_{T2}x_{T3}$) pair:

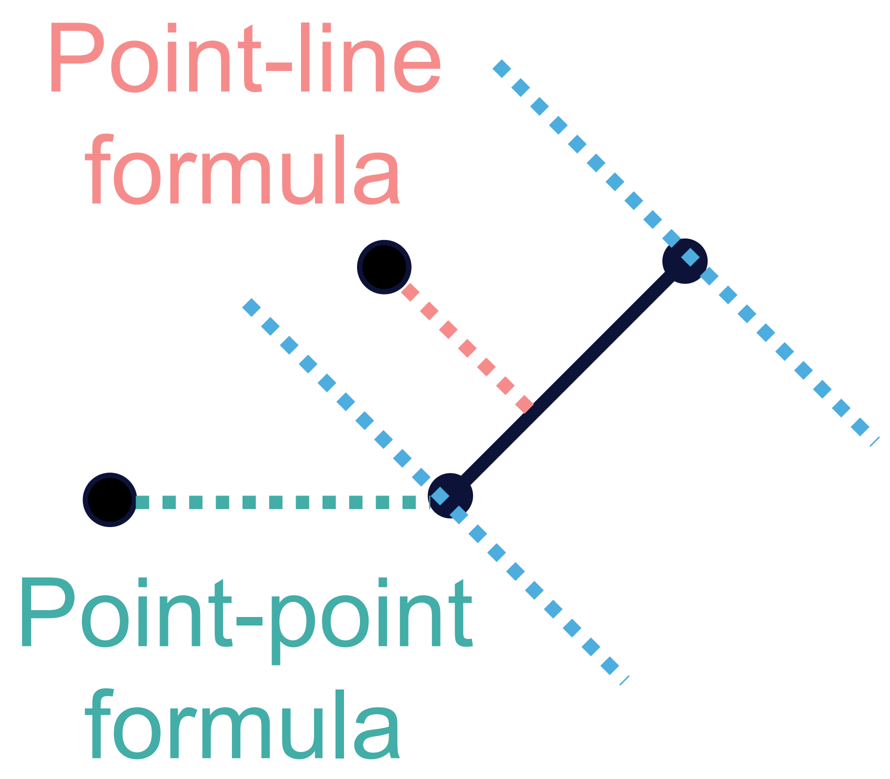
$$D_k^{PT}(x) = \min_{\beta_1,\beta_2} ||x_P - (x_{T1} + \beta_1(x_{T2} - x_{T1}) + \beta_2(x_{T3} - x_{T1}))|| \quad s.t. \quad \beta_1, \beta_2 \geq 0 \quad \beta_1 + \beta_2 \leq 1$$

k-th edge ($x_{11}x_{12}$) - edge ($x_{21}x_{22}$) pair:

$$D_k^{EE}(x) = \min_{\gamma_1,\gamma_2} ||x_{11} + \gamma_1(x_{12} - x_{11}) - (x_{21} + \gamma_2(x_{22} - x_{21}))|| \quad s.t. \quad 0 \leq \gamma_1, \gamma_2 \leq 0$$



— both are piecewise smooth analytical functions



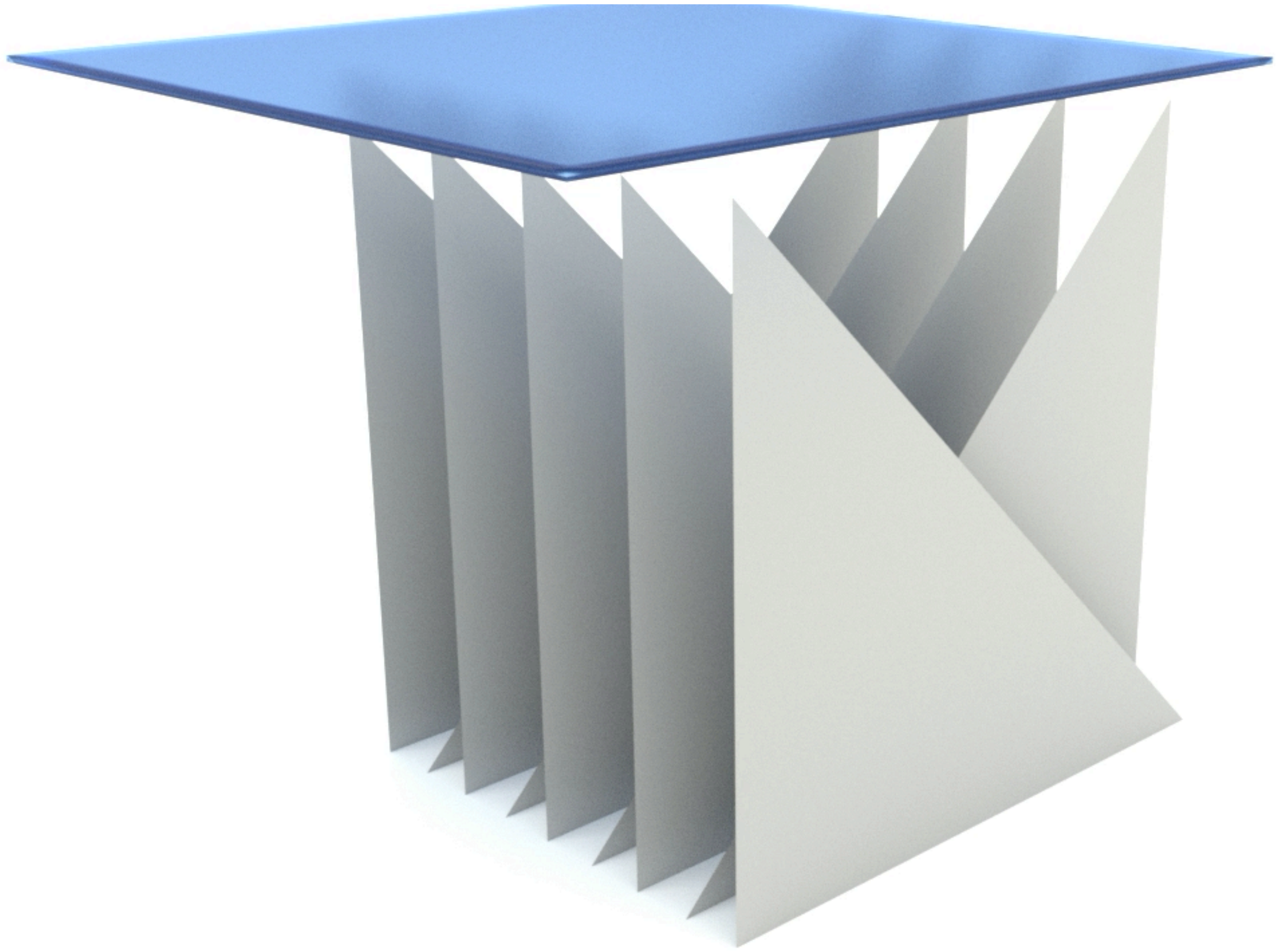
$g(x) = D(x) \geq 0$ always holds… Use $g(x) = D(x) > 0$:

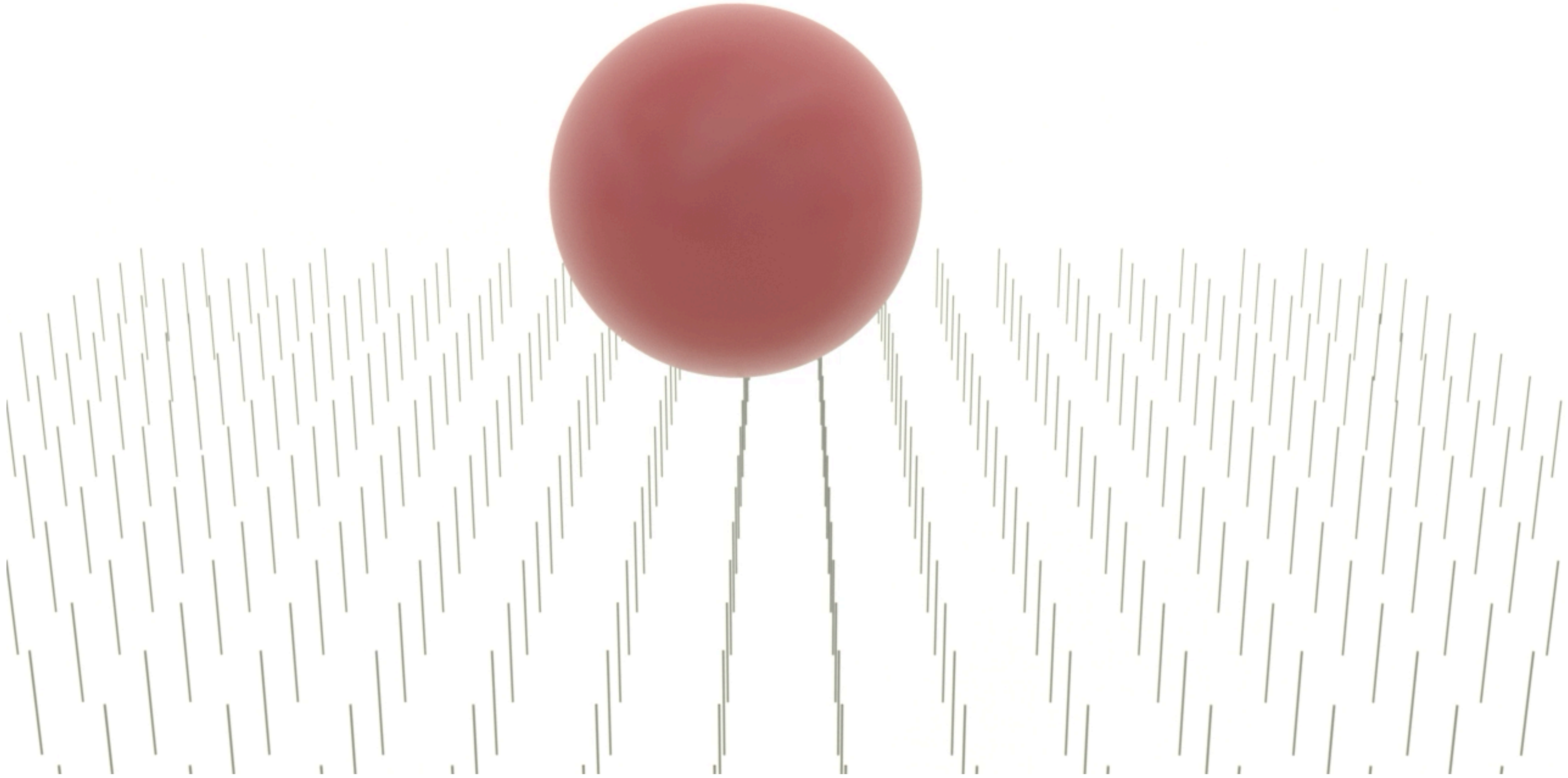
IPC Constraint definition: (with intersection-free $x^{0,0}$)
$\forall \alpha \in [0,1]$, time step t, iteration i, contact primitive pair k:

$$D_k^{PT}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0, \quad D_k^{EE}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0$$

CODIMENSIONAL CONTACT EXAMPLE

IPC's constraint definition with unsigned distances naturally supports codimensional contact:





CCD LINE SEARCH AND BARRIER FORMULATION

IPC Constraint definition: (with intersection-free $x^{0,0}$)

$\forall \alpha \in [0,1]$, time step t, iteration i, contact primitive pair k:

$$D_k^{PT}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0, \quad D_k^{EE}(\alpha x^{t,i} + (1-\alpha)x^{t,i+1}) > 0$$

IPC model problem becomes:

CCD in each iteration:

$\alpha_{CCD} \leftarrow CCD(x^{t,i}, x^{t,i+1})$

$x^{t,i+1} \leftarrow \alpha x^{t,i} + (1-\alpha)x^{t,i+1}$ with $\alpha \in (0, \alpha_{CCD})$

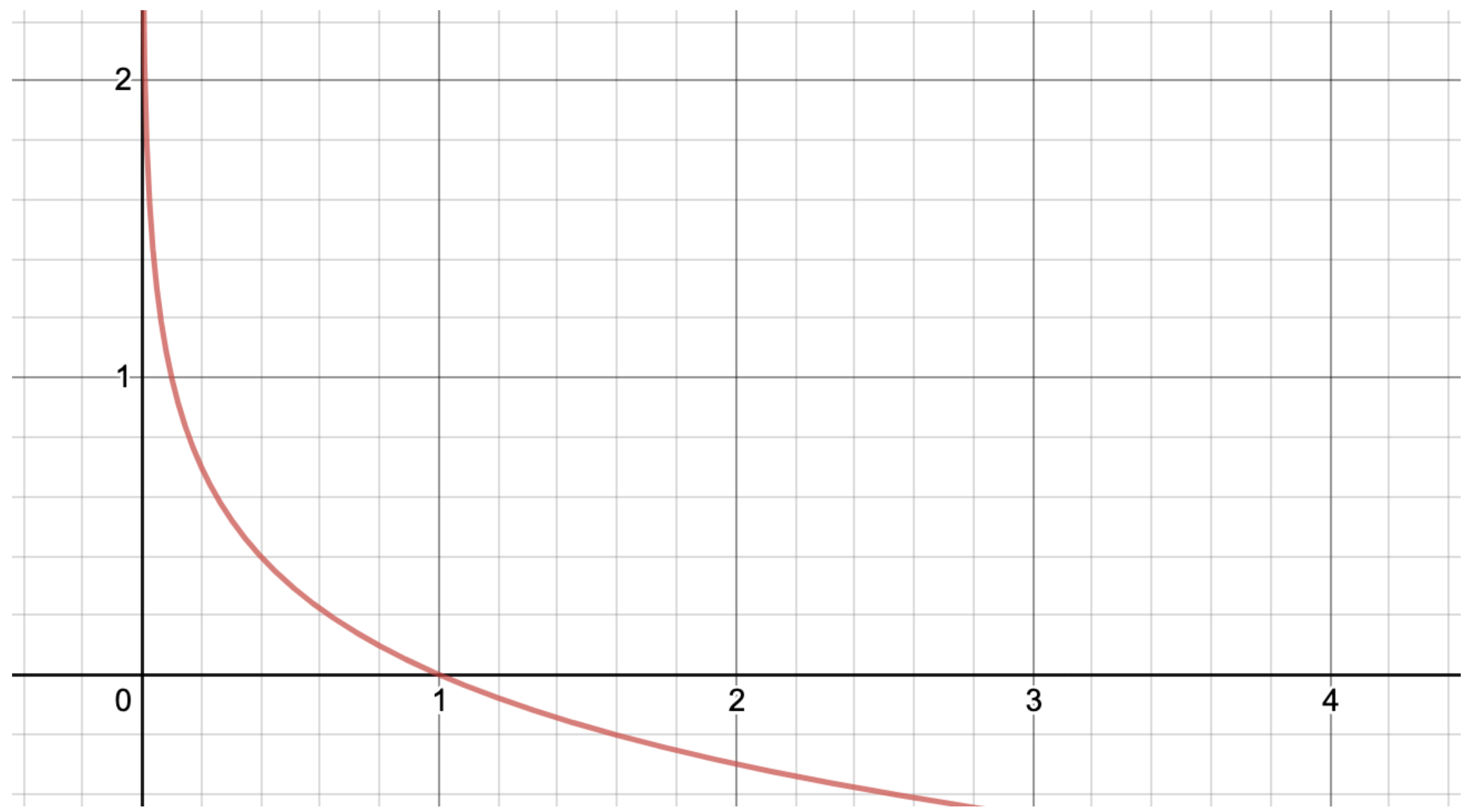
$x^{t+1} = \text{argmin}_x E(x) \quad s.t. \quad \forall k, \quad D_k(x) > 0$

Barrier method

$$x^{t+1} = \text{argmin}_x \left(E(x) + \kappa \sum_k b(D_k(x)) \right)$$

unconstrained!

Barrier function example: $b(D) = -\log(D)$



Newton iteration **quadratically** approximate $E(x) + \kappa \sum_k b(g_k(x))$ but not forming

$$\min_x x^T \nabla^2 E(x^i) x + 2x^T \nabla E(x^i)$$
$$s.t. \quad \forall k, \quad g_k(x^i) + x^T \nabla g_k(x^i) \geq 0$$

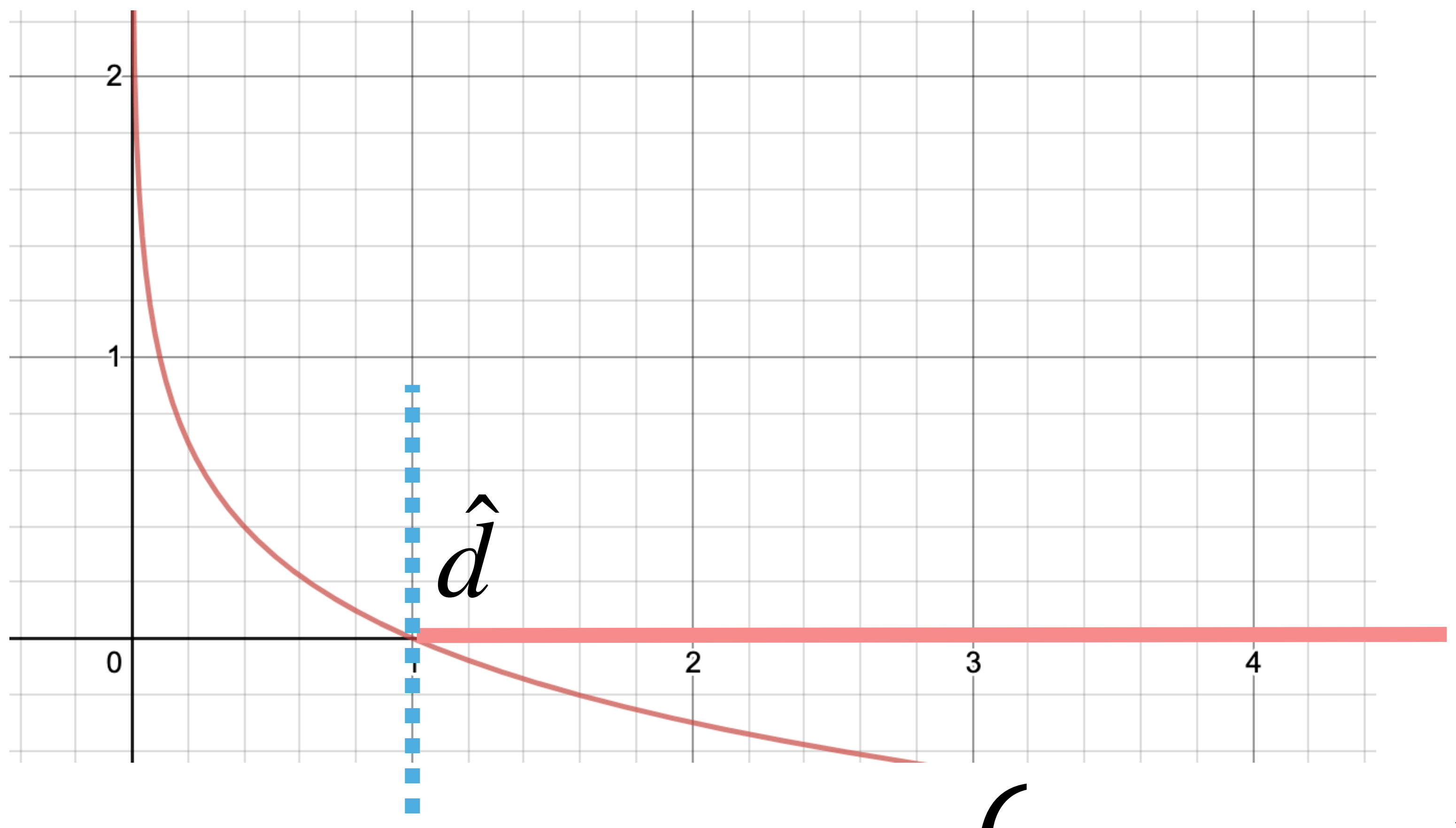
Hence no infeasible subproblems!

SMOOTHLY CLAMPED BARRIER FOR SCALABILITY

$$x^{t+1} = \text{argmin}_x \left(E(x) + \kappa \sum_k b(D_k(x)) \right)$$

quadratically increases with # surface primitives

Traditional barrier function: $b(D) = -\log(D)$

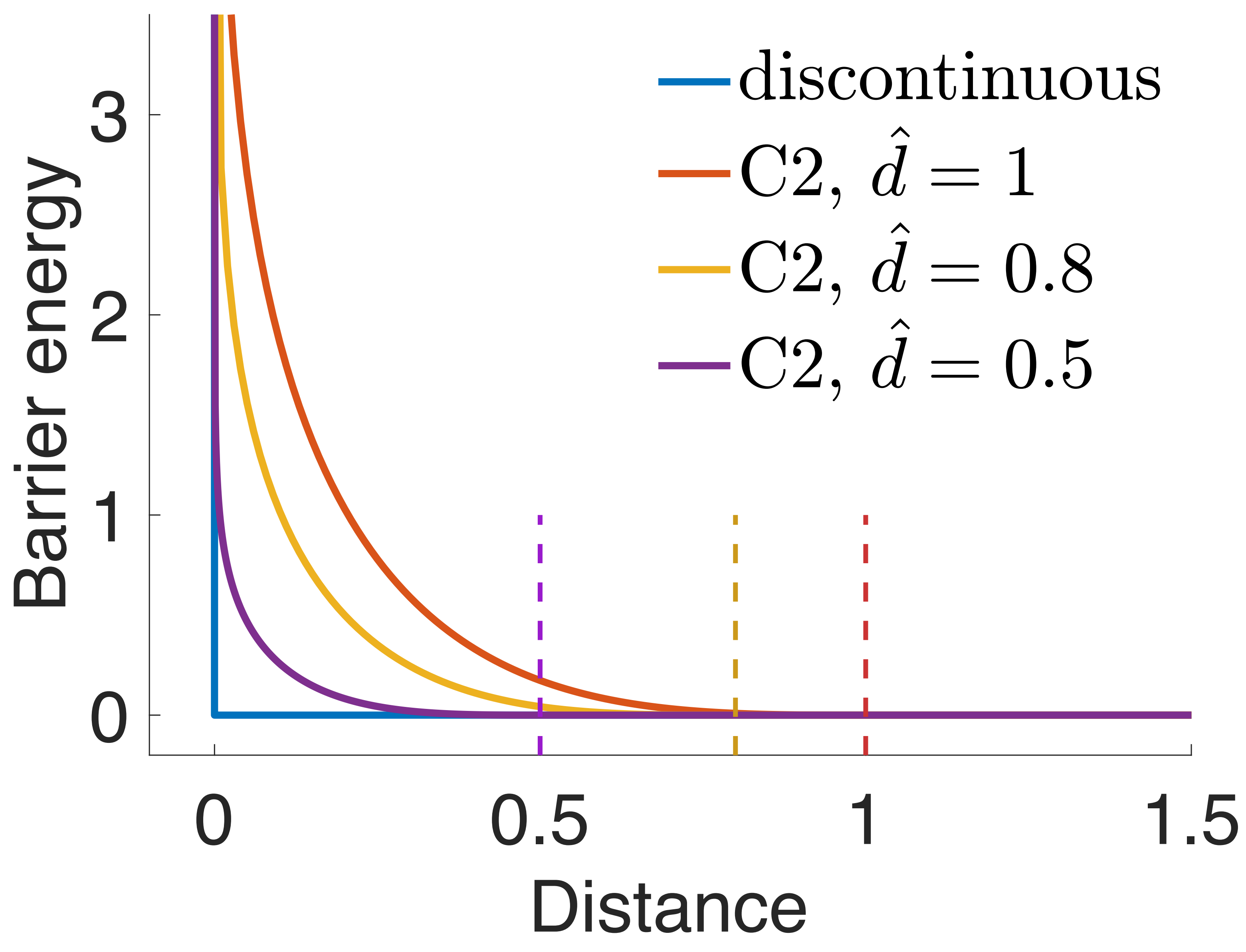


C0 clamping $b(D, \hat{d}) = \begin{cases} -\log(D/\hat{d}) & \text{if } D < \hat{d} \\ 0 & \text{if } D \geq \hat{d} \end{cases}$

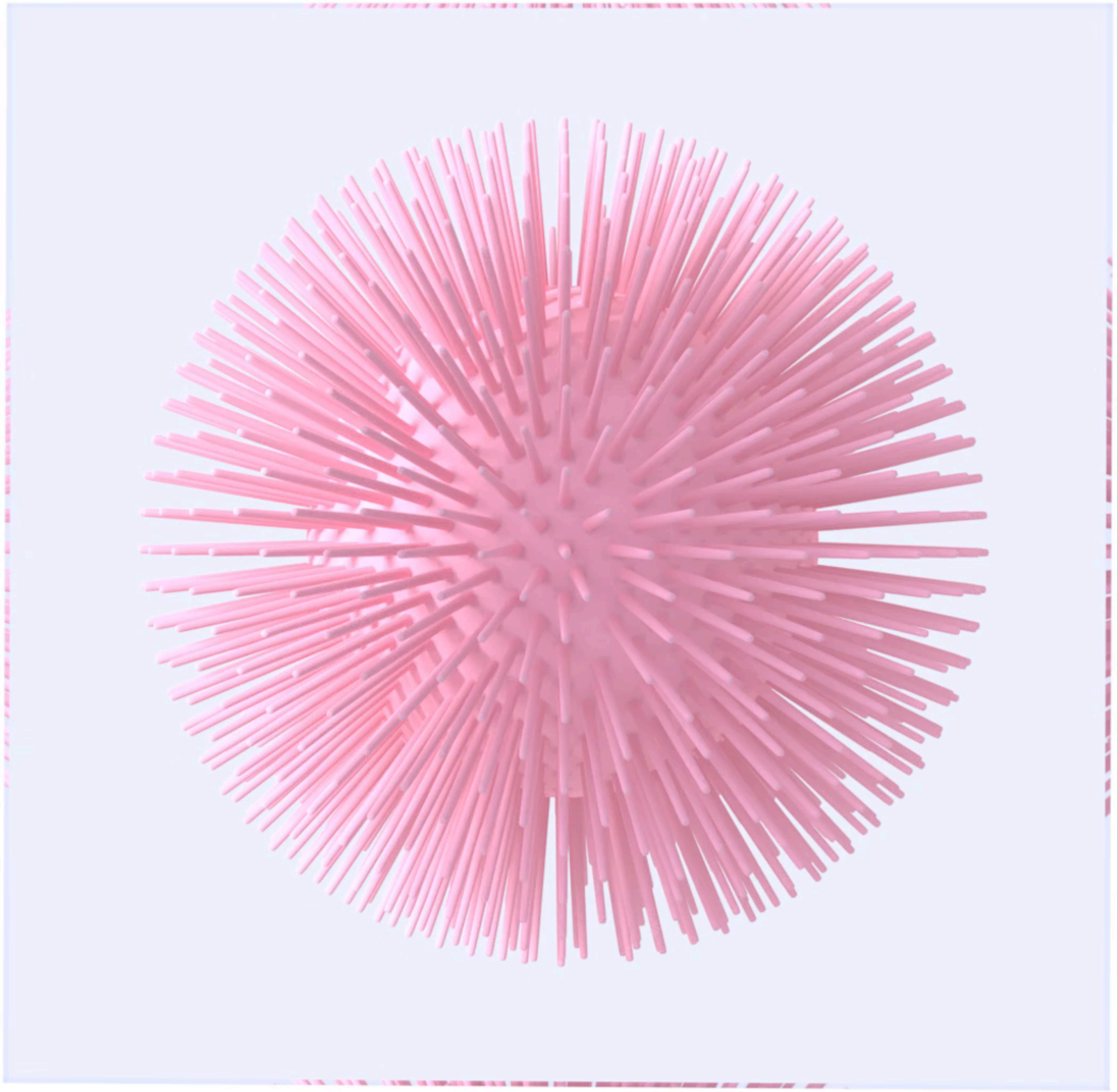
harms convergence!

IPC's C2 clamping:

$$b(D, \hat{d}) = \begin{cases} -(D - \hat{d})^2 \log(D/\hat{d}) & \text{if } D < \hat{d} \\ 0 & \text{if } D \geq \hat{d} \end{cases}$$



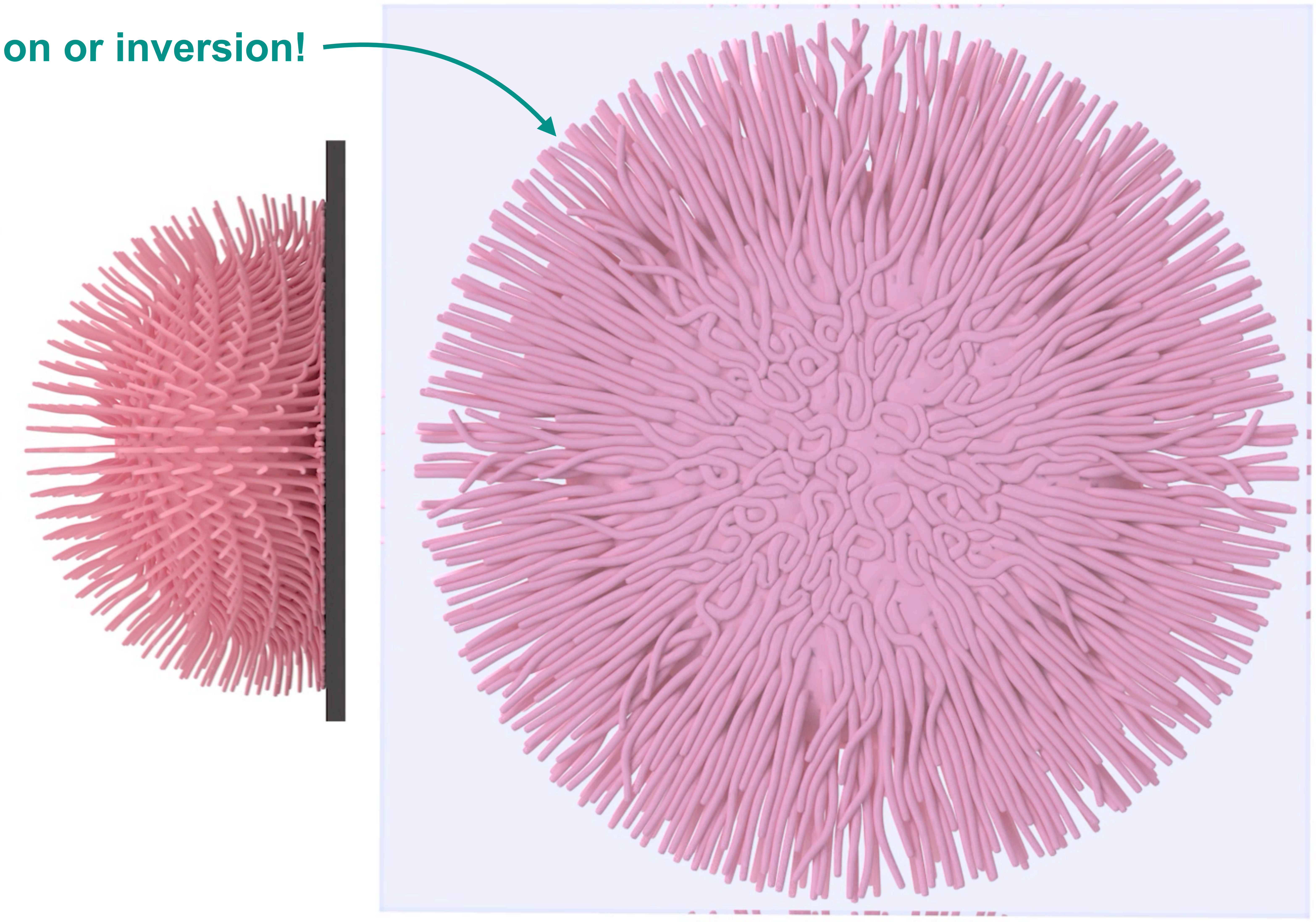
LARGE-SCALE SIMULATION EXAMPLE



tetrahedra: **2314K**
contacts per step (max): **105K**
h: 0.001s
runtime: 328.3s / step
0.1x playback speed

LARGE-SCALE SIMULATION EXAMPLE

No intersection or inversion!



tetrahedra: **2314K**
contacts per step (max): **105K**
h: 0.001s
runtime: 328.3s / step
0.1x playback speed

IPC PSEUDO-CODE

Initialization: $x \leftarrow x^t$, $x_{prev} \leftarrow x$, $E_{prev} \leftarrow E(x)$

Newton loop (no contact):

Do

$H \leftarrow$ SPDProject($\nabla_x^2 E(x)$)

$p \leftarrow -H^{-1} \nabla_x E(x)$

$\alpha \leftarrow 1$

Do

$x \leftarrow x_{prev} + \alpha p$

$\alpha \leftarrow \alpha/2$

While $E(x) > E_{prev}$

$E_{prev} \leftarrow E(x)$, $x_{prev} \leftarrow x$

While $\|p\|_\infty / h > \epsilon_d$

with IPC

$\hat{C} \leftarrow$ ComputeConstraintSet$(x, \hat{d})$, $E_{prev} \leftarrow B_t(x, \hat{d}, \hat{C})$

Newton loop (IPC): $B_t(x, \hat{d}, \hat{C}) = E(x) + \kappa \sum_{k \in \hat{C}} b(D_k(x), \hat{d})$

Do

$H \leftarrow$ SPDProject($\nabla_x^2 B_t(x, \hat{d}, \hat{C})$)

$p \leftarrow -H^{-1} \nabla_x B_t(x, \hat{d}, \hat{C})$

$\alpha \leftarrow \min(1, CCD(x, x+p))$

Do

$x \leftarrow x_{prev} + \alpha p$

$\hat{C} \leftarrow$ ComputeConstraintSet$(x, \hat{d})$

$\alpha \leftarrow \alpha/2$

While $B_t(x, \hat{d}, \hat{C}) > E_{prev}$

$E_{prev} \leftarrow B_t(x, \hat{d}, \hat{C})$, $x_{prev} \leftarrow x$

Update κ, boundary conditions, etc

While $\|p\|_\infty / h > \epsilon_d$

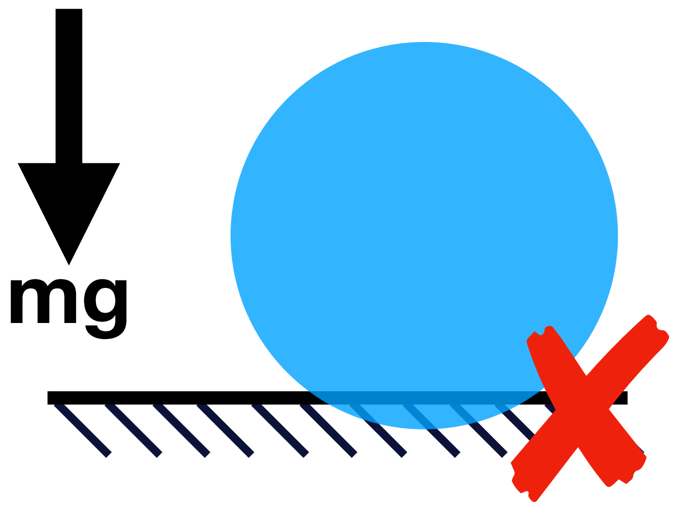
ACCURACY

KKT conditions for inequality constrained optimization:

Primal feasibility

$$\forall k, \quad D_k > 0$$

Interpenetration-free
(Also inversion-free when needed)

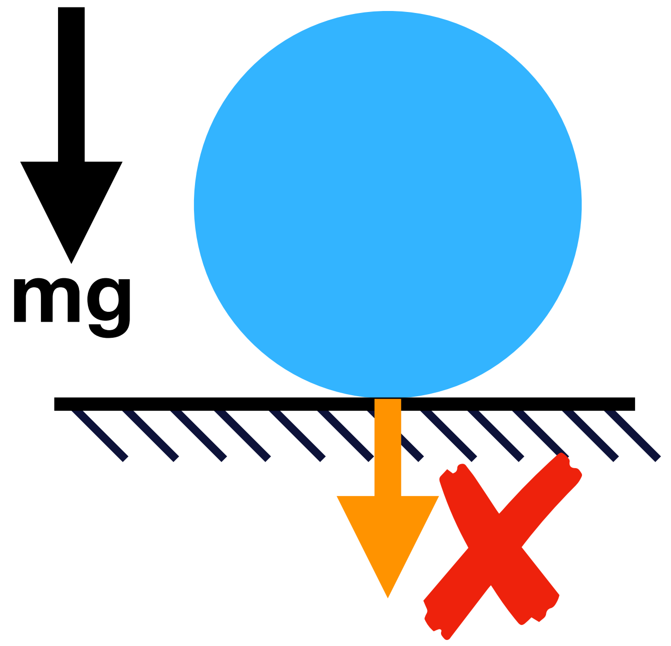


Satisfied by construction!

Dual feasibility

$$\forall k, \quad -\kappa \frac{\partial b}{\partial D_k} > 0$$

Contact force only push but not pull

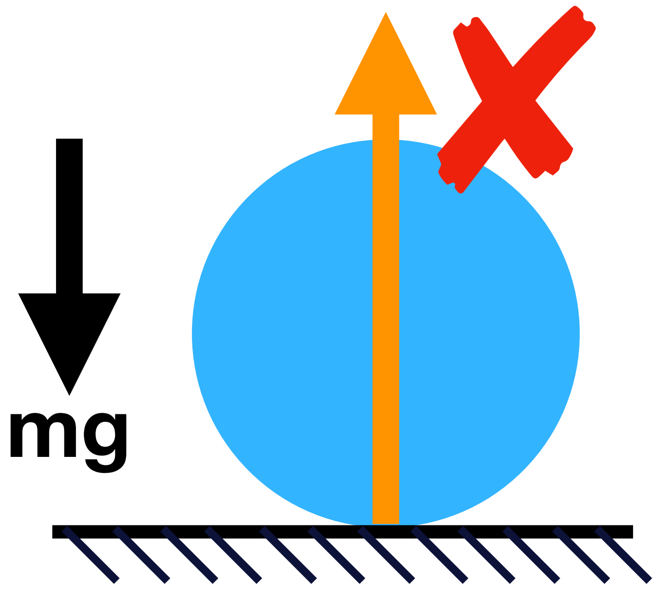


Satisfied by construction!

Stationarity

$$\nabla E(x) + \kappa \sum_k \nabla b(D_k(x), \hat{d}) = 0$$

Momentum balance

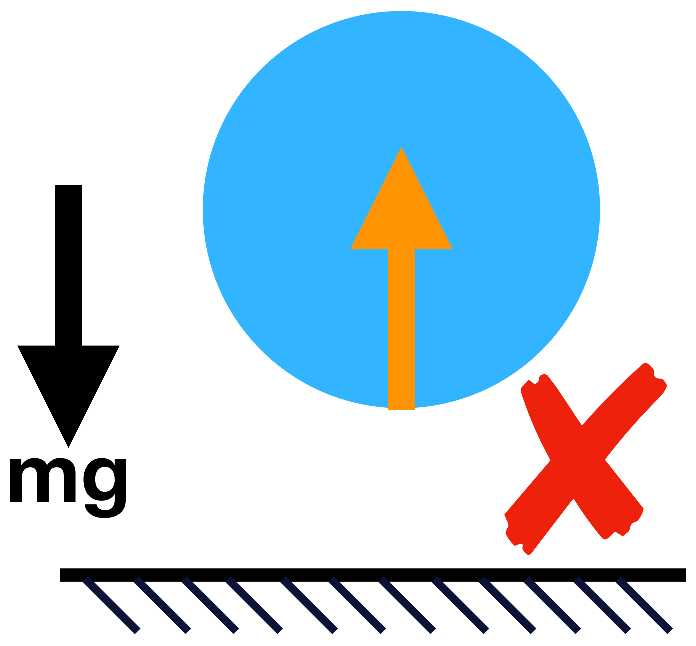


Accuracy controlled by Newton tolerance ϵ_d

Complementarity slackness

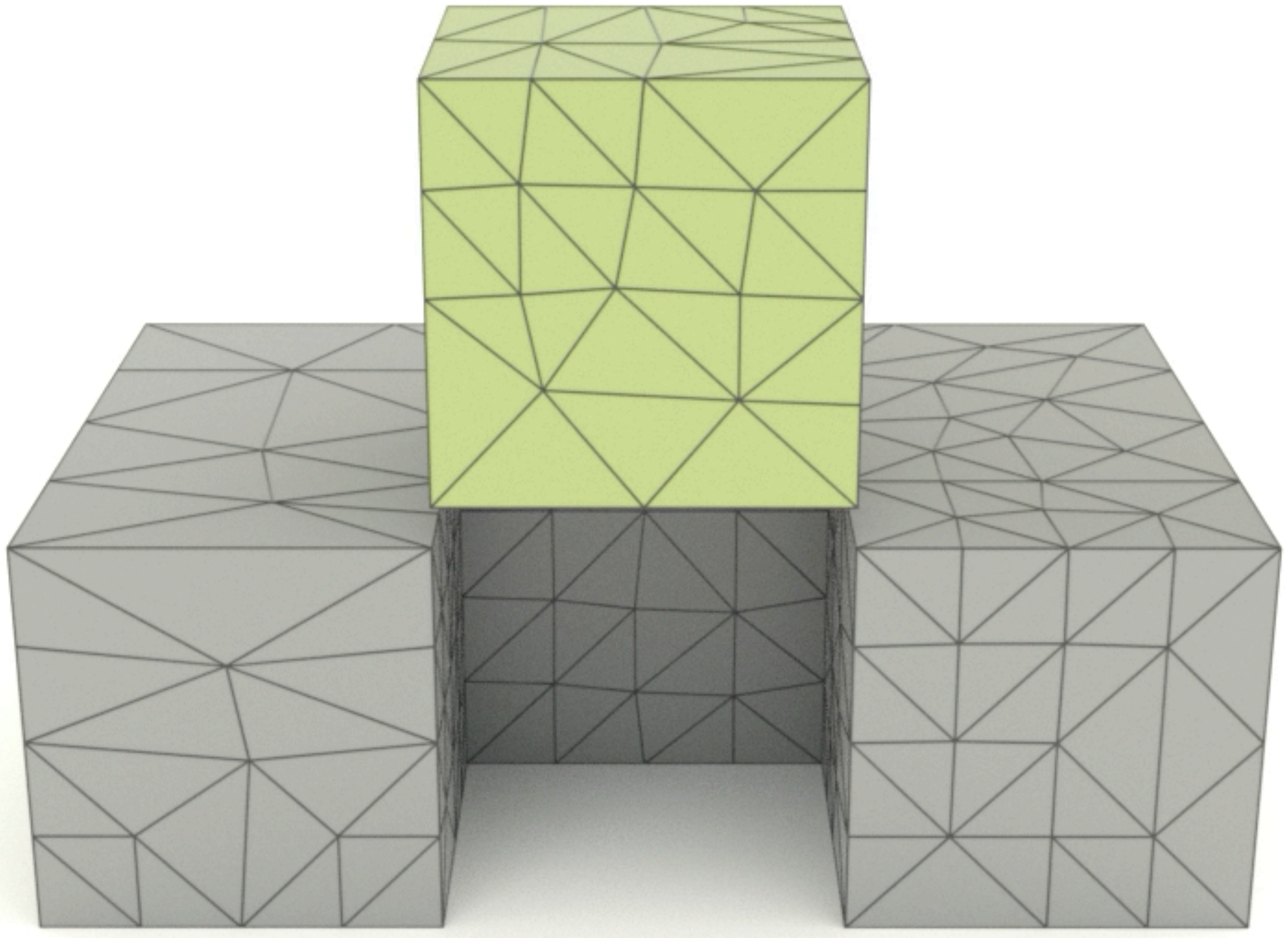
$$\sum_k D_k(-\kappa \frac{\partial b}{\partial D_k}) = 0$$

Contact force only applied on between touching regions



Accuracy controlled by clamping threshold $\hat{d}$

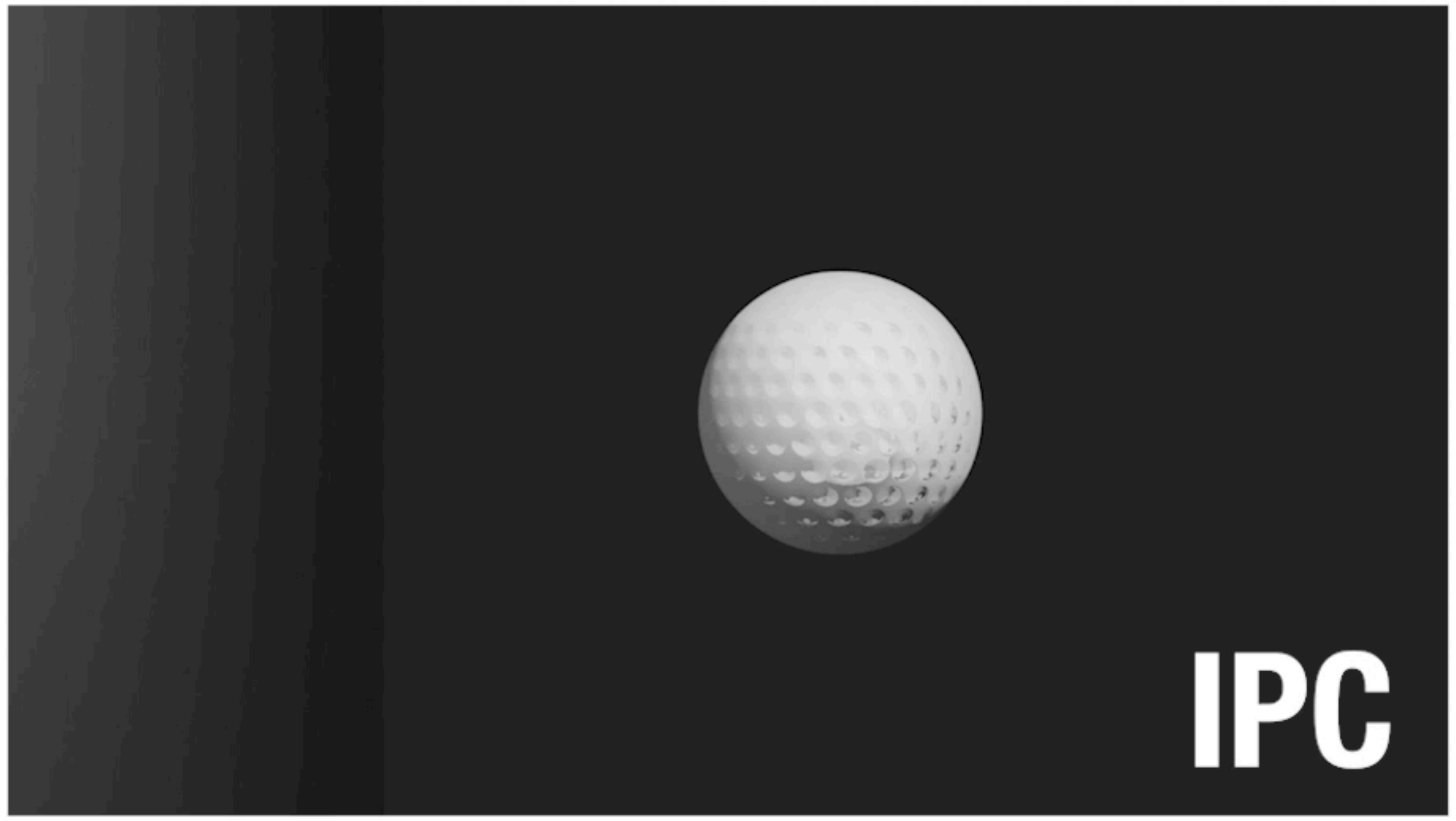
TIGHT CONFORMING CONTACT EXAMPLE



Cube size: $1m$

Slot size: $1m + 2\mu m$

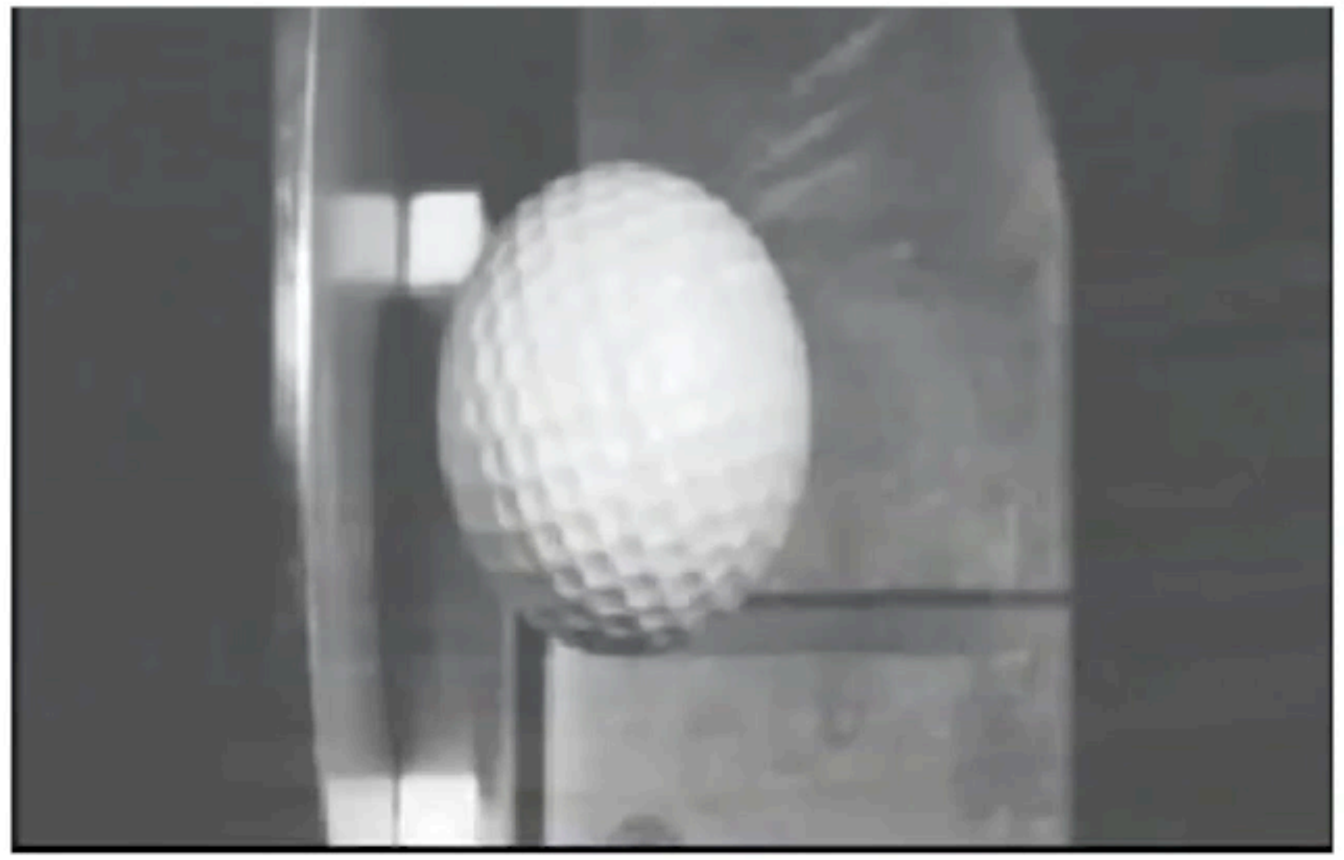
HIGH-SPEED IMPACT EXAMPLE



Initial velocity: 67m/s

IPC Velocity Magnitude

5e-4X

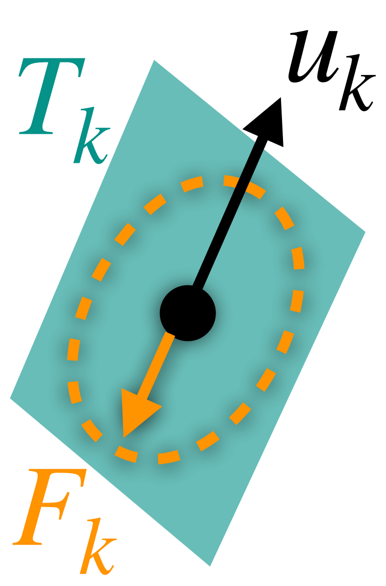


Recorded Footage

FRICTION

Maximum Dissipation Principle (MDP) [Moreau 1973]

With $u_k = T_k(x)^T(x - x^t)$, the local relative sliding displacement at contact k:

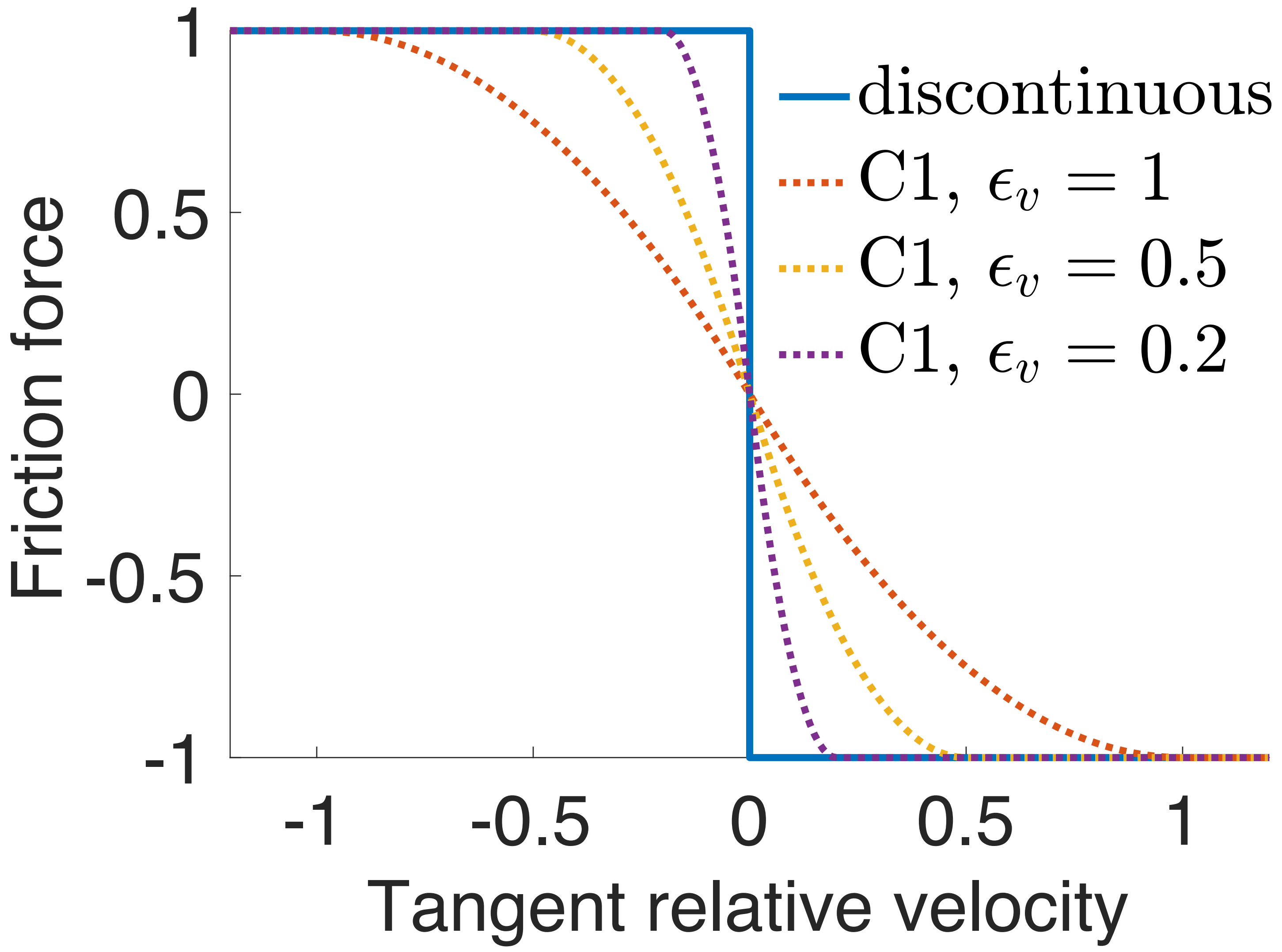


Friction force $F_k = \begin{cases} -\mu\lambda_k T_k u_k / \|u_k\| & \|u_k\| > 0 \\ -\mu\lambda_k T_k f & \|u_k\| = 0 \end{cases}$

λ_k is the normal force magnitude

f can take any vector with $\|f\| \leq 1$

IPC Friction smoothing:



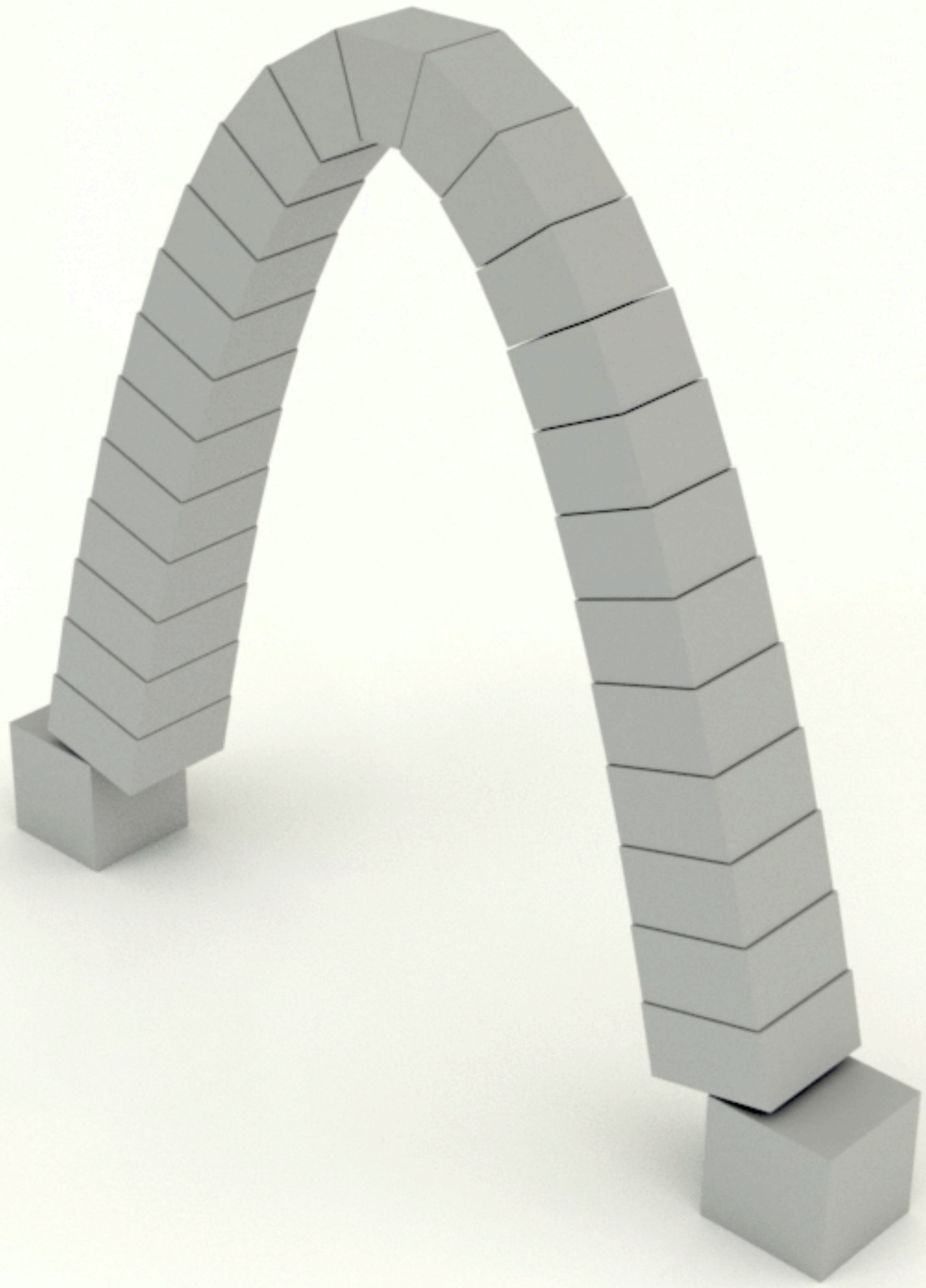
Challenges:

1. When $\|u_k\| = 0$, F_k can have non-smooth jumps
2. Displacement alone is not enough to F_k
3. No well-defined $P_k(x)$ where $-\partial P_k(x)/\partial x = F_k$

IPC lags λ_k and T_k to last time step or last nonlinear solve to approximately define variational friction

$F_k(x) = -\mu\lambda_k(x^j)T_k(x^j)f(x)$

MASONRY ARCH



Cement material:
  Density: 2300 kg/m^3
  Young’s modulus: 20 Gpa
  Poisson’s ratio: 0.2
h: 0.01s

HIT STIFF BOARD HOUSE

Young’s modulus: 0.1 Gpa
Poisson’s ratio: 0.4
h: 0.02s
0.4x playback speed

OPEN SOURCE

IPC is fully open sourced on https://github.com/ipc-sim/IPC with scripts to run all the examples in our paper including all the comparisons

Reusable components for quick integration: https://github.com/ipc-sim/ipc-toolkit

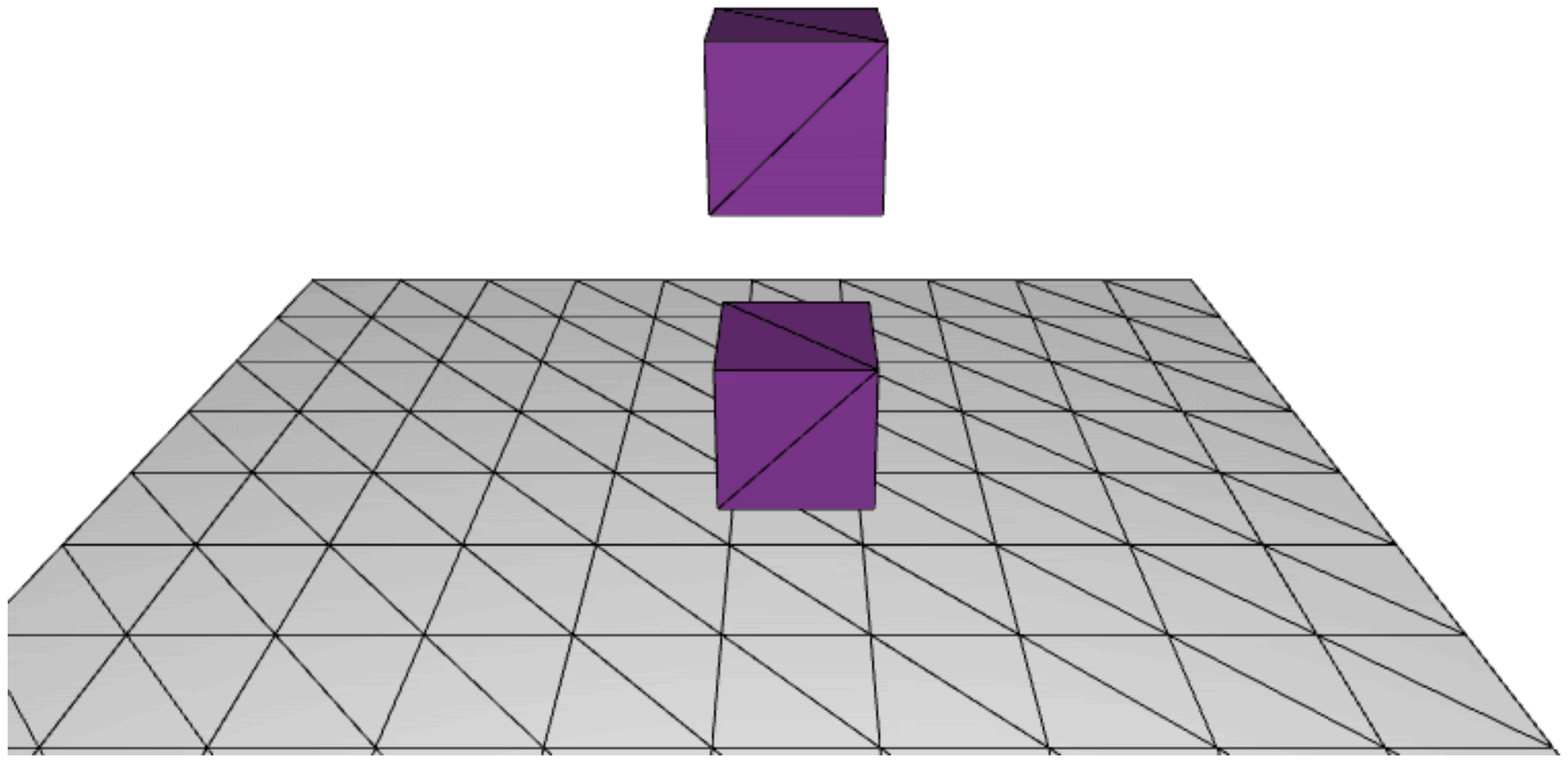
Quick start guide: https://github.com/ipc-sim/IPC/wiki

```
shapes input 2
input/tetMeshes/cube.msh  0 3 0  0 0 0  1 1 1
input/tetMeshes/cube.msh  0 1 0  0 0 0  1 1 1

selfFric 0.1

ground 0.1 0
```

Hello world script

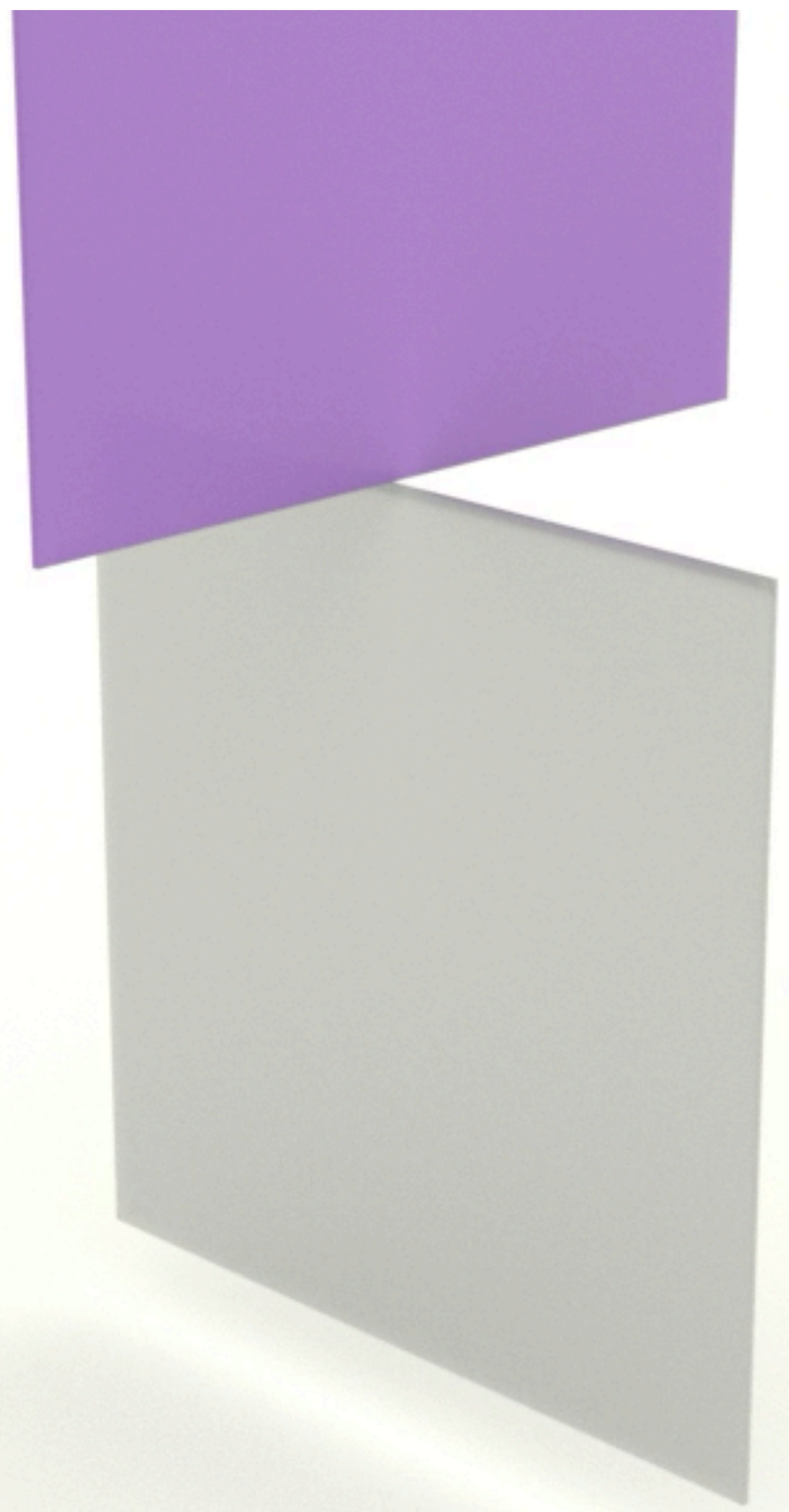


Hello world simulation

MORE RESULTS

SOFT-STIFF COUPLING

Young's modulus: 10^4 **Pa** 10^8 **Pa**

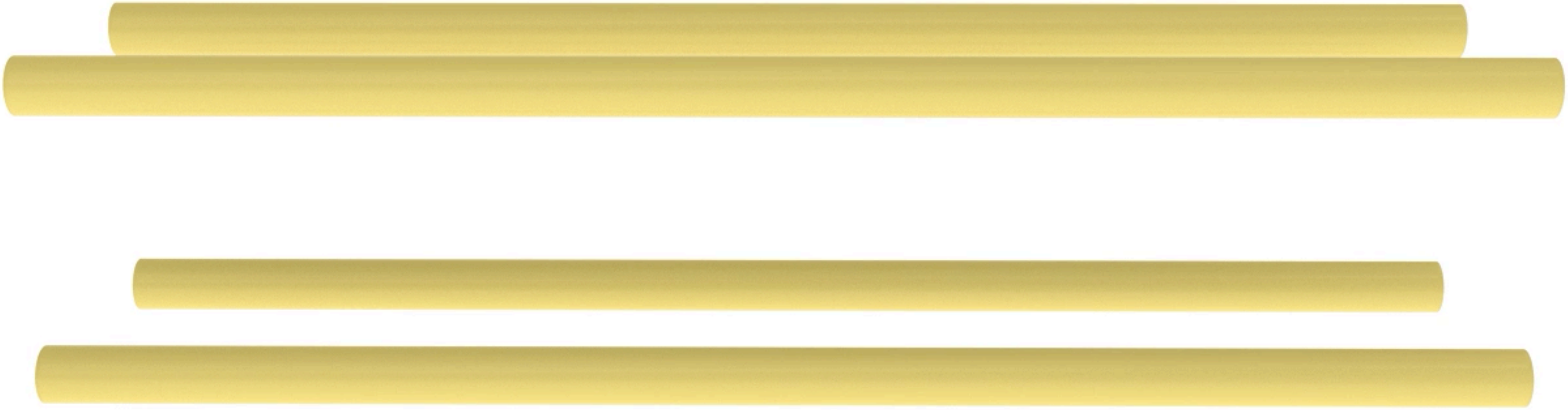


CHAINS

10 links:

100 links:

RODS TWISTING (100 SECONDS)



Rods Twist (100s)
53K nodes
202K tets
h: 0.025s
4x playback speed

VARYING TIME STEP SIZES (RODS TWISTING)

h (s)	# constraints avg (max)	per time step		total	
		t (s)	# iters	t (s)	# iters
0.002	137 (430)	0.29	2.12	862	6351
0.005	194 (584)	0.36	2.37	435	2843
0.01	269 (707)	0.38	2.65	229	1591
0.025	435 (1.0K)	0.38	2.69	91	645
0.05	551 (1.2K)	0.46	3.06	56	368
0.1	597 (1.3K)	0.73	4.75	44	285
0.2	607 (1.2K)	1.79	14.37	54	431
0.5	653 (1.4K)	11.39	100.58	137	1207
1	708 (1.3K)	18.41	188.17	110	1129
2	843 (1.3K)	52.02	522.00	156	1566

CONCLUSION

CONCLUSION

Incremental Potential Contact (IPC)

- Robust and accurate
- No parameter tuning
- Guarantees intersection- and inversion-free
- Differentiable
- Support various
  - material stiffnesses (with neo-Hookean even at E = 2e10Pa),
  - time-step sizes (even at h = 2s and quasi-static),
  - velocities (even at 1000m/s),
  - deformation severity.

Future works

- Faster IPC
- More accurate friction
- etc

THANK YOU!

Questions?

ipc-sim.github.io