Medial Elastics

Efficient and Collision-ready Deformation Via Medial Axis Transform

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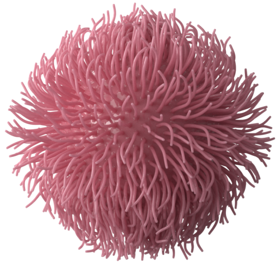
Deformable Objects

- High-quality deformable simulation is important for realism
- Well-known problem but computational expensive

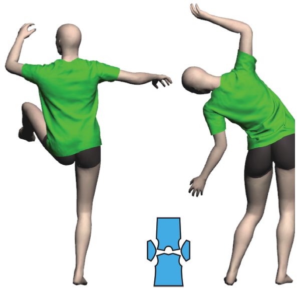
$$\mathbf{M}\ddot{\mathbf{u}} = \mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}, t)$$

  - PDE/ODE with a large number of DOFs
  - Frequent interactions like collision

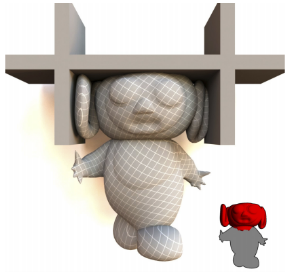


[Zheng & James, 2012]

[Zhao & Barbič, 2013]



[Xu et al, 2014]



[Teng et al, 2015]

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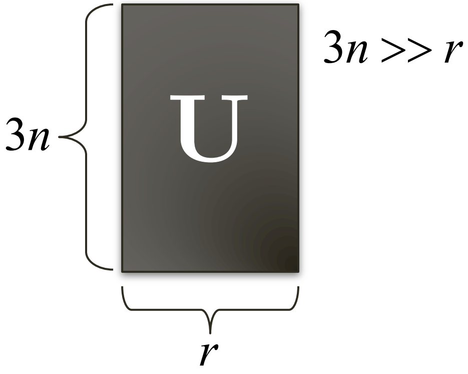
Reduced Simulation

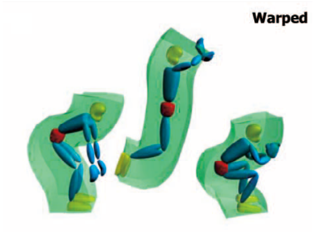
- A widely-used strategy, also known as (linear) **model reduction**

$$\mathbf{M}\ddot{\mathbf{u}} = \mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}, t) \qquad \mathbf{u} = \mathbf{U}\mathbf{q}$$

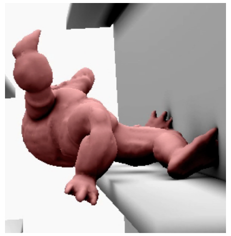
$$\mathbf{M}_q\ddot{\mathbf{q}} = \mathbf{U}^\top \mathbf{f}(\mathbf{U}\mathbf{q}, \mathbf{U}\dot{\mathbf{q}}, t), \qquad \mathbf{M}_q = \mathbf{U}^\top \mathbf{M}\mathbf{U}$$

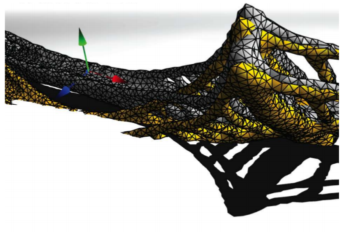


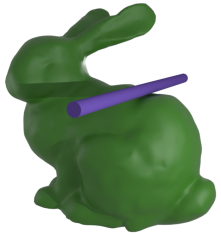
- Leveraged in many existing interactive simulation systems

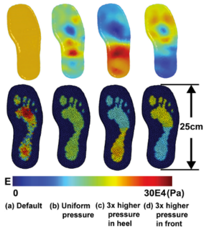


[Choi & Ko, 2005]


[Kim & James, 2011]


[Barbič et al, 2012]


[Harmon & Zorin, 2013]



[Xu et al, 2015]

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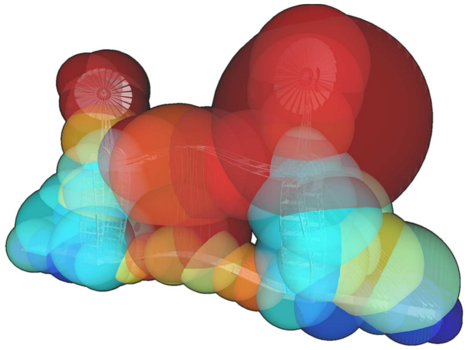
Reduction + Collision

- Collision culling/detection is another troubling issue
  - Culling is required as exhaustive triangle-triangle test is not scalable
- **Both seek for good shape approximate**

Collision culling



[James & Pai, 2004]

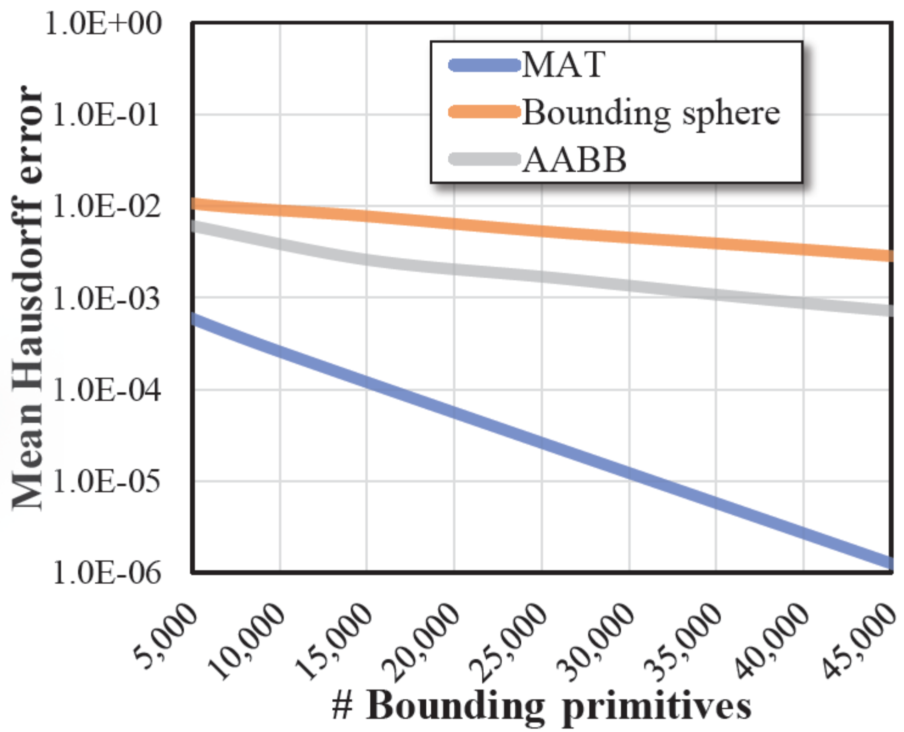
[Capell et al, 2002]

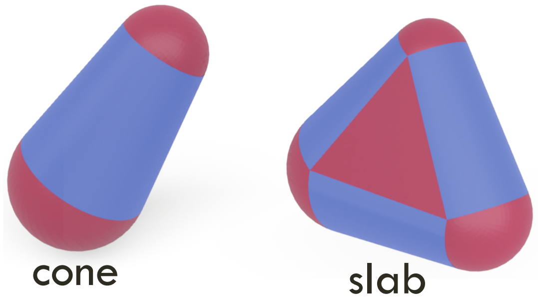
Reduced sim.

- Why not use a consistent representation for both tasks

Medial Elastics: MAT-based Reduction

- Our choice is **medial axis transform** (MAT)
- A set of maximally-inscribed spheres
  - Each has at least two closest points on its boundary
  - Known as the topology skeleton
  - Tight shape enclosure
- Could be discretized as medial mesh (MM)
  - Approximate original model with medial primitives
  - **Interpolated** spheres





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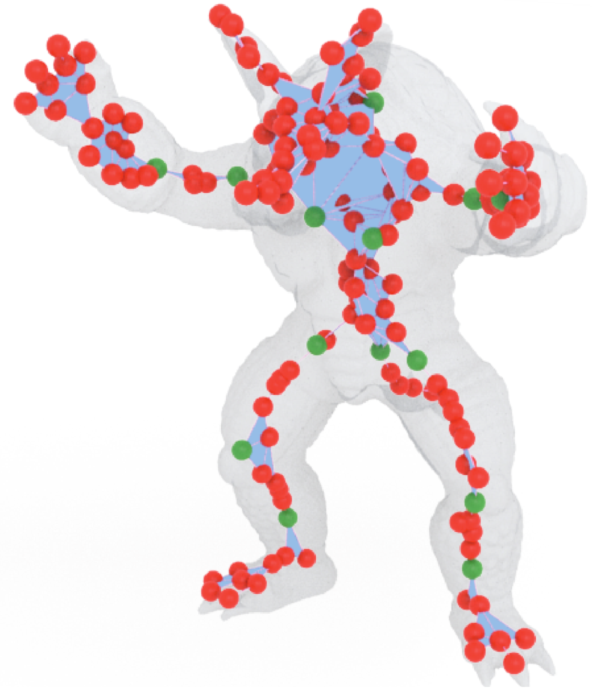
Spatial Reduction with MAT

- We place **handles** at MM vertices
  - A skinning-like framework, blended with harmonics weight
- In addition to linear handles, we also have **quadratic** handles to avoid locking
  - Each handle houses simulation DOFs
  - A linear handle has 12 DOFs (9 linear + 3 translation)
  - A quadratic handle has 30 DOFs (additional 18 DOFs)

$$\mathbf{U}(\mathbf{x}) = \left[\mathbf{U}_t, \mathbf{U}_a, \widetilde{\mathbf{U}}, \widehat{\mathbf{U}}\right] = \left[\mathbf{I}, \mathbf{I}\otimes\mathbf{x}^\top, \mathbf{I}\otimes\widetilde{\mathbf{x}}^\top, \mathbf{I}\otimes\widehat{\mathbf{x}}^\top\right] \in \mathbb{R}^{3\times 30}$$

$$\begin{cases} \widetilde{\mathbf{x}} = [x_1^2, x_2^2, x_3^2]^\top \\ \widehat{\mathbf{x}} = [x_1x_2, x_2x_3, x_1x_3]^\top \end{cases}$$



● Linear handle

● Quadratic handle

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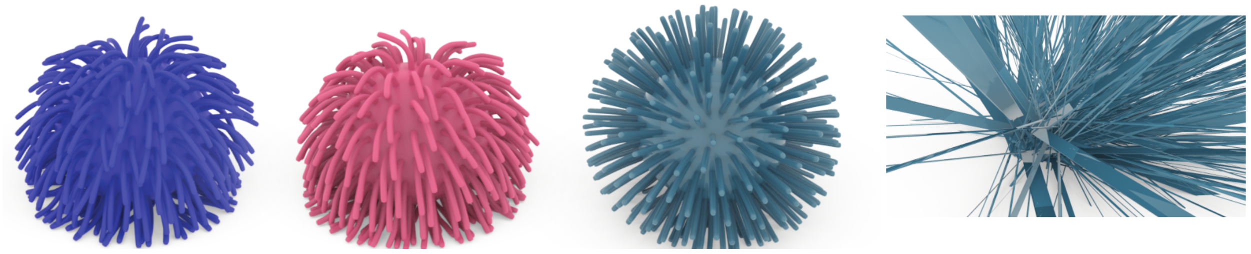
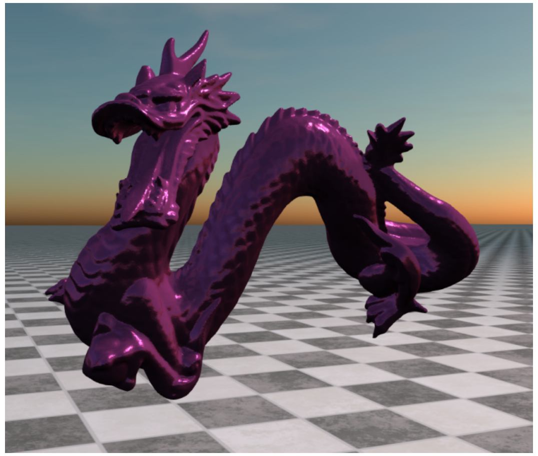
Semi-reduced Projective Dynamics

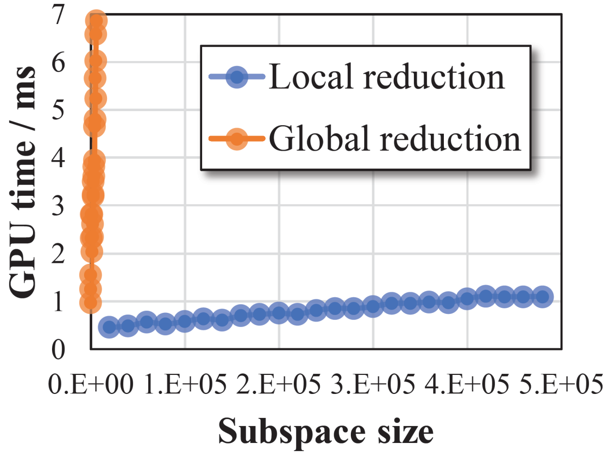
- Project dynamics [Bouaziz et al, 2014] is a fast simulation solution
  - Global: error measure, solve a fixed linear system
  - Local: error correction, project constraint
- Local reduction is less profitable
  - Massive parallelization of GPU is less sensitive to the reduction
  - An aggressive local reduction leads to artifact (constraint error)



Full simulation | Our method 1,000 handles | Hyper reduction 10,000 handles | Hyper reduction 1,000 handles



[Brandt et al, 2018]



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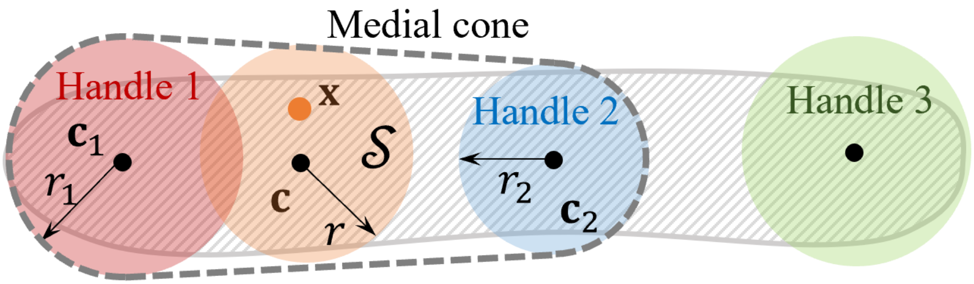
Global Reduction VS. Local Reduction

Efficient CC/D with MAT

- Assumption: triangles within a medial primitive do not collide
  - We only need to focus on inter-primitive collision
- Before simulation, the rest-shape model is enclosed by MAT
- During simulation, we need to make sure the deformed model is also enclosed by “deformed” MAT

Bounding Update of MAT

- Lazy solution: update vertices of MM
  - Extreme deformation is penalized
  - Miss collisions under large deformation



- What is the difference between deformed model and MAT

$$\mathbf{x}+\mathbf{u}-\mathbf{c}' = \mathbf{x}+\mathbf{u}-[t_1(\mathbf{c}_1+\mathbf{u}_1)+t_2(\mathbf{c}_2+\mathbf{u}_2)] = \mathbf{x}-\mathbf{c}+\mathbf{u}-t_1\mathbf{u}_1-t_2\mathbf{u}_2.$$

$$\left\|\mathbf{x}+\mathbf{u}-\mathbf{c}'\right\| = \|\mathbf{x}-\mathbf{c}+\Delta\mathbf{u}_1+\Delta\mathbf{u}_2\| \leq r+\|\Delta\mathbf{u}_1\|+\|\Delta\mathbf{u}_2\|$$

$$\begin{cases} \Delta\mathbf{u}_1 = w_1 T_1(\mathbf{x}) - t_1 T_1(\mathbf{c}_1) \\ \Delta\mathbf{u}_2 = w_2 T_2(\mathbf{x}) - t_2 T_2(\mathbf{c}_2) \end{cases}$$

→ How to bound displacement difference

$$\Delta \mathbf{u}_1 = \left(w_1 \mathbf{U}_t(\mathbf{x}) - t_1 \mathbf{U}_t(\mathbf{c}_1)\right) \mathbf{t}^1 + \left(w_1 \mathbf{U}_a(\mathbf{x} - t_1 \mathbf{U}_a(\mathbf{c}_1)\right) \mathbf{a}^1 + \left(w_1 \widetilde{\mathbf{U}}(\mathbf{x}) - t_1 \widetilde{\mathbf{U}}(\mathbf{c}_1)\right) \widetilde{\mathbf{q}}^1 + \left(w_1 \widehat{\mathbf{U}}(\mathbf{x}) - t_1 \widehat{\mathbf{U}}(\mathbf{c}_1)\right) \widehat{\mathbf{q}}^1$$

$$\left\| \left(w_1 \mathbf{U}_t(\mathbf{x}) - t_1 \mathbf{U}_t(\mathbf{c}_1)\right) \mathbf{t}^1 \right\| = \left\| (w_1 - t_1) \cdot \mathbf{t}^1 \right\| \leq \max\{|w_1 - t_1|\} \cdot \left\| \mathbf{t}^1 \right\|$$

Translational disp.

$$\left\| \left(w_1 \mathbf{U}_a(\mathbf{x}) - t_1 \mathbf{U}_a(\mathbf{c}_1)\right) \mathbf{a}^1 \right\| = \left\| \left[\mathbf{I} \otimes (w_1 \mathbf{x} - t_1 \mathbf{c}_1)^\top \right] \mathbf{a}^1 \right\| \leq \max\{\|w_1 \mathbf{x} - t_1 \mathbf{c}_1\|\} \cdot \rho^{1/2} \left(\sum_{i=1}^{3} \mathbf{a}_i^1 \mathbf{a}_i^{1\top} \right)$$

Linear disp.

$$\left\| \left(w_1 \widetilde{\mathbf{U}}(\mathbf{x}) - t_1 \widetilde{\mathbf{U}}(\mathbf{c}_1)\right) \widetilde{\mathbf{q}}^1 \right\| \leq \max\{\|w_1 \widetilde{\mathbf{x}} - t_1 \widetilde{\mathbf{c}}_1\|\} \cdot \rho^{1/2} \left(\sum_{i=1}^{3} \widetilde{\mathbf{q}}_i^1 \widetilde{\mathbf{q}}_i^{1\top} \right)$$

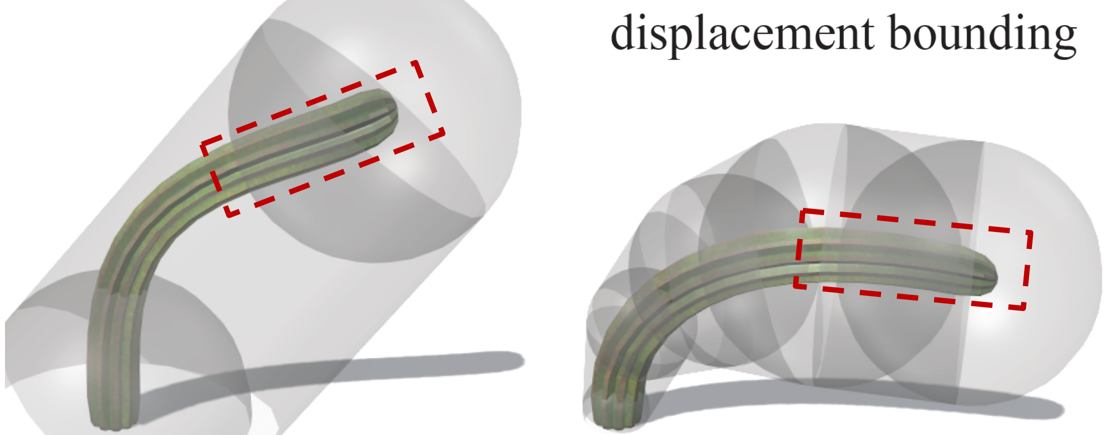
$$\left\| \left(w_1 \widehat{\mathbf{U}}(\mathbf{x}) - t_1 \widehat{\mathbf{U}}(\mathbf{c}_1)\right) \widehat{\mathbf{q}}^1 \right\| \leq \max\{\|w_1 \widehat{\mathbf{x}} - t_1 \widehat{\mathbf{c}}_1\|\} \cdot \rho^{1/2} \left(\sum_{i=1}^{3} \widehat{\mathbf{q}}_i^1 \widehat{\mathbf{q}}_i^{1\top} \right)$$

Quadratic disp.

Displacement and Deformation Bounding

- What is wrong with this formula
  - Large scaling must be applied to incorporate rigid body motion
  - The actual deformation could be small
- Deformation bounding
  - Remove rigid body motion per primitive
  - Bound deformation within a local frame

$$
\begin{aligned}
\mathbf{d}^* &= \overline{\mathbf{R}}^\top \mathbf{d} = \overline{\mathbf{R}}^\top \left(\mathbf{x} + \sum w_j T^j(\mathbf{x}) - \overline{\mathbf{R}}(\mathbf{x} - \overline{\mathbf{c}}) - \overline{\mathbf{c}} - \Delta\overline{\mathbf{c}}\right) \\
&= \overline{\mathbf{R}}^\top \left(\sum w_j \mathbf{A}^j + \mathbf{I}\right)\mathbf{x} + \overline{\mathbf{R}}^\top \sum w_j \mathbf{t}^j - (\mathbf{x} - \overline{\mathbf{c}}) - \overline{\mathbf{R}}^\top (\overline{\mathbf{c}} + \Delta\overline{\mathbf{c}}) \\
&= \underbrace{\left(\overline{\mathbf{R}}^\top \sum w_j \mathbf{A}^j \overline{\mathbf{R}} + \mathbf{I} - \overline{\mathbf{R}}\right)}_{\mathbf{A}^*} \mathbf{x}^* + \underbrace{\overline{\mathbf{R}}^\top \sum w_j \left(\mathbf{A}^j \overline{\mathbf{c}} + \mathbf{t}^j\right) - \overline{\mathbf{R}}^\top \Delta\overline{\mathbf{c}}}_{\mathbf{t}^*}
\end{aligned}
$$



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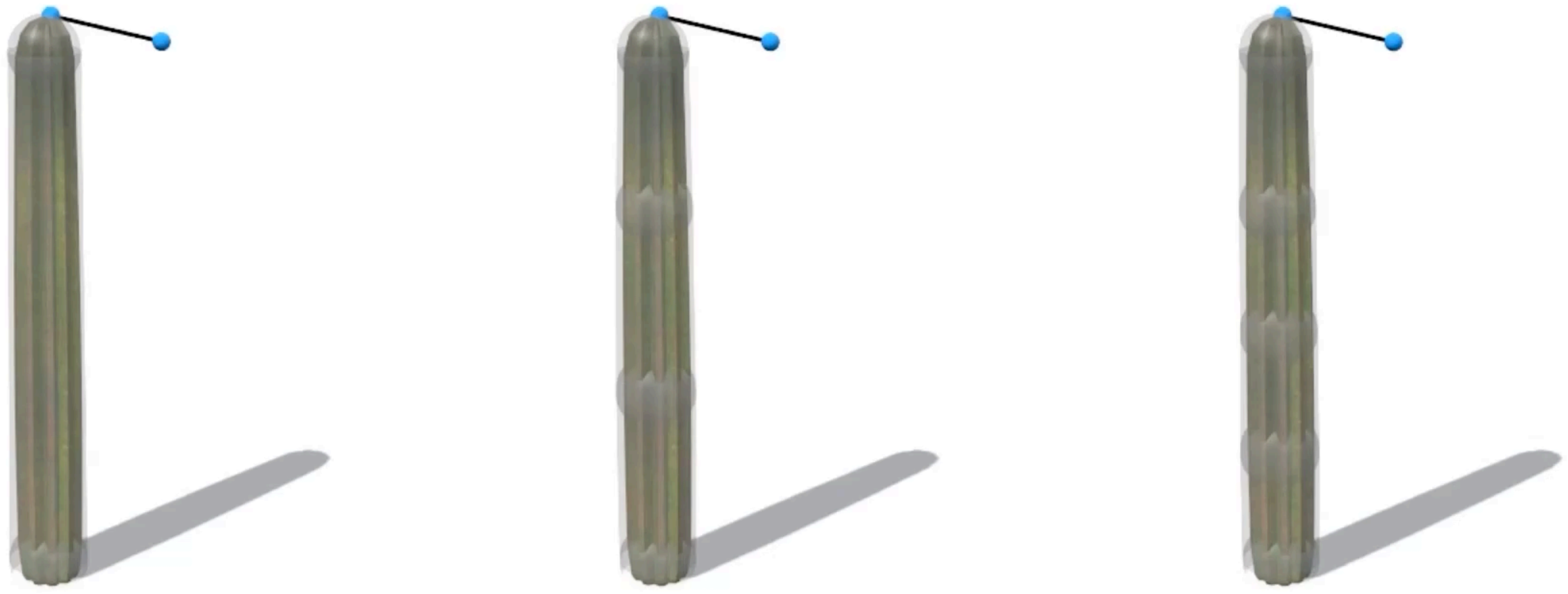
One Handle | Three Handles | Four Handles



Culling Effectiveness

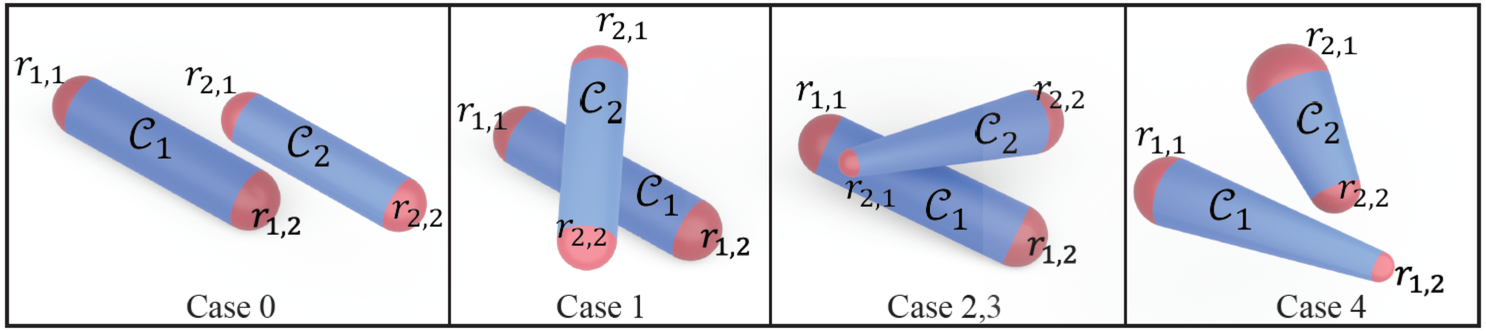
Efficient CC/D with MAT

ALGORITHM 1: Cone-cone collision culling.

Input: $A' = A - R_1^2$, $B' = B - R_1R_2$, $C' = C - R_3^2$, $D' = D - R_1R_3$, $E' = E - R_2R_3$, $F' = F - R_3^2$ for C_1 and C_2
Output: if C_1 and C_2 collide with each other or not

if $g(0,0) \leq 0$ or $g(0,1) \leq 0$ or $g(1,0) \leq 0$ or $g(1,1) \leq 0$ **then**
  return True
end
solve $A't_1^2 + D't_1 + F' = 0$; // set $t_2 = 0$
if $t_1 \in [0,1]$ **then**
  return True
end
solve $A't_1^2 + (B' + D')t_1 + C' + E' + F' = 0$; // set $t_2 = 1$
if $t_1 \in [0,1]$ **then**
  return True
end
solve $C't_2^2 + E't_1 + F' = 0$; // set $t_1 = 0$
if $t_2 \in [0,1]$ **then**
  return True
end
solve $C't_2^2 + (B' + E')t_2 + A' + D' + F' = 0$; // set $t_1 = 1$
if $t_2 \in [0,1]$ **then**
  return True
end
/* $g(t_1, t_2) = 0$ must be an ellipse */
$(t_x, t_y) \leftarrow$ center of $g(t_1, t_2) = 0$;
if $t_x, t_y \in [0,1]$ and $g(t_x, t_y) < 0$ **then**
  return True
end
return False

$$\begin{aligned} \min \quad & f(t_1, t_2) = \|\mathbf{c}_1 - \mathbf{c}_2\| - (r_1 + r_2) = \sqrt{S} - (R_1t_1 + R_2t_2 + R_3) \\ \text{s.t.} \quad & 0 \leq t_1 \leq 1, \quad 0 \leq t_2 \leq 1, \end{aligned}$$



Case 0,1: Two cones have constant radii

Case 2,3: One cone has a constant radius, and the other does not

Case 4: Both cones have varying radii

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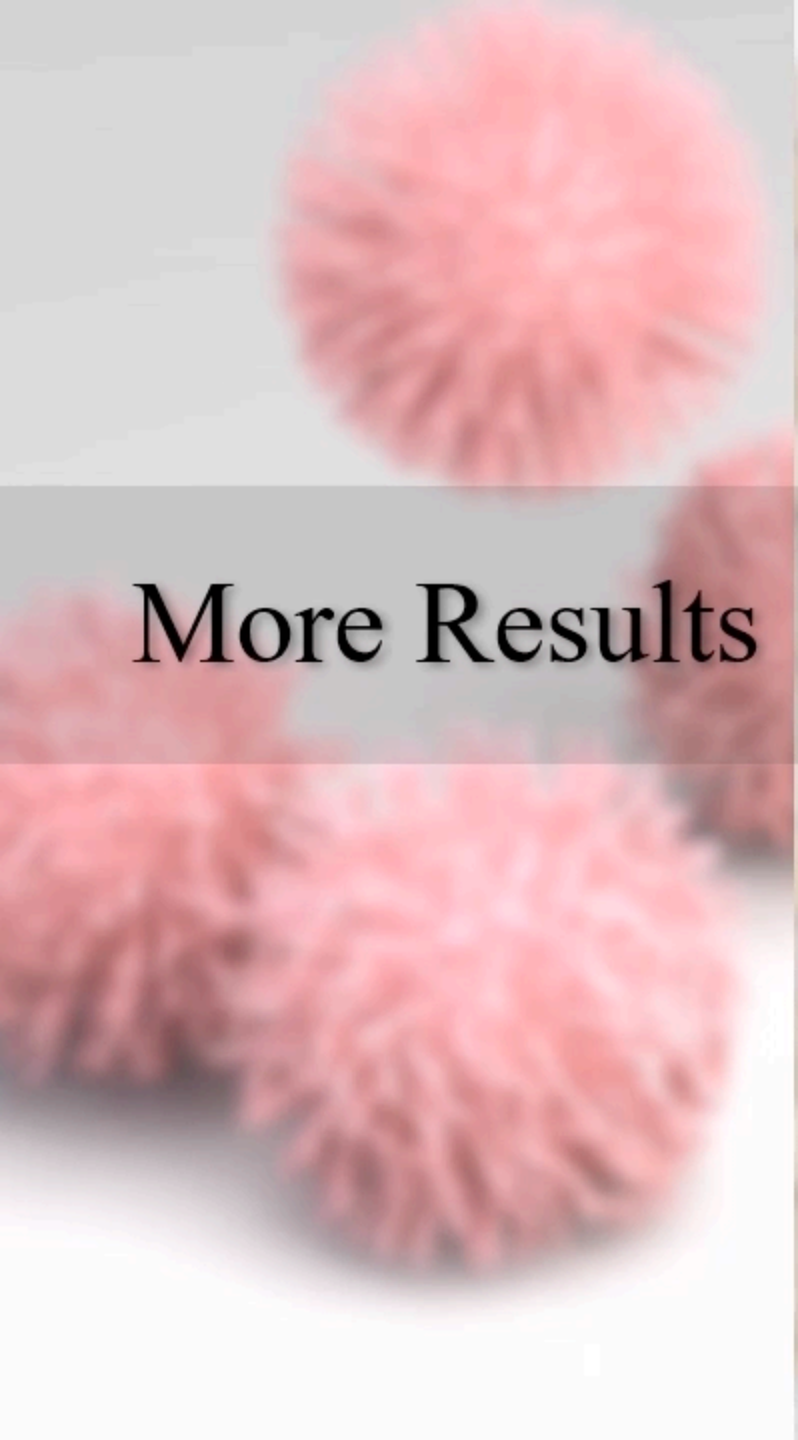
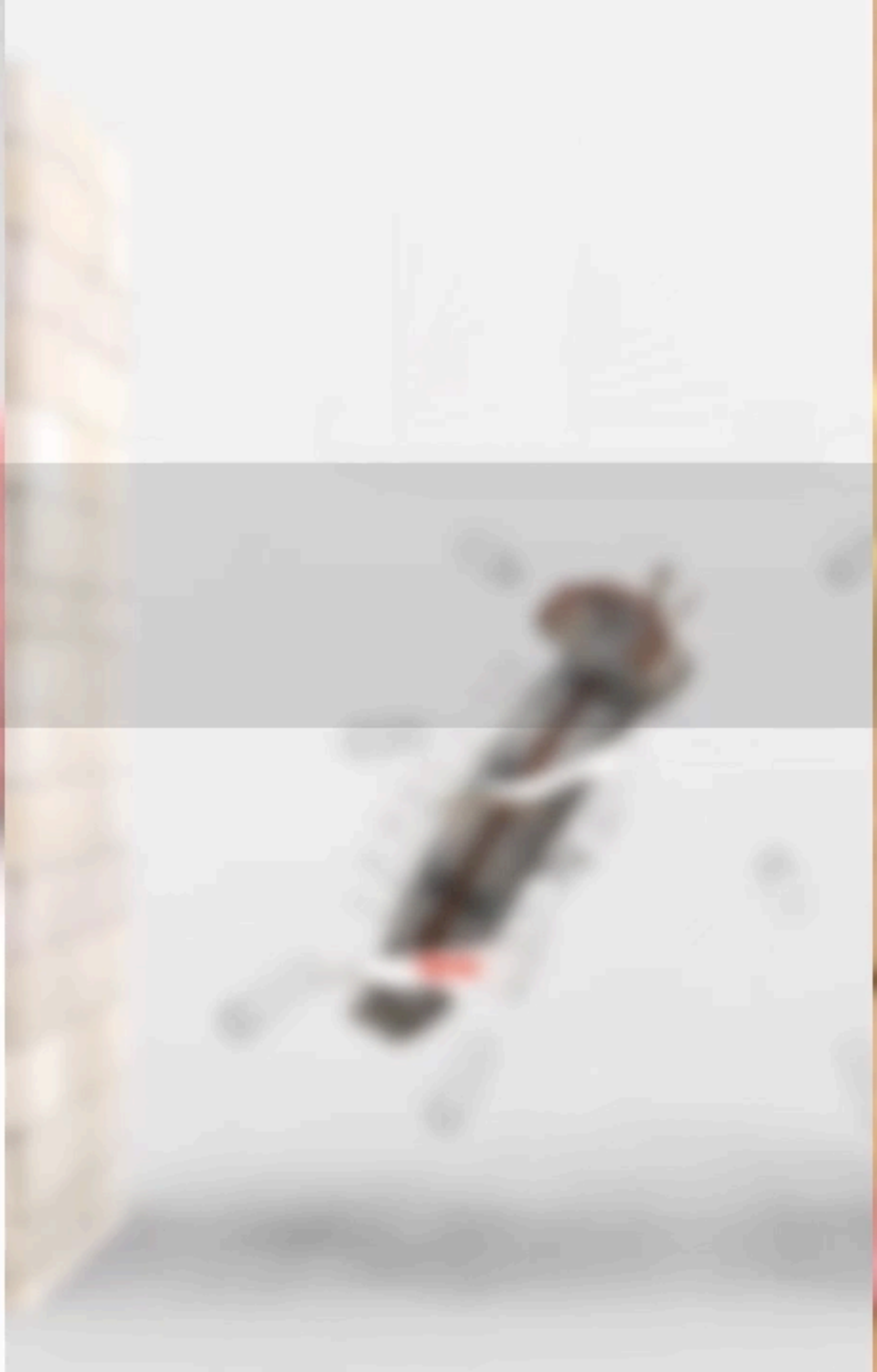
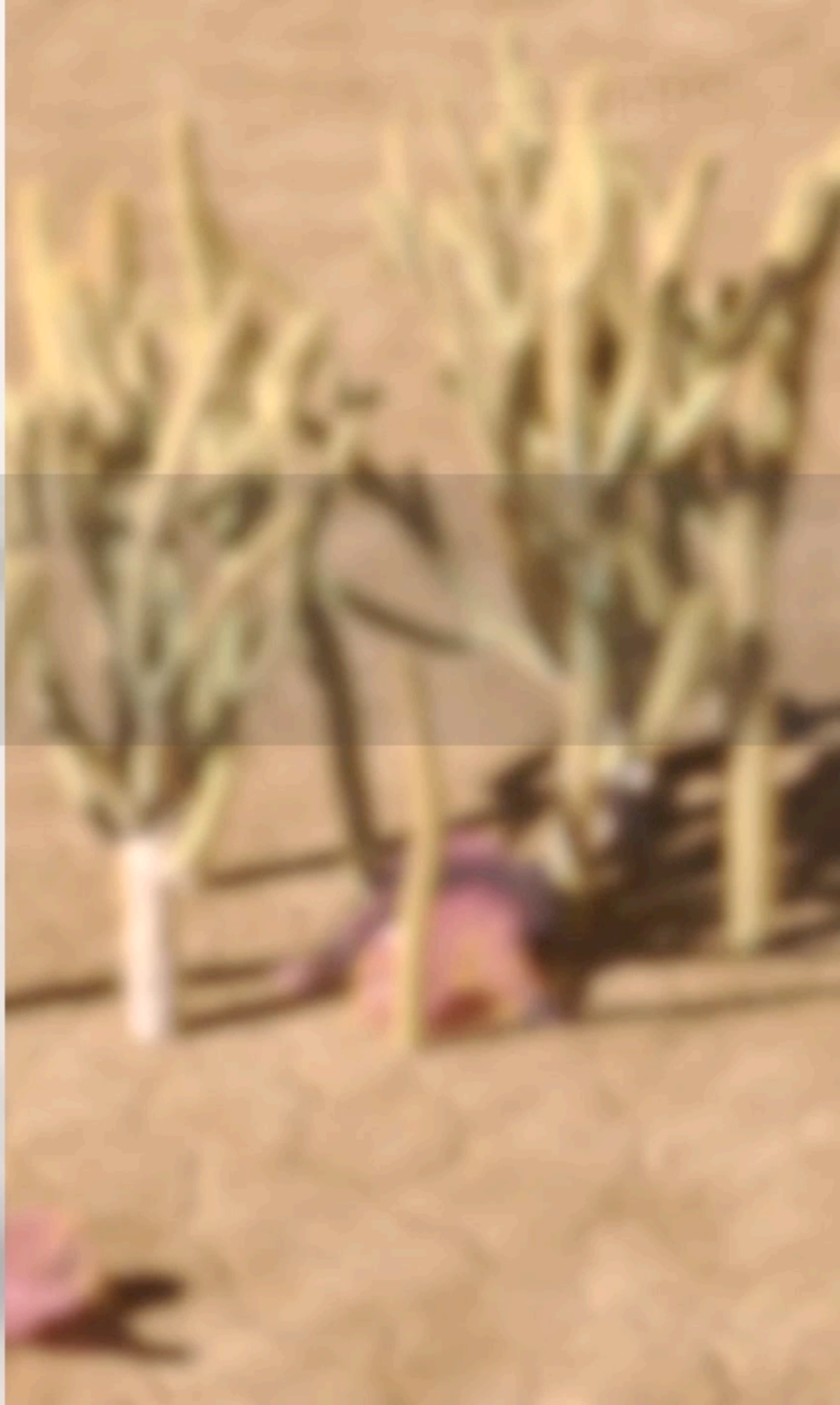
More Results







Thank You