CPPM: Chi-squared Progressive Photon Mapping

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Outline

• Background
• Challenge
• Key Idea
• Algorithm
• Results
• Limitation
• Summary
• Photon Mapping [Jensen 1996] can solve S-D-S paths:
  • Light sources emit photons
  • Collect photons to estimate radiance
• SPPM [Hachisuka and Jensen 2009] converges to the correct pixel measurement:
  • Multiple iterations
  • Bandwidth converges to infinitesimal
Background

Progressive Photon Mapping: A Probabilistic Approach
[Knaus and Zwicker 2011]

Adaptive Progressive Photon Mapping
[Kaplanyan and Dachsbacher 2013]

Deep Kernel Density Estimation for Photon Mapping
[Zhu et al. 2020]
Challenge
Challenge

• Challenge: Blur and Noise
  • Blur:
    • Caused by bias
    • Bandwidth too large
  • Noise:
    • Caused by variance
    • Bandwidth too small
• Difficult to eliminate them at the same time
Key Idea: Benefits of Uniform Distribution

• Uniformly distributed photons have advantages
• Unnecessary to use a smaller bandwidth
Key Idea: Chi-squared Test on Photons

\[ R_i + 1 = R_i \]

Reduce \( R_i \)

Keep \( R_i \)

Chi-squared Test

Non-uniform Distribution

Uniform Distribution
Key Idea: Bandwidth Reduction Scheme

• A novel bandwidth reduction scheme to work with chi-squared test
Algorithm: Pipeline

Photon Pass

- photon tracing and collection
- photon distribution chi-squared test
- flux accumulation
- bandwidth reduction
- Pixel measurement estimation

START
Eye Pass

Distributed Ray Tracing Pass

Next iteration
• How to define a uniform distribution on multiple searching areas?
• A unified space can be a solution
• How to define a uniform distribution on multiple searching areas?
• A unified space can be a solution
• Align the searching areas
• How to define a uniform distribution on multiple searching areas?
• A unified space can be a solution
• Align the searching areas
• How to define a uniform distribution on multiple searching areas?
• A unified space can be a solution
• Align the searching areas
Algorithm: Theoretical Foundation

- How to define a uniform distribution on multiple searching areas?
- A unified space can be a solution
- Align the searching areas
Algorithm: Chi-squared Test on Photons

- Partition the disc
- Count photons in sectors
- Calculate chi-squared statistic to identify uniform distribution

\[ R_i \]

Sector

Annulus
Algorithm: Conditional Bandwidth Reduction

Photons

- Uniformly distributed
  \[ R_{i+1} = R_i \]
- Not uniformly distributed
  \[ R_{i+1} < R_i \]
Algorithm: Conditional Bandwidth Reduction

Photons

Uniformly distributed

Not uniformly distributed

\[ R_{i+1} = R_i \]

\[ R_{i+1} < R_i \]

• How to get enough samples for the chi-squared test?

![Graph showing Bandwidth vs Iteration]

- SPPM
Photons

Minimum sample size

Enough Photons

Uniformly distributed

Not uniformly distributed

$R_{i+1} = R_i$

$R_{i+1} < R_i$

Algorithm: Bandwidth Reduction Scheme

Uniform distribution

Bandwidth

Iteration

SPPM

CPPM
CPPM Pipeline

Enough Photons? (Yes/No)

Conditional Bandwidth Reduction

Uniform Distribution? (Yes/No)

Bandwidth Reduction

Yes

No
Algorithm: Convergence

• Worst case:

\[ R_N = O\left( N^{-\frac{1}{2}} \log \beta \frac{1}{k} \right) \]

• equivalent to SPPM

• Best case:

Bias = 0
Variance = \( O(N^{-1}) \)
Exceptional Cases of the Chi-squared Test

• The chi-squared test may get wrong results:

1. Reject a uniform distribution
   - Acceptable

2. Not reject a non-uniform distribution
   - Critical

Solution: keep testing
Results

(a) Bandwidth (point A)

- SPPM
- CPPM
- APPM

Bandwidth vs. Iteration graph.
Results

(b) Error (Point A)
Chi-squared test fails to reject a non-uniform distribution
Results

(d) Error (Point B)
Results

• How many pixels can find a uniform distribution?
Results

• How many pixels can find a uniform distribution?
• Bandwidth of SPPM:
Results

• How many pixels can find a uniform distribution?
• Bandwidth of CPPM:
• Most pixels can find a uniform distribution
Results

APPM (red)  APPM (green)  APPM (yellow)

CPPM (red)  CPPM (green)  CPPM (yellow)
Limitation

- Uniformly distributed photons are not always good
- If the contribution of the photons depends on their location

Textured Spotlight:
• If the contribution of the photons is related to their location
• Emit the photons proportional to the luminance:
Summary

Contributions:

+ Unnecessary to converge the bandwidth to infinitesimal
+ Reveal the benefits of uniformly distributed photons
+ Introduce the chi-squared test to check the photons
+ Propose a pipeline to robustly find a desired bandwidth

Future Work:

★ Take contribution of photons into account
★ Make bandwidth expansion possible
★ Combine APPM and CPPM in a hybrid manner
★ Integrate CPPM with other sampling-based techniques
Q & A

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Thank you!

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