A Harmonic Balance Approach for Designing Compliant Mechanical Systems with Nonlinear Periodic Motions

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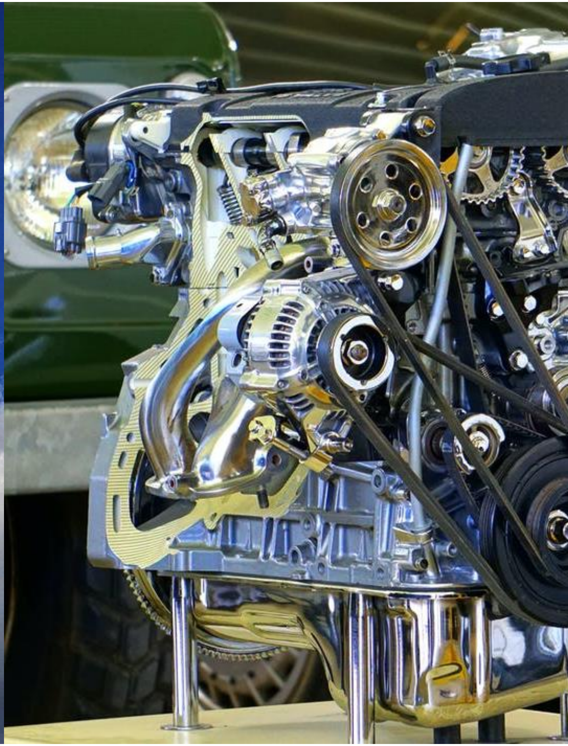
Oscillations are Everywhere

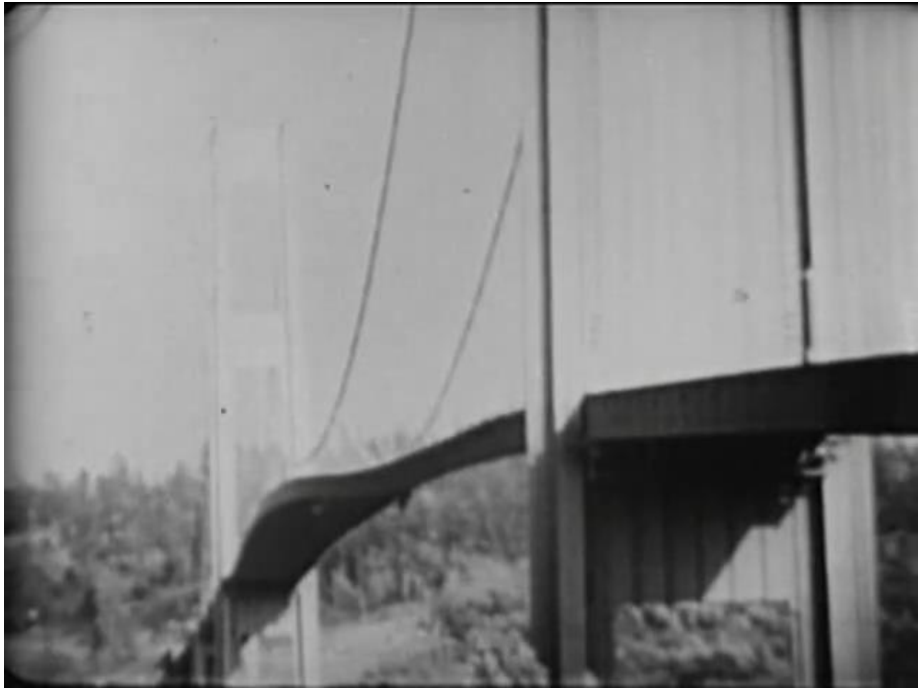


Oscillations in Engineering



Oscillations in Engineering

Collapse of Tacoma Narrows Bridge (1940)



https://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_(1940)

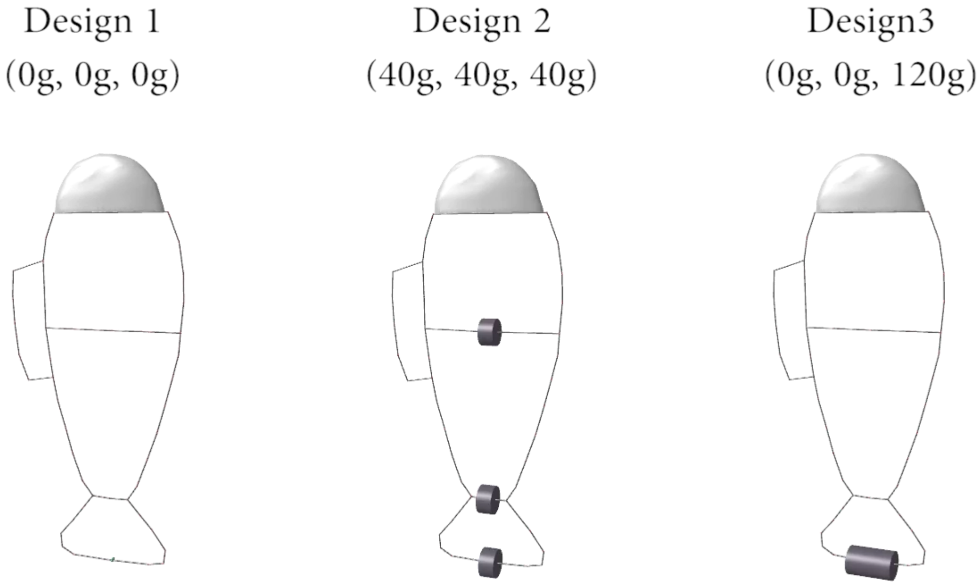
Large-Amplitude Oscillations in the Wild

Large-amplitude Oscillations in the Lab

[FastRunner from ihmc]

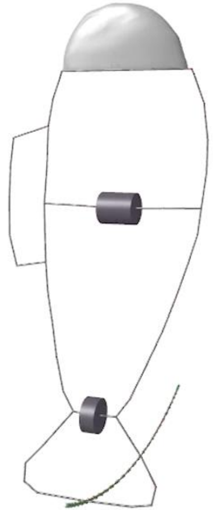
Flappy Hummingbird
[Fei et al. 2019]

Designing for Large-Amplitude Oscillations



Designing for Large-Amplitude Oscillations

Optimized Design
(80g, 31.4g, 0g)



Physical Prototype

Goal

Develop a computational design tool for nonlinear mechanical systems that exhibit desired large-amplitude oscillations.

Challenges

- How to model nonlinear periodic motions?

  → Frequency-space approach based on Harmonic Balance Method (HBM)

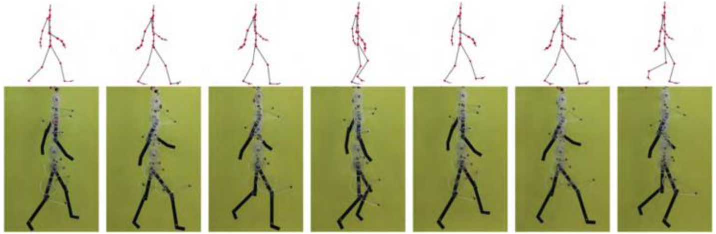
- How do design parameters affect nonlinear periodic motion?

  → Frequency-space sensitivity analysis

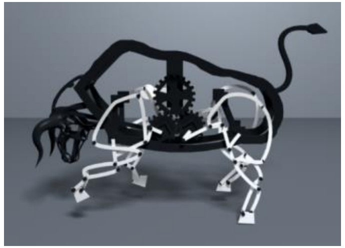
- How to find design parameters that lead to desired motion?

  → Forward exploration & inverse design tools

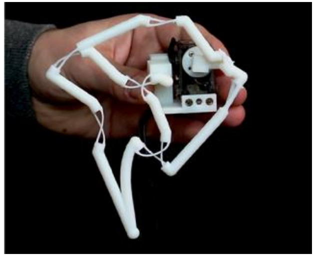
Designing Mechanical Motion



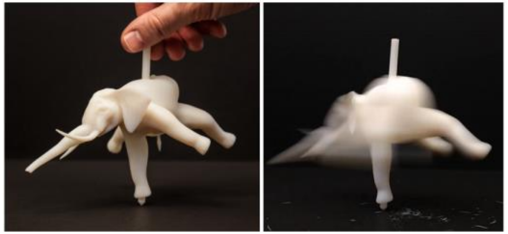
Designing and Fabricating Mechanical Automata from Mocap Sequences [Ceylan et al. 2013]



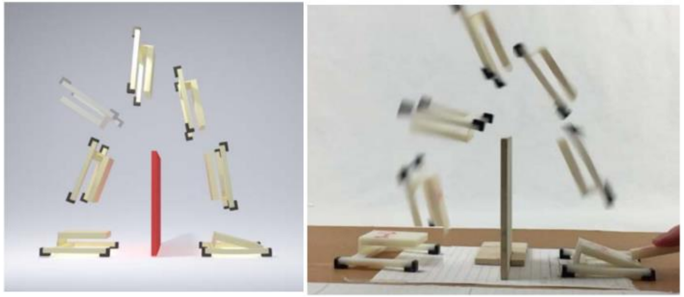
Computational Design of Linkage-Based Characters [Thomaszewski et al. 2014]



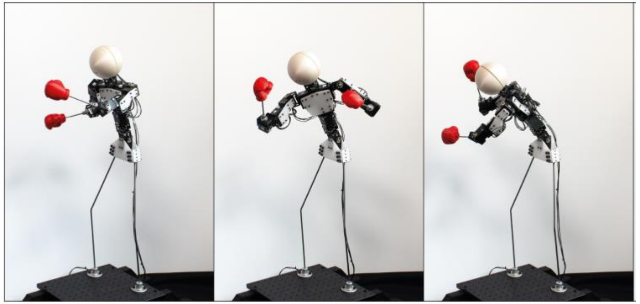
A Computational Design Tool for Compliant Mechanisms [Megaro et al. 2017]



Spin-It: Optimizing Moment of Inertia for Spinnable Objects [Bächer et al. 2014]

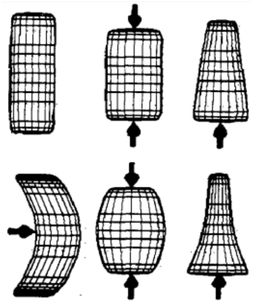


Dynamics-Aware Numerical Coarsening for Fabrication Design [Chen et al. 2017]

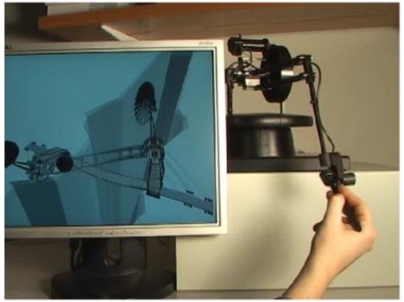


Vibration-Minimizing Motion Retargeting for Robotic Characters [Hoshyari et al. 2019]

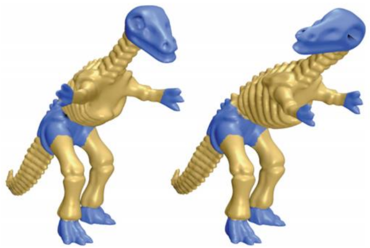
Modal Subspaces for Animation



Good vibrations: Modal dynamics for graphics and animation
[Pentland et al. 1989]



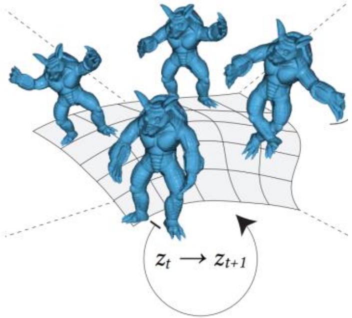
Real-Time Subspace Integration for St.Venant-Kirchhoff Deformable Models
[Barbič et al. 2005]



Interactive Surface Modeling Using Modal Analysis
[[Hildebrandt et al. 2011]

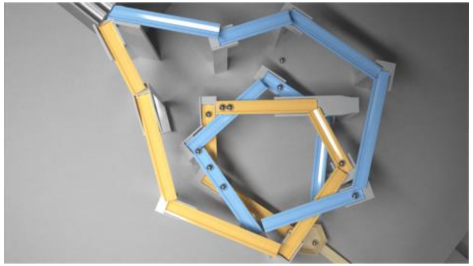
Subspace Dynamic Simulation Using Rotation-Strain Coordinates
[Pan et al. 2015]

Subspace Clothing Simulation Using Adaptive Bases
[Hahn et al. 2014]

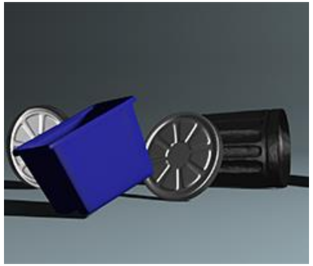


Latent-space Dynamics for Reduced Deformable Simulation
[Fulton et al. 2019]

Audible Vibrations



Toward High-Quality Modal Contact Sound
[Zheng et al. 2011]



Harmonic shells: a practical nonlinear sound model for near-rigid thin shell
[Chadwick et al. 2009]

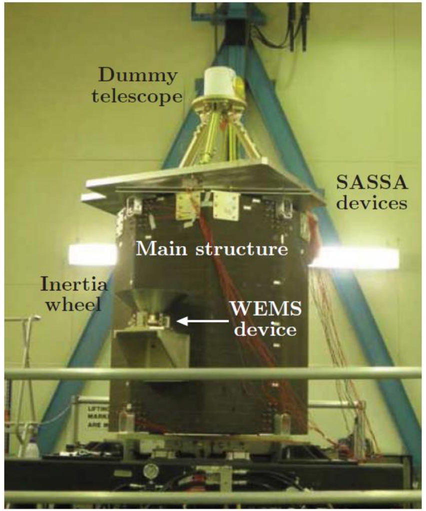
Multi-scale simulation of nonlinear thin-shell sound with wave turbulence
[Cirio et al. 2018]

Computational Design of Metallophone Contact Sounds
[Bharaj et al. 2015]

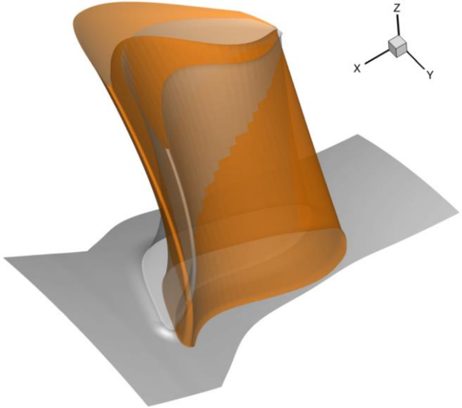
Printone: Interactive Resonance Simulation for Free-Form Print-Wind Instrument Design
[Umetani et al.]

Acoustic Voxels: Computational Optimization of Modular Acoustic Filters
[Li et al. 2016]

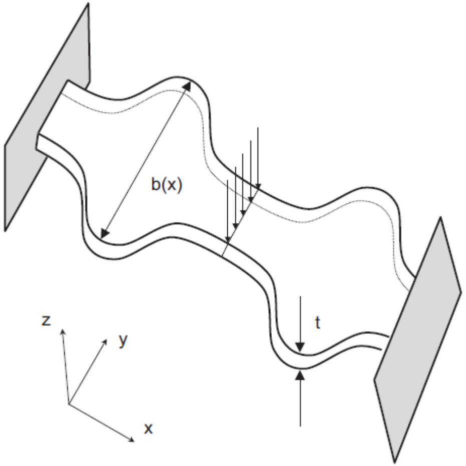
Nonlinear Vibrations in Engineering



The Harmonic Balance Method for Advanced Analysis and Design of Nonlinear Mechanical Systems
[Detroux et al. 2014]



Forced Response Sensitivity Analysis Using an Adjoint Harmonic Balance Solver
[Engels-Putzka et al. 2019]



Optimization of nonlinear structural resonance using the incremental harmonic balance method
[Dou and Jensen 2015]

Overview

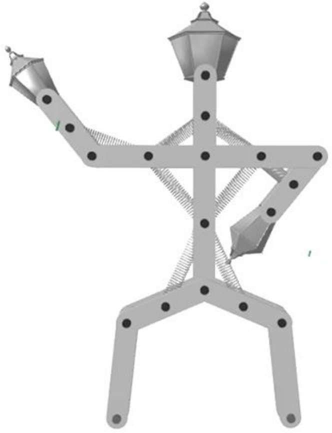
Harmonic Balance Method (HBM)

$$\mathbf{f} = \mathbf{M}\mathbf{a}$$
$$\rightarrow$$
$$\mathbf{A}(\omega)\mathbf{z} - \mathbf{b}(\mathbf{z}, \omega) = 0$$

Computational Design

$$\frac{df}{d\mathbf{p}} = \frac{d\mathbf{z}}{d\mathbf{p}}^T \frac{\partial f}{\partial \mathbf{z}} + \frac{\partial f}{\partial \mathbf{p}}$$

Examples



Harmonic Balance Method

Equations of Motion in Time Domain

- Dynamic equilibrium equations in the time domain

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} = \mathbf{f}_{\mathrm{int}}(\mathbf{x}) + \mathbf{f}_{\mathrm{ext}}(\mathbf{x}, \omega, \mathrm{t}) = \mathbf{f}(\mathbf{x}, \omega, \mathrm{t})$$

- Periodic solution can be expressed as Fourier series

$$\mathbf{x}(t) = \mathbf{c}_0^x + \sum_{k=1}^{\infty} \left(\mathbf{s}_k^x \sin(k\omega t) + \mathbf{c}_k^x \cos(k\omega t)\right)$$

- Truncation gives

$$\mathbf{x}(t) \approx \mathbf{c}_0^x + \sum_{k=1}^{N_H} \left(\mathbf{s}_k^x \sin(k\omega t) + \mathbf{c}_k^x \cos(k\omega t)\right)$$

Equations of Motion in Frequency Space

- Rewrite as

$$\mathbf{x}(t) = (\mathbf{Q}(t) \otimes \mathbf{I}_n)\mathbf{z} \quad \text{and} \quad \mathbf{f}(t) = (\mathbf{Q}(t) \otimes \mathbf{I}_n)\mathbf{b}$$

- Orthogonal trigonometric basis

$$\mathbf{Q}(t) = [1 \quad \sin(\omega t) \quad \cos(\omega t) \cdots \quad \sin(N_H \omega t) \quad \cos(N_H \omega t)]$$

- Fourier coefficients

$$\mathbf{z} = \left[(\mathbf{c}_0^x)^\mathrm{T} \quad (\mathbf{s}_1^x)^\mathrm{T} \quad (\boldsymbol{s}_1^x)^\mathrm{T} \ \cdots \quad (\boldsymbol{s}_{N_H}^x)^\mathrm{T} \quad (\mathbf{c}_{N_H}^x)^\mathrm{T}\right] \ \mathbf{c}_i^x, \mathbf{s}_i^x \in \mathbf{R}^{3n}$$

$$\mathbf{b} = \left[(\mathbf{c}_0^f)^\mathrm{T} \quad (\mathbf{s}_1^f)^\mathrm{T} \quad (\boldsymbol{s}_1^f)^\mathrm{T} \ \cdots \quad (\boldsymbol{s}_{N_H}^f)^\mathrm{T} \quad (\mathbf{c}_{N_H}^f)^\mathrm{T}\right] \ \mathbf{c}_i^f, \mathbf{s}_i^f \in \mathbf{R}^{3n}$$

Equations of Motion in Frequency Space

Galerkin projection

- Insert truncated series into time-domain equations of motion
- Integrate over period
- Project onto $\mathbf{Q}(t)$ $\rightarrow$ time-dependence disappears

Equations of motion in frequency space:

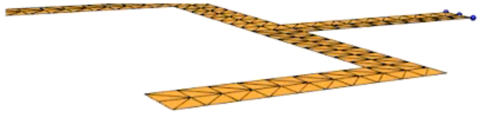
$$\mathbf{h}(\mathbf{z},\omega) \equiv \mathbf{A}(\omega)\mathbf{z} - \mathbf{b}(\mathbf{z},\omega) = 0$$

Inertia and damping forces

Nonlinear forces

Time-Domain vs. Frequency-Domain Simulation

Nonlinear Time Domain
(Newmark)



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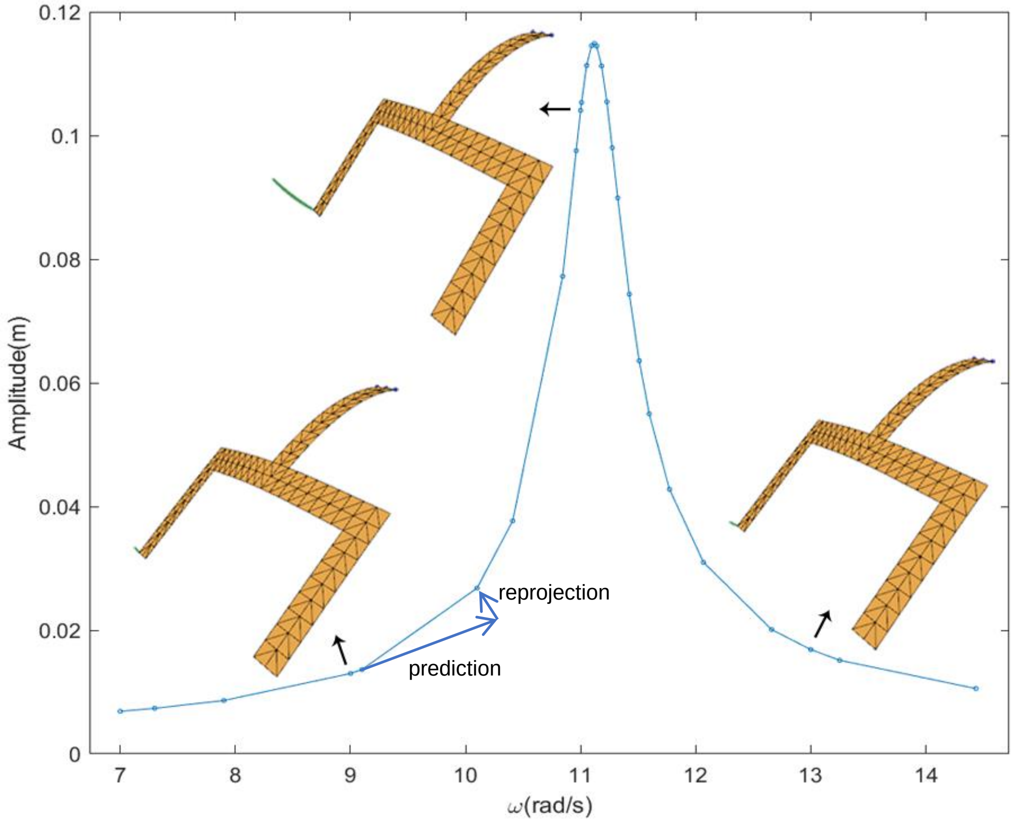
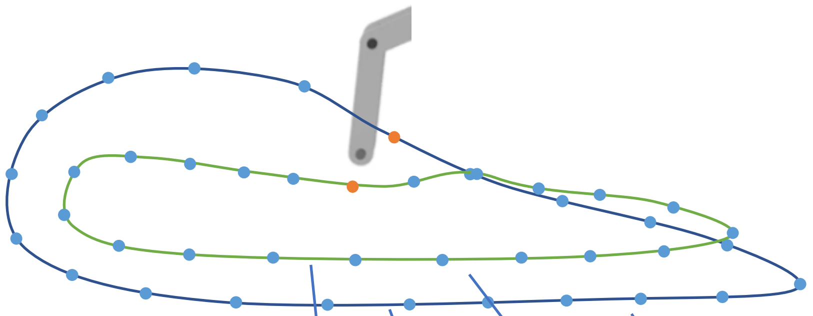
Wall-clock simulation time

Frequency Response Curves



Computational Design

Design Objectives – Trajectory



Distance between Fourier coefficients

$$f_{Dist}(\mathbf{z}) = \sum_{i=1}^{N_H} \sum_{j=1}^{3} \left(\left(\mathbf{s}_i^{3k+j} - \hat{\mathbf{s}}_i^{j} \right)^2 + \left(\mathbf{c}_i^{3k+j} - \hat{\mathbf{c}}_i^{j} \right)^2 \right)$$

Design Objectives – Amplitude

- How do we quantify motion magnitude?

$$A_i(\mathbf{z}) = \int_0^T \|\mathbf{v}_i(t)\|\, dt$$

$$\mathbf{x} = \Gamma_x \mathbf{z} \quad \rightarrow \quad A_i(\mathbf{z}) = \sum_j \left| \mathbf{x}_i^{j+1} - \mathbf{x}_i^j \right|$$

- Amplitude objective

$$f_{Ampl}(\mathbf{z}) = (A_i(\mathbf{z}) - \bar{A}_i)^2$$

Design Sensitivity

- Optimal design: minimize objective function $f(\mathbf{z}(\mathbf{p}), \mathbf{p})$ wrt. $\mathbf{p}$
- Requires gradient

$$\frac{df}{d\mathbf{p}} = \frac{d\mathbf{z}}{d\mathbf{p}}^T \frac{\partial f}{\partial \mathbf{z}} + \frac{\partial f}{\partial \mathbf{p}}$$

- Dynamic Equilibrium

$$\mathbf{h}(\mathbf{z}, \mathbf{p}, \omega) = \mathbf{A}(\mathbf{p}, \omega)\mathbf{z} - \mathbf{b}(\mathbf{p}, \mathbf{z}, \omega) = 0$$

$$\frac{d\mathbf{h}}{d\mathbf{p}} = 0 \qquad \rightarrow \frac{\partial \mathbf{h}}{\partial \mathbf{z}}\frac{d\mathbf{z}}{d\mathbf{p}} + \frac{\partial \mathbf{h}}{\partial \mathbf{p}} = \mathbf{0} \qquad \rightarrow$$

Sensitivity matrix

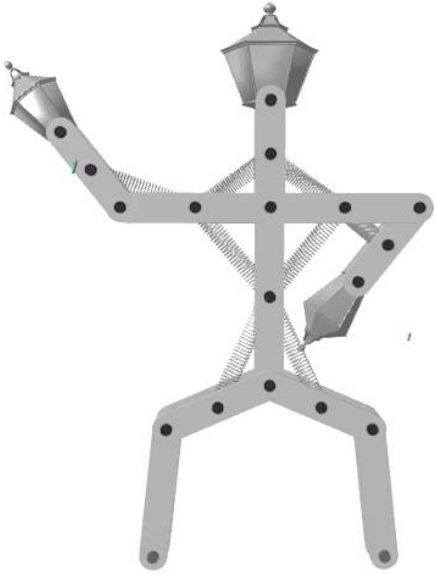
$$\mathbf{S} = \frac{d\mathbf{z}}{d\mathbf{p}} = -\frac{\partial \mathbf{h}}{\partial \mathbf{z}}^{-1} \frac{\partial \mathbf{h}}{\partial \mathbf{p}}$$

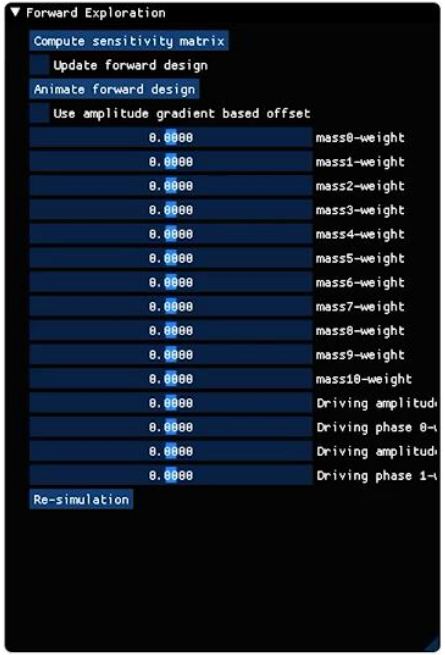
Examples

Forward Sensitivity Exploration

- First-order prediction for new equilibrium state

$$\mathbf{z}_p = \mathbf{z} + \frac{d\mathbf{z}}{d\mathbf{p}}\Delta\mathbf{p}$$



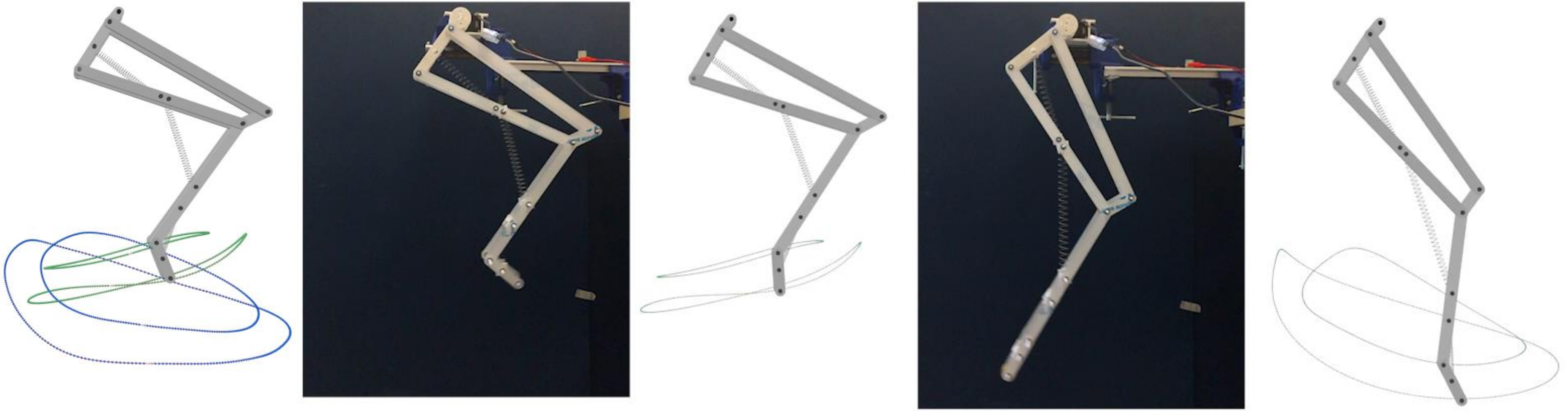


Inverse Design – Trajectory

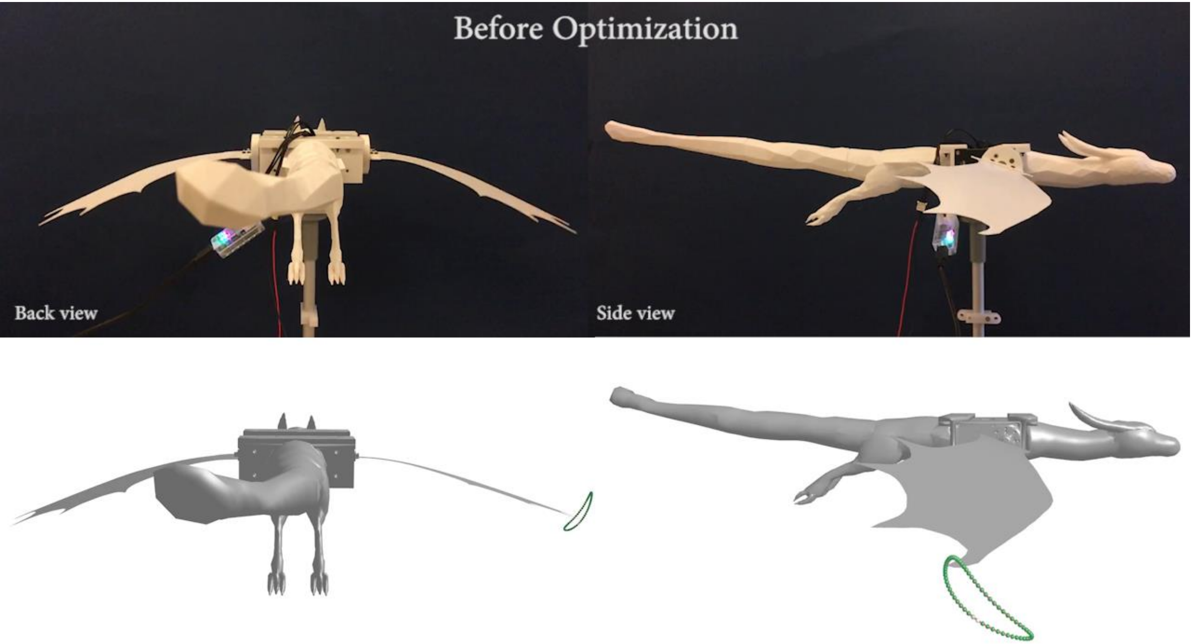
Target

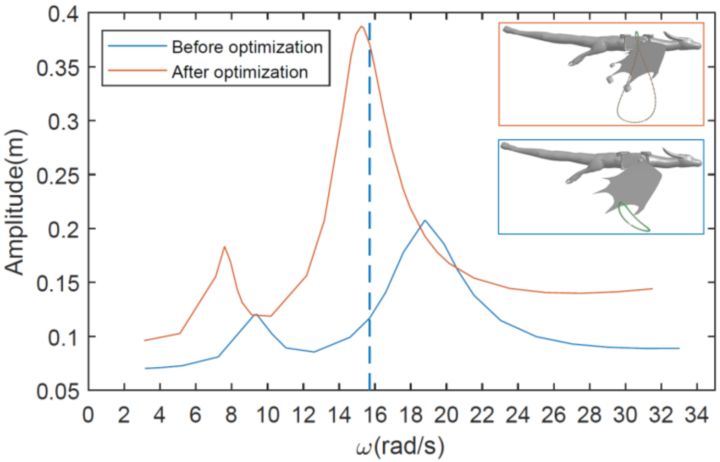
Before Optimization

After Optimization

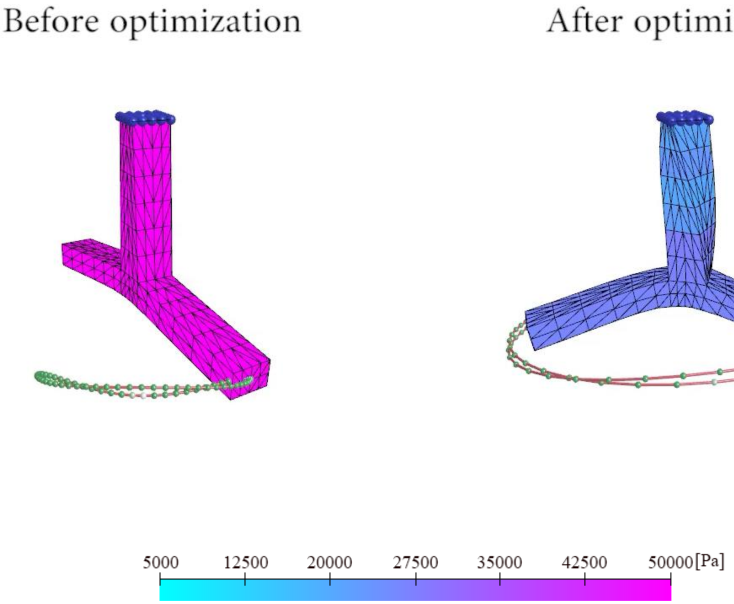


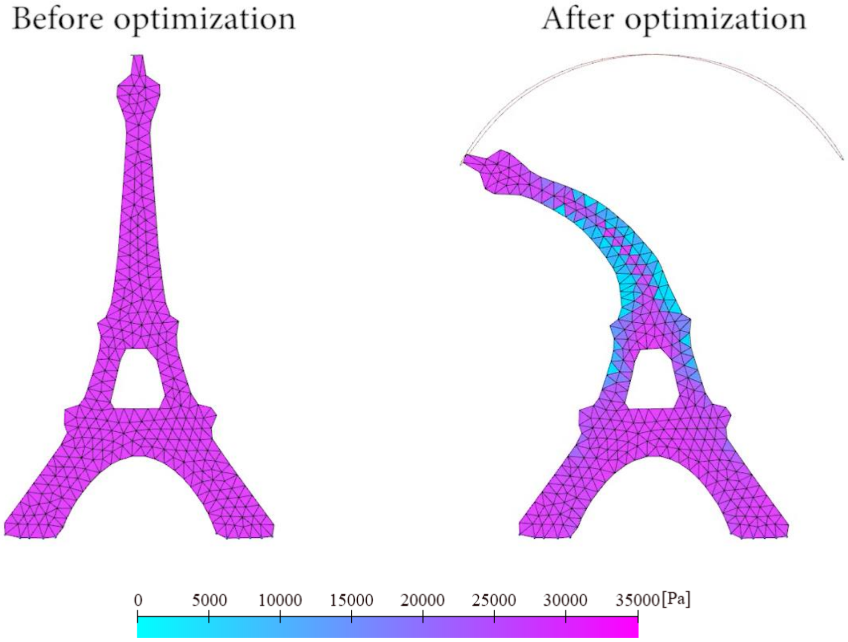
Inverse Design – Amplitude





Inverse Design – Amplitude





Conclusions

HBM + Sensitivity Analysis = Efficient and powerful approach for designing nonlinear mechanical systems with large-amplitude motion

Limitations & Future Work

- More accurate damping
- Contact and friction
- Subspace HBM

Thank you!