SIGGRAPH 2021

ANTITHETIC SAMPLING FOR MONTE CARLO DIFFERENTIABLE RENDERING

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STOCHASTIC SAMPLING FOR FORWARD RENDERING

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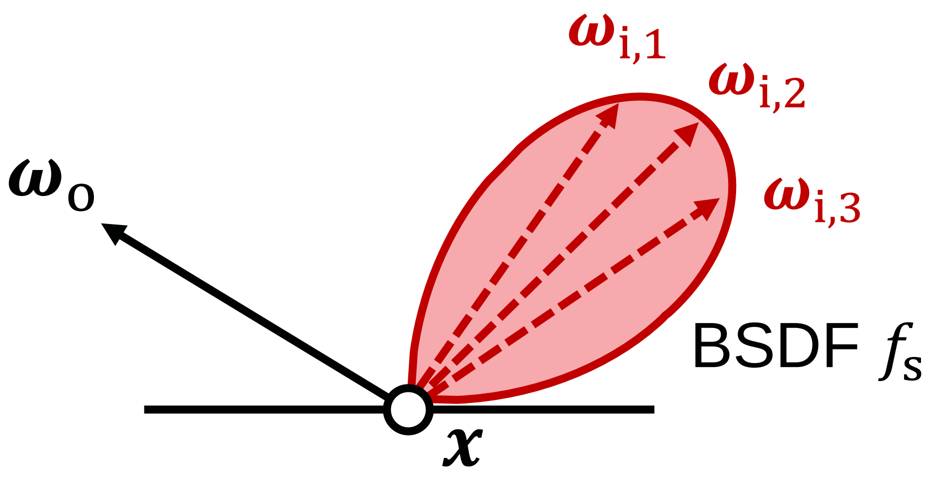
渲染方程 Rendering equation

$$L_r(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{o}) = \int_{\mathbb{S}^2} L_\mathrm{i}(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{i}) f_\mathrm{s}(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{i},\boldsymbol{\omega}_\mathrm{o})\,\mathrm{d}\boldsymbol{\omega}_\mathrm{i}^{\perp}$$

蒙特卡洛方法
Monte Carlo integration

$$L_r(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{o}) \approx \frac{1}{N}\sum_{n=1}^{N} \frac{L_\mathrm{i}(\boldsymbol{x},\boldsymbol{\omega}_{\mathrm{i},n}) f_\mathrm{s}(\boldsymbol{x},\boldsymbol{\omega}_{\mathrm{i},n},\boldsymbol{\omega}_\mathrm{o})}{p(\boldsymbol{\omega}_{\mathrm{i},n})}$$



BSDF (重要度) 采样

DIFFERENTIABLE RENDERING

渲染方程 Rendering equation

$$L_r(\boldsymbol{x}, \boldsymbol{\omega}_\mathrm{o}) = \int_{\mathbb{S}^2} L_\mathrm{i}(\boldsymbol{x}, \boldsymbol{\omega}_\mathrm{i}) f_\mathrm{s}(\boldsymbol{x}, \boldsymbol{\omega}_\mathrm{i}, \boldsymbol{\omega}_\mathrm{o})\, \mathrm{d}{\boldsymbol{\omega}_\mathrm{i}}^\perp$$

求导

$$\frac{\mathrm{d}L_r}{\mathrm{d}\theta} = \underbrace{\boxed{\int_{\mathbb{S}^2} \frac{\mathrm{d}(L_\mathrm{i} f_\mathrm{s})}{\mathrm{d}\theta}\, \mathrm{d}{\boldsymbol{\omega}_\mathrm{i}}^\perp}}_{\text{interior term}} + \underbrace{\boxed{\int_{\Delta\mathbb{S}^2} g(\boldsymbol{\omega}_\mathrm{i})\, \mathrm{d}\ell(\boldsymbol{\omega}_\mathrm{i})}}_{\text{boundary term}}$$

BSDF SAMPLING
FOR FORWARD RENDERING

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渲染方程 Rendering equation

$$L_r(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{o}) = \int_{\mathbb{S}^2} L_\mathrm{i}(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{i})\, f_\mathrm{s}(\boldsymbol{x},\boldsymbol{\omega}_\mathrm{i},\boldsymbol{\omega}_\mathrm{o})\, \mathrm{d}{\boldsymbol{\omega}_\mathrm{i}}^{\perp}$$

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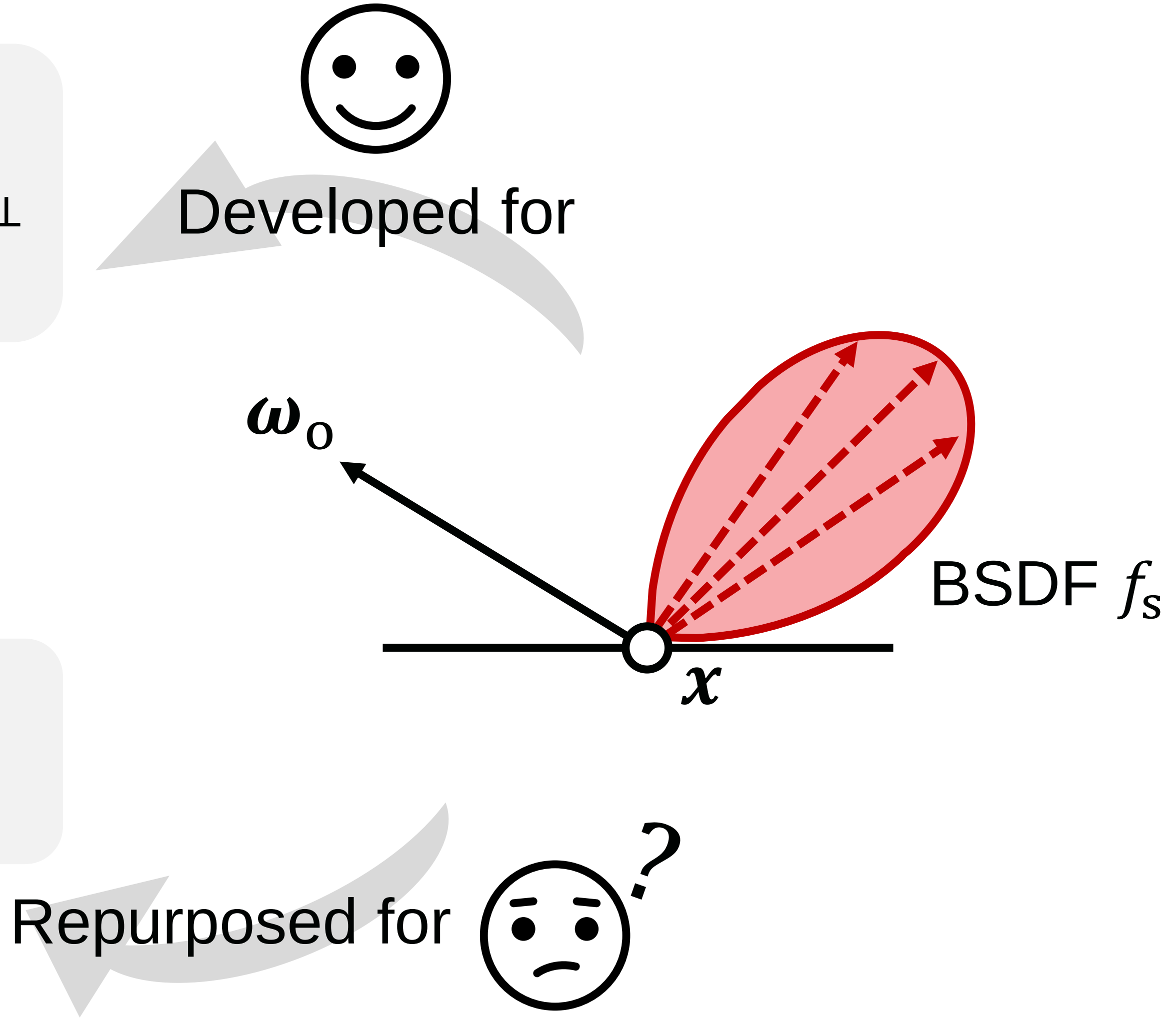
interior term boundary term



PRIOR WORK IN DIFFERENTIABLE RENDERING

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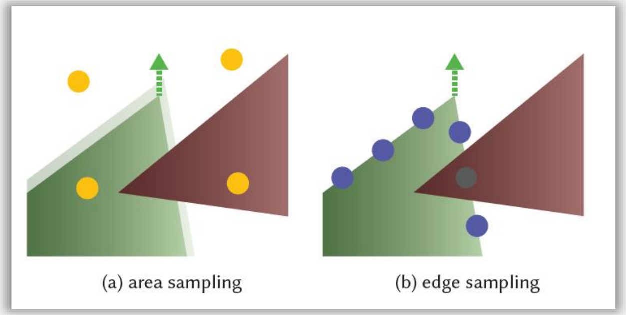
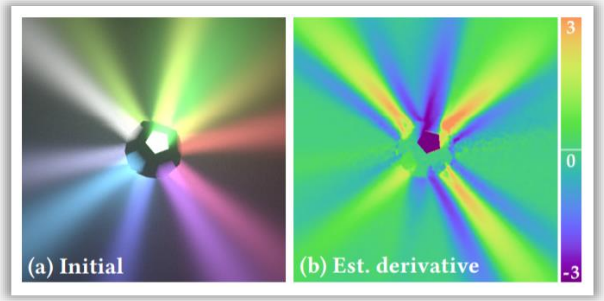
Edge-sampling methods

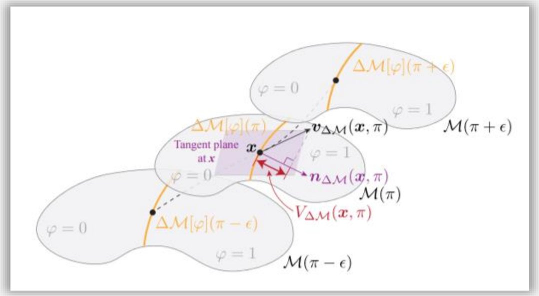


Li et al. 2018

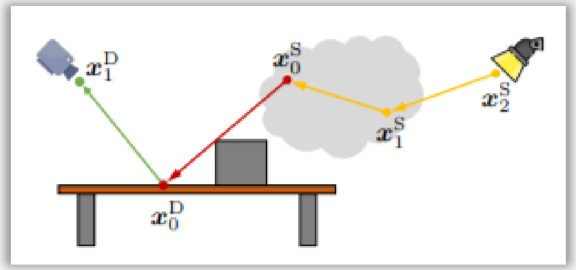


Zhang et al. 2019

Path-space methods

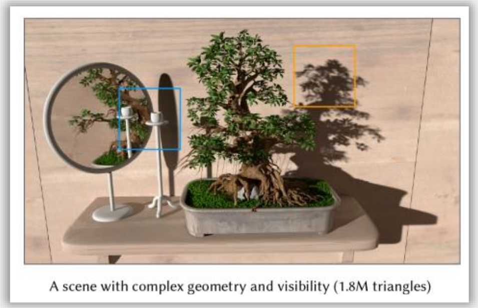


Zhang et al. 2020

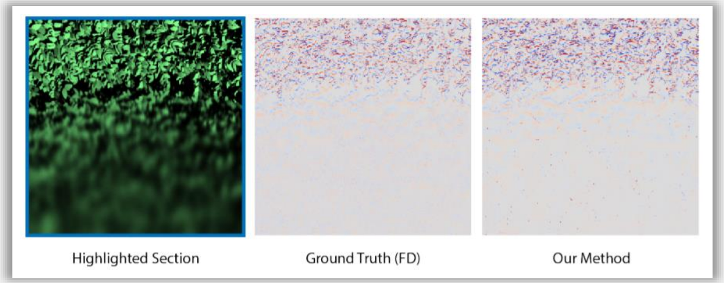


Zhang et al. 2021

Convolution-based methods



Loubet et al. 2019



Bangaru et. al. 2020

Most, if not all, prior work rely on **standard** BSDF sampling

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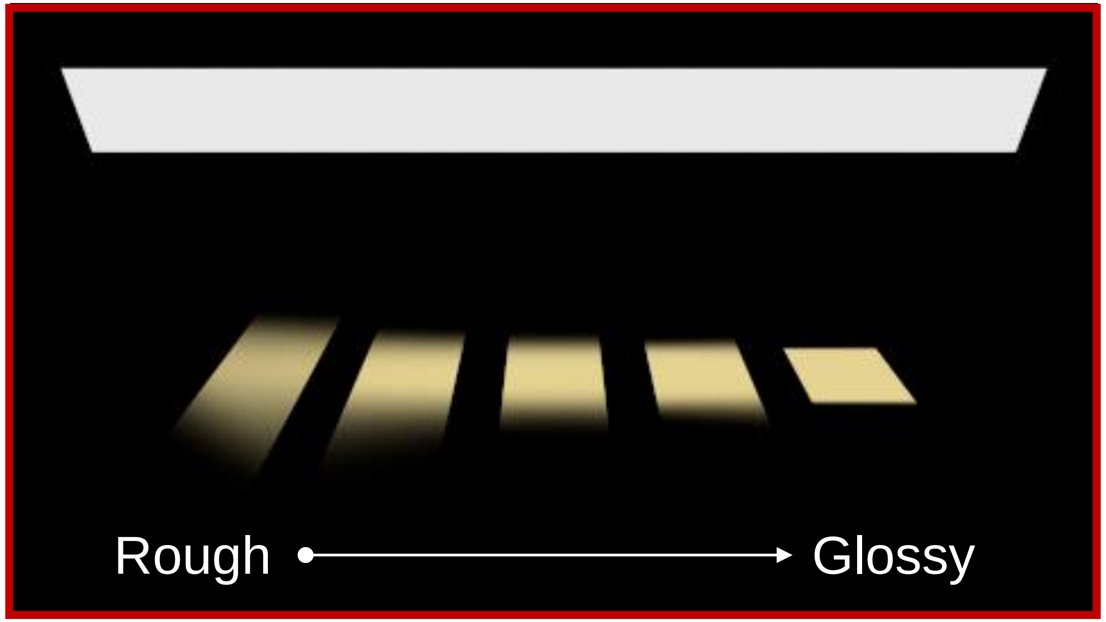
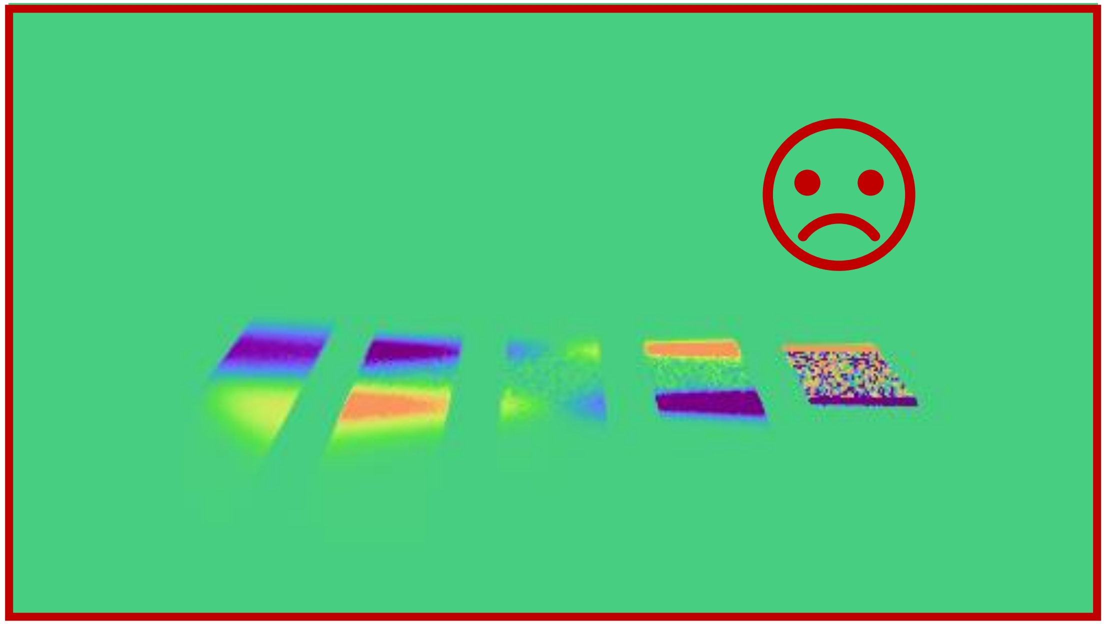
Observation

Standard BSDF sampling may lead to high variance for **glossy** material

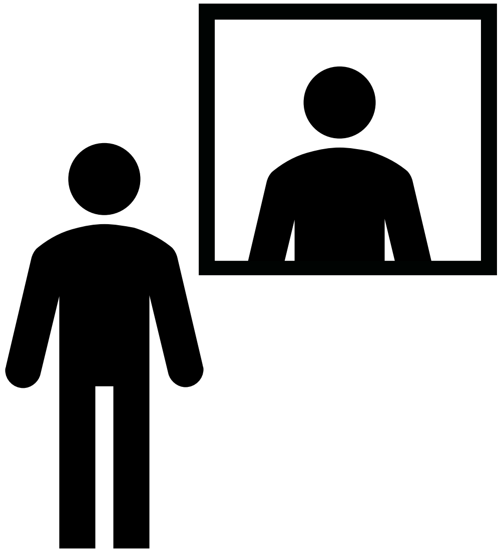


Original Image



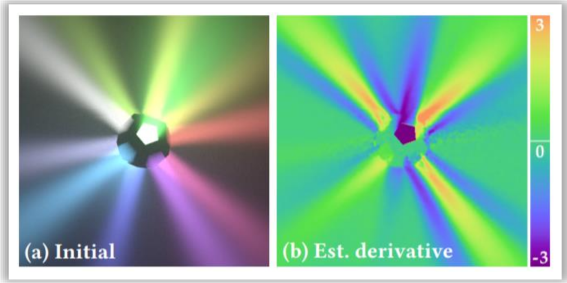
Derivative Image
(w.r.t. horizontal rotation)

OUR CONTRIBUTIONS

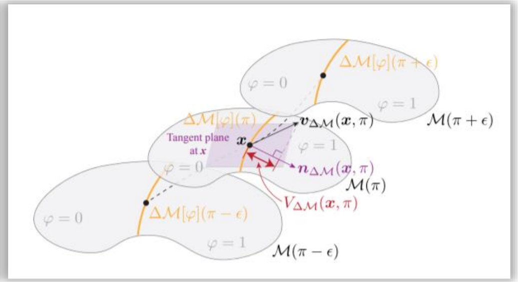


Handling geometric derivative of glossy surfaces

Edge-sampling methods



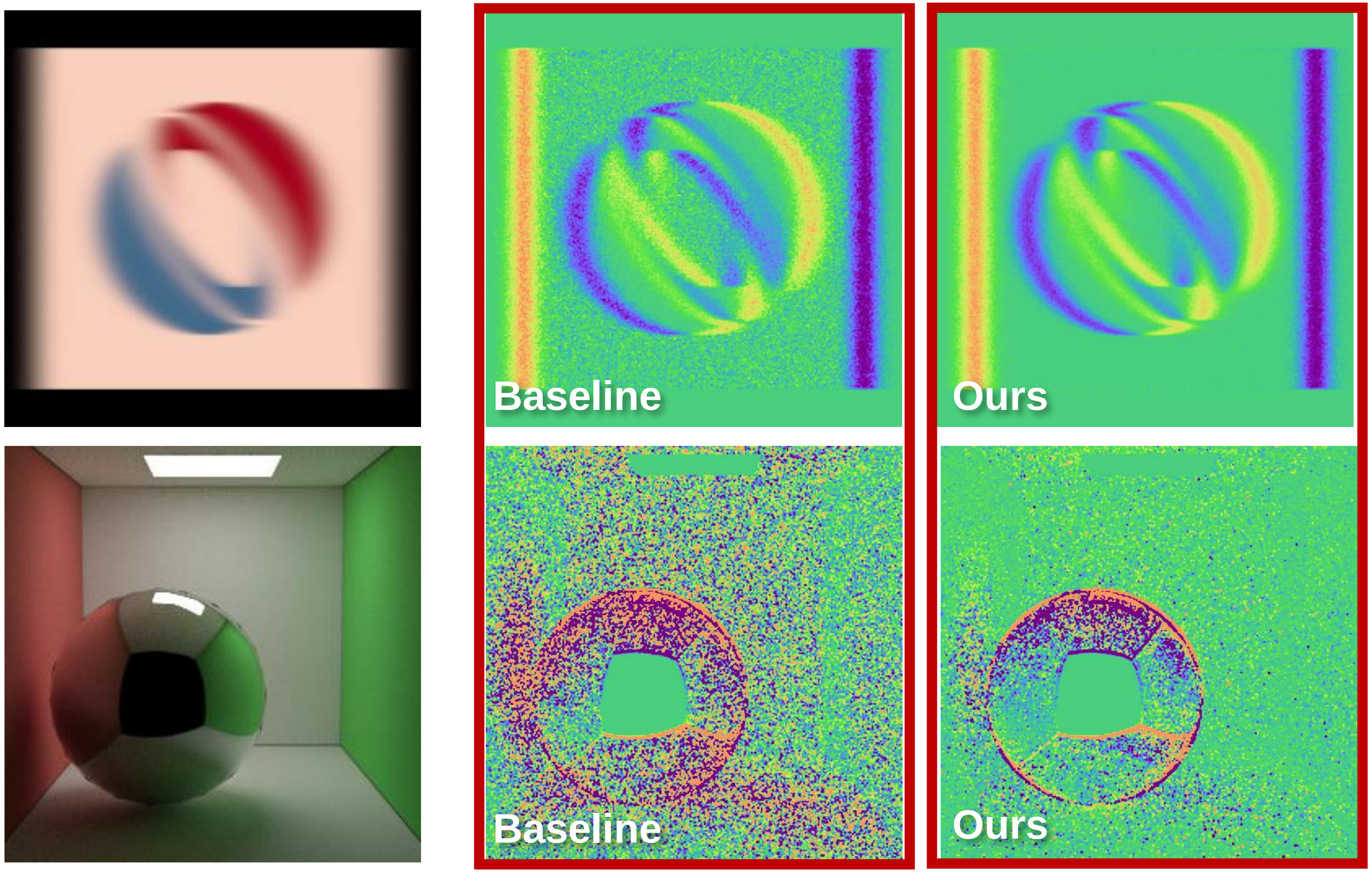
Path-space methods



Parameterization independent
(i.e., applicable to different formulations)

PREVIEW OF OUR RESULTS

Equal-time Comparison



Compute derivative w.r.t.
rotation angle of the reflector around a horizontal axis

Edge-sampling
[Li et al. 2018] [Zhang et al. 2019]

Compute derivative w.r.t.
vertical position of the sphere

PSDR
[Zhang et al. 2020]

OUR TECHNIQUE

ANTITHETIC SAMPLING

ANTITHETIC SAMPLING

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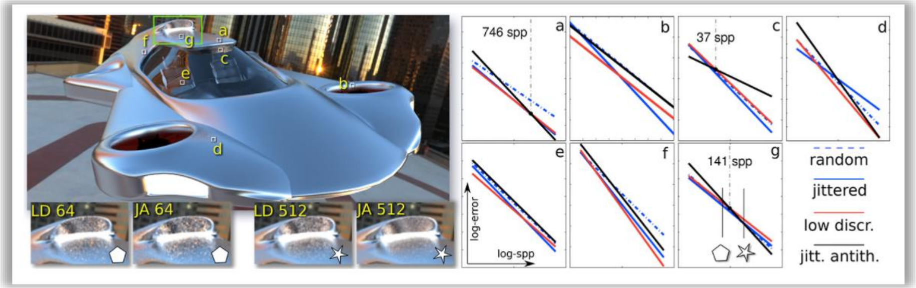
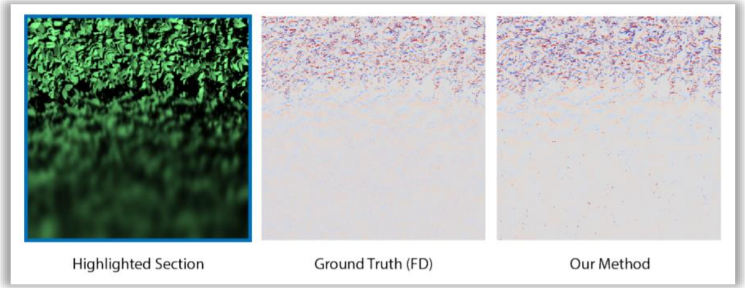
- Classic variance reduction framework for Monte Carlo estimation
- Limited uses in computer graphics



Subr et al. 2014



Bangaru et. al. 2020

- **Core idea:** using **negatively correlated （负相关）** samples

ANTITHETIC SAMPLING (1D EXAMPLE)

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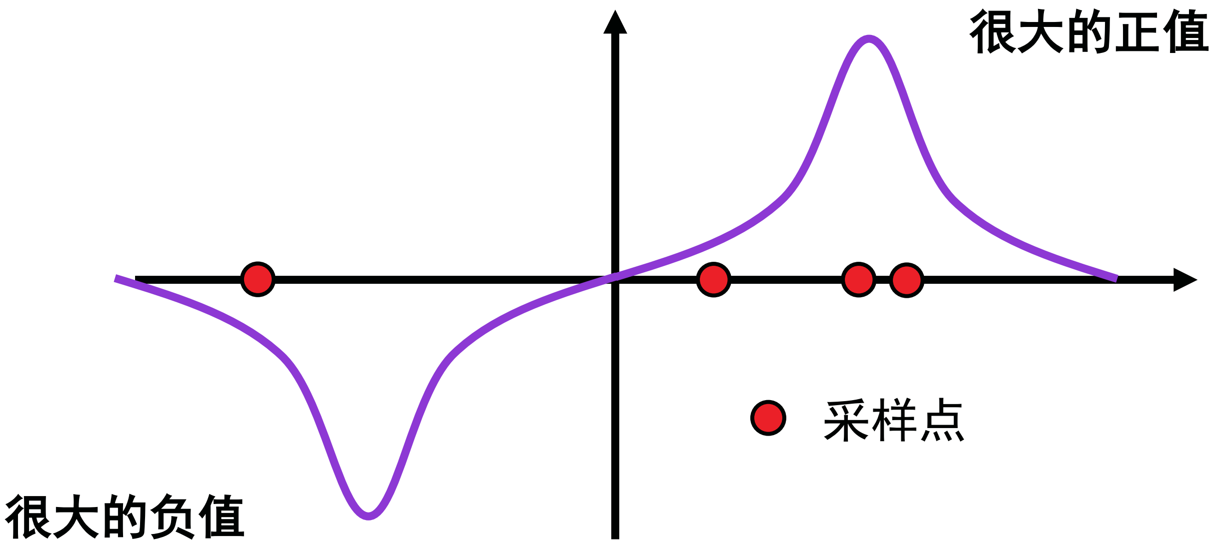
1D example: Estimating integral

$$I := \int_{-\infty}^{\infty} F(x)\,\mathrm{d}x$$

where the integrand F is approximately an odd function with $F(x) \approx -F(-x)$



Monte Carlo integration

- **Independent (独立)** samples

  Slow convergence if F contains high-magnitude positive and negative regions

ANTITHETIC SAMPLING (1D EXAMPLE)

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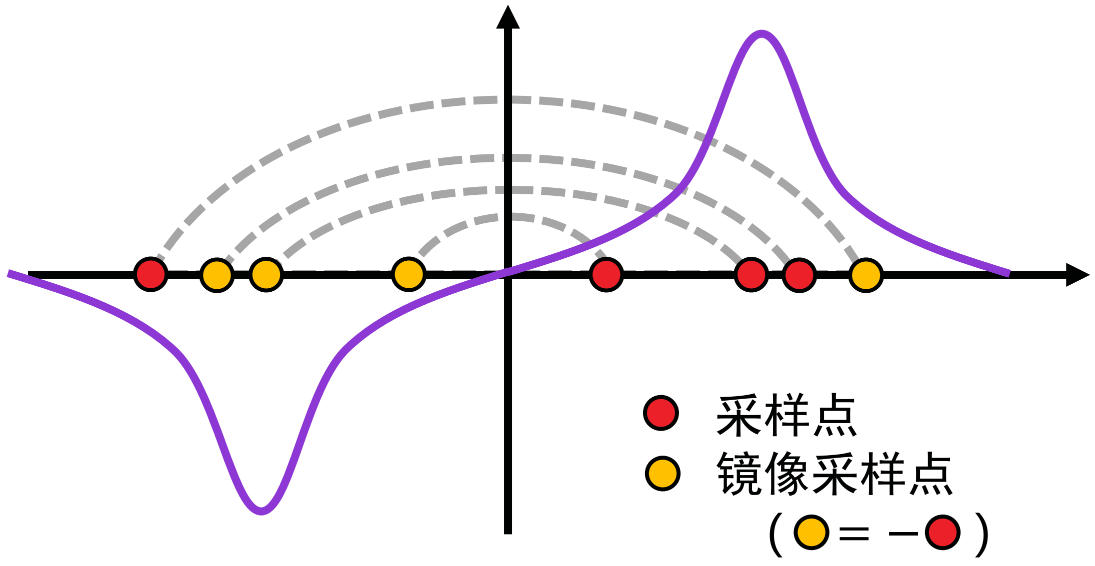


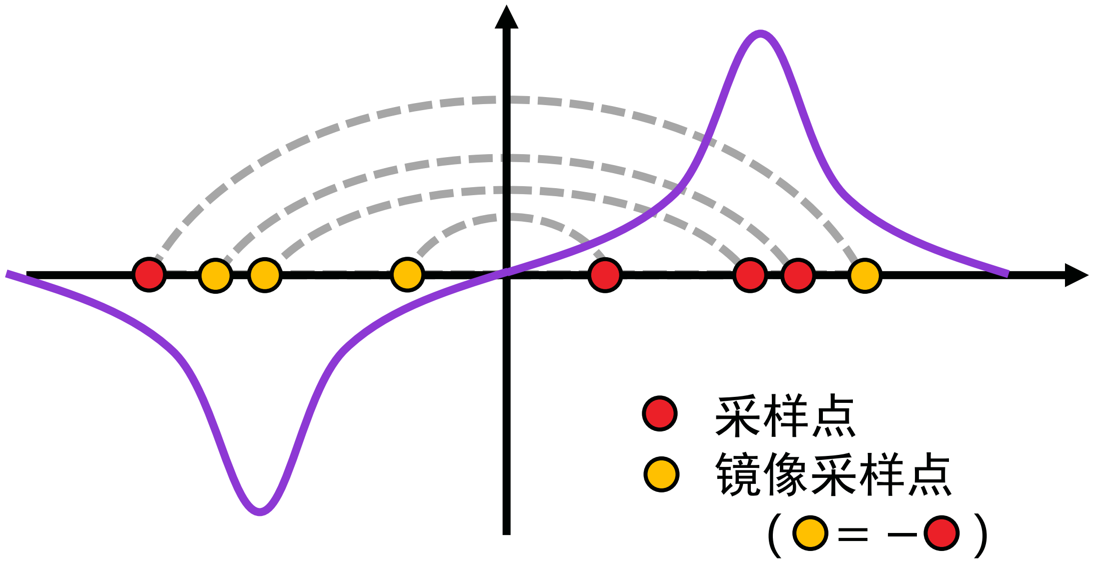
Monte Carlo integration

- **Independent (独立)** samples

  Slow convergence if F contains high-magnitude positive and negative regions
- **Negatively correlated (负相关)** samples

  Faster convergence since $F(\bullet) + F(\circ) \approx 0$

ANTITHETIC SAMPLING (BSDF SAMPLING)



interior term $\int_{\mathbb{S}^2} \frac{\mathrm{d}\,(\,L_\mathrm{i}\, f_\mathrm{s}\,)}{\mathrm{d}\theta}\, \mathrm{d}\boldsymbol{\omega}_\mathrm{i}{}^{\perp}$

Differentiable rendering

To apply *antithetic sampling*:
Finding the **point symmetry (对称性)** is the key!

L_i = incident radiance

f_s = BSDF

ANTITHETIC SAMPLING (BSDF SAMPLING)

Microfacet BSDF

$$f_s(\omega_i, \omega_o) = D(\omega_h)\, f_s^{\,0}(\omega_i, \omega_o)$$

All other factors
(i.e., Fresnel, shadowing/masking)

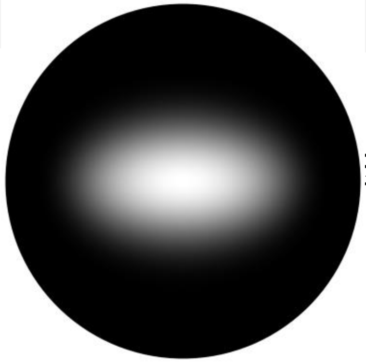
$$\frac{\mathrm{d}\,(L_i\, f_s)}{\mathrm{d}\theta} = f_s \frac{\mathrm{d}L_i}{\mathrm{d}\theta} + D(\omega_h) \frac{\mathrm{d}f_s^{\,0}}{\mathrm{d}\theta} L_i + \frac{\mathrm{d}D(\omega_h)}{\mathrm{d}\omega_h} \frac{\mathrm{d}\omega_h}{\mathrm{d}\theta} f_s^{\,0} L_i$$

Approximately
point symmetric

...rm $\int_{\mathbb{S}^2} \frac{\mathrm{d}\,(L_i\, f_s)}{\mathrm{d}\theta}\, \mathrm{d}\boldsymbol{\omega}_i^{\perp}$

vary
slowly

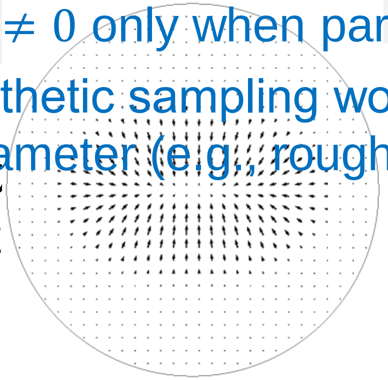
- $\frac{\mathrm{d}\omega_h}{\mathrm{d}\theta} \neq 0$ only when parameter θ controls geometry
- Antithetic sampling won't help if θ is a material parameter (e.g., roughness)



...法线分布函数 D
...tribution funct...

半程向量

Half-way vector $\boldsymbol{\omega}_h = \frac{\boldsymbol{\omega}_i + \boldsymbol{\omega}_o}{\|\boldsymbol{\omega}_i + \boldsymbol{\omega}_o\|}$



$D(\boldsymbol{\omega}_h)$ **point symmetric**
around surface normal

$\frac{\mathrm{d}D(\boldsymbol{\omega}_h)}{\mathrm{d}\boldsymbol{\omega}_h}$ **point symmetric**
around surface normal

ANTITHETIC SAMPLING (BSDF SAMPLING)

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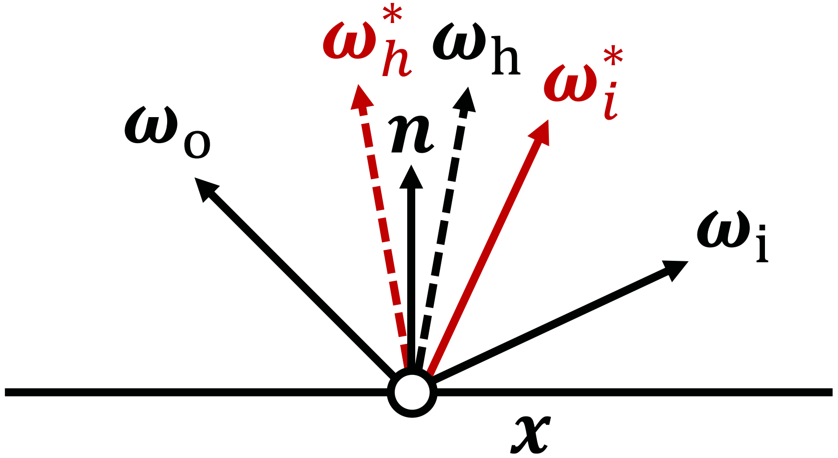
Properly sampled by NDF

$$\frac{\mathrm{d}(L_\mathrm{i} f_\mathrm{s})}{\mathrm{d}\theta} = f_\mathrm{s}\frac{\mathrm{d}L_\mathrm{i}}{\mathrm{d}\theta} + D(\omega_\mathrm{h})\frac{\mathrm{d}f_\mathrm{s}^0}{\mathrm{d}\theta}L_\mathrm{i} + \frac{\mathrm{d}D(\omega_\mathrm{h})}{\mathrm{d}\theta} f_\mathrm{s}^0 L_\mathrm{i}$$

$$:= \frac{\mathrm{d}D(\omega_\mathrm{h})\,\mathrm{d}\omega_\mathrm{h}}{F\,\mathrm{d}\omega_\mathrm{h}}$$

$$F(\omega_\mathrm{h}) + F(\boldsymbol{\omega}_h^*) \approx 0$$



AntitheticBSDFSample

```
fun AntitheticBSDFSample(x, ωo):
  Draw ωh = [xh, yh, zh] ~ ph
  Compute ωi based on ωh and ωo
  Compute pi based on ph(ωh)
  Set ωh* = [−xh, −yh, zh]
  Compute ωi* based on ωh* and ωo
  Compute pi* based on ph(ωh*)
  return (ωi, pi, ωi*, pi*)
```

ANTITHETIC SAMPLING (BSDF SAMPLING)

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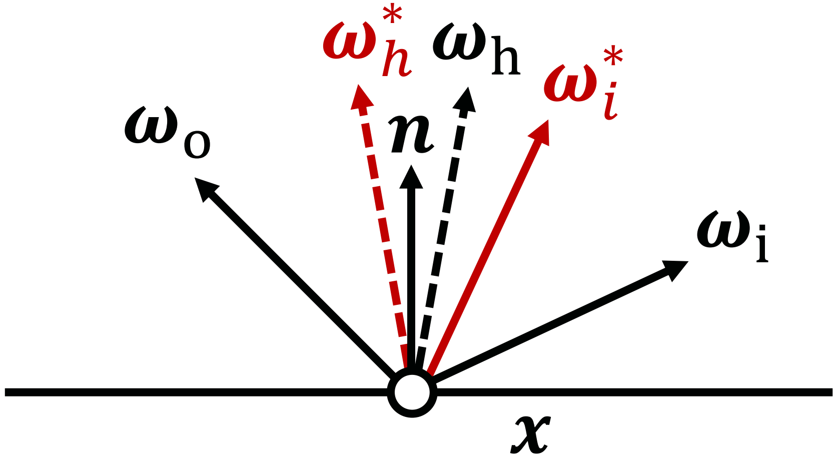


AntitheticBSDFSample

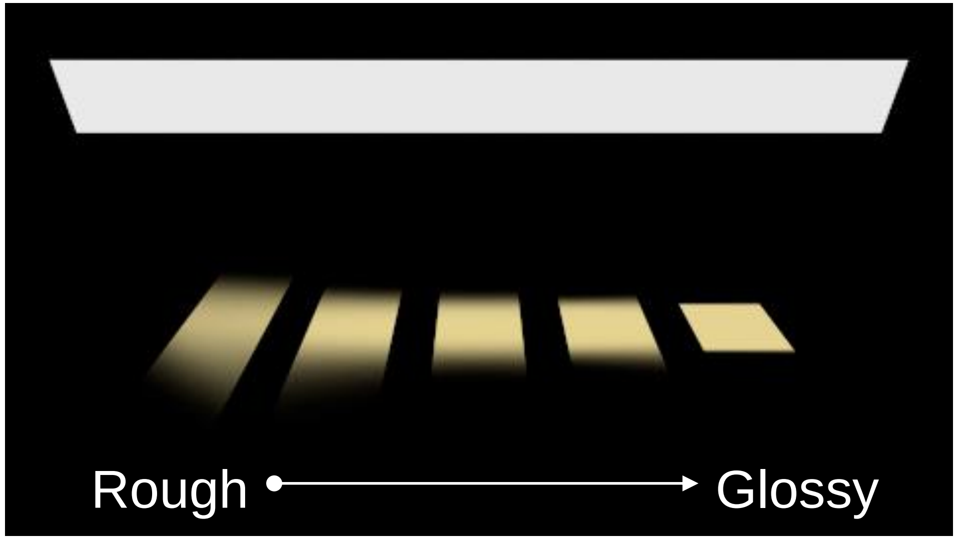
Algorithm: Differential path tracing

```
fun diffPT(x, ω_o):
    L = L_e(x, ω_o)
    (ω_i, p) = BSDFSample(x, ω_o)
    y = rayTrace(x, ω_i)
    L += diffPT(y, −ω_i) f_s^⊥(x, ω_i, ω_o)/p
    return L
```

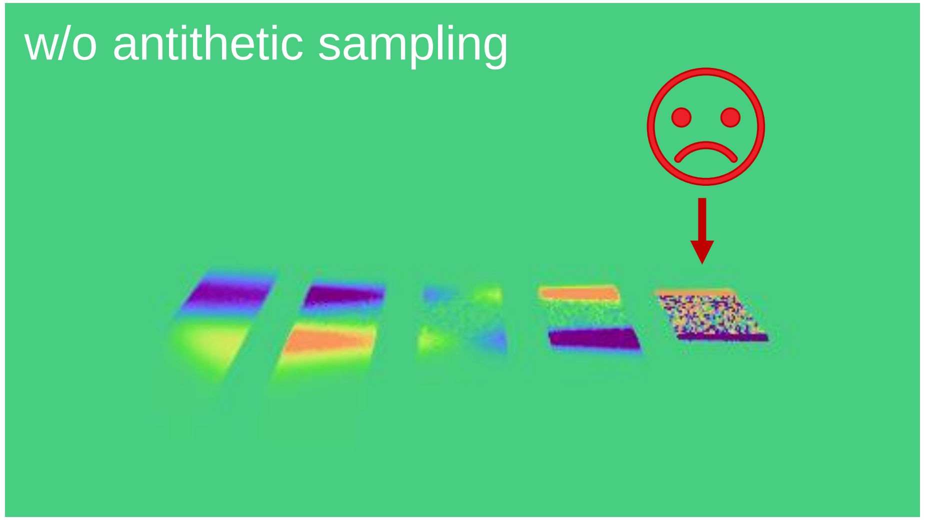
Algorithm: Differential path tracing with antithetic sampling

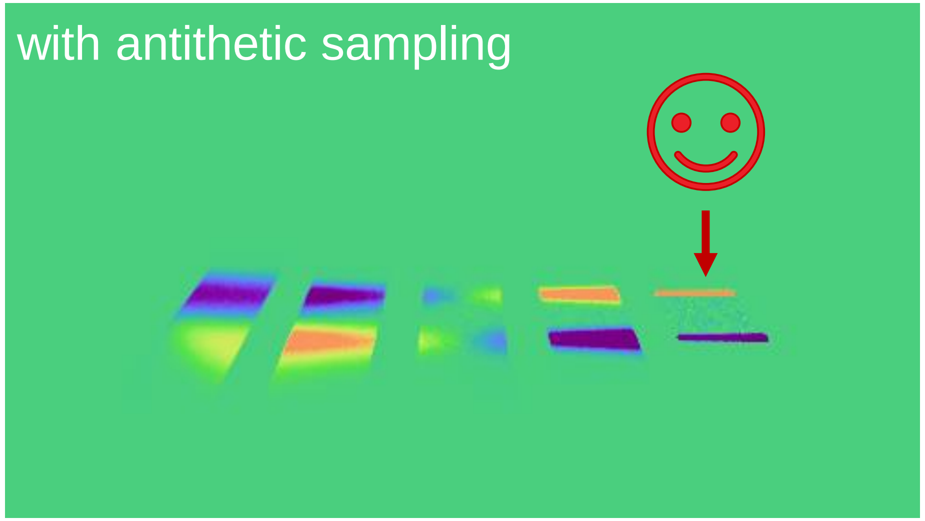
```
fun diffPT(x, ω_o):
    L = L_e(x, ω_o);
    (ω_i, ω_i^*, p, p^*) = AntitheticBSDFSample(x, ω_o)
    y = rayTrace(x, ω_i)
    L += diffPT(y, −ω_i) f_s^⊥(x, ω_i, ω_o)/(p + p^*)
    y^* = rayTrace(x, ω_i^*)
    L += diffPT(y^*, −ω_i^*) f_s^⊥(x, ω_i^*, ω_o)/(p + p^*)
    return L
```

ANTITHETIC SAMPLING (BSDF SAMPLING)



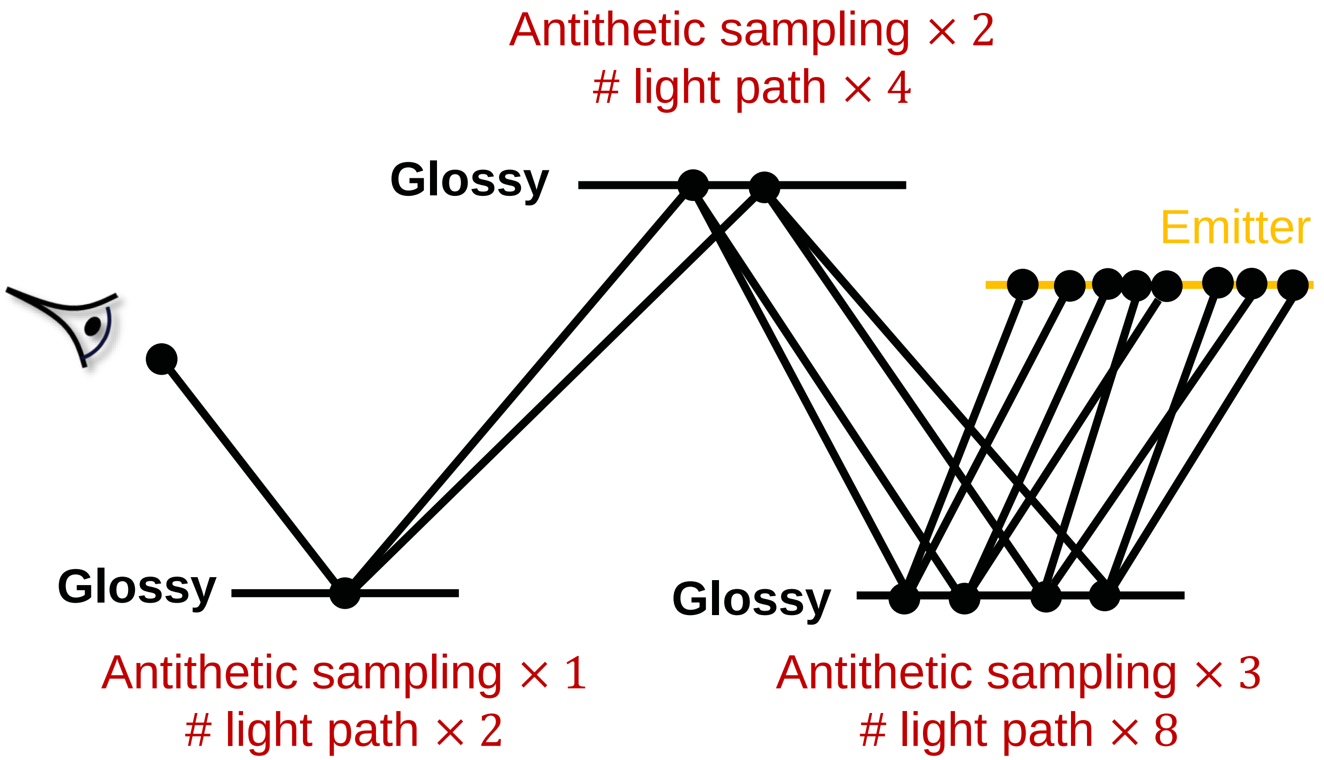
Original Image





Derivative Image
(w.r.t. horizontal rotation)

EXPONENTIAL BRANCHING



Problem: **Exponential branching** of light paths for **mostly glossy** scene

ANTITHETIC SAMPLING (PATH LEVEL)

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- Key idea: Decompose the derivative of light path contribution (via product rule)
- One antithetic path per glossy vertex
- Antithetic & ordinary path cancels out

Glo

More details

- Gradient-domain path tracing (GDPT) maximizes consistency between antithetic and ordinary path
- Can be adopted for BDPT (双向路径追踪)

拆解前：多个BSDF的乘积

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(f_s^1\, f_s^2\, f_s^3\right) = \frac{\mathrm{d}f_s^1}{\mathrm{d}\theta} f_s^2 f_s^3 + \frac{\mathrm{d}f_s^2}{\mathrm{d}\theta} f_s^1 f_s^3 + \frac{\mathrm{d}f_s^3}{\mathrm{d}\theta} f_s^1 f_s^2$$

拆解后：每项只有一个BSDF导数

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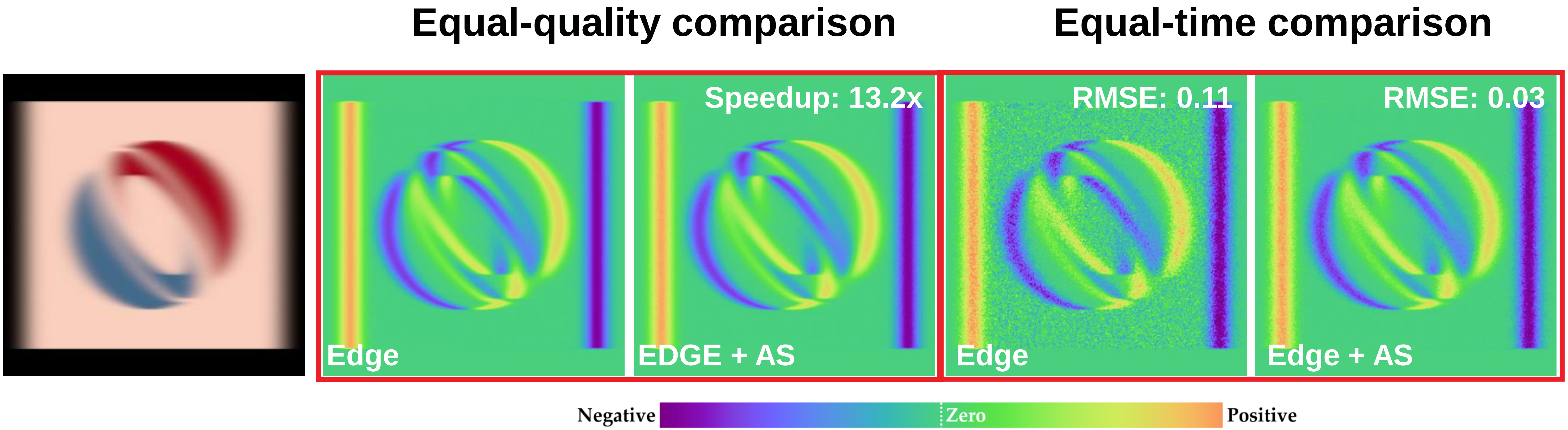
RESULTS

RESULTS: DIFFERENTIABLE-RENDERING COMPARISONS



RESULTS: DIFFERENTIABLE-RENDERING COMPARISONS

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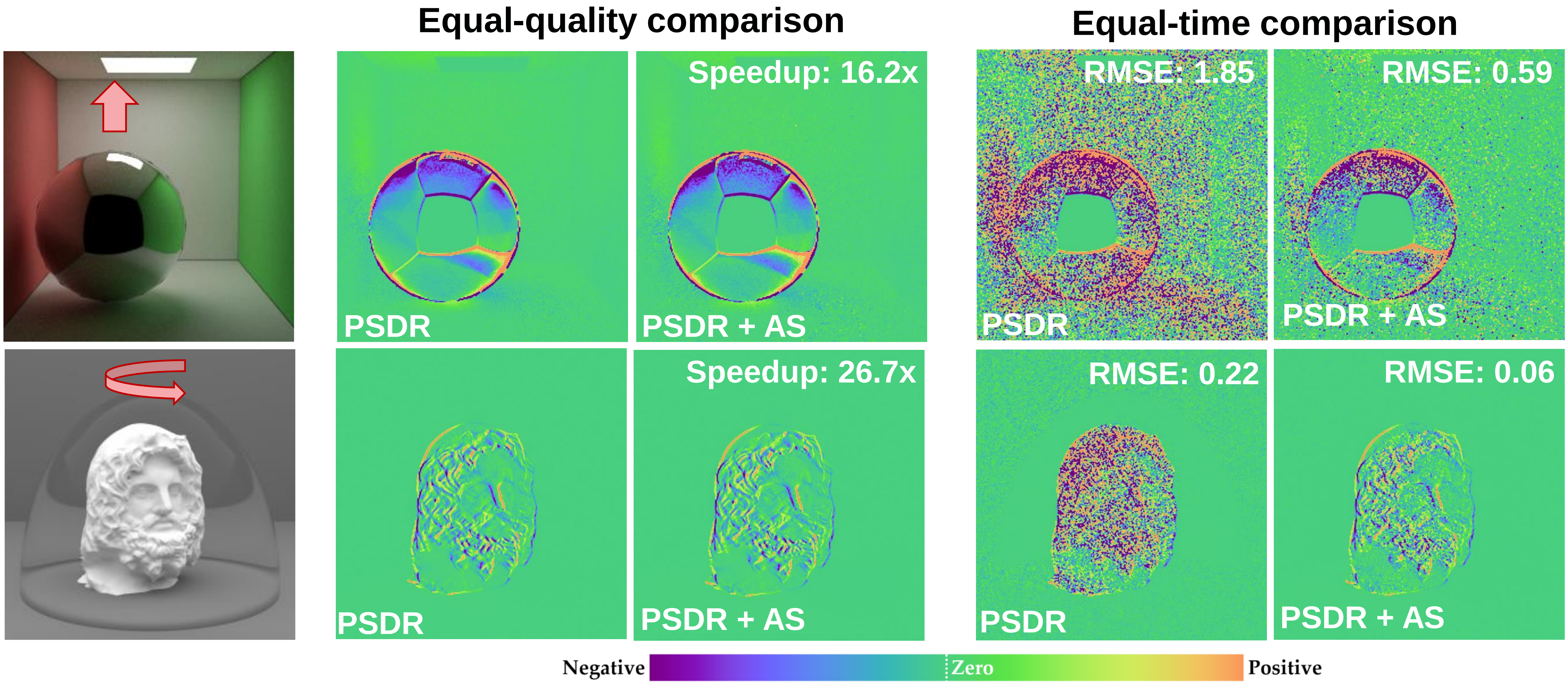
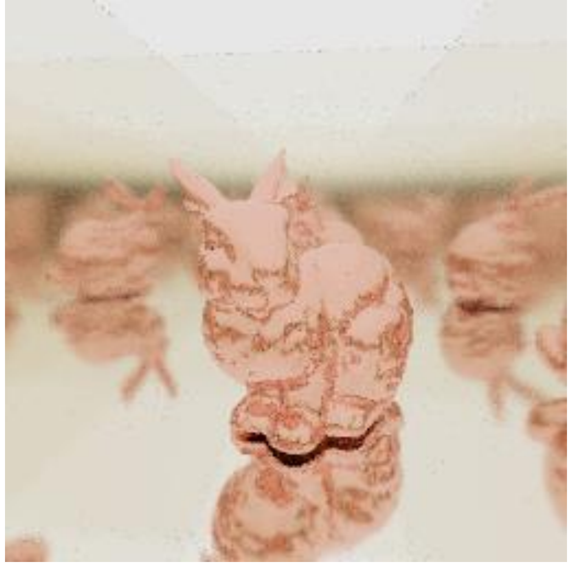
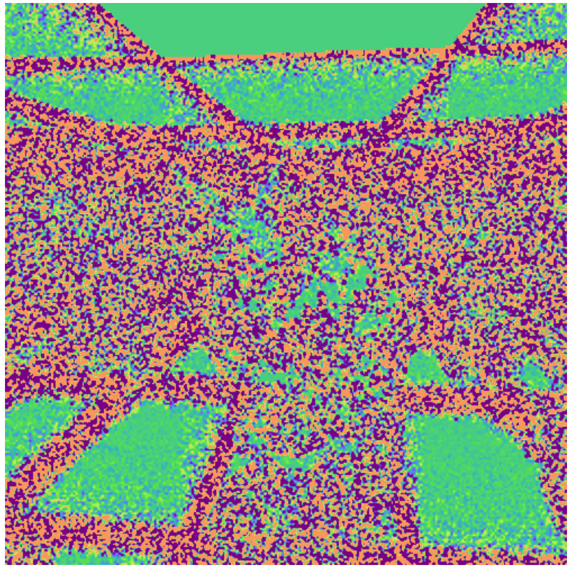


RESULTS: PATH-SPACE ANTITHETIC SAMPLING

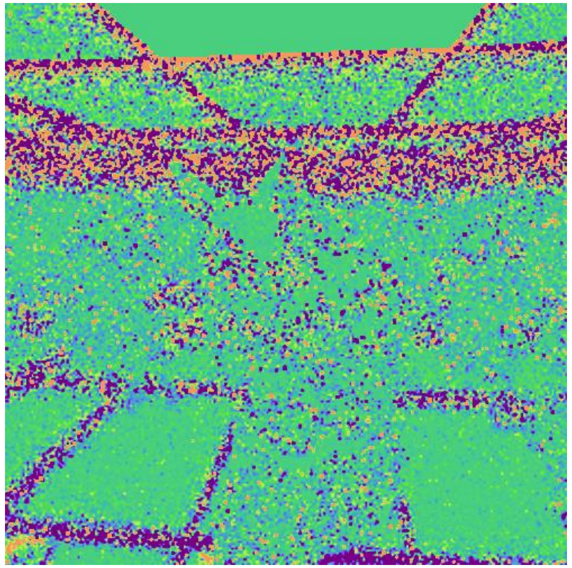
Equal-time comparison



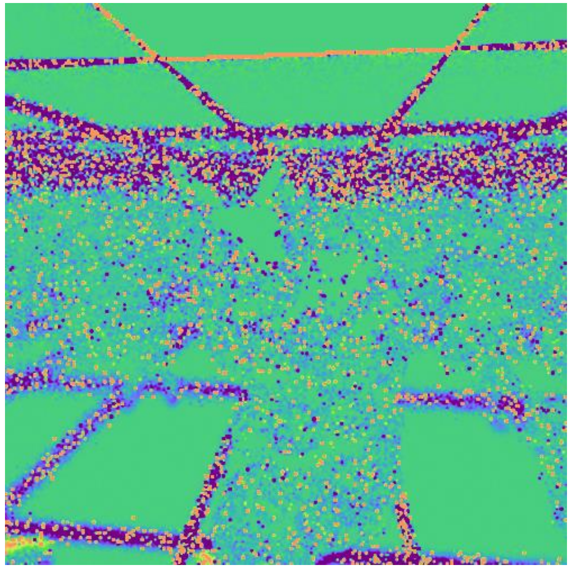
Original



PSDR



PSDR
+ AS (branching)



PSDR
+ AS (path level)

Negative


Positive

INVERSE RENDERING

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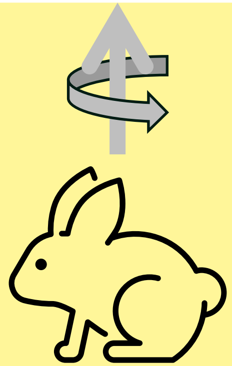
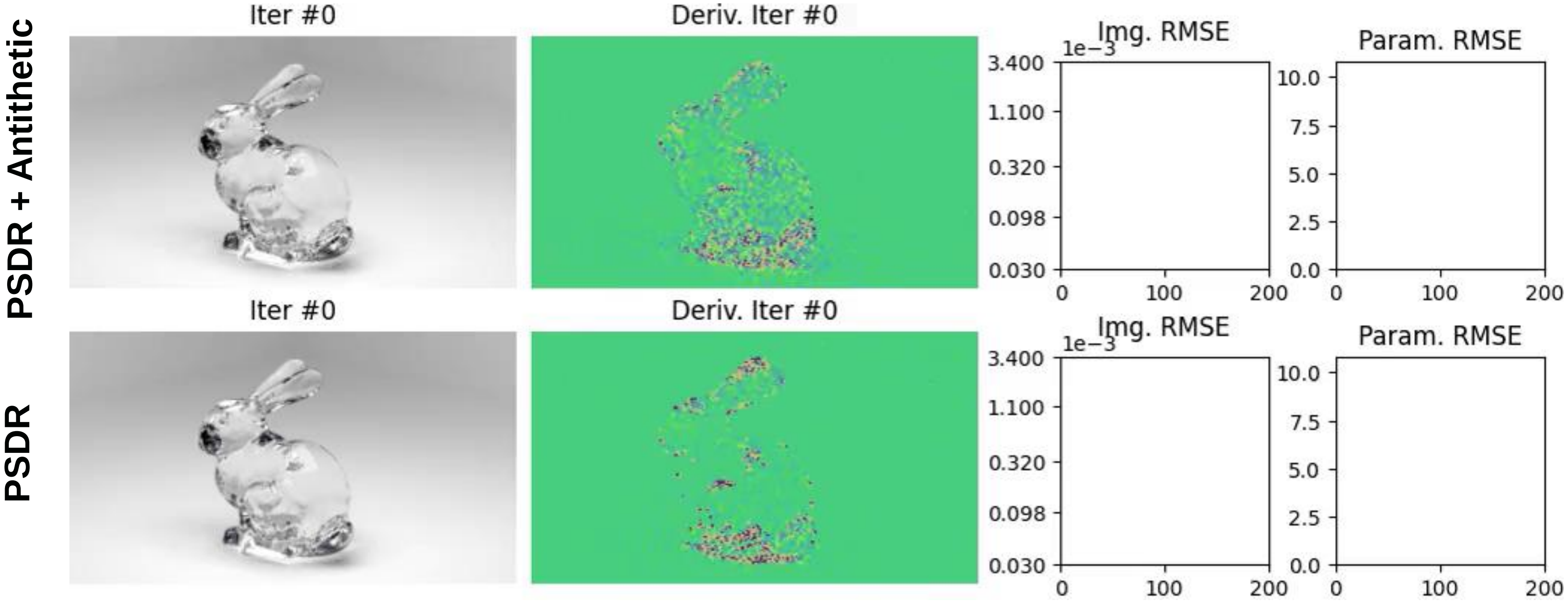
- Inverse rendering optimization comparisons
- To ensure fairness
  - All optimization parameters are fixed (e.g., optimizer, initial state, learning rate)
  - Derivative computation is under equal time
  - Only varying factor: antithetic sampling

INVERSE RENDERING

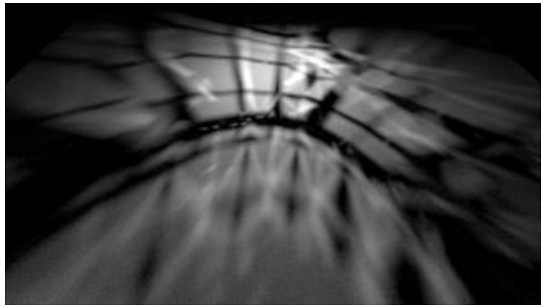
Target

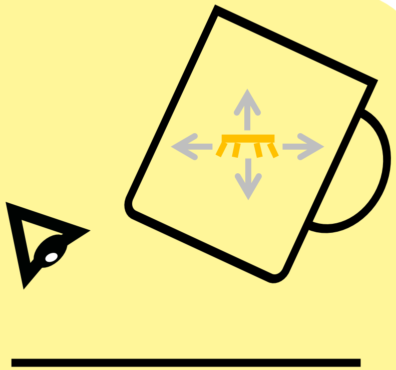


Opt. config.



Target





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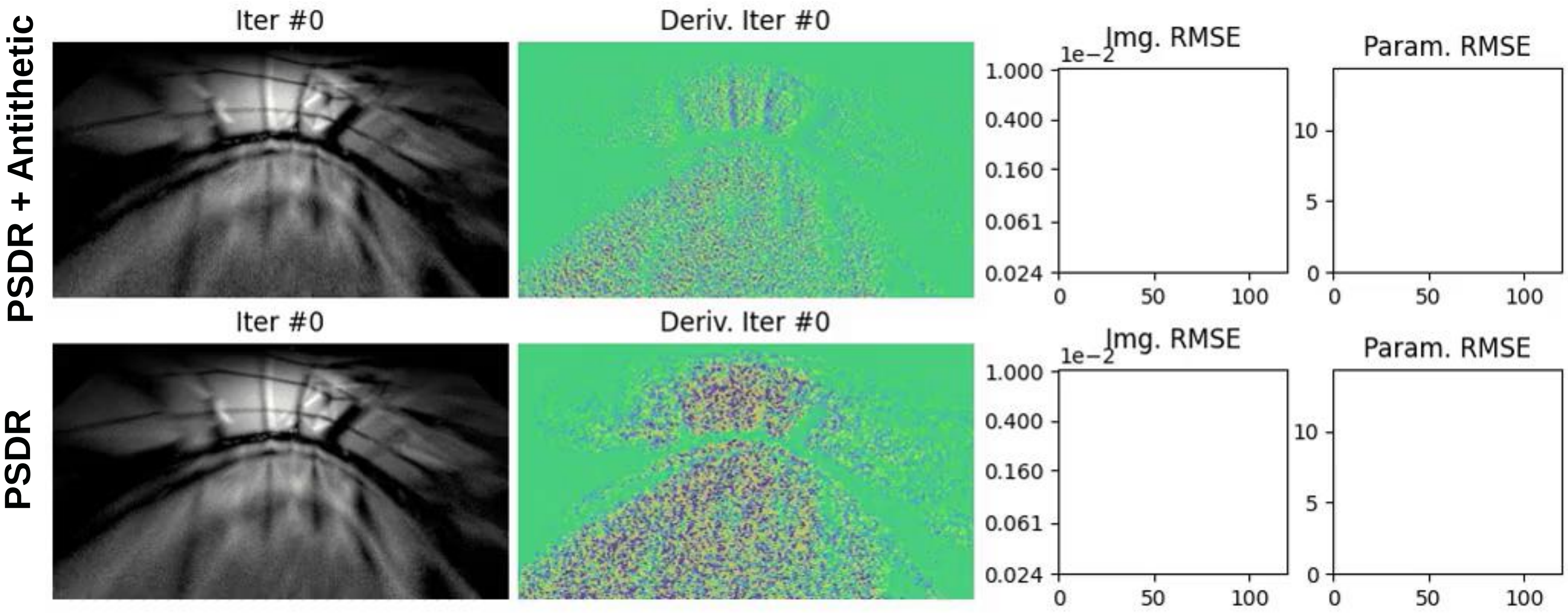
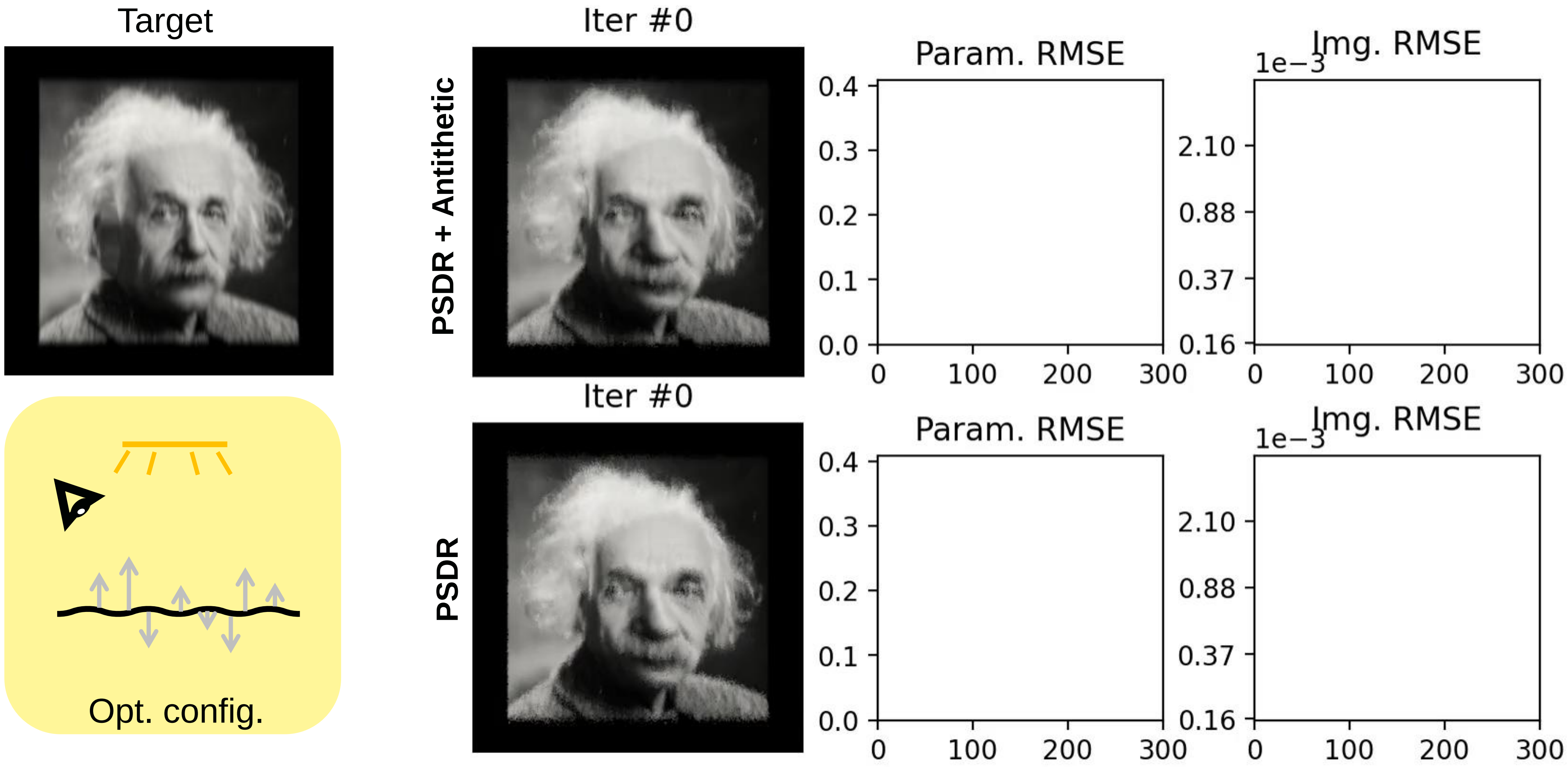


INVERSE RENDERING



LIMITATIONS & FUTURE WORK

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- Glossy-to-glossy interactions
  - Active research topic in forward rendering
  - Combine with advanced sampling methods
- We focus on interior term in differentiable rendering
  - Improving efficiency of boundary-integral estimation for glossy materials

CONCLUSION

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- We introduced antithetic sampling to Monte Carlo differentiable rendering
  - Efficient estimation of geometric derivative for glossy surfaces
  - Individual BSDFs and full light transport path
  - Parameterization independent
  - Applicable to most differentiable-rendering frameworks

Project webpage
https://rb.gy/7e6dzt