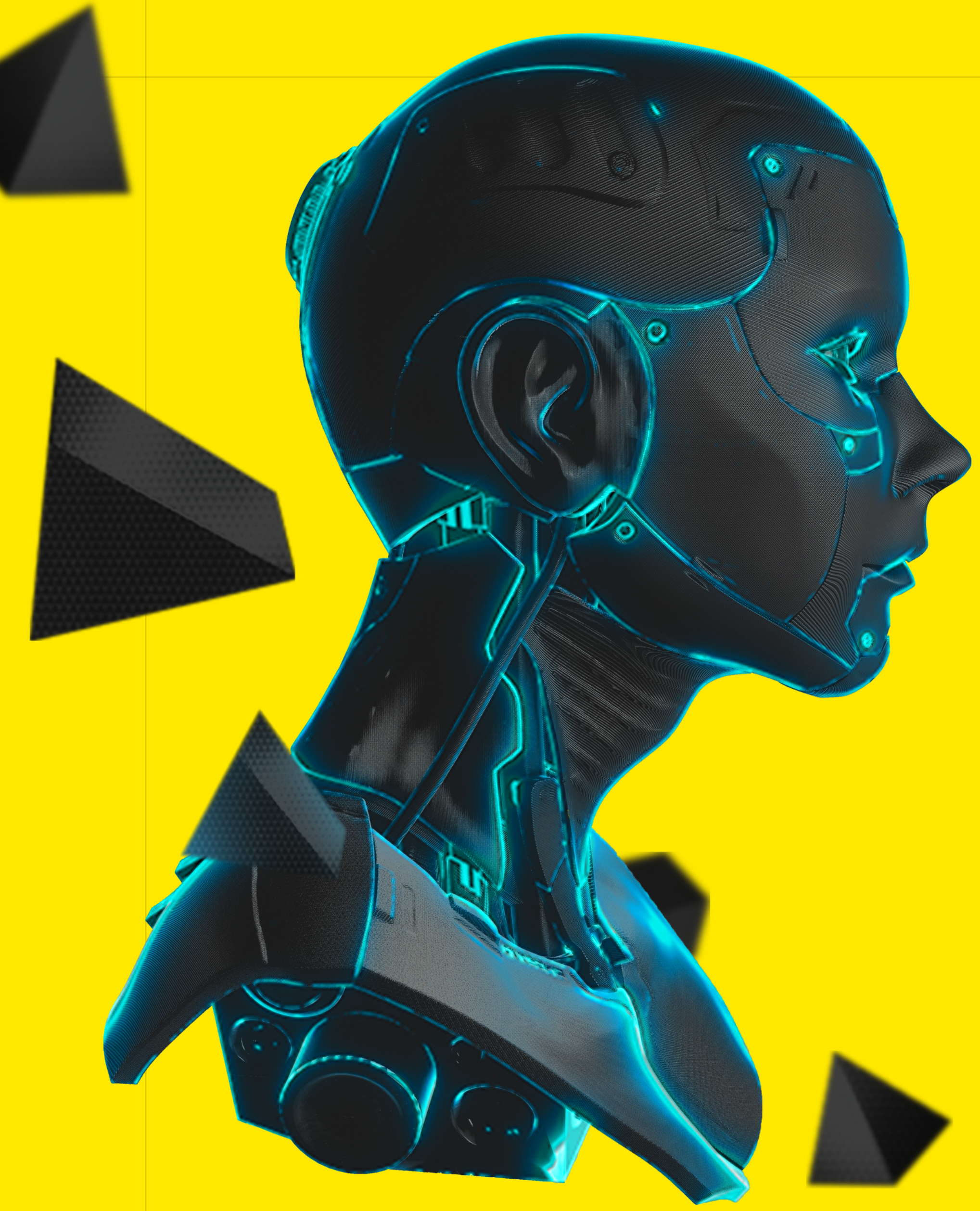

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PATH-SPACE DIFFERENTIABLE RENDERING OF PARTICIPATING MEDIA

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UNIVERSITY OF CALIFORNIA, IRVINE, USA

(*: EQUAL CONTRIBUTION)

WHY WE NEED DIFFERENTIABLE RENDERING?

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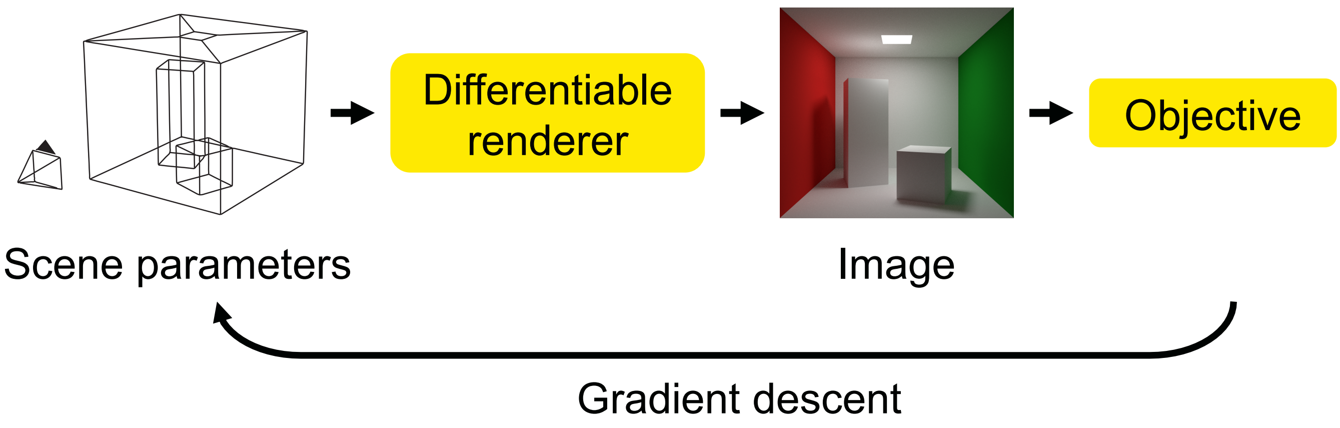
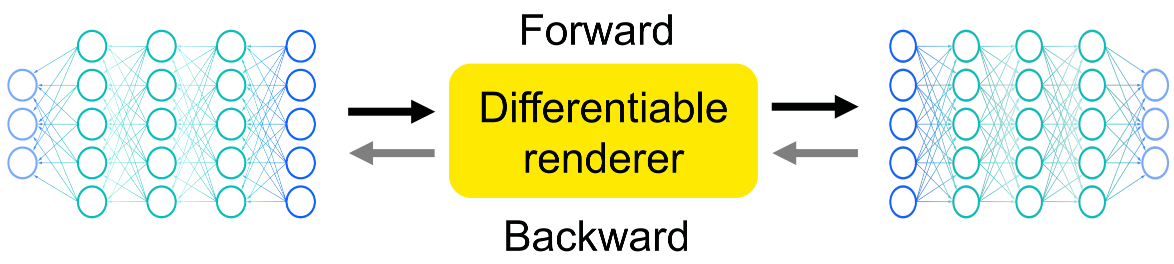
- Inverse rendering
  - Enabling gradient-based optimization



- Machine learning
  - Incorporating light transport simulation
  - Backpropagation through rendering



PRIOR WORK IN DIFFERENTIABLE RENDERING

Li et al. 2018

Loubet et al. 2019 Bangaru et. al. 2020

Surface-only frameworks

Cannot handle volumetric light transport

A differential theory of radiative transfer (DTRT) [Zhang et al. 2019]

Cannot handle complex light transport effects (BDPT)
Cannot handle complex geometries

Our method addresses those challenges efficiently!

OUR CONTRIBUTIONS

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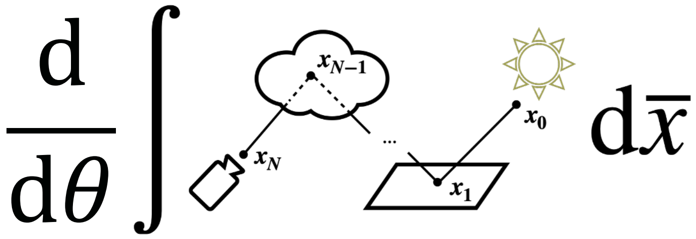
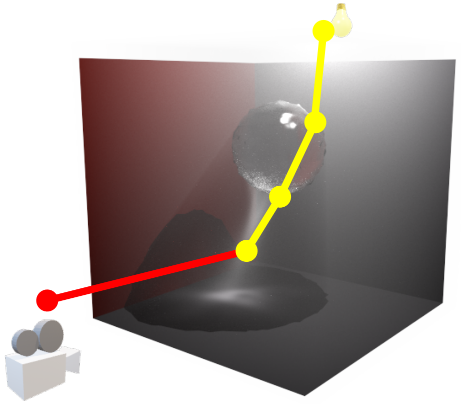
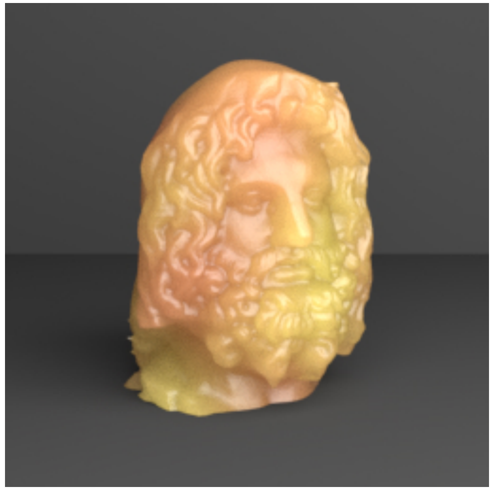


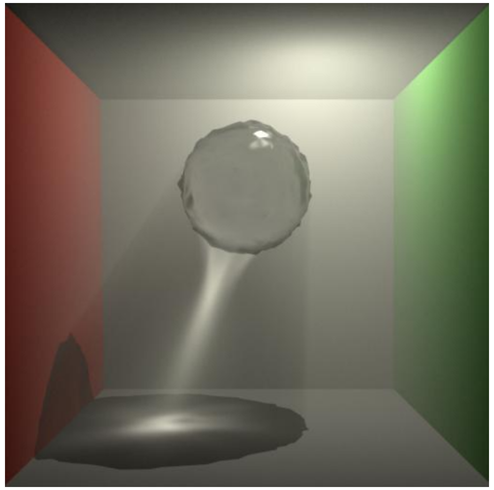
Generalized differential path integral



Monte Carlo estimators

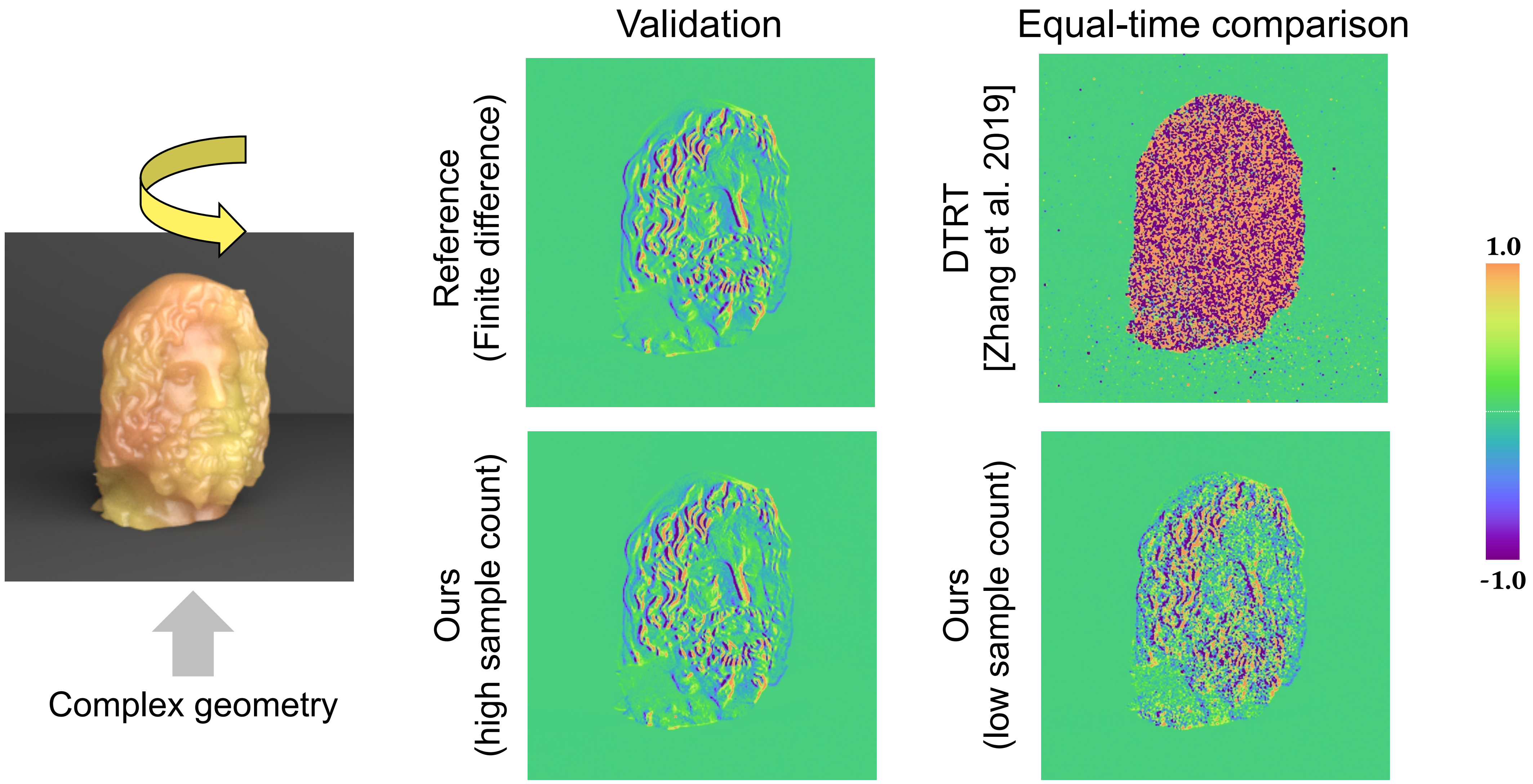


Complex geometry

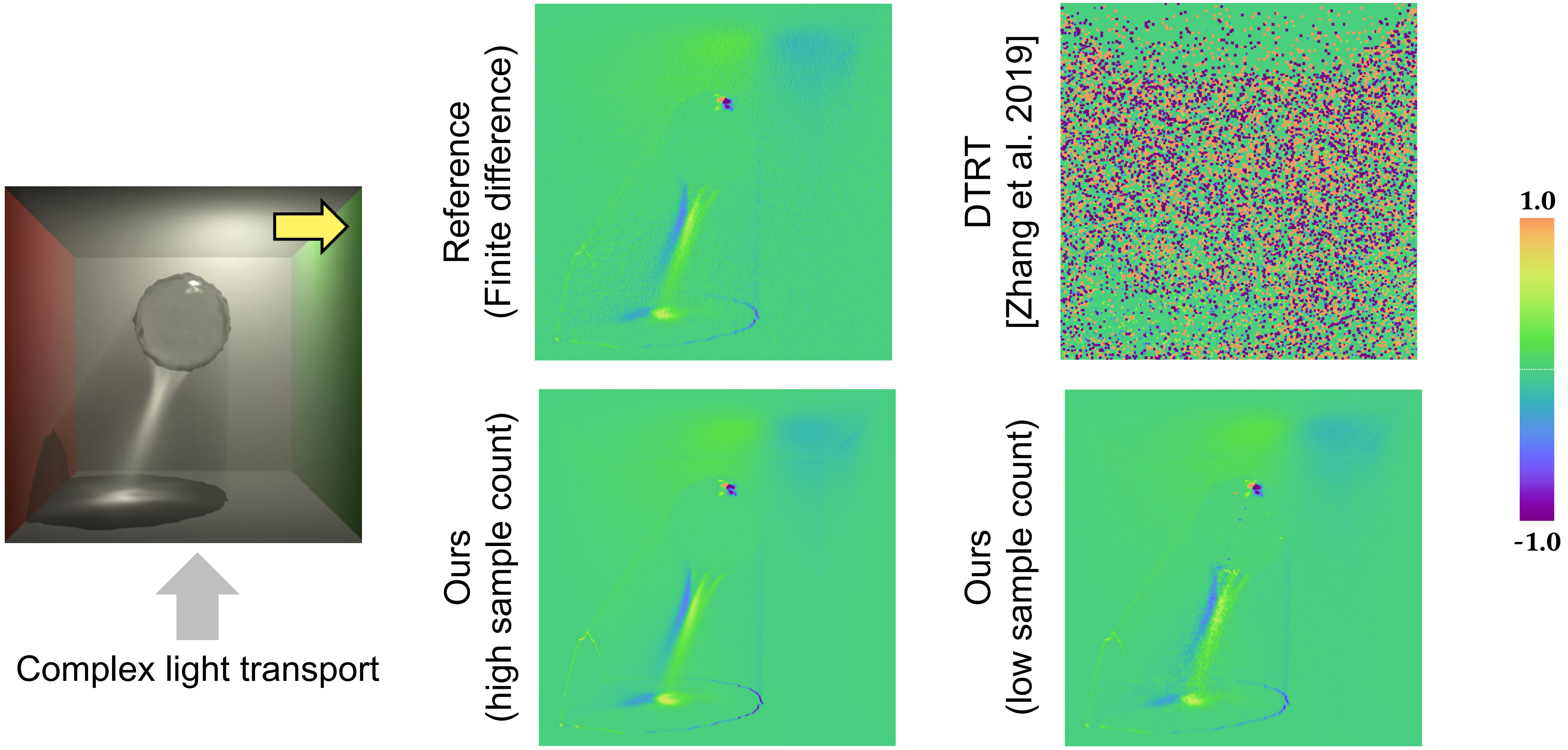


Complex light transport

PREVIEW OF OUR RESULTS



PREVIEW OF OUR RESULTS



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PRELIMINARIES

ON PATH-SPACE
DIFFERENTIABLE RENDERING

PATH INTEGRAL

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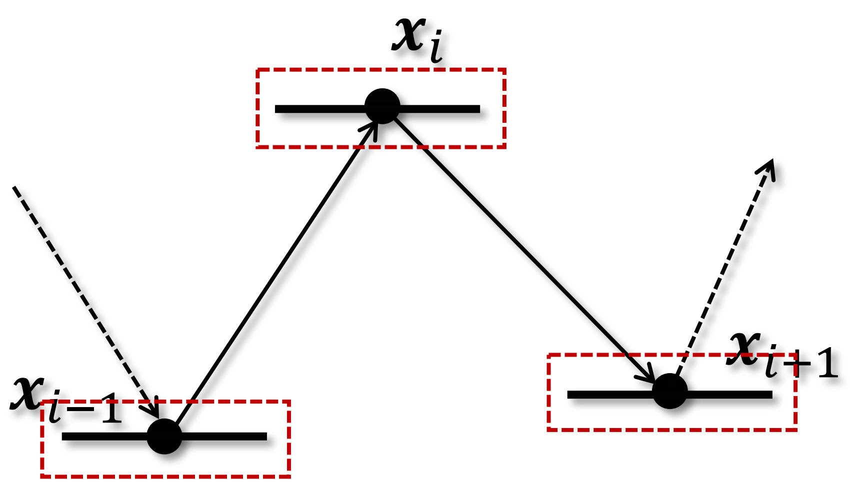
Measurement contribution

$$I = \int_{\Omega} f(\overline{\boldsymbol{x}})\, \mathrm{d}\mu(\overline{\boldsymbol{x}})$$

Path space

Area-product measure

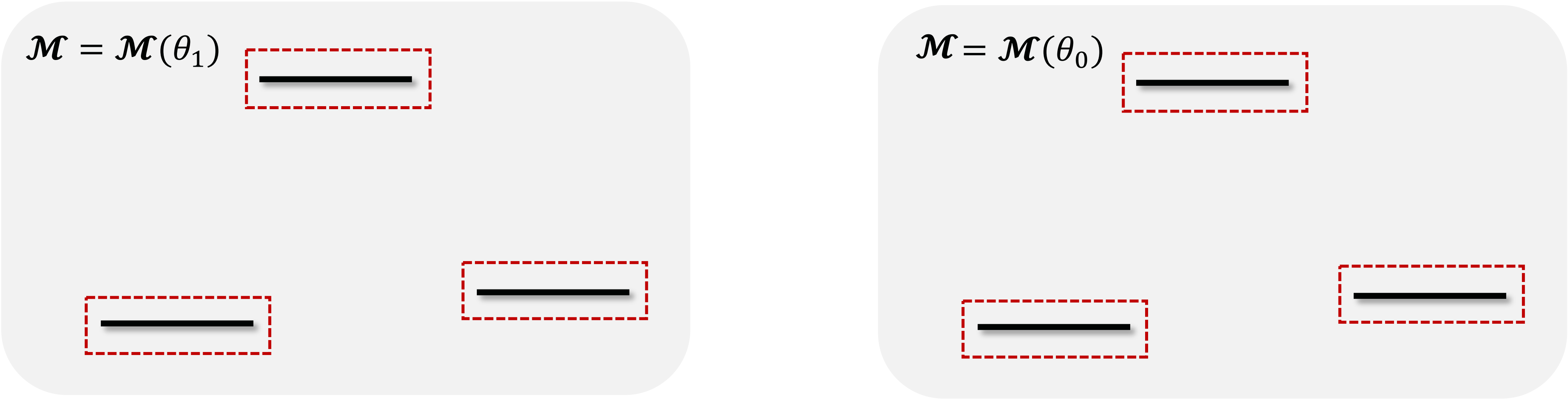
- Introduced by Veach [1997]
- Foundation of sophisticated Monte Carlo algorithms (e.g., BDPT, MCMC rendering)



Light path $\overline{\boldsymbol{x}} = (\cdots, \boldsymbol{x}_{i-1}, \boldsymbol{x}_i, \boldsymbol{x}_{i+1}, \cdots)$

Object surfaces $\boldsymbol{\mathcal{M}}$

Path space $\Omega = \cup_{N=1}^{\infty} \boldsymbol{\mathcal{M}}^{N+1}$



Evolving surface $\mathcal{M} \coloneqq \mathcal{M}(\theta)$

$$\Omega = \bigcup_{N=1}^{\infty} \mathcal{M}^{N+1}$$

Hard to differentiate the path integral wrt. θ

θ-dependent path space $\Omega \coloneqq \Omega(\theta)$

MATERIAL-FORM REPARAMETERIZATION

Path integral

$$I = \int_{\Omega(\theta)} f(\overline{\boldsymbol{x}})\, \mathrm{d}\mu(\overline{\boldsymbol{x}})$$

Dependent of θ

(Material-form) reparam.

Material-form path integral

$$I = \int_{\widehat{\Omega}} f(\overline{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})} \right| \mathrm{d}\mu(\overline{\boldsymbol{p}})$$

Independent of θ

Path-space Differentiable Rendering (PSDR)
Zhang et al. 2020

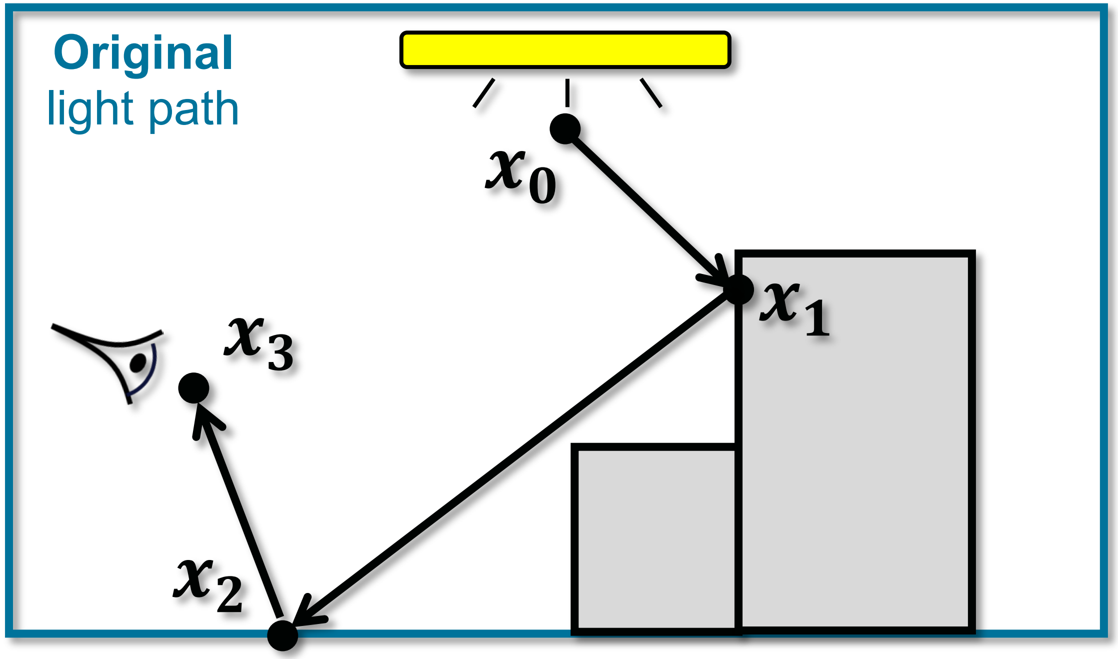
Material-form path integral

$$I = \int_{\hat{\Omega}} f(\overline{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})} \right| \mathrm{d}\mu(\overline{\boldsymbol{p}})$$

Differentiate

Material-form differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \underbrace{\int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta}\left(f(\overline{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})} \right| \right) \mathrm{d}\mu(\overline{\boldsymbol{p}})}_{\text{Interior integral}} + \underbrace{\int_{\partial\hat{\Omega}} g(\overline{\boldsymbol{p}})\mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})}_{\text{Boundary Integral}}$$



- Similar to the ordinary path integral
  - Integrate over the material path space
  - Differentiated integrand

Path-space Differentiable Rendering (PSDR)
Zhang et al. 2020

MATERIAL-FORM DIFFERENTIAL PATH INTEGRAL

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Material-form path integral

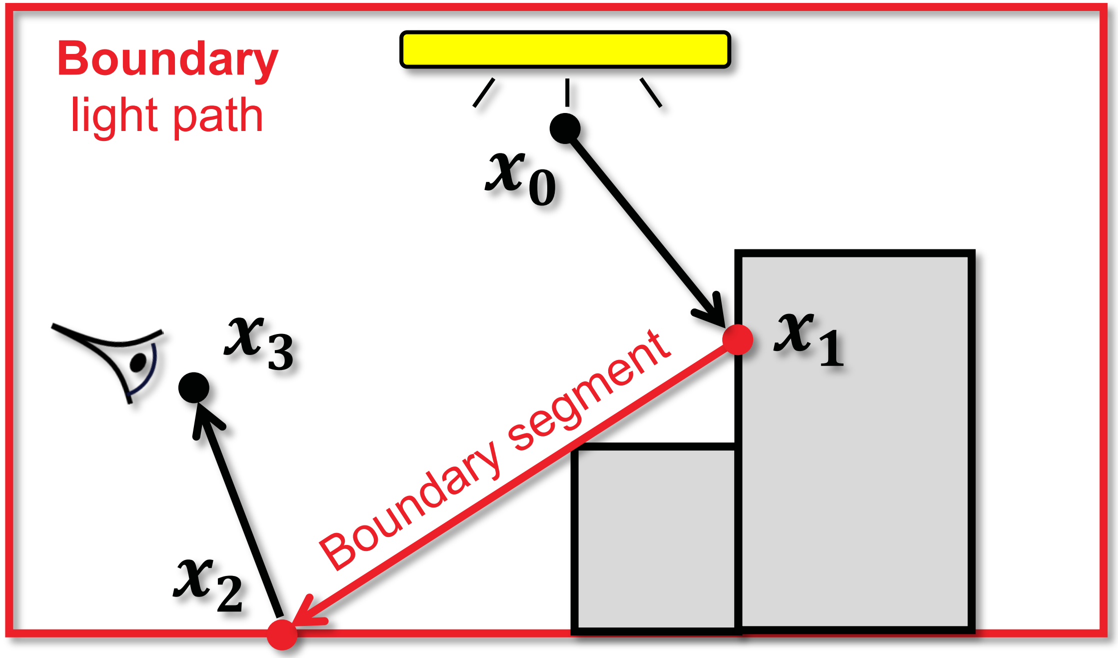
$$I = \int_{\hat{\Omega}} f(\overline{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})} \right| \mathrm{d}\mu(\overline{\boldsymbol{p}})$$

Differentiate

Material-form differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \left(f(\overline{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})} \right| \right) \mathrm{d}\mu(\overline{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} g(\overline{\boldsymbol{p}}) \mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})$$

Boundary Integral



x_1 lies on the visibility boundary with respect to x_2

- Different from the ordinary path integral
  - Integrate over boundary path space
  - Unique to differentiable rendering
  - Exactly one boundary segment

Path-space Differentiable Rendering (PSDR)
Zhang et al. 2020

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Path integral

$$I = \int_{\Omega} f(\bar{\boldsymbol{x}})\, \mathrm{d}\mu(\bar{\boldsymbol{x}})$$

(Material-form) **reparam.**

Material-form path integral

$$I = \int_{\widehat{\Omega}} f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right| \mathrm{d}\mu(\bar{\boldsymbol{p}})$$

Differentiation

Material-form differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \int_{\widehat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta}\left(f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right| \right) \mathrm{d}\mu(\bar{\boldsymbol{p}}) + \int_{\partial\widehat{\Omega}} g(\bar{\boldsymbol{p}})\mathrm{d}\dot{\mu}(\bar{\boldsymbol{p}})$$

OUR TECHNIQUE

OVERVIEW

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We **generalize** this framework to fully support **volumetric** light transport

Path integral

Path integral

$$I = \int_{\Omega} f(\bar{\boldsymbol{x}})\, \mathrm{d}\mu(\bar{\boldsymbol{x}})$$

(Material-form) **reparam.**

(Material-form) **reparam.**

Material-form path integral

Material-form path integral

$$I = \int_{\hat{\Omega}} f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right| \mathrm{d}\mu(\bar{\boldsymbol{p}})$$

Differentiation

Differentiation

Material-form differential path integral

Material-form differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta}\left(f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right| \right) \mathrm{d}\mu(\bar{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} g(\bar{\boldsymbol{p}})\, \mathrm{d}\dot{\mu}(\bar{\boldsymbol{p}})$$

GENERALIZED PATH INTEGRAL

- Generalized path integral [Pauly et al. 2000]

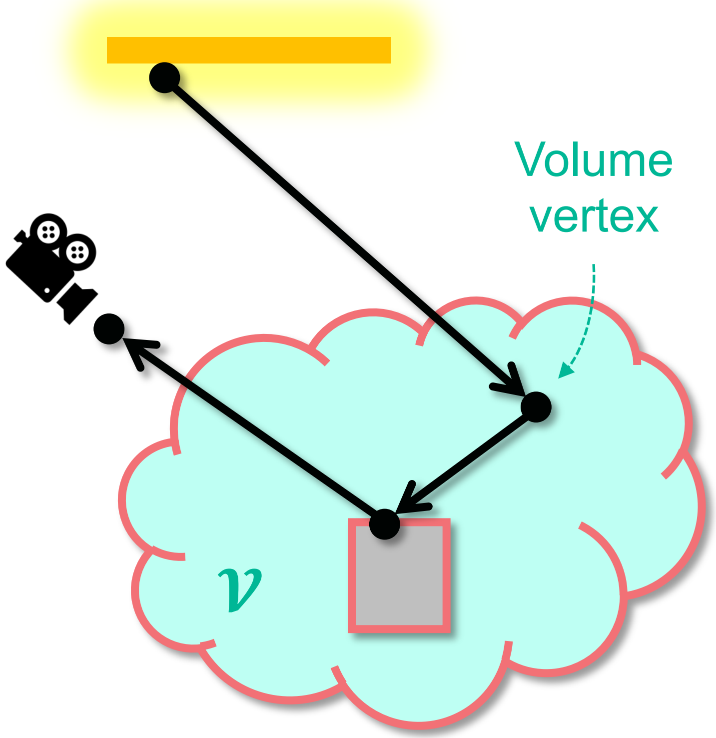
Measurement contribution

$$I = \int_{\Omega} f(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

Path space

  - Similar form as Veach's (surface-only) version
  - Allows **volume vertices** $x \in \mathcal{V}$ (capturing subsurface scattering)
  - Path space $\Omega = \bigcup_{N=1}^{\infty} (\mathcal{M} \cup \mathcal{V})^{N+1}$

Surfaces Volumes



GENERALIZED (MATERIAL-FORM) REPARAMETERIZATION

SIGGRAPH 2021

- Both surfaces $\mathcal{M}$ and volumes $\mathcal{V}$ may be controlled by some θ
  - Example: a translating rectangle

Objective:
Making the path space *independent* of θ

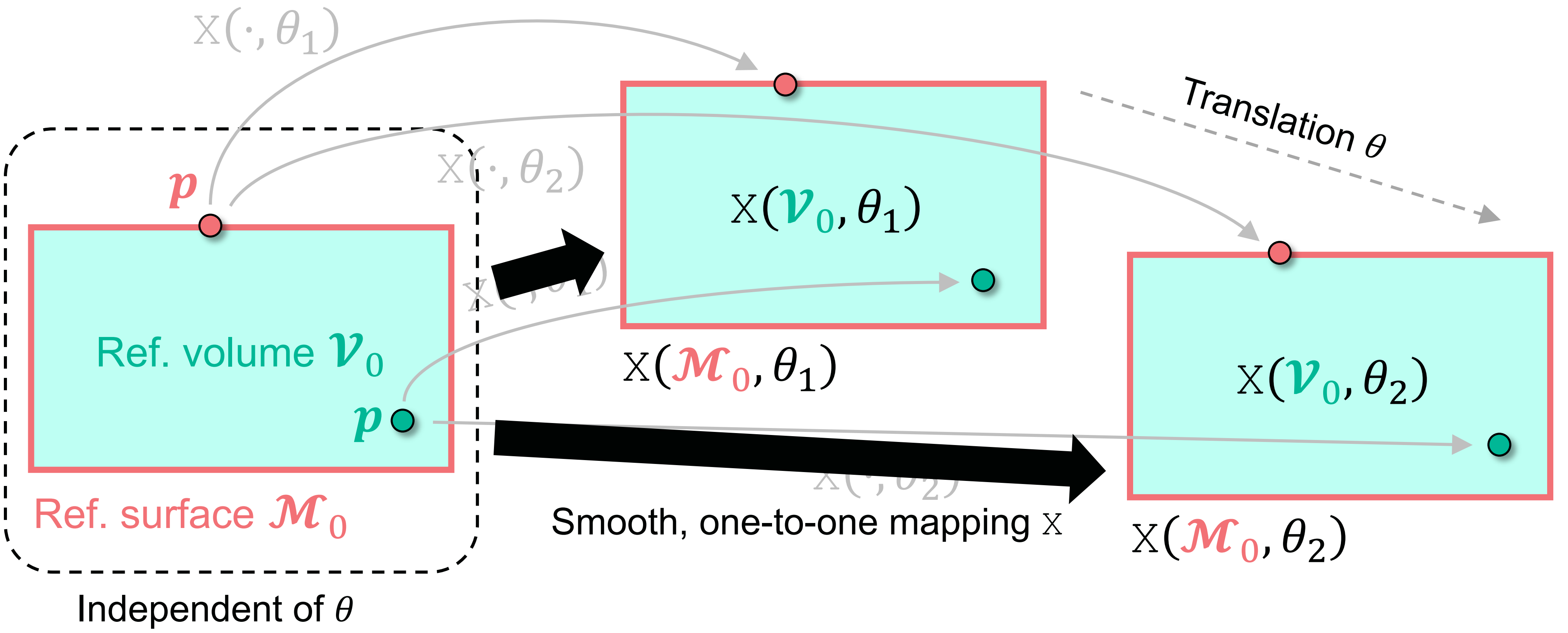
Solution:
Parameterizing surfaces $\mathcal{M}$ and volumes $\mathcal{V}$

- The path space $\Omega = \bigcup_{N=1}^{\infty} (\mathcal{M} \cup \mathcal{V})^{N+1}$ also depends on θ

GENERALIZED (MATERIAL-FORM) REPARAMETERIZATION

SIGGRAPH 2021

- Both surfaces $\mathcal{M}$ and volumes $\mathcal{V}$ may be controlled by some θ



OUTLINE
- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

GENERALIZED (MATERIAL-FORM) REPARAMETERIZATION

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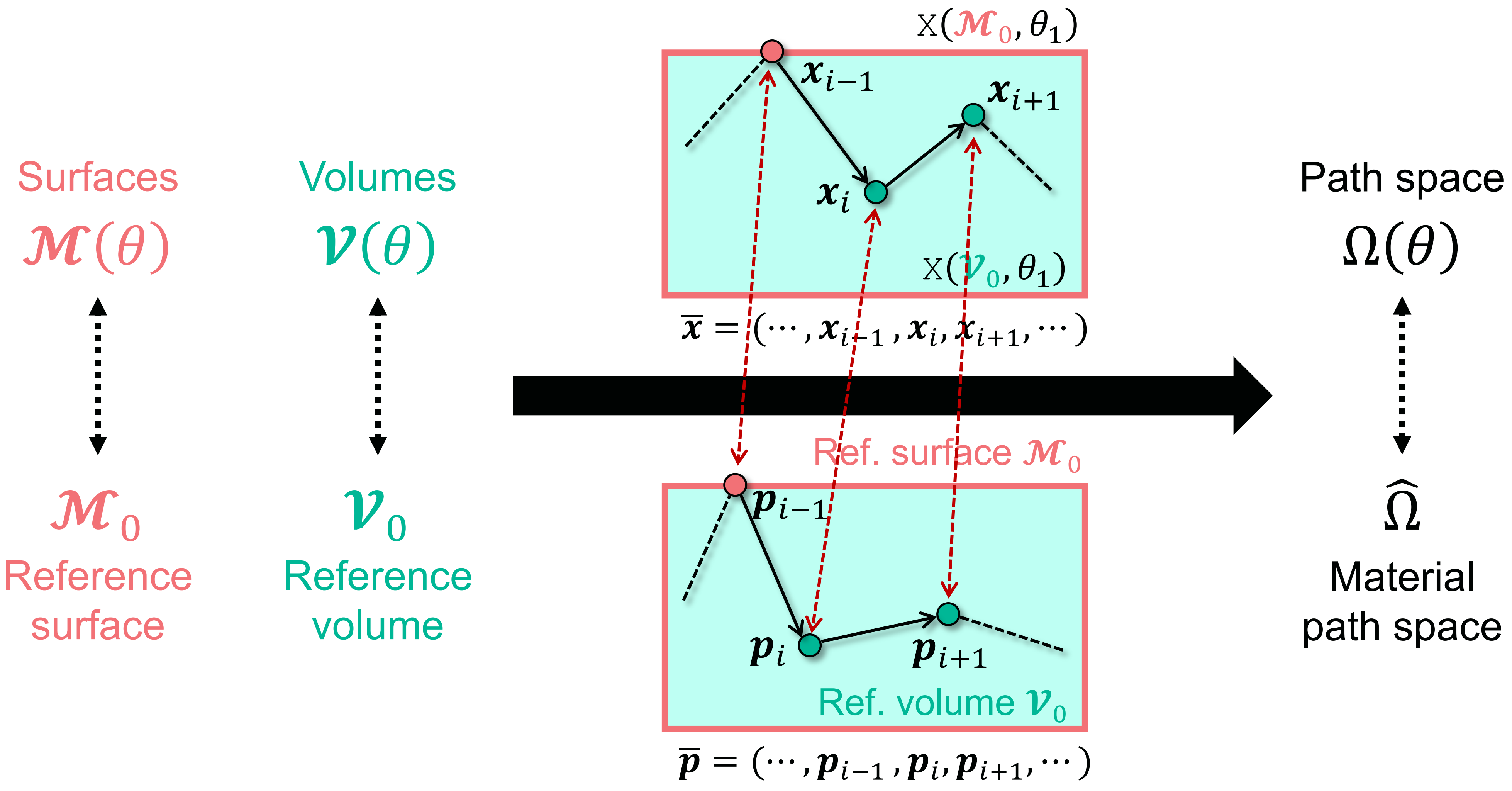

OUTLINE

- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

MATERIAL-FORM GENERALIZED PATH INTEGRAL

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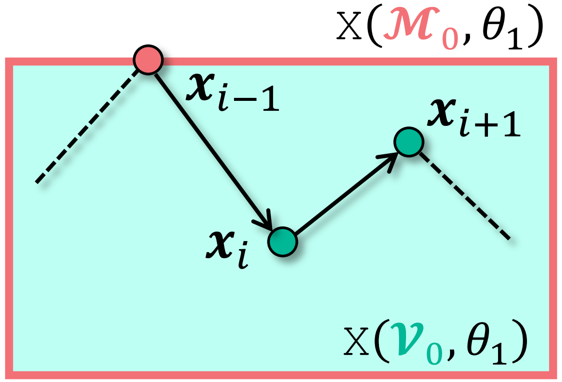
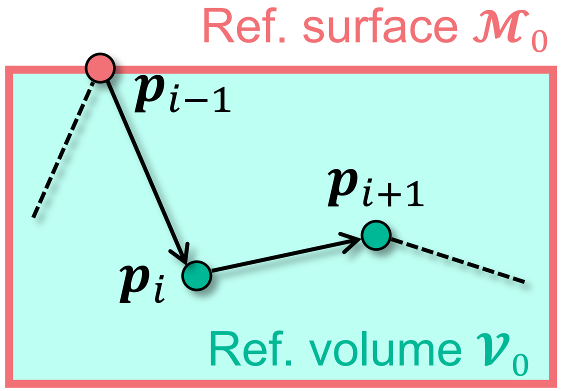


$\bar{x} = (\cdots, x_{i-1}, x_i, x_{i+1}, \cdots)$



$\bar{p} = (\cdots, p_{i-1}, p_i, p_{i+1}, \cdots)$

Generalized path integral

$$I = \int_{\Omega(\theta)} f(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

$$I = \int_{\hat{\Omega}} f(\bar{x}) \boxed{\left| \frac{\mathrm{d}\mu(\bar{x})}{\mathrm{d}\mu(\bar{p})} \right|} \mathrm{d}\mu(\bar{p})$$

Material-form
generalized path integral

Domain of integration

Path space

$\Omega(\theta)$

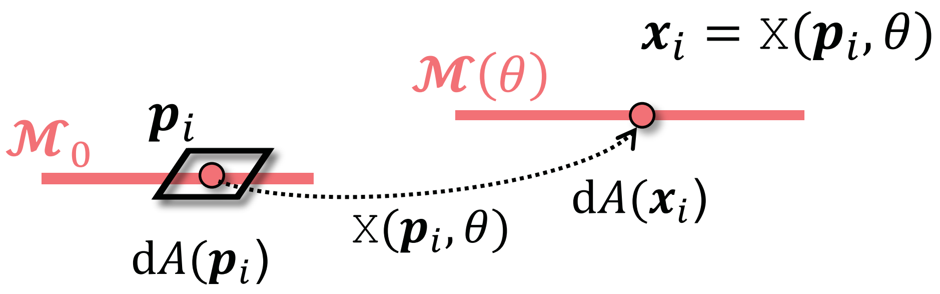
$\hat{\Omega}$

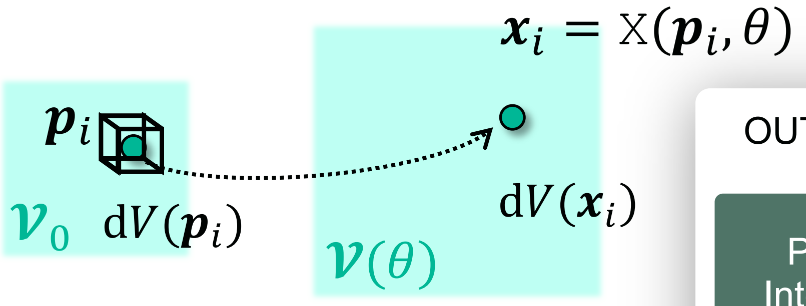
Material path space

OUTLINE

- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

MATERIAL-FORM GENERALIZED PATH INTEGRAL





$$\left|\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}\boldsymbol{p}_i}\right| = \left|\frac{\mathrm{d}A(\boldsymbol{x}_i)}{\mathrm{d}A(\boldsymbol{p}_i)}\right| \Leftarrow \prod_i \left|\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}\boldsymbol{p}_i}\right| \Rightarrow \left|\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}\boldsymbol{p}_i}\right| = \left|\frac{\mathrm{d}V(\boldsymbol{x}_i)}{\mathrm{d}V(\boldsymbol{p}_i)}\right|$$

Capturing area stretch

Capturing volume stretch

$$\text{II}$$

$$I = \int_{\hat{\Omega}} f(\overline{\boldsymbol{x}}) \boxed{\left|\frac{\mathrm{d}\mu(\overline{\boldsymbol{x}})}{\mathrm{d}\mu(\overline{\boldsymbol{p}})}\right|} \mathrm{d}\mu(\overline{\boldsymbol{p}})$$

Material-form
generalized path integral

GENERALIZED DIFFERENTIAL PATH INTEGRAL

SIGGRAPH 2021

Material-form generalized path integral

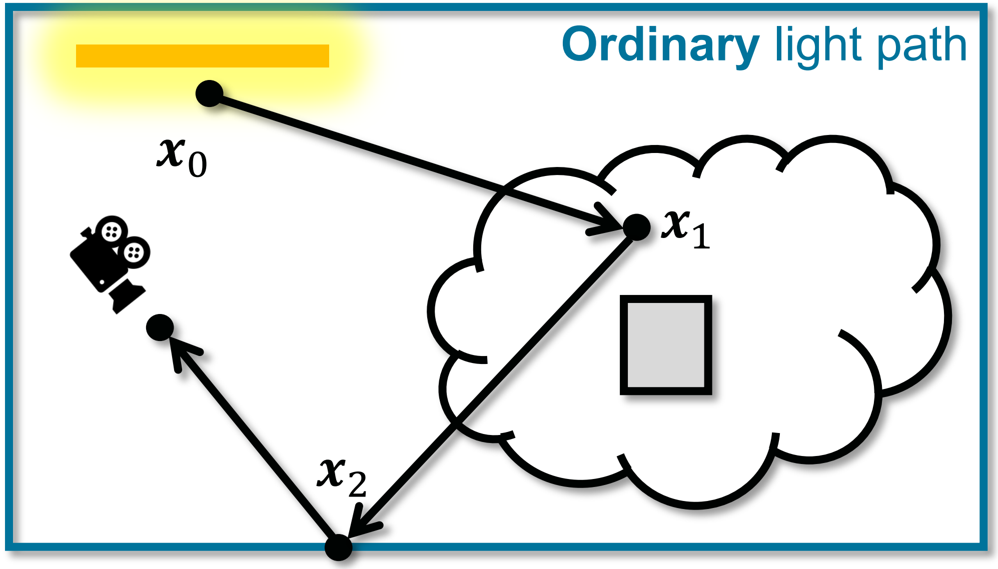
$$I = \int_{\hat{\Omega}} \underbrace{f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right|}_{\hat{f}(\bar{\boldsymbol{p}})} \mathrm{d}\mu(\bar{\boldsymbol{p}})$$

Differentiate

Material-form generalized differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \underbrace{\int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\bar{\boldsymbol{p}}) \mathrm{d}\mu(\bar{\boldsymbol{p}})}_{\text{Interior integral}} + \underbrace{\int_{\partial\hat{\Omega}} g(\bar{\boldsymbol{p}}) \mathrm{d}\dot{\mu}(\bar{\boldsymbol{p}})}_{\text{Boundary Integral}}$$

- **Interior integral:**
  - Over the same material path space as the ordinary path integral



OUTLINE
- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

GENERALIZED DIFFERENTIAL PATH INTEGRAL

Material-form
generalized path integral

$$I = \int_{\hat{\Omega}} \underbrace{f(\bar{\boldsymbol{x}}) \left| \frac{\mathrm{d}\mu(\bar{\boldsymbol{x}})}{\mathrm{d}\mu(\bar{\boldsymbol{p}})} \right|}_{\hat{f}(\bar{\boldsymbol{p}})} \mathrm{d}\mu(\bar{\boldsymbol{p}})$$

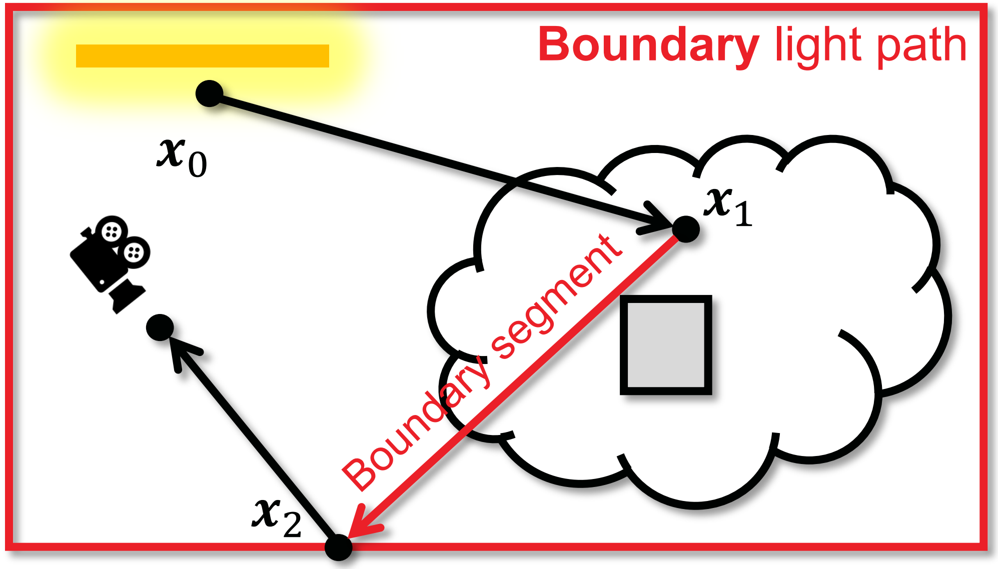
Differentiate

Material-form
generalized differential path integral

$$\frac{\mathrm{d}I}{\mathrm{d}\theta} = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\bar{\boldsymbol{p}}) \mathrm{d}\mu(\bar{\boldsymbol{p}}) + \boxed{\int_{\partial\hat{\Omega}} g(\bar{\boldsymbol{p}}) \mathrm{d}\dot{\mu}(\bar{\boldsymbol{p}})}$$

Boundary Integral

- **Boundary integral:**
  - Unique to differentiable rendering
  - Over the **boundary path space**
  - Exactly one **boundary segment**

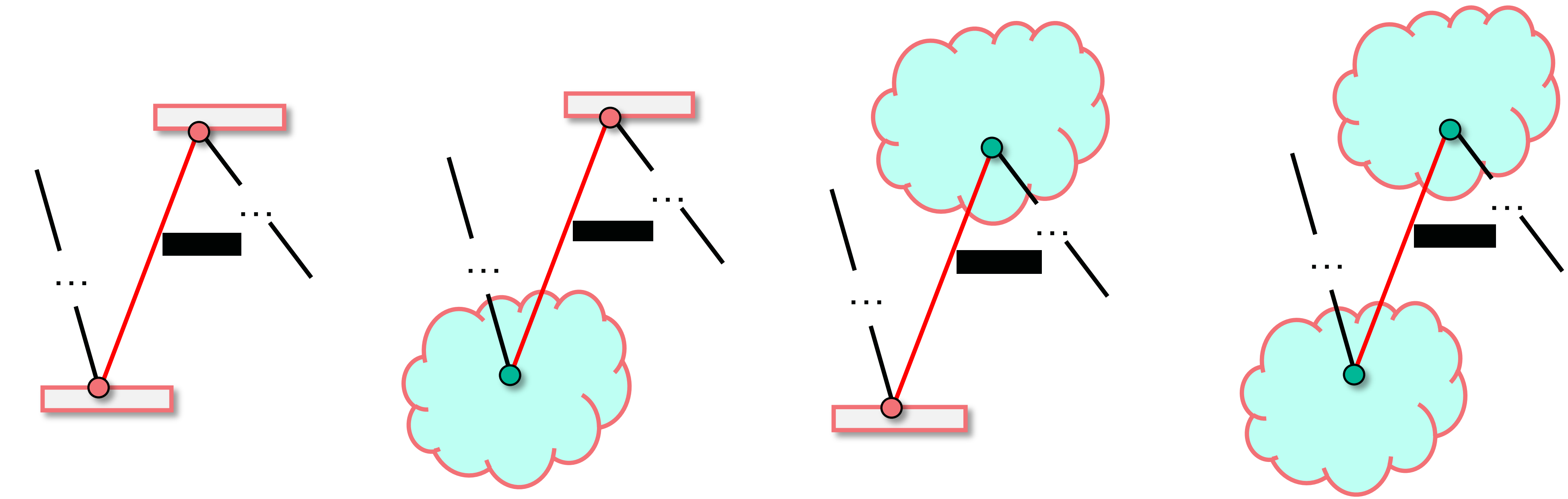


OUTLINE

- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

BOUNDARY SEGMENT

4 types of boundary segment:



Surface to surface | Volume to surface | Surface to volume | Volume to volume

[Zhang et al. 2020]

BOUNDARY INTEGRAL

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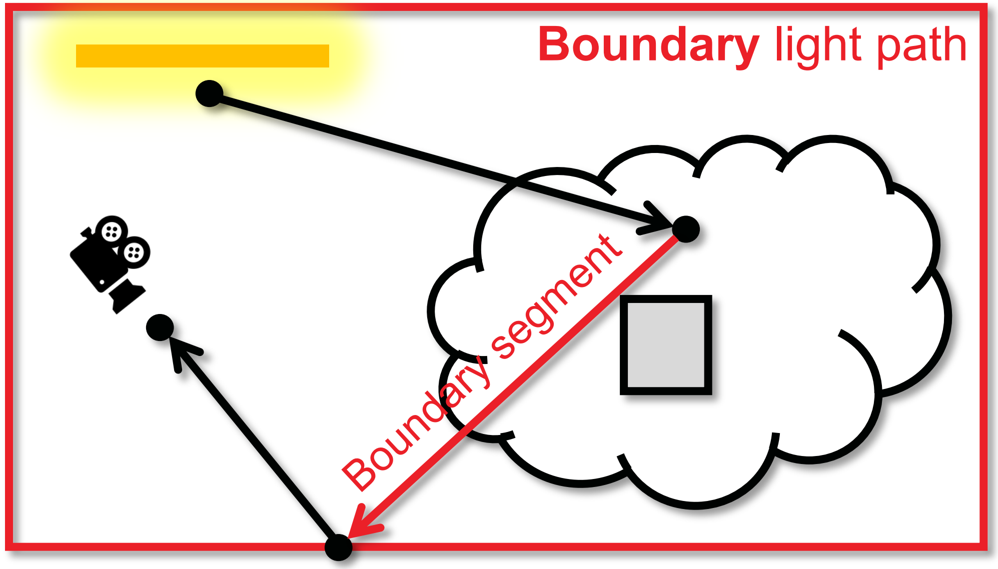
$$\frac{\mathrm{d}}{\mathrm{d}\theta} I = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\overline{\boldsymbol{p}}) \mathrm{d}\mu(\overline{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} \Delta\hat{f}(\overline{\boldsymbol{p}})\, v(\boldsymbol{p}_K)\, \mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})$$

Boundary Integral

- Two key terms
  - $\Delta\hat{f}(\overline{\boldsymbol{p}})$: Difference in $\hat{f}(\overline{\boldsymbol{p}})$ across discontinuity boundaries
  - $v(\boldsymbol{p}_K)$: Evolution "speed" of discontinuity boundaries



OUTLINE

- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

DIFFERENCE OF MEASUREMENT CONTRIBUTION

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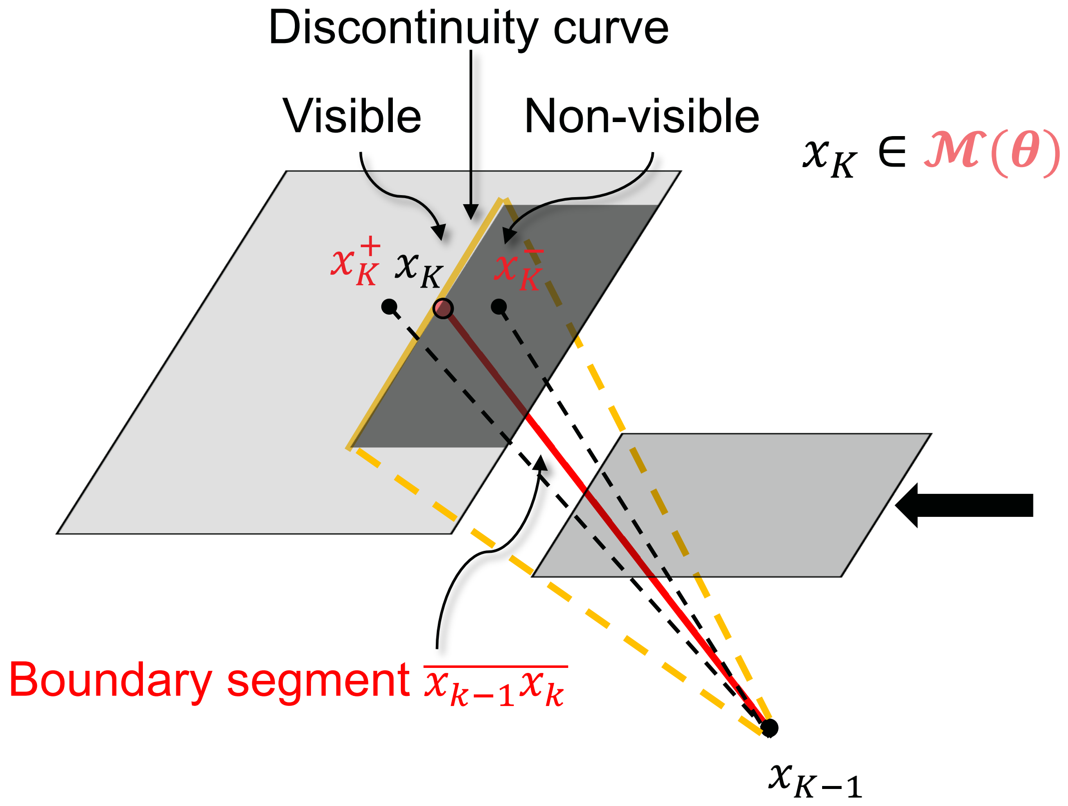
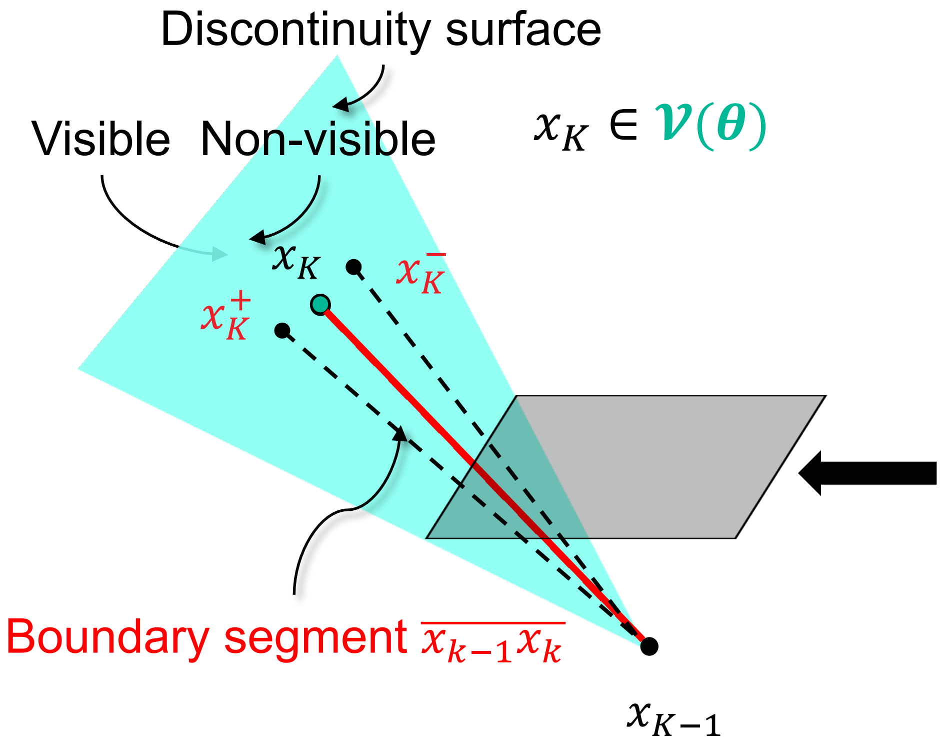
$$\frac{\mathrm{d}}{\mathrm{d}\theta} I = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\overline{\boldsymbol{p}}) \mathrm{d}\mu(\overline{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} \Delta \hat{f}(\overline{\boldsymbol{p}})\, v(\boldsymbol{p}_K)\, \mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})$$





OUTLINE
- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

EVOLUTION OF DISCONTINUITY BOUNDARIES

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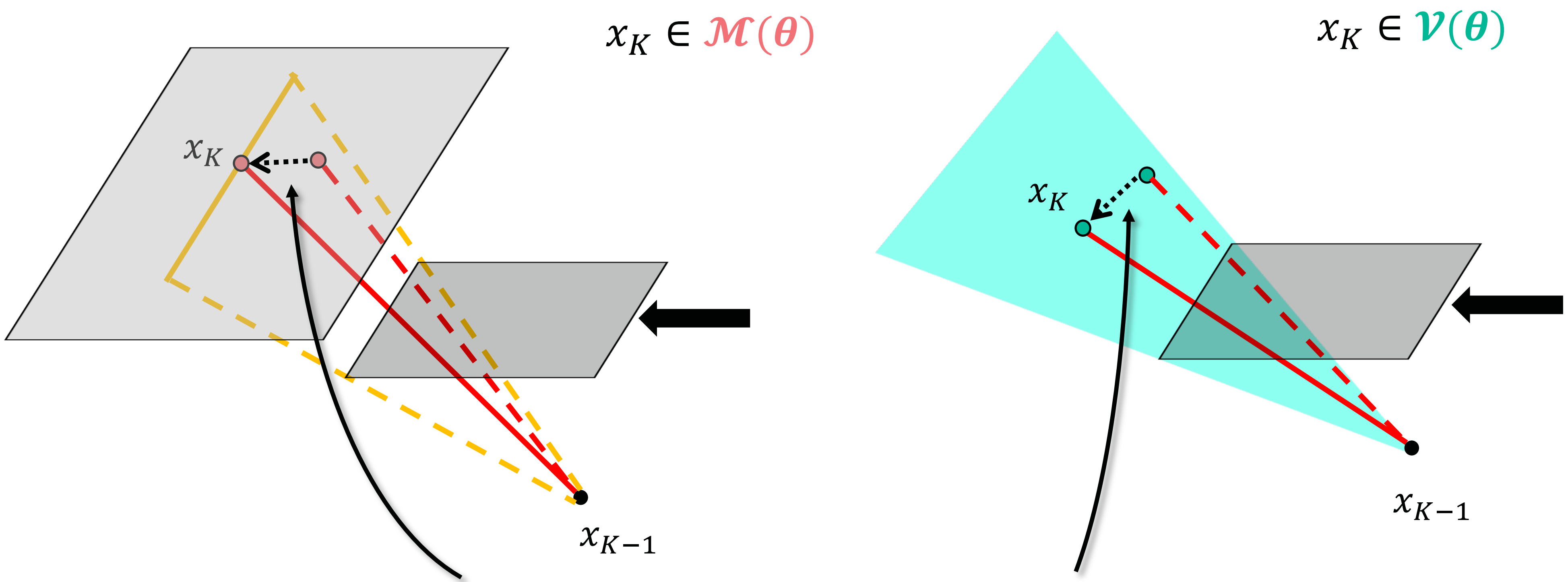
$$\frac{\mathrm{d}}{\mathrm{d}\theta} I = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\overline{\boldsymbol{p}}) \mathrm{d}\mu(\overline{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} \Delta \hat{f}(\overline{\boldsymbol{p}})\, v(\boldsymbol{p}_K)\, \mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})$$



$v(\boldsymbol{p}_k)$ capture the normal velocity

OUTLINE

- Path Integral
- **Reparam.**
- Mat-form PI
- **Diff.**
- Mat-form Diff. PI

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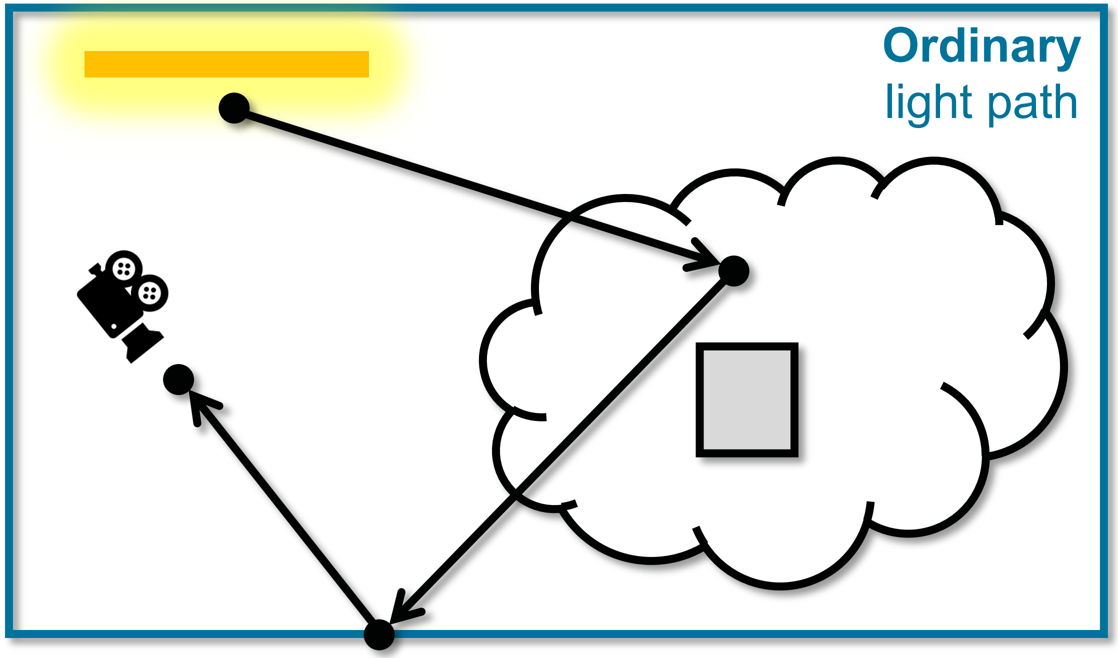
MONTE CARLO ESTIMATOR

ESTIMATING INTERIOR INTEGRAL

Generalized differential path Integral

$$\frac{\mathrm{d}}{\mathrm{d}\theta} I = \underbrace{\int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta} \hat{f}(\overline{\boldsymbol{p}}) \mathrm{d}\mu(\overline{\boldsymbol{p}})}_{\text{Interior Integral}} + \underbrace{\int_{\partial\hat{\Omega}} \Delta \hat{f}(\overline{\boldsymbol{p}}) v(\boldsymbol{p}_K) \mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})}_{\text{Boundary Integral}}$$



- Can be estimated using identical path sampling strategies as forward rendering

  Different MC estimators
  - Unidirectional volumetric path tracing
  - Bidirectional volumetric path tracing
  - …

ESTIMATING BOUNDARY INTEGRAL

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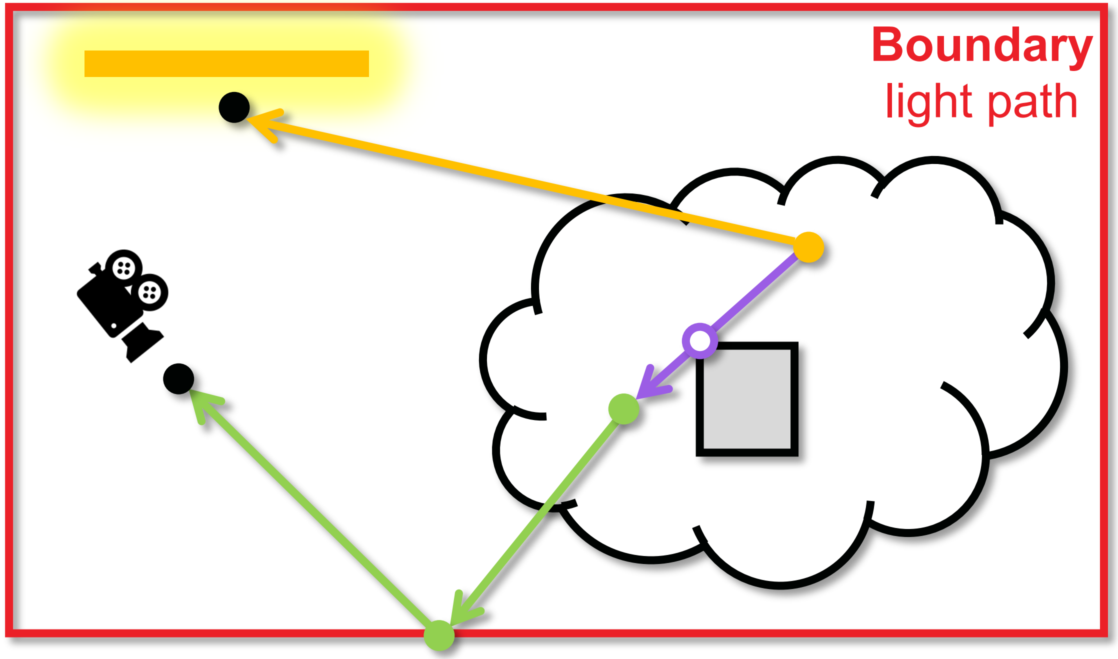
Generalized
differential path Integral

$$\frac{\mathrm{d}}{\mathrm{d}\theta}I = \int_{\hat{\Omega}} \frac{\mathrm{d}}{\mathrm{d}\theta}\hat{f}(\overline{\boldsymbol{p}})\mathrm{d}\mu(\overline{\boldsymbol{p}}) + \int_{\partial\hat{\Omega}} \Delta\hat{f}(\overline{\boldsymbol{p}})v(\boldsymbol{p}_K)\mathrm{d}\dot{\mu}(\overline{\boldsymbol{p}})$$

Boundary Integral

- Multi-directional sampling
  - Construct boundary segment
  - Construct sensor and source subpaths



SAMPLING BOUNDARY SEGMENT

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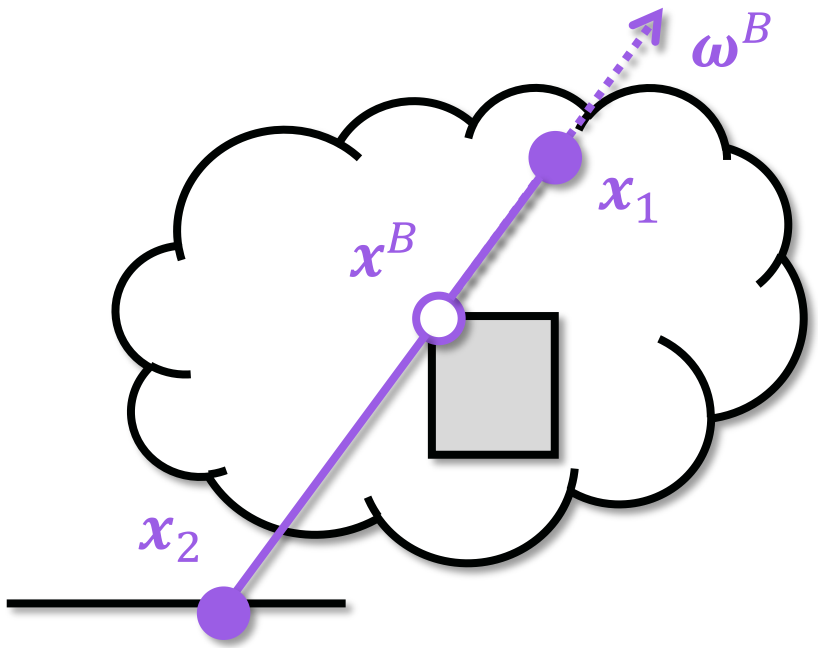
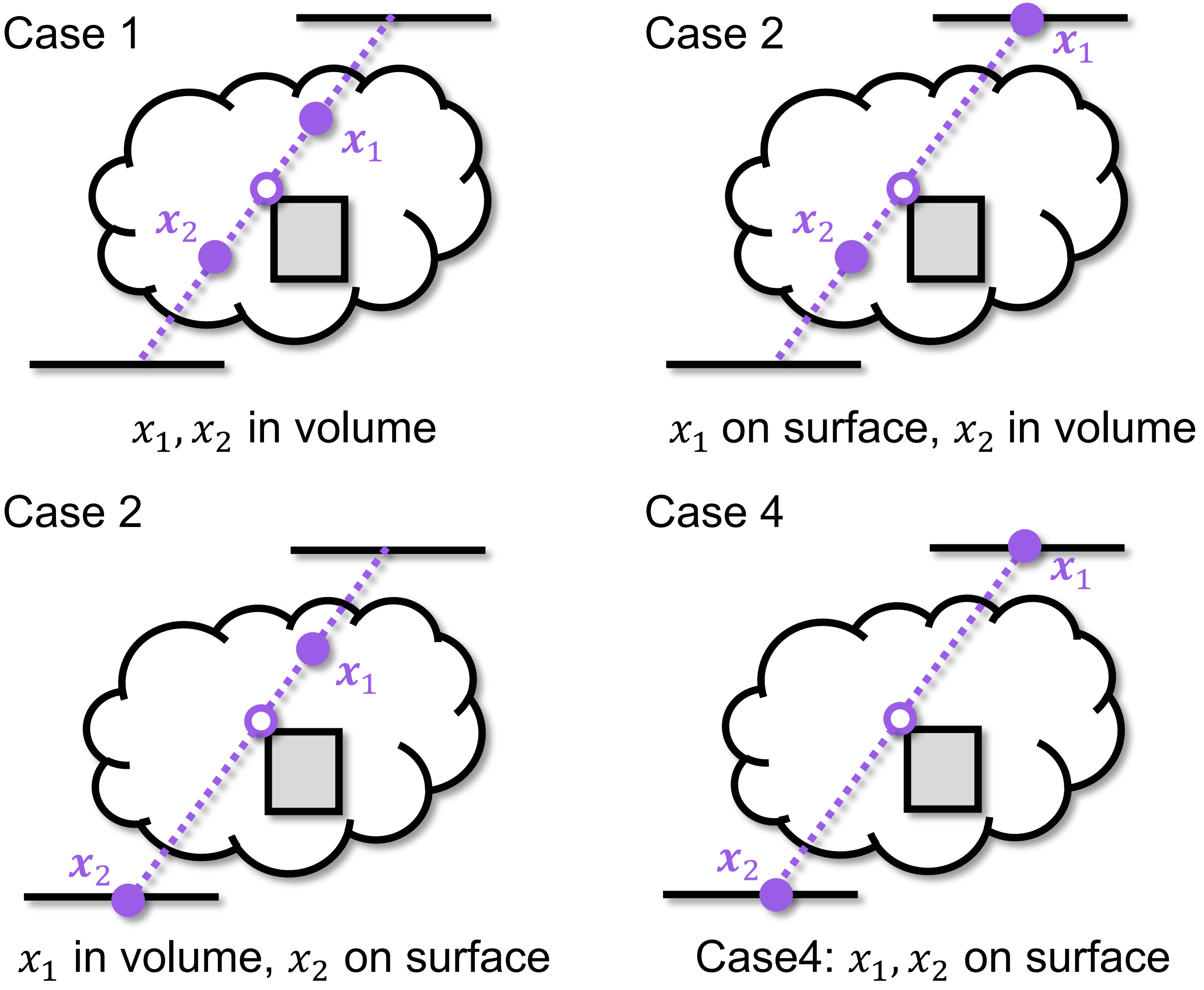
- To avoid explicit silhouette detection, we draw a boundary segment by
  - Sample $\boldsymbol{x}_B$ on a face edge
  - Sample ray direction $\boldsymbol{\omega}_B$
  - Sample $\boldsymbol{x}_1, \boldsymbol{x}_2$ along $\boldsymbol{\omega}_B$ and $-\boldsymbol{\omega}_B$



SAMPLING BOUNDARY SEGMENT



fun sampleBoundarySegment($\boldsymbol{x}, \boldsymbol{\omega}_o$):
  Draw $(\boldsymbol{x}_B, \omega_B)$
  Sample distance along ω_B, get $\boldsymbol{x}_1$
  Sample distance along $-\omega_B$, get $\boldsymbol{x}_2$
  If $\boldsymbol{x}_1$ in volume:
    If $\boldsymbol{x}_2$ in volume: case 1
    Else: case 2
  Else:
    If $\boldsymbol{x}_2$ in volume: case 3
    Else: case 4
  return $(\boldsymbol{x}_1, \boldsymbol{x}_2)$

Jacobian for the change of variables!

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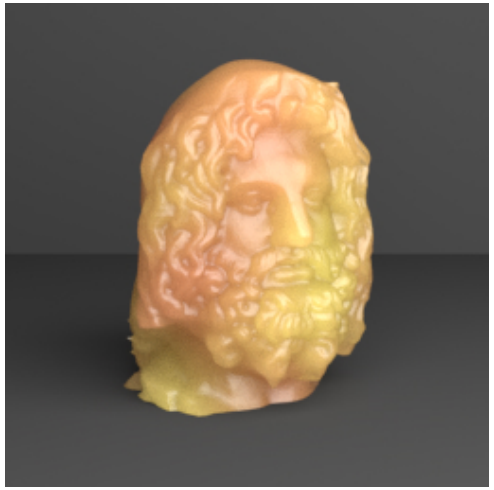
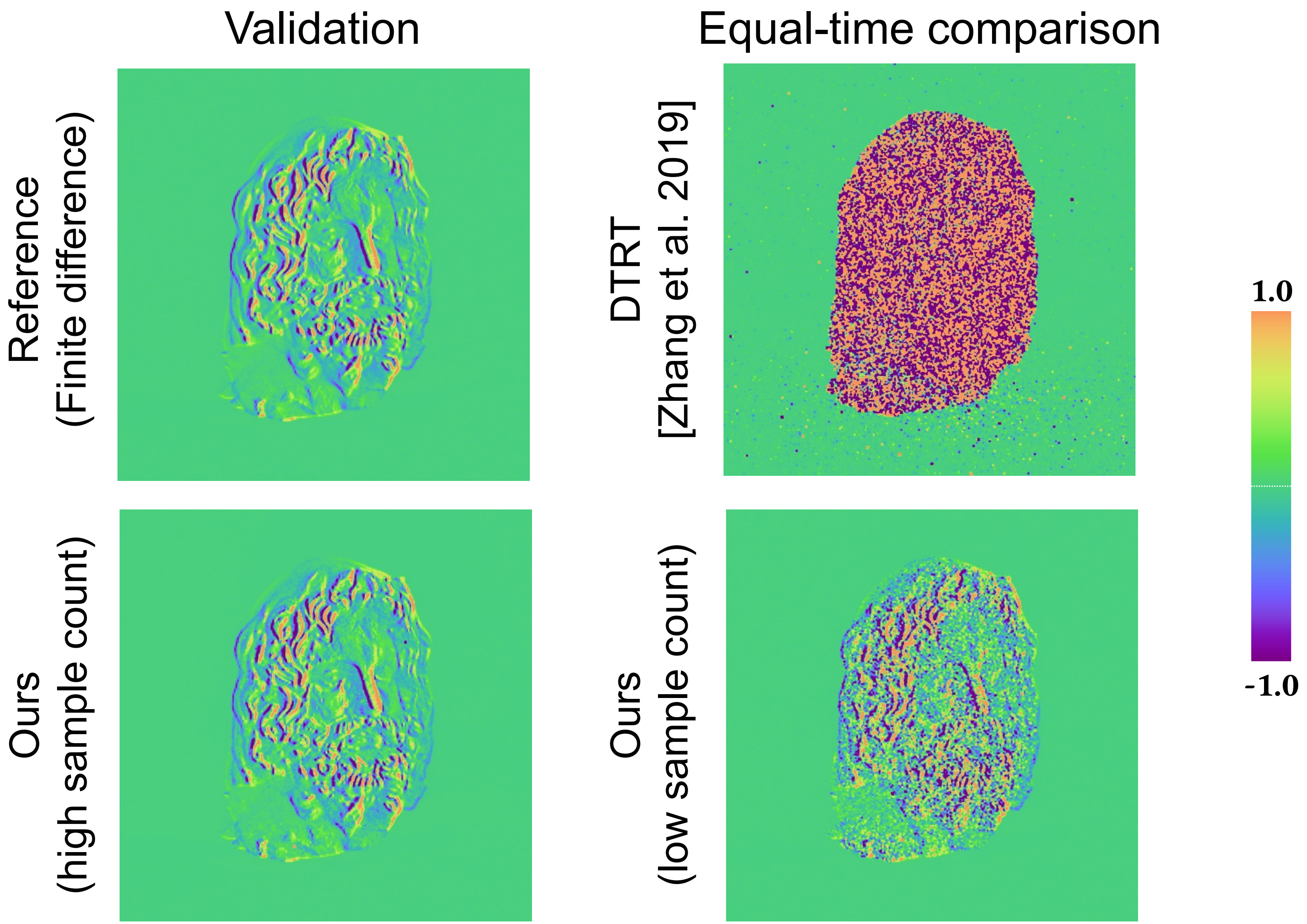
RESULTS

RESULTS

Complex geometry





RESULTS

Complex geometry

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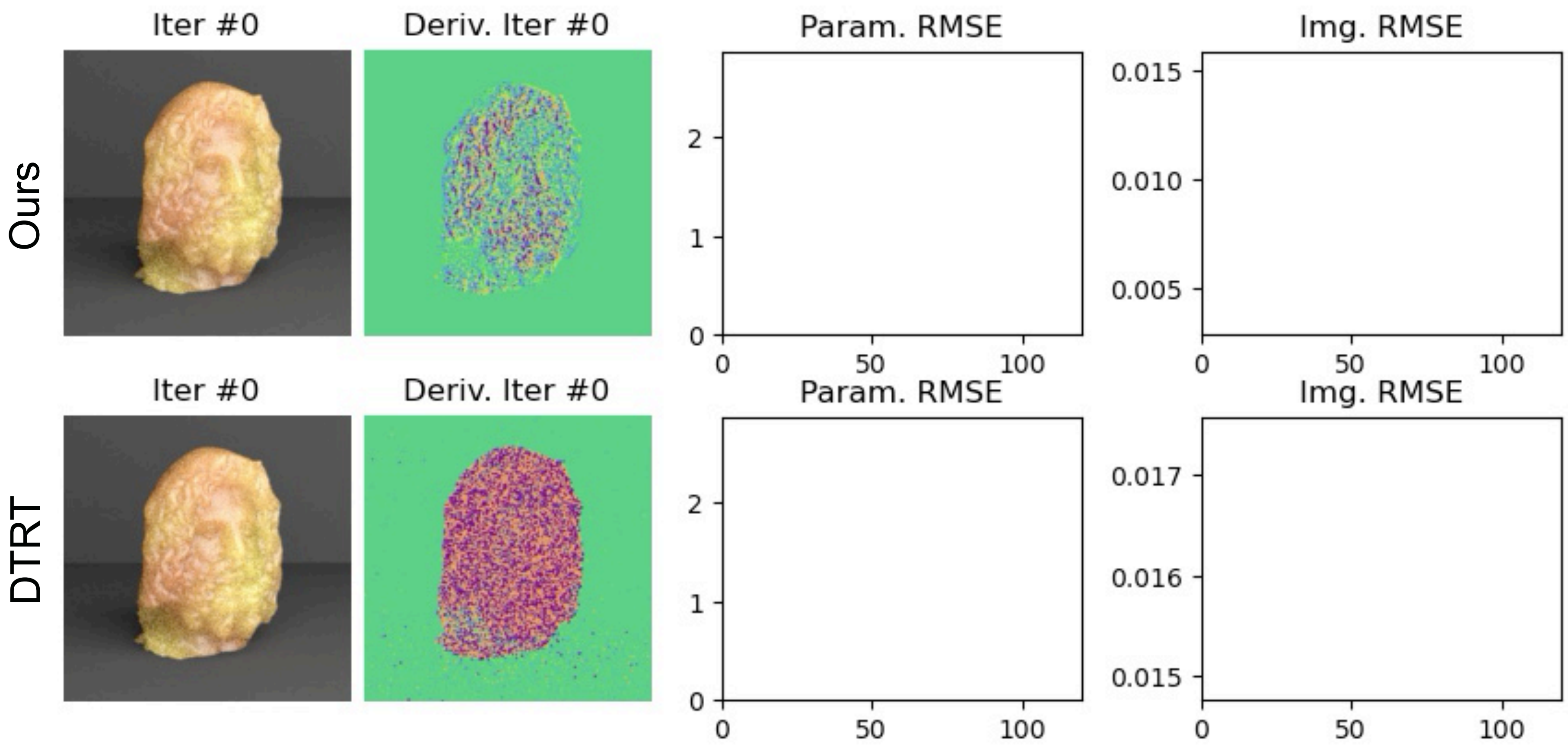
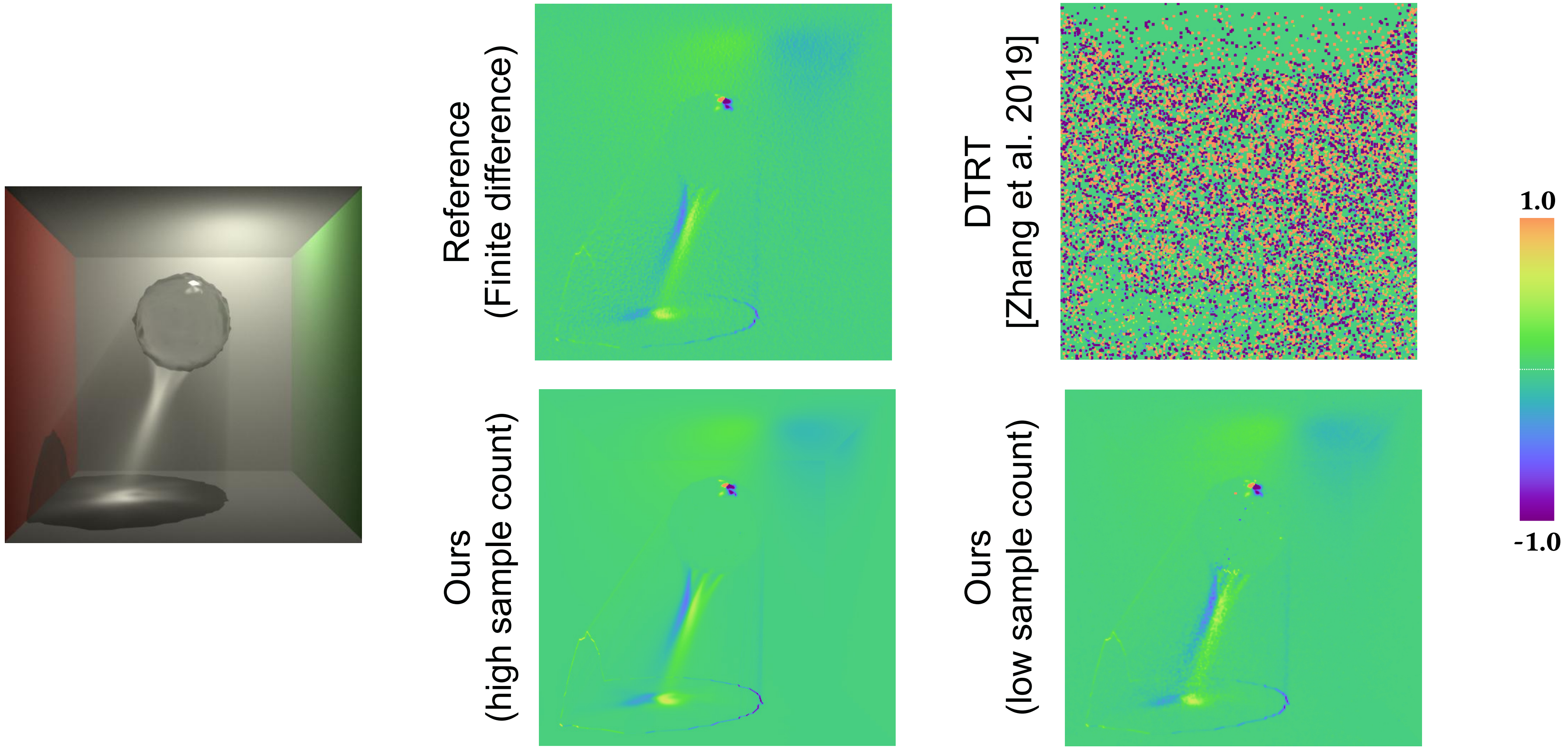
Target image

- Optimize *rotation angle*
- Equal-time per iteration
- Identical optimization setting
  - Learning rate (Adam)
  - Initializations



RESULTS

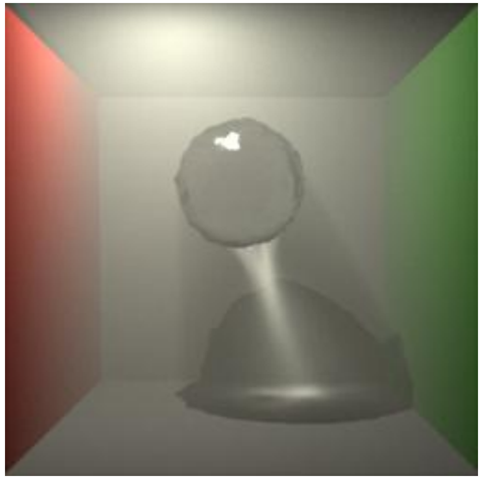
Complex light transport effect



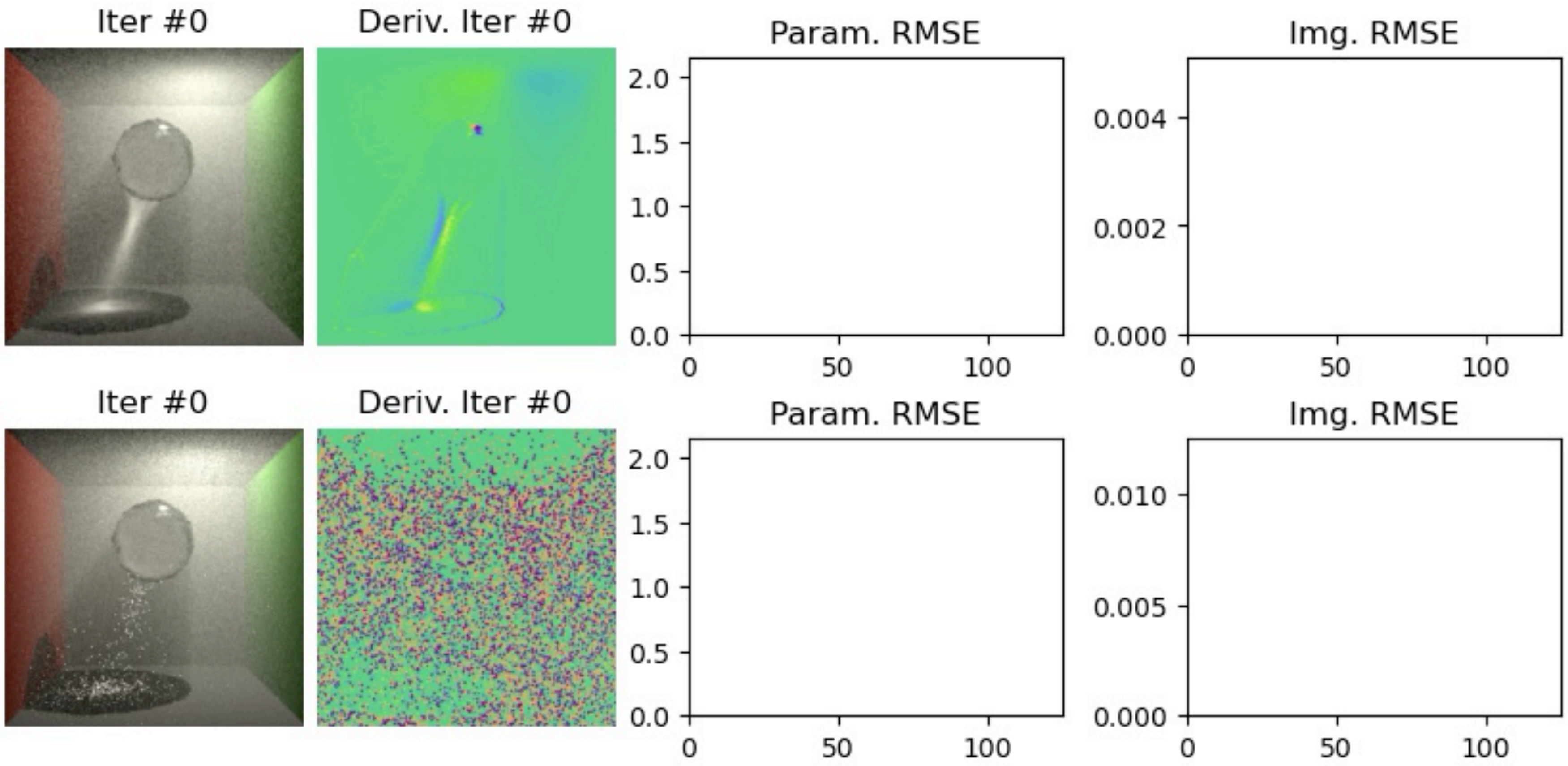
RESULTS

Complex light transport effect

Target image



- Optimize *area light position*
- Equal-time per iteration
- Identical optimization setting
  - Learning rate (Adam)
  - Initializations



RESULTS

Joint optimization of geometry and material

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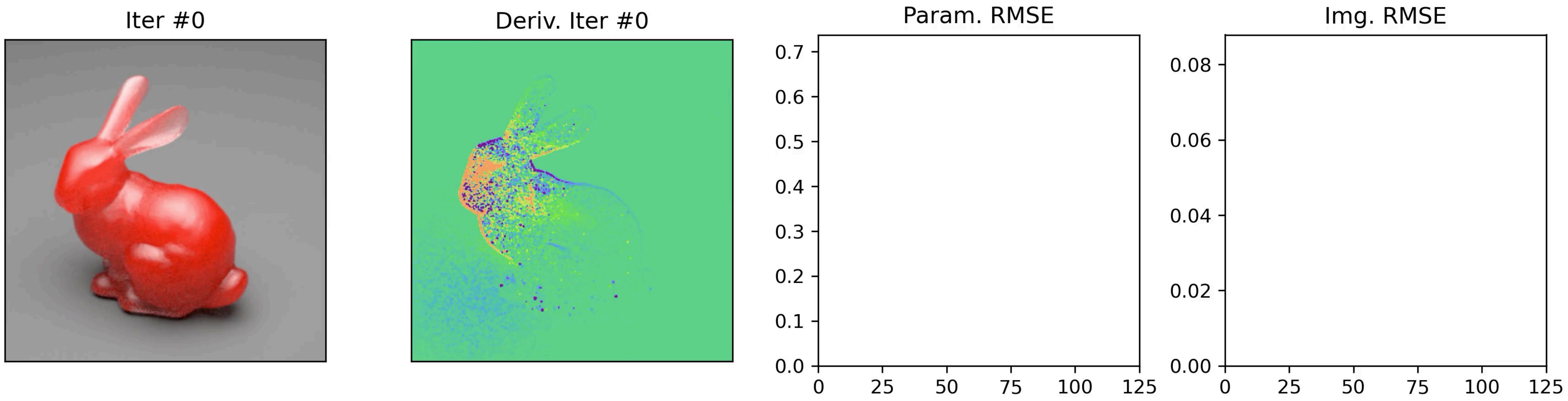
Init image

Target image



LIMITATIONS AND FUTURE WORK

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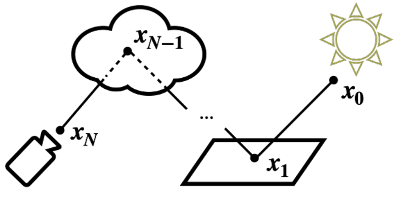
- Current implementation: CPU only
  - Can not handle large number of parameters (~millions)
  - GPU implementation for real-world application
- Implicit scene geometry
  - Signed distance field (SDF)
  - Topology changes
- Primary-sample-space (PSS)

CONCLUSION

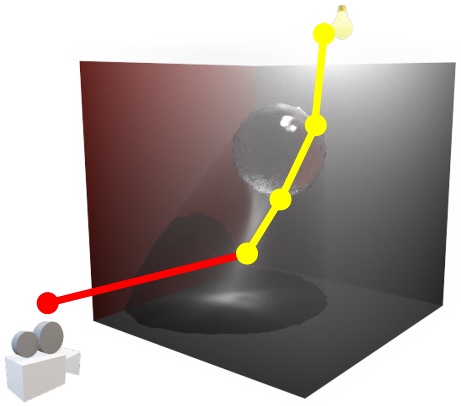
- Generalized differential path integral
  - Interfacial and volumetric light transport

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\int$$



$$\mathrm{d}\bar{x}$$

- Monte Carlo methods
  - Handle interior and boundary separately
  - Complex geometry
  - Complex light transport effects (multiple scatterings, volumetric caustics)



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SIGGRAPH 2021

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Project webpage
https://rb.gy/chvkme