

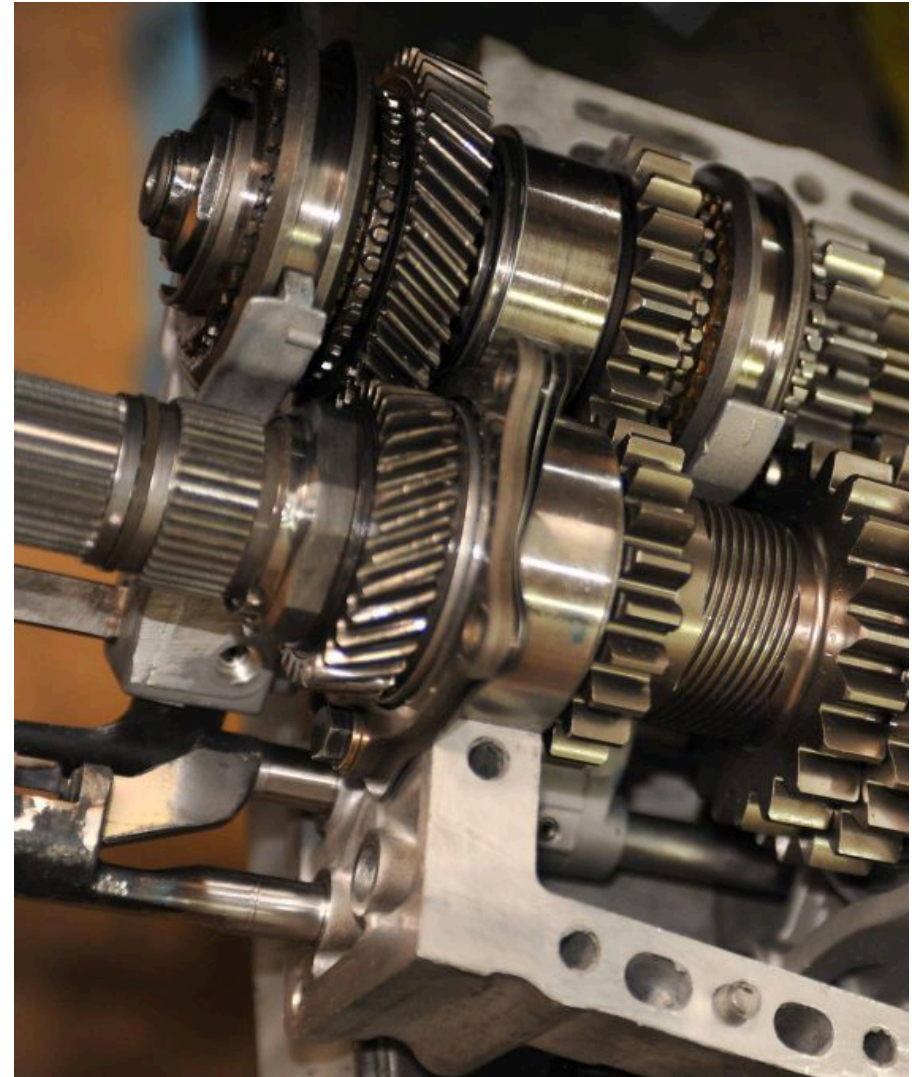


**SIGGRAPH 2021**

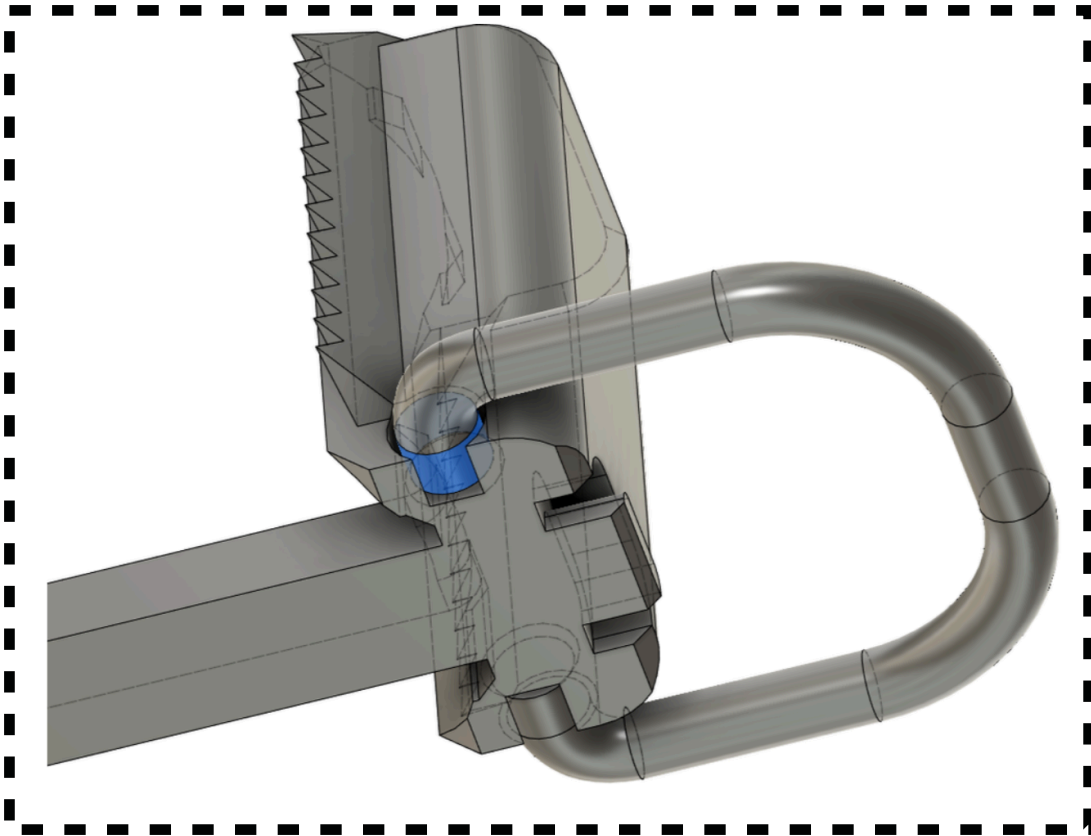
# **GUARANTEED GLOBALLY INJECTIVE 3D DEFORMATION PROCESSING**

Yu Fang\*, Minchen Li\*, Chenfanfu Jiang,  
Danny M. Kaufman

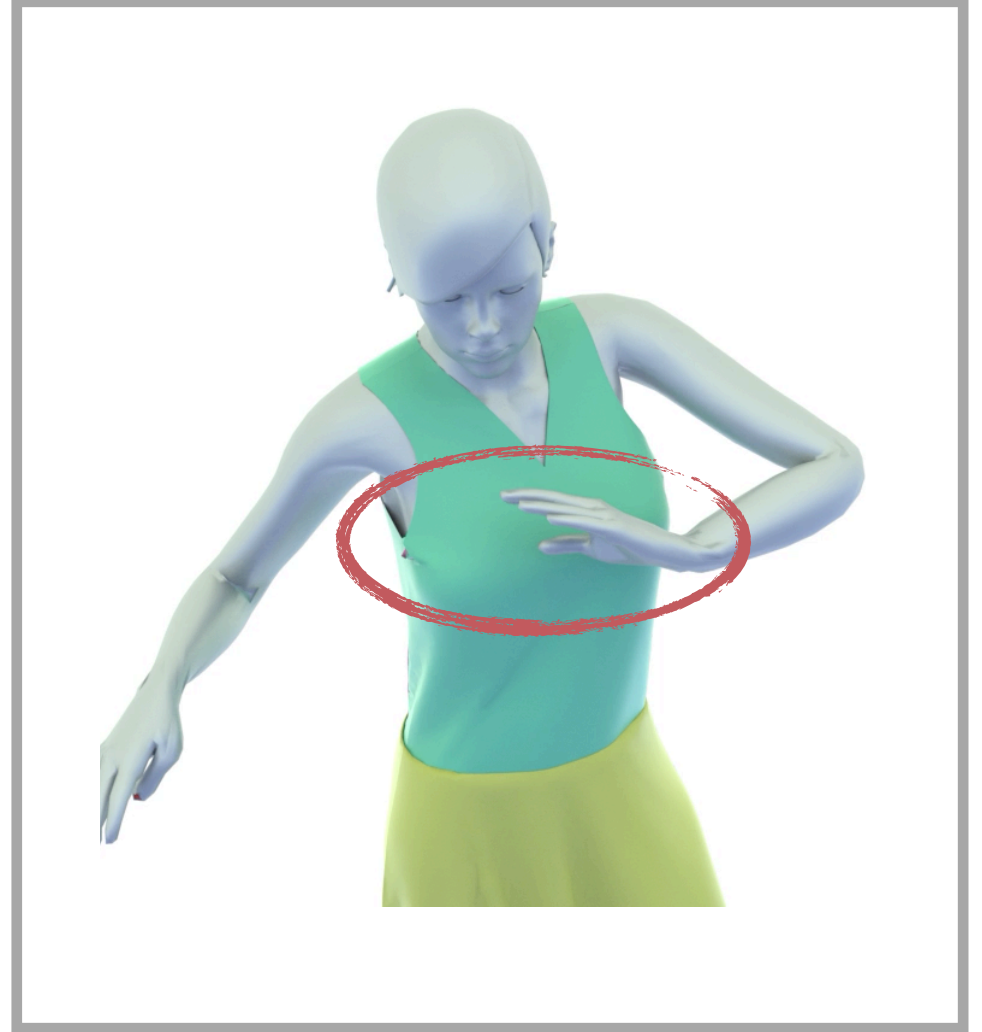
Guaranteed Globally  
Injective 3D  
Deformation Processing



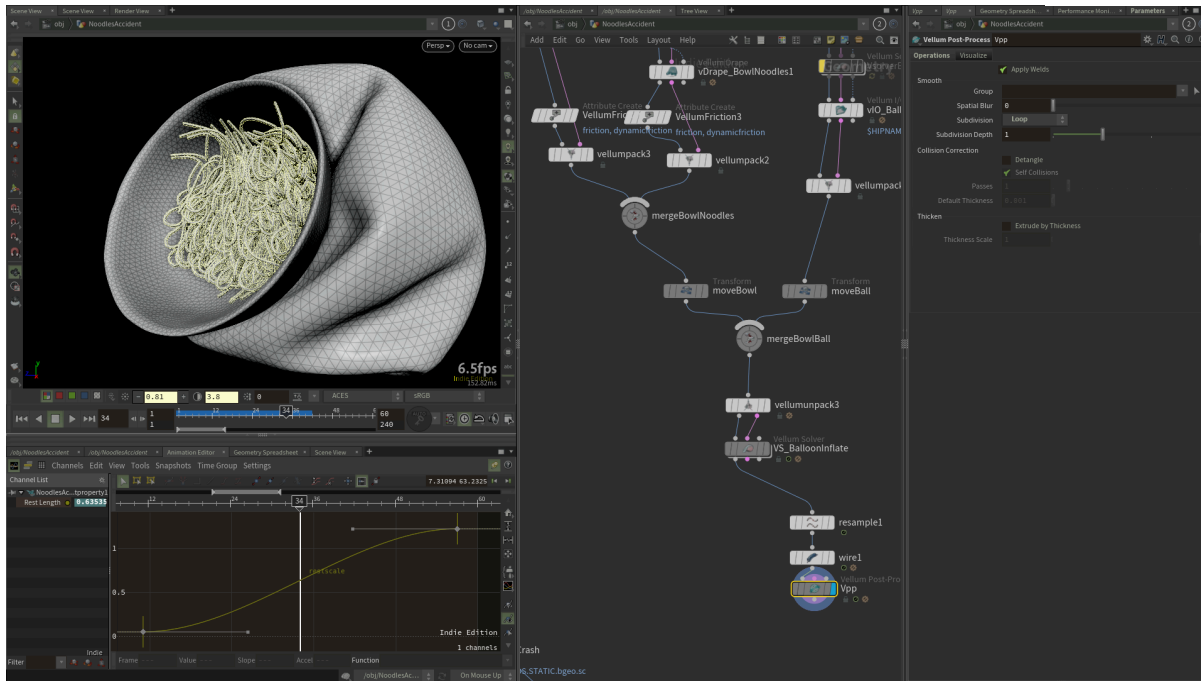
## Contacts in Real-life



## Fabrication



Animation



# 3D Modeling

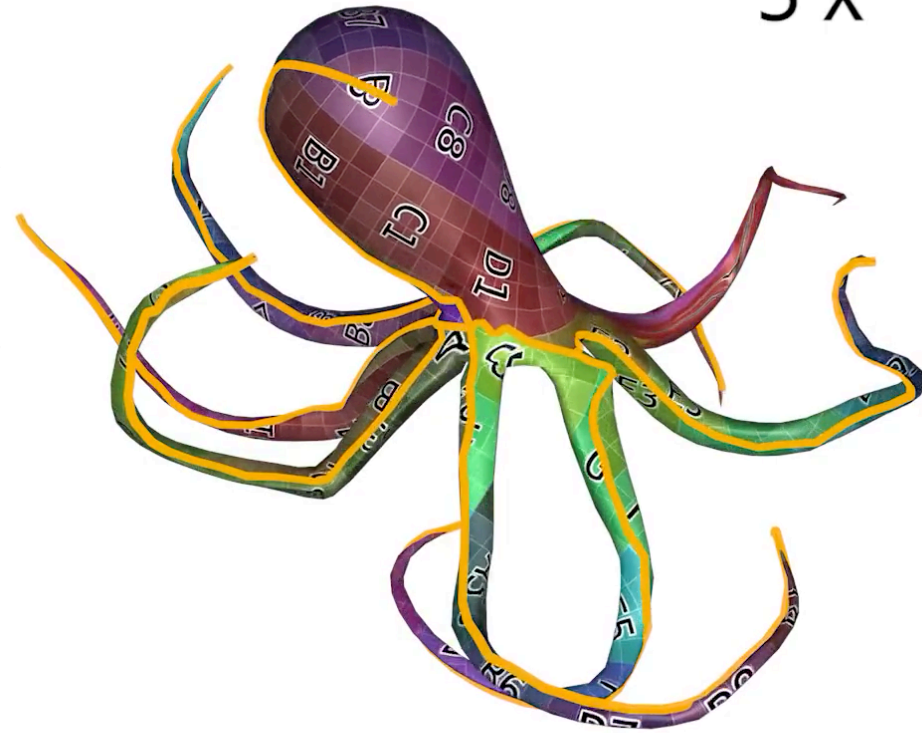
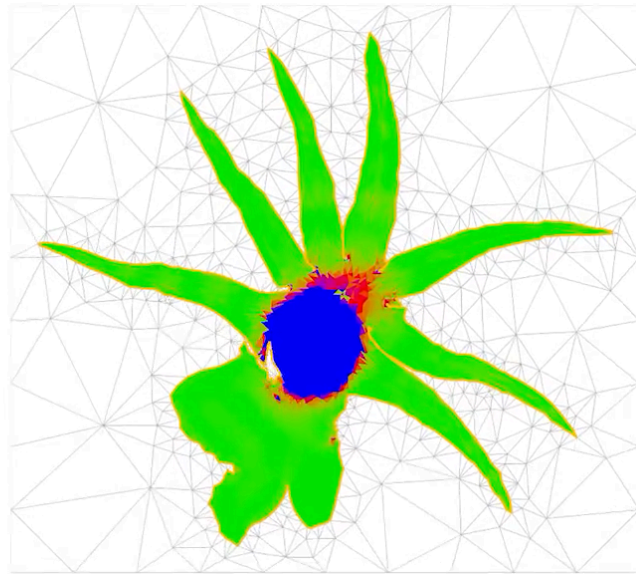
Octopus Vertex #: 3002  $b_d = 4.1$

▶▶  
5x

$E_d = 8.5$



$E_d = 4.0$

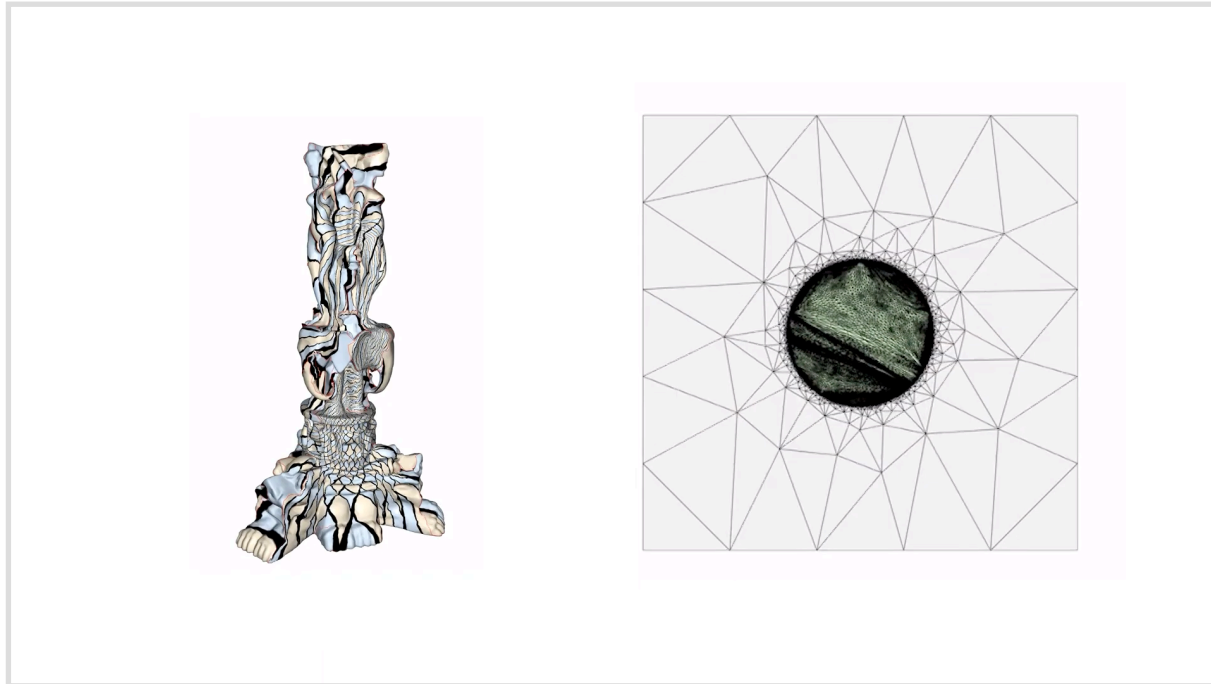


[Li et al. 2018]

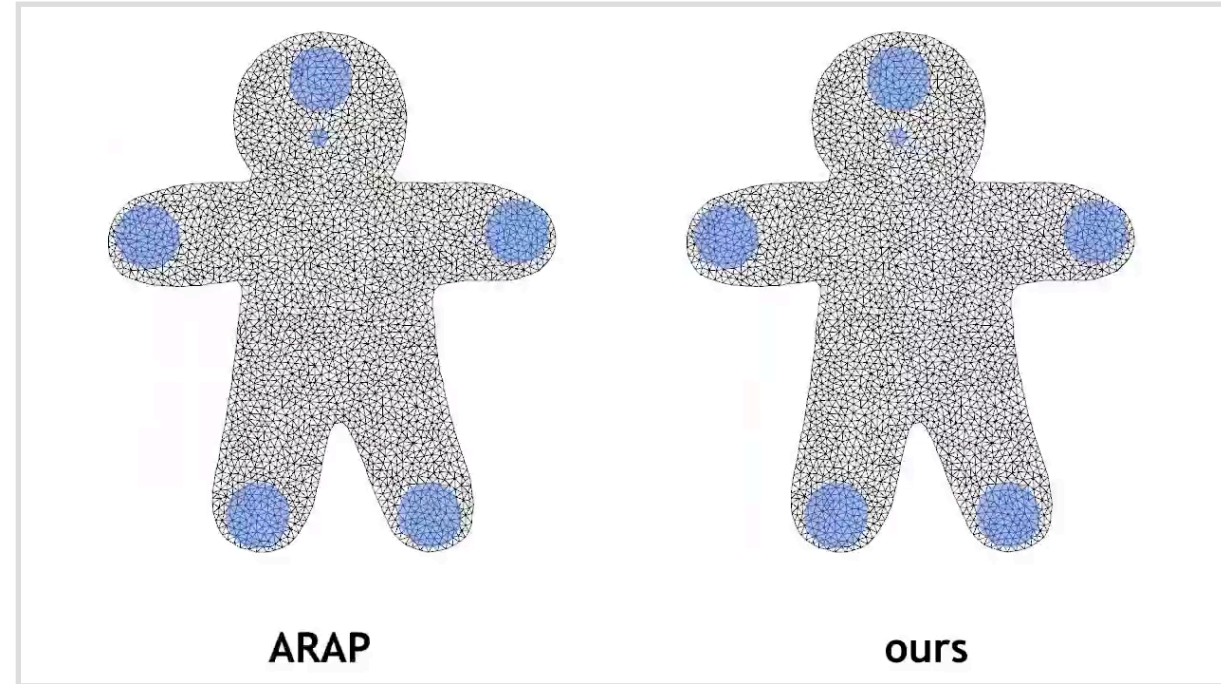
## Mapping

# Prior Work 2D

[Schüller et al. 2013, Smith and Schaefer 2015, Kovalsky et al. 2016, Jiang et al. 2017, Zhu et al. 2018, Su et al. 2020, ...]



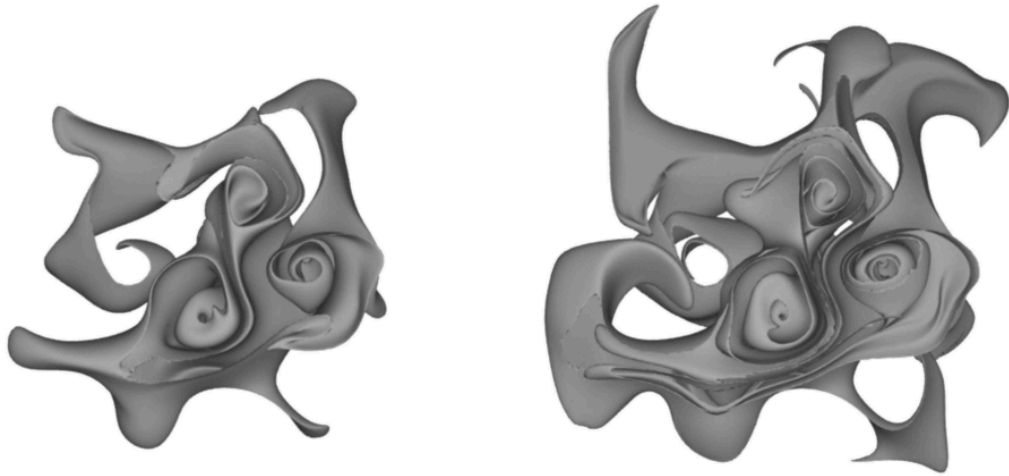
[Jiang et al. 2017]



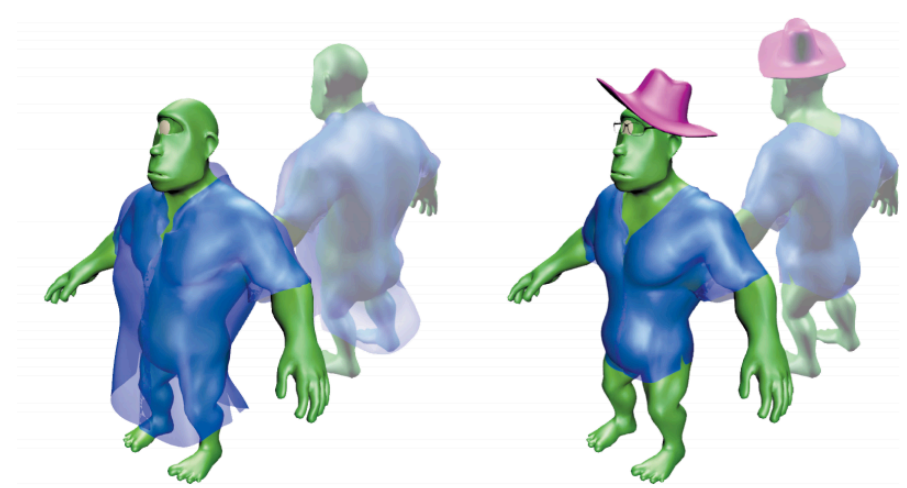
[Schüller et al. 2013]



# Iterative Contact Processing Methods



[Brochu and Bridson 2009]



[Harmon et al. 2011]



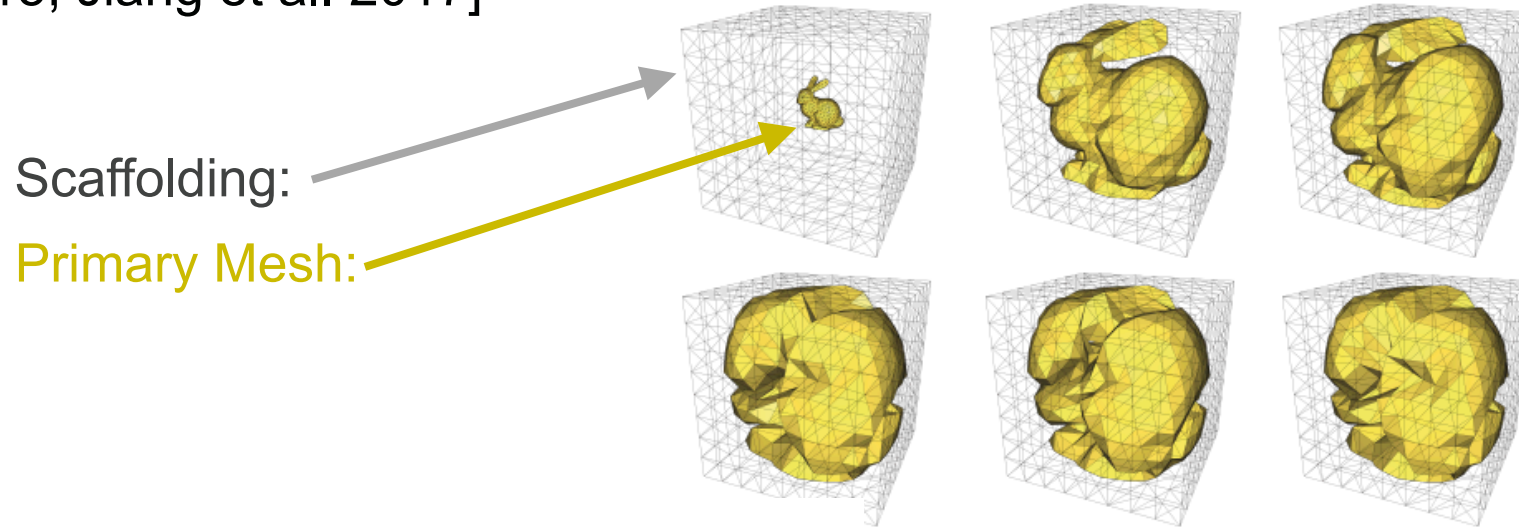
[Jiang et al. 2017]

Iterative Contact Processing Failure (intersection)



# Scaffolding Methods

[Müller et al. 2015, Jiang et al. 2017]



Locking behavior:



Scaffolding Framework [Jiang et al. 2017] result.

# Problem

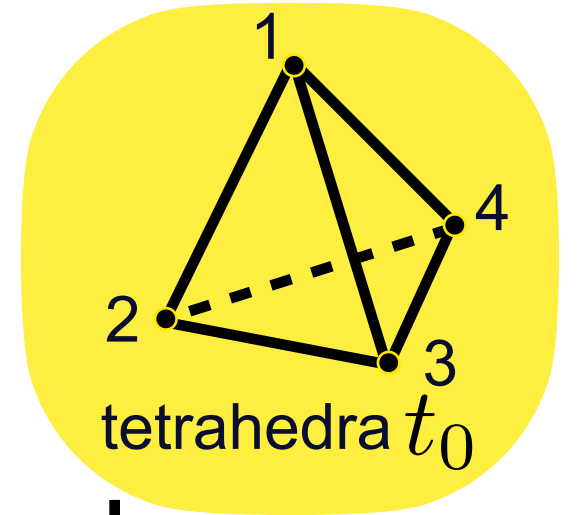
$$x^* = \arg \min_x E_{\bar{x}}(x)$$

**Energy:**  $E_d(x, \bar{x}) = \sum_{t \in T} v_t \Psi(F_t)$

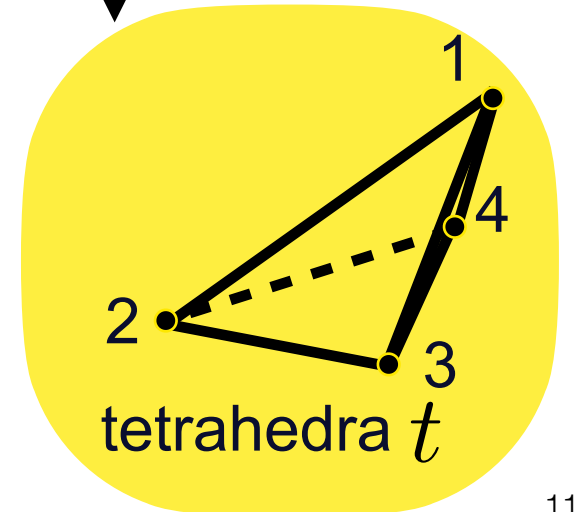
SVD

**Neo-Hookean:**  $E = \frac{\mu}{2} (|\sigma|^2 - 3) - (\mu - \frac{\lambda}{2} \log(\sigma_1 \sigma_2 \sigma_3)) \times \log(\sigma_1 \sigma_2 \sigma_3)$

**Symmetric Dirichlet:**  $E = \kappa (\sigma_1^2 + \frac{1}{\sigma_1} + \sigma_2^2 + \frac{1}{\sigma_2} + \sigma_3^2 + \frac{1}{\sigma_3})$



Deformation Gradient  $F_t$



# IPC Barrier Energy

$$x^* = \arg \min_x E_{\bar{x}}(x)$$

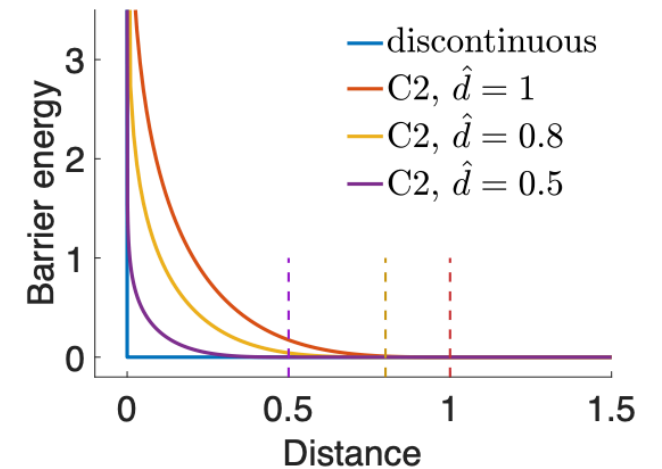
IPC Barrier:  $B(x) = \kappa \sum_{i \in C} b(d_i(x), \hat{d})$

All Pairs (those far away contributes zero)

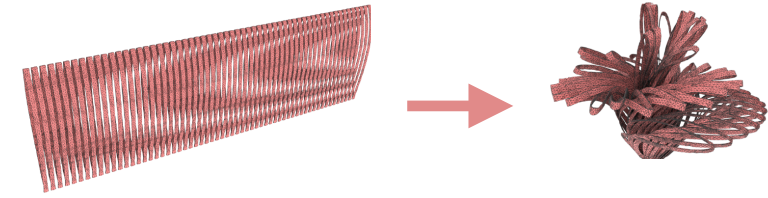
$$b(d, \hat{d}) = \begin{cases} -(d - \hat{d})^2 \ln\left(\frac{d}{\hat{d}}\right), & 0 < d < \hat{d} \\ 0 & d \geq \hat{d} \end{cases}$$

$b()$  is C2 Continuous

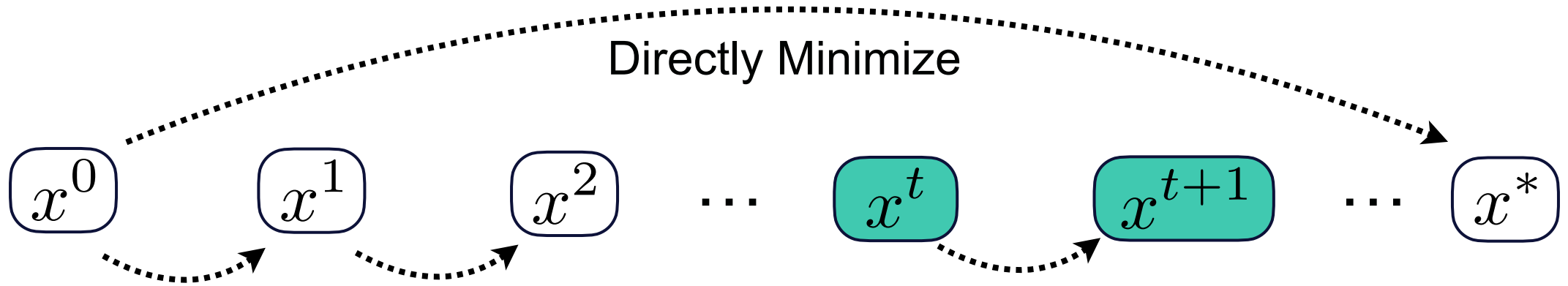
$d()$  may be C0 continuous (Edge-Edge case), mollification is introduced!



# Artificial Time Stepping



$$x^* = \arg \min_x E_{\bar{x}}(x)$$

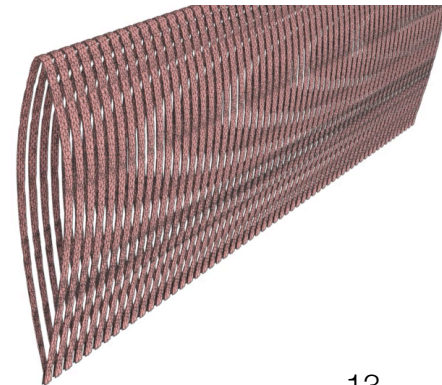


Time step  $M\dot{x} = -\nabla E(x)$  to progressively optimize

Time discretization (implicit Euler):

$$x^{t+1} = \arg \min_x \frac{1}{2} \|x - x^t\|_M^2 + h E(x, \bar{x}, u(t))$$

Artificial step size
Mass matrix



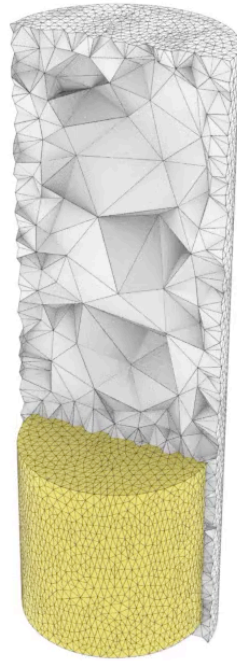
# Scaffolding Comparison



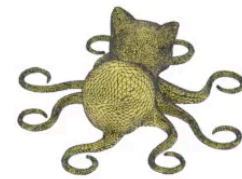
**IDP**



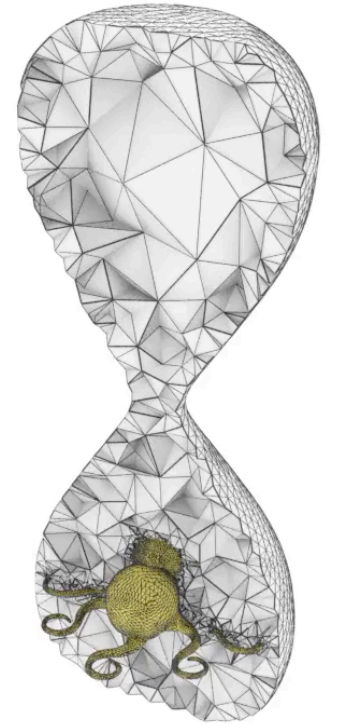
**SCAF [Jiang et al. 2017]**



**IDP**



**SCAF [Jiang et al. 2017]**



# Time-Varying Boundary Conditions

**Goal:** a set of positionally constrained  $\mathcal{B} \subset [1, n]$

Satisfy moving Dirichlet boundary condition  $x_k = \tilde{x}_k, k \in \mathcal{B}$

**Penalty term:**  $P(x, t) = \frac{\kappa_{\mathcal{B}}}{2} m_k \|x_k - \tilde{x}_k^{t+1}\|^2$

**Solution:**

Solver with static BC

**While**  $\|x_k - \tilde{x}_k^{t+1}\|_{\infty} > 10^{-6}$  **do**

**Solve**  $E = \frac{1}{2}\|x - x^t\|_M^2 + hE_{\bar{x}}(x)$

$g, H = \nabla E, \nabla^2 E$

$x_+ = \alpha(-H^{-1}g), \alpha$  is from  
line search

Solver with moving BC

**While**  $\|x_k - \tilde{x}_k^{t+1}\|_{\infty} > 10^{-6}$  **do**

**Solve**  $E = \frac{1}{2}\|x - x^t\|_M^2 + hE_{\bar{x}}(x) + P(x, t)$

$g, H = \nabla E, \nabla^2 E$

$x_+ = \alpha(-H^{-1}g), \alpha$  is from line search

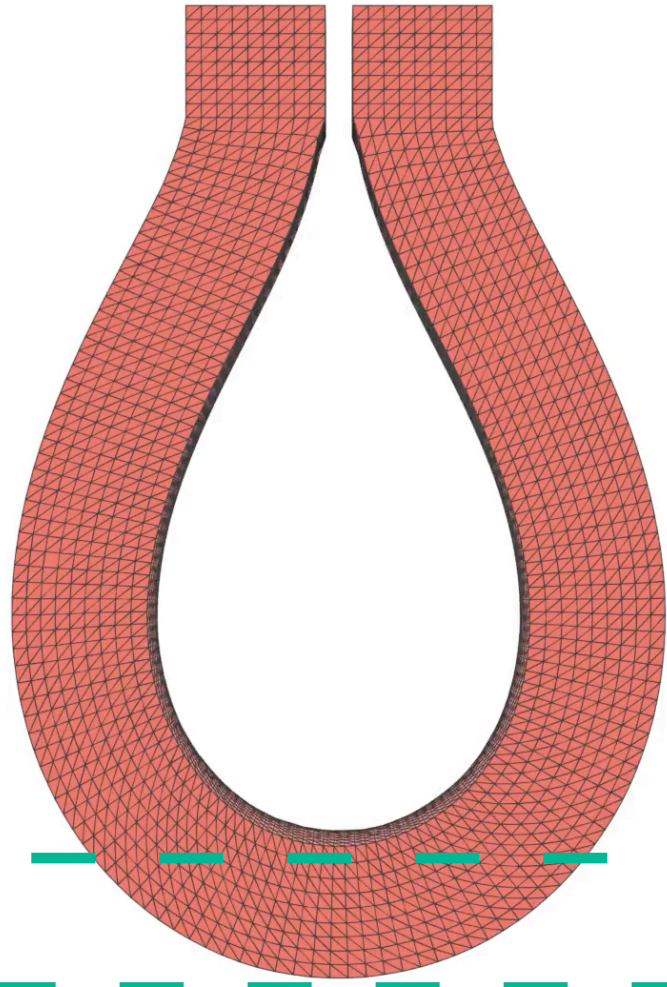
$\kappa_{\mathcal{B}} = 2\kappa_{\mathcal{B}}$

**Solve**  $E = \frac{1}{2}\|x - x^t\|_M^2 + hE_{\bar{x}}(x), \text{s.t. } x_k = \tilde{x}_k, k \in \mathcal{B}$

$g, H = \nabla E, \nabla^2 E$

$x_+ = \alpha(-H^{-1}g), \alpha$  is from line search

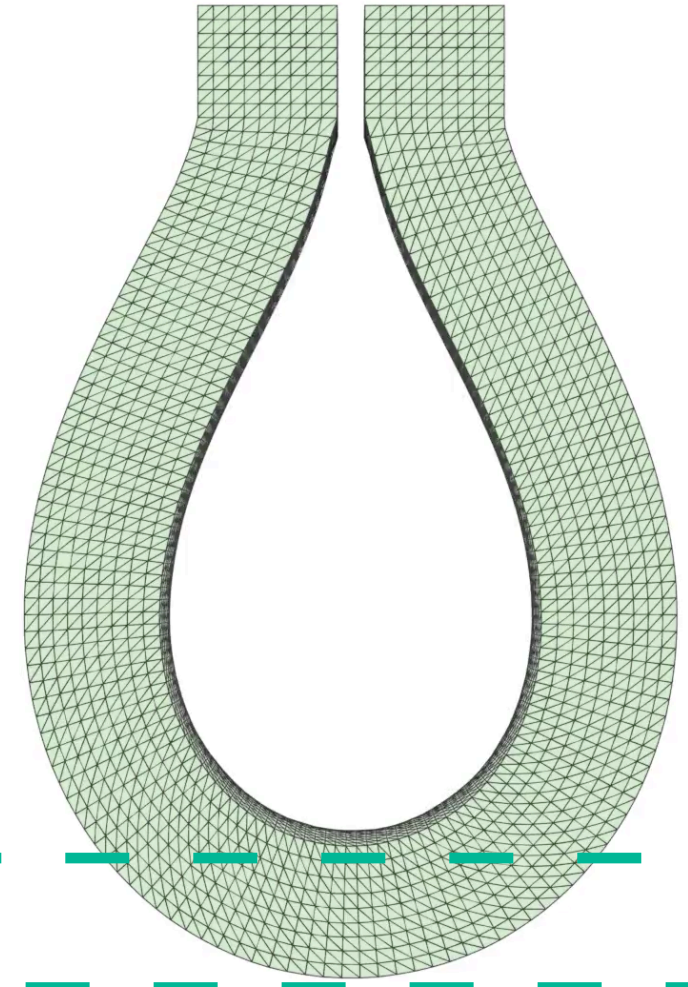
# Time-Varying Boundary Conditions



**Plastic + Elastic + Barrier**



**Elastic + Barrier**



**Elastic**

*B*



# Applications

# (1) Normal Flow

$$E(x, N^t) = B(x) - \gamma \frac{1}{2} x^T L x - \beta x^T N^t$$

Flow Speed

Normal Direction



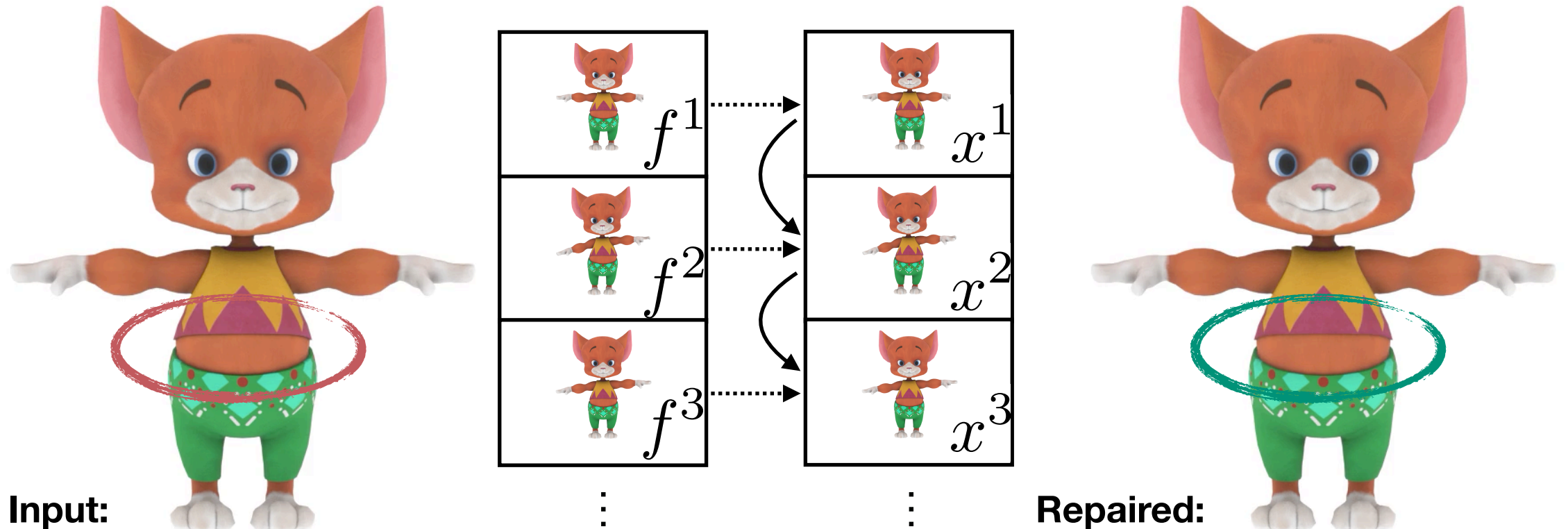
## (2) Animation Repair

$$x^s = \arg \min_x \frac{1}{2} \|x - f^s\|_M^2 + hE(x, f^s)$$

Repaired output animation frame Input animation frame

$$E(x, \bar{x}) = \Psi_{\text{memb}}(x, \bar{x}) + \Psi_{\text{bend}}(x, \bar{x}) + B(x)$$

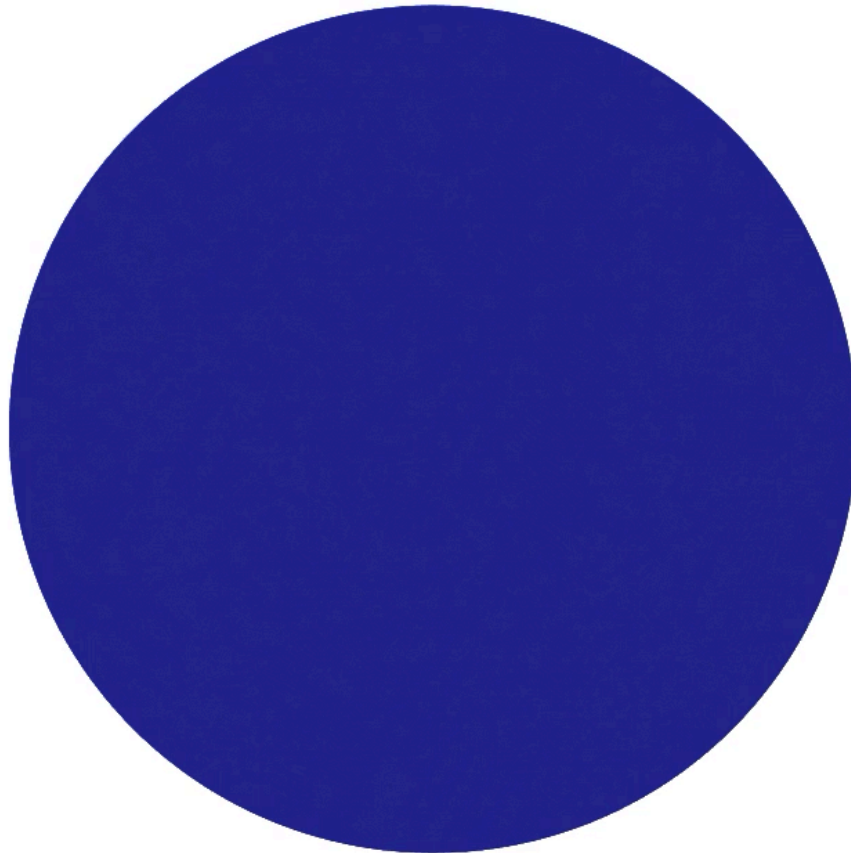
Membrane Energy Shell Bending Energy



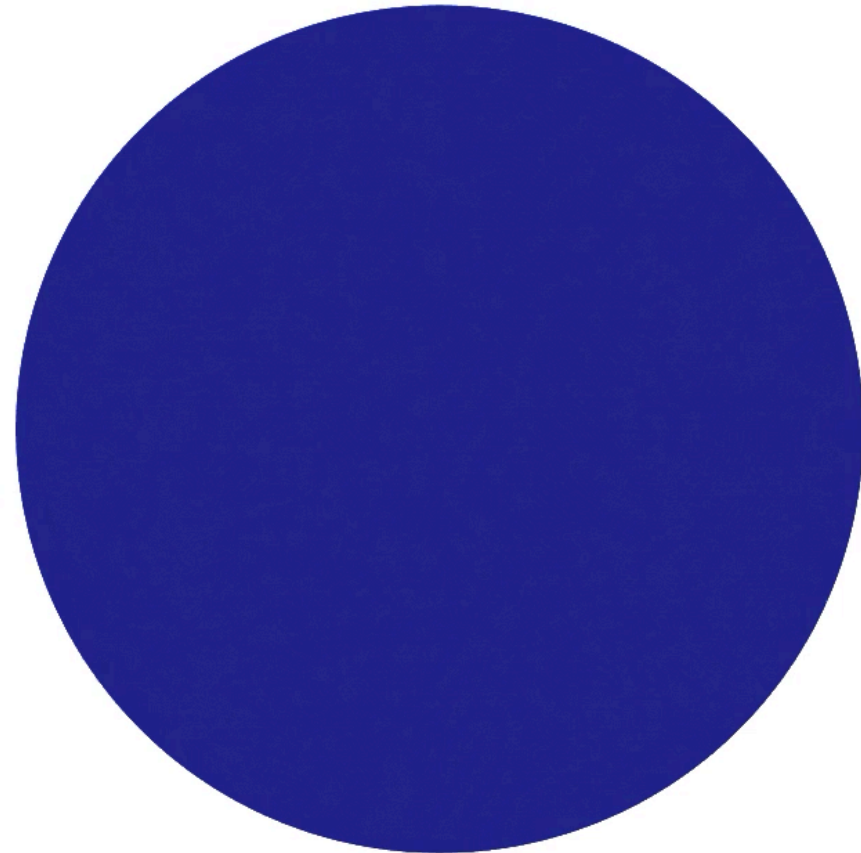
### (3) Min-max distortion – 2D

$$E_p(x, \bar{x}) = r(p) \sum_{t \in T} v_t (\Psi(F_t))^{p}, \quad r(p) = \frac{1}{p} (\Psi(I))^{(1-p)}$$

Exponential Factor

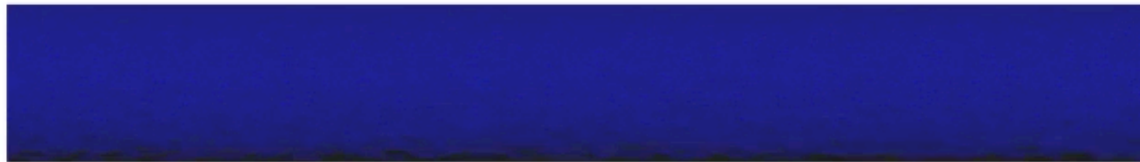


**p = 1**



**p = 10**

### (3) Min-max distortion—3D



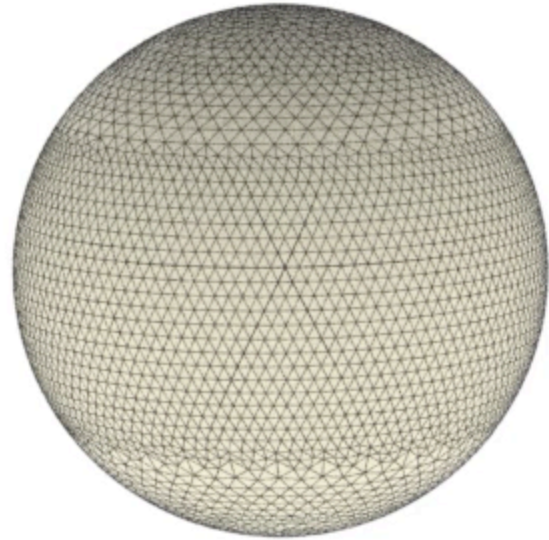
$p = 1$



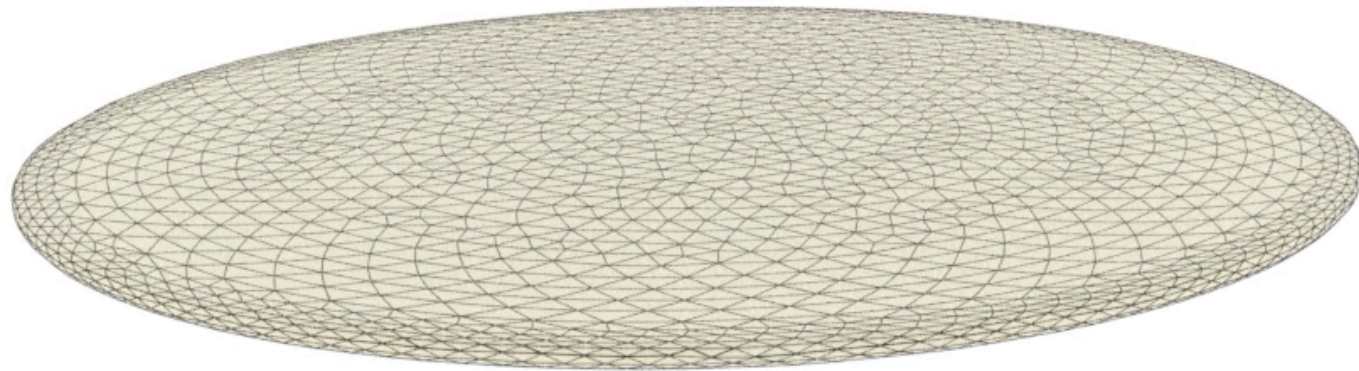
$p = 5$

## (4) Modeling—Squash

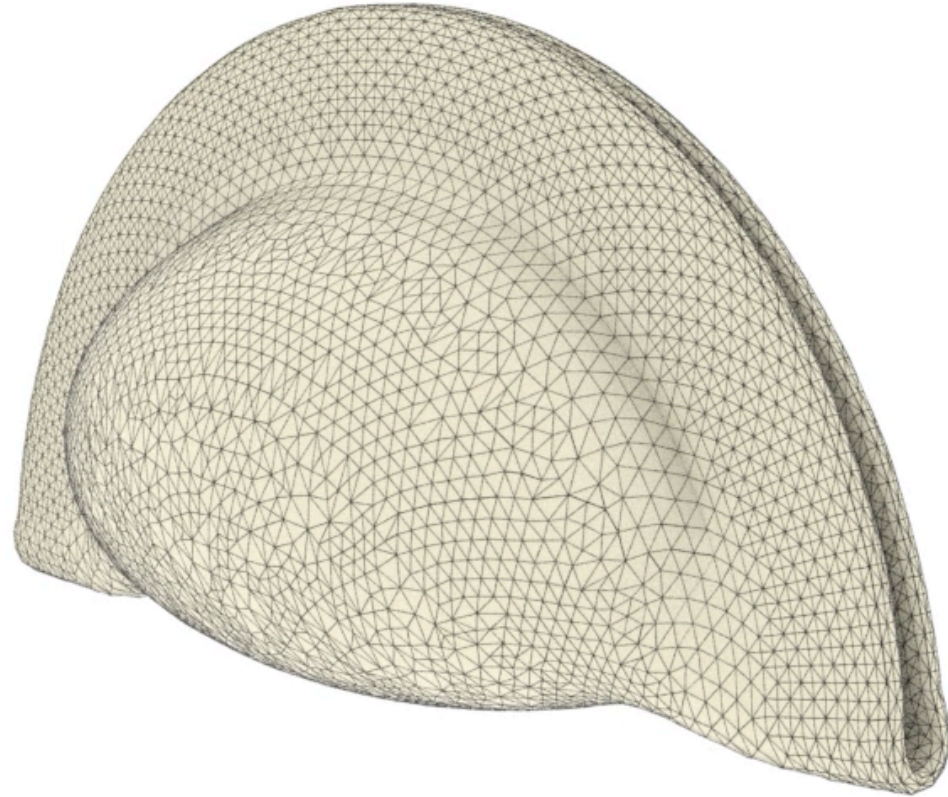
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## (4) Modeling—Wrap

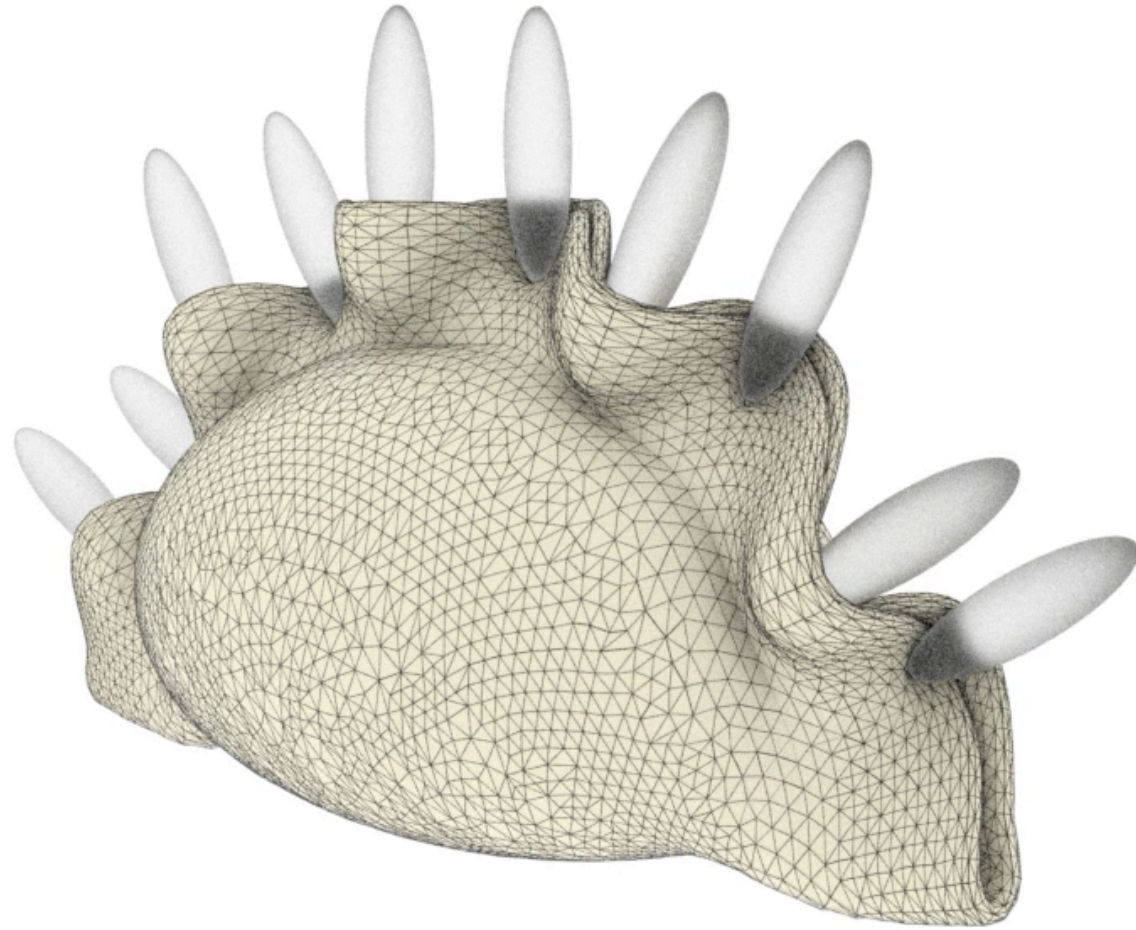


## (4) Modeling—Pinch

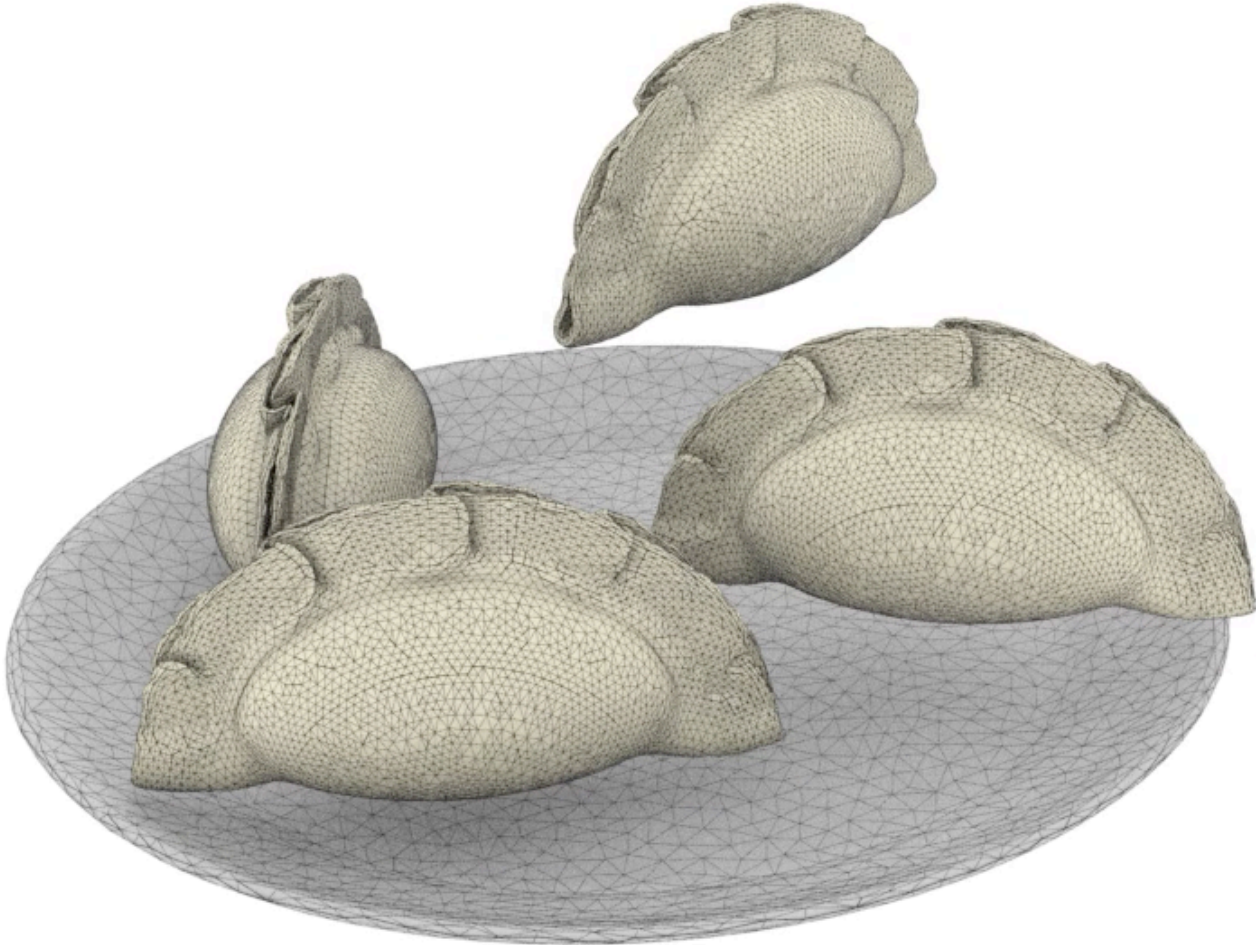




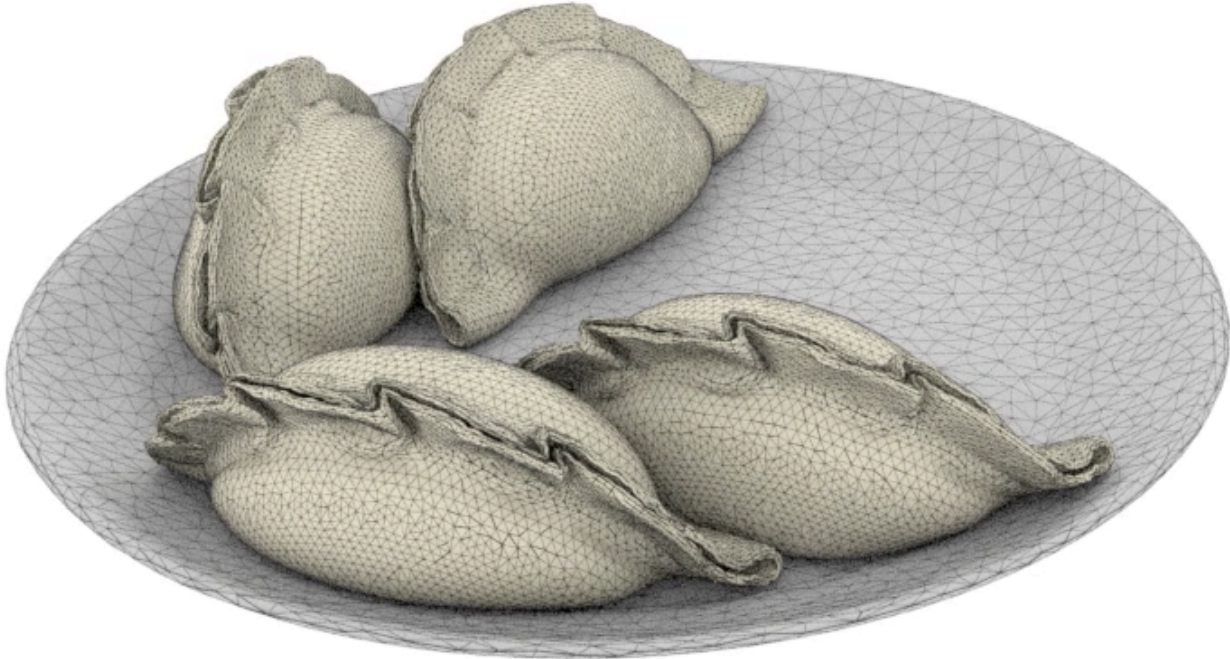
## (4) Modeling—Squeeze



# (4) Modeling—Layout



# (4) Modeling—Layout



Enjoy

Dumplings



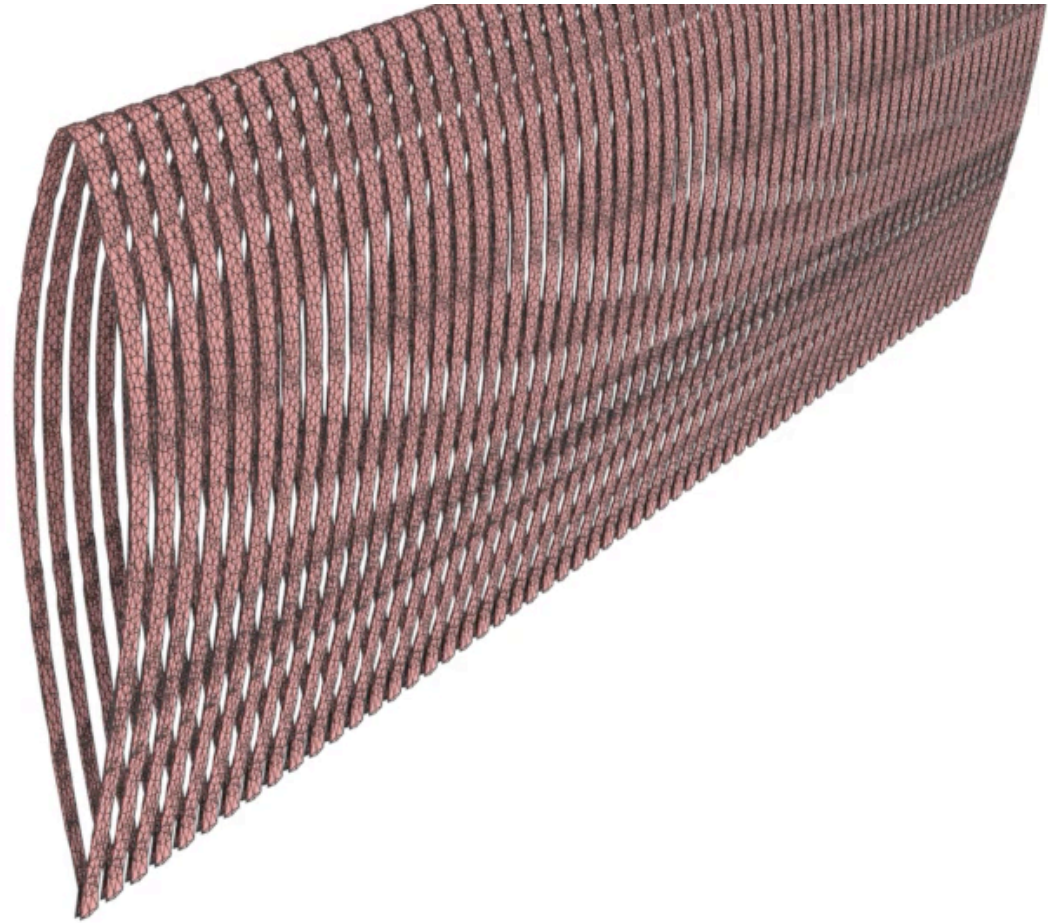
# (4) Modeling – Bend



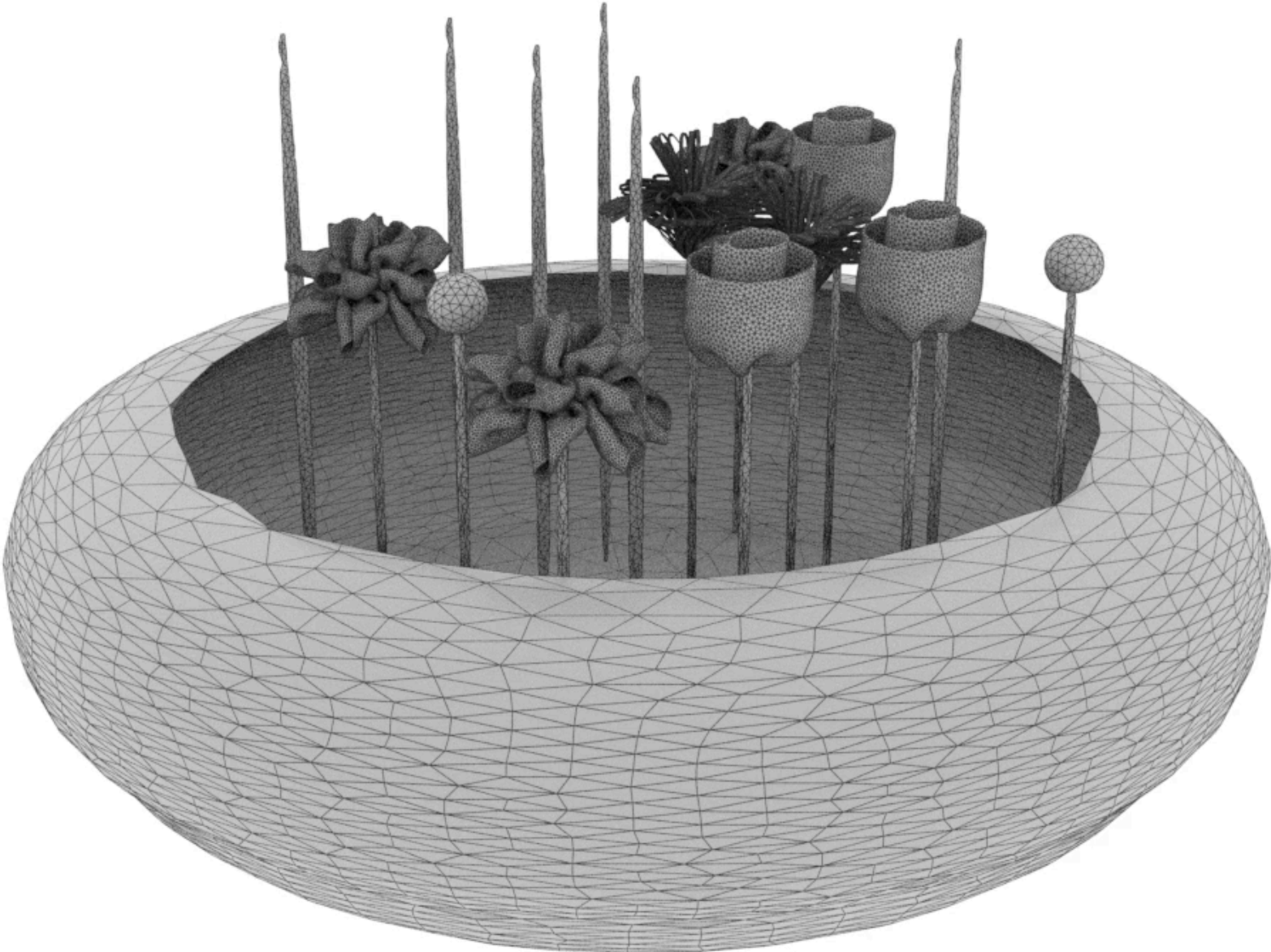
## (4) Modeling—Copy Paste



## (4) Modeling—Wind



# (4) Modeling – Flower

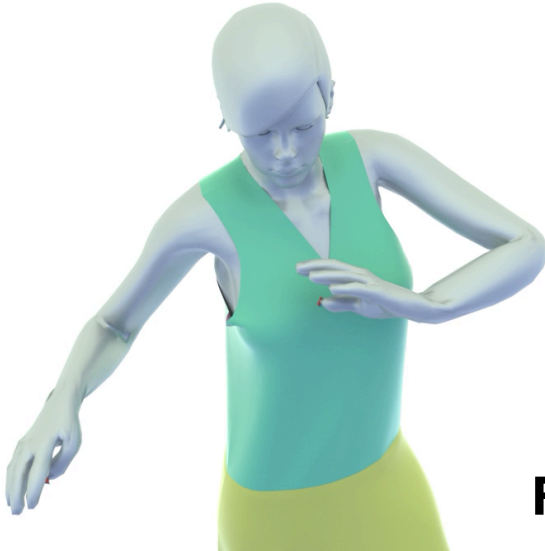
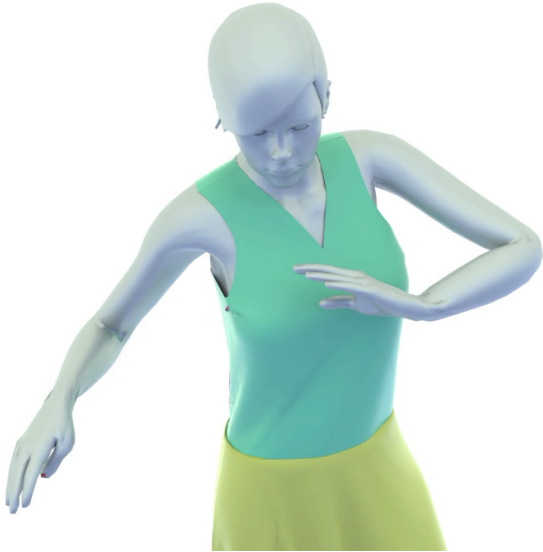
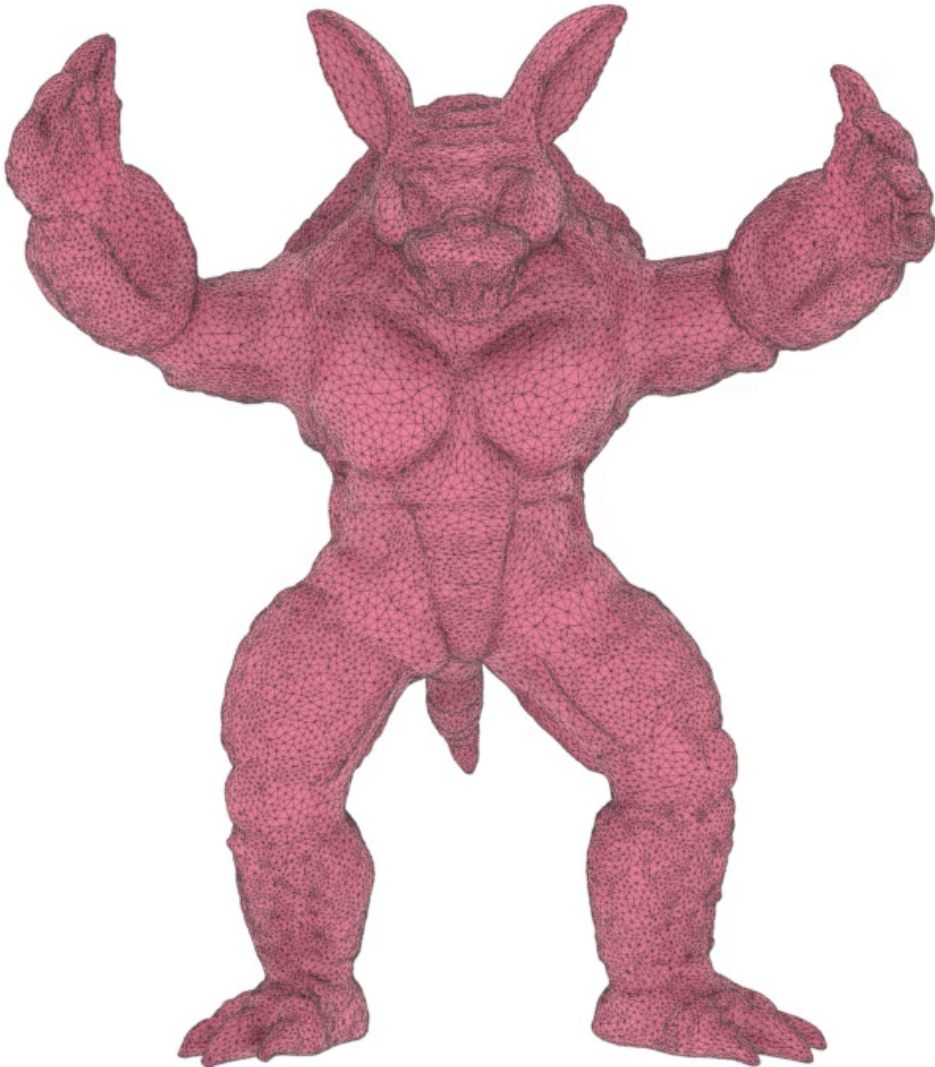




**Enjoy**



# Conclusion



Thanks!

谢谢!