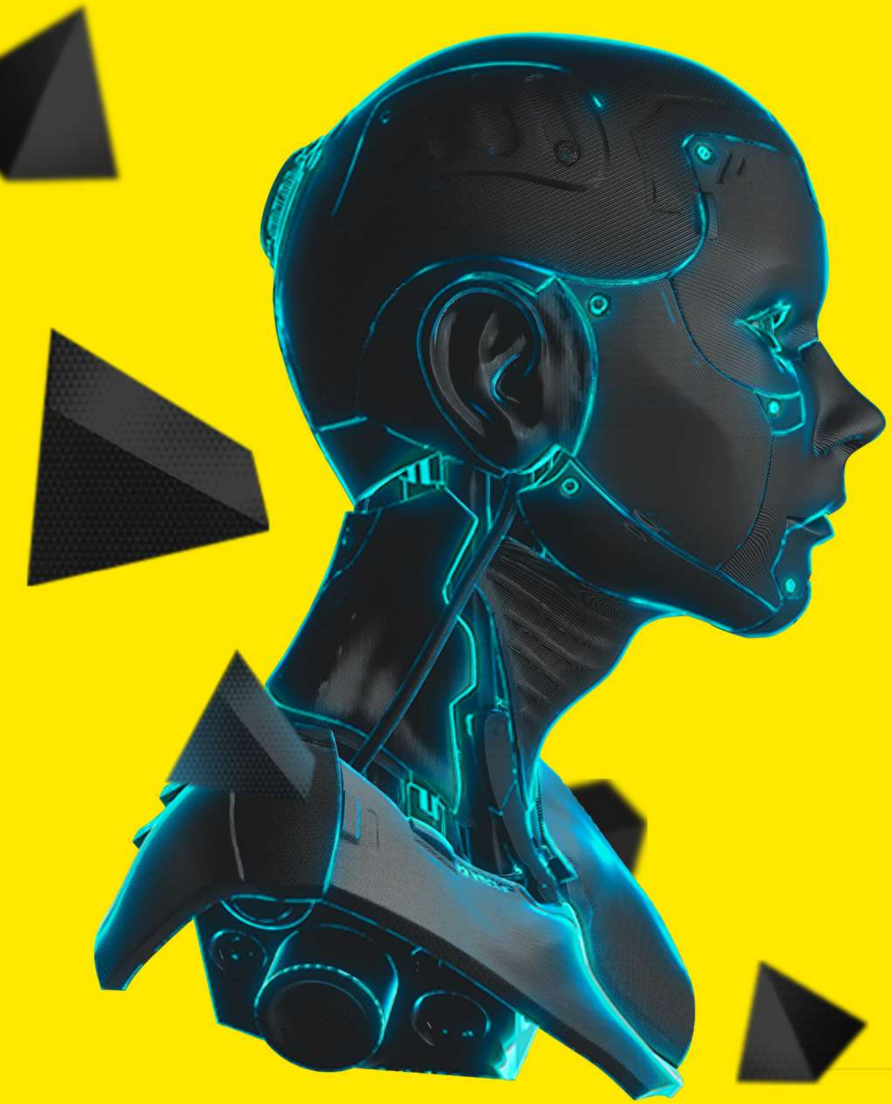

SIGGRAPH 2021

THIN-FILM SMOOTHED PARTICLE HYDRODYNAMICS FLUID

MENGDI WANG, YITONG DENG, XIANGXIN KONG, ADITYA H. PRASAD, SHIYING XIONG, BO ZHU

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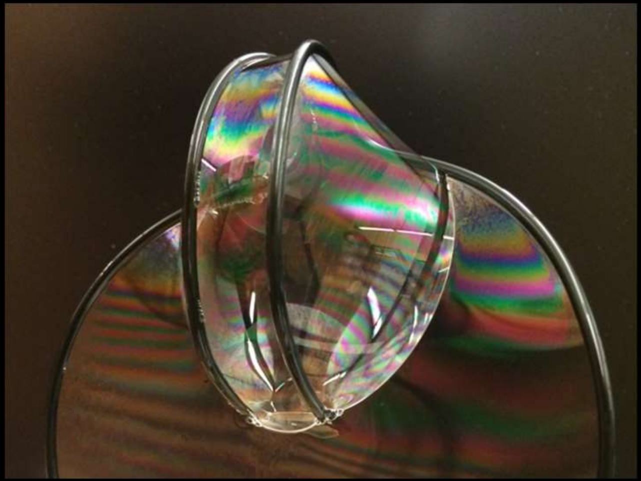
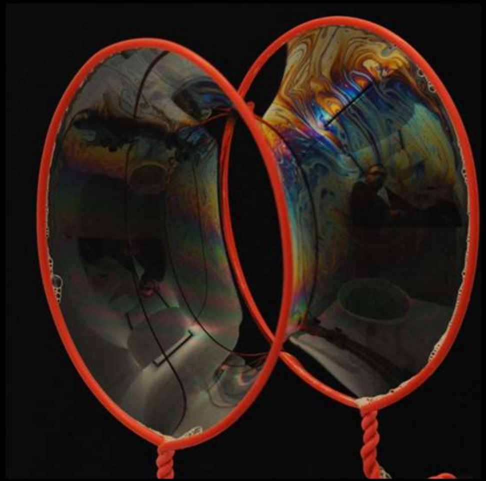
Motivation: Thin Soap Films



Giusteri et al. 2017[1]



https://www.soapbubble.dk/en/articles/former

https://www.sandiegofamily.com/for-the-kids/family-science/bubble-science

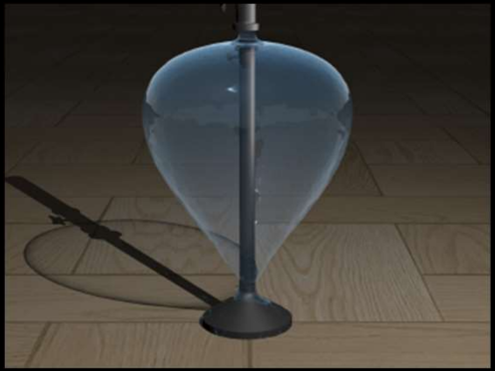
[1] Giusteri, G.G., Lussardi, L. & Fried, E. Solution of the Kirchhoff–Plateau Problem. J Nonlinear Sci 27, 1043–1063 (2017). https://doi.org/10.1007/s00332-017-9359-4

Motivation: Thin Soap Films

https://www.animations.physics.unsw.edu.au/jw/light/soap-bubbles.htm

https://www.youtube.com/watch?v=WTxDyYHaYAI

Related Works: Mesh Representation



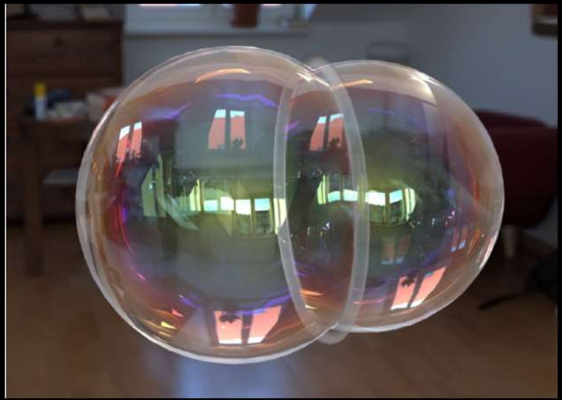
Zhu et al. 2014[1]

Ishida et al. 2020[2]

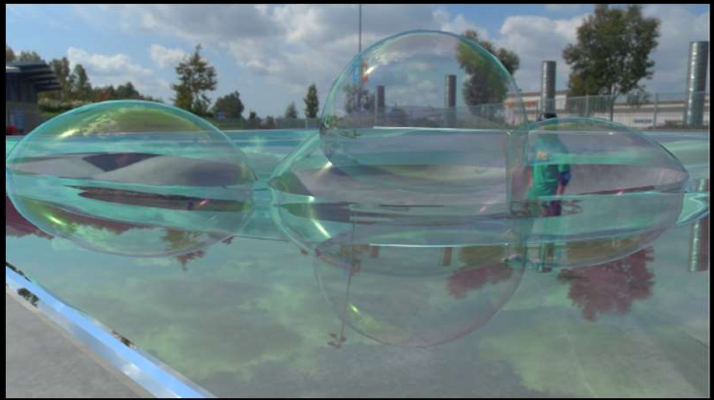
Huang et al. 2020[3]

[1] Bo Zhu, Ed Quigley, Matthew Cong, Justin Solomon, and Ronald Fedkiw. 2014. Codimensional surface tension flow on simplicial complexes. ACM Transactions on Graphics (TOG) 33, 4 (2014), 1–11.
[2] Sadashige Ishida, Peter Synak, Fumiya Narita, Toshiya Hachisuka, and Chris Wojtan. 2020. A Model for Soap Film Dynamics with Evolving Thickness. ACM Transactions on Graphics 39, 4 (2020), 31:1–31:11.
[3] Weizhen Huang, Julian Iseringhausen, Tom Kneiphof, Ziyin Qu, Chenfanfu Jiang, and Matthias B. Hullin. 2020. Chemomechanical Simulation of Soap Film Flow on Spherical Bubbles. ACM Transactions on Graphics 39, 4 (2020).

Related Works: Mesh Representation



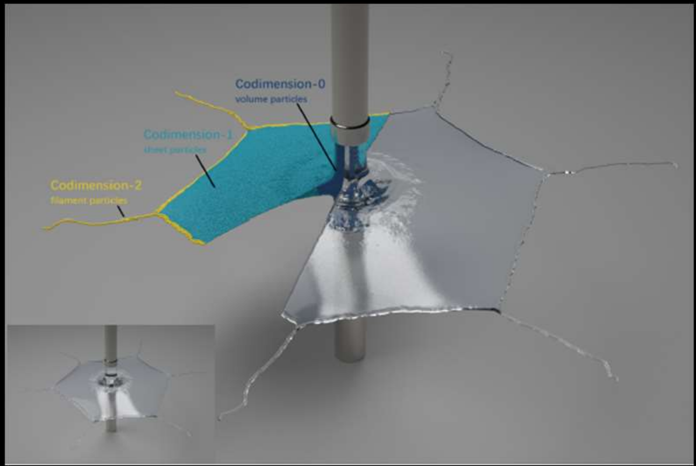
Da et al. 2015[1]



Ishida et al. 2017[2]

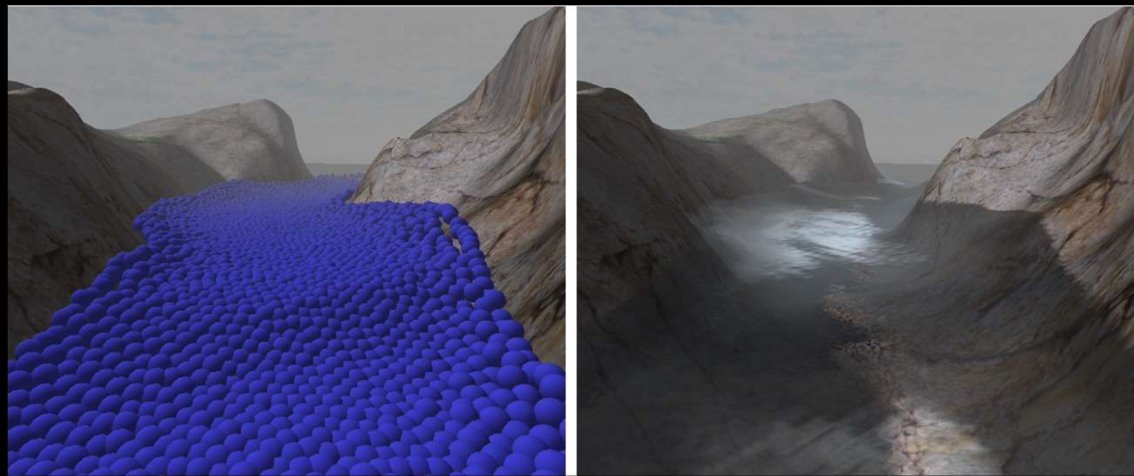
[1] Fang Da, Christopher Batty, Chris Wojtan, and Eitan Grinspun. 2015. Double bubbles sans toil and trouble: Discrete circulation-preserving vortex sheets for soap films and foams. ACM Transactions on Graphics (TOG) 34, 4 (2015), 1–9.
[2] Sadashige Ishida, Masafumi Yamamoto, Ryoichi Ando, and Toshiya Hachisuka. 2017. A hyperbolic geometric flow for evolving films and foams. ACM Transactions on Graphics (TOG) 36, 6 (2017), 1–11.

Related Works: Point-Set Surface



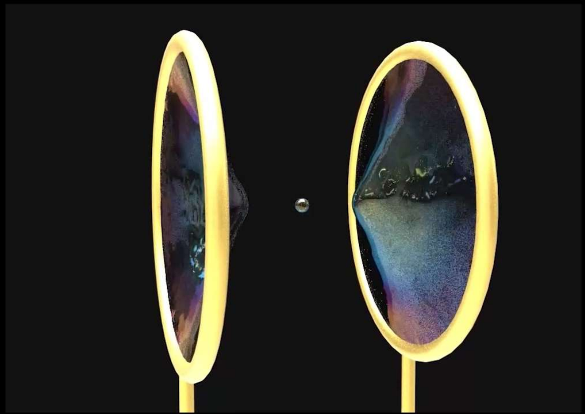
[Wang et al. 2020]

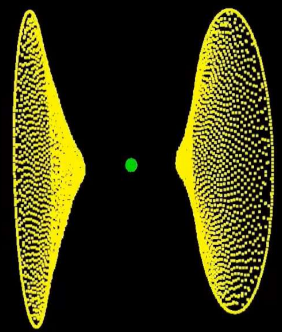
[Solenthaler et al. 2011]



Method Overview

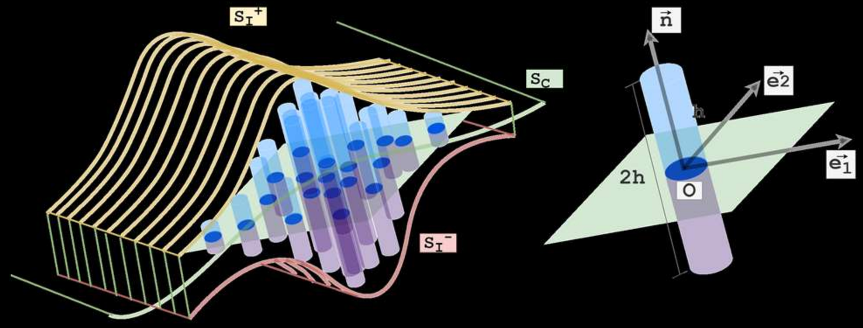
- Geometry Representation
  - Codimension particles represent a manifold thin film
  - Density => thickness, mass conservation=>thickness
- Physical Model
  - NS + surface tension
- Discretization
  - Codimension SPH operators
- Visualization
  - Interference model

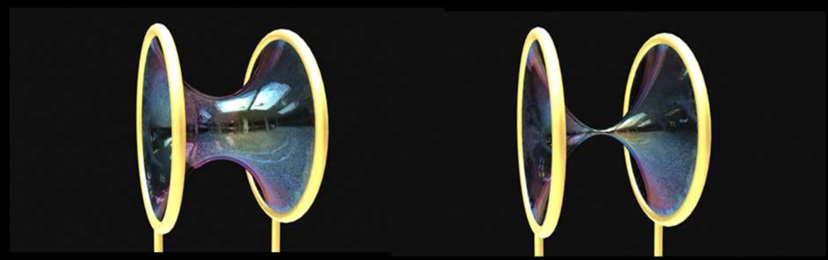


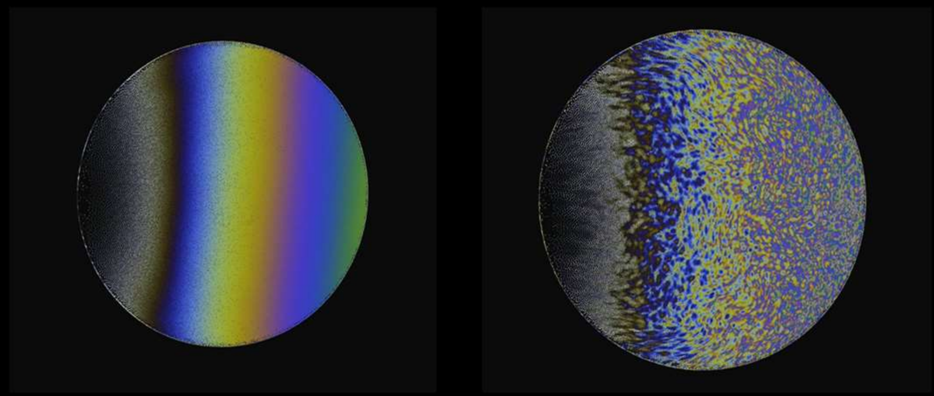


Our Method

- A codimensional SPH method simulating thin fluid films
  - **Geometry Representation**
  - Physical Model
  - SPH Discretization
  - Visualization

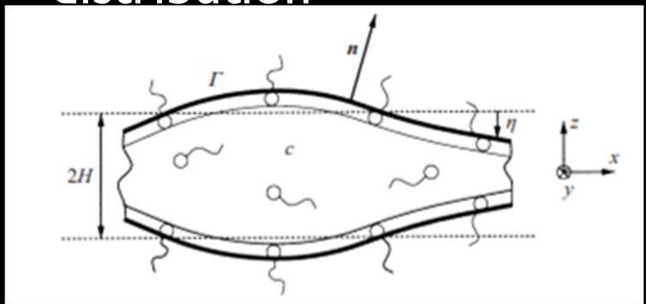




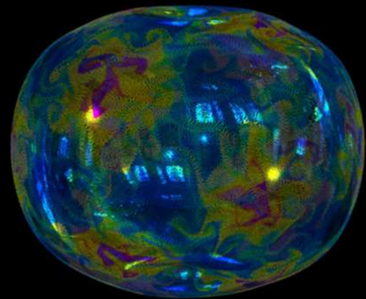


Geometric Representation: Two Aspects

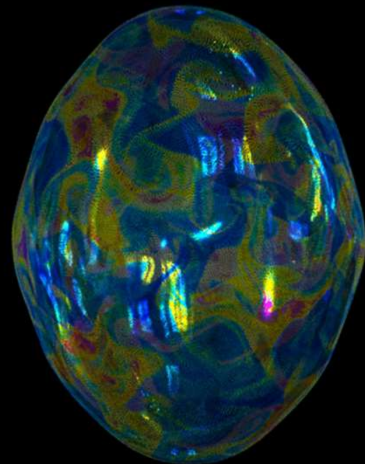
- Macroscopic (~ 100 mm)
  - Surface deformation
  - Represented by the particle locations and normals
- Microscopic (~ 100 nm)
  - Thickness variation
  - Represented by the particle distribution

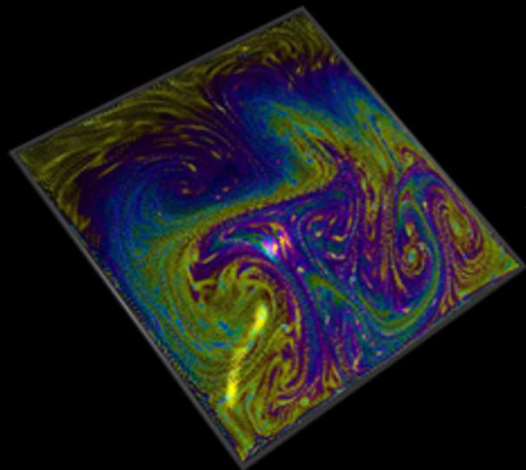


[Chomaz 2001]

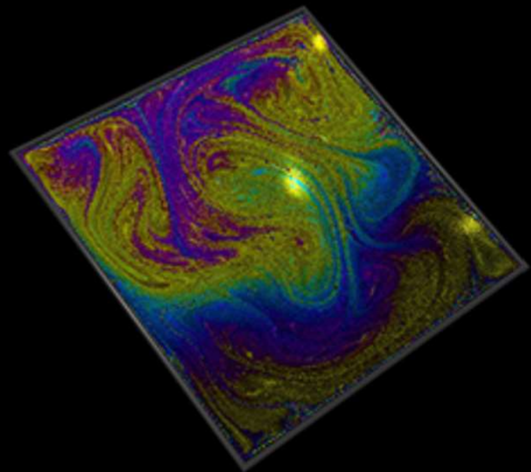


V.S.





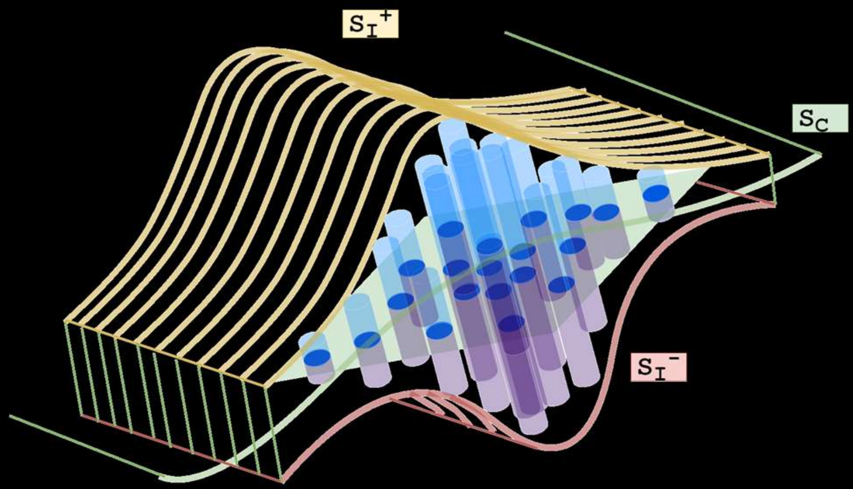
V.S.



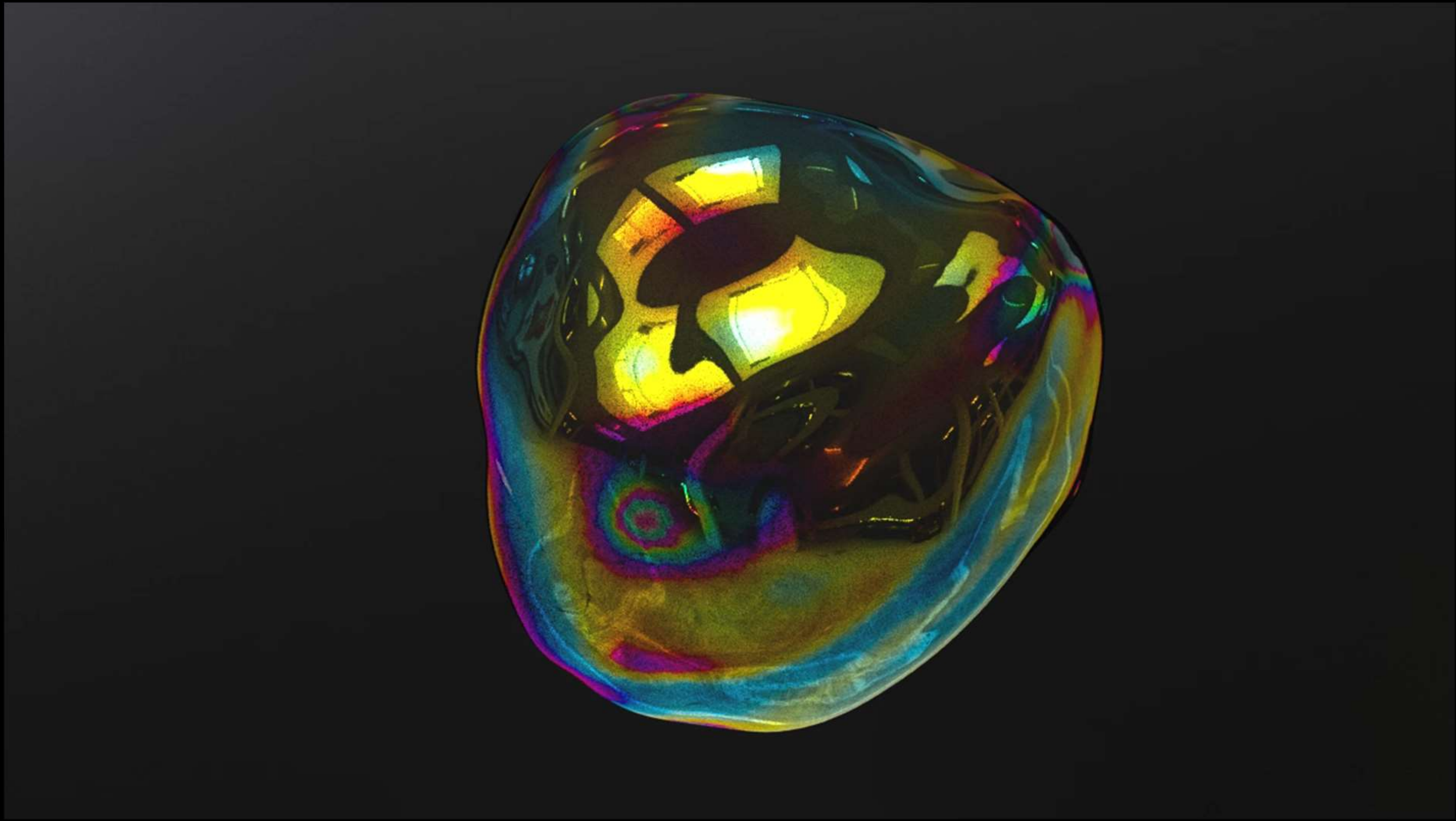
Geometric Representation: Particles

- Central surface + height field
- Information on particles
  - Local coordinate system by PCA
  - Height value
- Area computation:

$$A_i = \frac{V_i}{h_i}$$

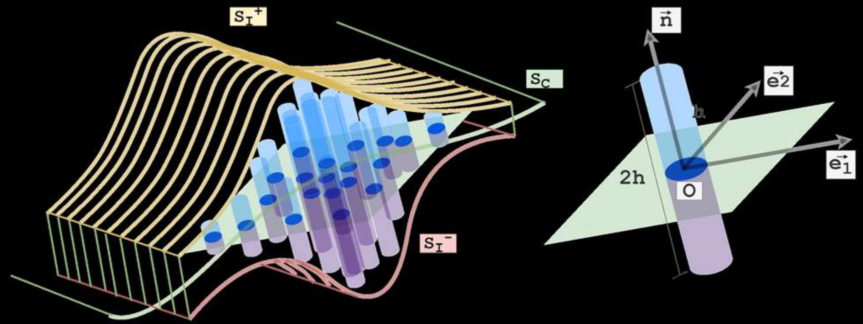


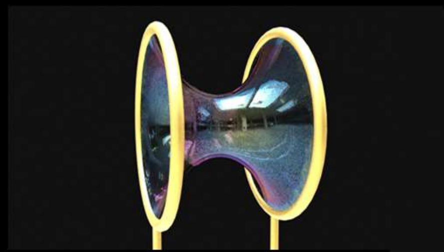
Example: Irregular Bubble

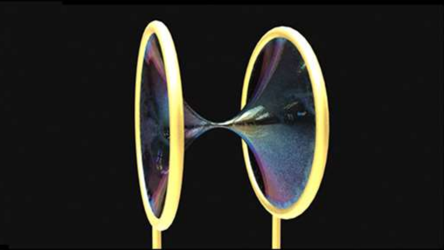


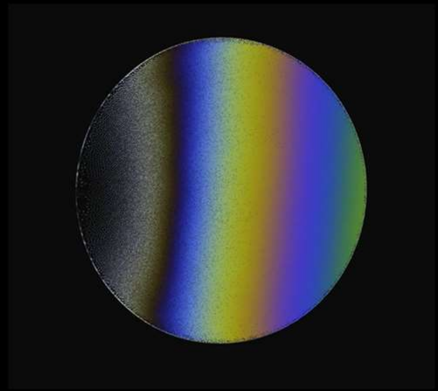
Our Method

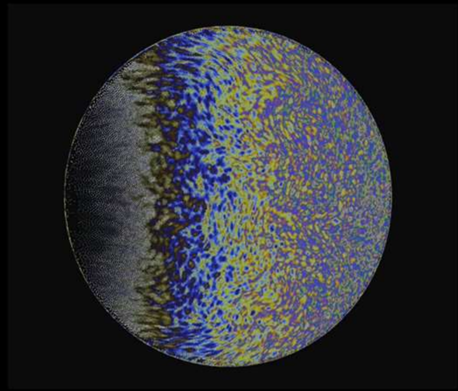
- A codimensional SPH method simulating thin fluid films
  - Geometry Representation
  - **Physical Model**
  - SPH Discretization
  - Visualization











Physical Model: Surfactants

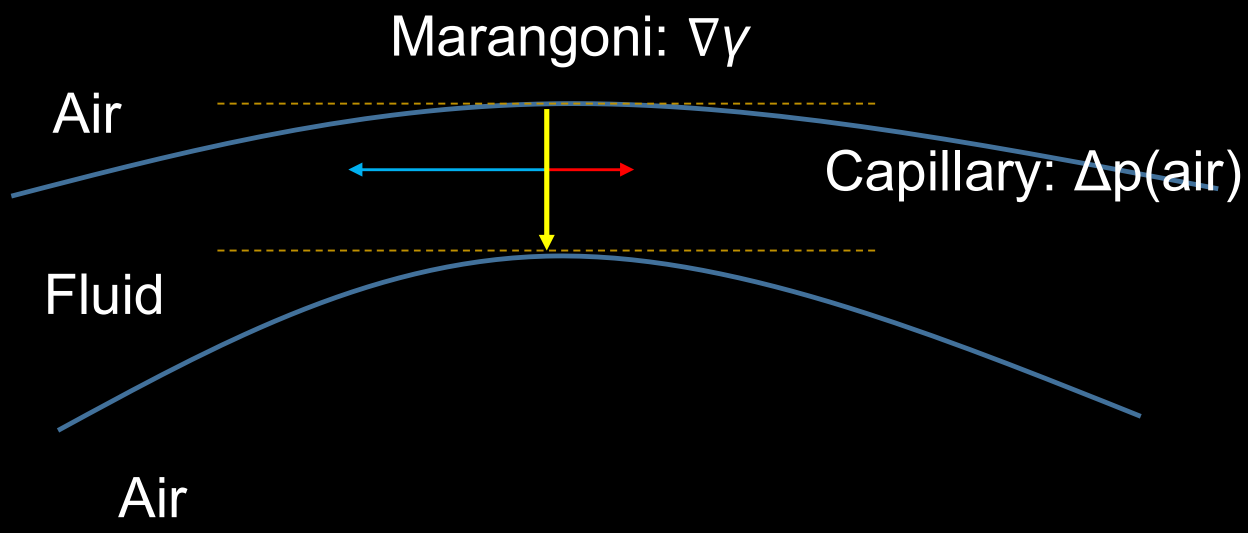
- Langmuir equation of state: $\gamma(\Gamma) = \gamma_0 + RT\Gamma_\infty \log\left(1 - \frac{\Gamma}{\Gamma_\infty}\right)$
- Linear Form: $\gamma(\Gamma) = \gamma_0 - RT\Gamma$

https://www.stevespanglerscience.com/lab/experiments/milk-color-explosion/

Physical Model: Surface Tension

- Young-Laplace Equation:
  $$\Delta p = 2\gamma\kappa$$

- Marangoni Force:
  $$\mathbf{f} = \frac{1}{h}\boldsymbol{\nabla}_s\gamma$$



Example: Marangoni Effect

Physical Model: Equations

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f}, \quad -h < z < h$$

+

$$(p - p_a + 2\kappa^{\pm}\gamma)\mathbf{n}^{\pm} = -\nabla_{s^{\pm}}\gamma + \mu[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]\cdot\mathbf{n}^{\pm}, \quad z = \pm h$$

=

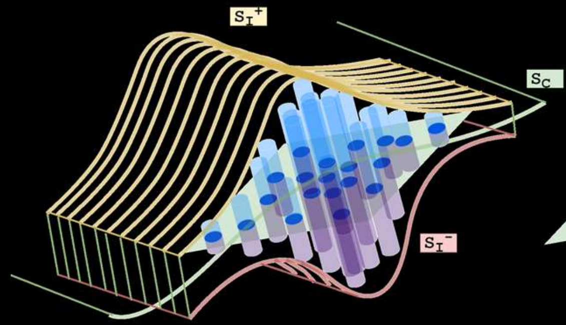
$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \underbrace{2\nabla_s(\kappa_h\gamma + \nabla_s\cdot\mathbf{u}_s)}_{\text{pressure}} + \underbrace{\frac{2\gamma}{h}\kappa_c\mathbf{n} + \frac{1}{h}\nabla_s\gamma}_{\text{surface tension}} + \underbrace{\mu\nabla_s^2\mathbf{u} + \mathbf{f}}_{\text{others}}$$

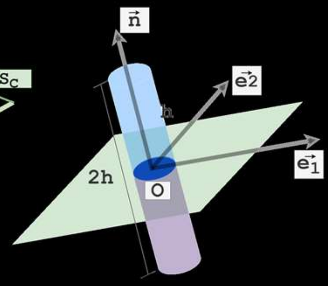
Physical Model: Governing Equations

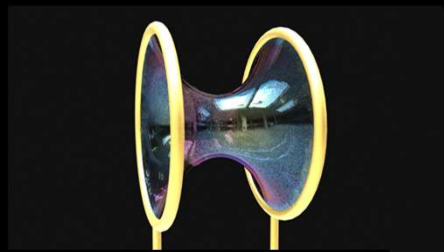
$$\begin{cases} \rho \dfrac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = 2\boldsymbol{\nabla}_s(\kappa_h\gamma + \boldsymbol{\nabla}_s \cdot \mathbf{u}_s) + \dfrac{2\gamma}{h}\kappa_c\mathbf{n} + \dfrac{1}{h}\boldsymbol{\nabla}_s\gamma + \mu\nabla_s^2\mathbf{u} + \mathbf{f}, \\ \dfrac{Dh}{Dt} = -(\boldsymbol{\nabla}_s \cdot \mathbf{u}_s)h, \\ \dfrac{\mathrm{D}\Gamma}{\mathrm{D}t} = \alpha_c\nabla_s^2\Gamma. \end{cases}$$

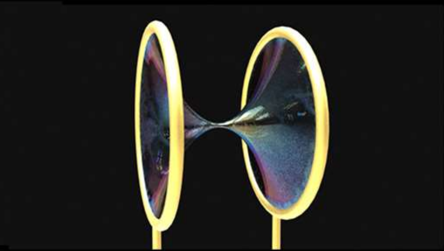
Our Method

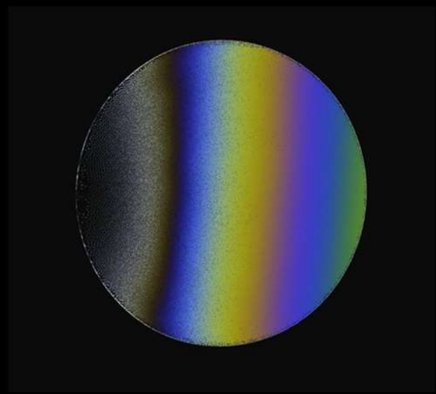
- A codimensional SPH method simulating thin fluid films
  - Geometry Representation
  - Physical Model
  - **SPH Discretization**
  - Visualization

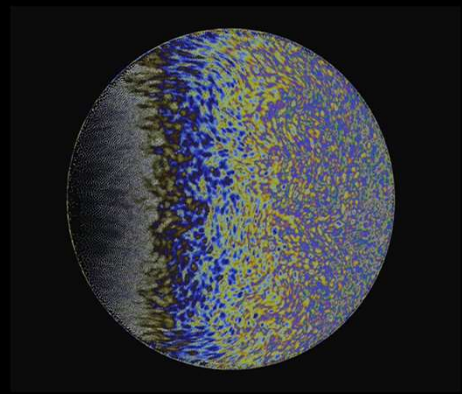






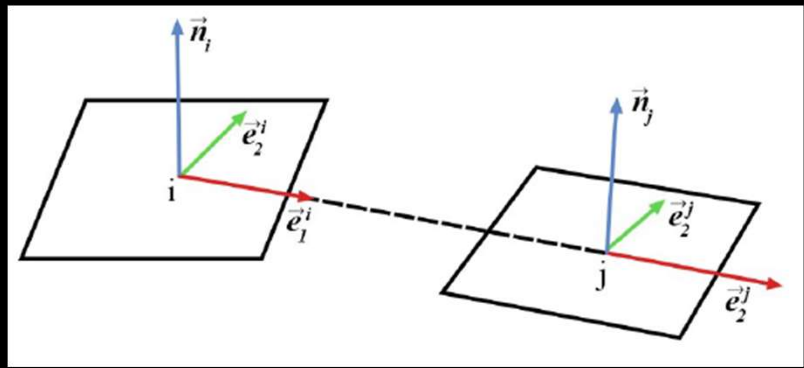






SPH Discretization: Differential Operators

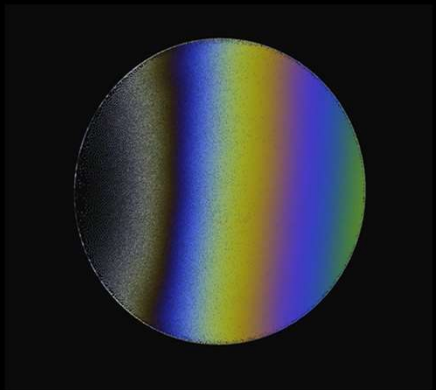
$$
\begin{cases}
(\nabla_s f)_i = \sum_j h_i V_j \left(\frac{f_i}{h_i^2} + \frac{f_j}{h_j^2} \right) \nabla_s W_{ij}, \text{(symmetric form)} \\
(\nabla_s f)_i = \sum_j \frac{V_j}{h_j} (f_j - f_i) \nabla_s W_{ij}, \text{(difference form)} \\
(\nabla_s \cdot \mathbf{u}_s)_i = \sum_j \frac{V_j}{h_j} \tilde{\mathbf{u}}_{ij} \cdot \nabla_s W_{ij}, \\
(\nabla_s^2 f)_i = \sum_j \frac{V_j}{h_j} (f_j - f_i) \frac{2|\nabla_s W_{ij}|}{|\tilde{\mathbf{r}}_{ij}|}.
\end{cases}
$$

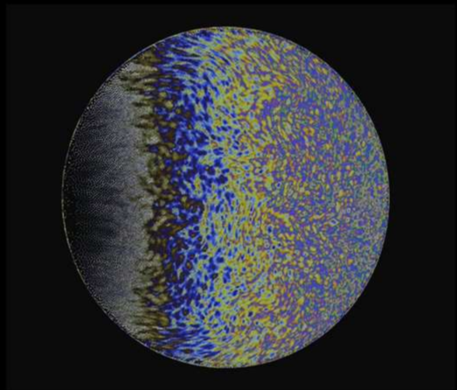


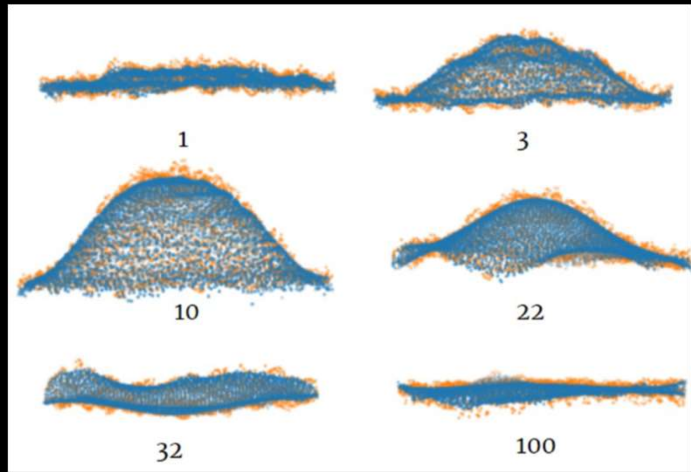
$$\tilde{\mathbf{u}}_{ij} = (u_j - u_i)\tilde{\mathbf{e}}_1^i + (v_j - v_i)\tilde{\mathbf{e}}_2^i$$

SPH Discretization : Particle Heights

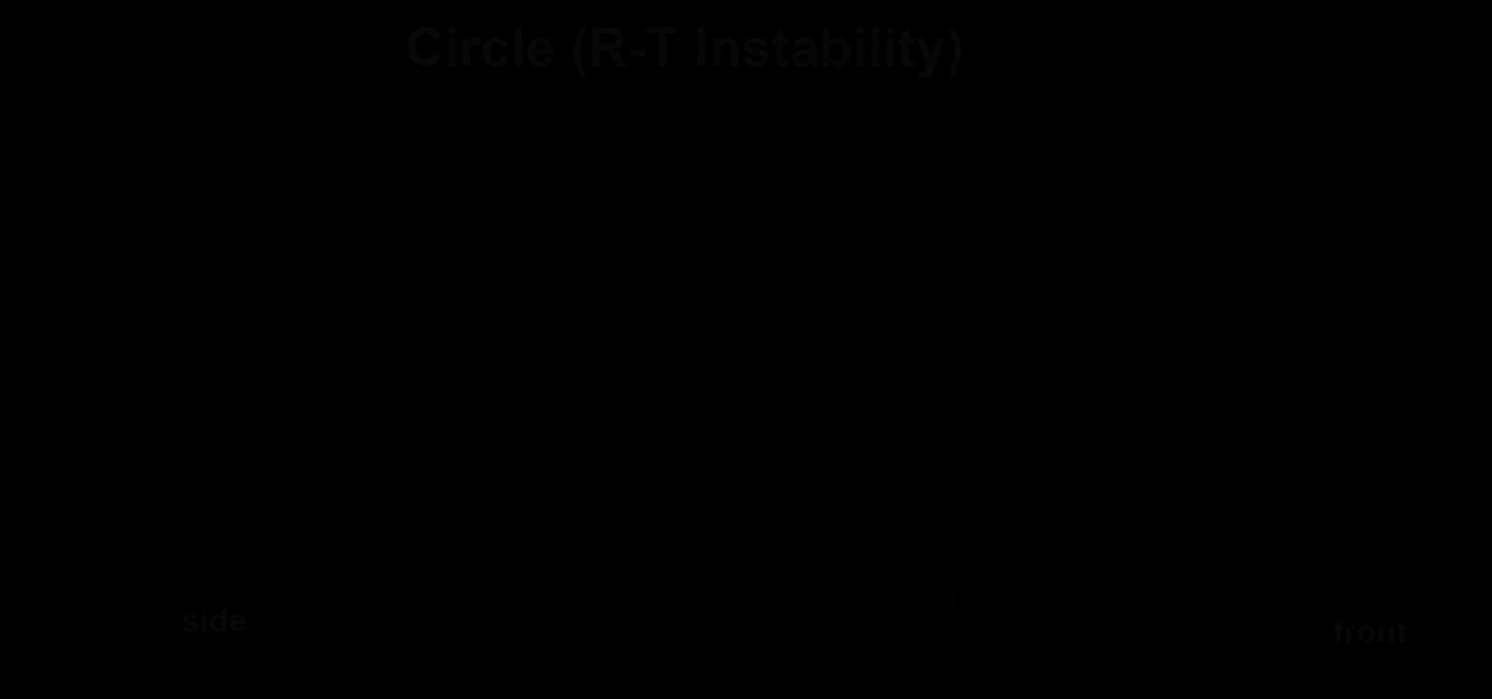
- Numerical height: $h_i = \sum_j V_j W_{ij}$
- Advected height: $\frac{\mathrm{D}\hat{h}}{\mathrm{D}t} = -(\nabla_s \cdot \mathbf{u}_s)\hat{h}$



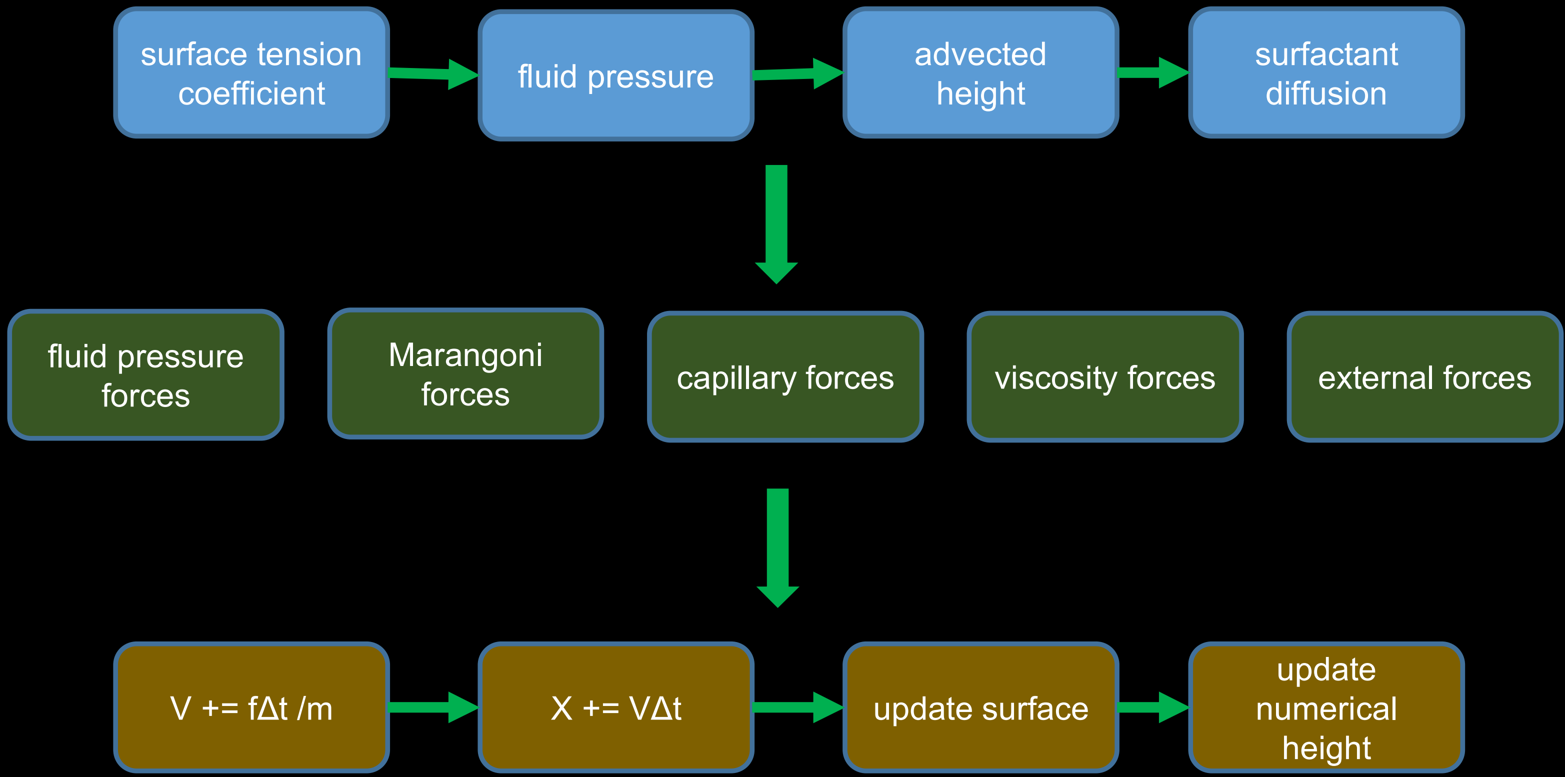




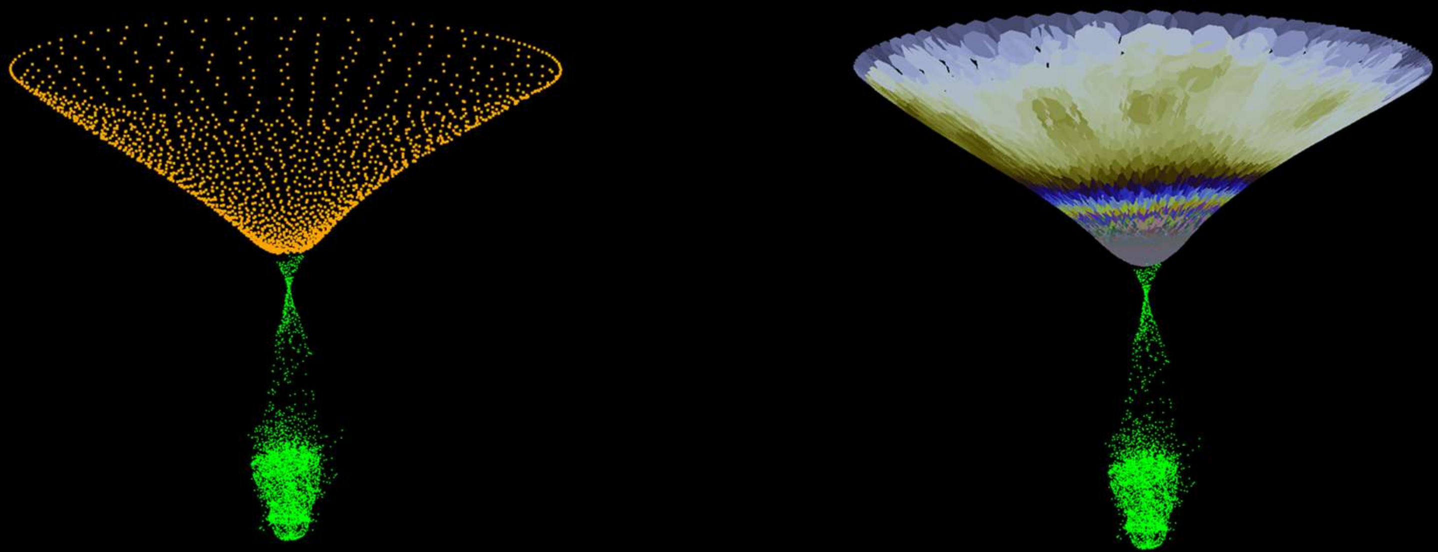
Example: R-T Instability (Circular)



SPH Discretization : Time Integration

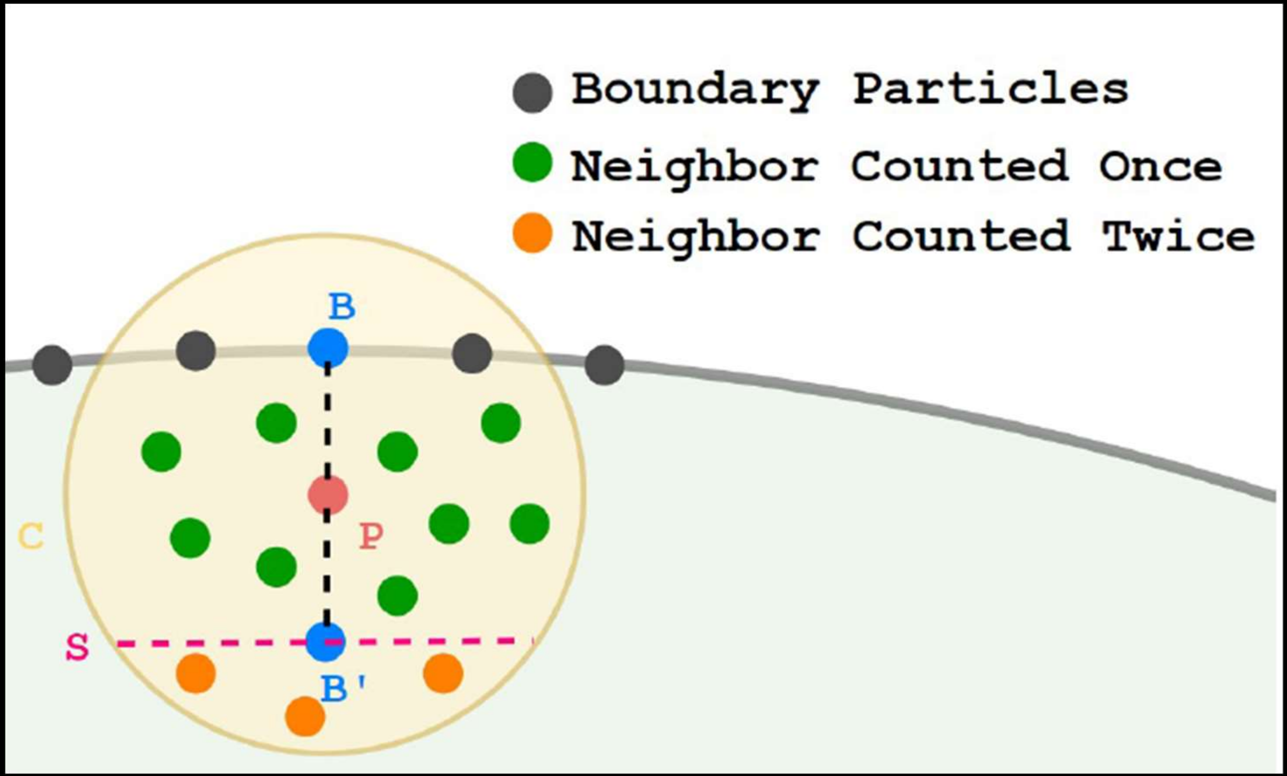


SPH Discretization : Codimension Transition

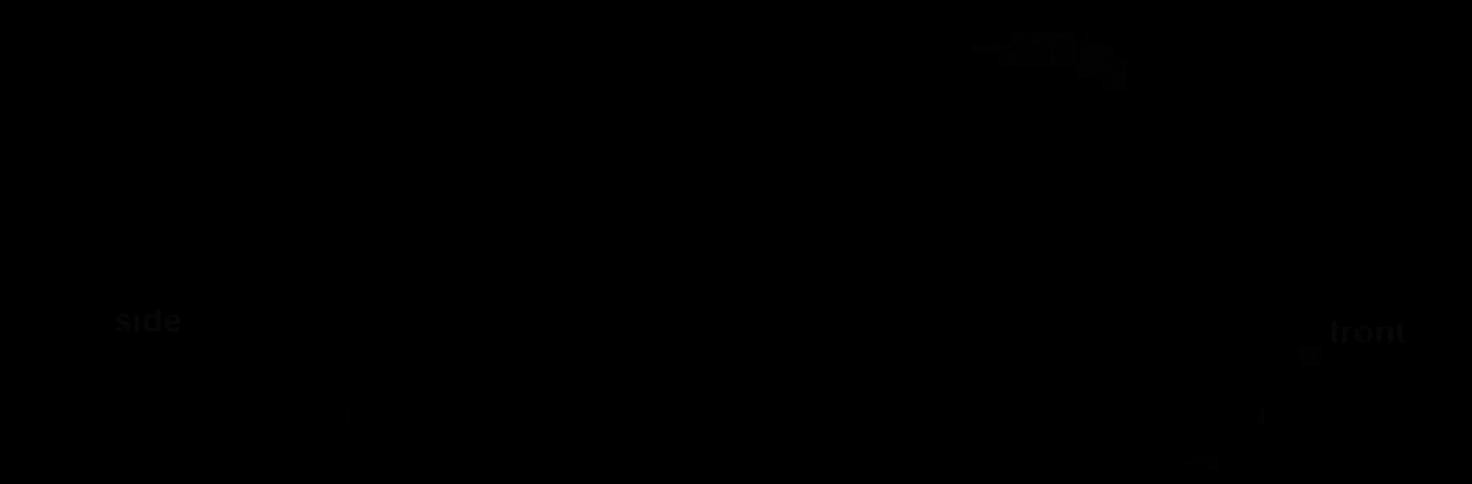


Example: Catenoid

SPH Discretization : Boundary Treatment

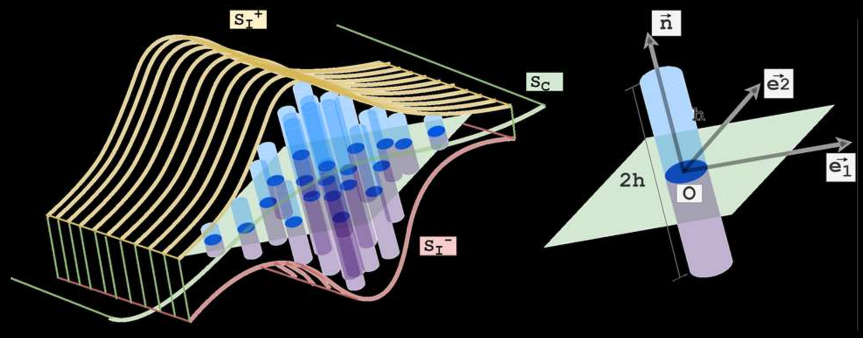


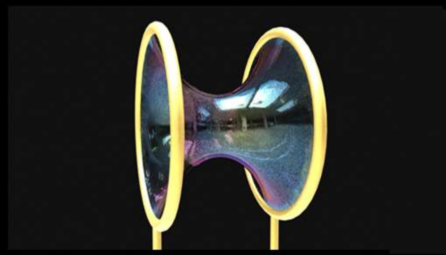
Example: R-T Instability (Square)

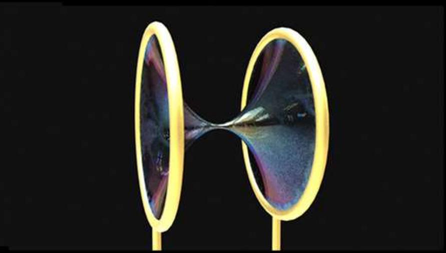


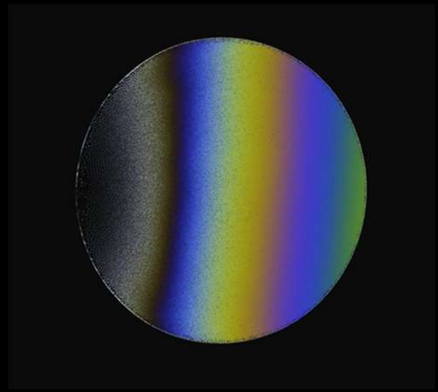
Our Method

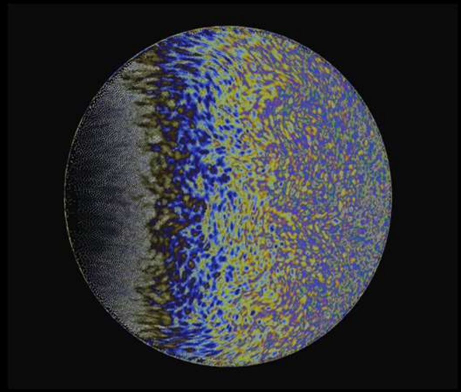
- A codimensional SPH method simulating thin fluid films
  - Geometry Representation
  - Physical Model
  - SPH Discretization
  - **Visualization**











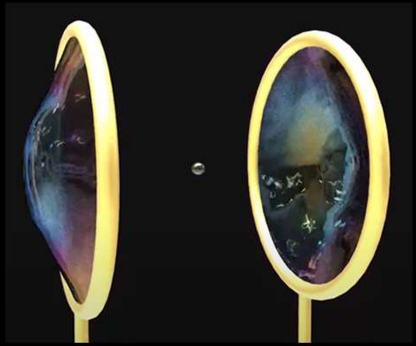
Visualization: Color Computation

- Standard lighting condition (CIE illuminant D65)
- Thin-film height
- Refraction of vapor and fluid
- Carried out via ColorPy

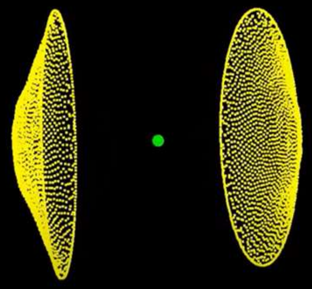
[ColorPy]

Visualization: Rendering

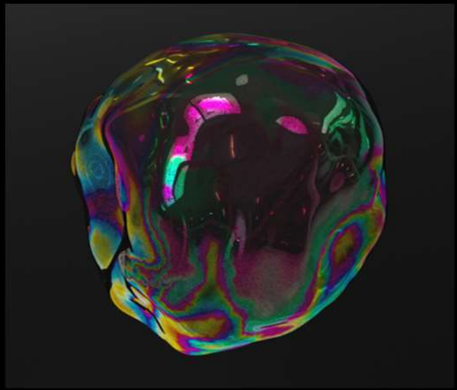
- OpenGL (schematic visualization)
- Houdini (realistic rendering)
  - Particle based rendering
  - Mesh based rendering



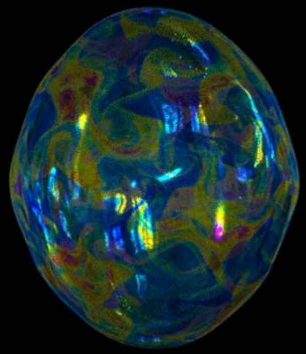
Houdini (mesh)



OpenGL



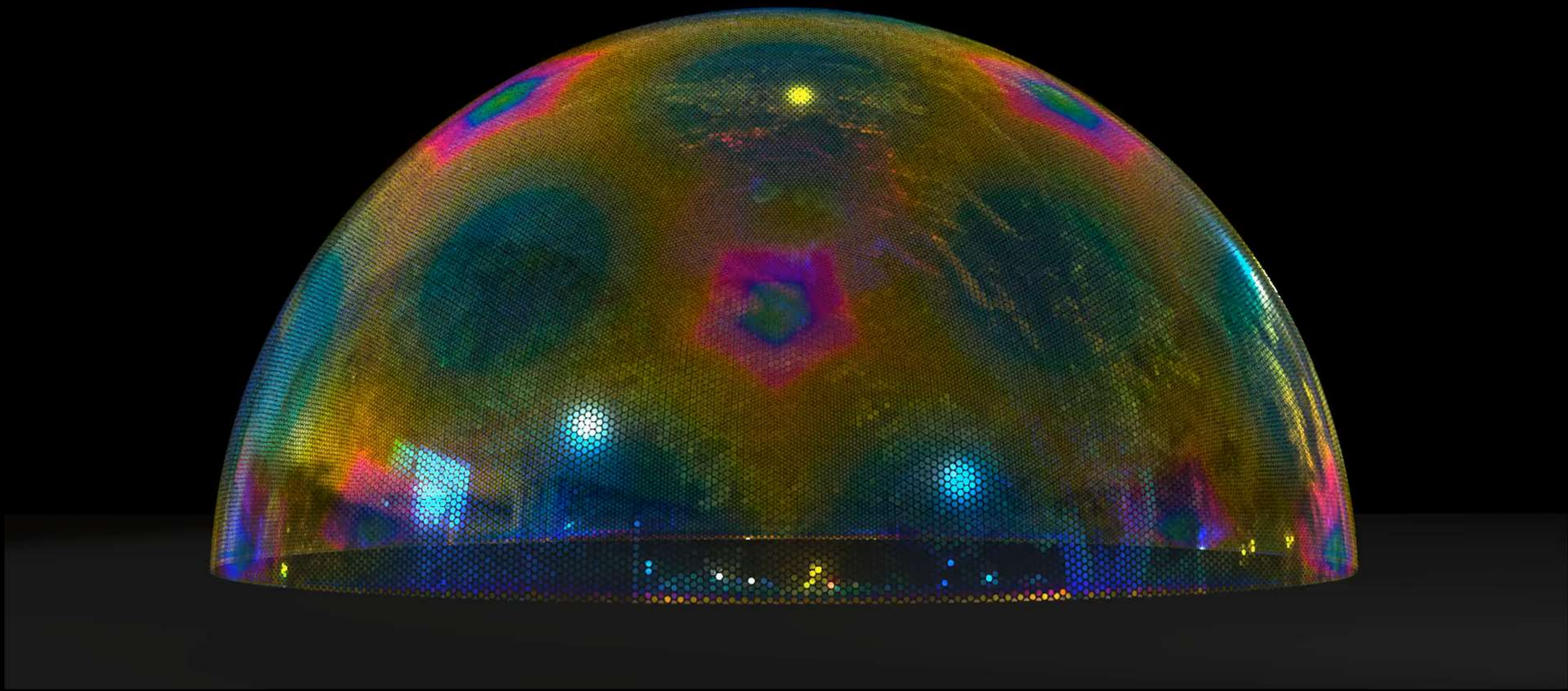
Houdini (mesh)



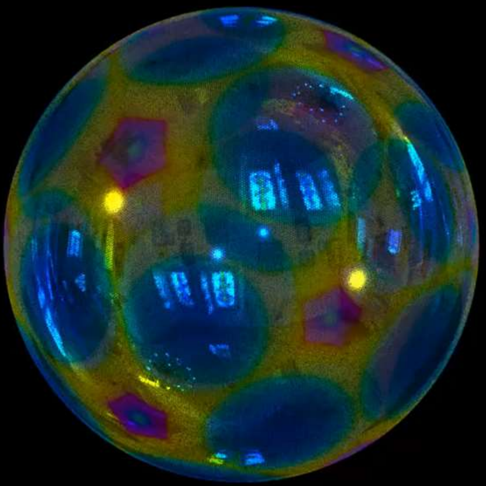
Houdini (particle)

Example: Wet-Film Dripping

Results: Half Bubble



Results: Sphere Bubble and Burst



Limitations

- Manifold
  - Point-set surface can’t deal with non-manifold intersections.
- Trade-off between density variation and numerical robustness
- Codimension transition criterion
  - Adding and deleting particles
- Limited number of particles

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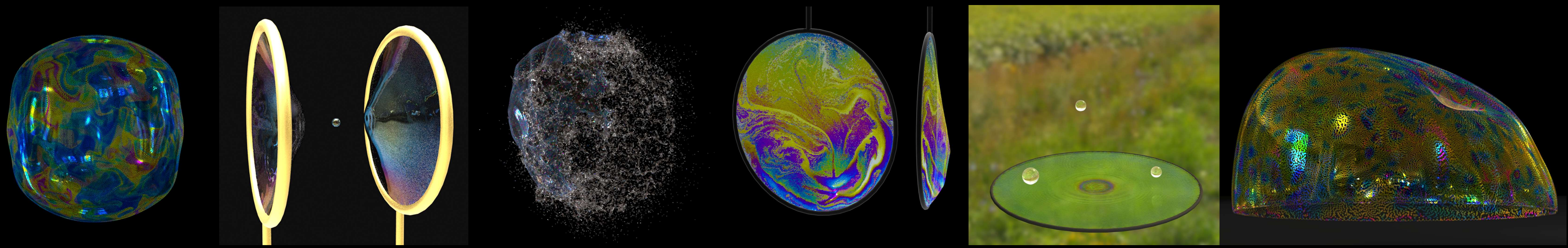
Thank you!



Project Page: https://wang-mengdi.github.io/proj/thin-film-sph/

YouTube Link: https://www.youtube.com/watch?v=__1VjKF-gTk

Related Papers: https://www.cs.dartmouth.edu/~bozhu/