



Shape As Points

A Differentiable Poisson Solver

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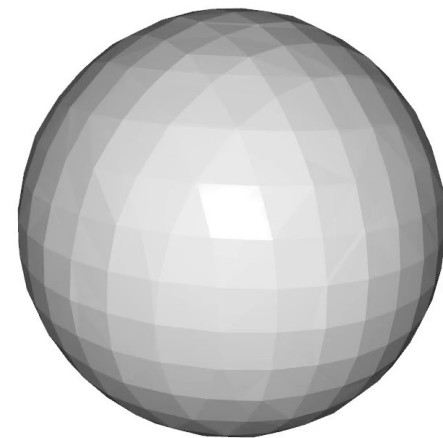
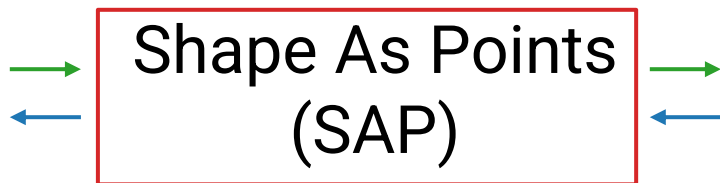
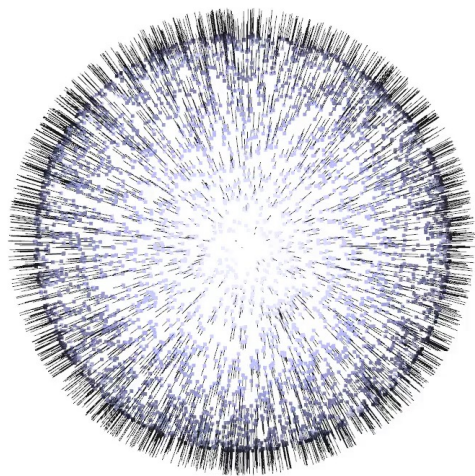


Marc Pollefeys



Andreas Geiger

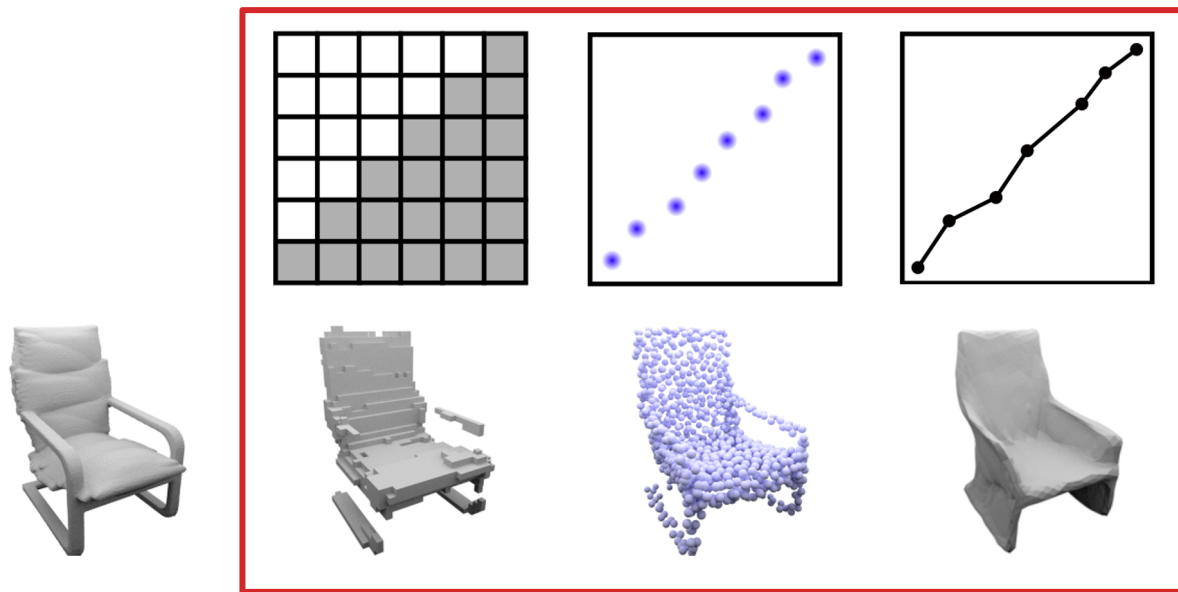




Duality between **oriented point clouds** and **3D dense geometry**

What is a good **3D shape representation**?

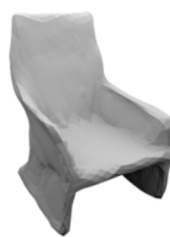
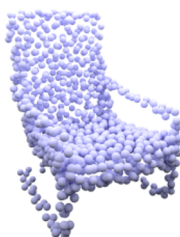
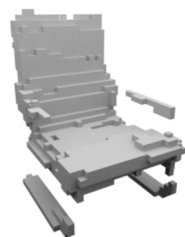
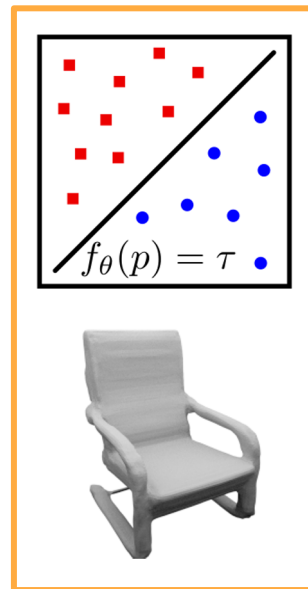
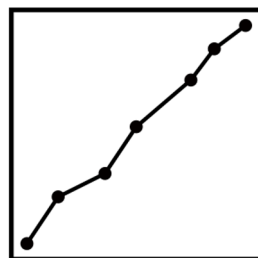
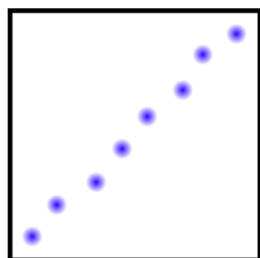
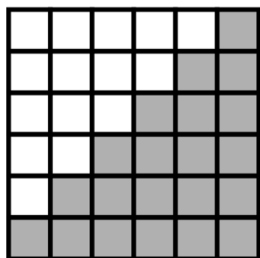
3D Shape Representations



Traditional Explicit Representations

- + Fast inference
- Discrete

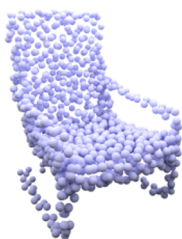
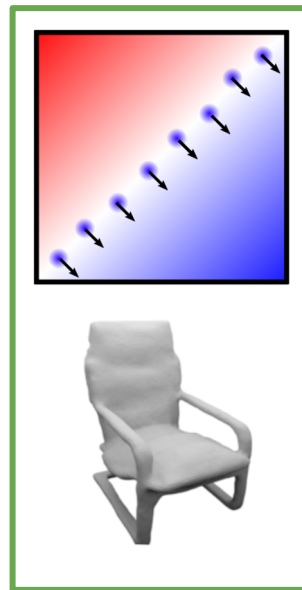
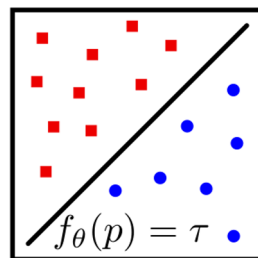
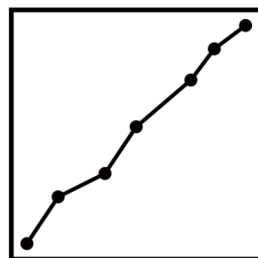
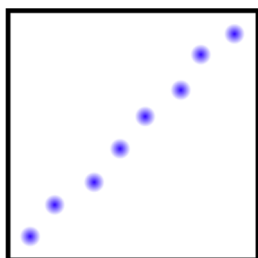
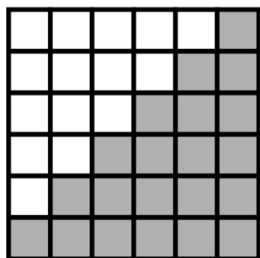
3D Shape Representations



Neural Implicit Representations

- + Continuous, watertight
- Slow inference
- Difficult to initialize

3D Shape Representations



Shape As Points (SAP) - Hybrid Representation

- + Discrete \Rightarrow Continuous
- + Fast inference
- + Easy initialization, topology-agnostic

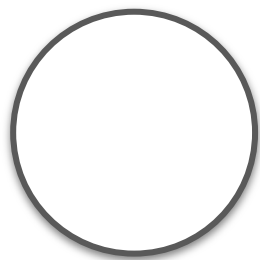
Method

Differentiable Poisson Solver



Intuition of Poisson Equation

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$



Shape



χ

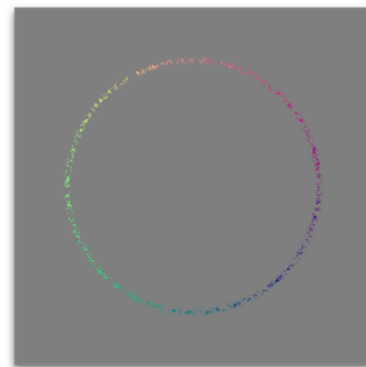
Indicator Function



$\nabla \chi$

Gradient

\approx



\mathbf{v}

Point Normals

Our Poisson Solver

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$

- **Discretization** allows to invert the divergence operator

$$\chi = (\nabla^2)^{-1} \nabla \cdot \mathbf{v}$$

- **Spectral methods** to solve the Poisson equation
 - Derivatives of signals in spectral domain are computed analytically
 - Fast Fourier Transform (FFT) are **highly optimized on GPUs/TPUs**
 - Only **25-line code**

$$\tilde{\mathbf{v}} = \text{FFT}(\mathbf{v}) \quad \longrightarrow \quad \tilde{\chi} = \tilde{g}_{\sigma,r}(\mathbf{u}) \odot \frac{i\mathbf{u} \cdot \tilde{\mathbf{v}}}{-2\pi\|\mathbf{u}\|^2} \quad \longrightarrow \quad \chi' = \text{IFFT}(\tilde{\chi})$$

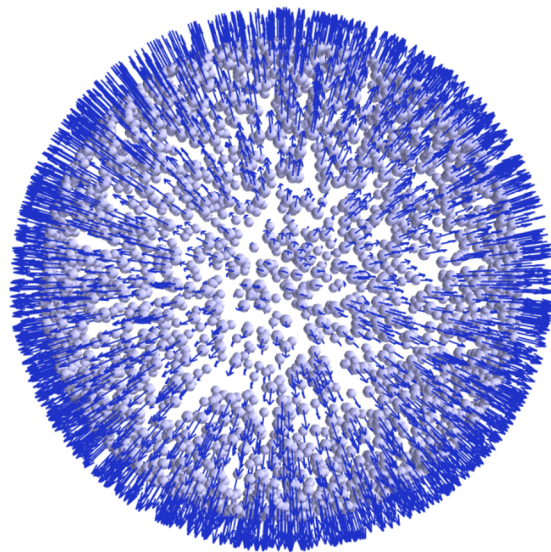
Surface Reconstruction from Unoriented Point Clouds

1. SAP for **Optimization-based** 3D Reconstruction
2. SAP for **Learning-based** 3D Reconstruction

SAP for Optimization-based 3D Reconstruction

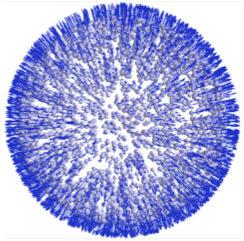
Pipeline - Forward Pass

Input an initial oriented point cloud
(noisy / incomplete observations)

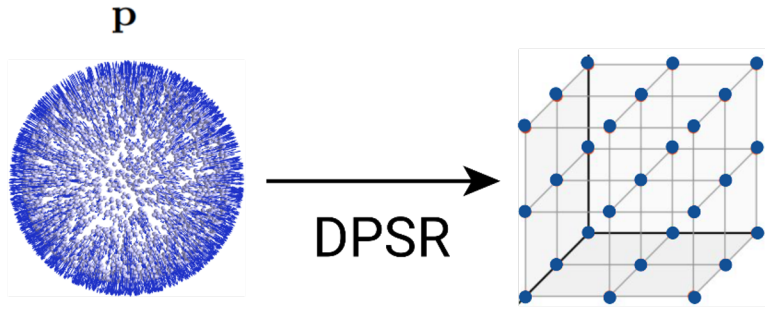


Pipeline - Forward Pass

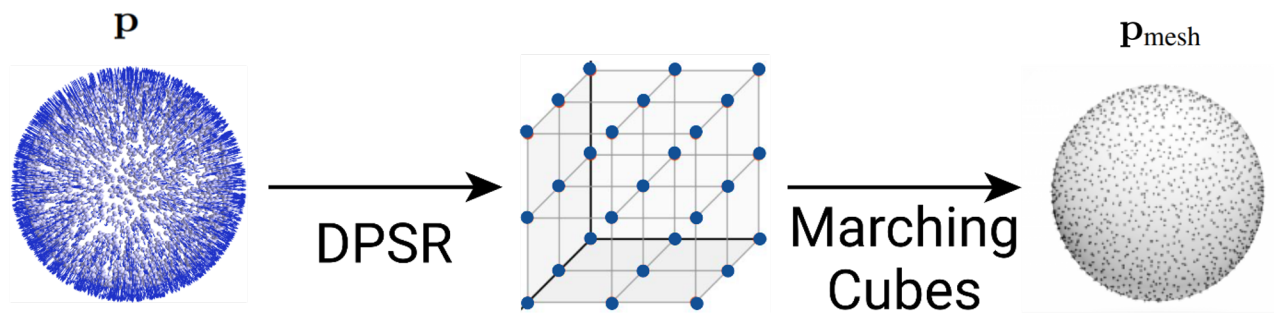
P



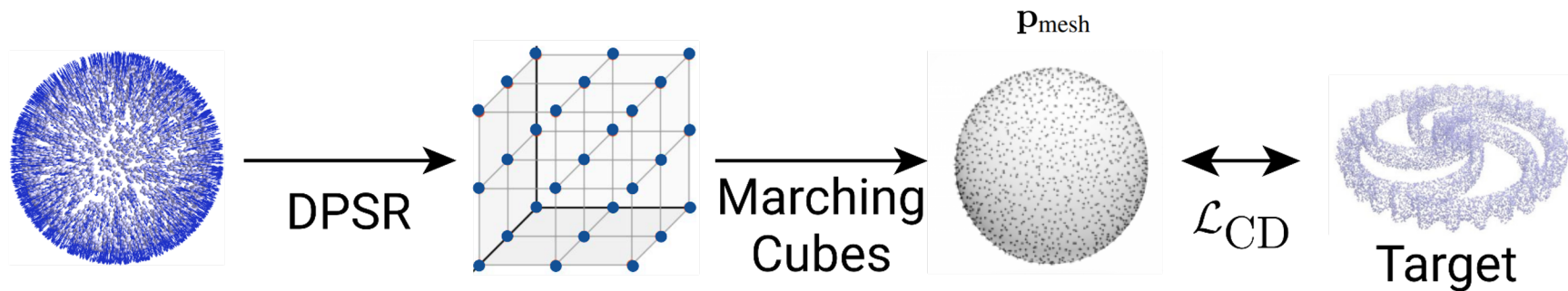
Pipeline - Forward Pass



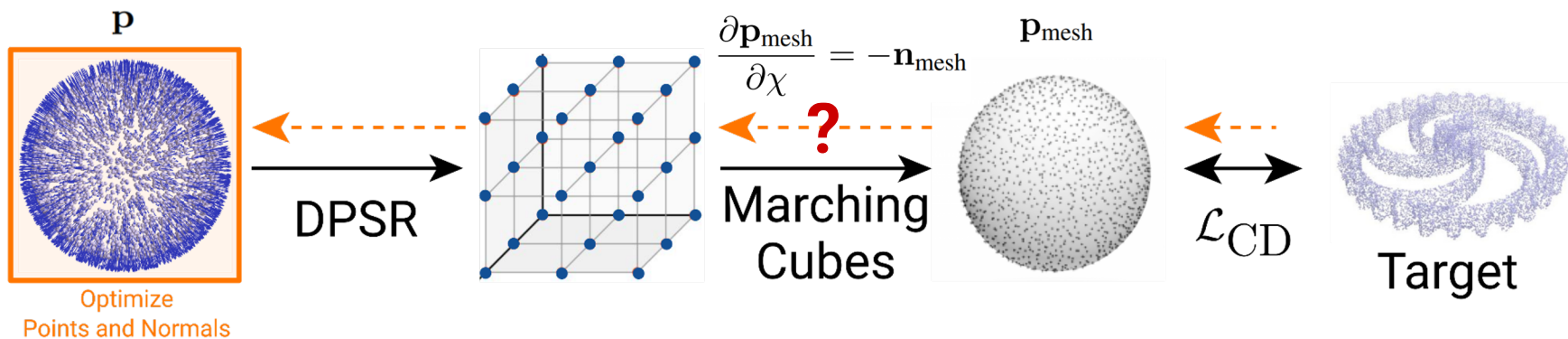
Pipeline - Forward Pass



Pipeline - Forward Pass

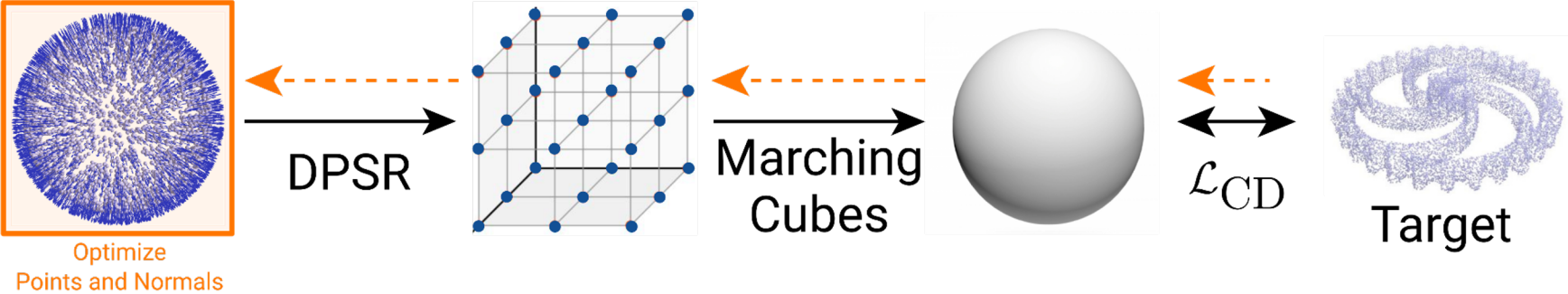


Pipeline - Backward Pass

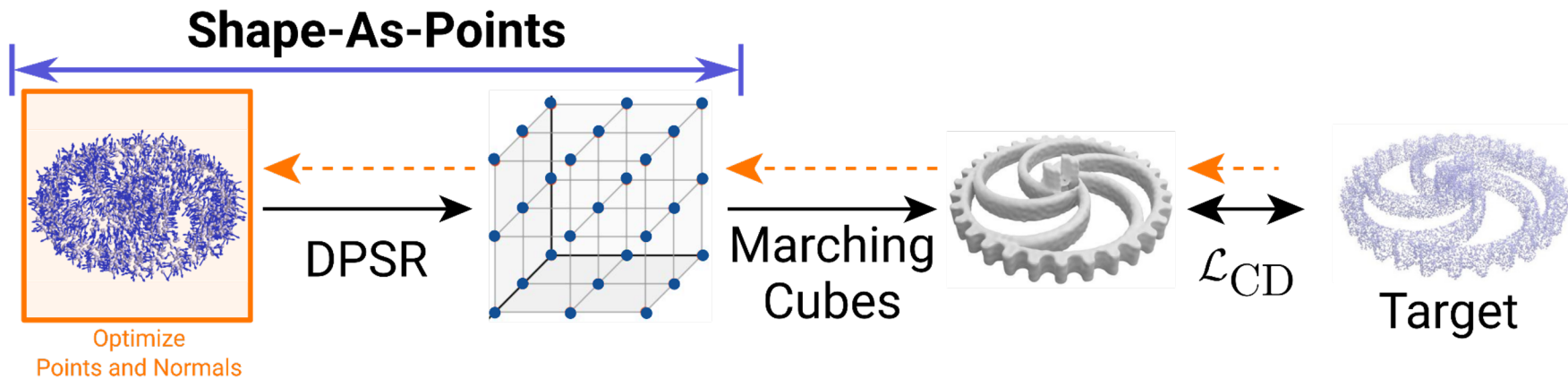


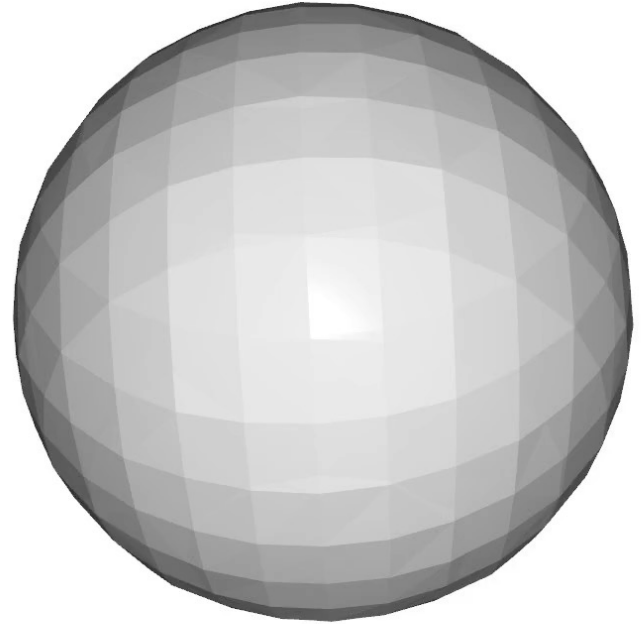
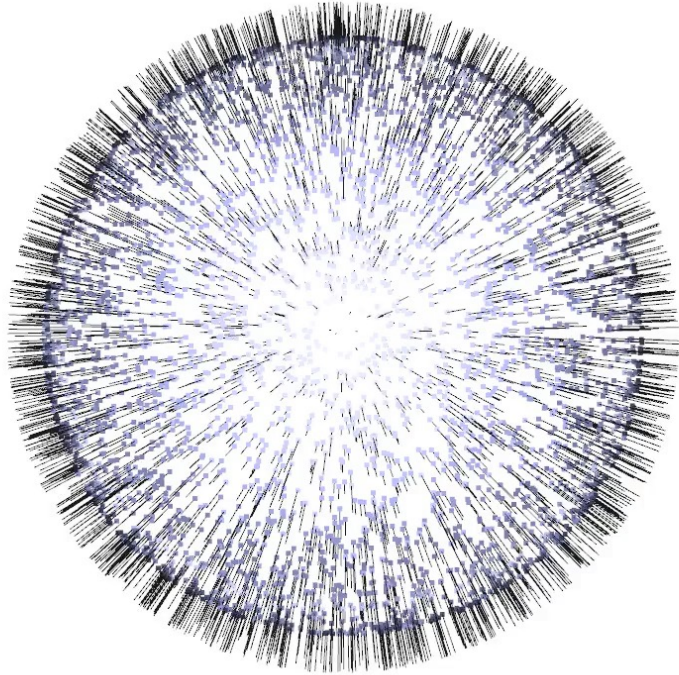
$$\frac{\partial \mathcal{L}_{CD}}{\partial \mathbf{p}} = \frac{\partial \mathcal{L}_{CD}}{\partial \mathbf{p}_{\text{mesh}}} \frac{\partial \mathbf{p}_{\text{mesh}}}{\partial \chi} \frac{\partial \chi}{\partial \mathbf{p}}$$

Pipeline

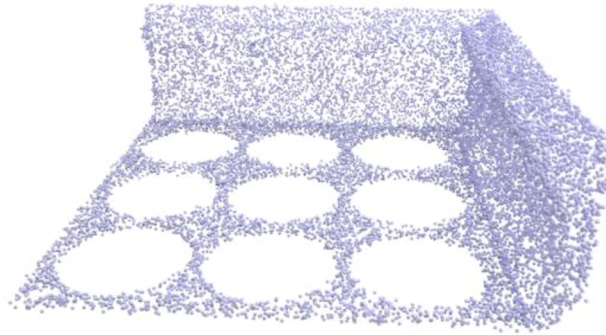


Pipeline





Comparison

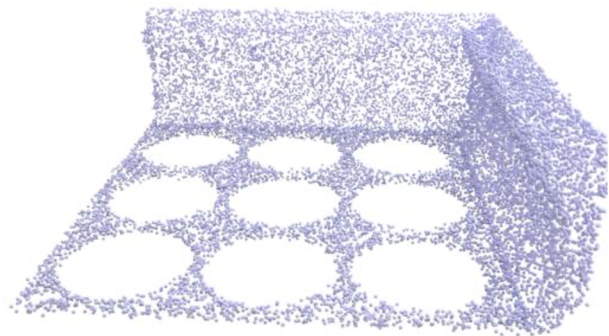


Unoriented Point Clouds

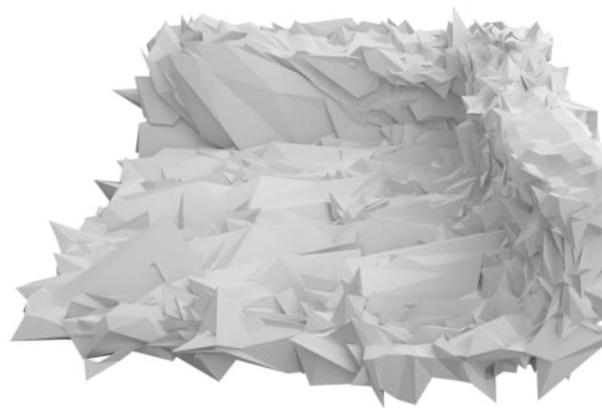


GT Mesh

Comparison



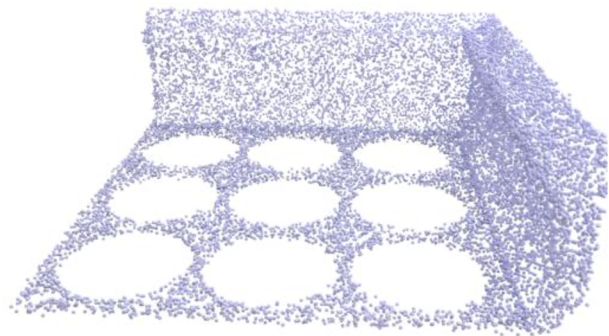
Unoriented Point Clouds



Point2Mesh

Runtime: 62 mins

Comparison



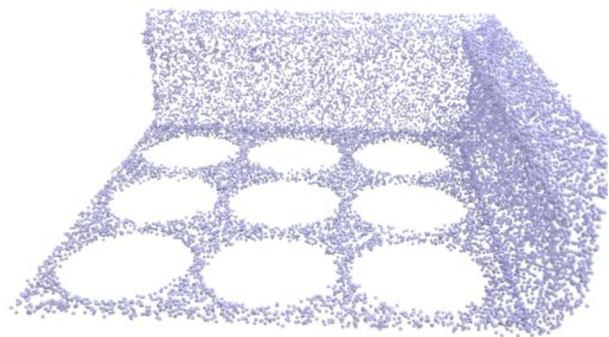
Unoriented Point Clouds



IGR

Runtime: 30 mins

Comparison



Unoriented Point Clouds



SAP

Runtime: ~6 mins

Comparison



SPSR

Runtime: ~9 sec



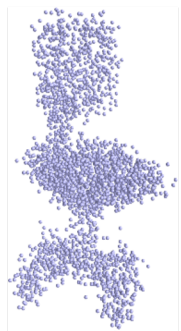
SAP

Runtime: ~6 mins

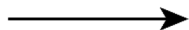
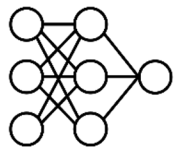
Can we further leverage the **differentiability** of the Poisson solver
for **deep neural networks**?

SAP for Learning-based 3D Reconstruction

Learning-based Pipeline

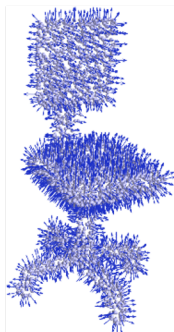


Noisy Input

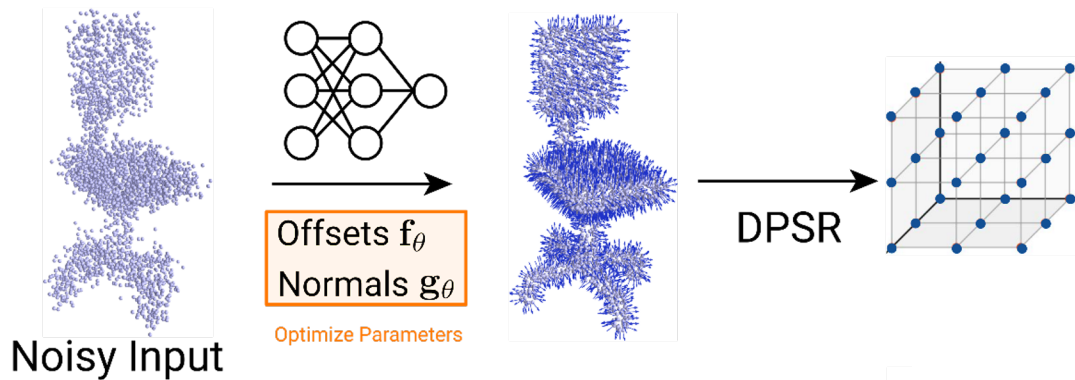


Offsets f_θ
Normals g_θ

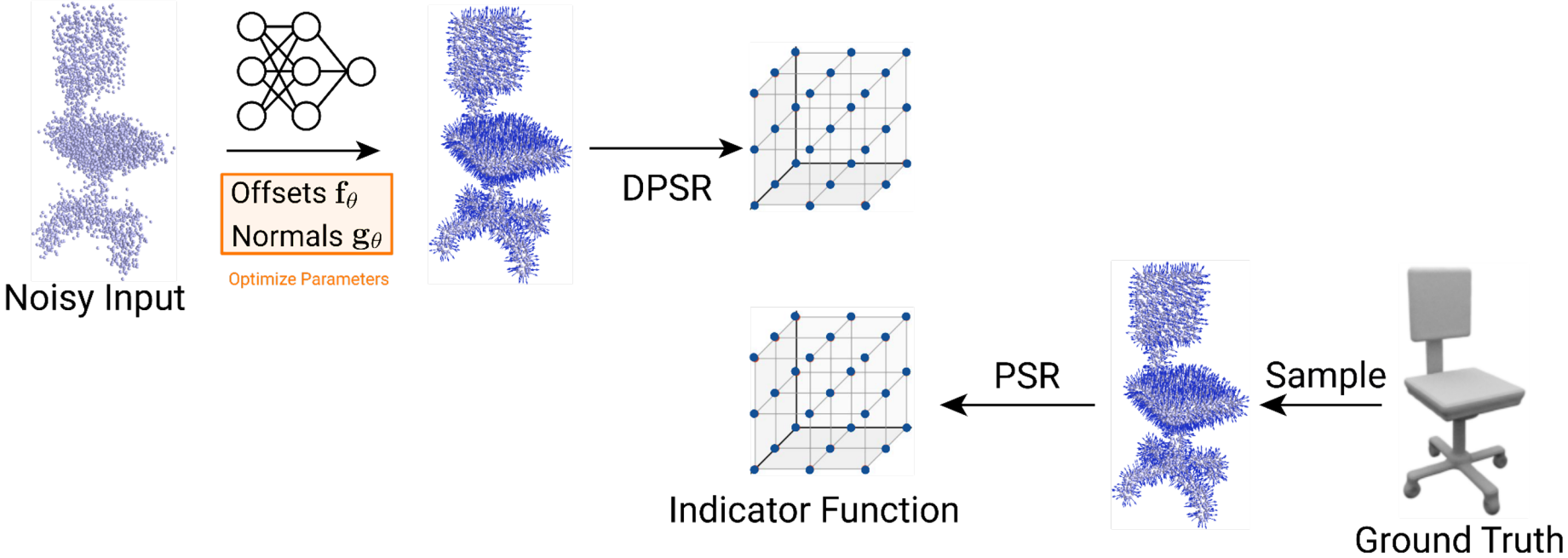
Optimize Parameters



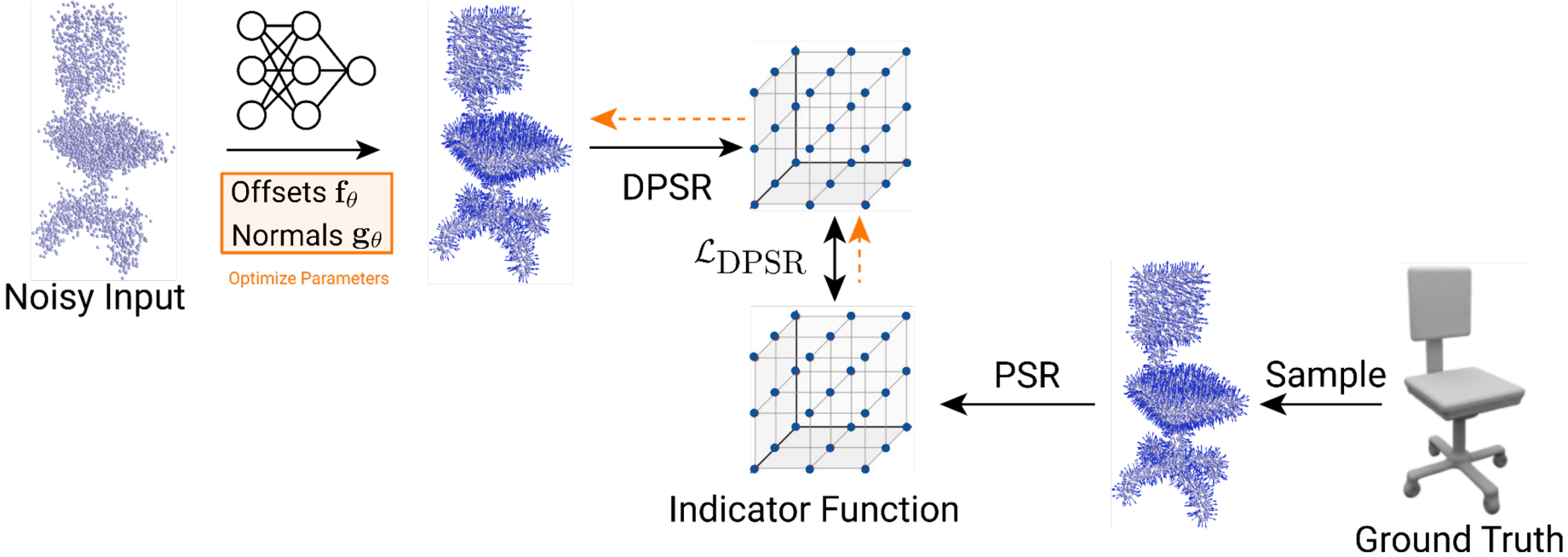
Learning-based Pipeline



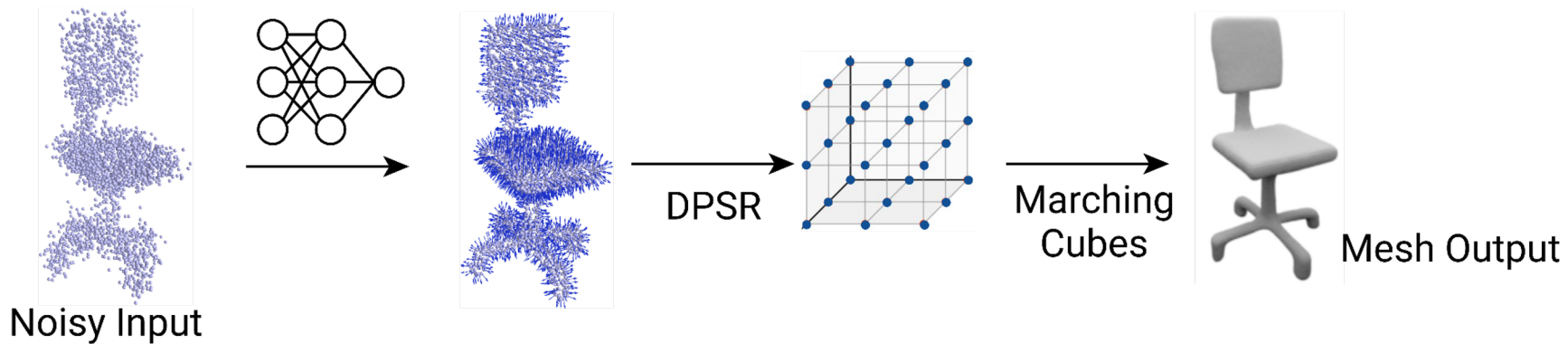
Learning-based Pipeline



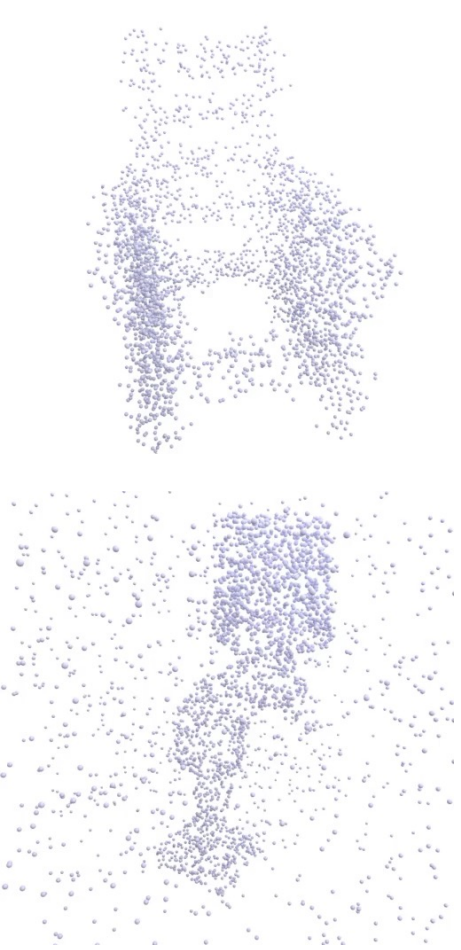
Learning-based Pipeline



Learning-based Pipeline



Results



Inputs



GT Mesh



Inputs



GT Mesh



R2N2

15 ms



AtlasNet

25 ms





Inputs

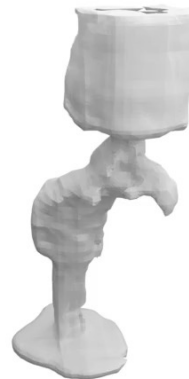


GT Mesh



ConvONet

327 ms





Inputs



GT Mesh



ConvONet

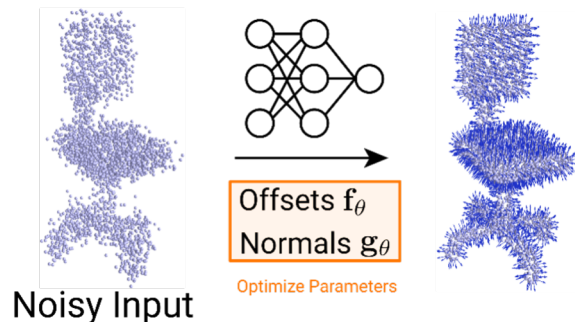
327 ms



Ours

64 ms

Benefit of Geometric Initialization



Chamfer distance over the training process

| Iterations | 10K | 50K | 100K | 200K | Best |
|------------|--------------|--------------|--------------|--------------|--------------|
| ConvONet | 0.082 | 0.058 | 0.055 | 0.050 | 0.044 |
| Ours | 0.041 | 0.036 | 0.035 | 0.034 | 0.034 |

SAP converges much faster!

Conclusions

- SAP is **interpretable, lightweight** and guarantees **HQ watertight meshes**
- SAP is also **topology agnostic**, enables **fast inference**
- Our Poisson solver is **differentiable** and **GPU-accelerated**

Limitation: Cubic memory requirements limits SAP for small scenes

Thank You!

<https://pengsongyou.github.io/sap>

